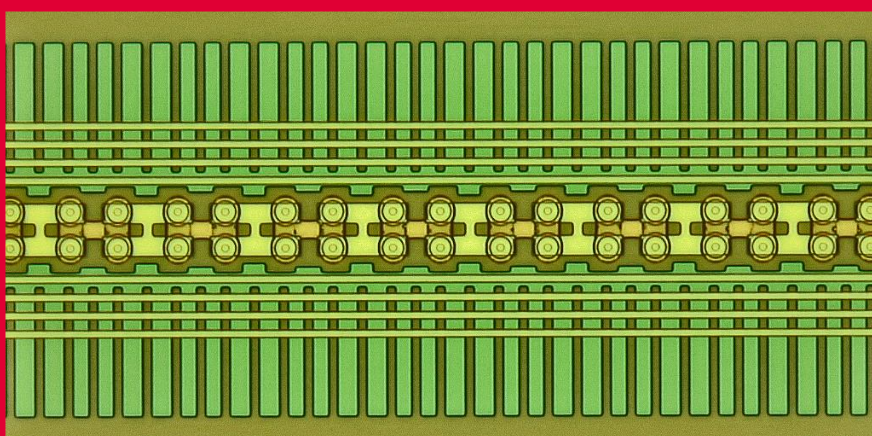
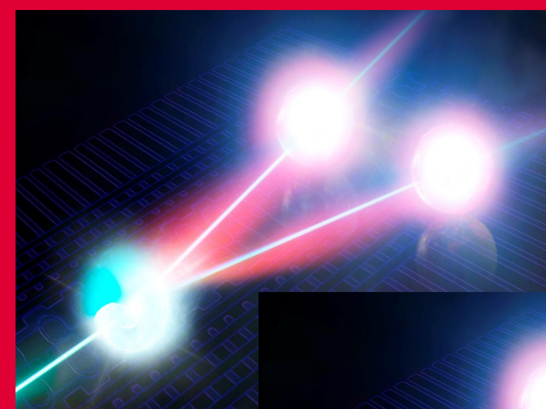
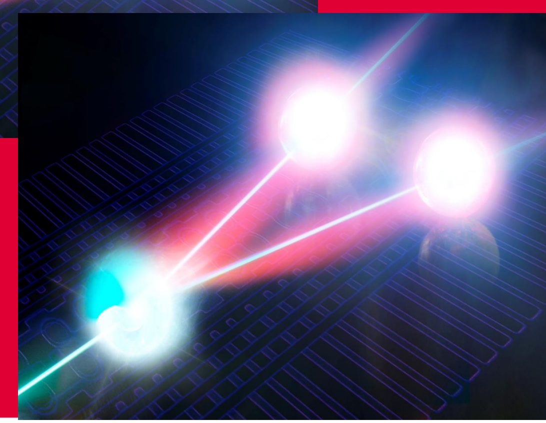
Aalto-yliopisto
Perustieteiden
korkeakoulu

Quantum vacuum, noise, and entanglement

Pertti Hakonen
Stockholm, November, 2018







OUTLINE

Introduction

- Concept of vacuum
- Mode correlations in quantum optics
- Entanglement

Dynamical Casimir effect

- Photon generation with a Josephson metamaterial
- Correlations: two mode squeezing

Vacuum fluctuations under double parametric pumping

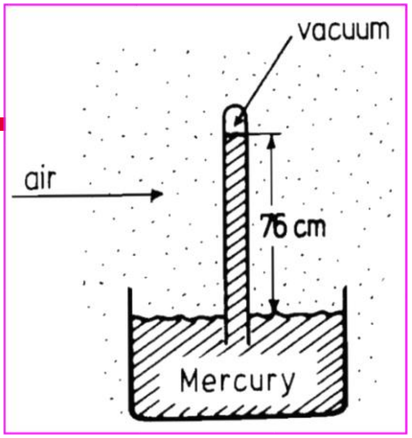
- New kind of correlations
- Which color information

Relativity and quantum noise

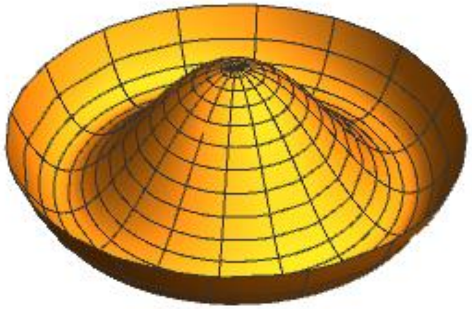
- Past – future correlations from 4-dim spacetime

Summary of “open problems”

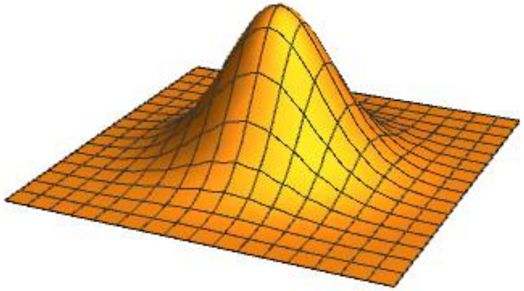
Introductory remarks



- **Toricellian vacuum**
(Evangelista Toricelli, 1643)
- **First vacuum pump, Magdeburg hemispheres**
(Otto von Guericke, 1654)

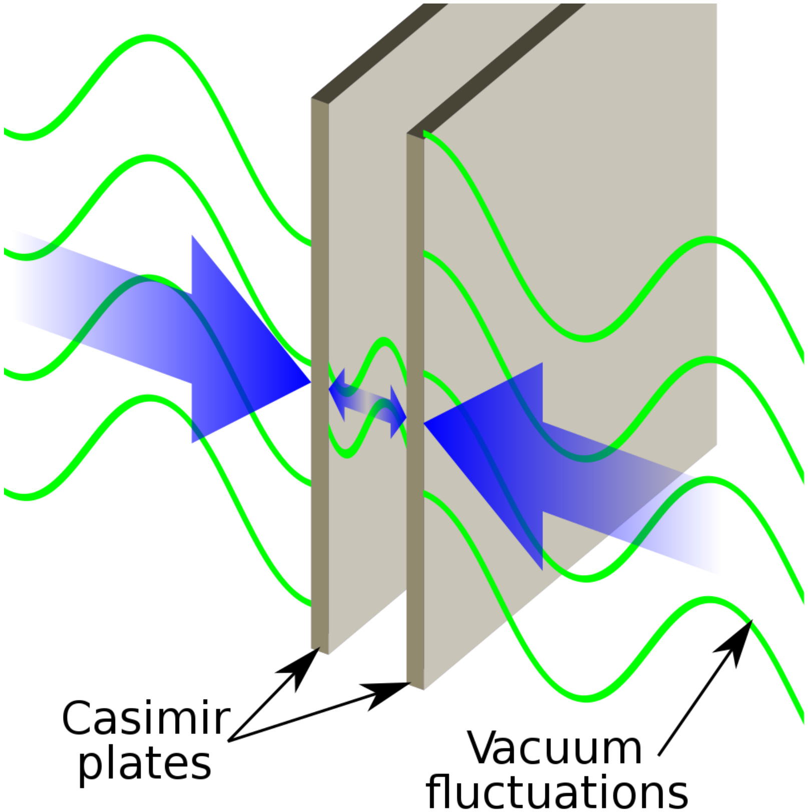


Modern view of vacuum
= quantum-mechanical ground state of a field
- Higgs vacuum
- BEC vacuum
- virtual particles, fluctuations



Effects related to vacuum:
- spontaneous emission
- Lamb shift
- static Casimir effect

Vacuum fluctuations: Casimir force



$$\frac{F_C}{A} = -\frac{d}{da}\frac{\langle E \rangle}{A} = -\frac{\pi^2 \hbar c}{240 a^4}$$

“Two ships should not be moored too close together because they are attracted one towards the other by a certain force of attraction.”

The Album of the Mariner
P. C. Caussée,1836

Nature, doi:10.1038/news060501-7

Measured by S. K. Lamoreaux in 1997

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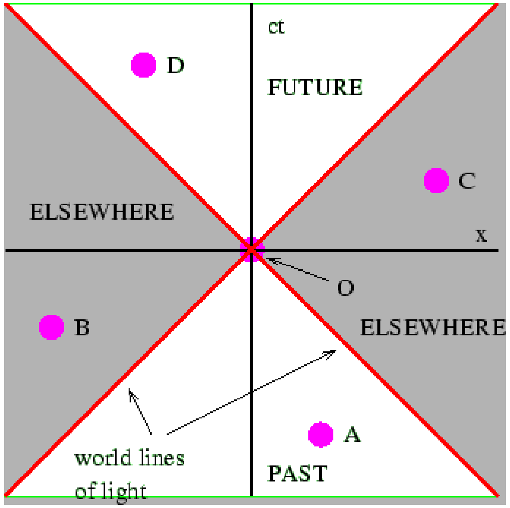
Exciting the vacuum

How to get something out of vacuum:

- use strong electric fields [Schwinger effect]
- change fast a boundary condition or the speed of light [dynamical Casimir effect]
- use a strong gravitational field [Hawking effect]
- accelerate the system [Unruh effect]

- Entanglement of virtual particles
- Entanglement transfer to qubits

- Past-future correlations



"Mode" observables: Quadratures

$$H_{\mathrm{k}} = \hbar\omega_{\mathrm{k}}(a_{\mathrm{k}}^{\dagger}a_{\mathrm{k}} + \frac{1}{2}))$$

Quadrature operators (like x and p):

$$X_1 = \frac{1}{\sqrt{2}}\left(a^{\dagger} + a\right)$$

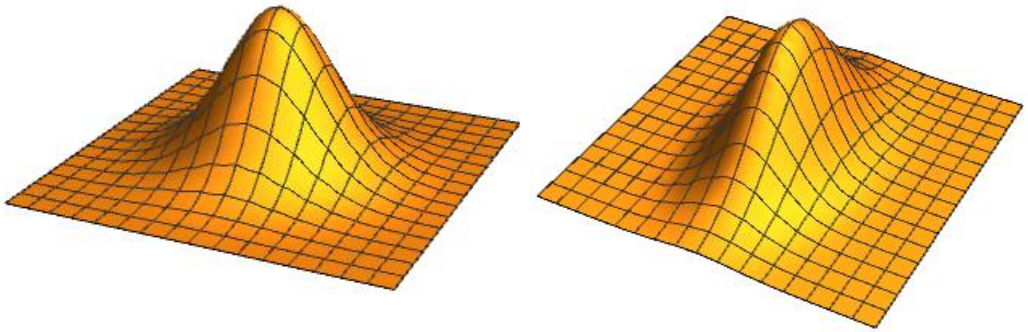
$$X_2 = \frac{i}{\sqrt{2}}\left(a^{\dagger} - a\right)$$

$$X_{\theta} = \frac{1}{\sqrt{2}}\left(ae^{-i\theta} + a^{\dagger}e^{i\theta}\right)$$

Since $\left[X_1, X_2\right] = i$**, there must be an uncertainty relation**

$$\Delta X_1 \Delta X_2 \geq \frac{1}{2}$$

Correlation of quadratures can be manipulated

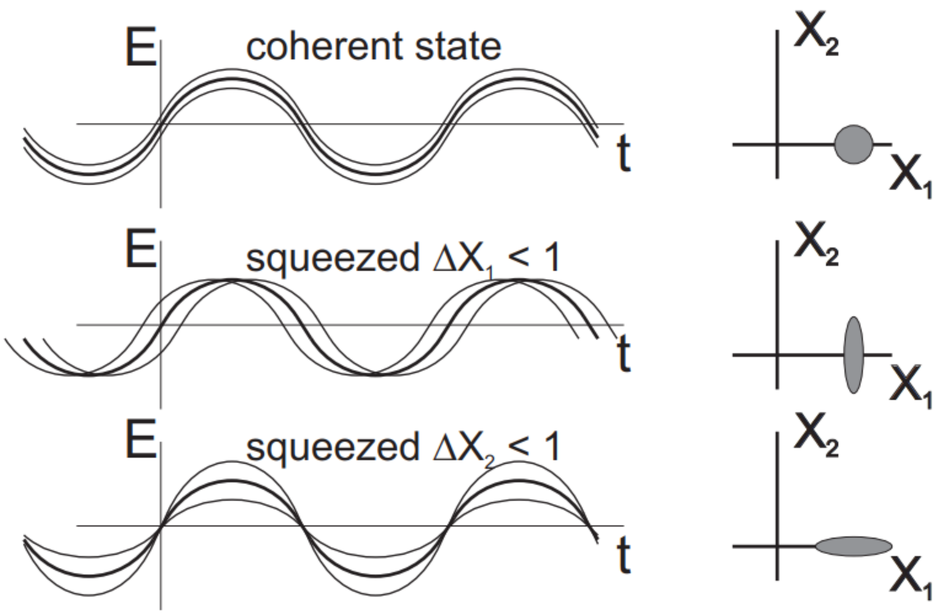


Single mode squeezing

Squeezing operator

$$S = \exp\left(\frac{1}{2}\xi a^{\dagger 2} - \frac{1}{2}\xi^* a^2\right) \qquad \xi = re^{i\theta} \qquad |\xi\rangle = S|0\rangle$$

$$\left.\begin{aligned}\langle \Delta X_1{}^2\rangle &= \frac{1}{2}e^{2r} \\ \langle \Delta X_2{}^2\rangle &= \frac{1}{2}e^{-2r}\end{aligned}\right\} \Delta X_1 \Delta X_2 = \frac{1}{2}$$



Basic correlator:

$$\langle aa\rangle = \cosh r \sinh r e^{i\theta}$$

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Two-mode squeezing

Two mode squeezing operator

$$S_2 = \exp\left(\xi^* ab - \xi a^\dagger b^\dagger\right) \qquad \xi = re^{i\theta}$$

