

LECTURE 2 - Quantum Magnetism with UCQG

Superexchange Interactions

Single Spin Impurity

Bound Magnons

AFM Order in the Fermi Hubbard Model

Probing Hidden AFM in 1D Hubbard Chains

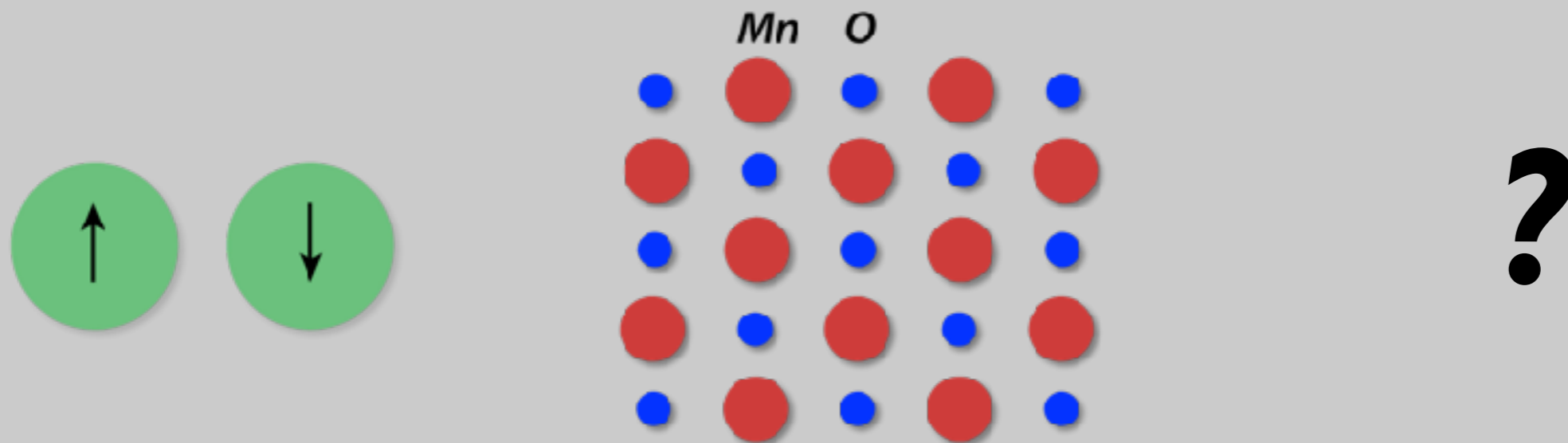
Direct Imaging of Spin-Charge Separation

Incommensurate AFM in 1D

Imaging Polarons - Charge Impurities in an AFM

Superexchange Interactions

Origin of Spin-Spin Interactions – Exchange Interactions –

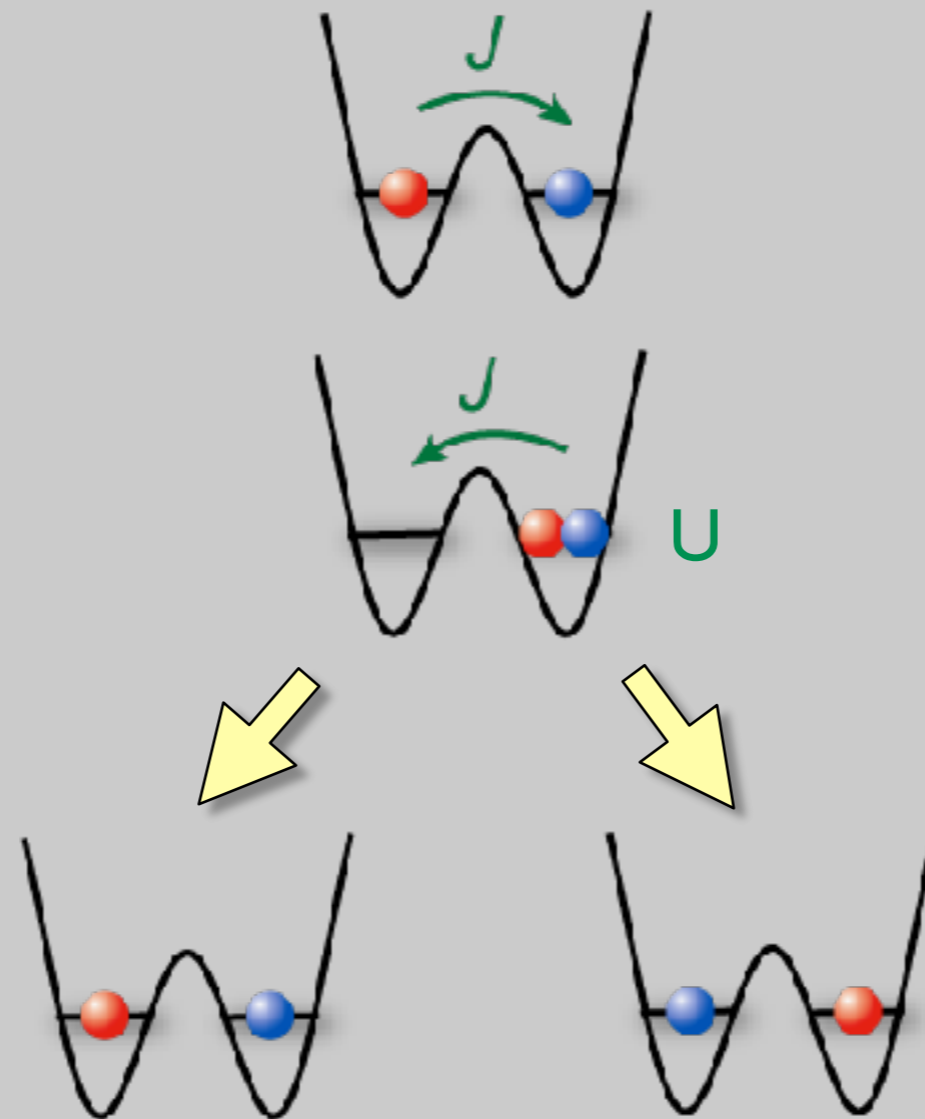


Important ionic solids with *no direct exchange* between magnetic ions show magnetic ordering (*MnO*, *CuO*)!

„*Super*“-exchange interactions must be at work!

Deriving the Effective Spin Hamiltonian (I)

How do we get from $-J \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \frac{1}{2} U \sum_i \hat{n}_i (\hat{n}_i - 1)$ to $H = -J_{ex} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$?



$$\hat{1} + \hat{X}_{LR}$$

Deriving the Effective Spin Hamiltonian (2)

Second order hopping can be written as

$$H = -2 \frac{J^2}{U} (1 + \hat{X}_{LR})$$

$$\hat{X}_{LR} \left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right] = - \left[\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} \right]$$

$$\hat{X}_{LR} \left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \right] = + \left[\frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}} \right]$$

$$H = -J_{ex} \hat{P}_{\text{triplet}}$$



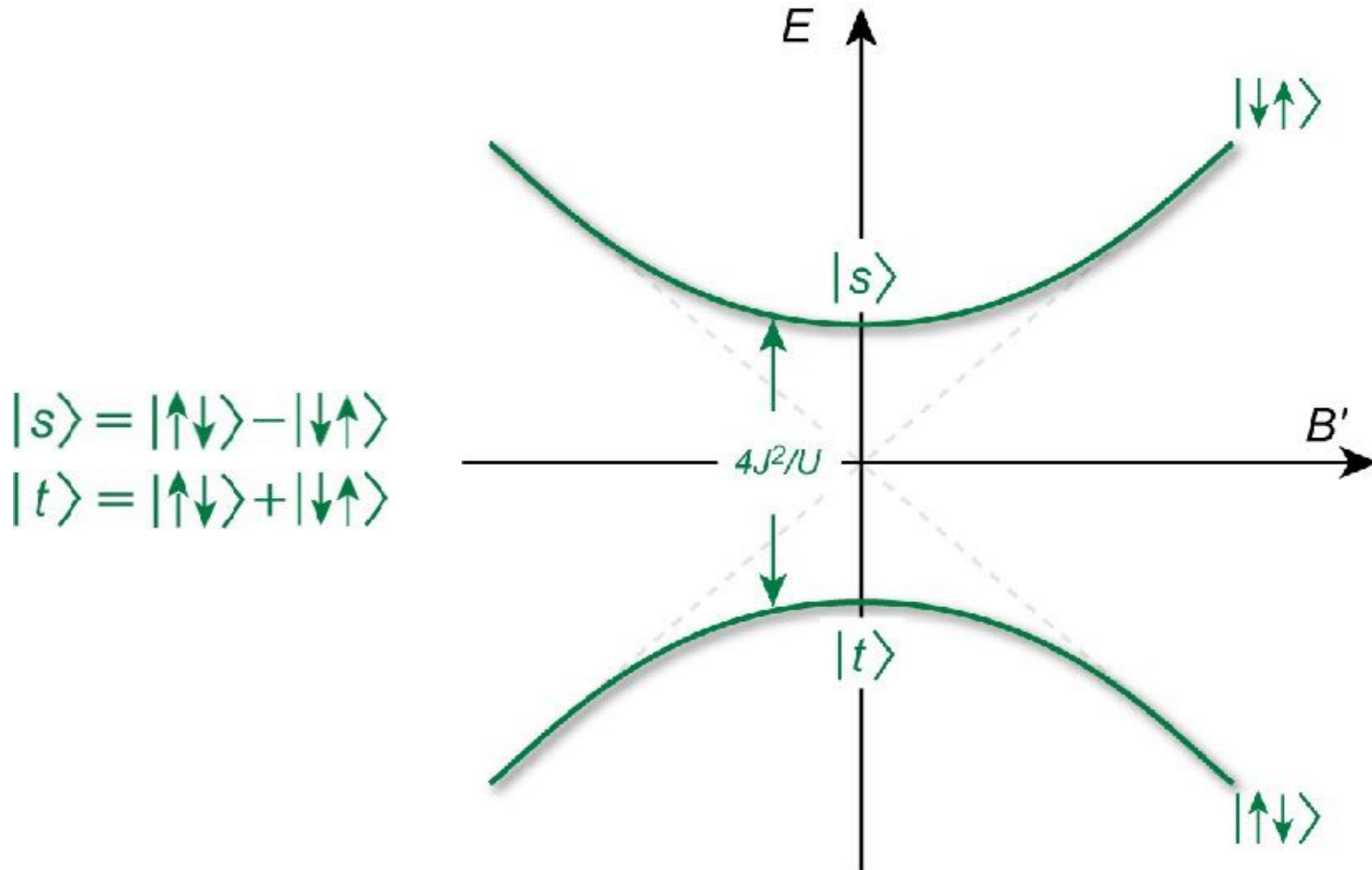
Deriving the Effective Spin Hamiltonian (3)

$$\hat{P}_{\text{triplet}} = \hat{P}_{S=1}$$

$$\begin{aligned}\mathbf{S}_L \cdot \mathbf{S}_R &= \frac{(\mathbf{S}_L + \mathbf{S}_R)^2}{2} - \frac{3}{4} \\ &= \frac{S(S+1)}{2} - \frac{3}{4} \\ &= \hat{P}_{S=1} - \frac{3}{4}\end{aligned}$$

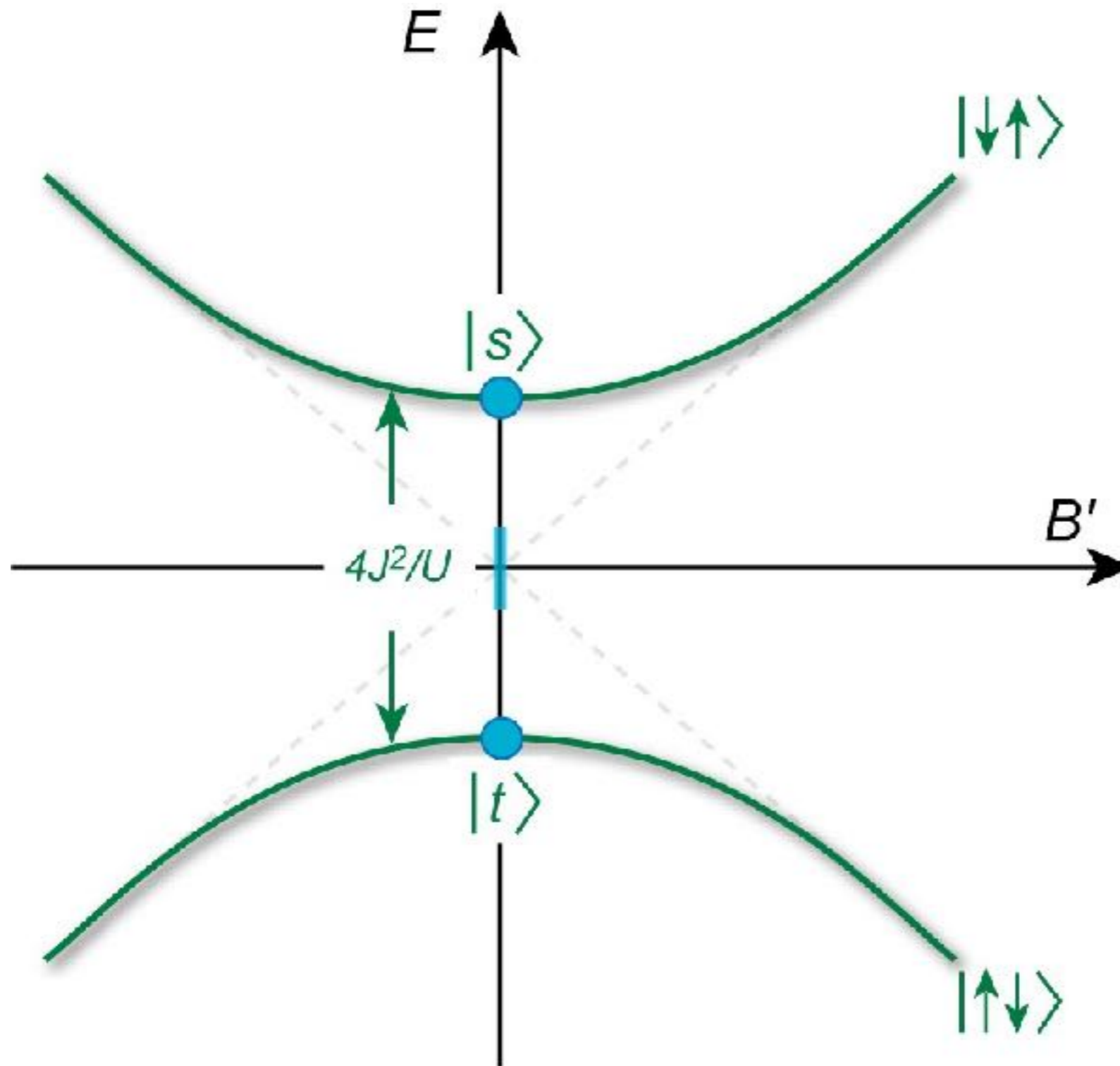
$$H = -J_{ex} \left(\mathbf{S}_L \cdot \mathbf{S}_R + \frac{3}{4} \right)$$

Direct Detection of Superexchange Interactions

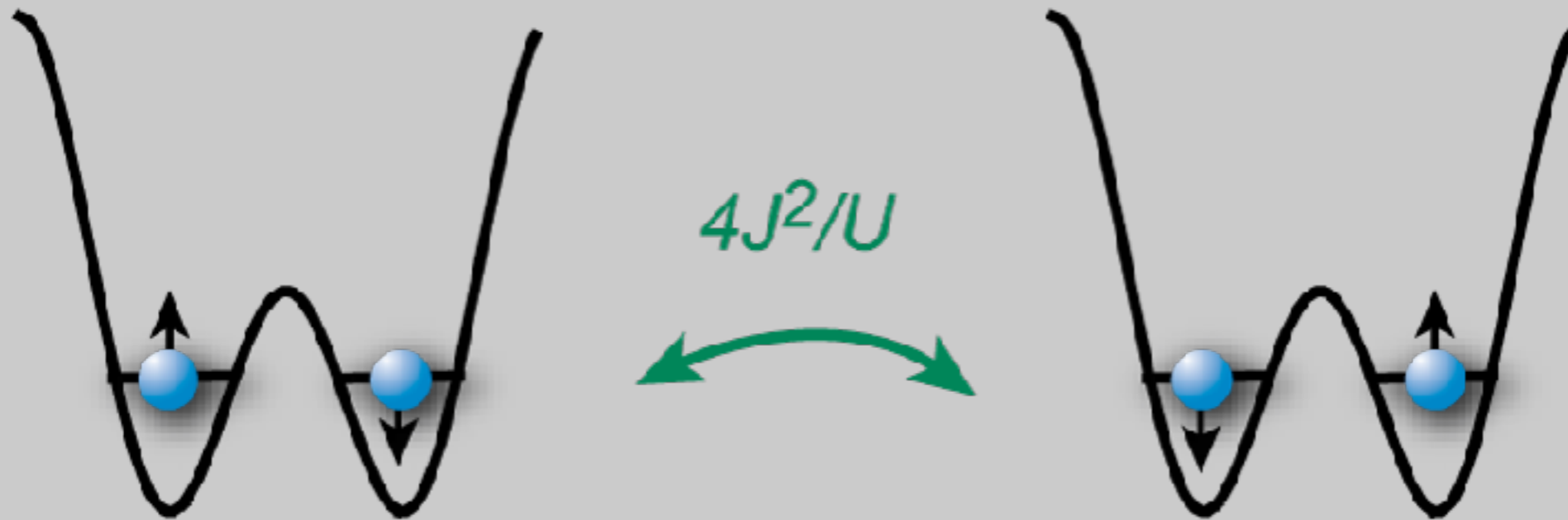


$$H_{eff} = -J_{ex} \vec{S}_L \cdot \vec{S}_R - \mu_B B' (S_{z,L} - S_{z,R})$$

Direct Detection of Superexchange Interactions (2)

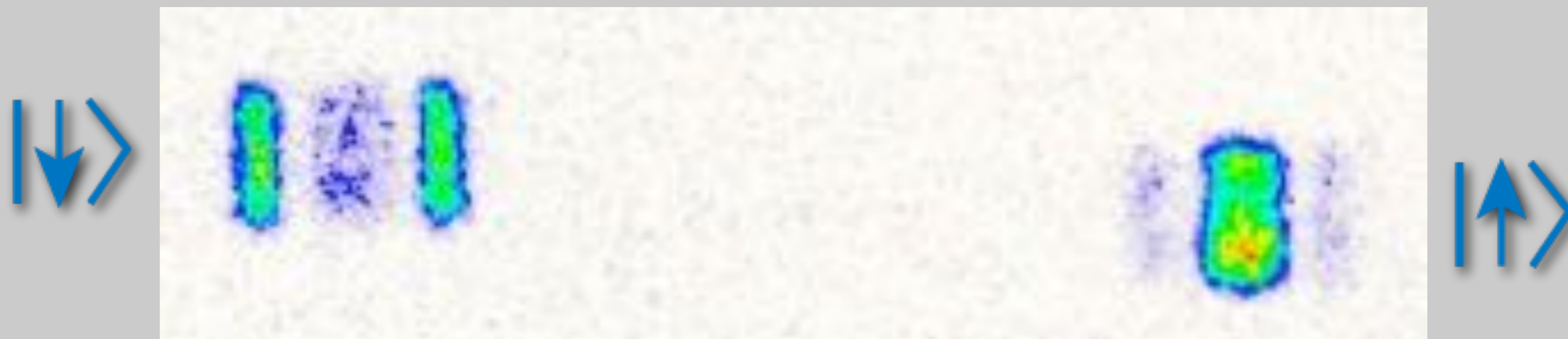
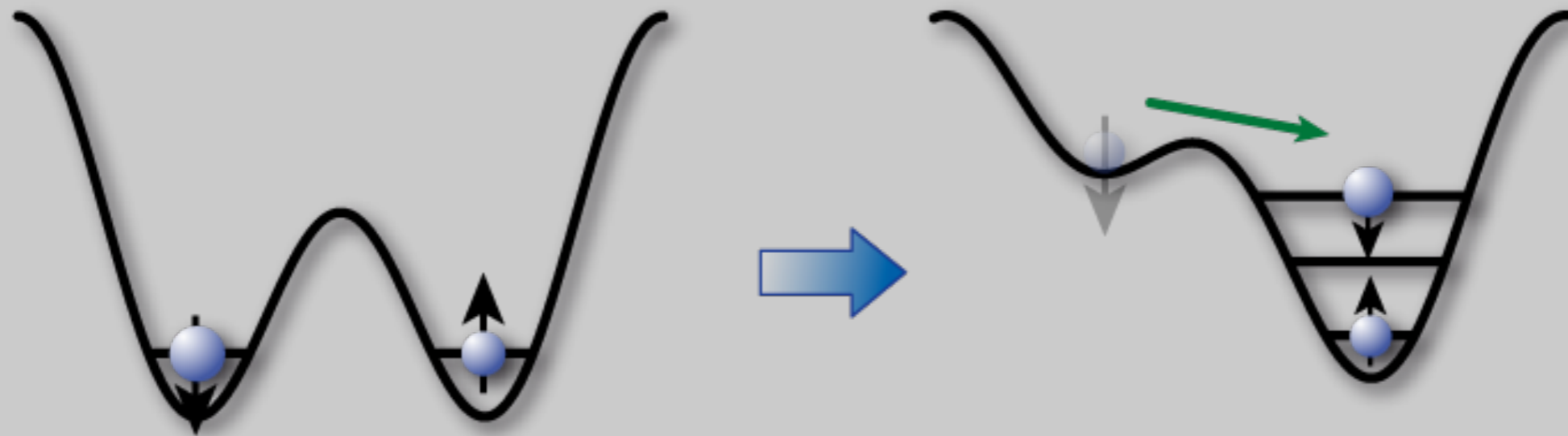


Superexchange induced flopping



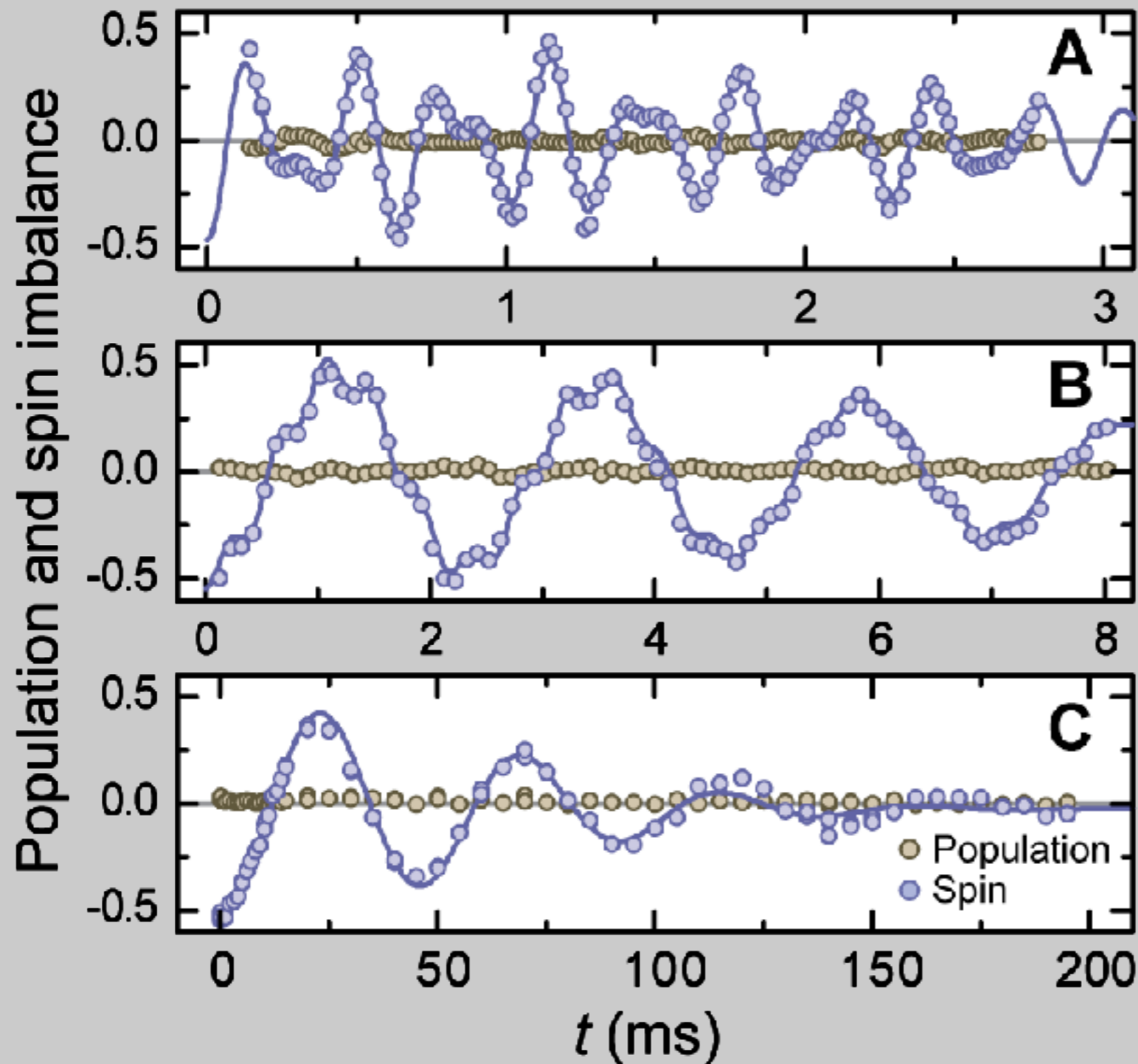
$$H_{eff} = -J_{ex} \vec{S}_i \cdot \vec{S}_j$$
$$= -\frac{J_{ex}}{2} \left(\hat{S}_i^+ \hat{S}_j^- + \hat{S}_i^- \hat{S}_j^+ \right) - J_{ex} \hat{S}_i^z \hat{S}_j^z$$

Mapping the Spins



Initial AF order verified in the experiment!

Superexchange induced flopping



$$J/U = 1.25$$
$$V_{\text{short}} = 6 E_r$$

$$J/U = 0.26$$
$$V_{\text{short}} = 11 E_r$$

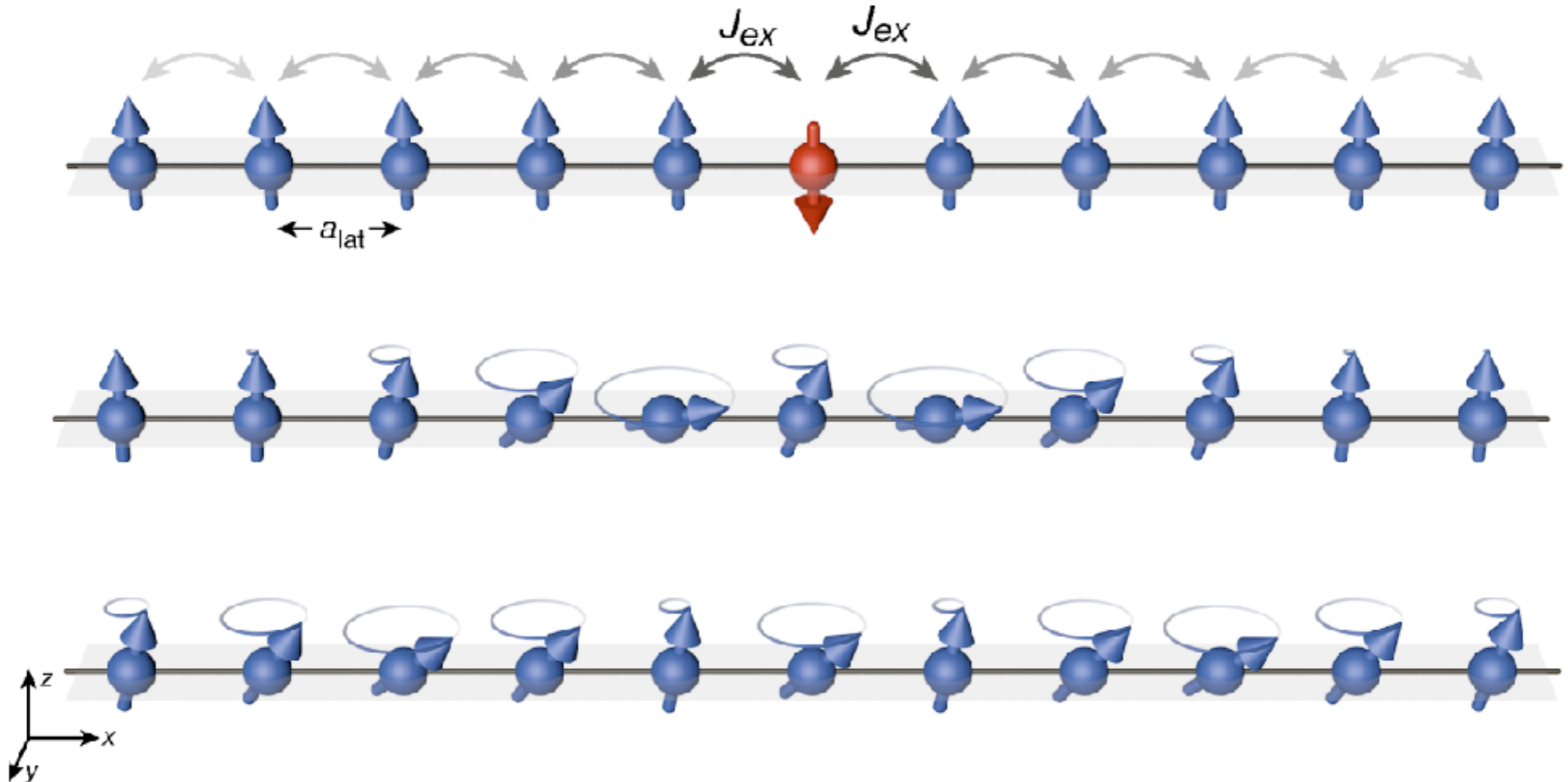
$$J/U = 0.05$$
$$V_{\text{short}} = 17 E_r$$

Quantum Dynamic of Mobile Single Spin Impurity

T. Fukuhara, M. Endres, M. Cheneau P. Schauss, Ch. Gross, I. Bloch, S. Kuhr,
U. Schollwöck, A. Kantian, Th. Giamarchi

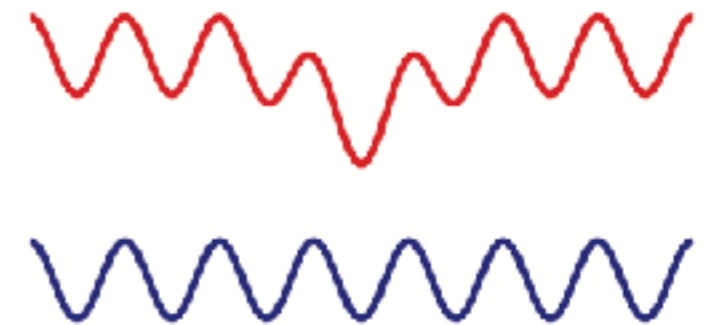
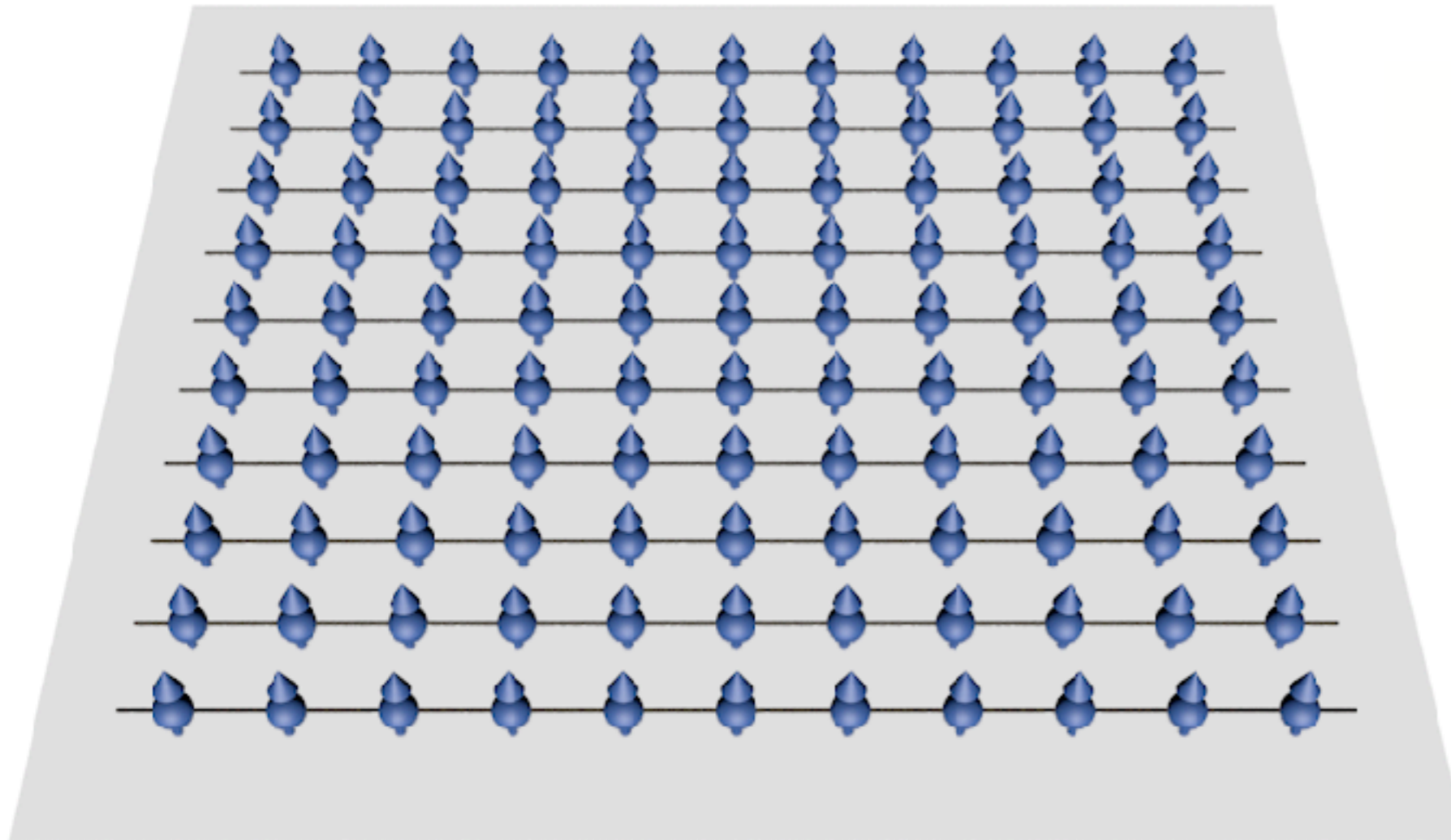
Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

www.quantum-munich.de



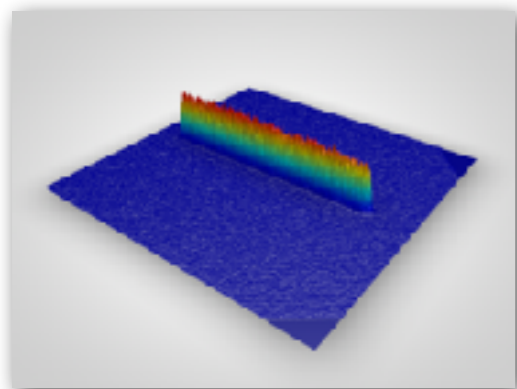
$$-J_{ex} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ferromagnetic Heisenberg Interaction

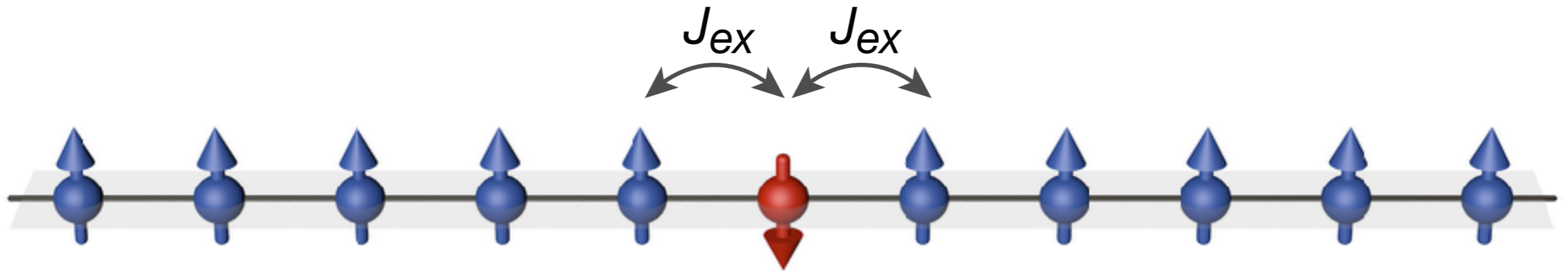


$$|2\rangle = |F=2, m_F=-2\rangle$$

$$|1\rangle = |F=1, m_F=-1\rangle$$



Line-shaped light field created with DMD SLM

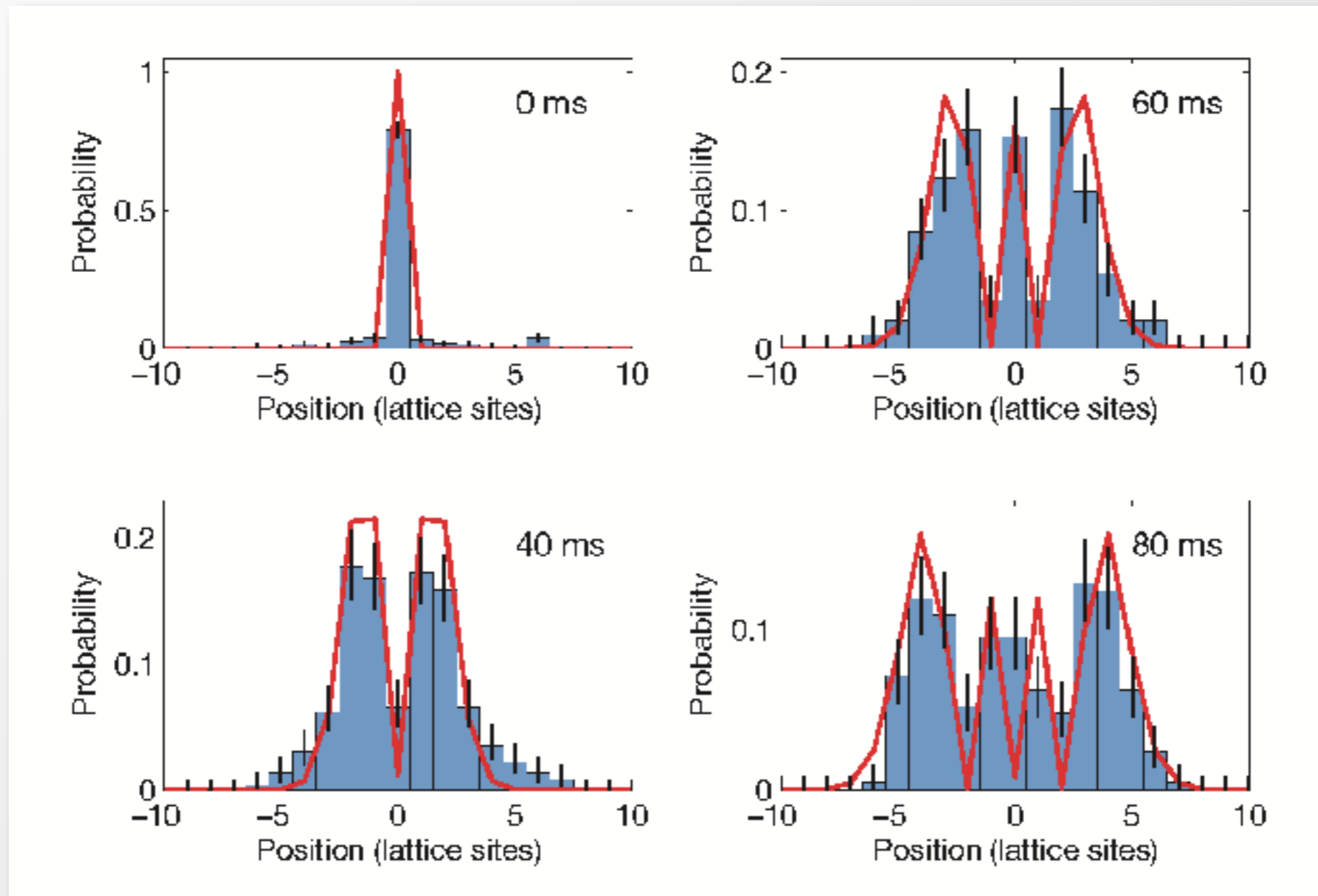


Heisenberg Hamiltonian

$$\begin{aligned}
 H &= -J_{ex} \sum \mathbf{S}_i \cdot \mathbf{S}_j = -J_{ex} \sum \left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right) \\
 &= -\frac{J_{ex}}{2} \sum \left(S_i^+ S_j^- + S_i^- S_j^+ \right) - \cancel{J_{ex} \sum S_i^z S_j^z}
 \end{aligned}$$

$$J_{ex} = 4 \frac{J^2}{U}$$

$$H = -J \sum \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger \right) \text{ single particle tunneling}$$



$$V = 10 E_r$$

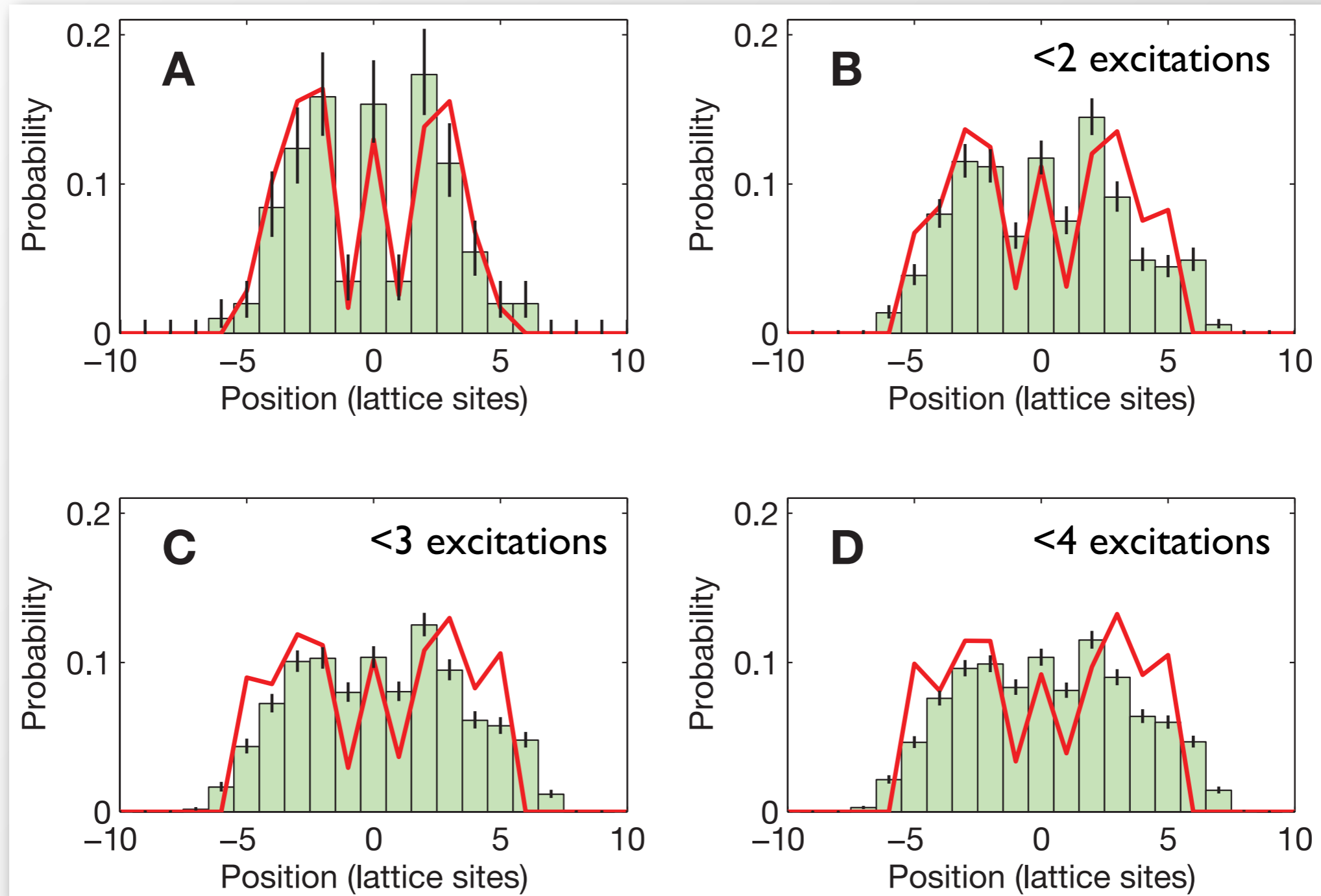
$$U/J = 19$$

$$H = -\frac{J_{ex}}{2} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$$

$$P_j(t) = \left[\mathcal{J}_j \left(\frac{J_{ext} t}{\hbar} \right) \right]^2$$

Bessel function of the first kind





$$V = 10 E_r$$

$$U/J = 19$$

- Only visibility goes down
- Spreading speed almost independent of holes



Quantum Dynamics of Interacting Atoms/Spins

- Effect of Temperature/Holes on Dynamics
- Dynamics of **Magnon bound states**
- Domain Walls
- Higher Dimensions (1D, 2D, 3D)
- Entropy Transport
- Probe for **Quantum Critical Transport**
- Direct measurement of **Green's function**

$$G(x_i, x_j, t) \propto \langle \uparrow | \hat{S}^{\dagger}(x_j, t) \hat{S}^{-}(x_i, 0) | \uparrow \rangle$$

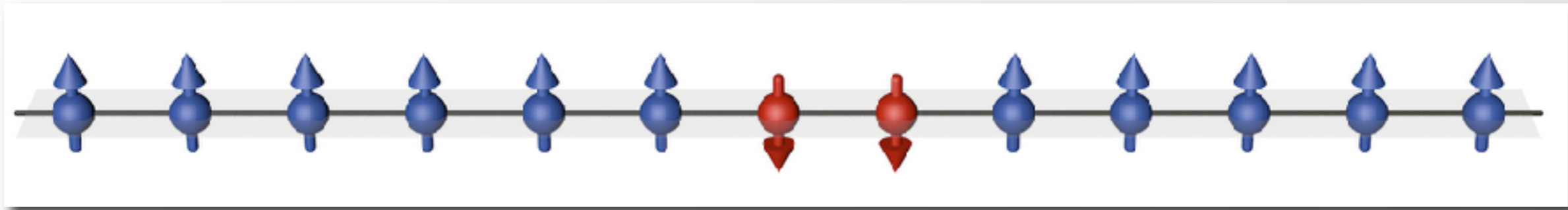


Direct Observation of Magnon Bound States

T. Fukuhara, P. Schauss, S. Hild, J. Zeiher, M. Cheneau, M. Endres, I. Bloch, Ch. Gross

T. Fukuhara et al., Nature **502**, 76 (2013)

for photons: O. Firstenberg et al., Nature **502**, 71 (2013)

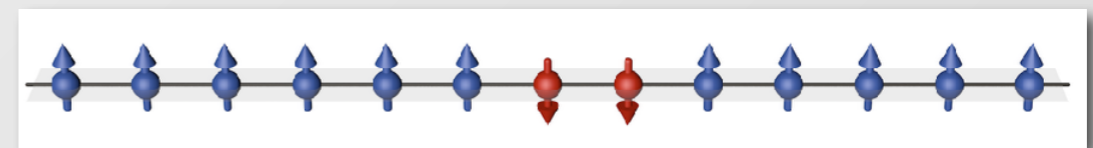


There can be bound states in a Heisenberg spin chain!
Development of **Bethe Ansatz**.

$$H = -J_{ex} \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

Hans Bethe
(1906-2005)

General
I-string bound states



H. Bethe, Z. Phys. (1931)

M. Wortis, Phys Rev. (1963)

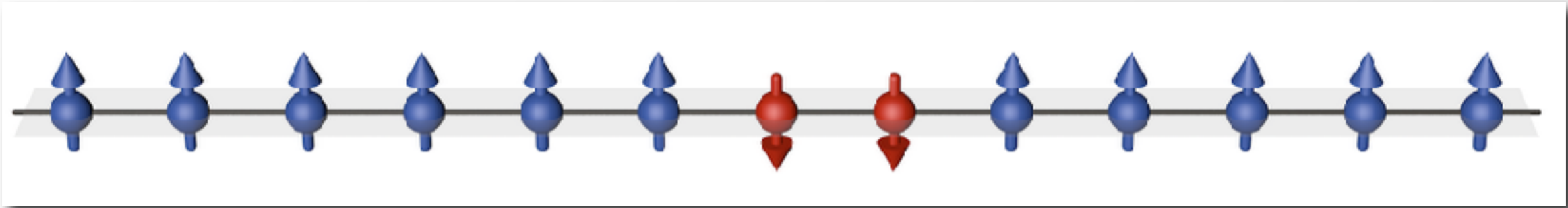
M. Takahashi & M. Suzuki Prog. Th. Phys. (1972)

M. Karbach, G. Müller (1997)

see also: **repulsively bound pairs & interacting atoms**

K. Winkler et al. Nature (2006); S. Fölling et al. Nature (2007); Y Lahini et al. PRA (2012)





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$$H = -J_{ex} \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

$$H = -\frac{J_{ex}}{2} \sum_i (\hat{S}_i^+ \hat{S}_{i+1}^- + \hat{S}_i^- \hat{S}_{i+1}^+) - \Delta \sum_i \hat{S}_i^z \hat{S}_{i+1}^z$$

H. Bethe, Z. Phys. (1931)

M. Wortis, Phys Rev. (1963)

M. Takahashi & M. Suzuki Prog. Th. Phys. (1972)

M. Karbach, G. Müller (1997)

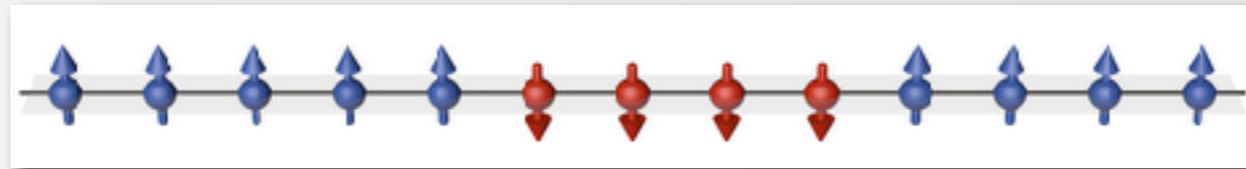
see also: **repulsively bound pairs & interacting atoms**

K. Winkler et al. Nature (2006); S. Fölling et al. Nature (2007); Y Lahini et al. PRA (2012)



Eigenenergies:

$$E(k) = -J_{ex} \frac{\sin(\nu)}{\sin(l\nu)} \left\{ \cos(l\nu) - (-1)^l \cos k \right\}$$



$$\Delta = \cos(\nu)$$

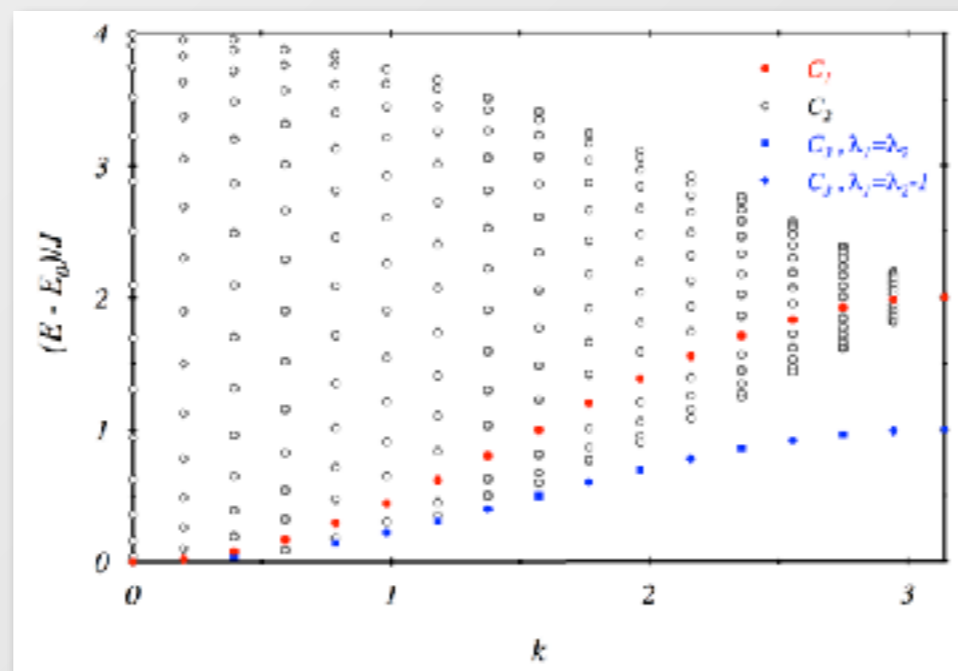
l-string

Maximum propagation velocity:

$$v_{max,l} = \frac{\sin(\nu)}{\sin(l\nu)}$$

$$v_{max,2} = \frac{J}{2\Delta}$$

Excitation spectrum:

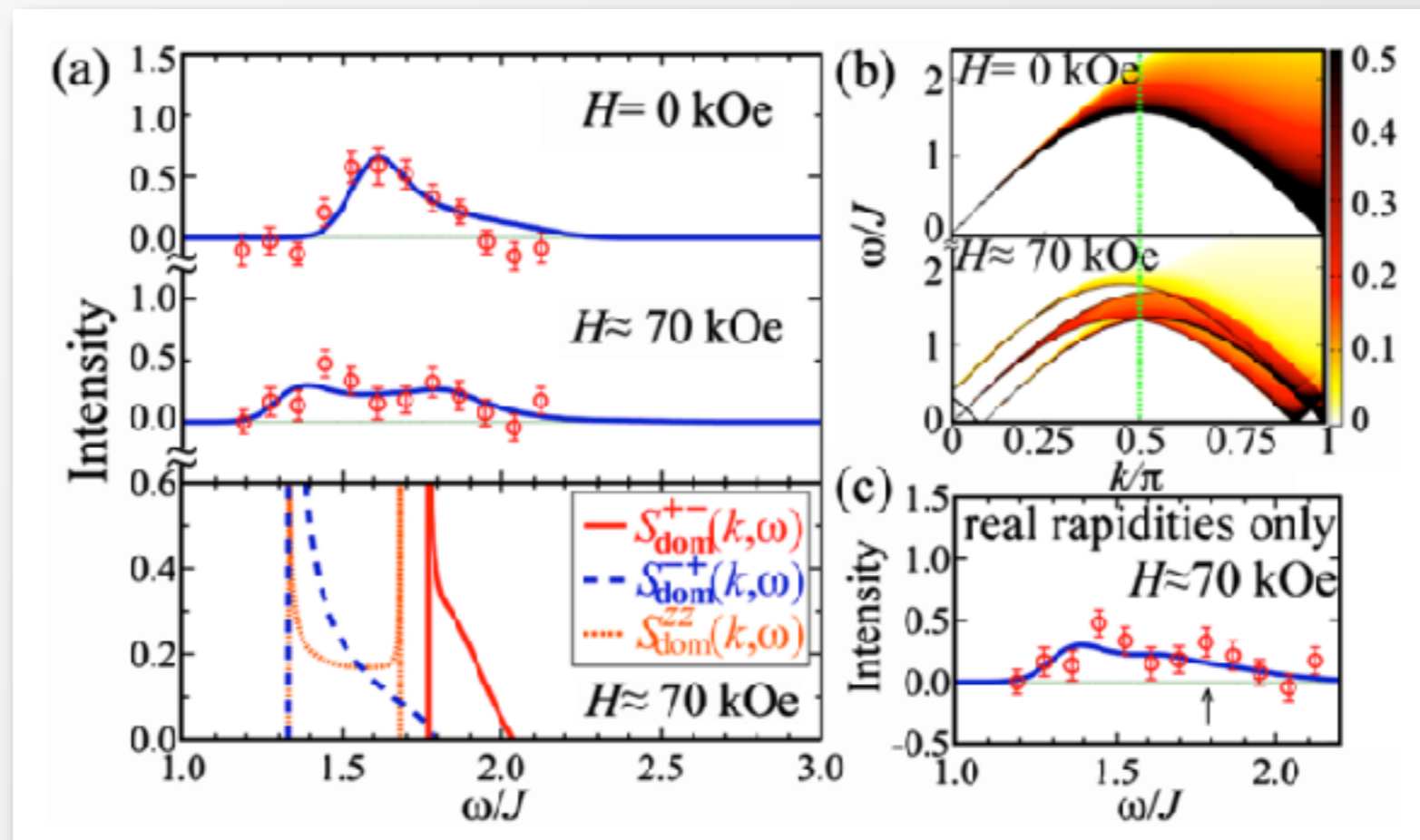


M. Karbach & G. Müller (1997)

Bound magnon



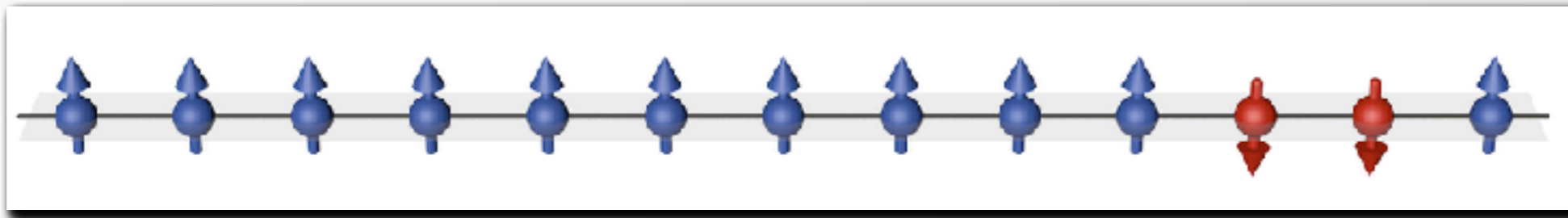
Very difficult to observe in spectroscopic data in real materials!



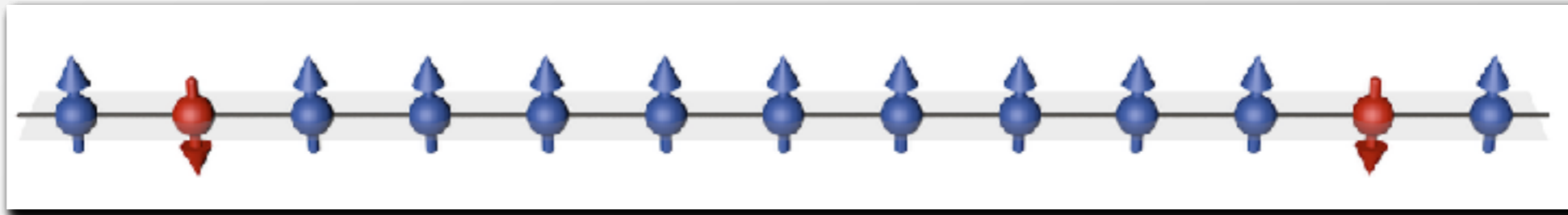
theory **with**
bound states

theory **without**
bound states

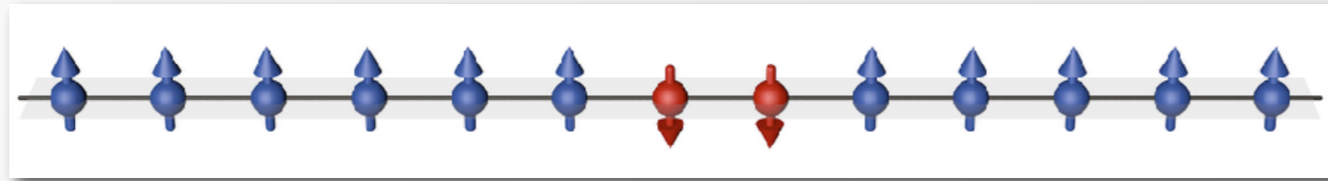
Bound Magnon Motion



Breakup and Single Spin Motion



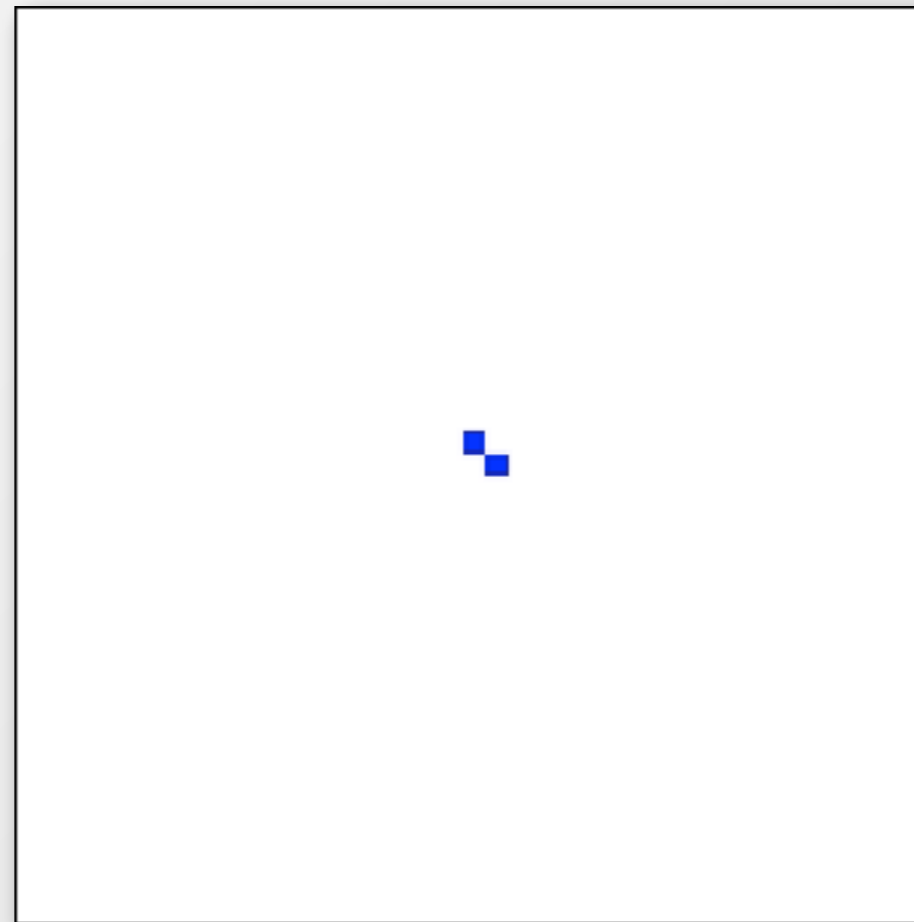
Initial State:



Pair distribution evolution

$$P(x_1, x_2)$$

x_1

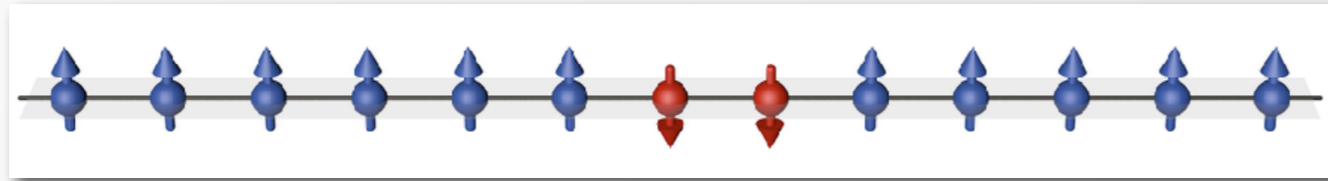


x_2

$\Delta = 0$
Non-Interacting

see also: two interacting atoms
Y. Lahini et al., PRA 86, 011603 (2012)

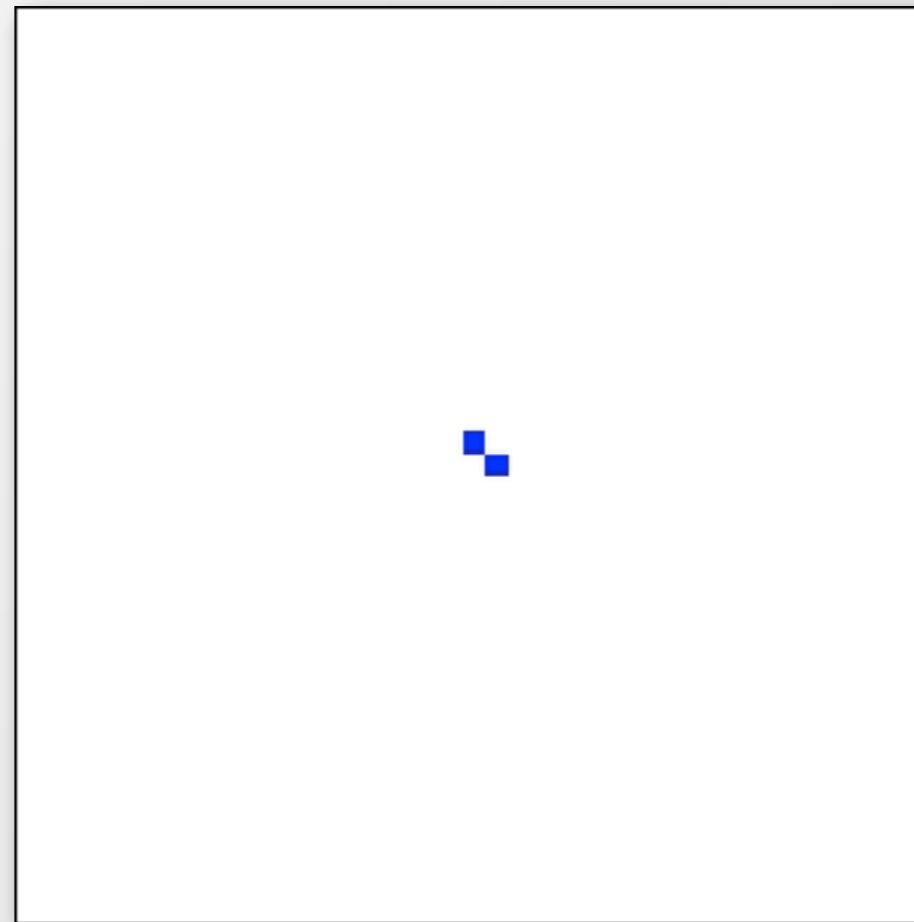
Initial State:



Pair distribution evolution

$$P(x_1, x_2)$$

x_1



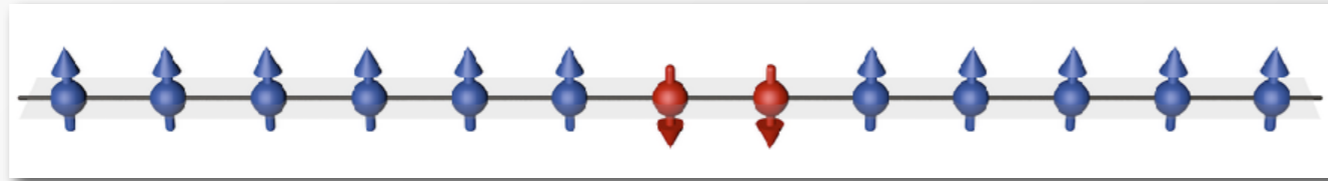
x_2

$$\Delta = 1$$

Interacting
Isotropic Heisenberg

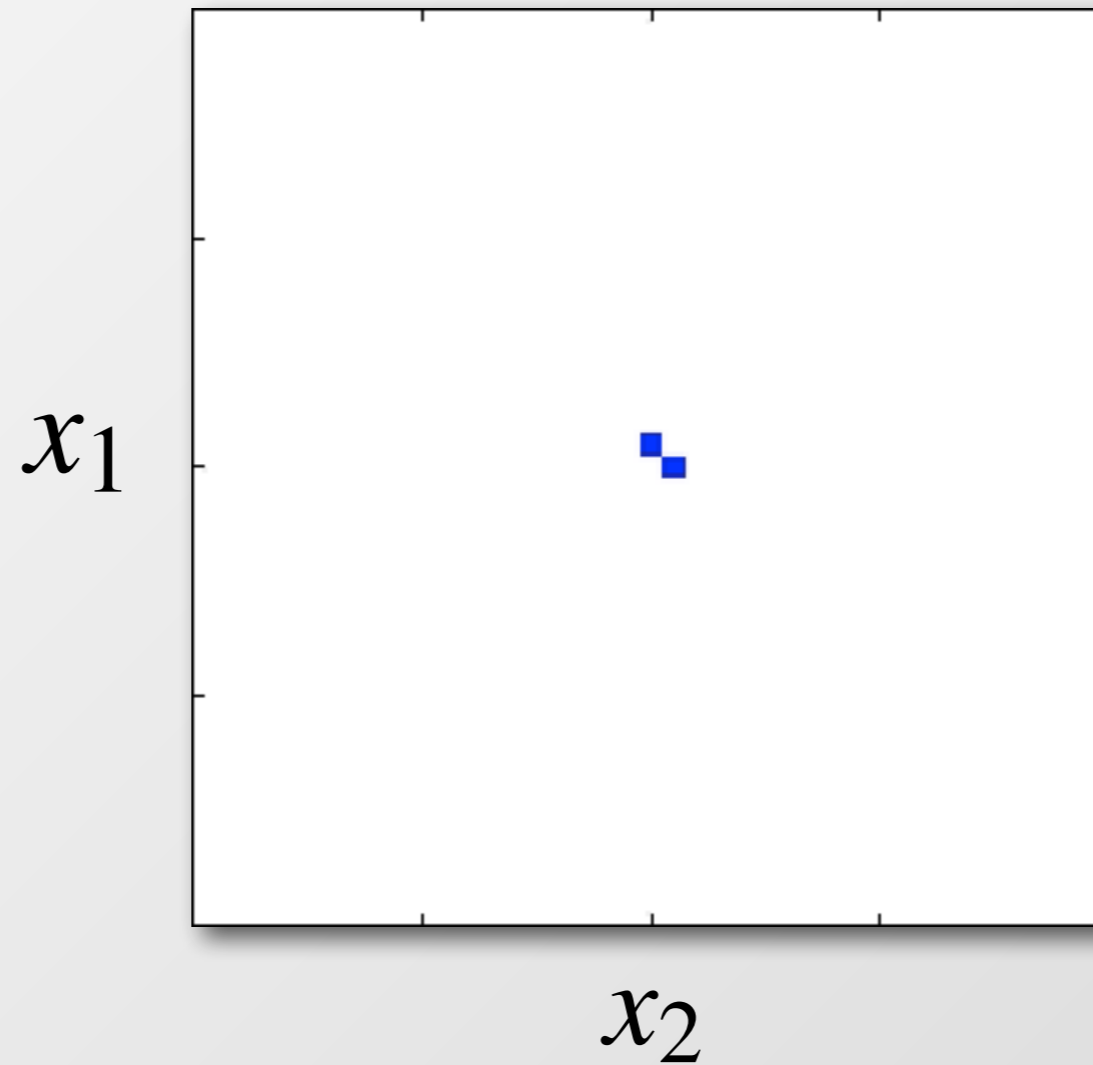


Initial State:



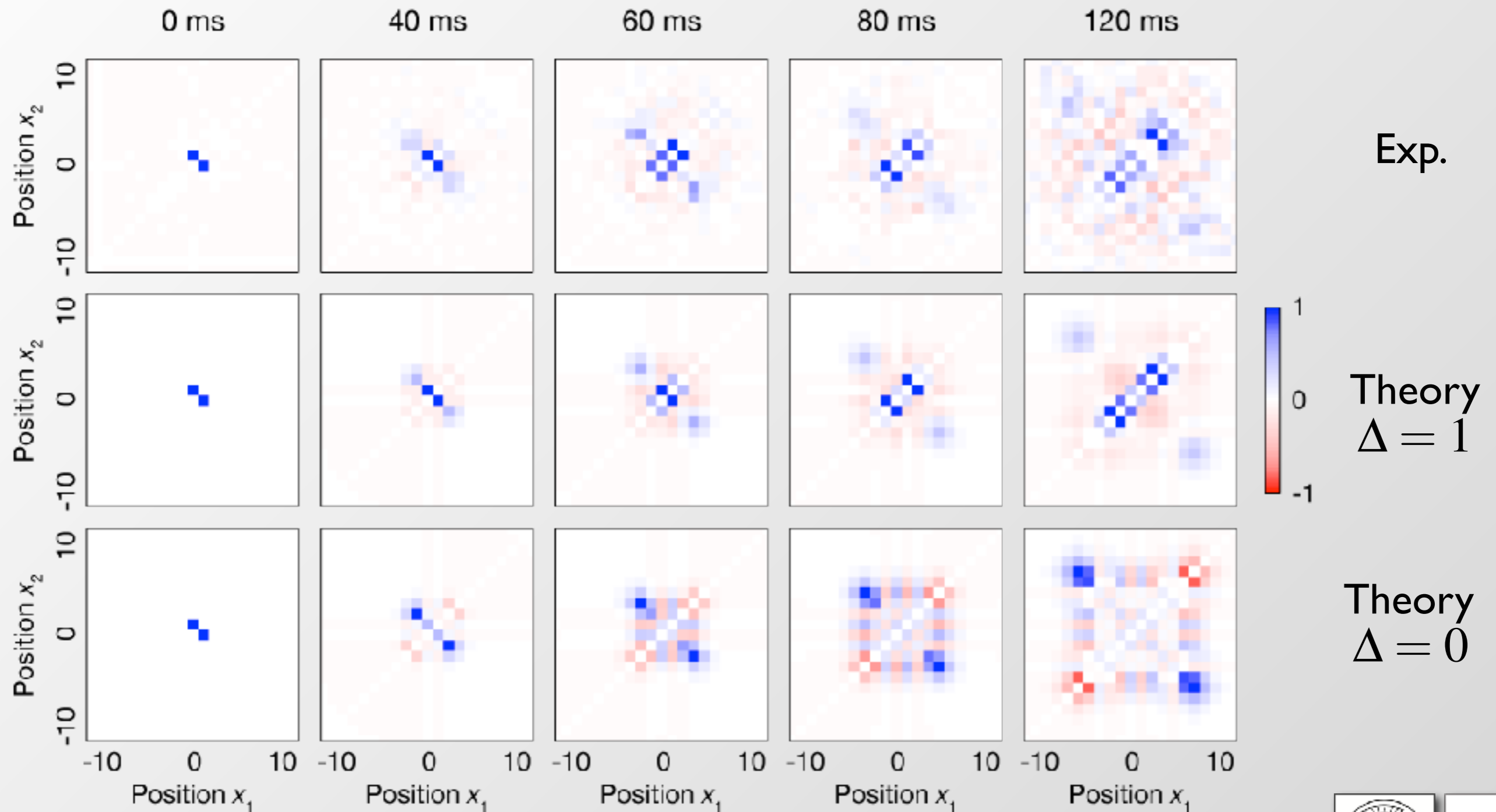
Pair distribution evolution

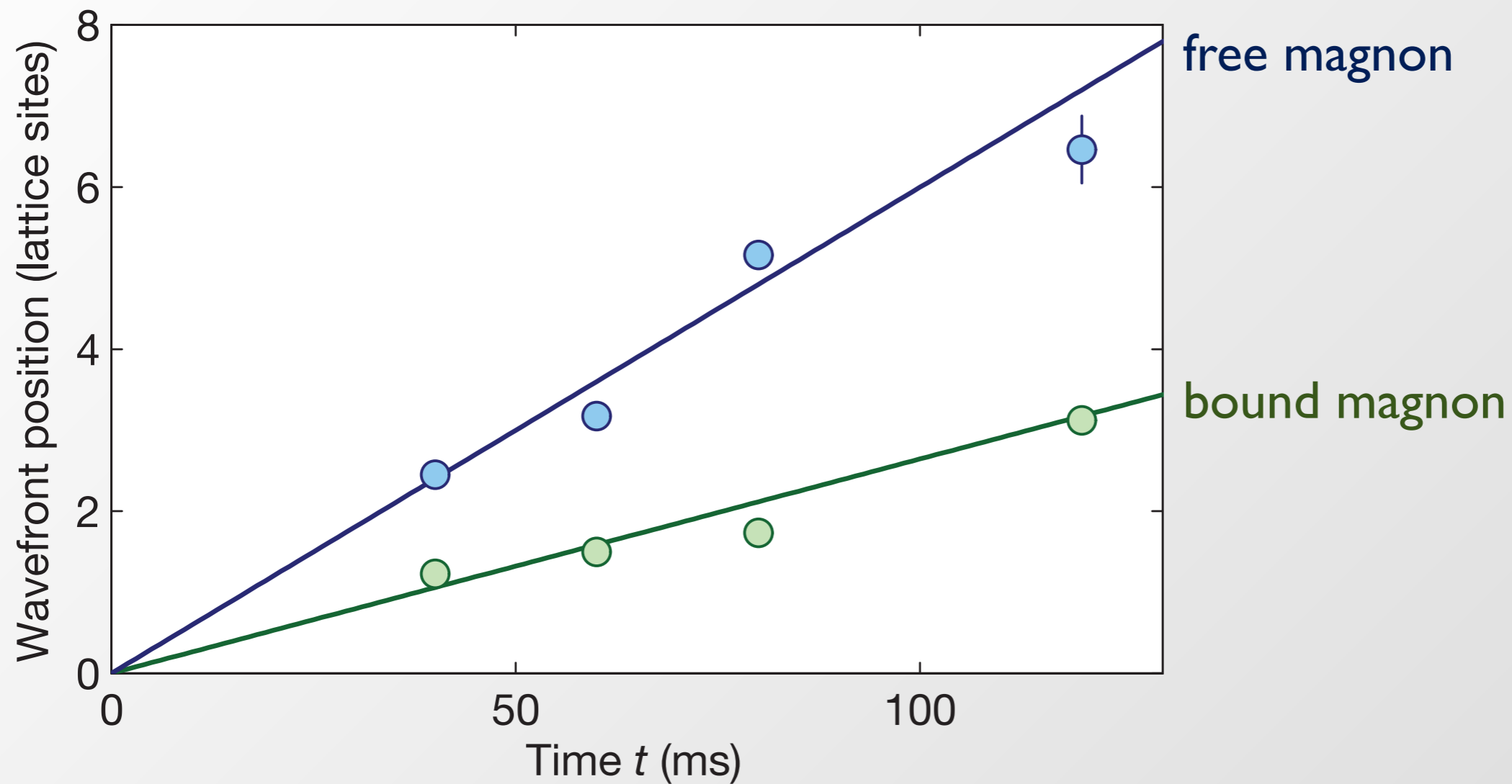
$$P(x_1, x_2)$$



$\Delta = 1.6$
Interacting
Heisenberg

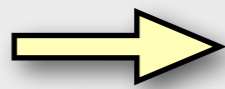
$$C(x_1, x_2) = P(x_1, x_2) - P(x_1)P(x_2)$$





$$v_f = \frac{J_{ex}a}{\hbar}$$

$$v_b = \frac{J_{ex}a}{2\hbar\Delta}$$



$$\frac{v_f}{v_b} = 2\Delta$$

We find: $\frac{v_f}{v_b} = 2.3(3)$



Quantum Dynamics of Interacting Atoms/Spins

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$$G(x_i, x_j, t) \propto \langle \uparrow \uparrow | \hat{S}^{\dagger}(x_j, t) \hat{S}^{-}(x_i, 0) | \uparrow \uparrow \rangle$$

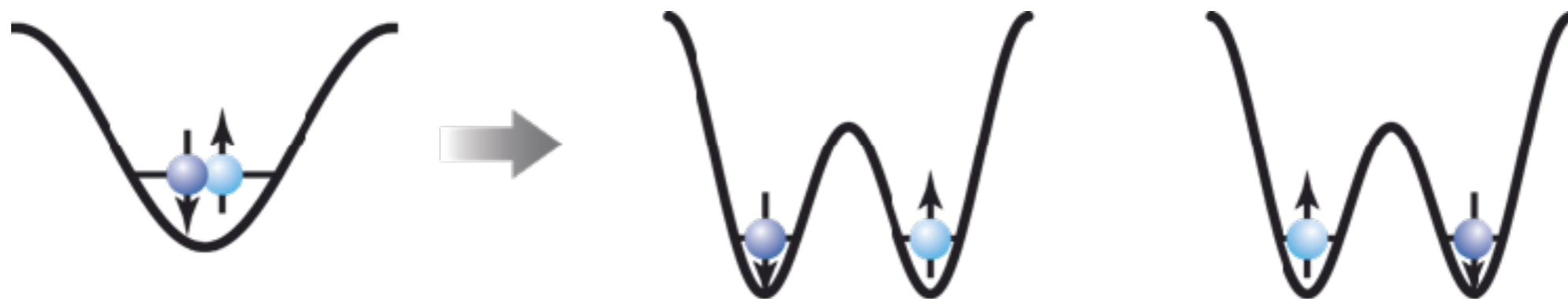
M. Knap et al. PRL **111**, 147205 (2013)



Splitting a spin pair

- **Spin pairs** in $|F = 1, m_F = \pm 1\rangle \equiv |\uparrow\rangle, |\downarrow\rangle$ (repulsive)
- Barrier raised *slowly* to split
→ Crossing a miniature Mott-transition: $n_{\text{Left}} = n_{\text{Right}} = 1$

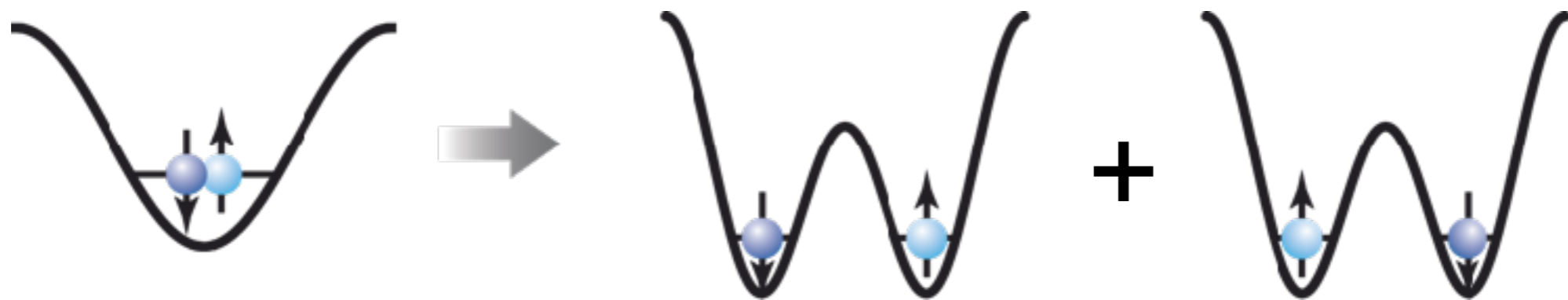
J. Sebby-Strabley et al., PRL 98 (2007)



*Details on the loading of the Spin-pairs:
S.T., P. Cheinet et al., Science 319 (2008)*

Splitting a spin pair

- **Spin pairs** in $|F = 1, m_F = \pm 1\rangle \equiv |\uparrow\rangle, |\downarrow\rangle$
- Barrier raised *slowly* to split *J. Sebby-Strabley et al., PRL 98 (2007)*
→ Crossing a miniature Mott-transition: $n_{\text{Left}} = n_{\text{Right}} = 1$

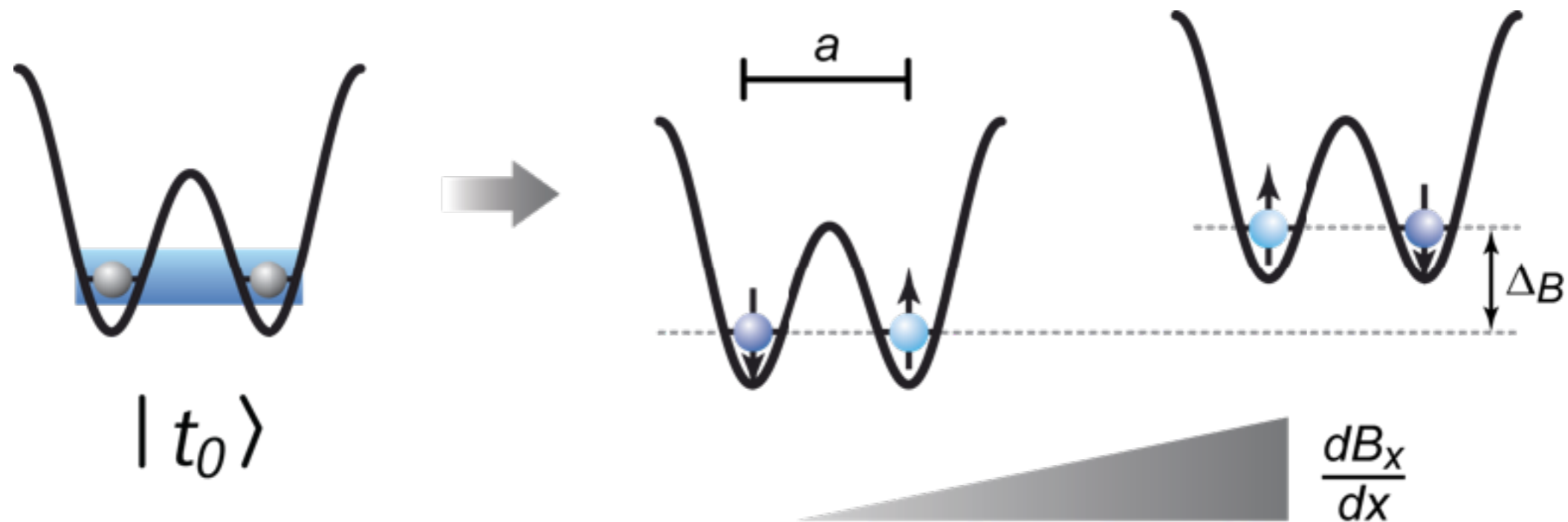


- **Bosons:** Symmetric wavefunction → Triplet $|t_0\rangle$
(Fermions: Antisymmetric wavefunction → Singlet $|s\rangle$)

Driving Triplet-Singlet oscillations

- Magnetic field gradient lifts degeneracy:

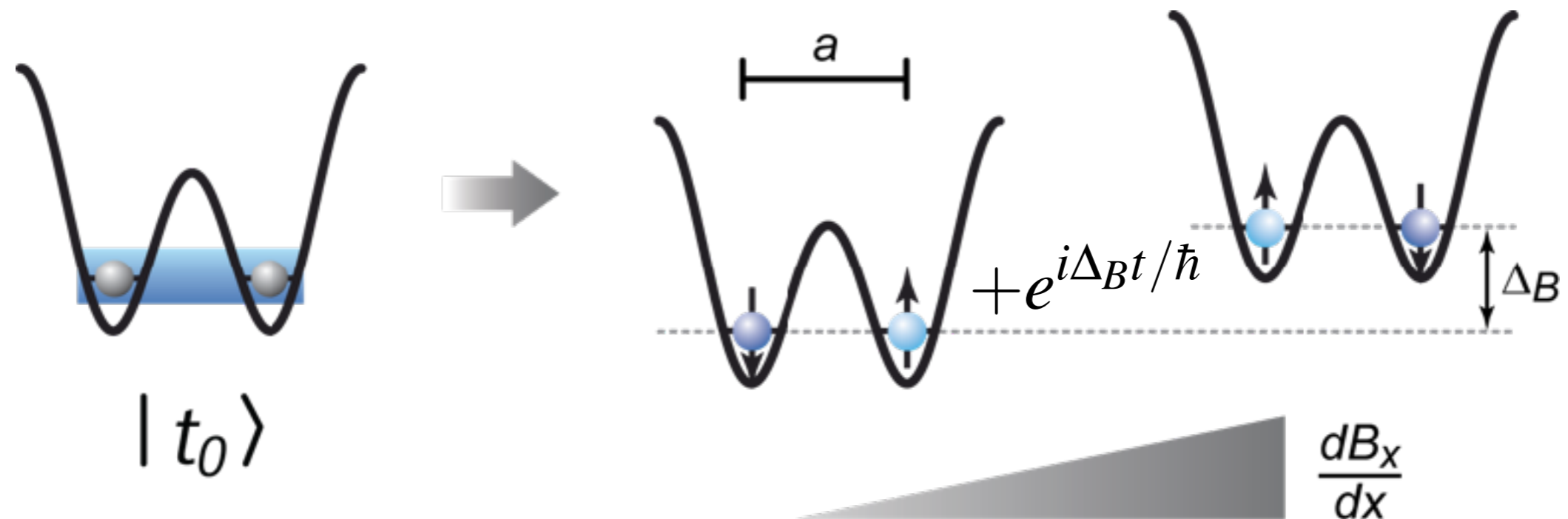
$$\Delta_B \propto a \cdot \partial_x B_x$$



Driving Triplet-Singlet oscillations

- **Magnetic field gradient** lifts degeneracy:

$$\Delta_B \propto a \cdot \partial_x B_x$$



- **Triplet-Singlet oscillations** with frequency Δ_B / \hbar

$$|t_0\rangle \leftrightarrow |S\rangle$$

How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
→ **Triplet**: both atoms reach the **ground state**



How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
 - **Triplet**: both atoms reach the **ground state**

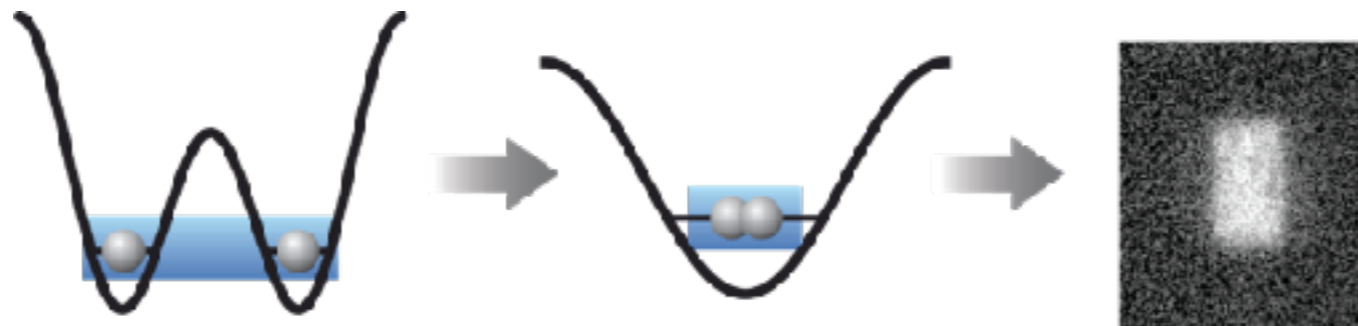


- **Singlet**: needs anti-symm. spatial wavefunction (Bosons)
 - One atom transferred to **higher vibrational band**

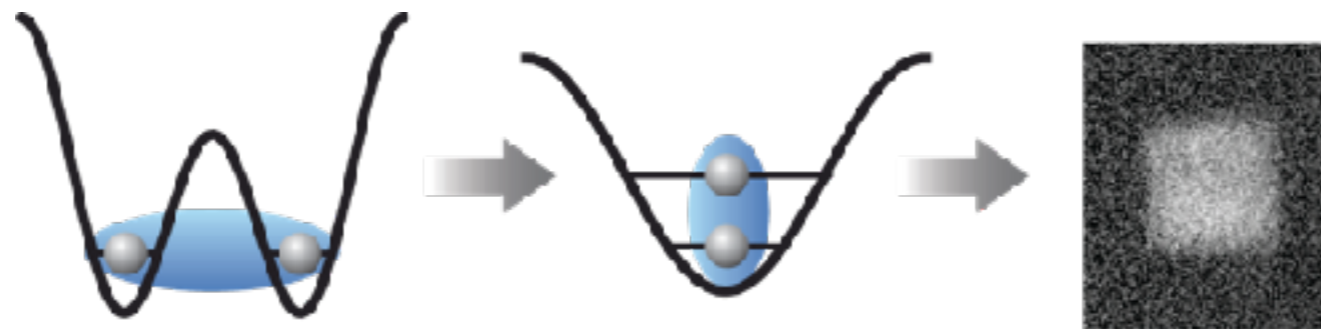


How to detect triplets and singlets

- Barrier lowered slowly to **merge** double-wells
→ **Triplet**: both atoms reach the **ground state**



- **Singlet**: needs anti-symm. spatial wavefunction (Bosons)
One atom transferred to **higher vibrational band**



Band-mapping reveals **singlet-contribution**
in higher Brillouin-Zone

A sensitive probe of next-neighbor spin-correlations in Mott-insulator type many-body systems

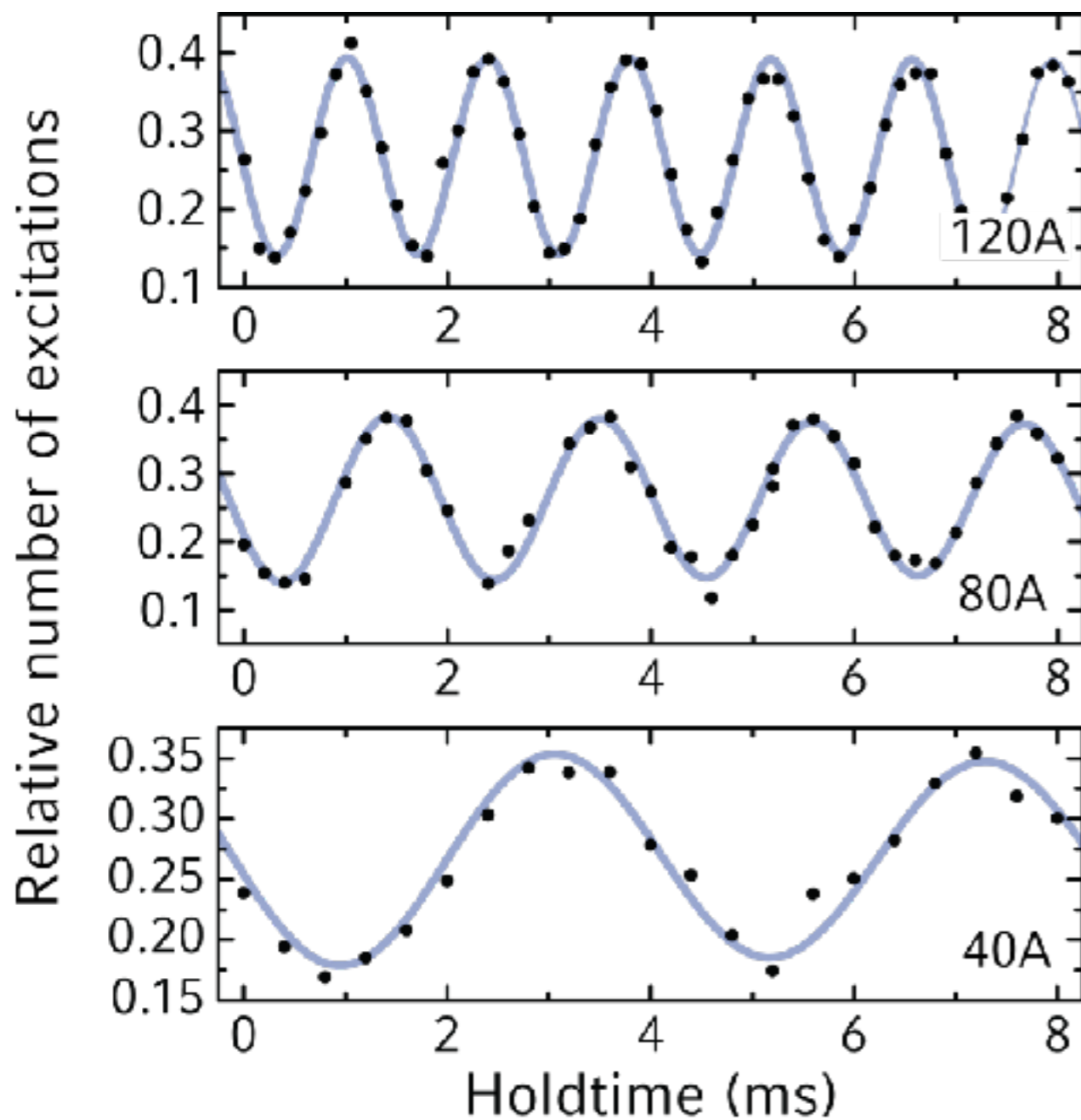
	band excitations		STO amplitude	
	bosons	fermions	bosons	fermions
$ t\rangle$	0%	50%	50%	50%
$ s\rangle$	50%	0%	50%	50%
$ \downarrow, \uparrow\rangle$	25%	25%	0%	0%
$ \uparrow, \downarrow\rangle$	25%	25%	0%	0%
$ \uparrow, \uparrow\rangle$	0%	50%	0%	0%
$ \downarrow, \downarrow\rangle$	0%	50%	0%	0%

→ Capable of probing spin-order in strongly correlated phases at low temperatures

Band-mapping reveals **singlet-contribution**
in higher Brillouin-Zone



Singlet-Triplet oscillations



- **Load** system and create **spin pairs**
- Split pairs into **triplets**
- Induce **STO** via gradient
- **Merging** and **band-mapping** for detection

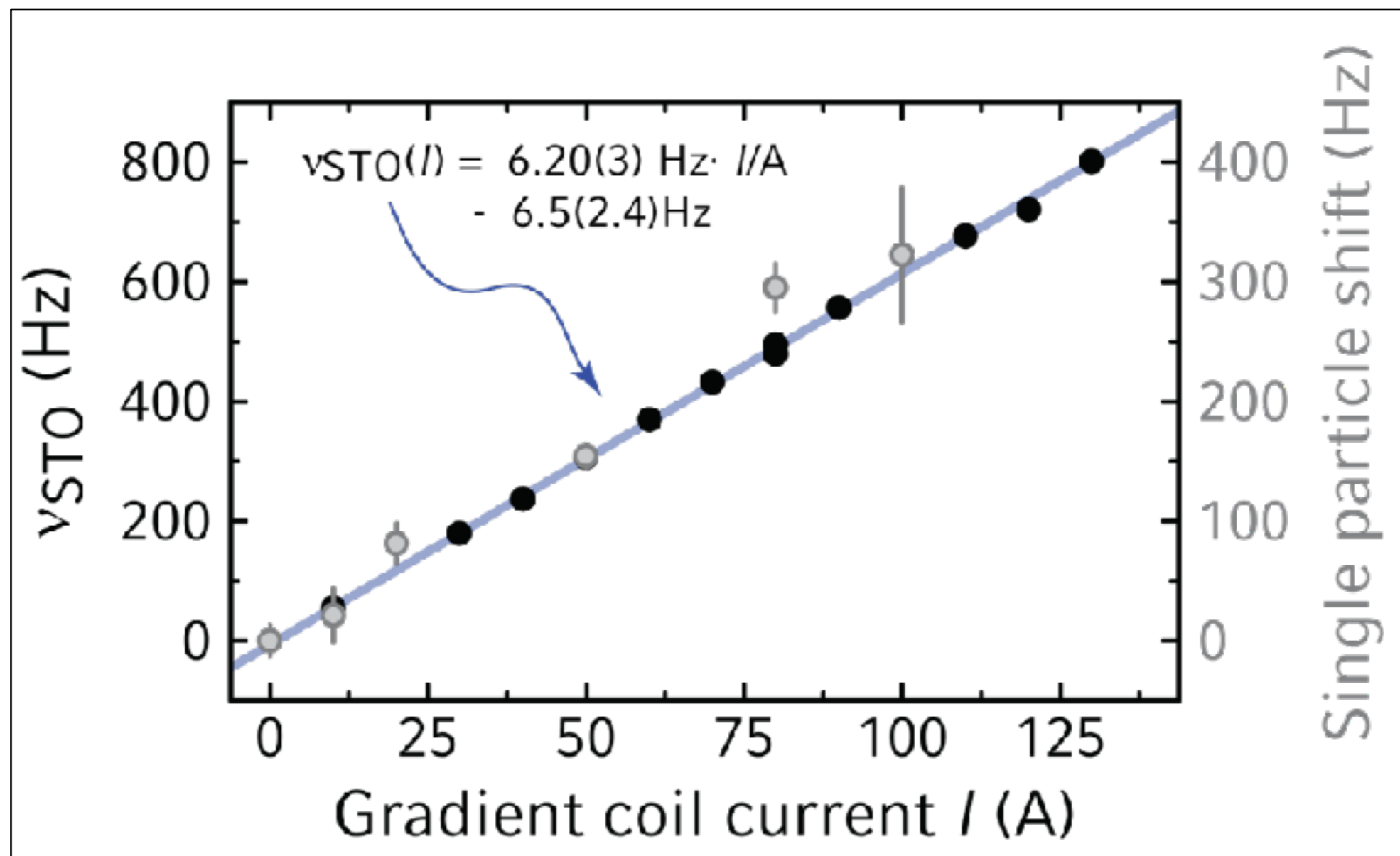
→ **Traces of STO** versus holdtime with gradient