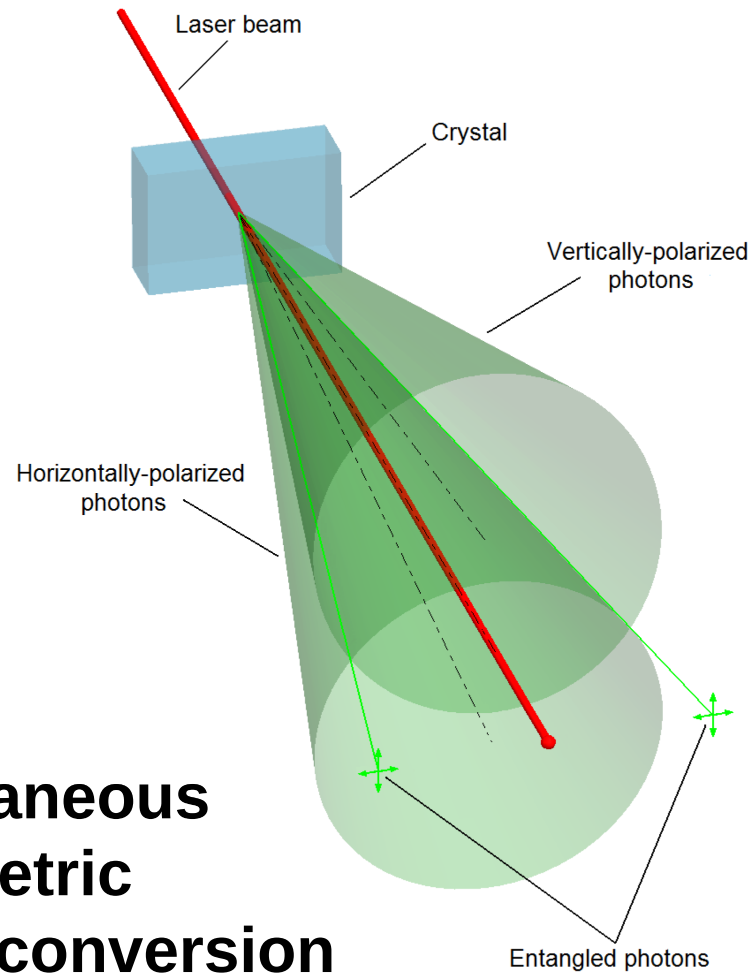
$$\langle ab \rangle = \cosh r \sinh r e^{i\theta} \qquad \langle ab^\dagger \rangle = 0$$

Maps to single mode case by defining operator

$$d = \frac{1}{\sqrt{2}}(a+b) \qquad \left[d, d^\dagger\right] = 1$$

$$X_\theta^d = \frac{1}{\sqrt{2}}\left(de^{-i\theta} + d^\dagger e^{i\theta}\right) \quad \left\langle \Delta X_1^{d\,2} \right\rangle = \frac{1}{2}e^{2r} \quad \left\langle \Delta X_2^{d\,2} \right\rangle = \frac{1}{2}e^{-2r}$$

Entanglement



Spontaneous parametric down-conversion

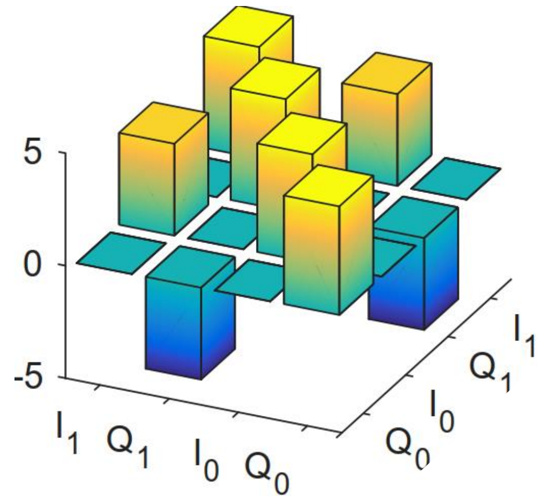
Polarization in optics

- vertically/horizontally

$$\left[|H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2 \right] / \sqrt{2}$$

Quadratures at microwaves

- in phase and out of phase

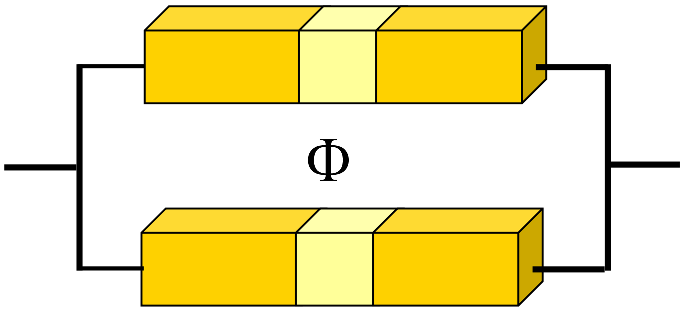


$$\sigma(x_i, x_j) = \langle x_i x_j \rangle$$

Quantum entanglement:

$$\sigma(x_+, x_+) - \sigma(x_+, x_-) < 1/4$$

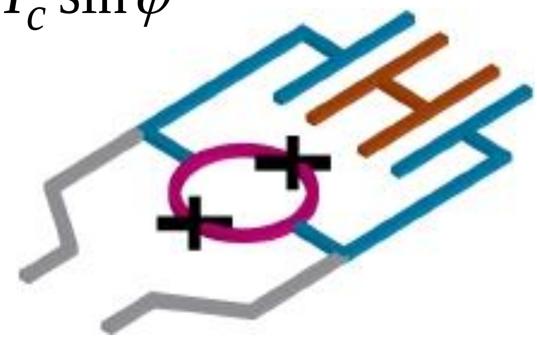
SQUID: A NONLINEAR *L*

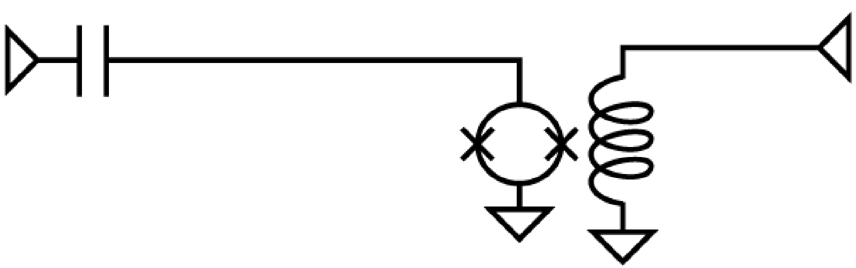


SQUID loop with

$$\varphi = \pi \frac{\Phi}{\Phi_0}$$

$$I = I_c \sin\varphi$$

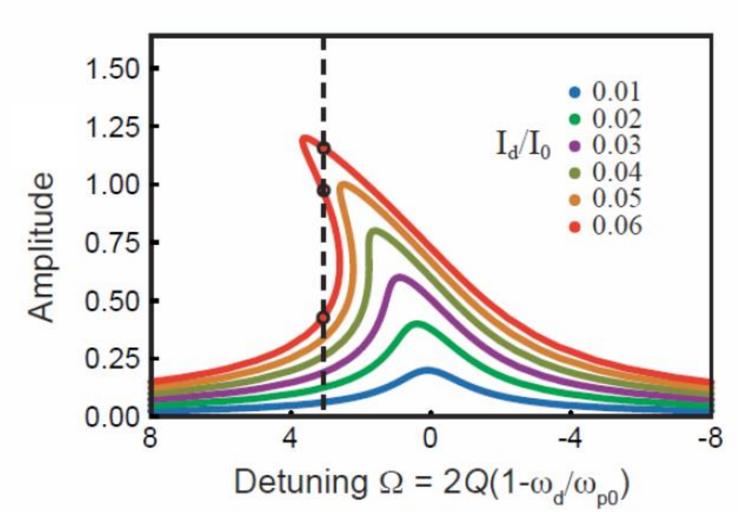




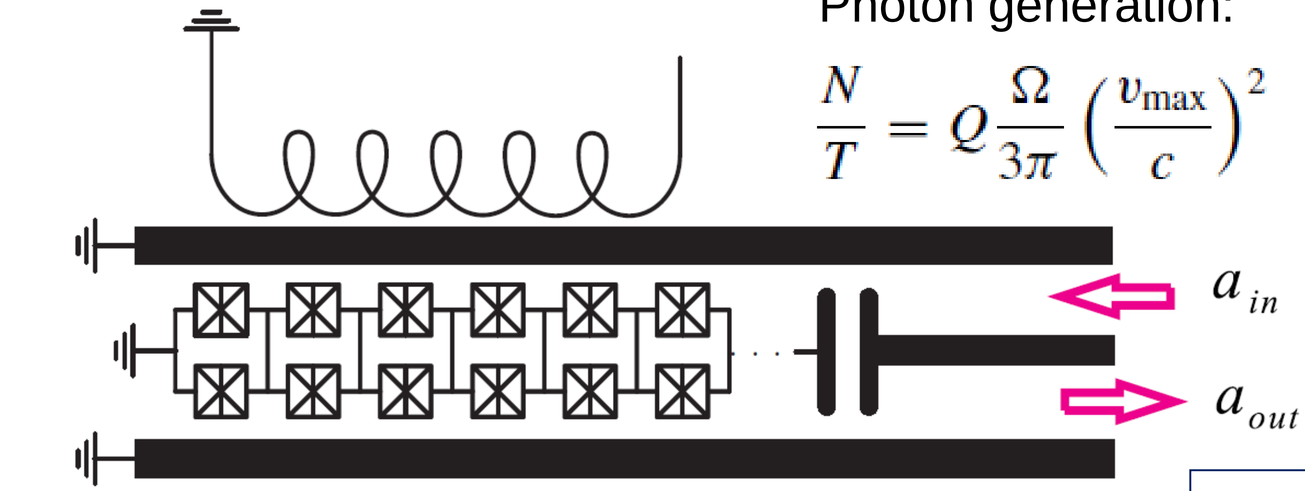
Josephson inductance

$$\frac{1}{L} = \left(\frac{2e}{\hbar}\right)^2 \frac{\partial^2 E}{\partial \varphi^2}$$

$$L_J = \frac{\hbar}{2eI_C} \frac{1}{\cos\varphi}$$



Analogy of dynamic Casimir effect (DCE)



Photon generation:

$$\frac{N}{T} = Q\frac{\Omega}{3\pi}\left(\frac{v_{\max}}{c}\right)^2$$

Propagation speed:

$$v = \frac{1}{\sqrt{lc}}$$

$$a_{\text{out}}(\omega) = e^{i\psi_\omega} a_{\text{in}}(\omega)$$

$$\psi_\omega = \arctan\frac{\kappa(\omega-\omega_{\text{res}})}{(\kappa/2)^2-(\omega-\omega_{\text{res}})^2}$$

G. T. Moore, J. Math. Phys. (N.Y.) 1970
E. Yablonovitch, PRL 1989
V. Dodonov, PRA 1993

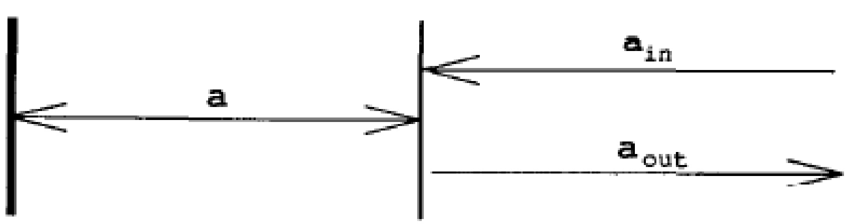
J. Johansson et al., PRL 2009, PRA 2010
C. Wilson et al., Nature 2011
P. Lähteenmäki et al., arXiv 2011

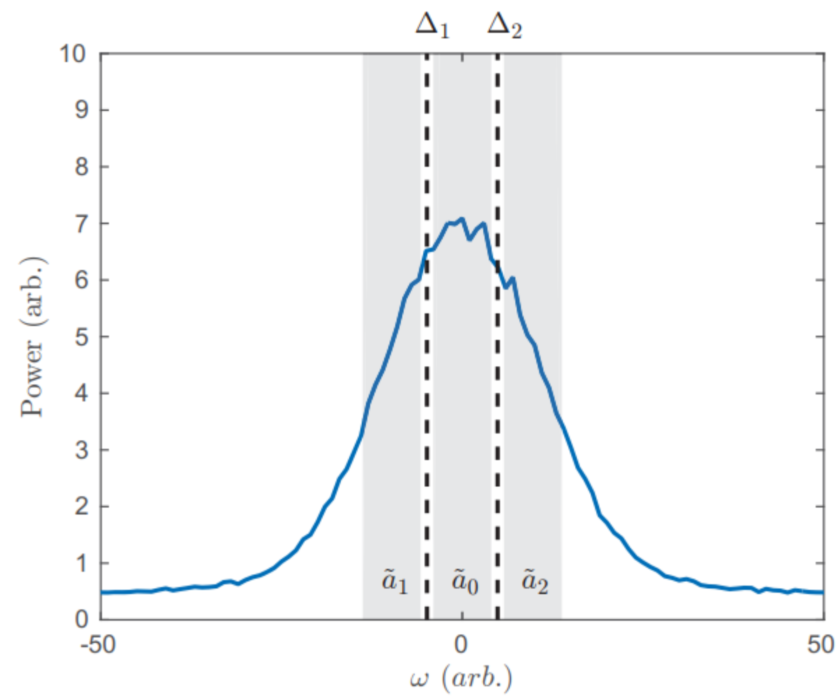
Semiclassical theory

$$H = \hbar\omega_{res} a^{\dagger} a + \frac{\hbar}{2i} \sum_{p=1,2} \left[\alpha_p^{*} e^{i\omega_p t} - \alpha_p e^{-i\omega_p t} \right] \left(a + a^{\dagger} \right)^2$$