- Vary gradient coil current

S. Trotzky et al., Phys. Rev. Lett. **105**, 265303 (2010) & D. Greif et al., Science **340**, 1307–1310 (2013)

Singlet-Triplet oscillations

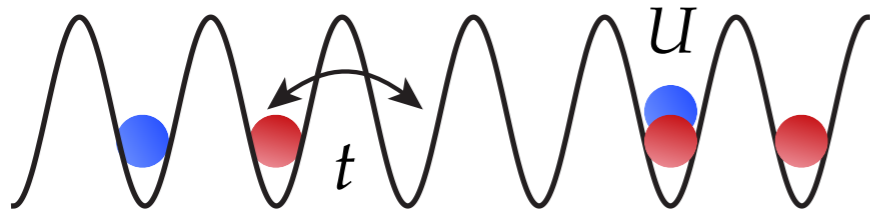
- **Linear increase** in Frequency with gradient strength
- Frequency = **2x single particle shift** (independently meas.)
→ confirms **2-particle nature** of oscillations



Controlling and Detecting Spin Correlation in Doped FHM

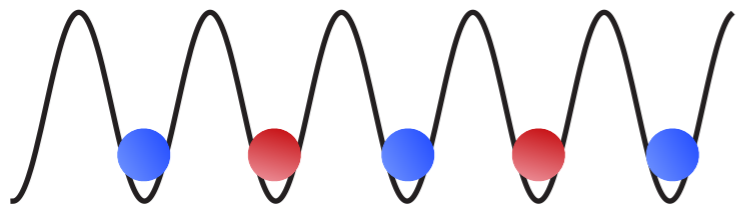
Fermi Hubbard Model (FHM)

Fermi-Hubbard Model



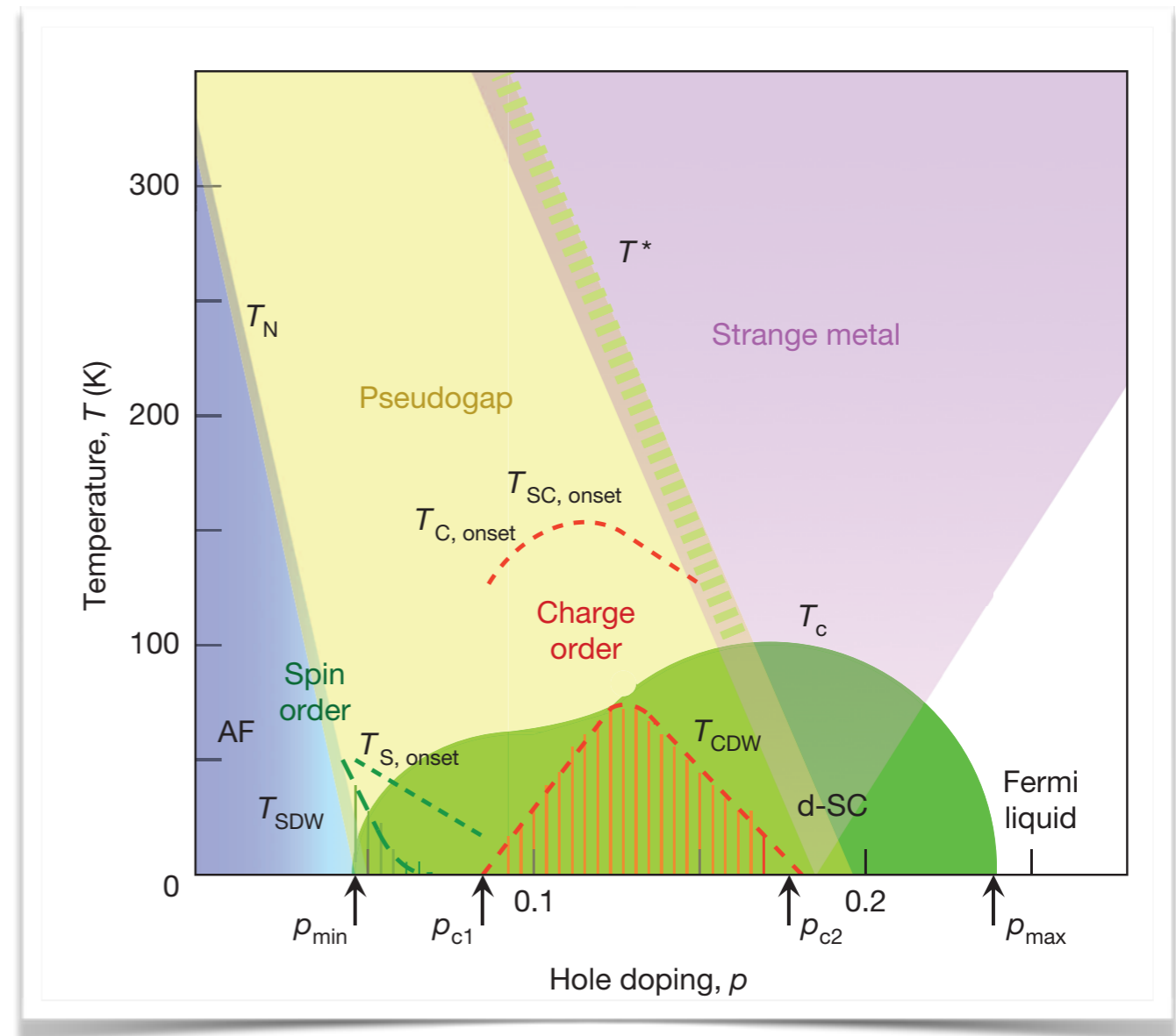
AFM Heisenberg Model

Half filling & strong interaction



$$H = J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j \quad J = \frac{4t^2}{U}$$

B. Keimer et al., Nature **518** 2015



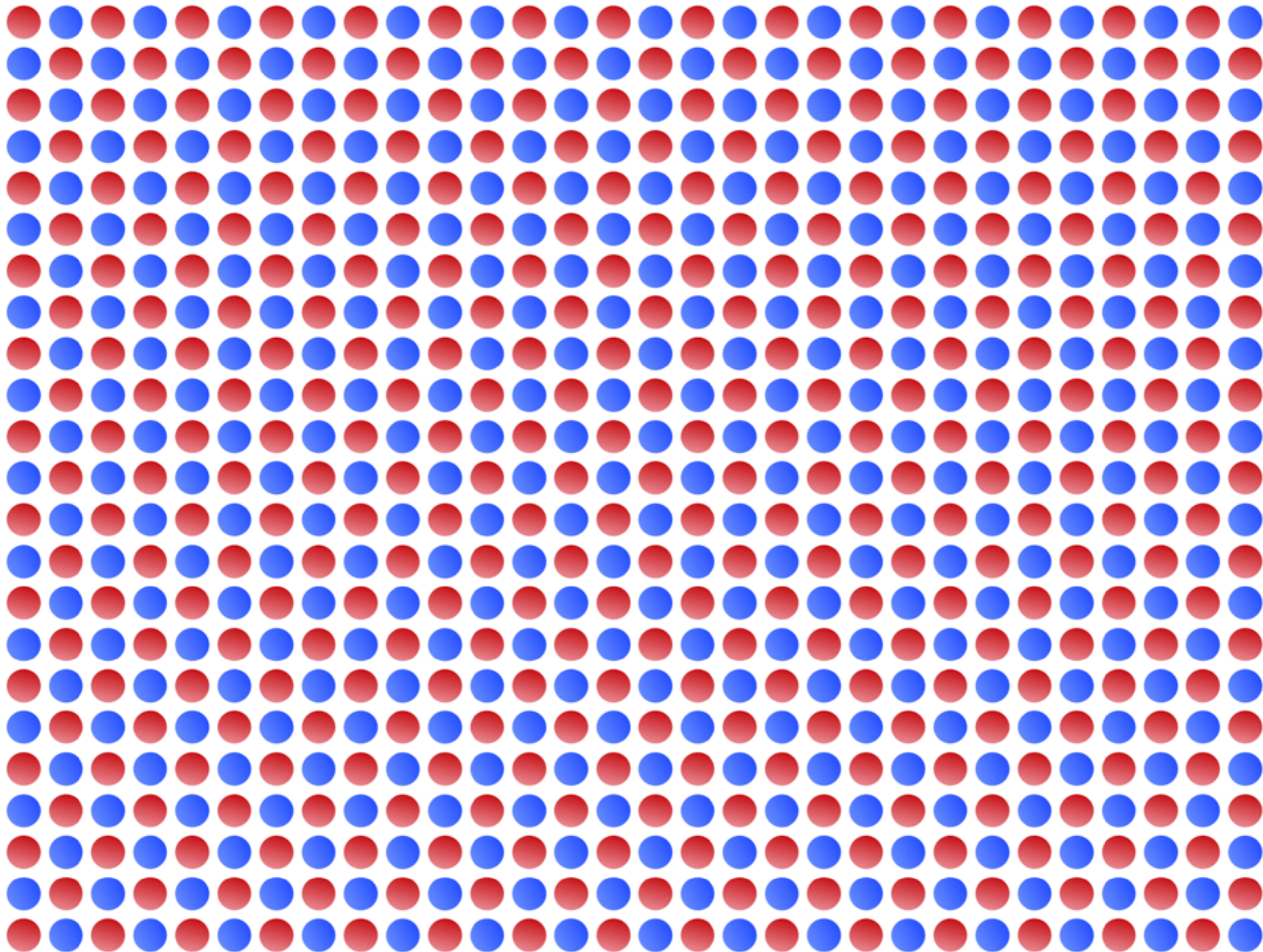
Away from half filling: **t-J model**

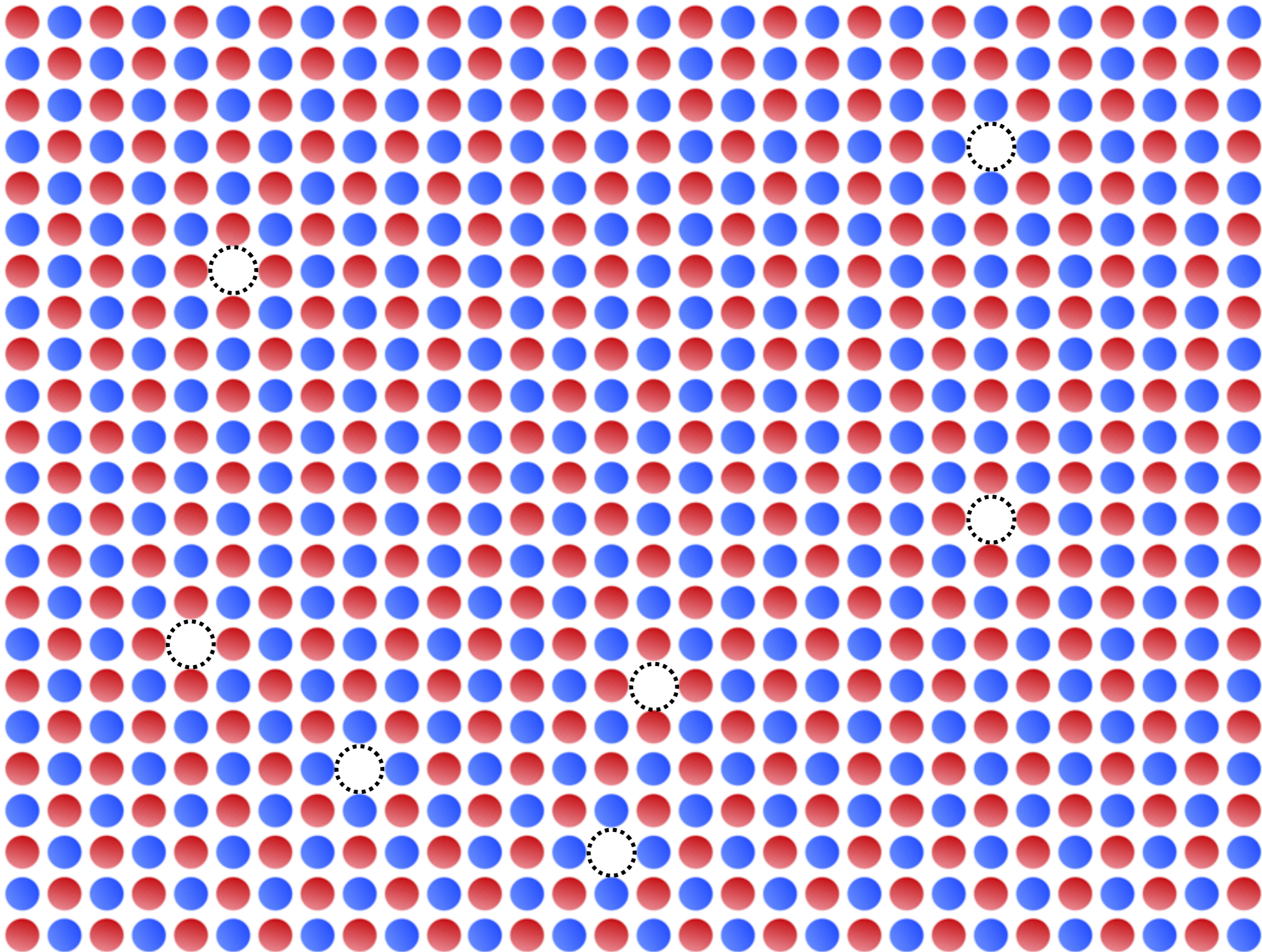
competition between

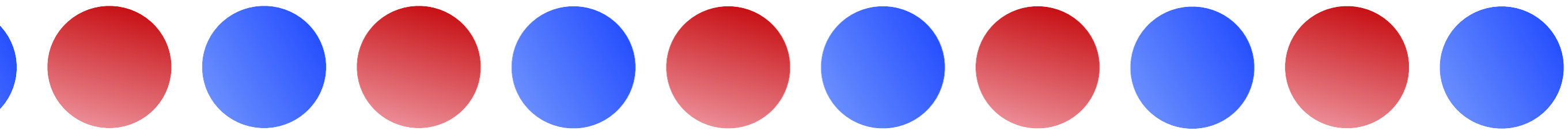
hole delocalization

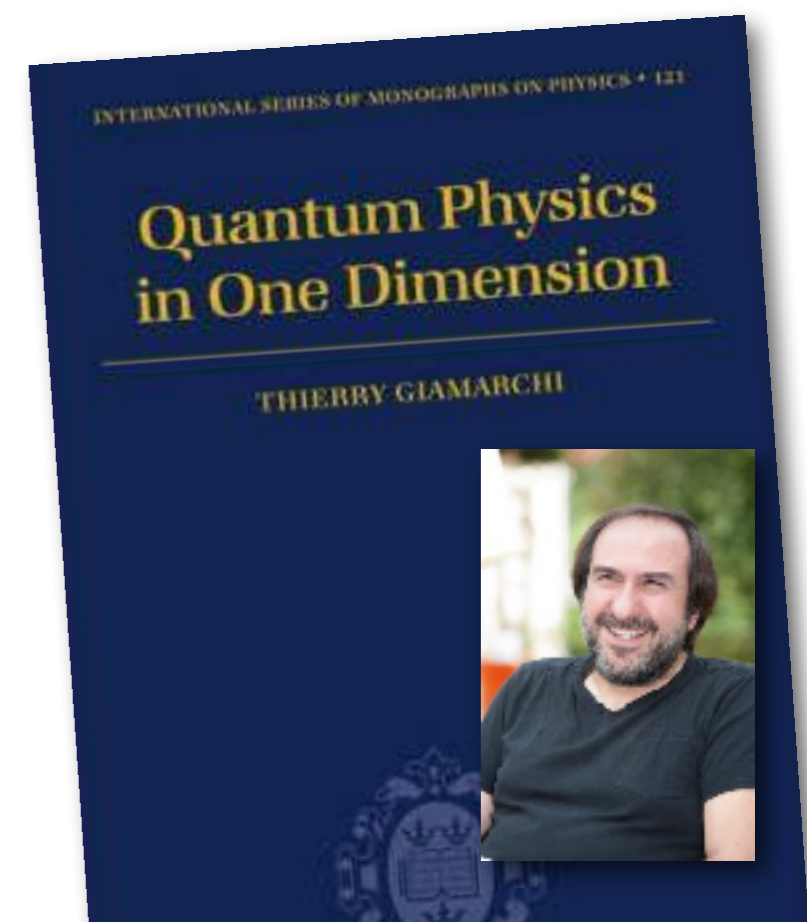
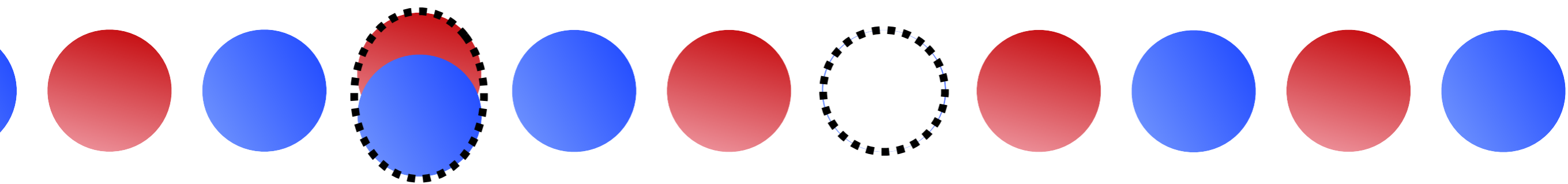


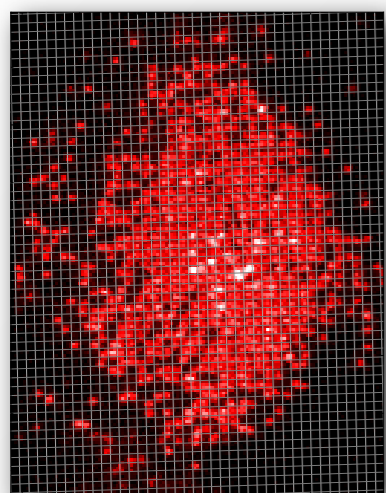
magnetic order



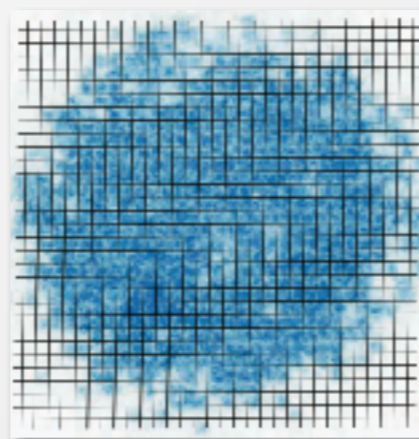




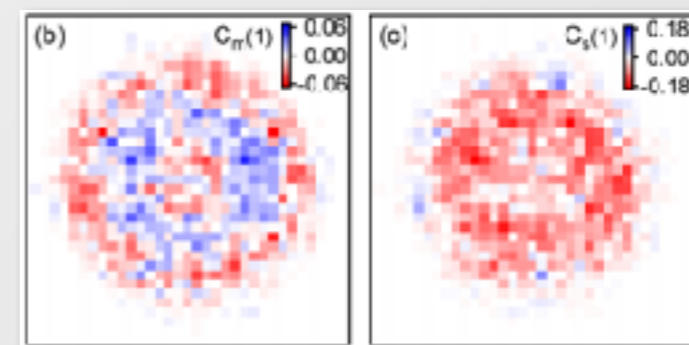




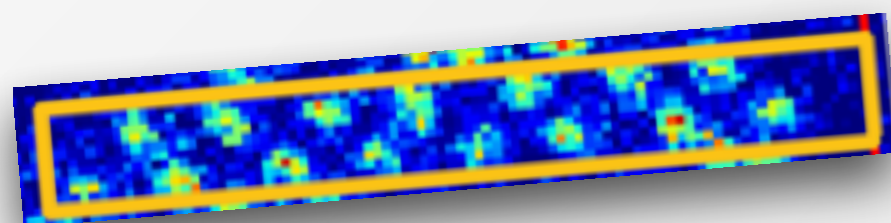
A. Omran *et al.* PRL (2015)



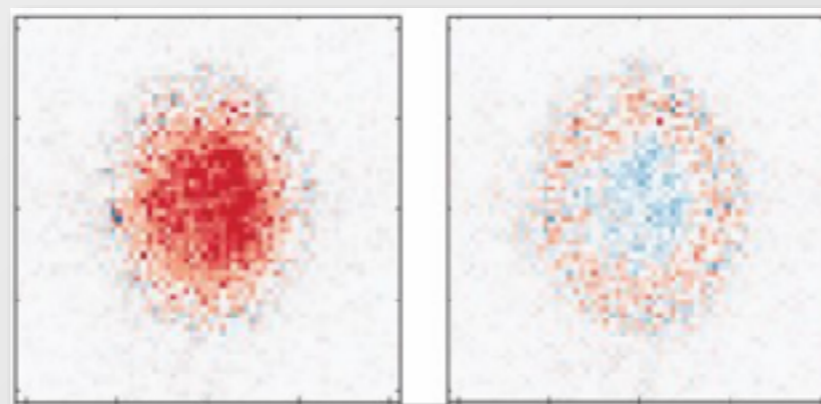
D. Greif *et al.* Science (2016)



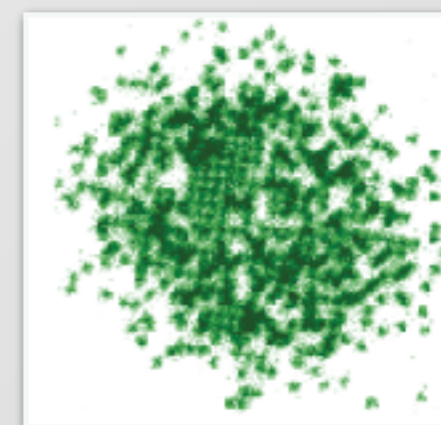
L. Cheuk *et al.* Science (2016)



M. Boll *et al.* Science (2016)



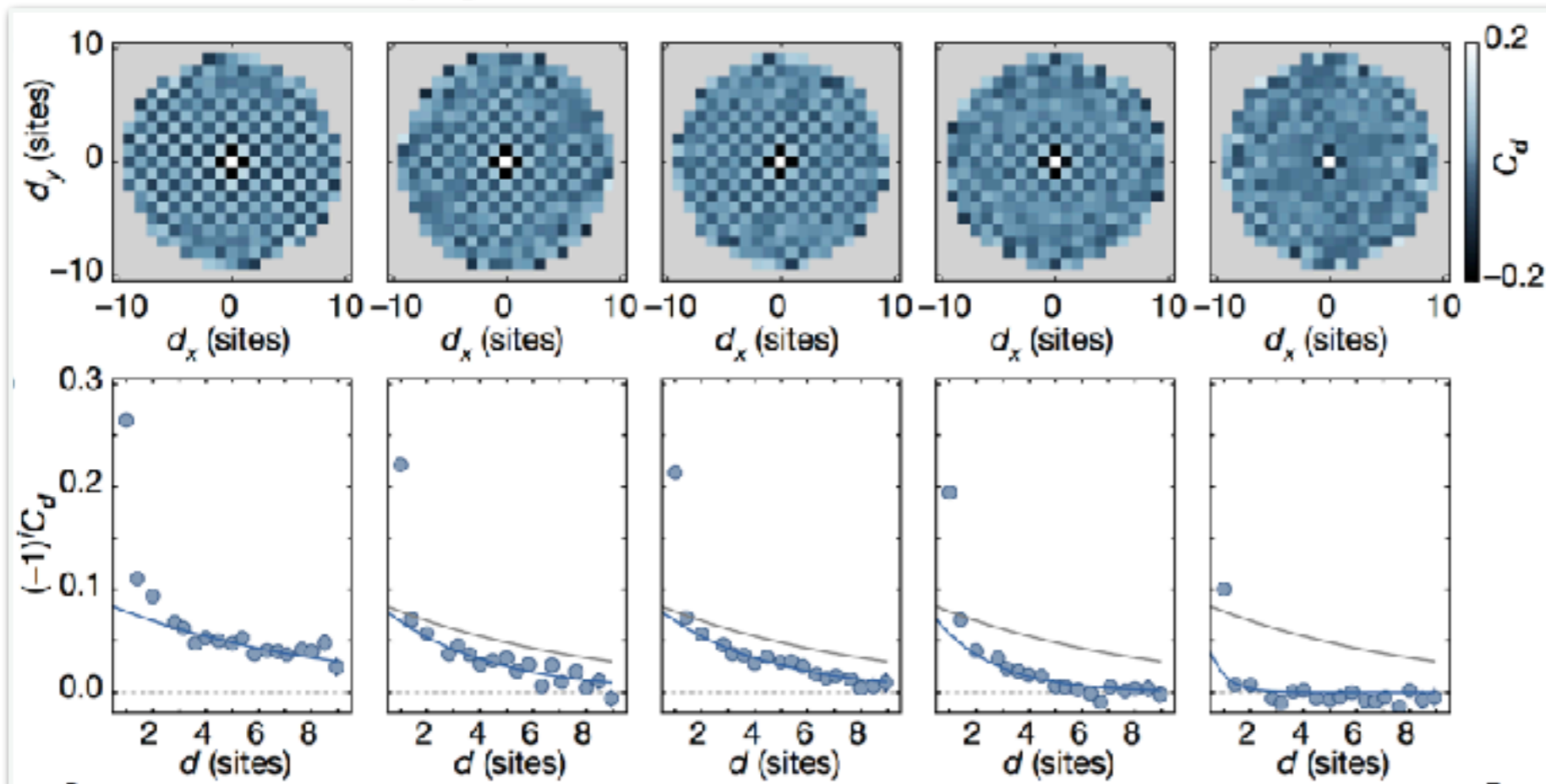
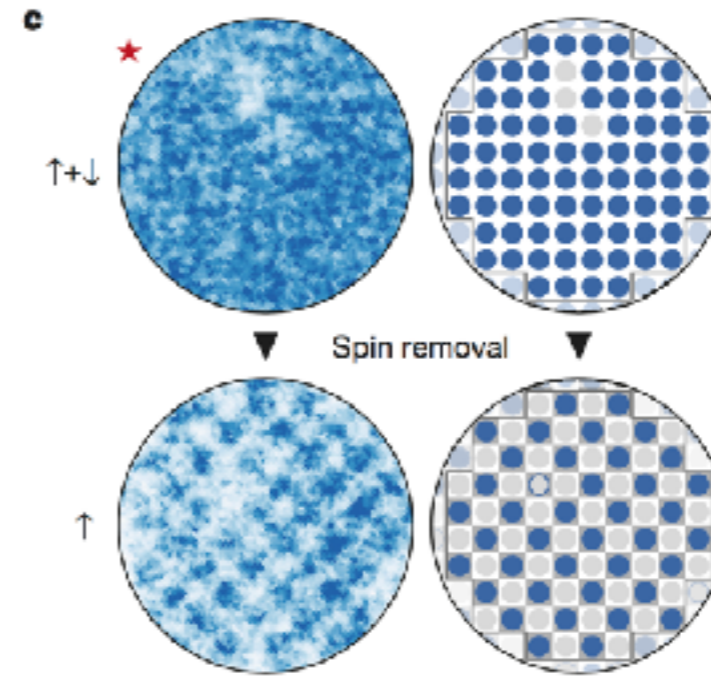
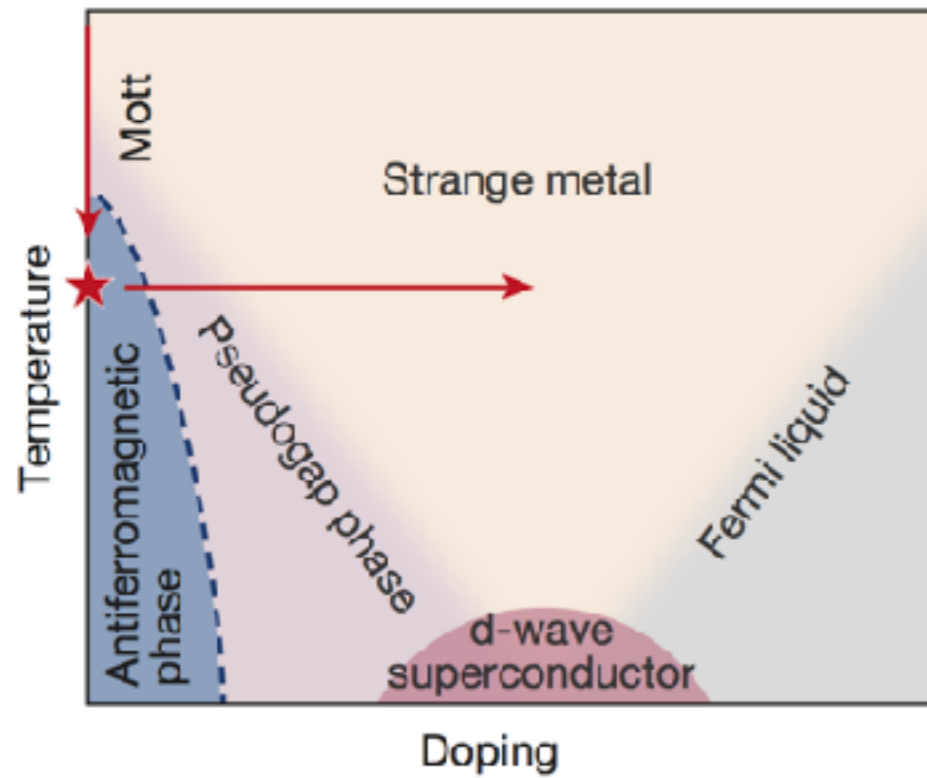
M. Parsons *et al.* Science (2016)



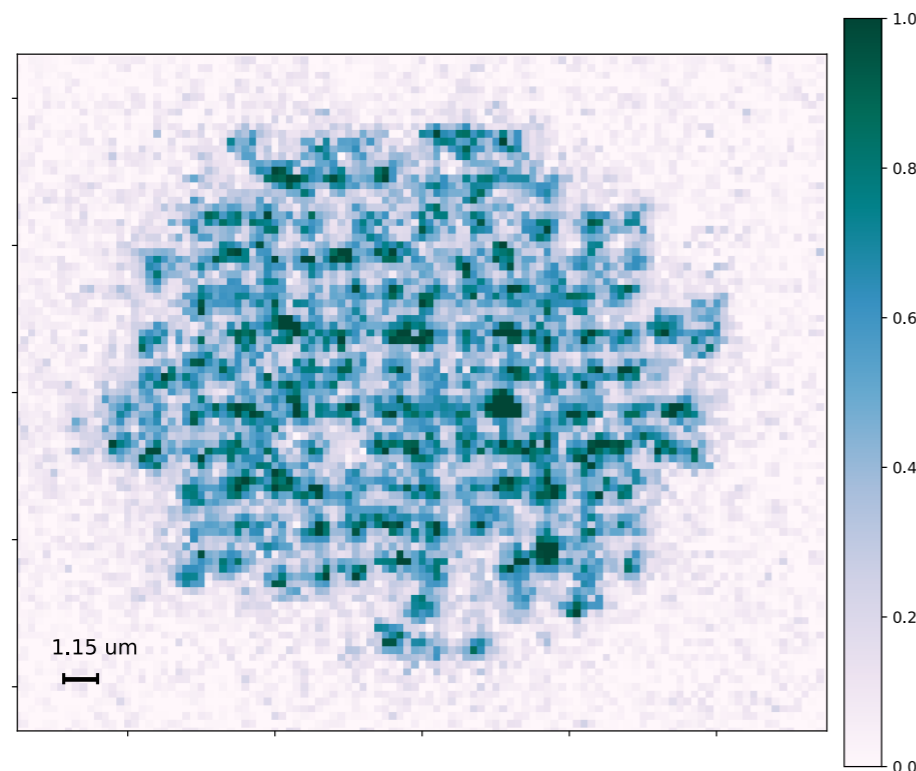
P. Brown *et al.* Science (2017)

AFM Correlations (Short, Medium & Long Ranged) now visible!