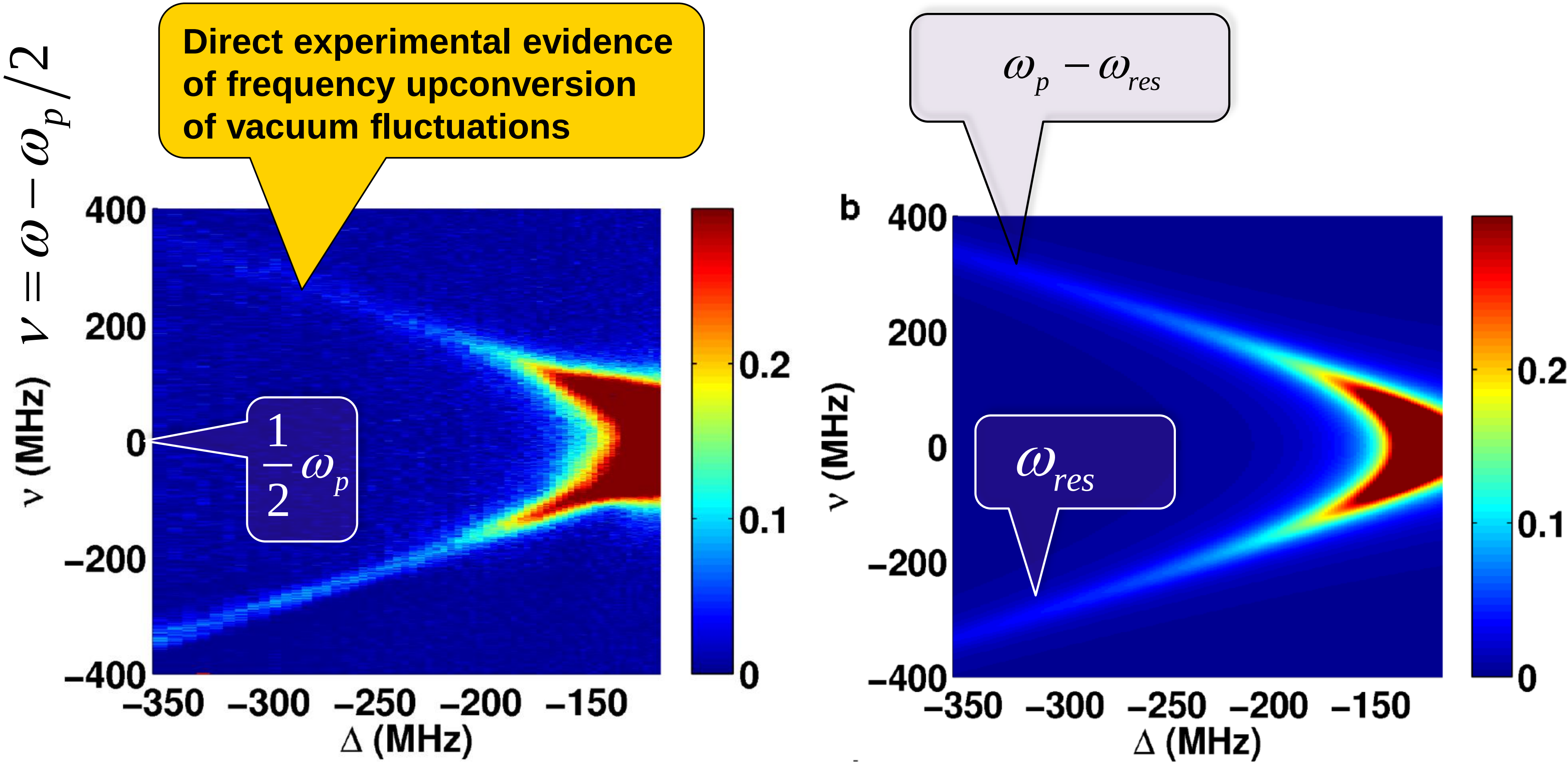
$$a(t) = \tilde{a}(t) \exp[-\omega_{res} t]$$
$$\Delta_p = \omega_p / 2 - \omega_{res}$$

$$\dot{\tilde{a}} = \sum_{p=1,2} \alpha_p e^{-2i\Delta_p t} \tilde{a}^{\dagger} - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\mathrm{in}}$$





Measurements at large detunings



$$\Delta = \omega_{res} - \omega_p/2$$

$$\left\langle \tilde{a}^{\dagger}_{\text{out}} \tilde{a}_{\text{out}} \right\rangle$$

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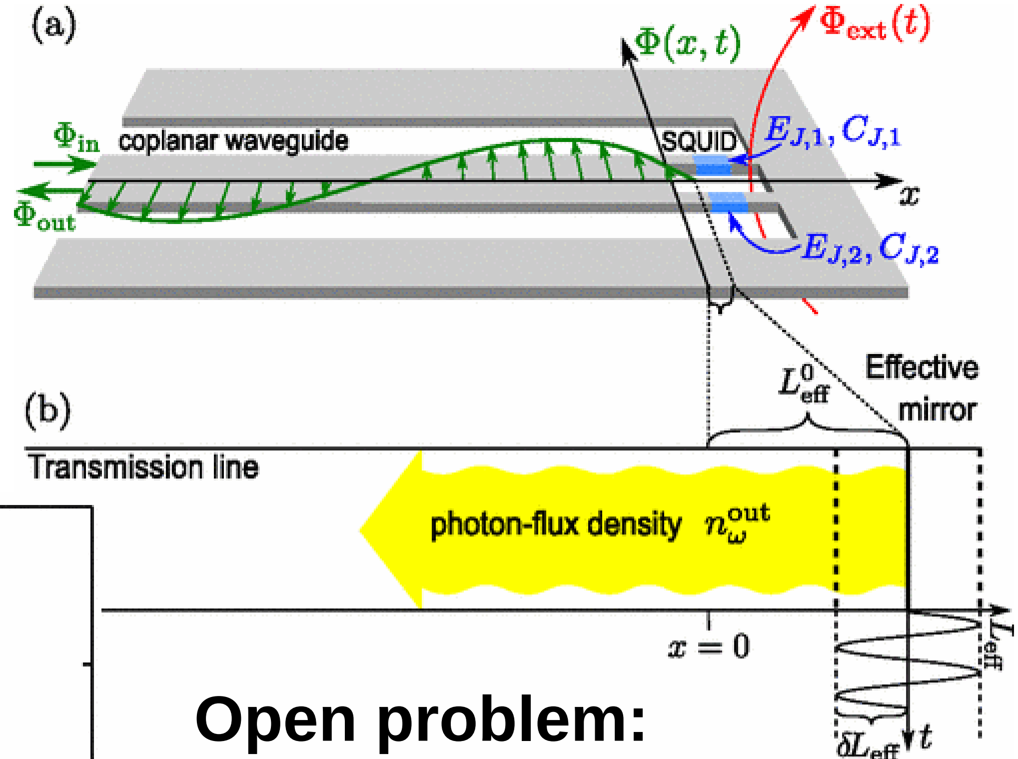
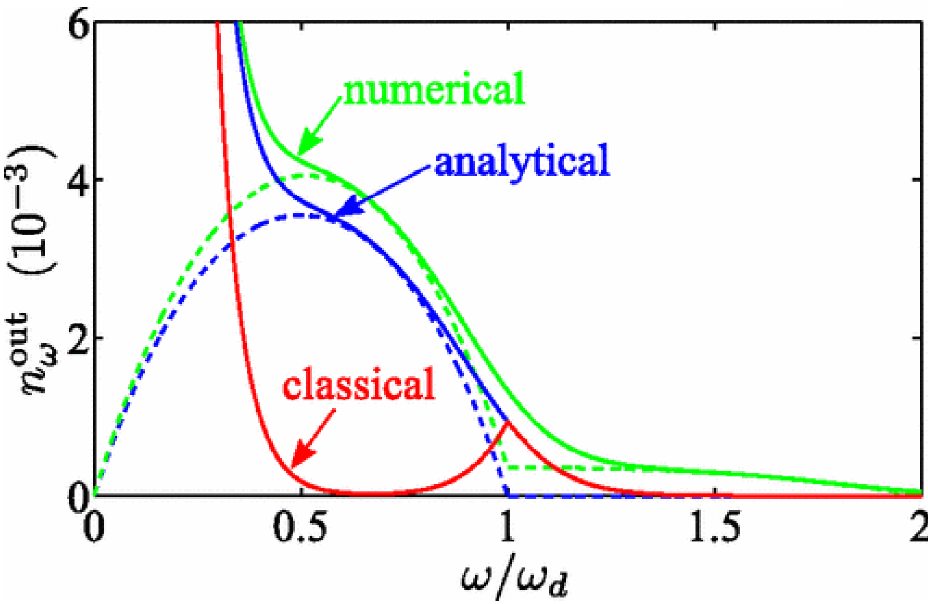
Lähteenmäki, P., Paraoanu, G. S., Hassel, J. & Hakonen, P. J. Proc. Natl. Acad. Sci. **110**, 4234–4238 (2013).

Intrinsic spectrum of DCE

Need a semi-infinite transmission line
- better sensitivity
- broad band

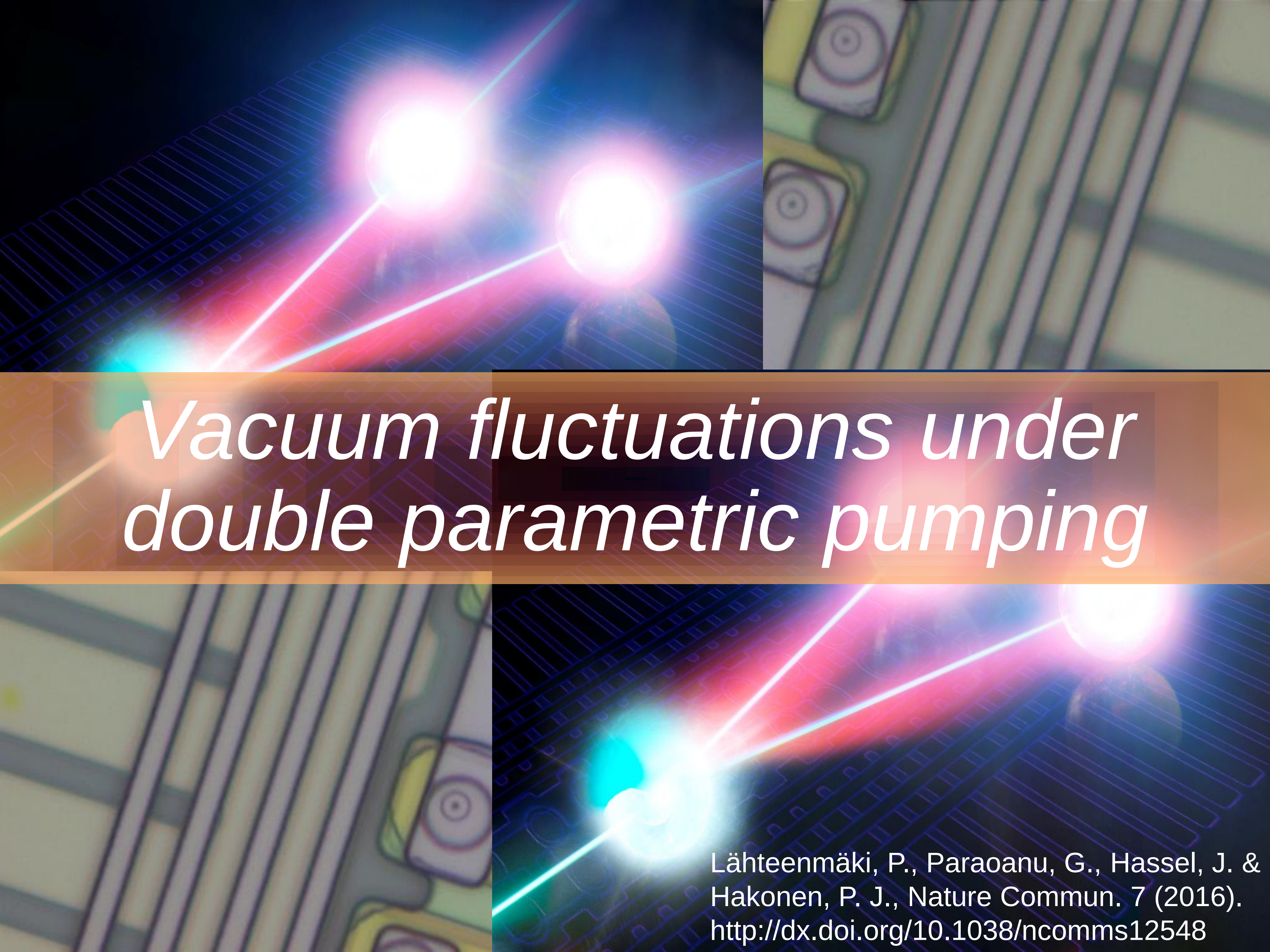
C. Wilson et al., Nature 2011





Open problem:
Specific parabolic spectrum expected

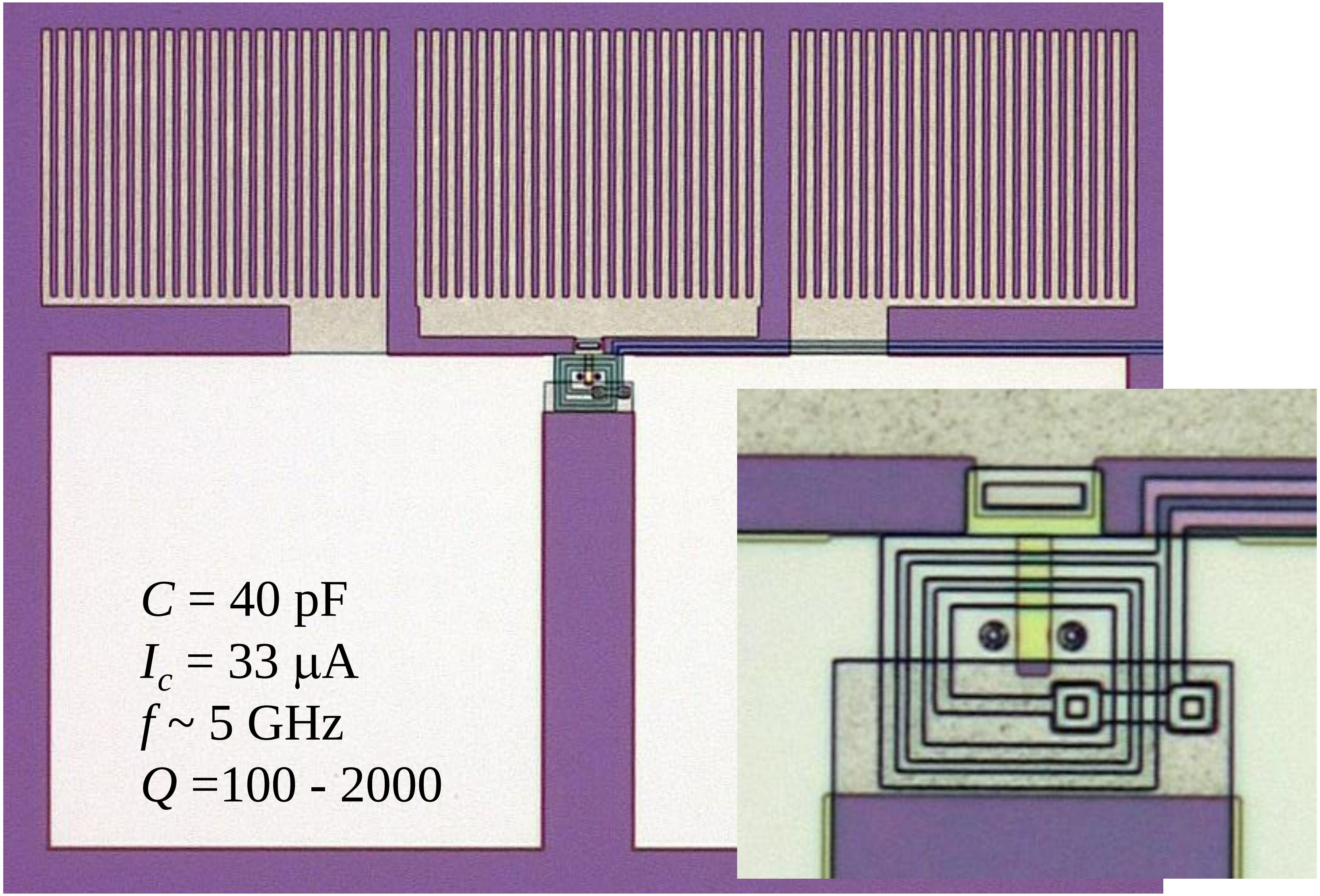
J. R. Johansson et al.,
PRL 103, 147003 (2009)



Vacuum fluctuations under double parametric pumping

Lähteenmäki, P., Paraoanu, G., Hassel, J. & Hakonen, P. J., Nature Commun. 7 (2016).
http://dx.doi.org/10.1038/ncomms12548

Lumped element parametric device



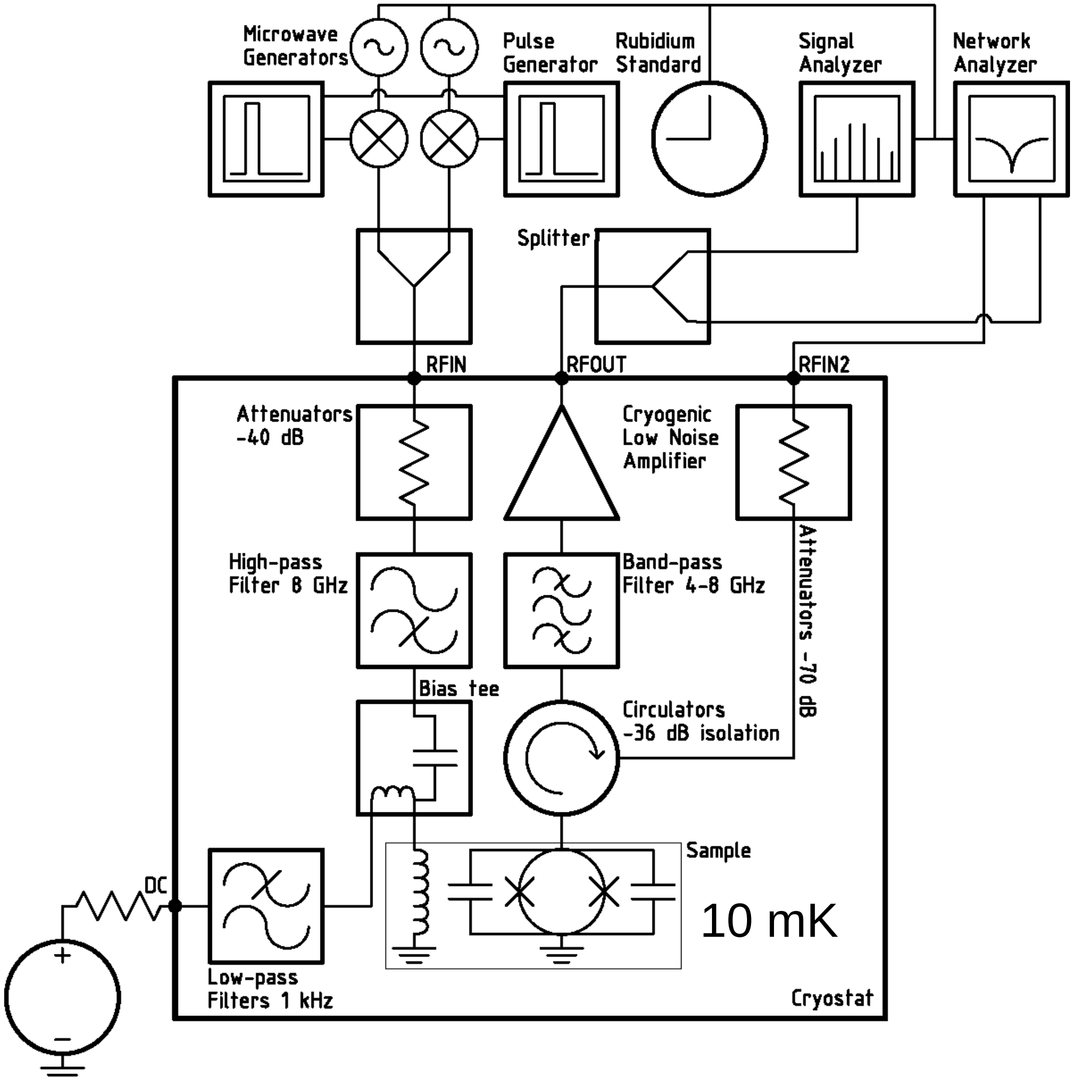
$C = 40$ pF

$I_c = 33$ μA

$f \sim 5$ GHz

$Q = 100 - 2000$

Experimental setup

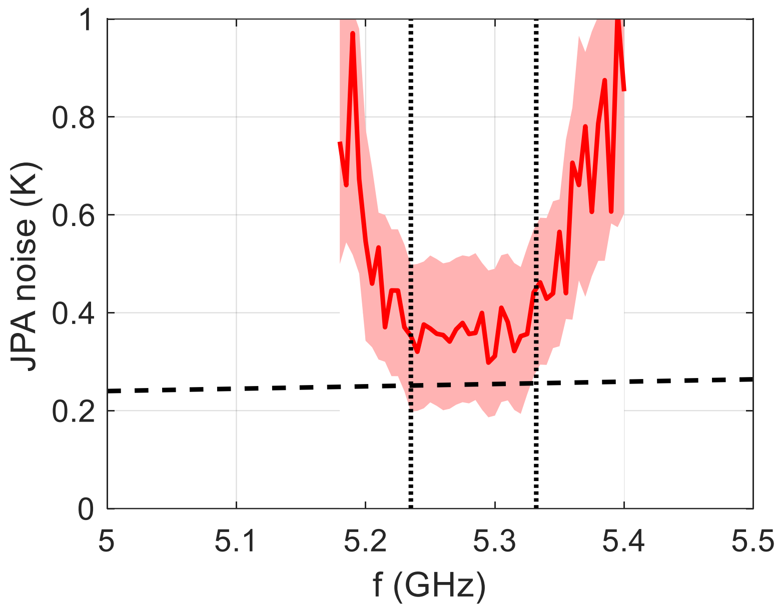


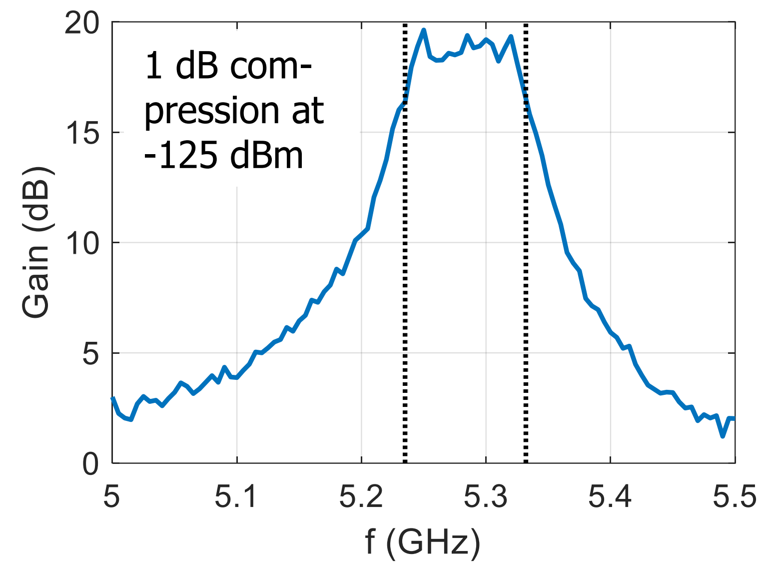
- Vector signal analyzer

- Quadrature components digitized at 50 MHz rate

- Digital band filtering and correlations with FFT

Gain and Noise Performance





- ❑ JPA noise using SNR improvement
  - ○ Vertical lines: 3 dB bandwidth
  - ○ Dashed line: quantum limit
  - ○ Shaded area: measurement error

- ❑ 100 MHz bandwidth
- ❑ Approching quantum limit

$$T_{QN} = \frac{\hbar\omega}{k_b}$$

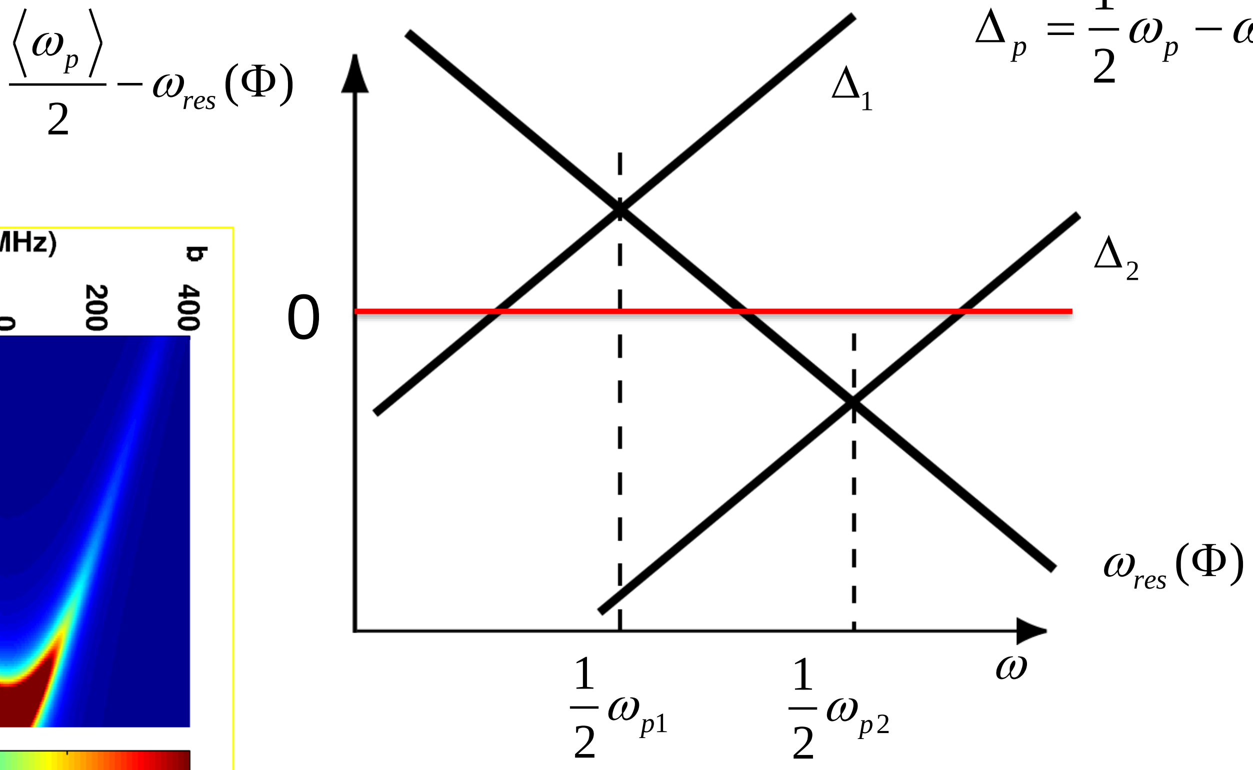
- ❑ Observation of quantum squeezing indicates low losses

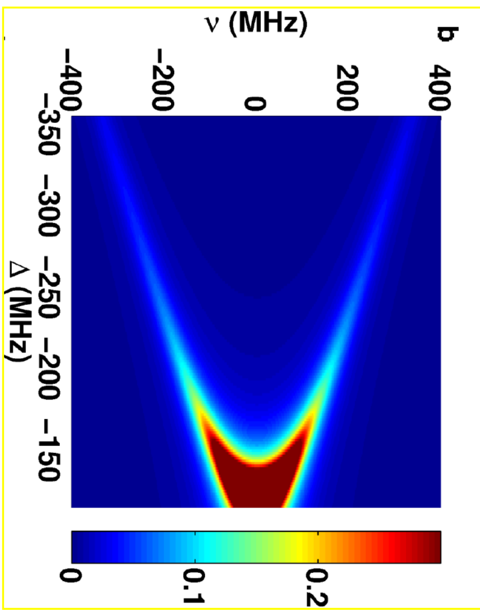
T. Elo, et al. 2018

Correlations in a two pump configuration

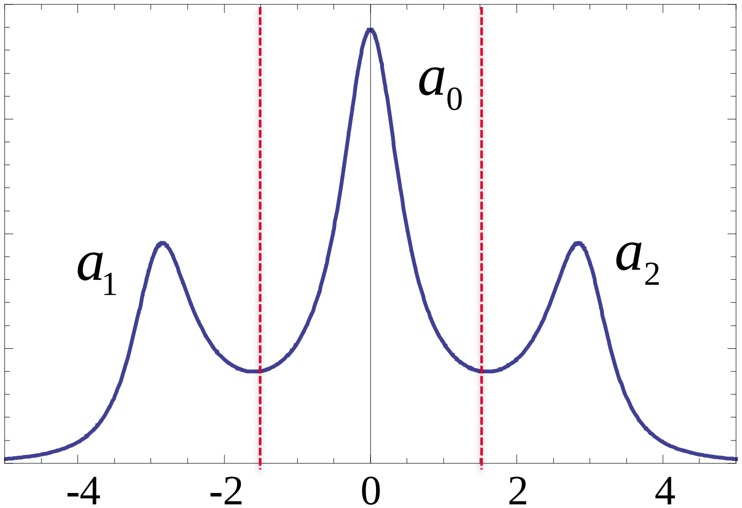
$$H = \hbar\omega_{res} a^{\dagger} a + \frac{\hbar}{2i} \sum_{p=1,2} \left[\alpha_p^{*} e^{i\omega_p t} - \alpha_p e^{-i\omega_p t} \right] \left(a + a^{\dagger} \right)^2$$

$$\Delta_p = \frac{1}{2}\omega_p - \omega_{res}$$





Bright and dark modes

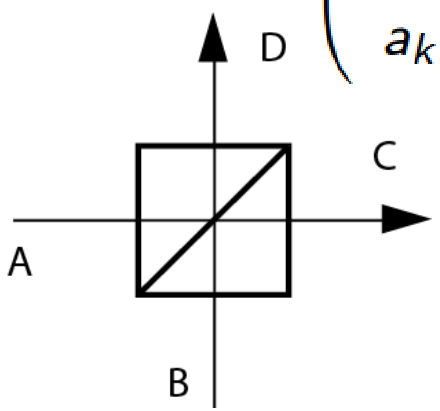


$$\left.\begin{matrix}\langle \tilde{a}_{\rm out}\tilde{d}_{\rm out}\rangle \\ \langle \tilde{b}_{\rm out}\tilde{d}_{\rm out}\rangle \\ \langle \tilde{d}^{\dagger}_{\rm out}\tilde{d}_{\rm out}\rangle\end{matrix}\right\} = 0$$

$$\begin{pmatrix} a_{k,C} \\ a_{k,D} \end{pmatrix} = U \begin{pmatrix} a_{k,A} \\ a_{k,B} \end{pmatrix}$$