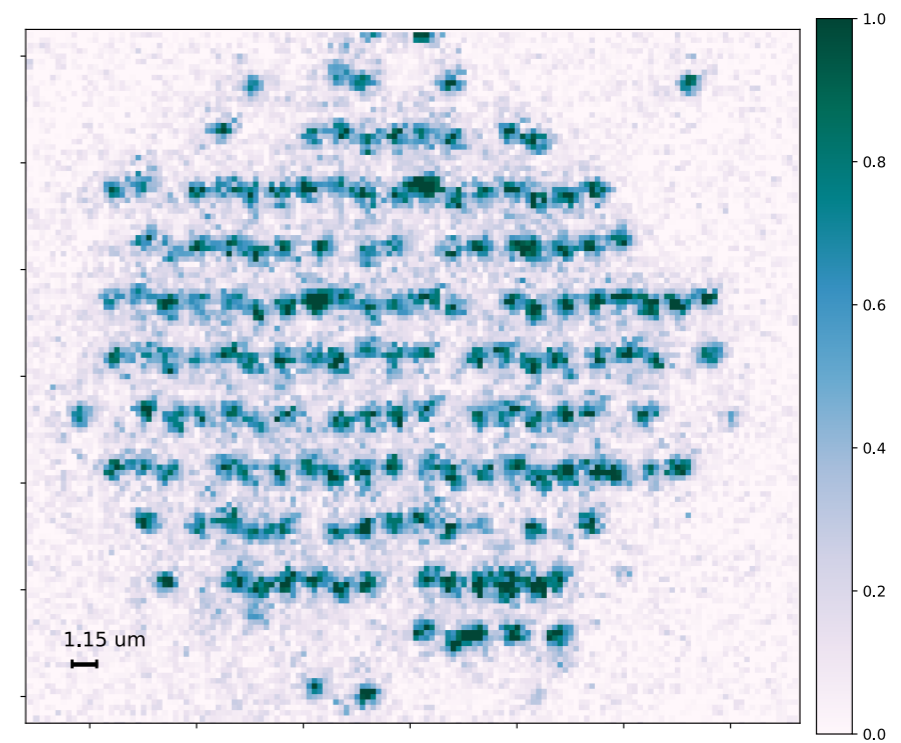
A. Maruzenko *et al.* Nature (2017), M. Boll *et al.* Science (2016), T. Hilker *et al.* Science (2017),
L. Cheuk *et al.* Science (2016), P. Brown *et al.* Science (2017)



A. Mazurenko et al.,
Nature **545**, 462

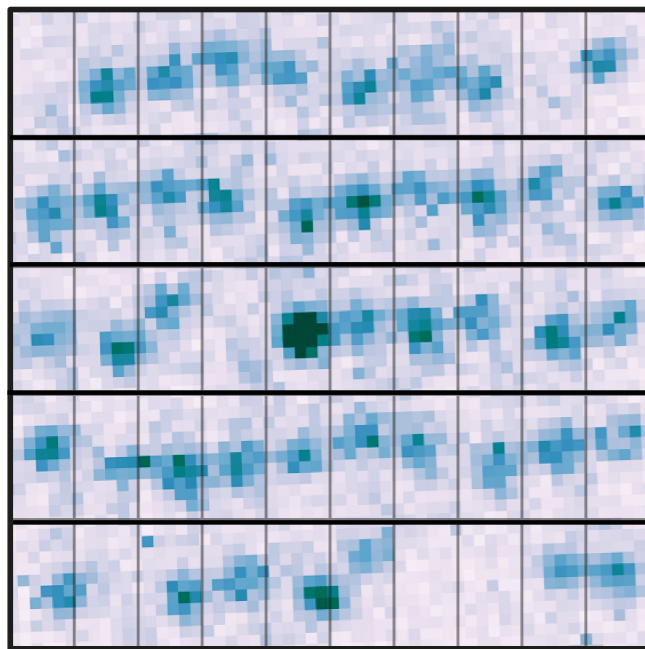
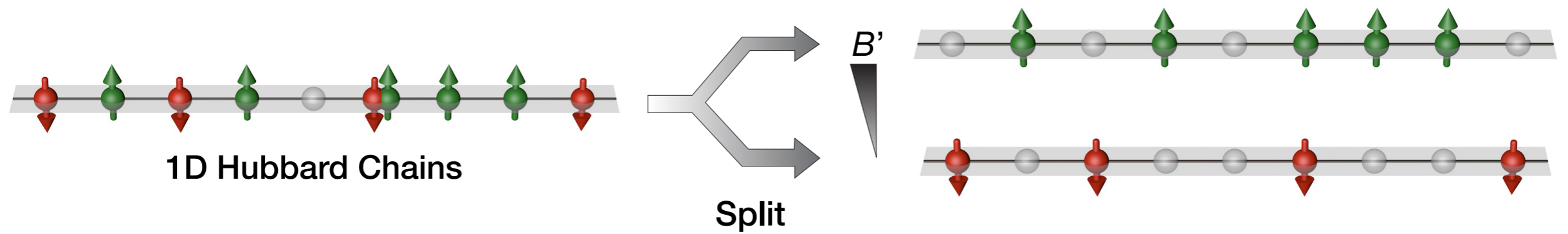


Mott Insulator
(Short-Short Lattice)

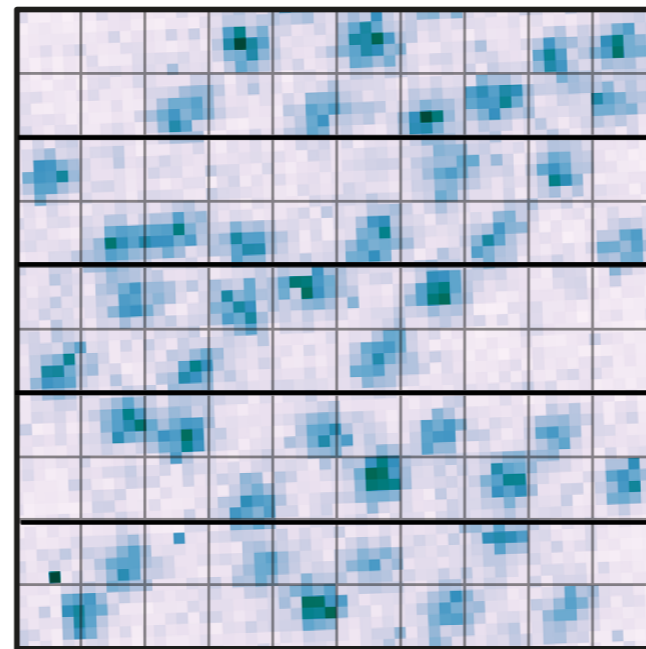


Mott Insulator
(Short-Long Lattice)

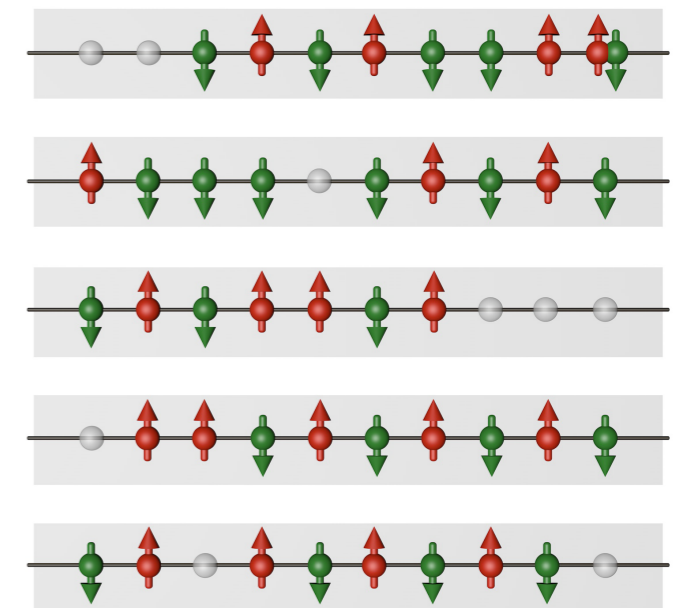
No Parity Projection!
Holes-Doublons Distinguishable



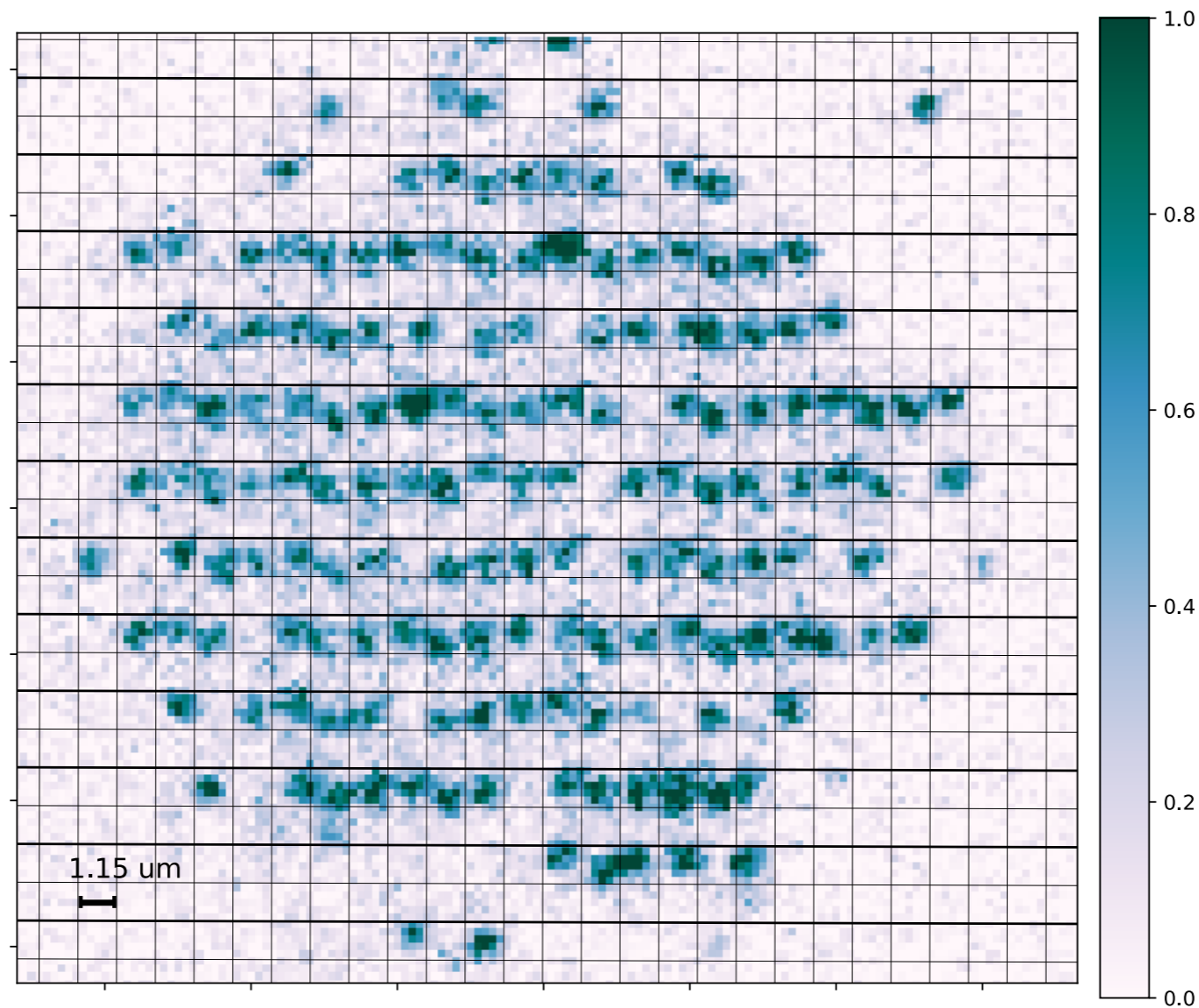
1D Chains (spin unresolved)



Split 1D Chains (spin resolved)



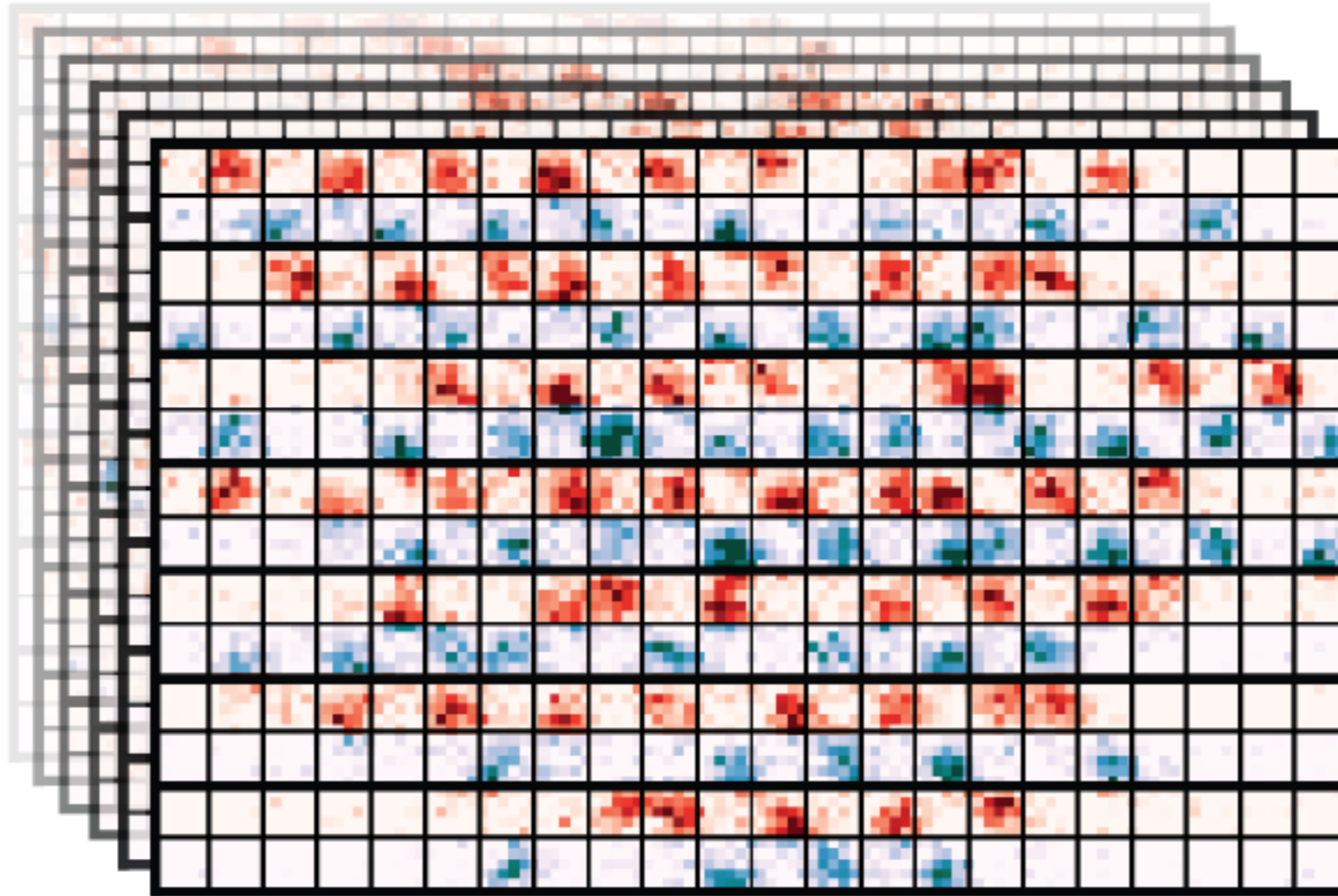
Reconstruction



Charge Resolved

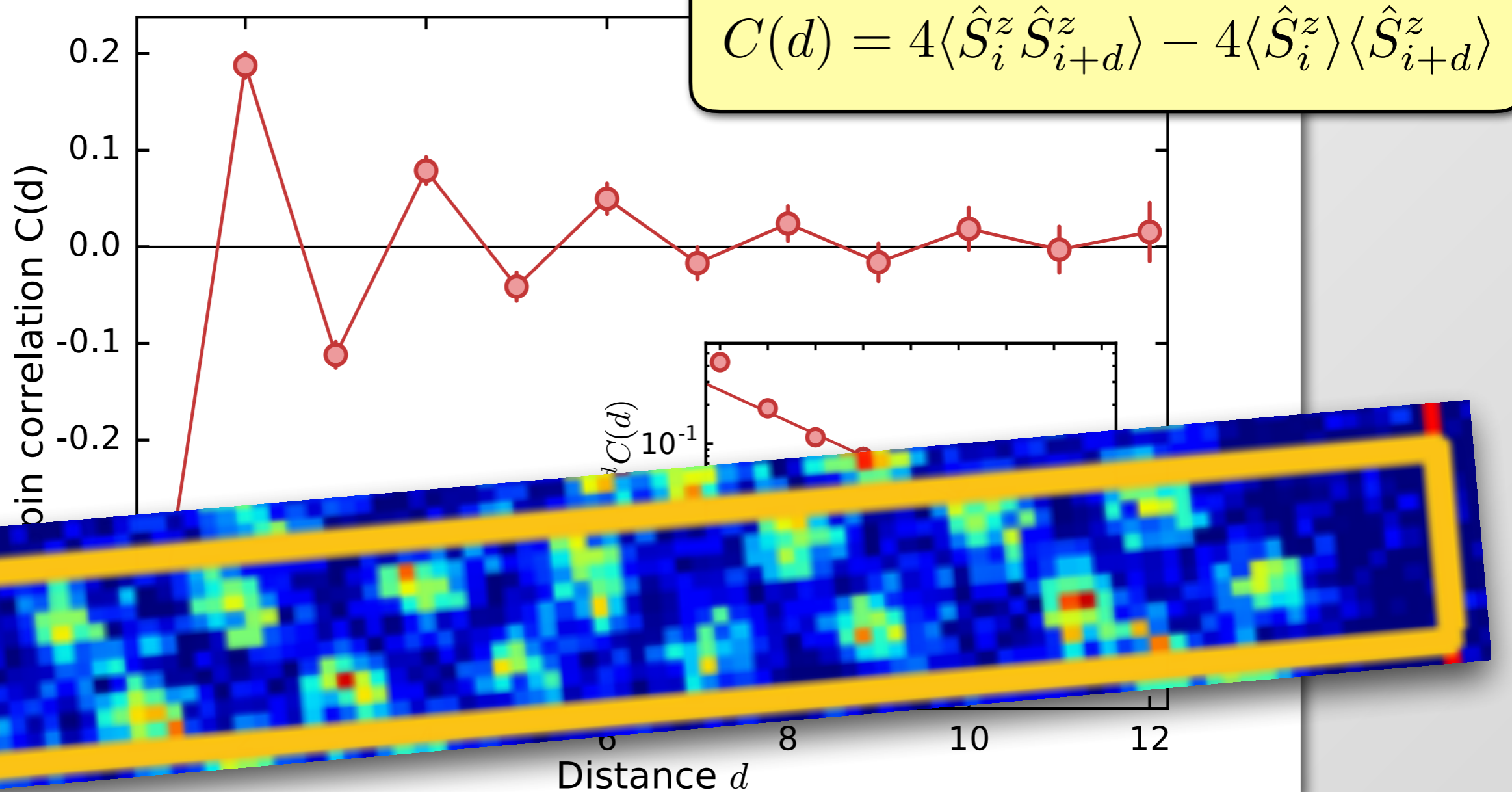
Spin Un-Resolved

Spin Resolved



Charge Resolved

Postselection to $M_z = 0$!



$\xi(0, T)$ Largest possible decays length !

Fractionalization & Dynamical Spin-Charge Separation

The Electron

Charge $-e$

Spin $1/2$

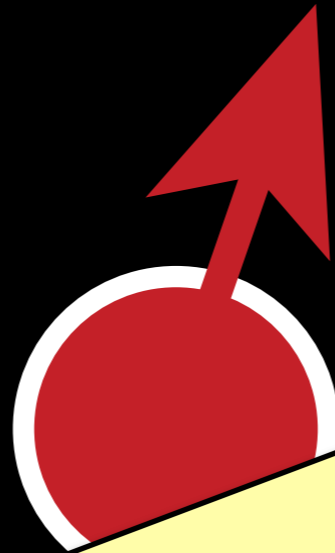
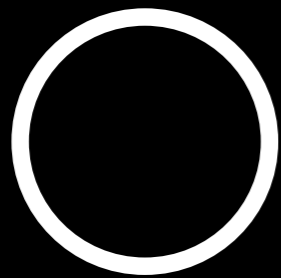
Impossible!

Charge $-e$

Spin $1/2$

Particle

Particle



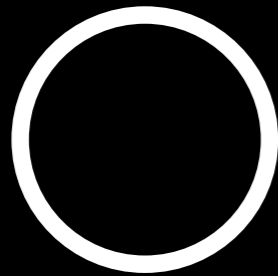
The Electron

Charge $-e$

Spin $1/2$

Fractionalization

Deconfinement of Quasi-particles
that make up the elementary particle



Charge $-e$

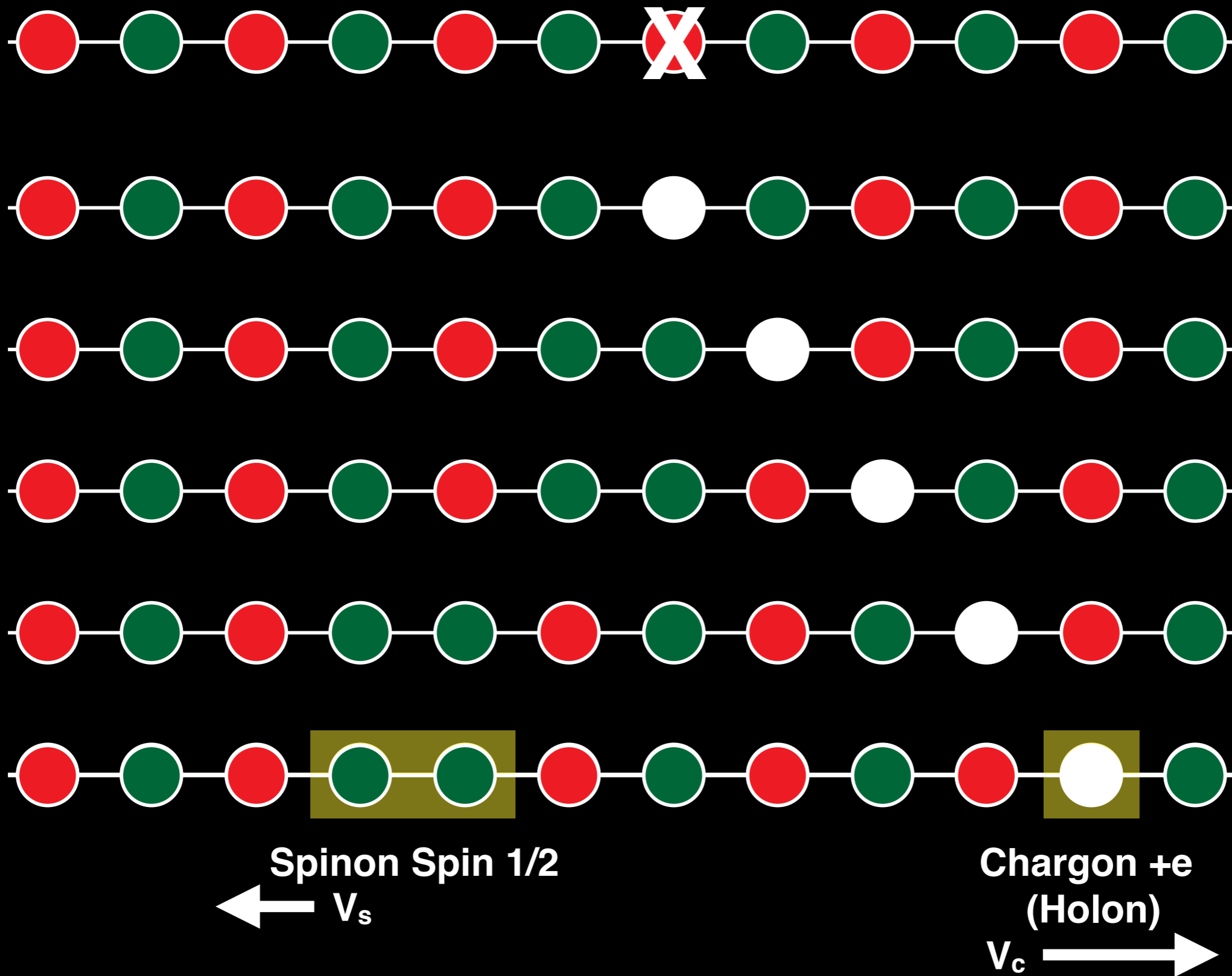
Quasi-Particle



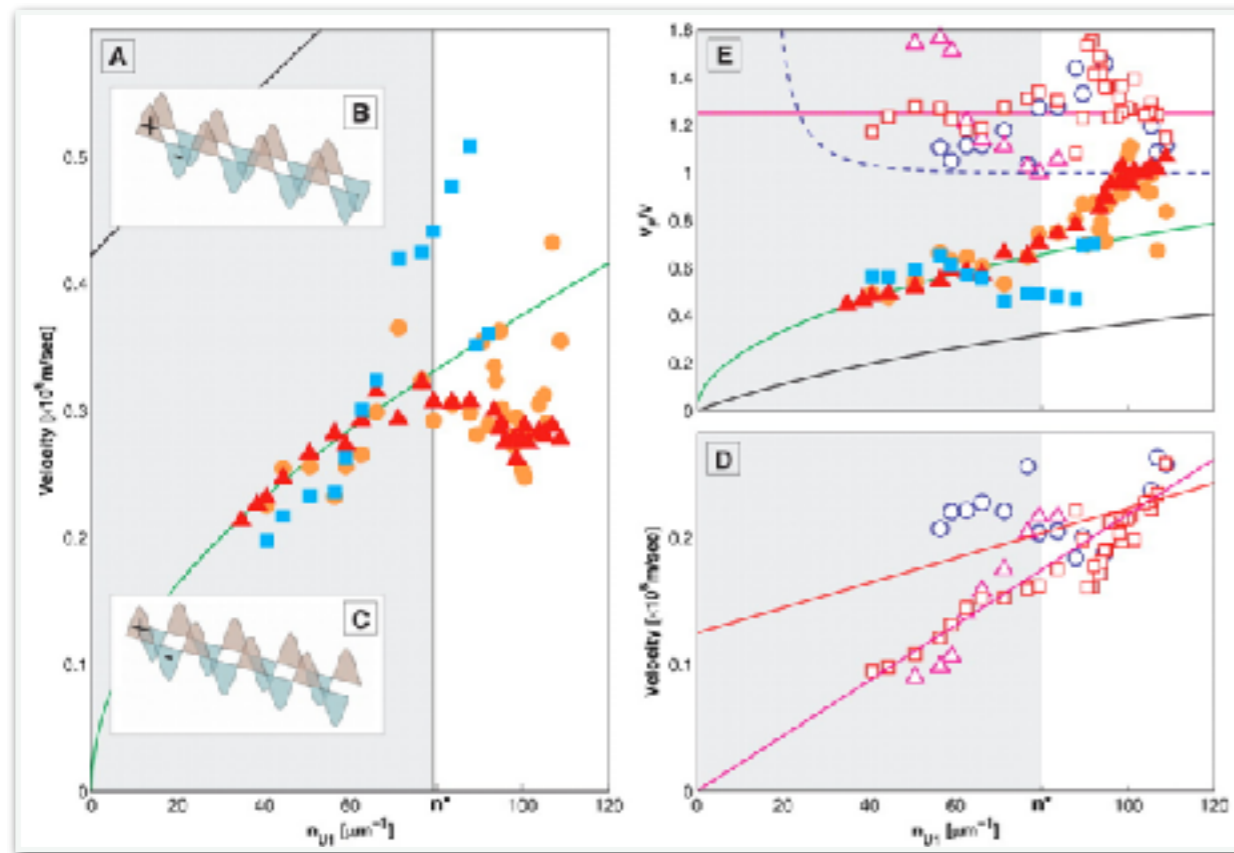
Spin $1/2$

Quasi-Particle

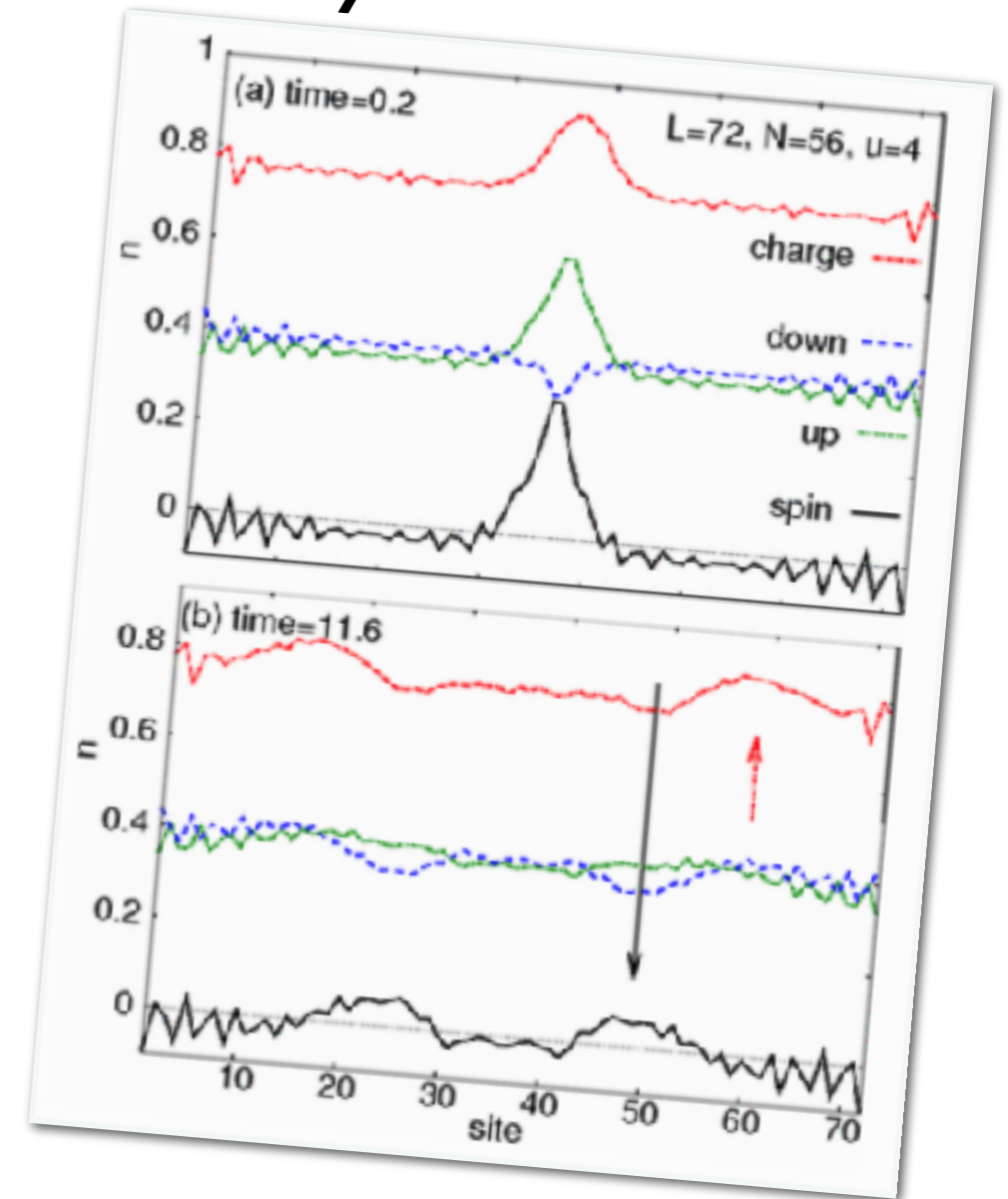
Fractionalization



Experiment



Theory



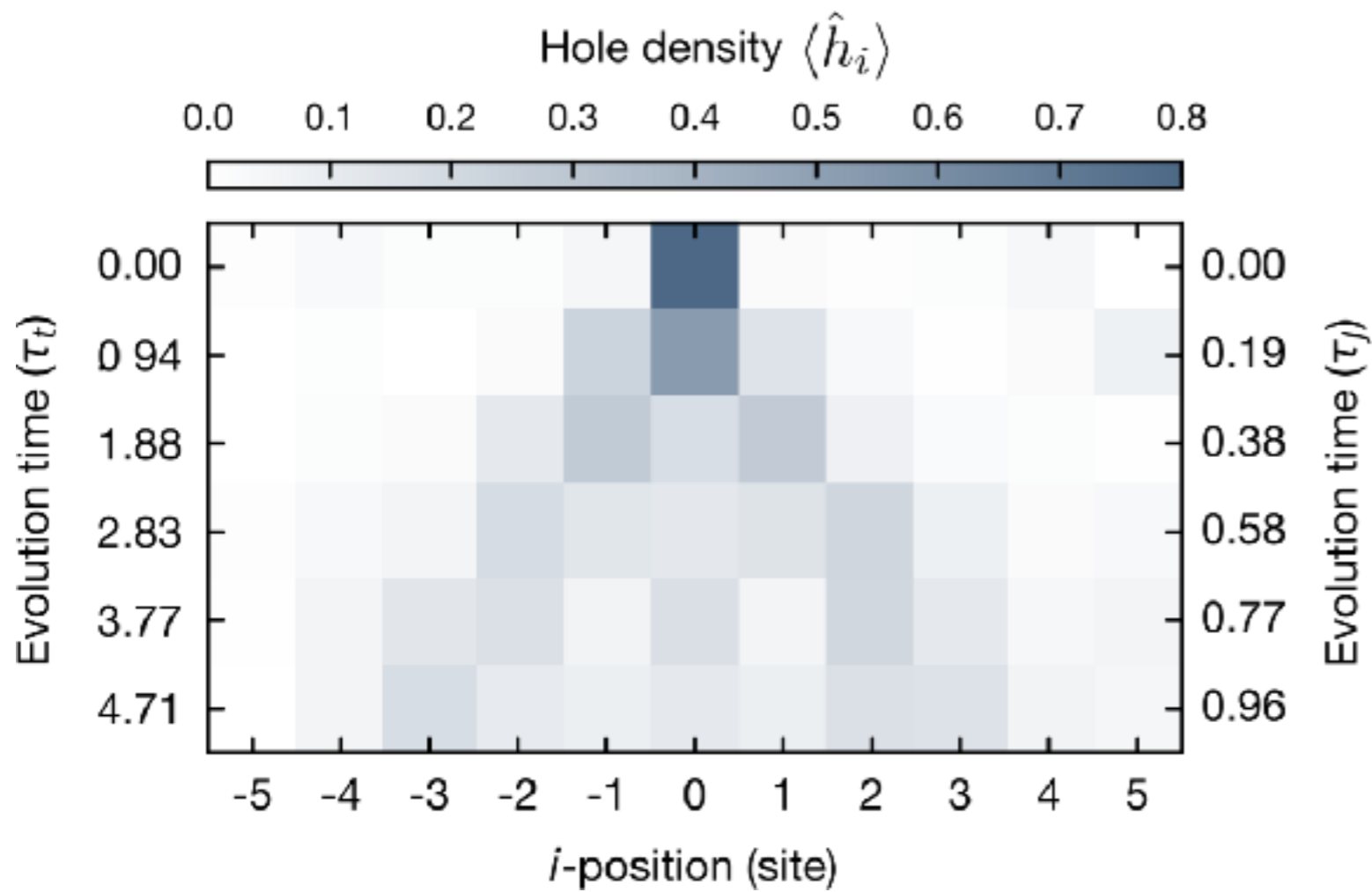
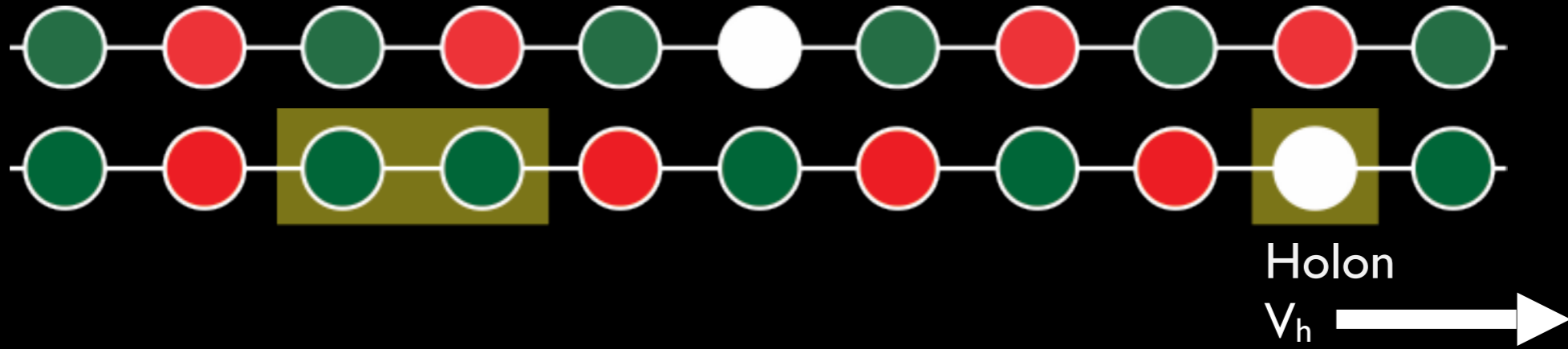
Spectroscopic determination:

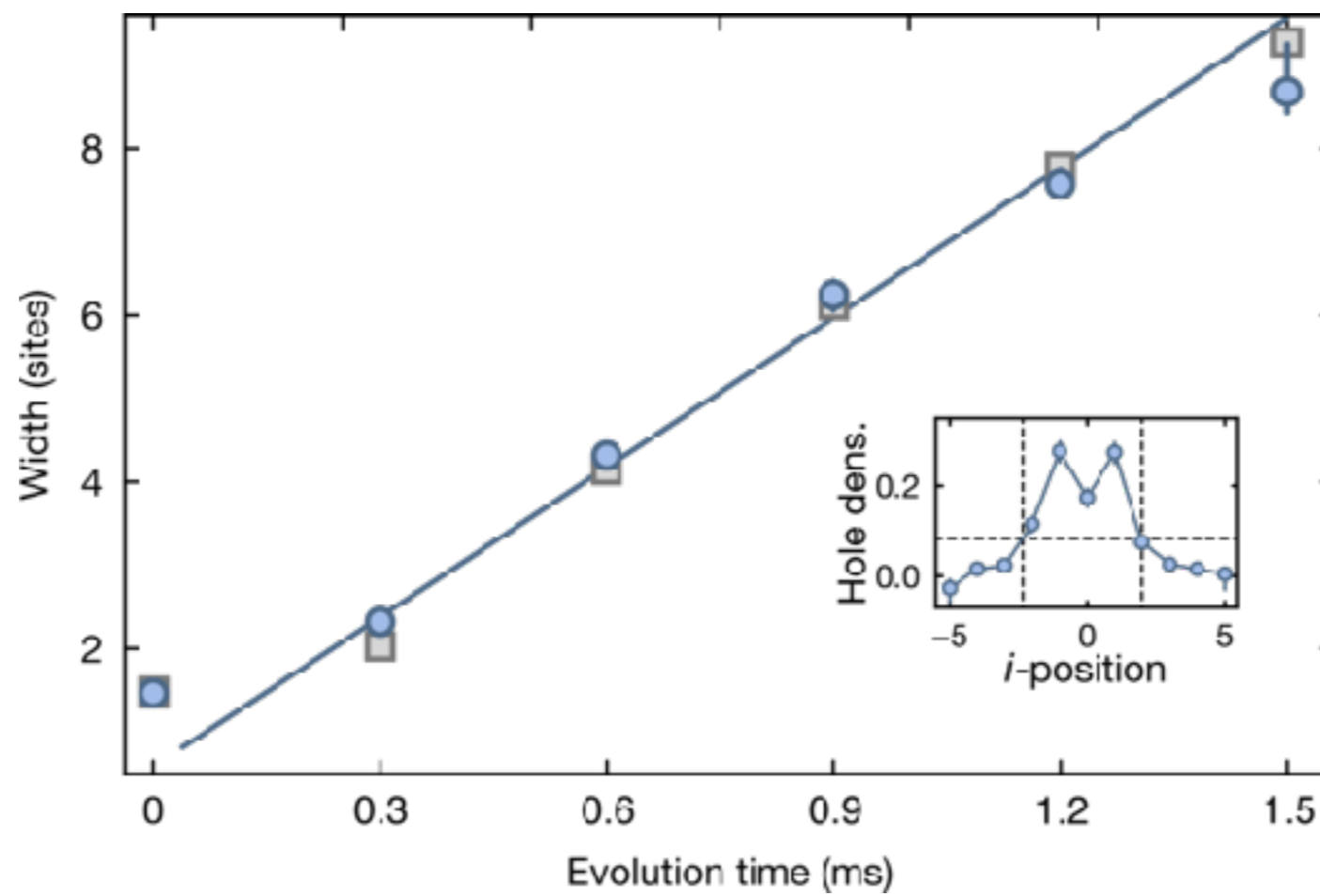
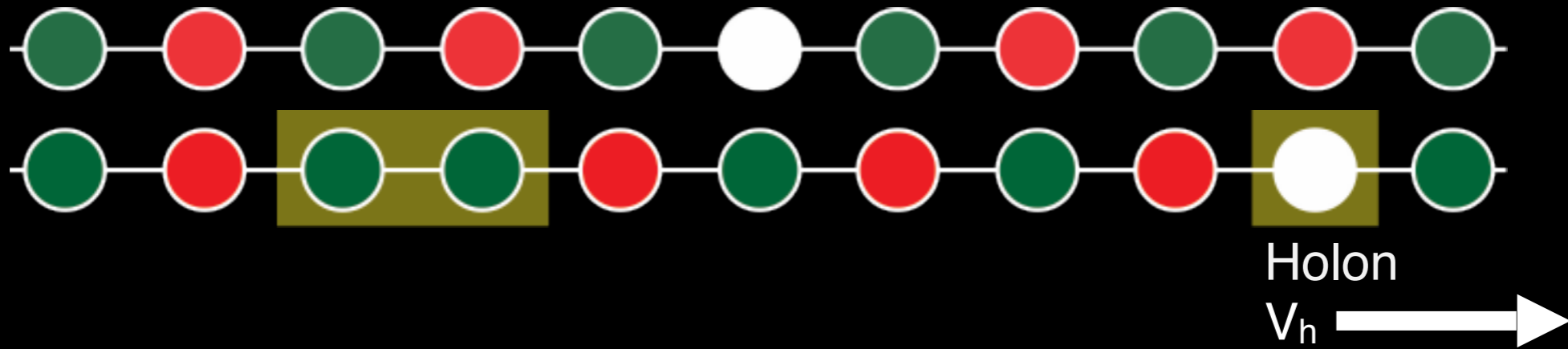
C. Kim, et al. Phys. Rev. Lett. **77**, 4054 (1996)
 O.M. Auslaender et al. Science **308**, 88 (2005)

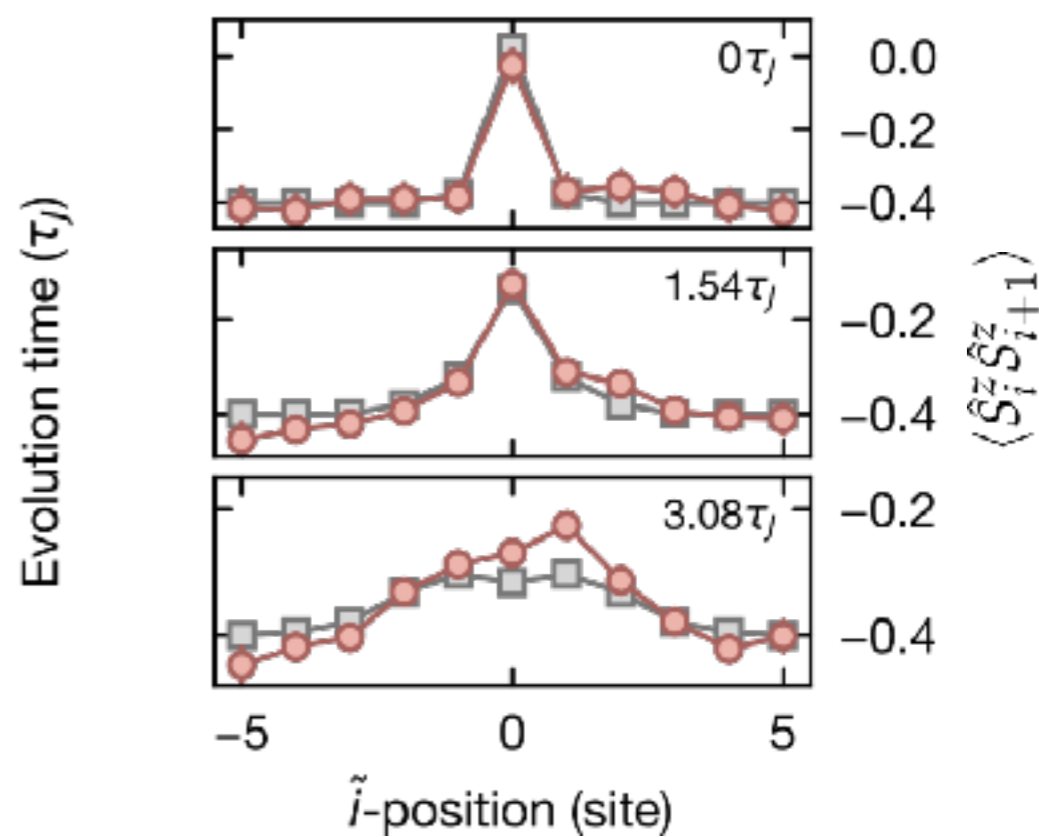
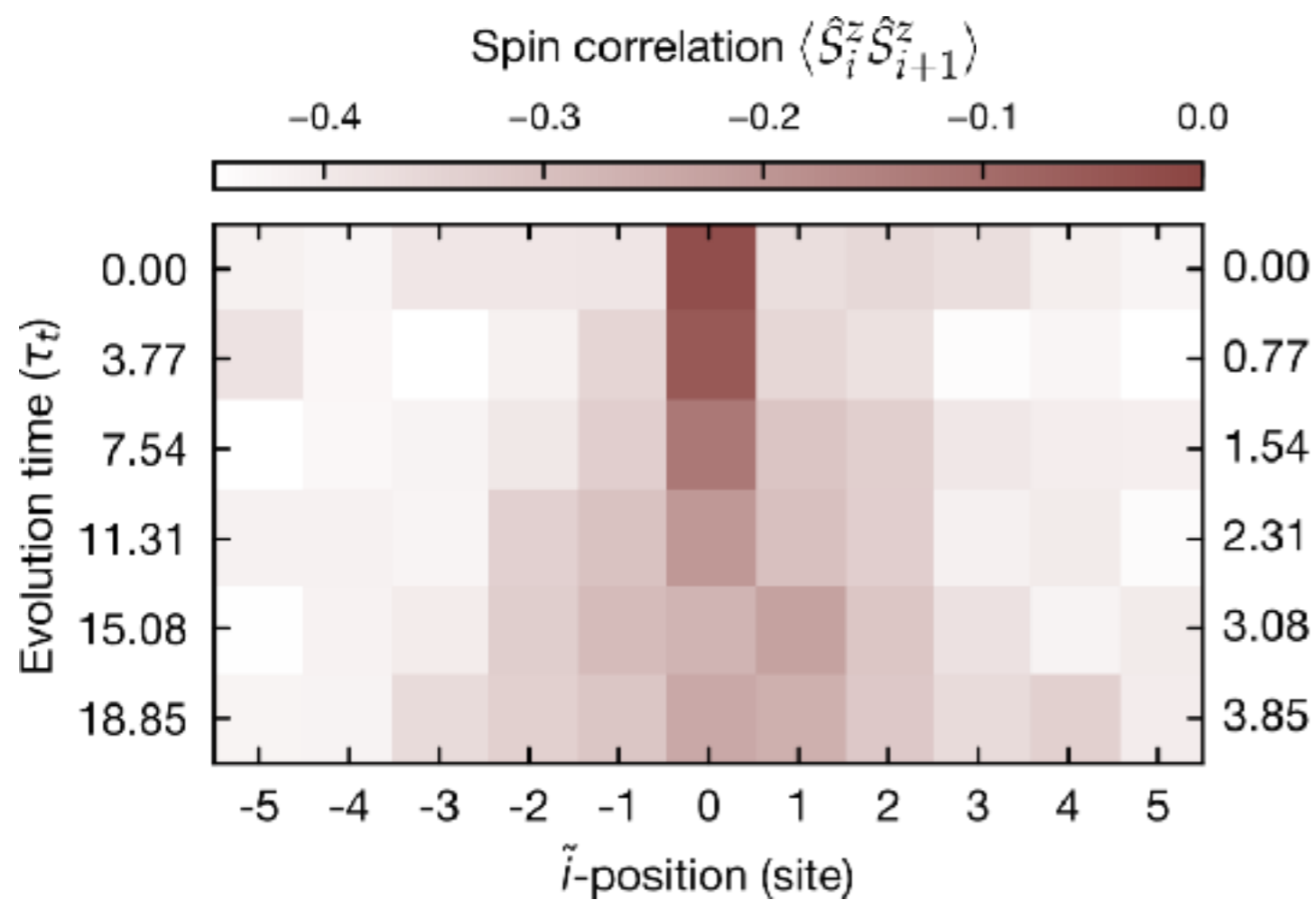
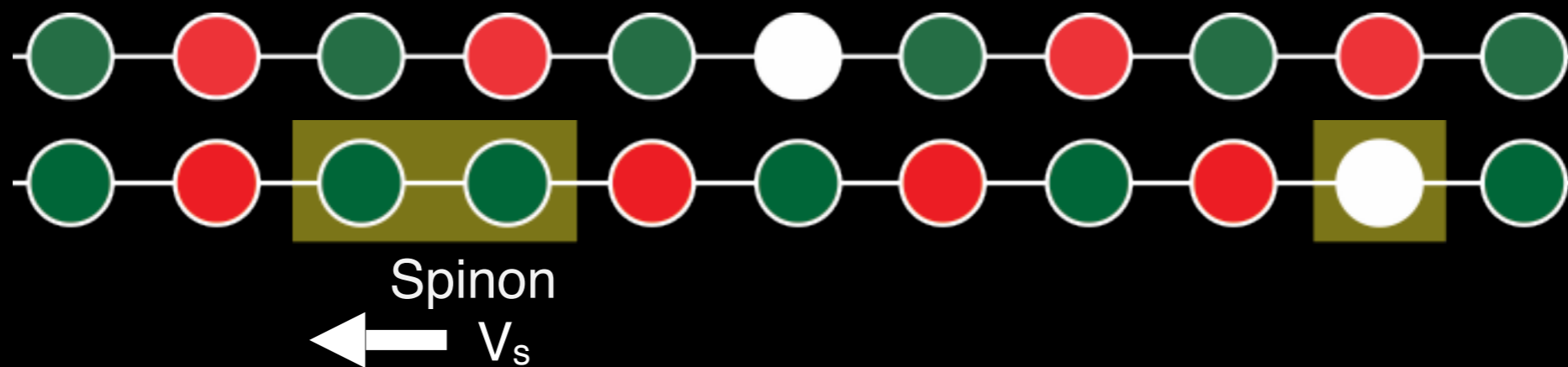
DMRG Simulation:

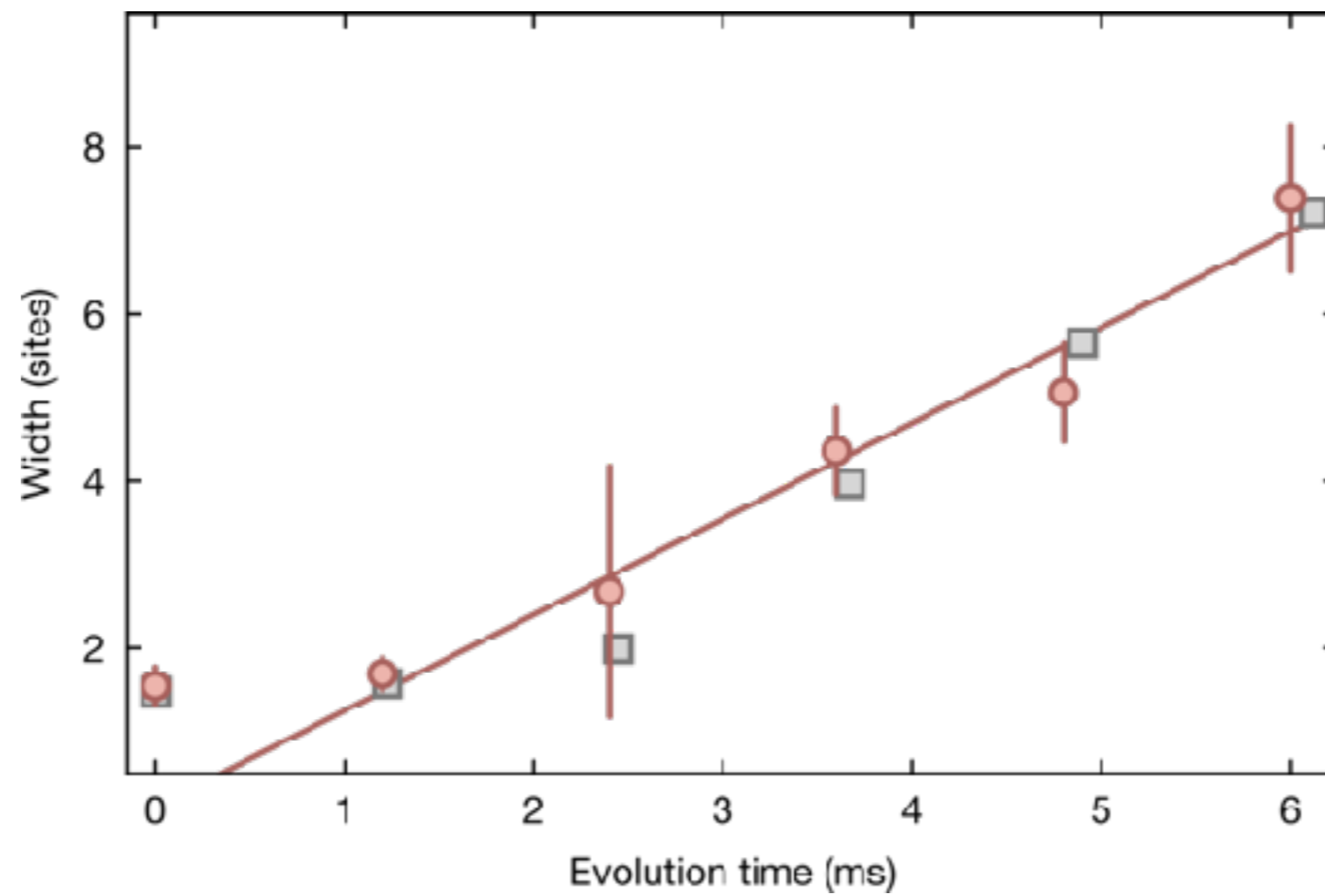
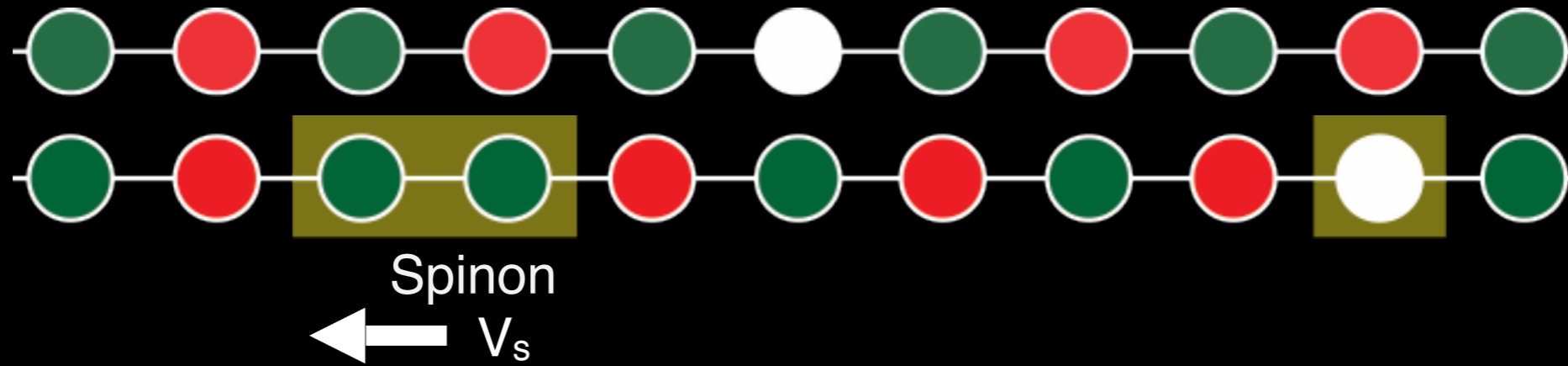
C. Kollath, U. Schollwöck, W. Zwerger
 Phys. Rev. Lett. **95**, 176401 (2005)

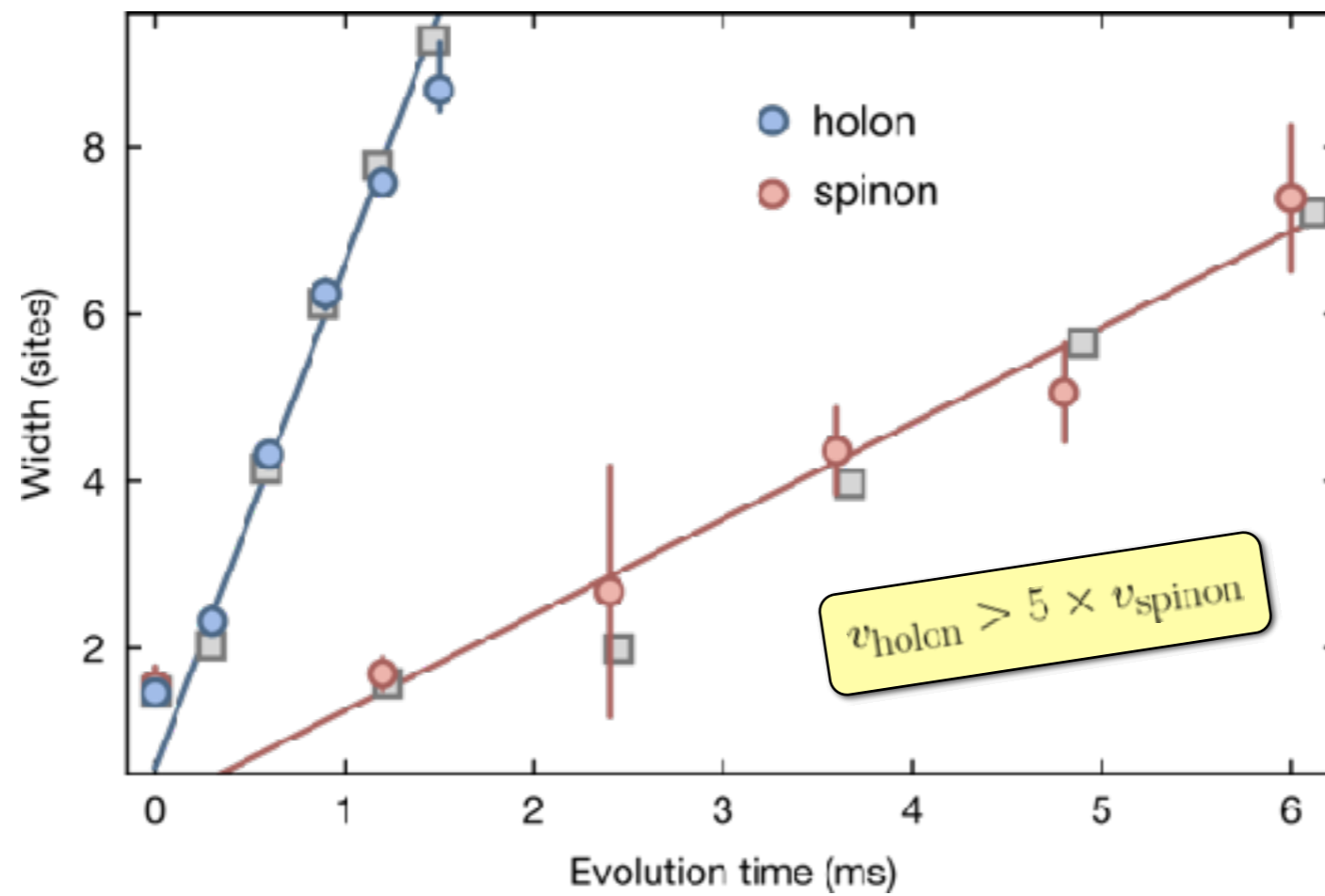
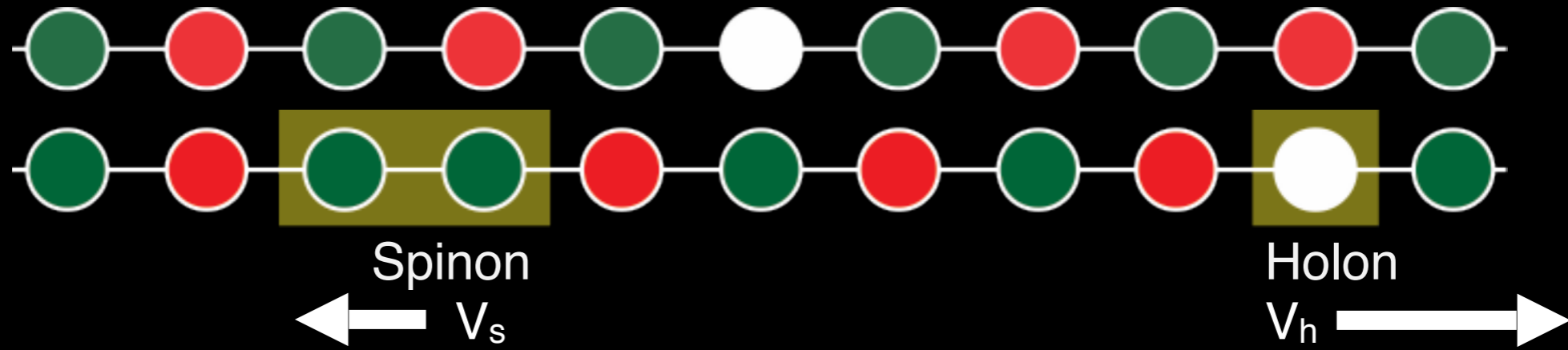






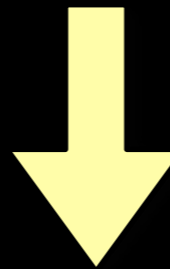
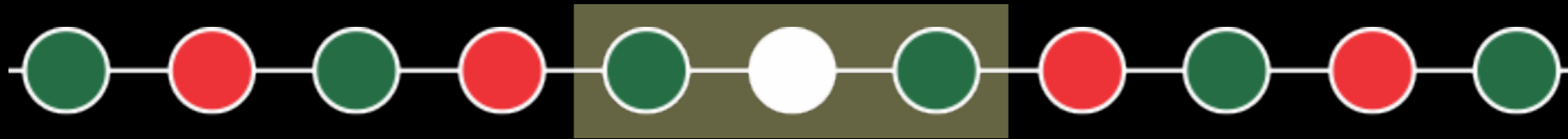