Bright state $$\tilde{b} = \frac{1}{\sqrt{2}}\left(\tilde{a}_1 + \tilde{a}_2\right)$$

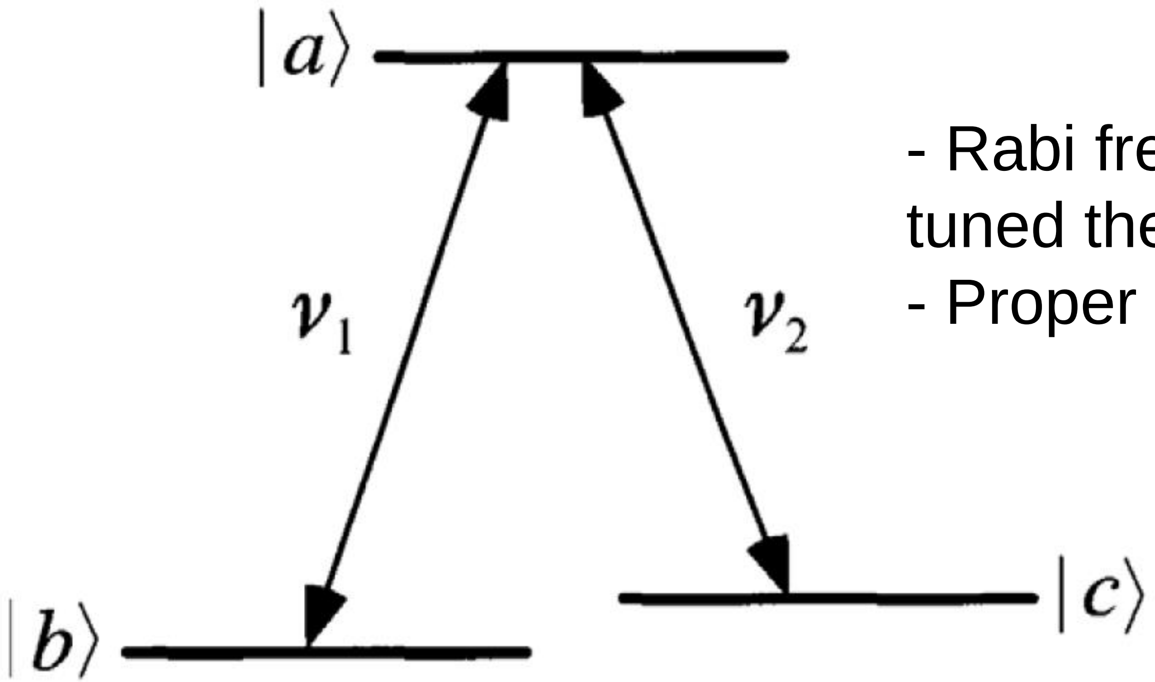
Dark state $$\tilde{d} = \frac{1}{\sqrt{2}}\left(\tilde{a}_1 - \tilde{a}_2\right)$$



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

- Coherence due to the same quantum fluctuation taking part in the generation of the pairs

Coherent population trapping (CPT)



- Rabi frequencies tuned the same
- Proper phases

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}|b\rangle + e^{-i\varphi}\frac{1}{\sqrt{2}}|c\rangle$$

Dark state:
- population trapped on $|b\rangle$ & $|c\rangle$
- no absorption

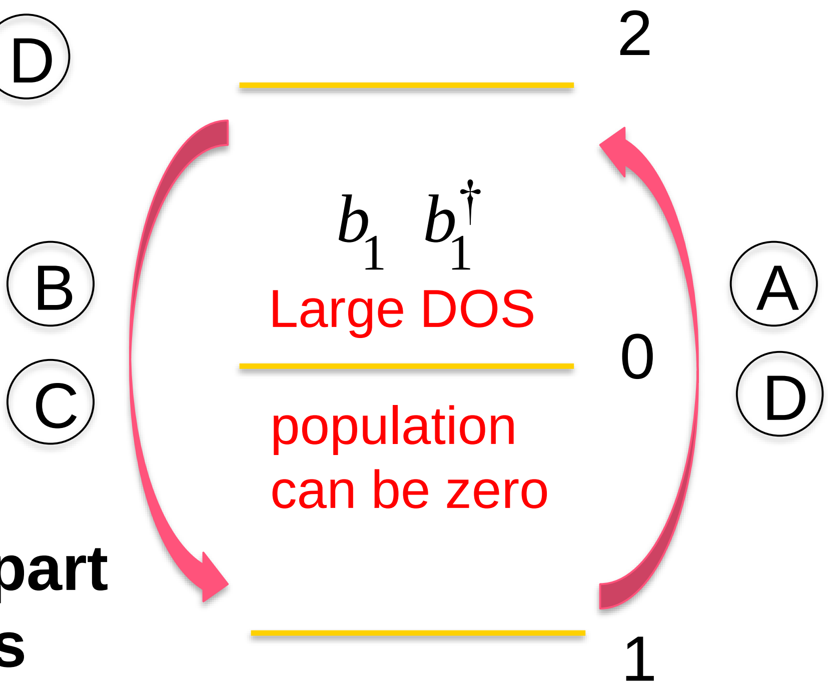
Vacuum induced coherence

Pump 1 $$\overset{(A)}{H_1 = b_1 a_2^\dagger a_0^\dagger} + \overset{(B)}{b_1^\dagger a_2 a_0}$$

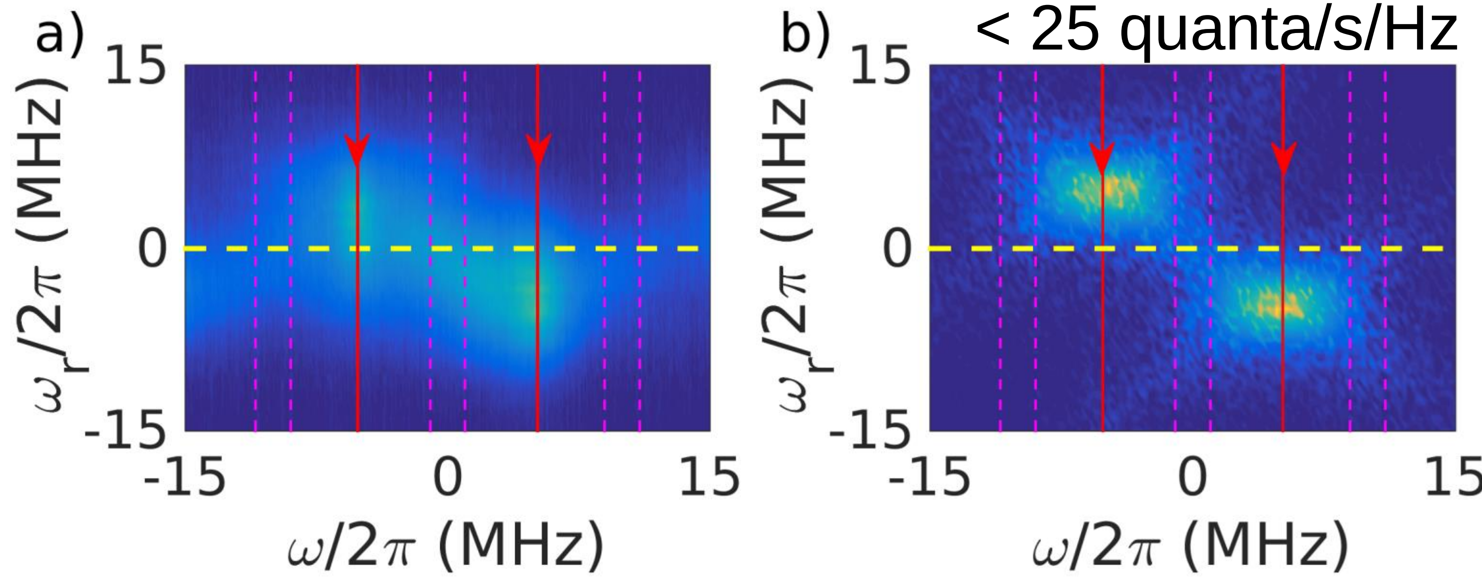
Pump 2 $$\underset{(C)}{H_2 = b_2 a_0^\dagger a_1^\dagger} + \underset{(D)}{b_2^\dagger a_0 a_1}$$

Correct phase:
no time development
i.e. dark state

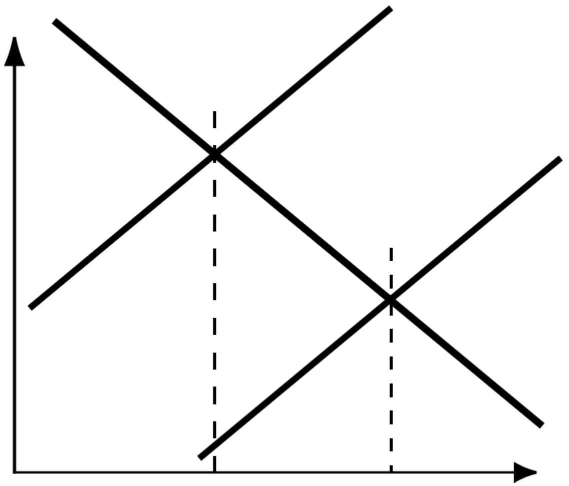
Coherence due to the same quantum fluctuation taking part in the generation of the pairs



Noise power measurements (low power)

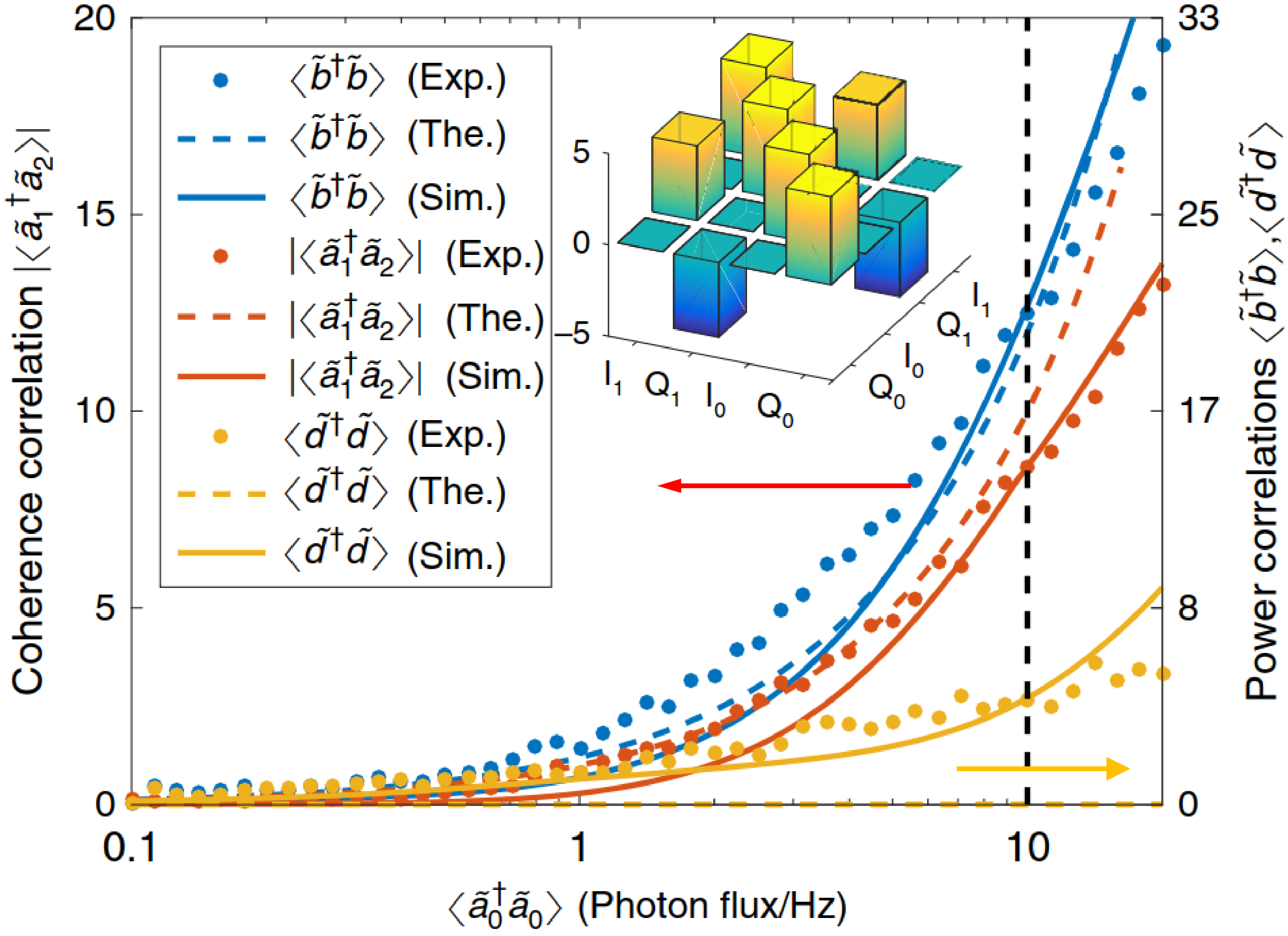


< 25 quanta/s/Hz $\omega_r = \omega_{res}$



$$\dot{\tilde{a}} = \sum_{p=1,2} \alpha_p e^{-2i\Delta_p t} \tilde{a}^\dagger - \frac{\kappa}{2}\tilde{a} - \sqrt{\kappa}\tilde{a}_{\text{in}}$$

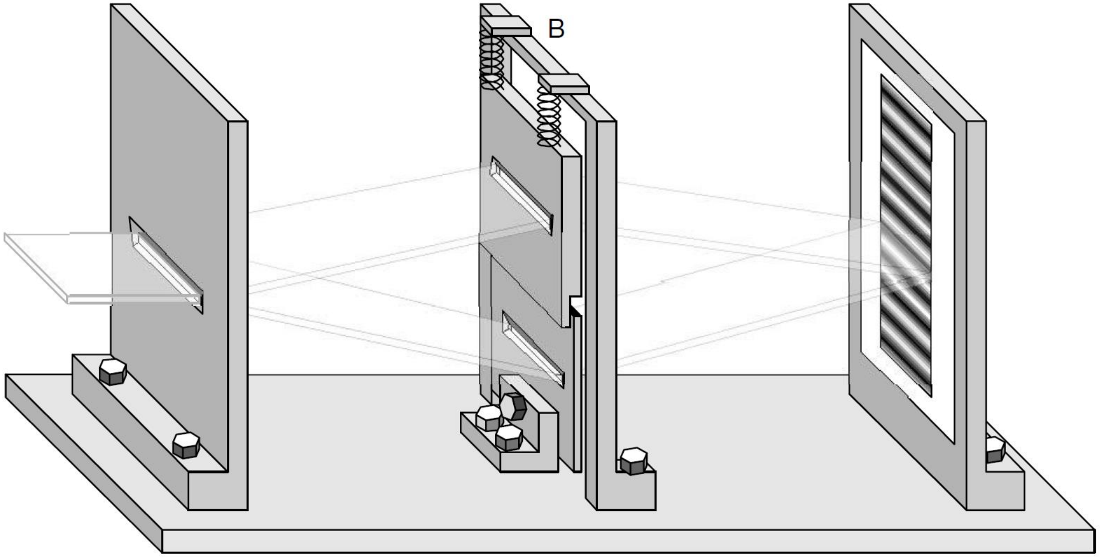
Mode correlators



Lähteenmäki, P., Paraoanu, G. S., Hassel, J. & Hakonen, P. J., Nature Communications **7** (2016). http://dx.doi.org/10.1038/ncomms12548

Which path - which color information

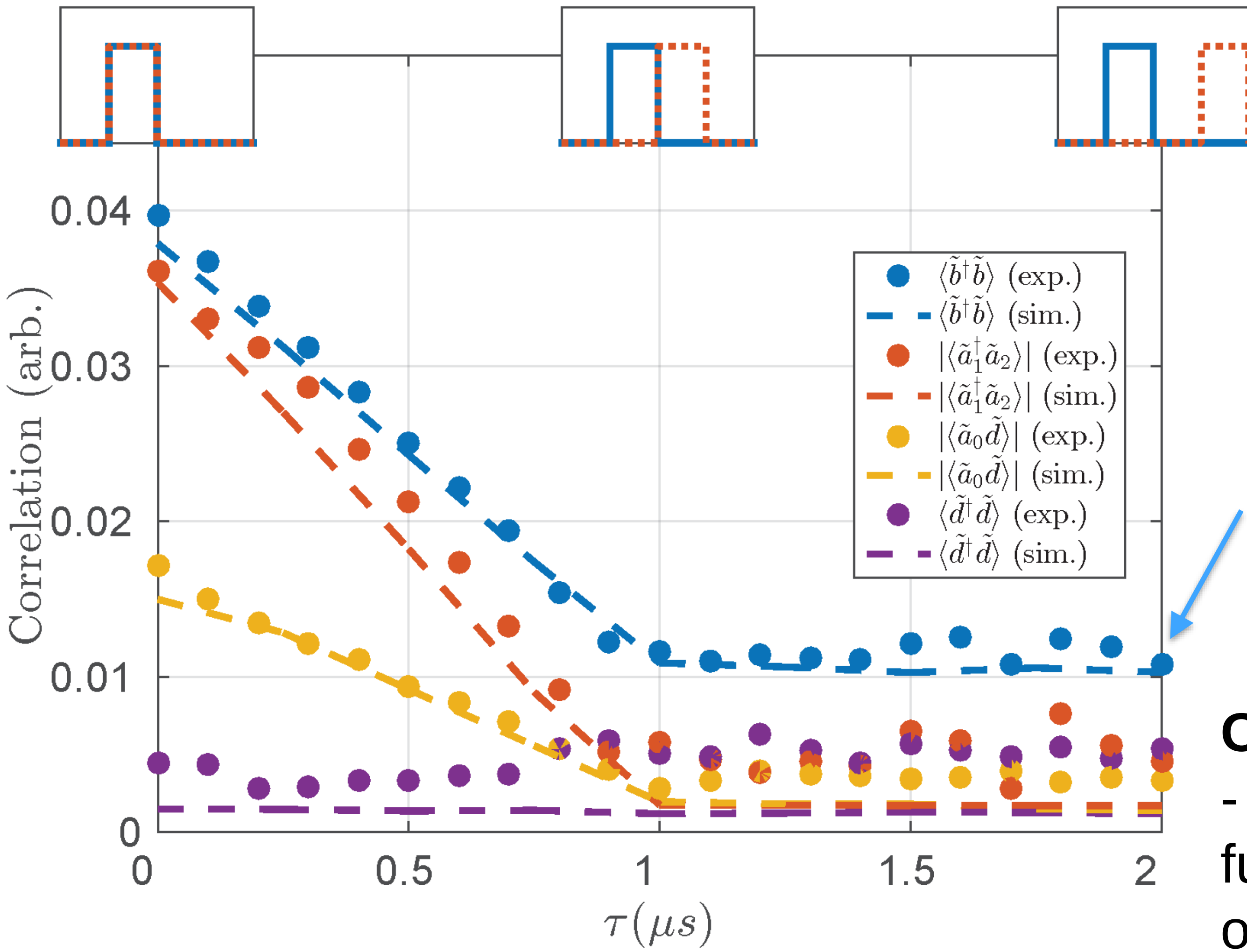
In our case: two **slits open when two pumps are on** – the system does not know from which pump the photon came



- Our which path is in **frequency space**
- Information can be obtained by varying pumps in time

P. Bertet, S. Osnaghi, A. Rauschenbeutel, G. Nogues, A. Auffeves, M. Brune, J. M. Raimond & S. Haroche, Nature **411**, 166 (2001)

Pulsed pumps with tuned overlap



$\kappa_I \approx 1\,\text{MHz}$
$\kappa_E \approx 2\,\text{MHz}$
$Q \approx 1700$

Base line for $b^\dagger b$ contains DCE power

Open problem:
- How correlation functions depend on time

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The vacuum and relativity

Minkowski metric

$$(\Delta s)^2 = (\Delta ct)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$$

- The past light cone contains all the events that could have a causal influence on O

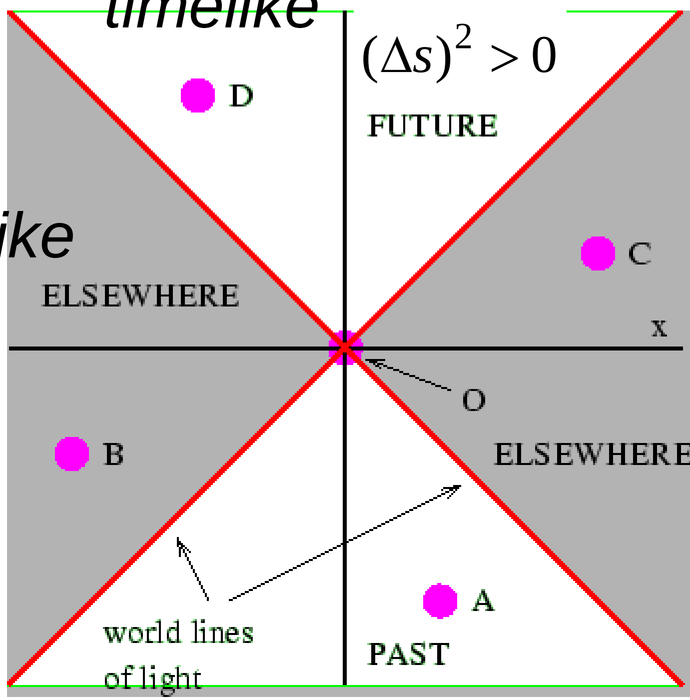
timelike

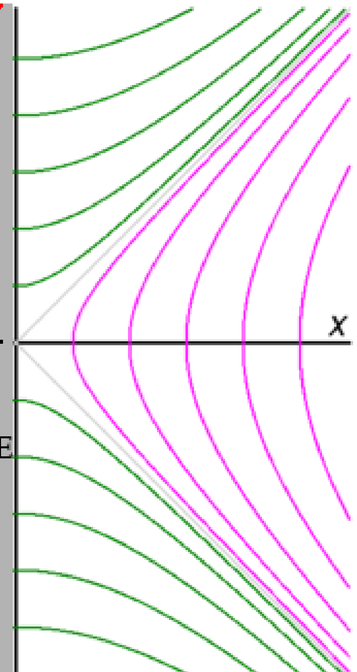
$x = \pm ct$

spacelike region

$(\Delta s)^2 < 0$

Light cone





Non-local correlations via quantum vacuum*

Also **past-future correlations**

Closely related to the **Unruh effect**

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*A. Valentini, Phys. Lett. A **153**, 321 (1991)
*B. Reznik, et al., Phys. Rev. A **71**, 042104 (2005)

The vacuum and the equivalence principle

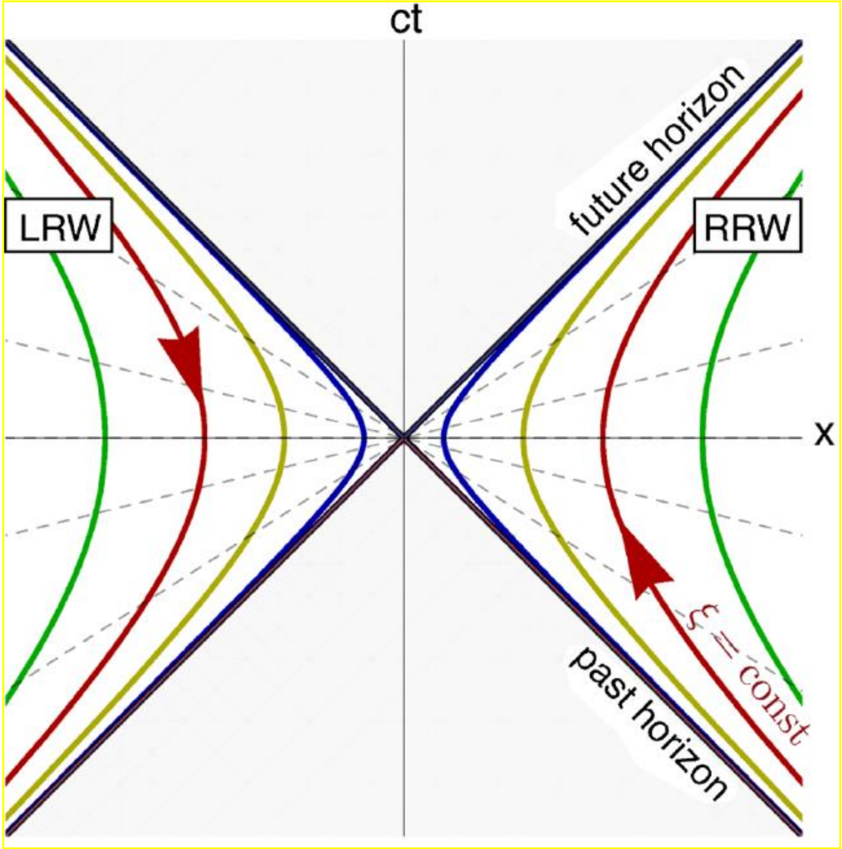
The Unruh effect:

- An accelerating observer will observe **blackbody radiation** where an inertial observer would observe none.

- The uniformly accelerating observer is out of causal contact with part of space time (having both positive and negative *f*)

$$k_B T_U = \frac{\hbar a}{2\pi c}$$

How to observe?

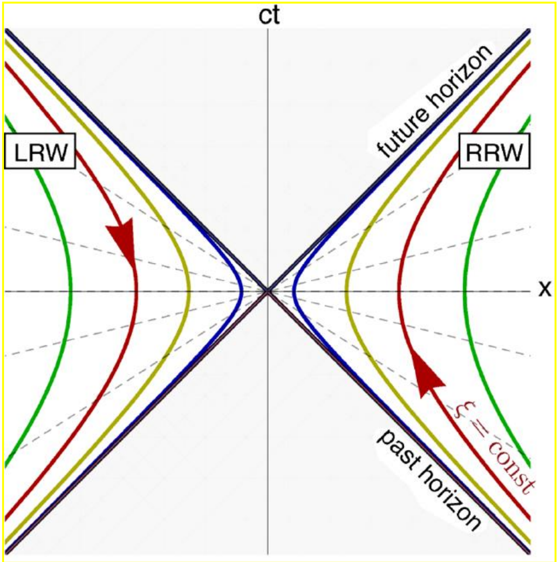


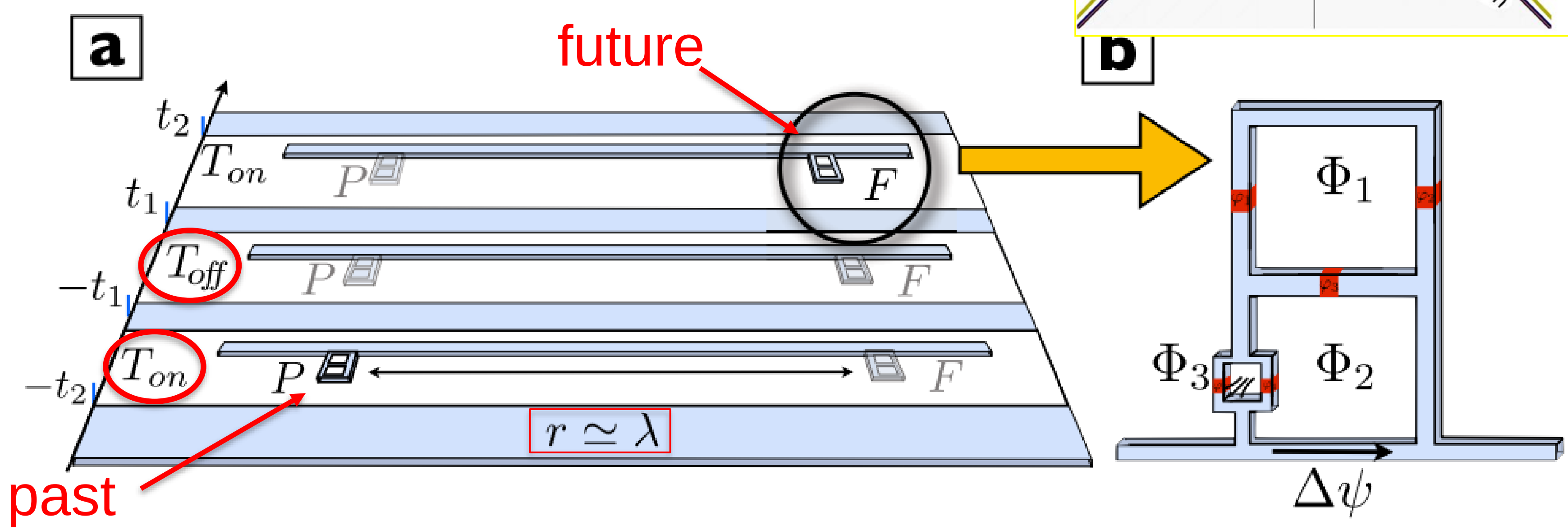
$$T_U = 1.2\times 10^{-19} K \quad a = 10\, m/s^2$$