Absence of binding between quasiparticles

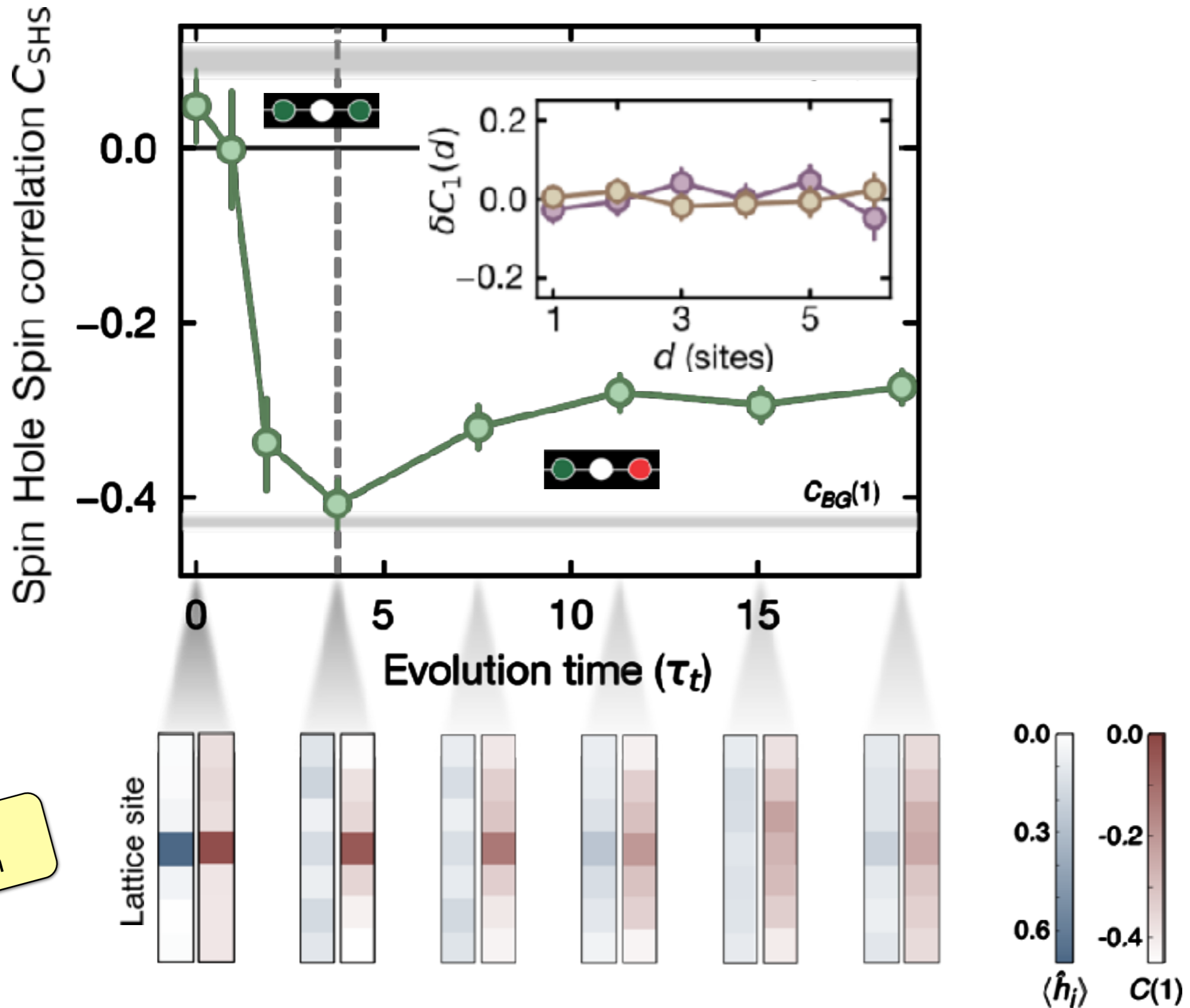
$$\langle \hat{S}_{i-1}^z \hat{h}_i \hat{S}_{i+1}^z \rangle > 0$$



$$\langle \hat{S}_{i-1}^z \hat{h}_i \hat{S}_{i+1}^z \rangle < 0$$

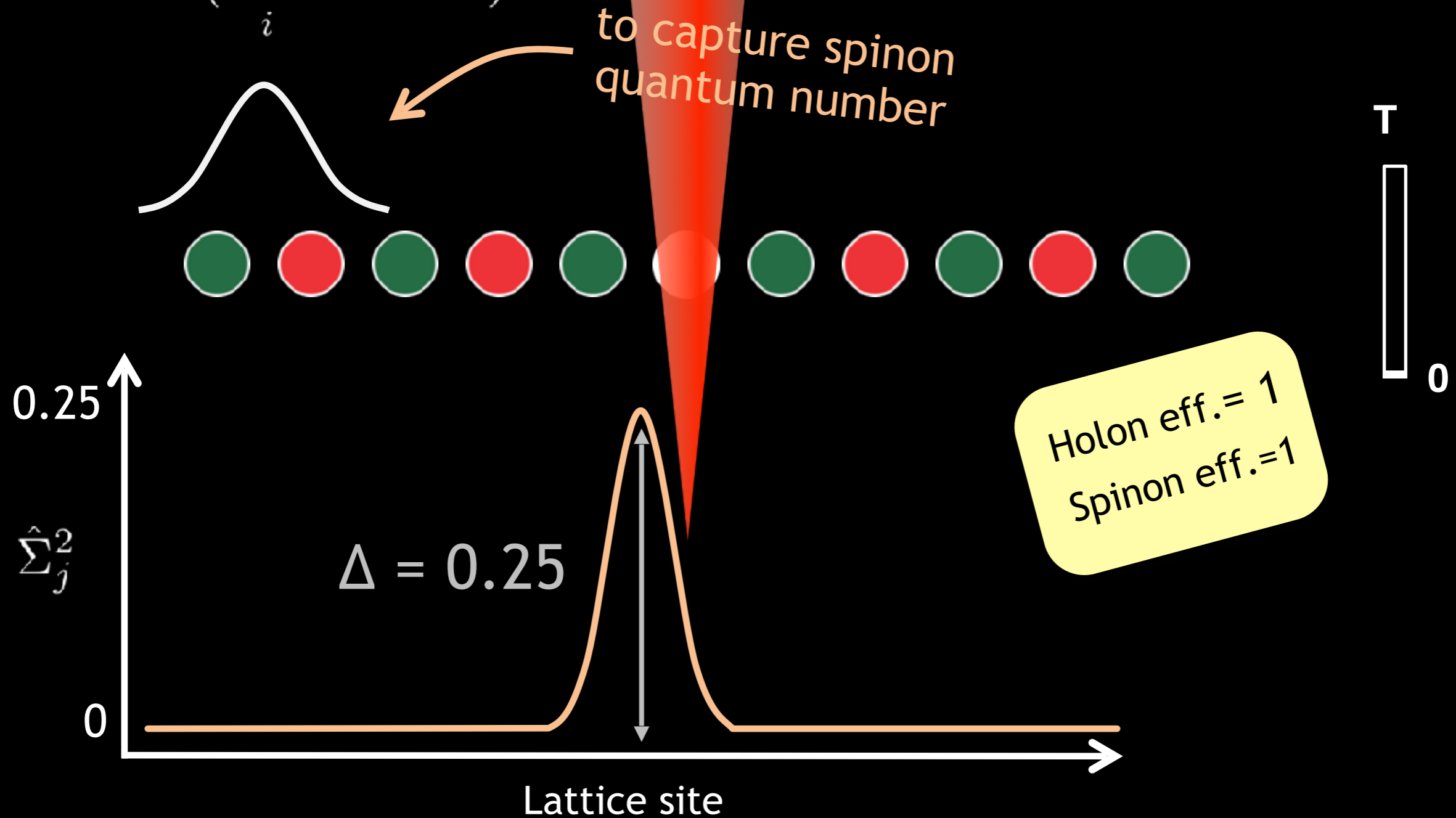


Hole gets rid of spinon

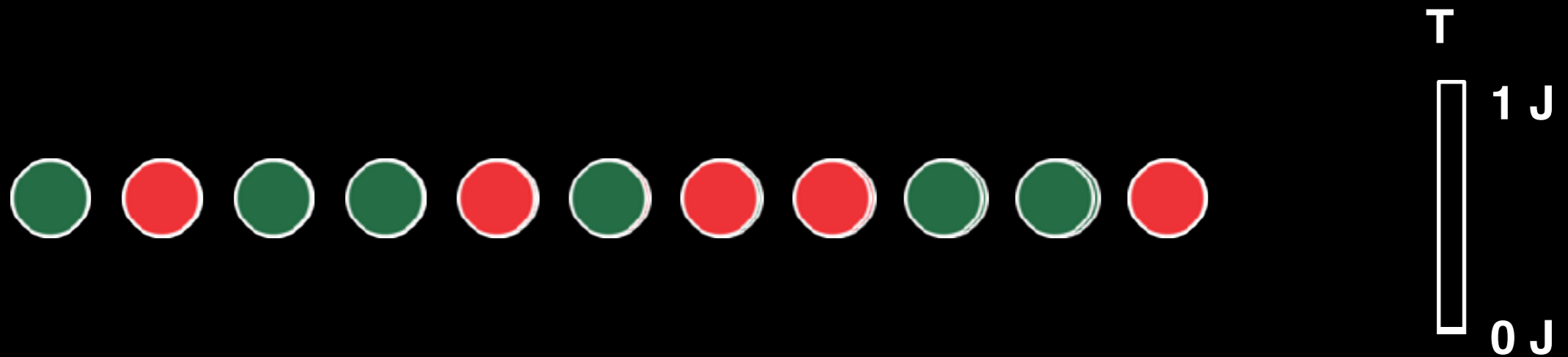


Fractionalization at zero temperature

$$\hat{\Sigma}_j^2 = \left(\sum_i \hat{S}_i^z f_j^\sigma(i) \right)^2$$

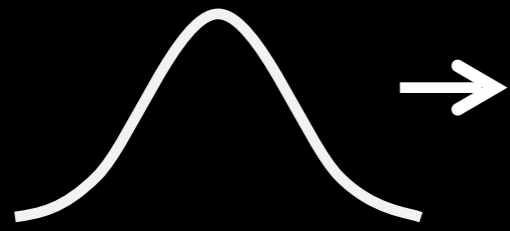


Fractionalization at non-zero temperature



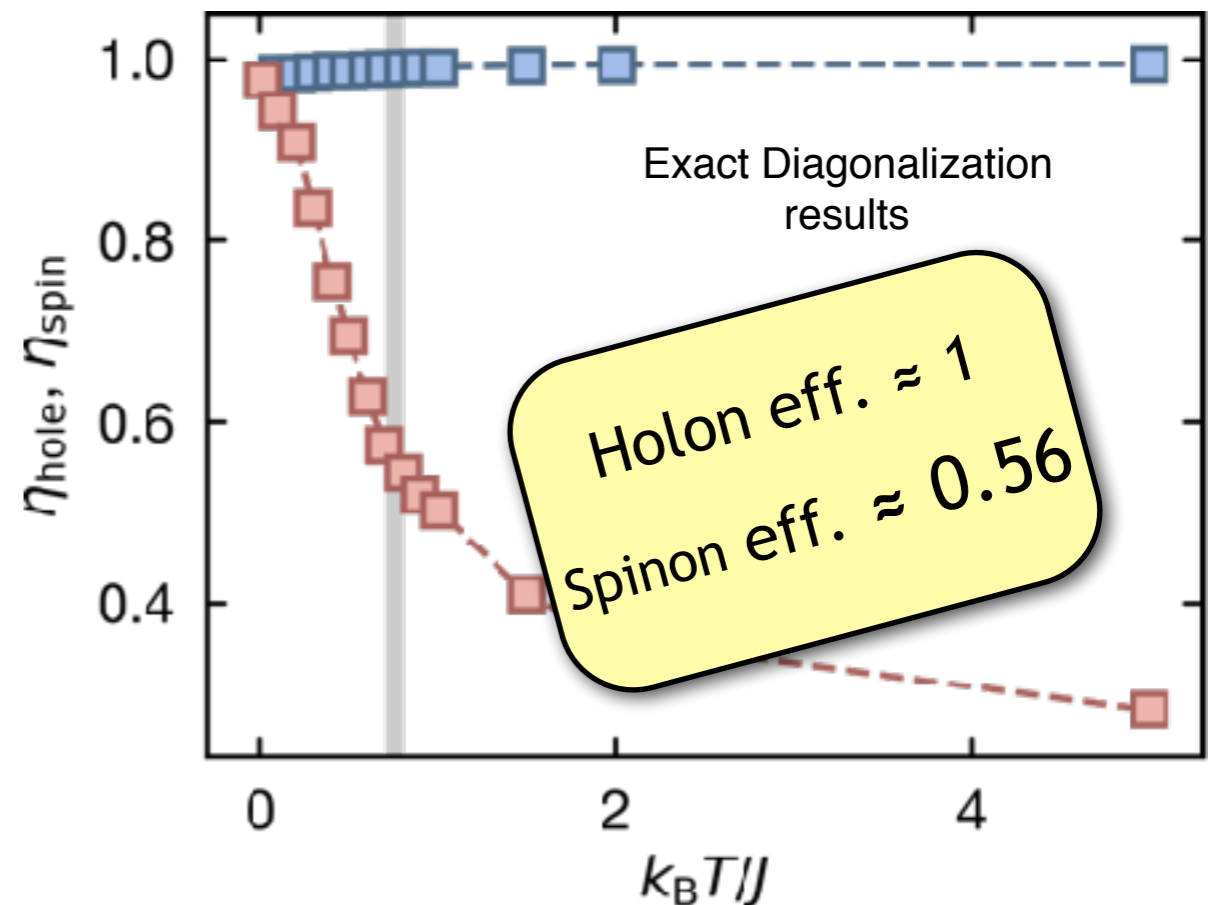
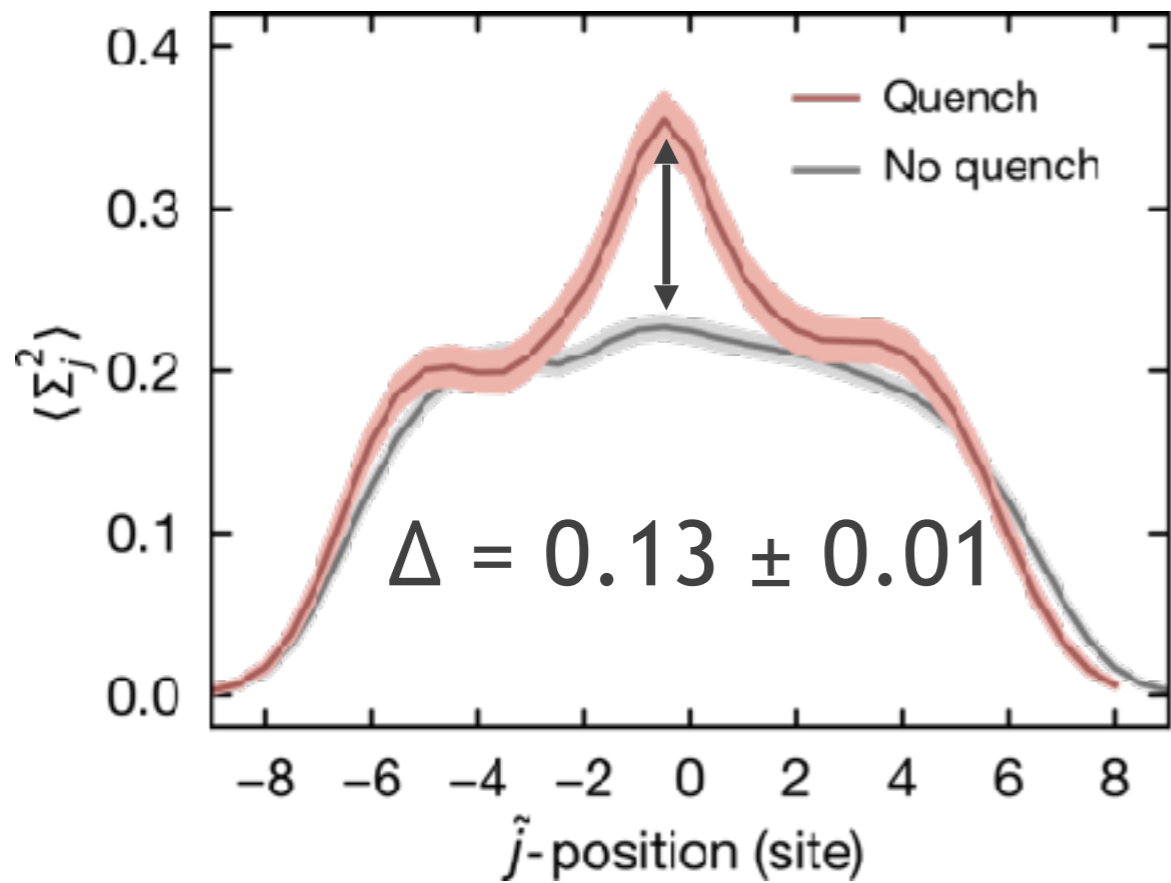
Fractionalization at non-zero temperature

$$\hat{\Sigma}_j^2 = \left(\sum_i \hat{S}_i^z f_j^\sigma(i) \right)^2$$



no spinon
created locally!

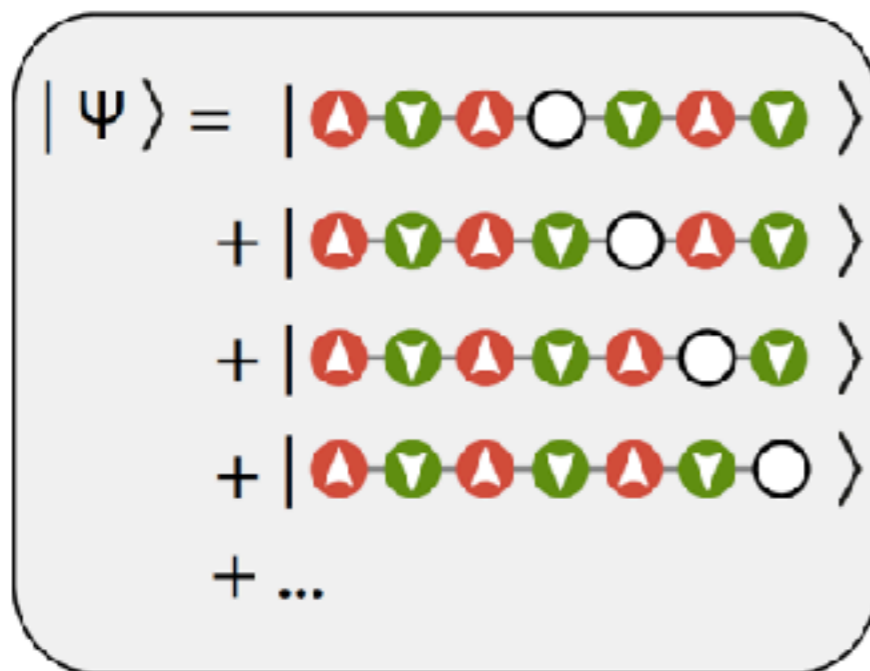
T
1 J



How is fractionalisation connected to hidden order in the ground state?

Minimize Energy \rightarrow Two Conditions

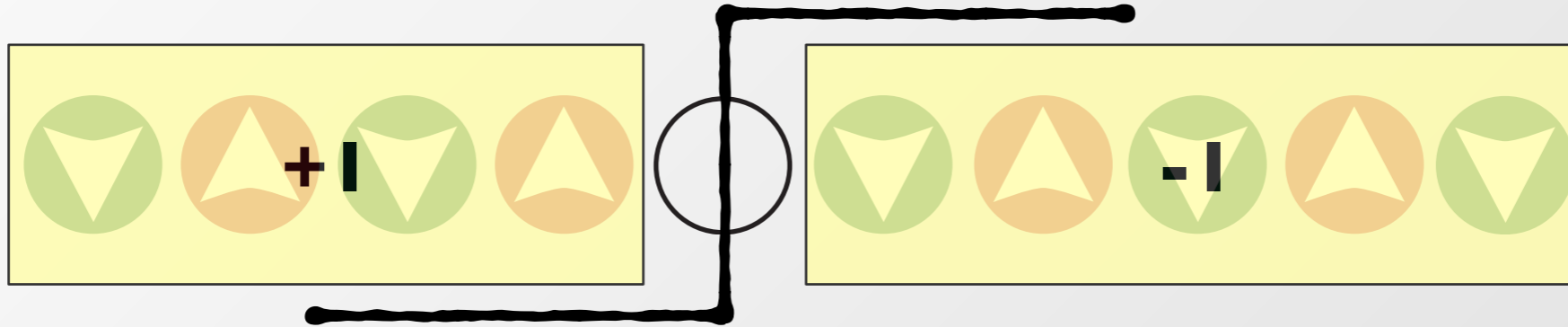
- ▶ Holes want to delocalise
- ▶ Spins want to align antiferromagnetically



Ground State



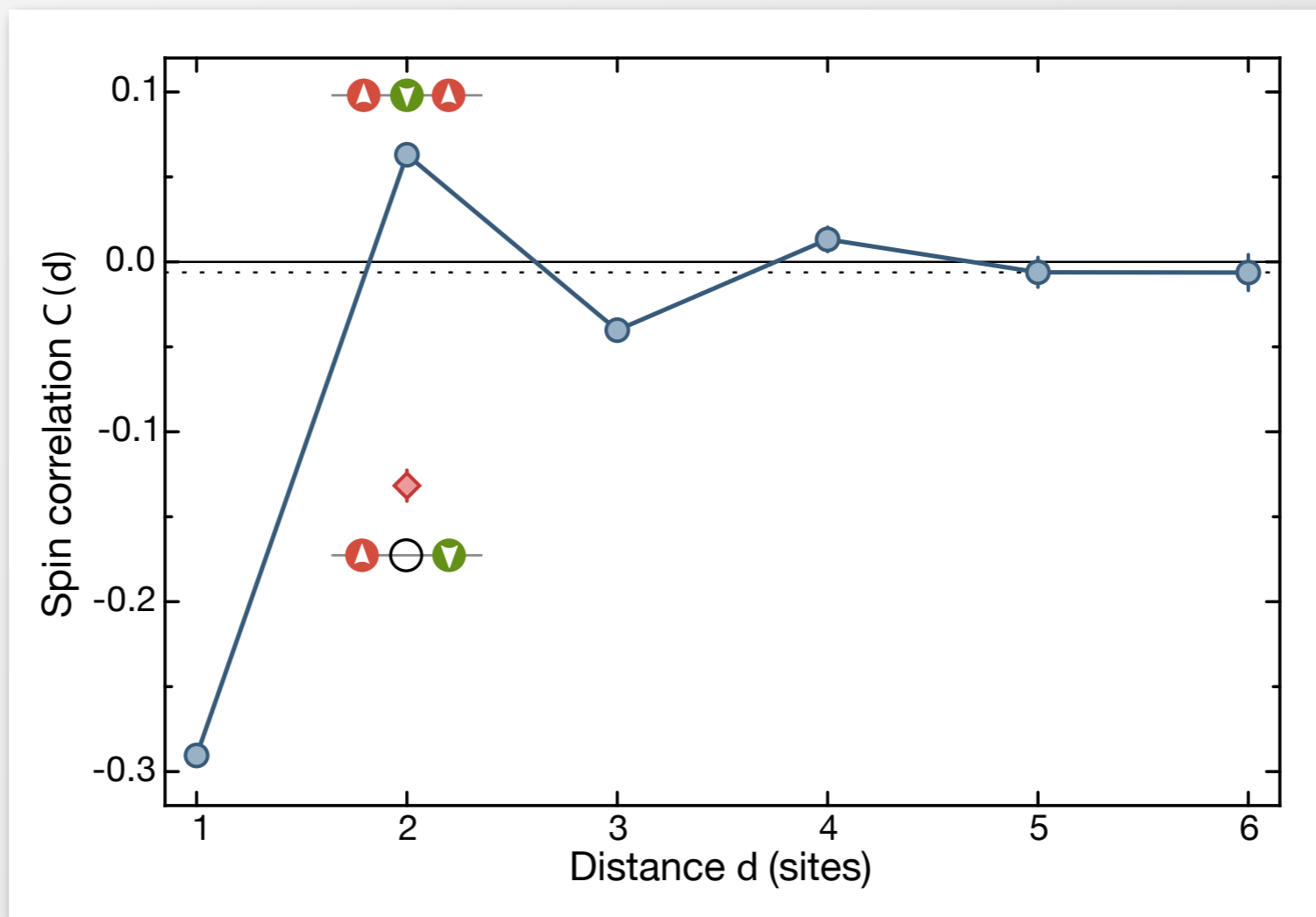
Excited State

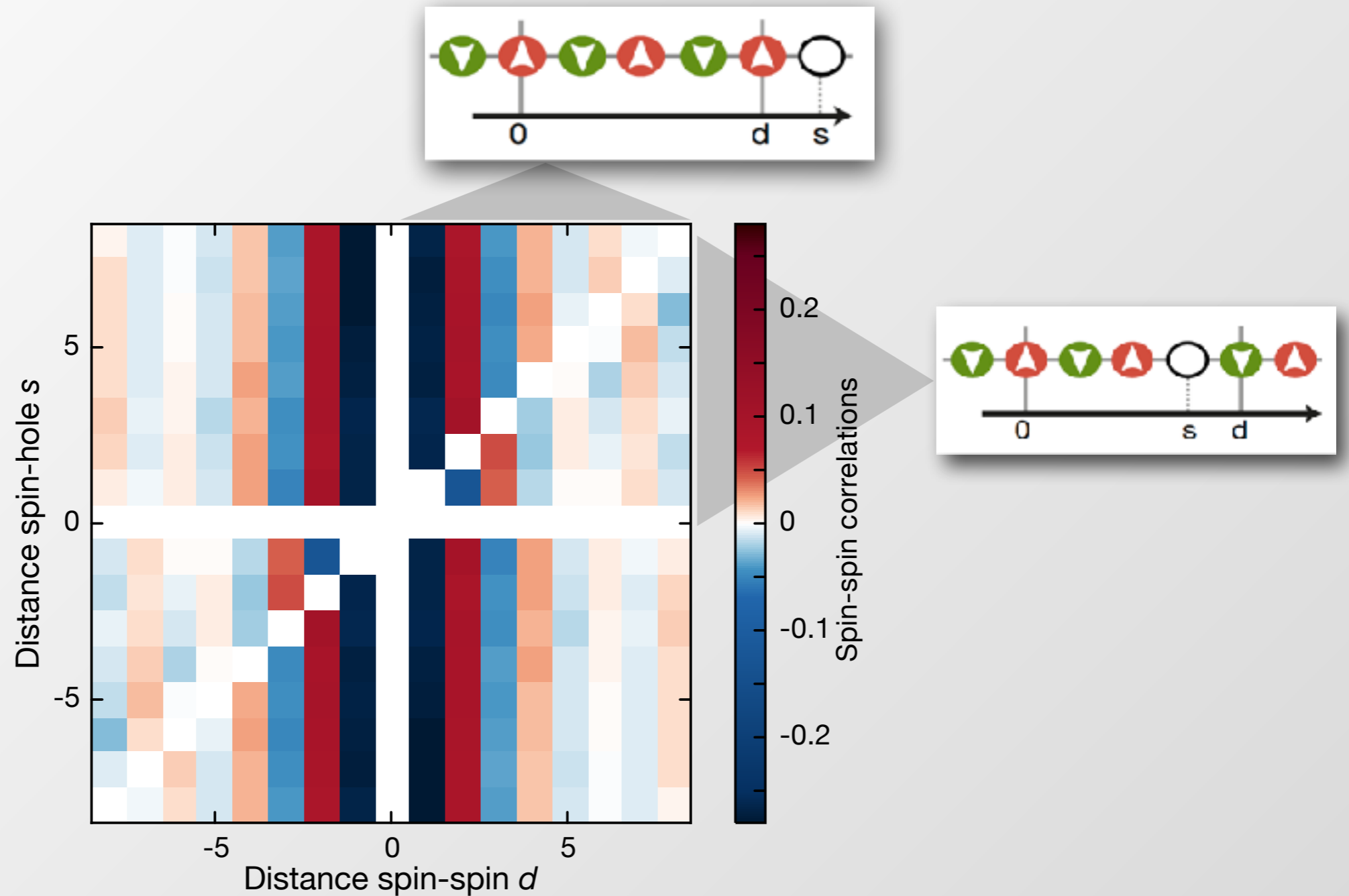


**Hole introduce domain wall
“parity” kinks in AFM background!**



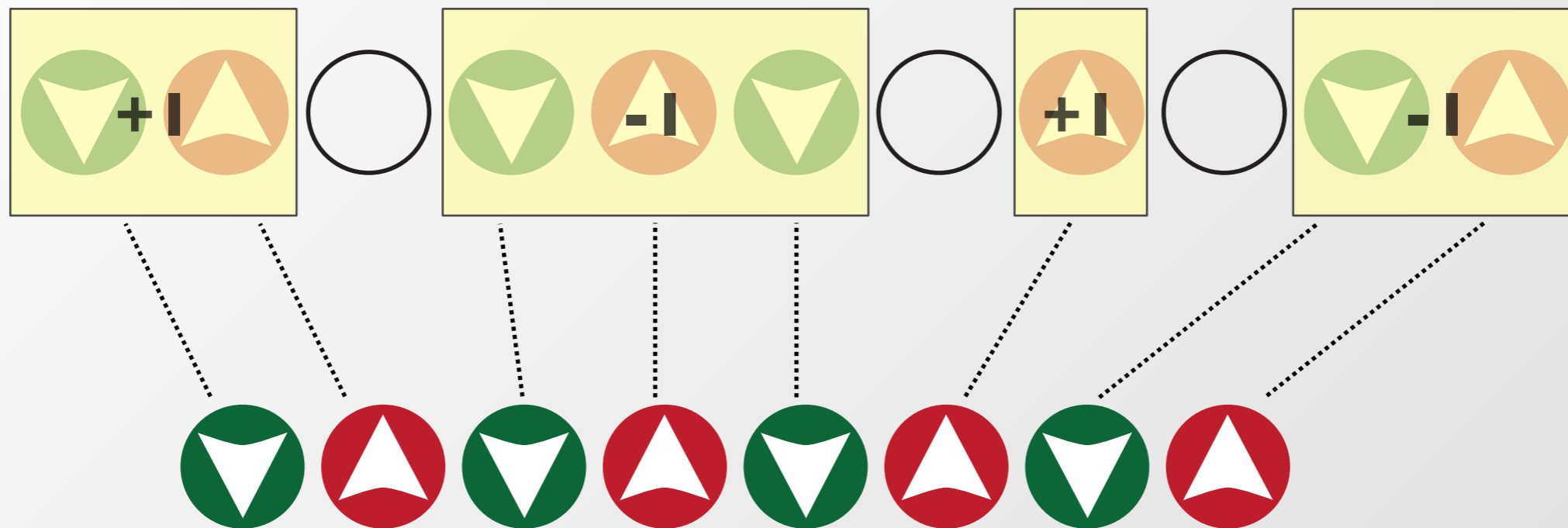
$$C_{s,h}(d=2) \langle \hat{S}_z(i) \hat{S}_z(i+2) | \hat{h}_{i+1} \rangle$$





$$C_{s,h}(s, d) = \langle \hat{S}_z(i) \hat{h}_{i+s} \hat{S}_z(i + d) \rangle$$





Heisenberg AFM in “Squeezed Space”

$$\Psi(x_1, \dots, x_N) = \Psi_{SF}(x_1, \dots, x_N) \Psi_{\text{Heis}}(y_1, \dots, y_M)$$

F. Woynarovich J. Phys. C (1982)
M. Ogata & H. Shiba Phys. Rev. B (1990)

String Correlator

$$O_{top} = \langle \hat{S}^z(0) (-1)^{\sum_{j=1}^{d-1} 1 - \hat{n}_j} \hat{S}_z(d) \rangle$$

Typical Order Parameter in Landau Paradigm of Phase Transition

$$\lim_{|\mathbf{x}-\mathbf{y}|\rightarrow\infty} \langle \hat{A}(\mathbf{x}) \hat{A}(\mathbf{y}) \rangle = c$$

Order Parameter:

Examples:

**General classification
scheme for
all phases of matter ???**

(Magnetism, AFM,...)

(Function)

Order Parameter Characterization of Ground State Correlations

Local ordering!



E.g. in 1D gapped systems where $\langle \hat{A}(\mathbf{x}) \hat{A}(\mathbf{y}) \rangle$ decays exponentially with distance

However, they can show hidden non-local order:

$$\lim_{|\mathbf{x}-\mathbf{y}| \rightarrow \infty} \langle \hat{A}(\mathbf{x}) \left(\prod_{\mathbf{z} \in S(\mathbf{x}, \mathbf{y})} \hat{B}(\mathbf{z}) \right) \hat{A}(\mathbf{y}) \rangle = c$$

We say the order is **hidden**, because a “**global view**” of the underlying state is required. (**Topological Order**: X.-G. Wen)

Allows us to characterize state only via its ground state correlations!