S. Fulling 1973, P. Davies 1975, and W. G. Unruh 1976

Past-Future Vacuum Correlations in Circuit QED

- Entanglement across O
- Qubits like Unruh de Witt detectors:
  operate detectors with ***varying splitting***
  instead of acceleration or ***at different t***
- Small level spacing -> long time scales
- Coherence times sufficient





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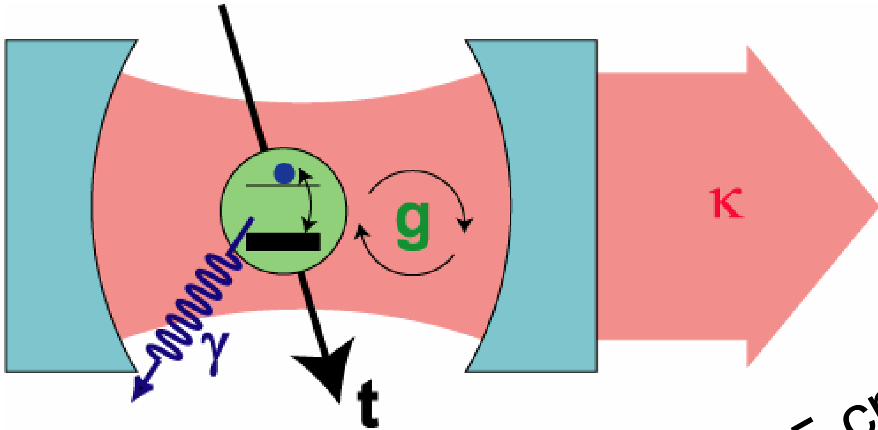
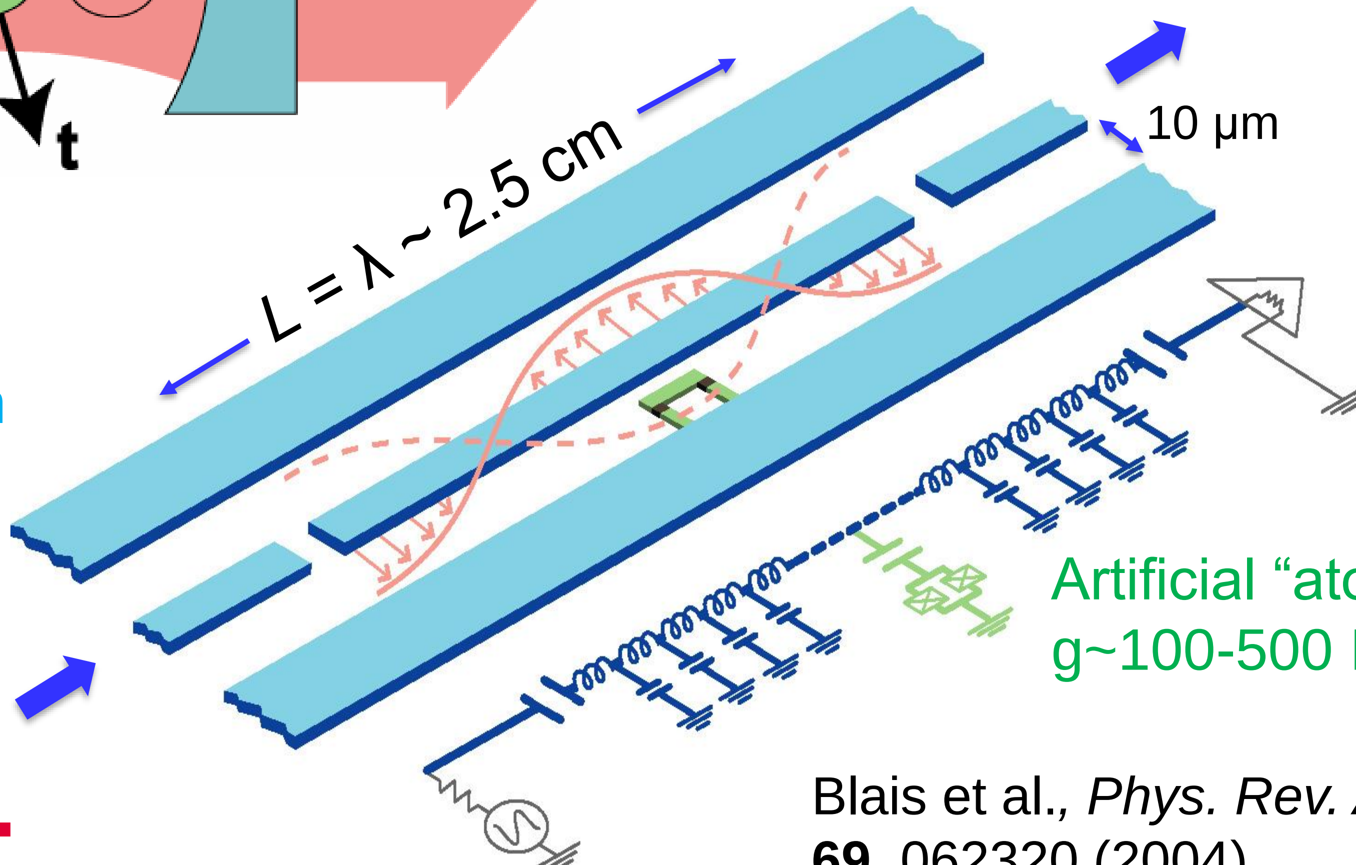
S. J. Olson and T. C. Ralph, Phys. Rev. A **85**, 012306 (2012)
C. Sabin, et al. PRL **109**, 033602 (2012)

Circuit QED



2g = vacuum Rabi frequency
κ = cavity decay rate
γ = “transverse” decay rate



transmission line “cavity”

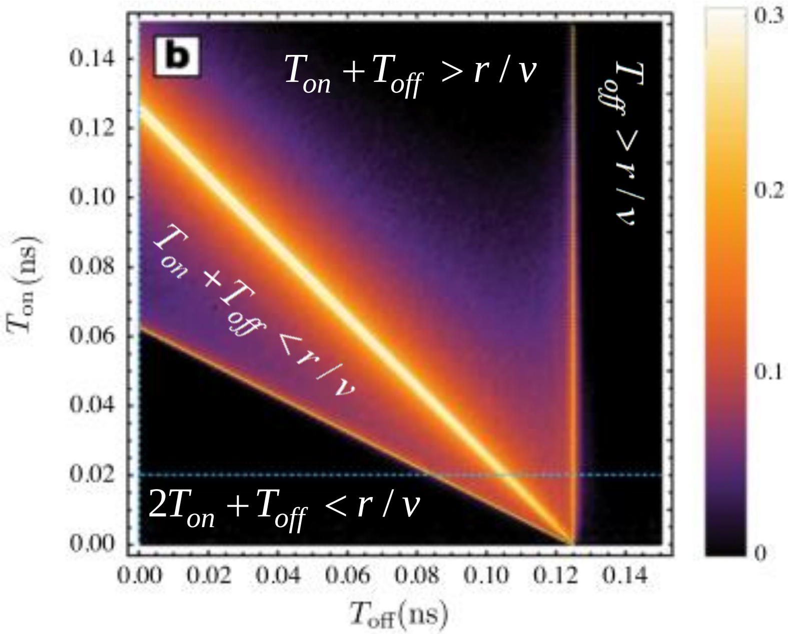
5 GHz

Artificial “atom”
g~100-500 MHz

Blais et al., *Phys. Rev. A* **69**, 062320 (2004)

Aalto University

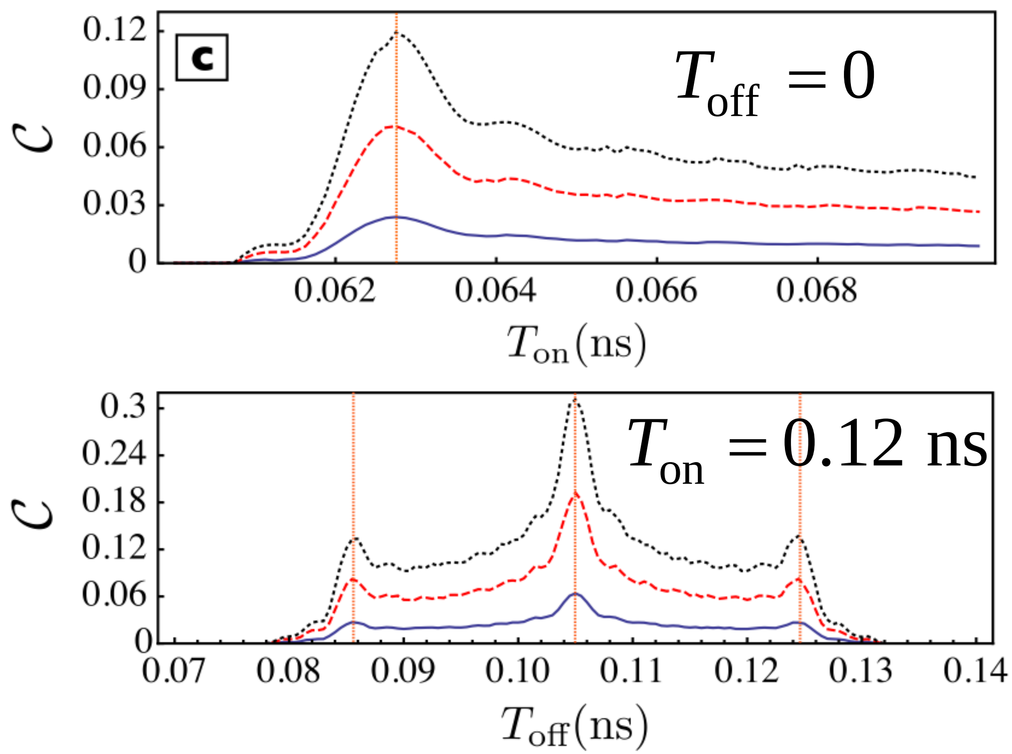
Past-Future Vacuum Correlations in Circuit QED



$\Omega_P = \Omega_F = 2\pi \times 1\text{GHz}$ Qubit splitting

$g/\Omega = 0.19$ Qubit coupling

$r/\lambda = 0.125$ Scaled distance



Challenges:

- Low electronic T
- Fast pulsing
- Rapid low-noise measurement

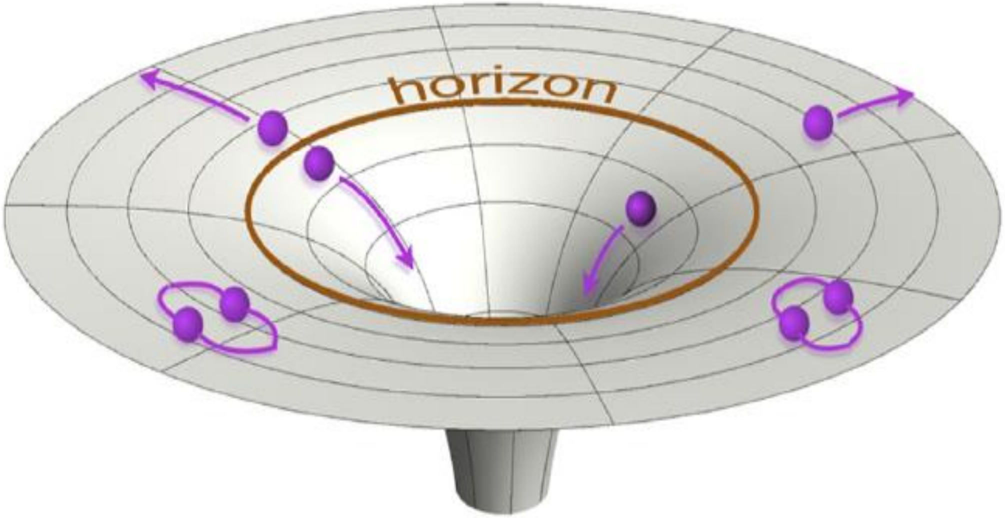
C. Sabin, et al. PRL **109**, 033602 (2012)

Gravitational effects and its analogs

The Hawking effect

(1974)

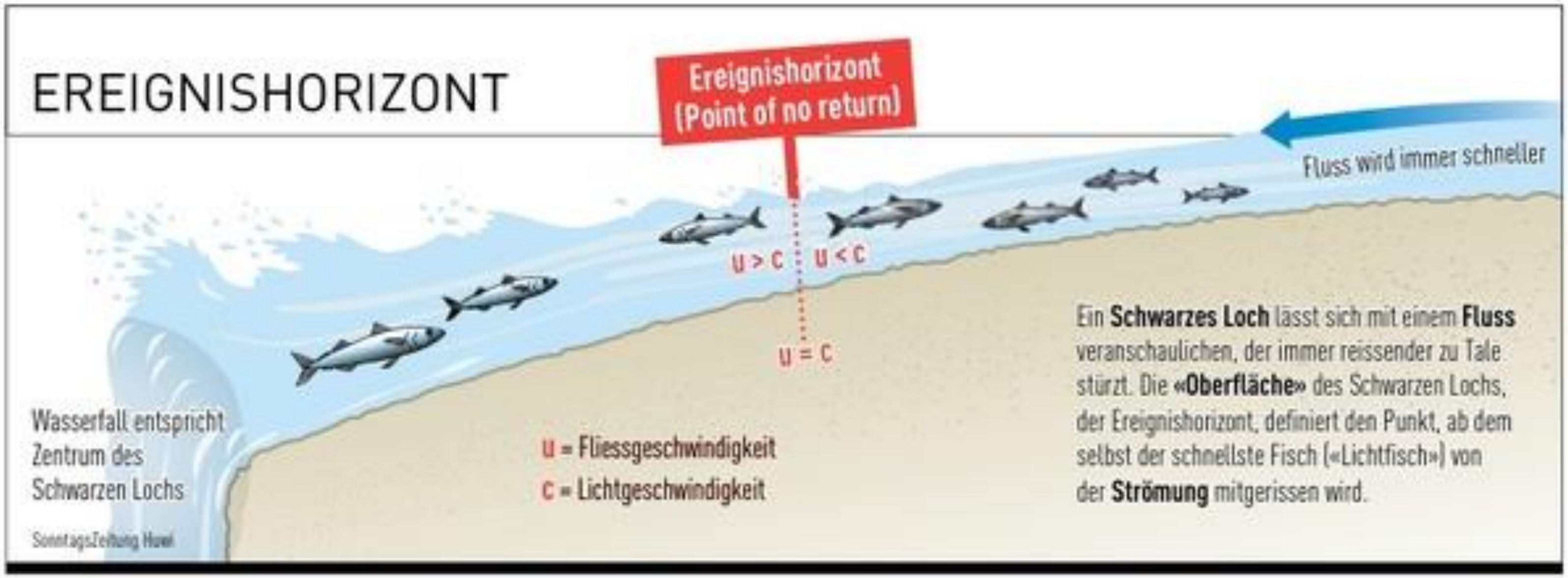
$$k_B T_H = \frac{\hbar g_h}{2\pi c} \qquad g_h = \frac{c^4}{4GM}$$



Estimate:

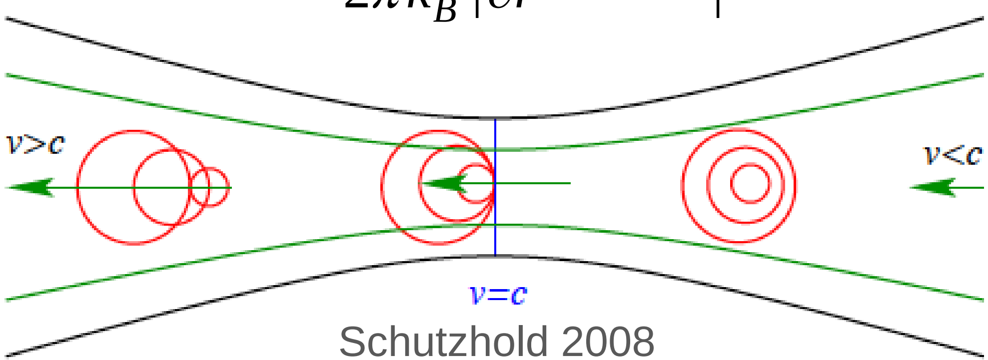
For a black hole with *M* = the solar mass
$T_H = 10^{-7} K$ **... but the c.m.b. is at 2.7 K**

Sonic analog of black holes



$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B}\left|\frac{\partial}{\partial r}(v_0 - c)\right|$$

Unruh 1980



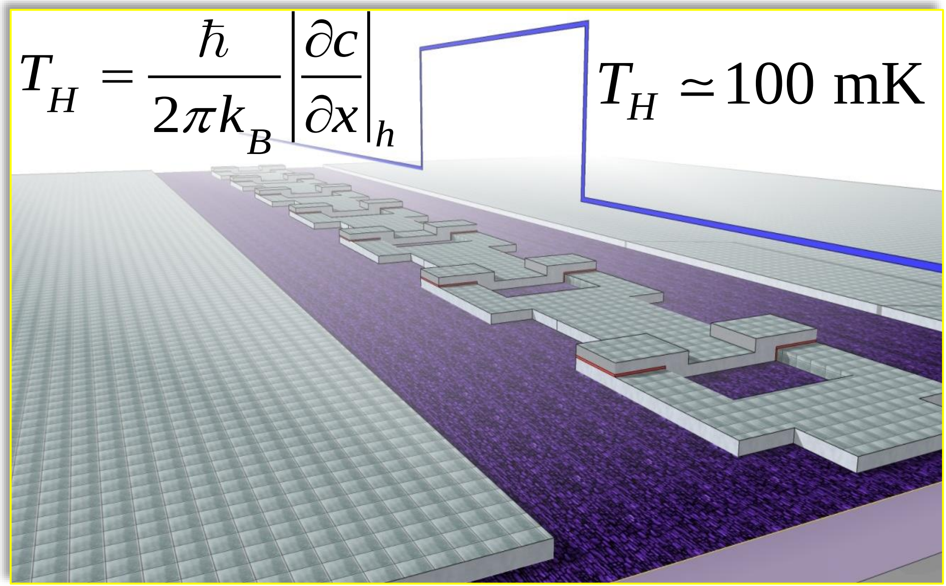
Schutzhold 2008

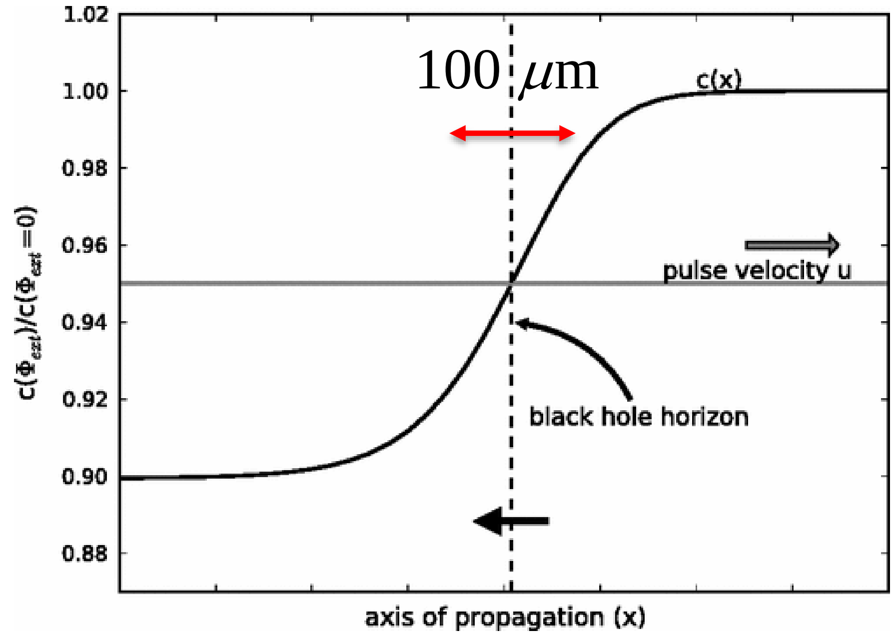
In Bose-Einstein condensates:

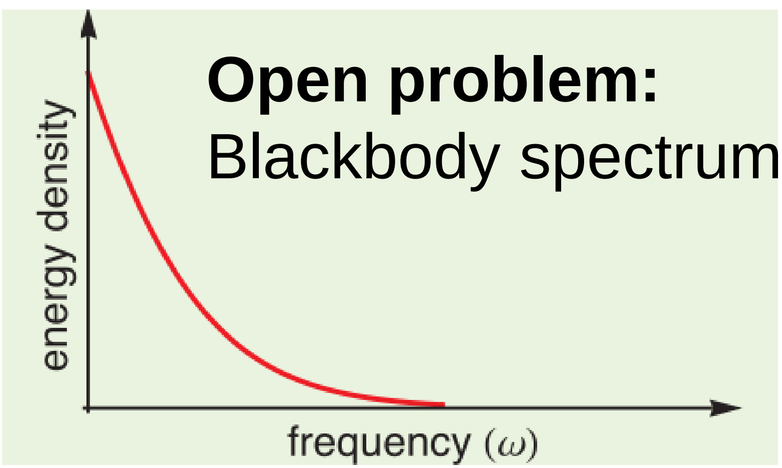
Observation of quantum Hawking radiation and its entanglement in an analogue black hole

J. Steinhauer
Nature Phys. **12**, 959 (2016)

Analog cosmological effects in SQUID arrays







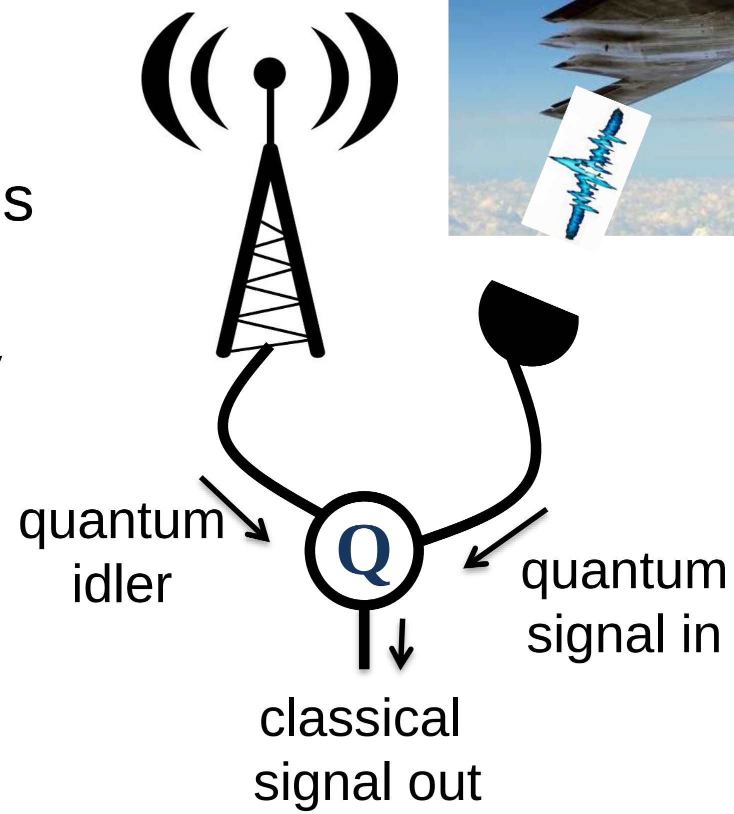
R. Schützhold, and W. G. Unruh, Phys. Rev. Lett. **95**, 031301 (2005)
D. Nation, M. P. Blencowe, A. J. Rimberg, and E. Buks, Phys. Rev. Lett. **103**, 087004 (2009)

Blackbody spectrum in acoustic systems?
Silke Weinfurtner, et al., Phys. Rev. Lett. **106**, 021302 (2011)

Aalto University

Entanglement as a resource: quantum radar

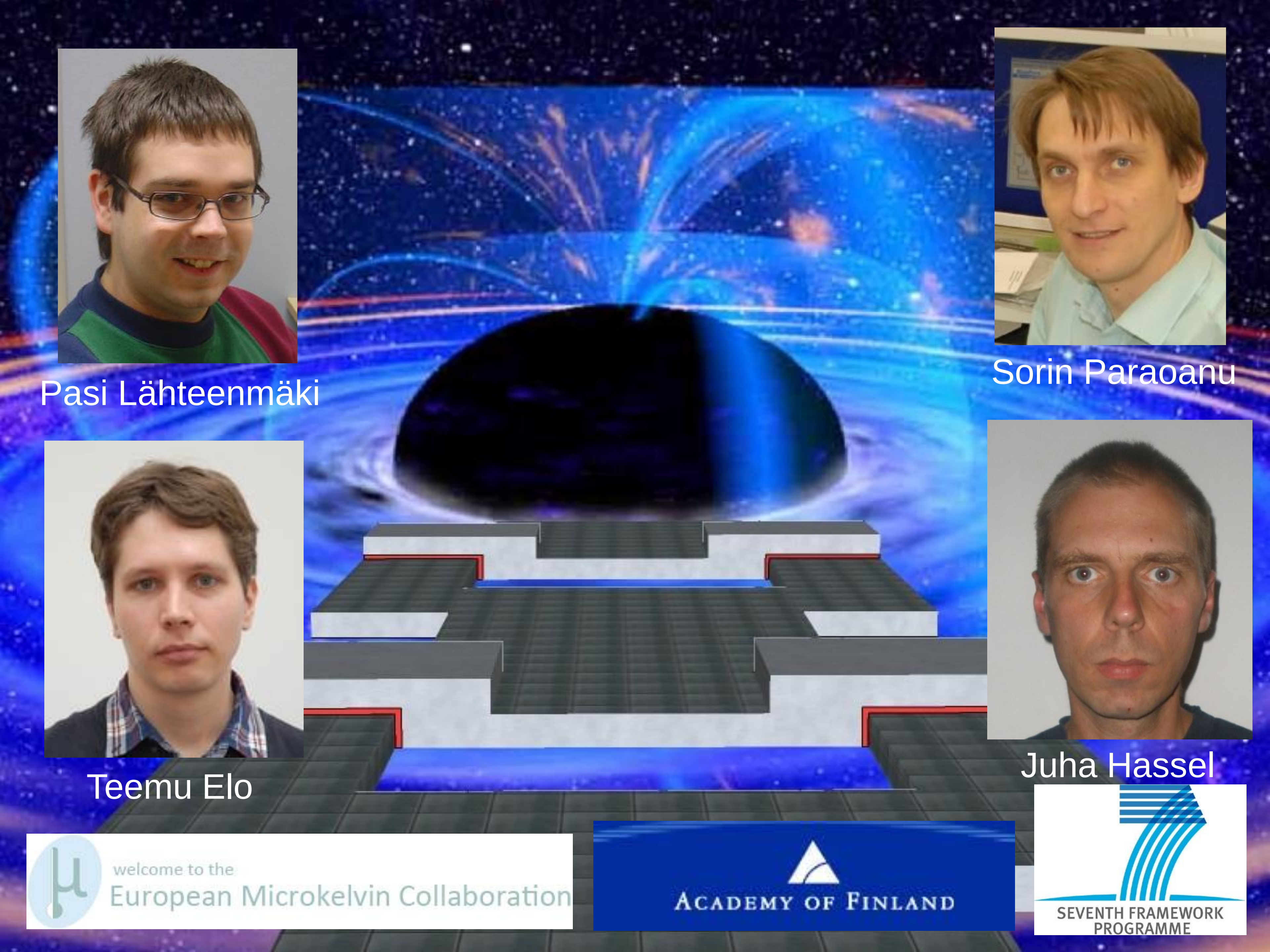
- Quantum correlations (entanglement) shared by transmitted and idler radiation
- Receiver distills the correlations from the incoming radiation
- Particularly useful in extremely lossy and noisy situations.



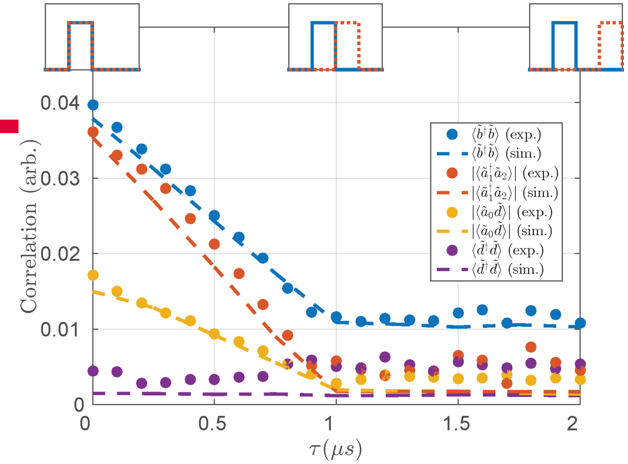
→ **Higher sensitivity with less power**

Application: detection of stealth aircrafts

"China's latest quantum radar won't just track stealth bombers, but ballistic missiles in space too"


Pasi Lähteenmäki
Sorin Paraoanu
Teemu Elo
Juha Hassel
welcome to the
European Microkelvin Collaboration
Academy of Finland
SEVENTH FRAMEWORK PROGRAMME

“Open problems” summary



1) *Casimir photon generation*
- *time dependent phenomena*
- *parabolic spectrum*

2) *Hawking radiation*
- *analog using electronic circuits*
- *blackbody spectrum*

3) *Past – future correlations*
- *entanglement transfer of quantum vacuum*
- *sub-nanosecond, low noise measurements*

4) *Quantum radar*
- *how to use entanglement to improve SNR*