M. den Nijs, K. Rommelse, Phys. Rev. B 40, 4709 (1989).

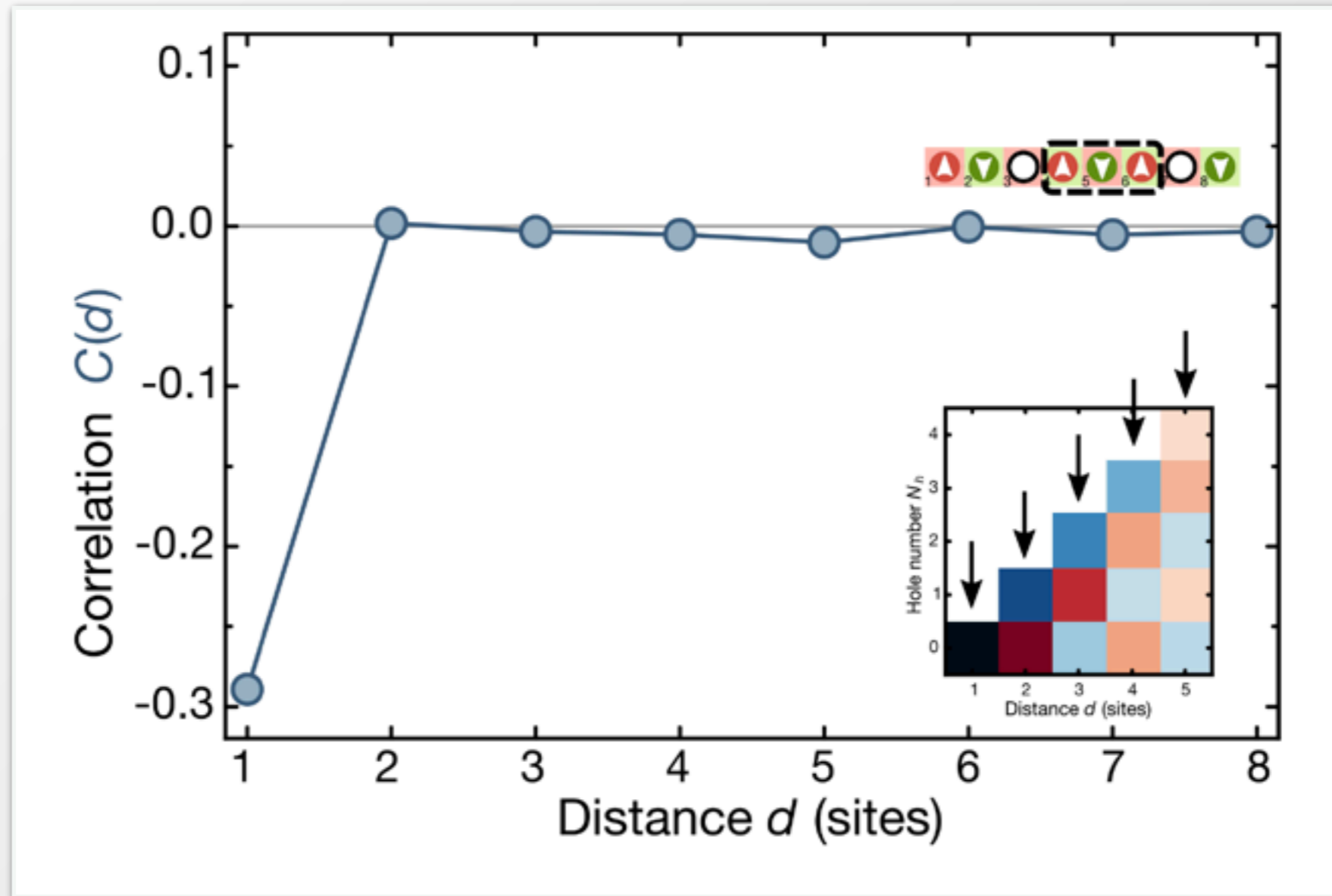
E. Kim, G. Fath, J. Solyom, D. Scalapino, Phys. Rev. B 62, 14965 (2000)

E. G. Dalla Torre, E. Berg, E. Altman, Phys. Rev. Lett. 97, 260401 (2006)

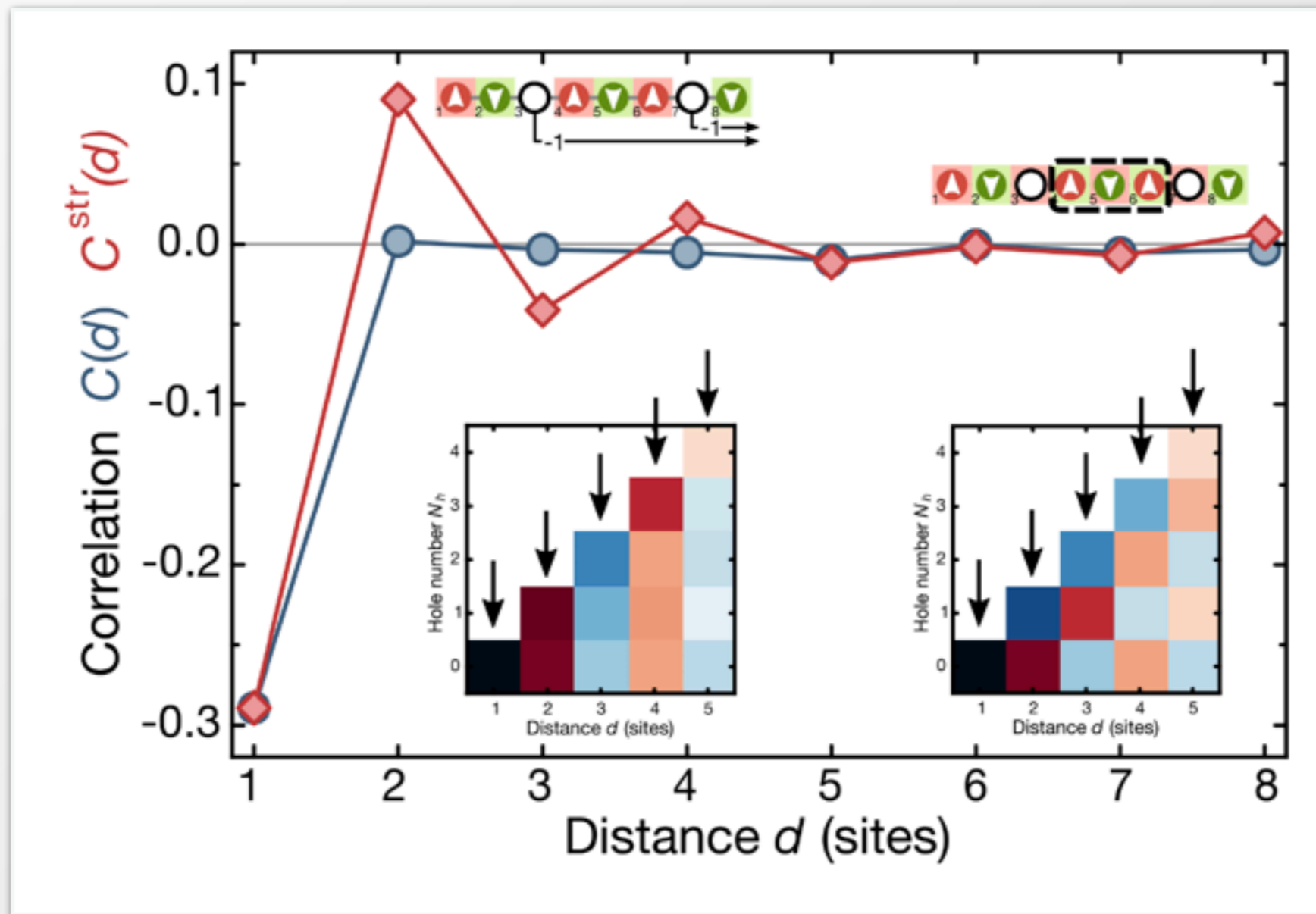
F. Anfuso, A. Rosch, Phys. Rev. B 75, 144420 (2007)

E. Berg, E. Dalla Torre, T. Giamarchi, E. Altman, Phys. Rev. B 77, 245119 (2008)





Two Point Correlator - Doped Chains



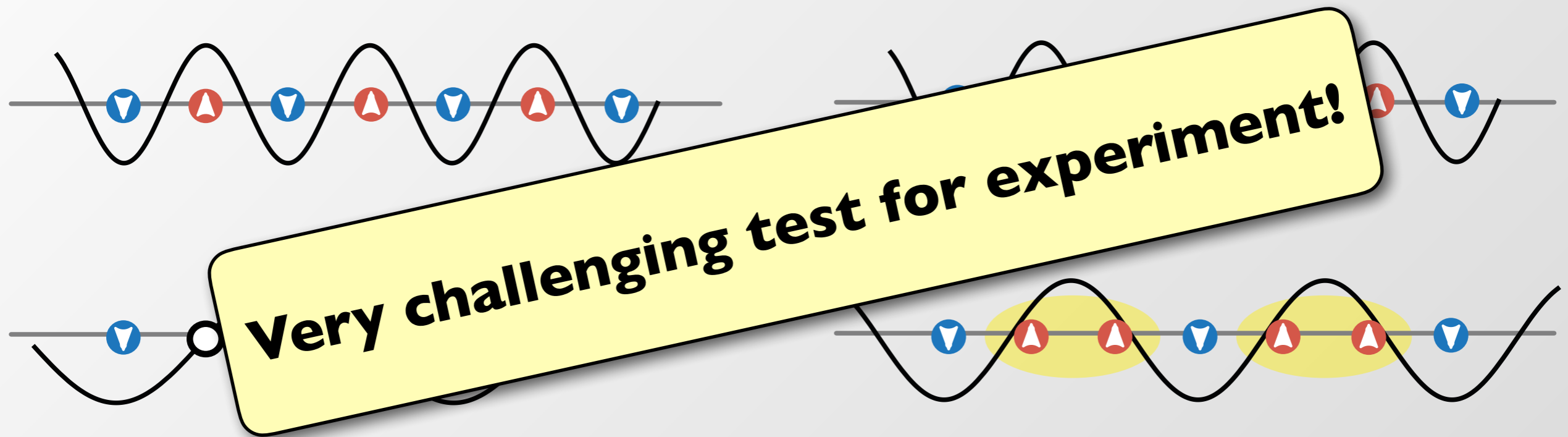
Incommensurate AFM in 1D

Density Doping

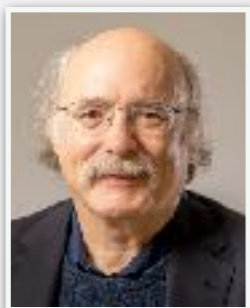
$$\langle S_0^z S_d^z \rangle \simeq A_n e^{-d/\xi_n} \cos(\pi n d)$$

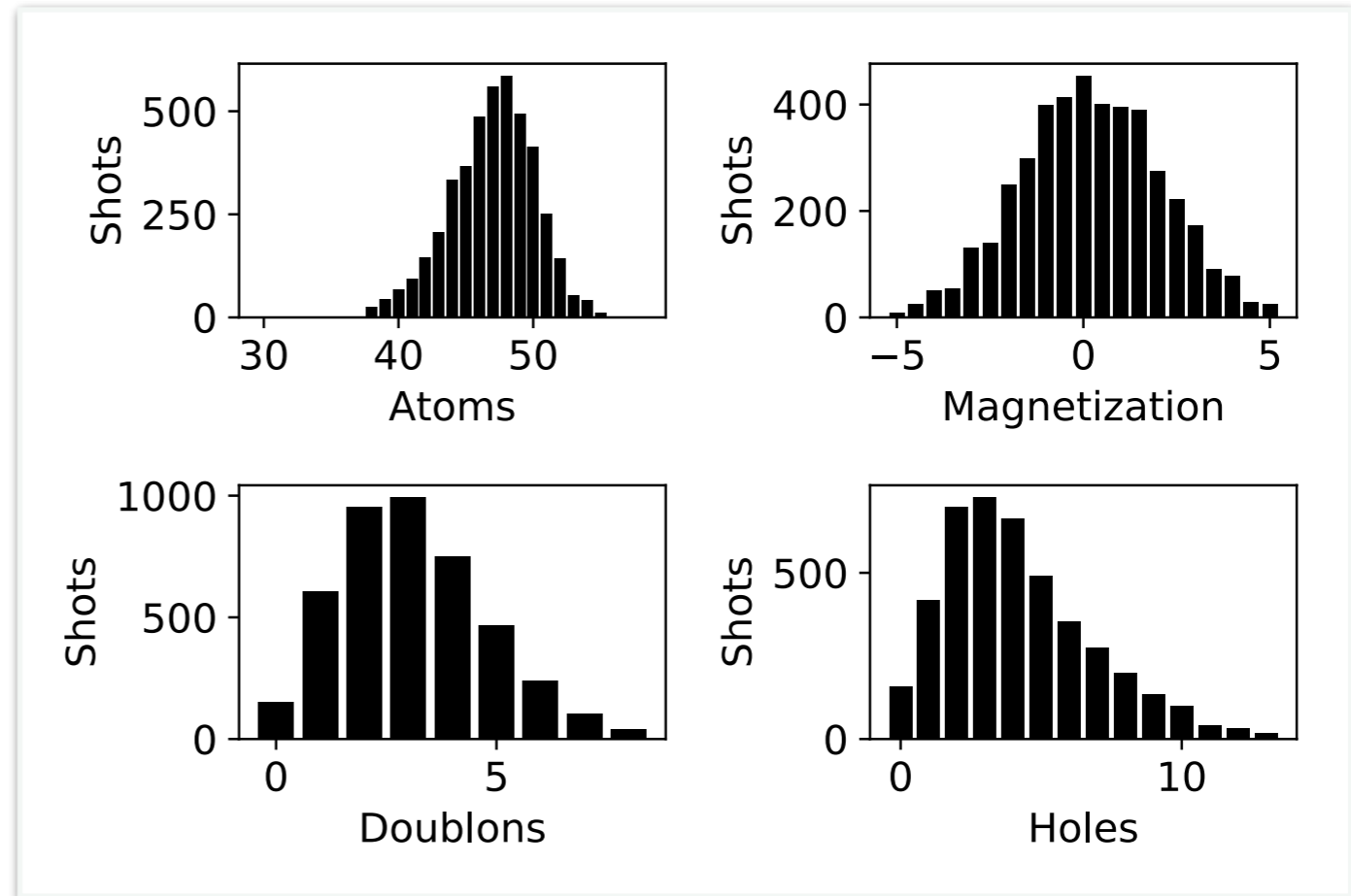
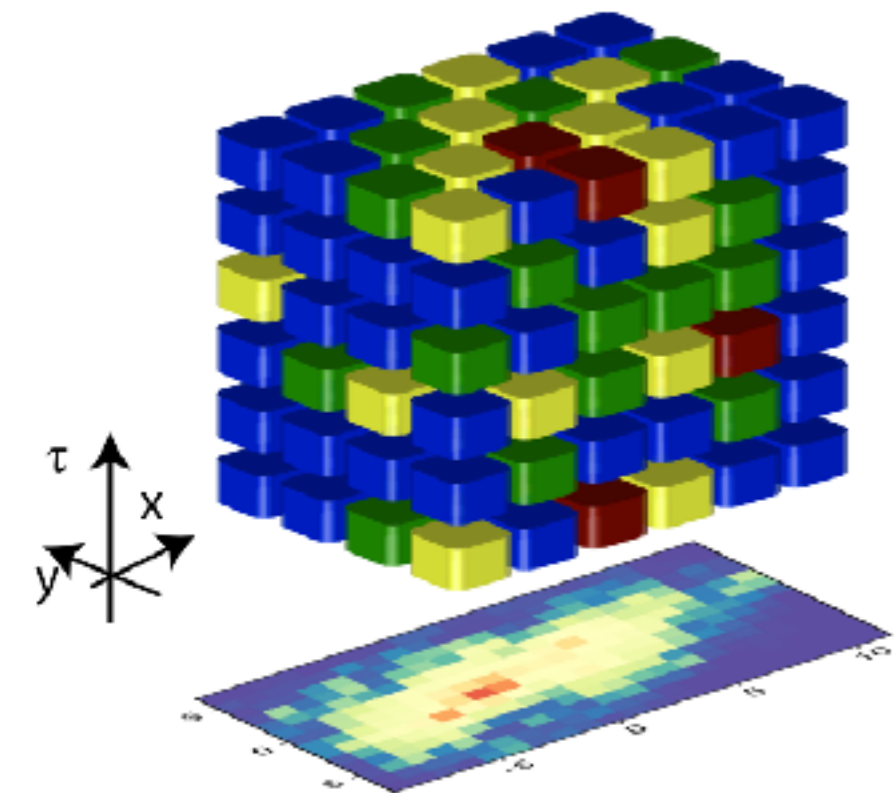
Finite Magnetization

$$\langle S_0^z S_d^z \rangle \simeq A_m e^{-d/\xi_m} \cos(\pi(1 - 2m)d)$$



Luttinger Liquid Theory

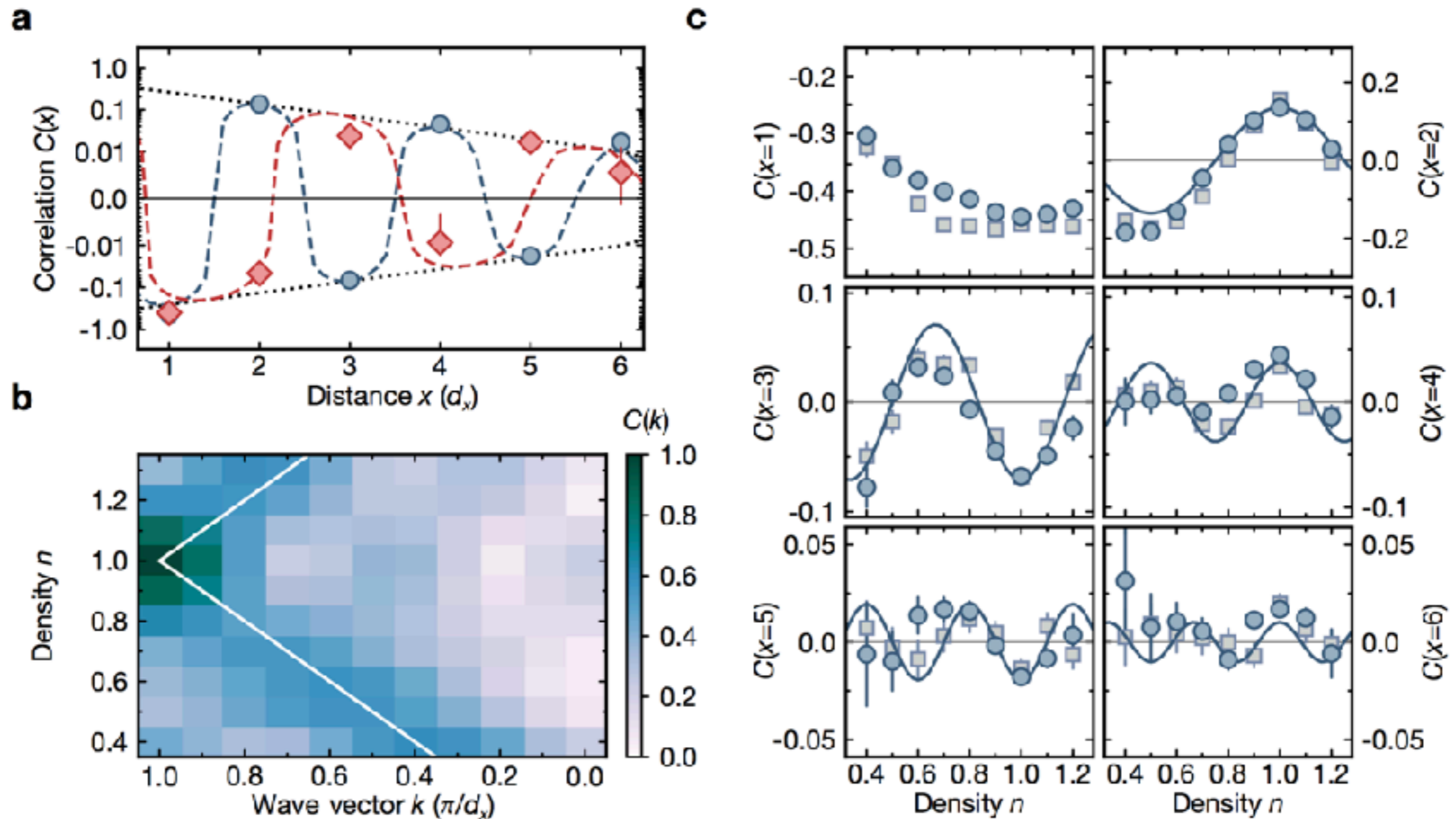




**Single Spin and
Single Atom Sensitivity !**

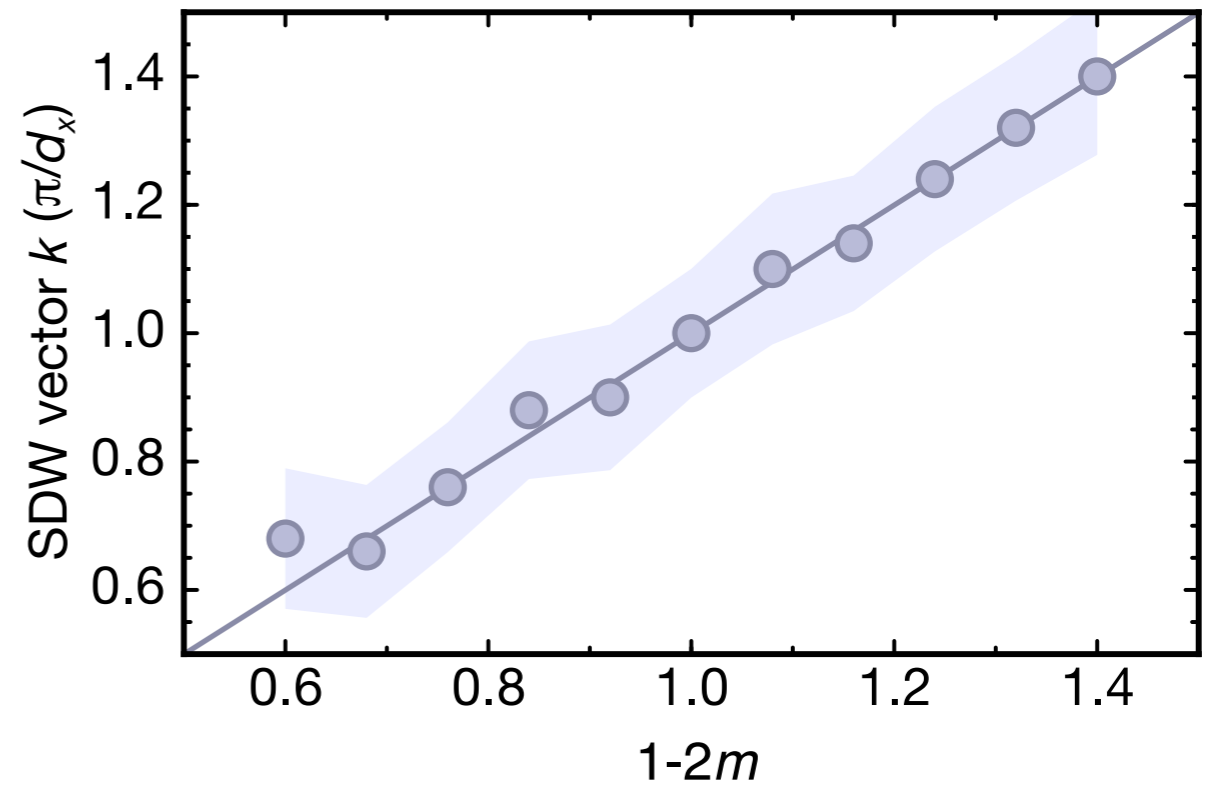
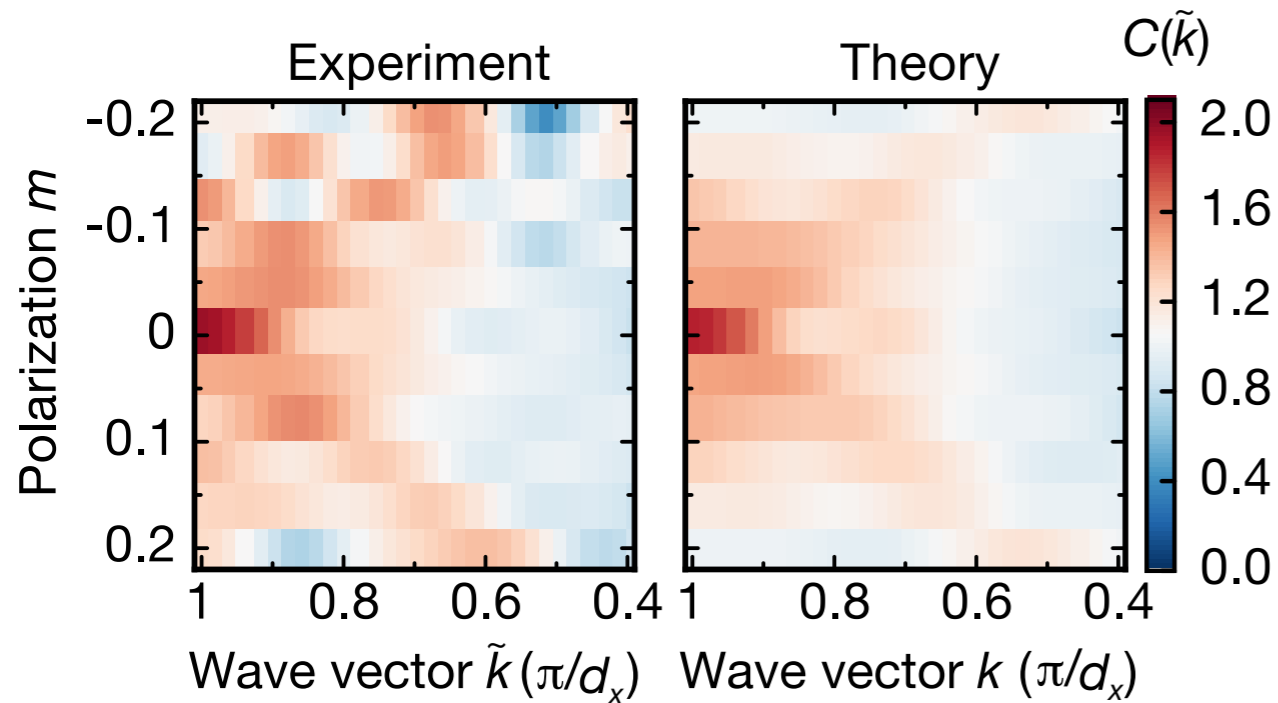
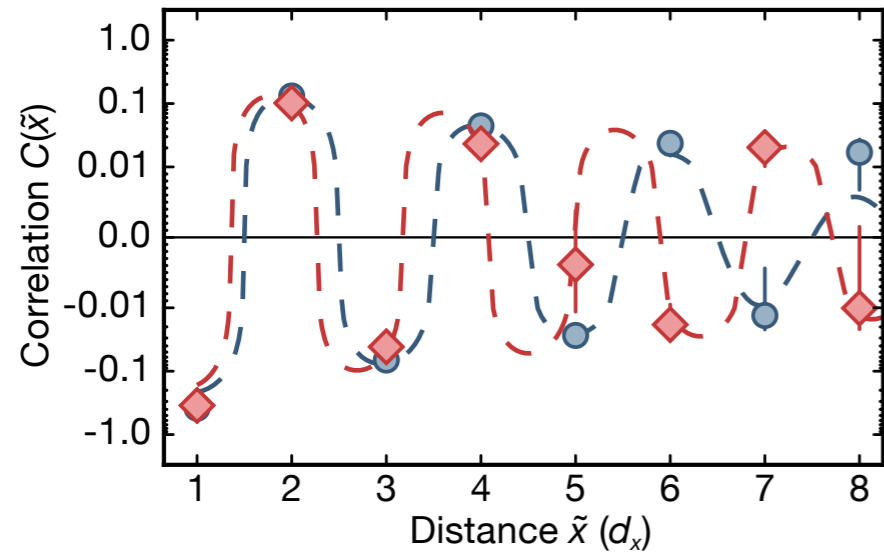
*Select shots with defined
magnetisation and/or density !*





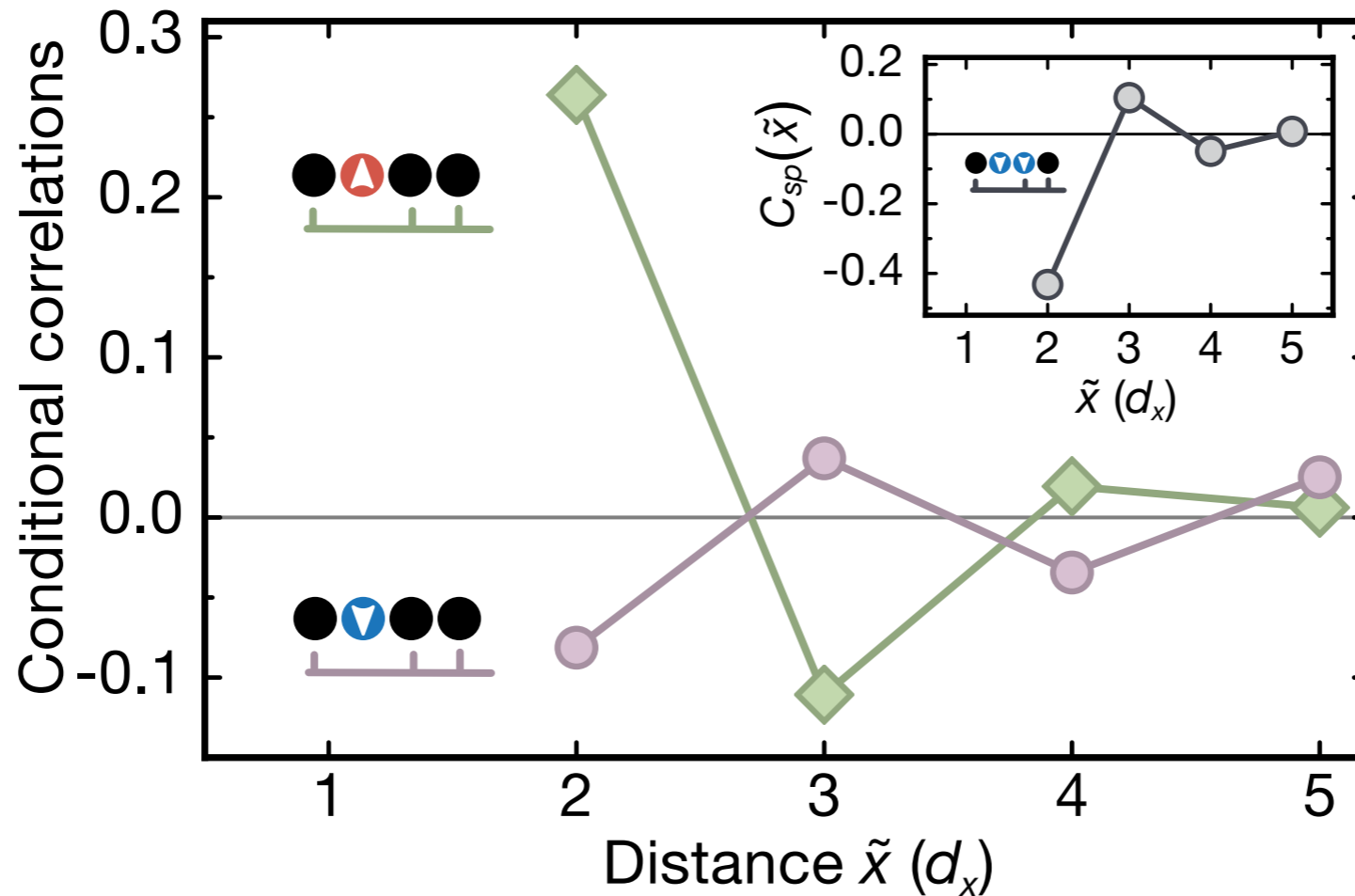
$$\langle S_0^z S_d^z \rangle \simeq A_n e^{-d/\xi_n} \cos(\pi n d)$$



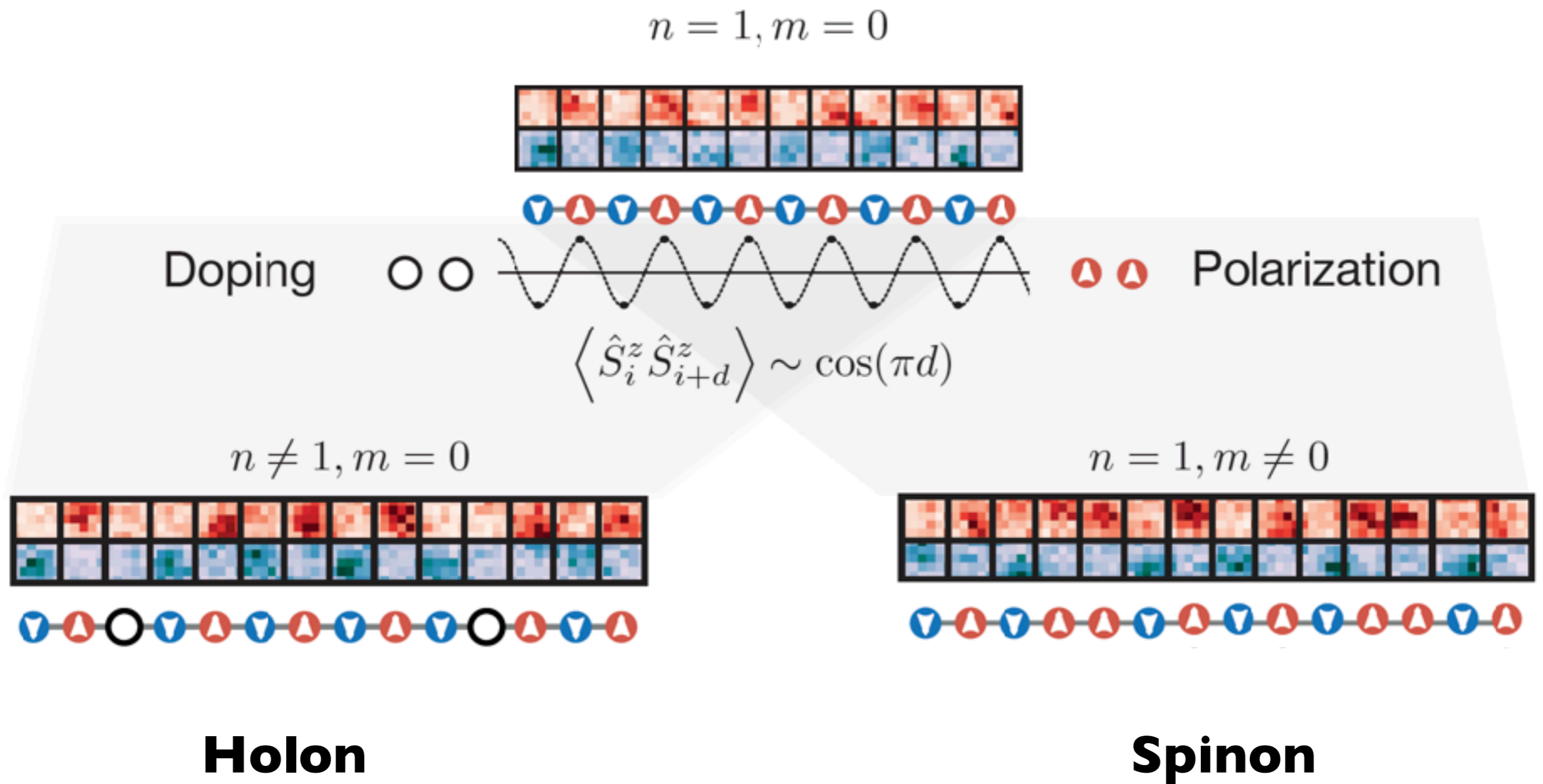


$$\langle S_0^z S_d^z \rangle \simeq A_m e^{-d/\xi_m} \cos(\pi(1 - 2m)d)$$



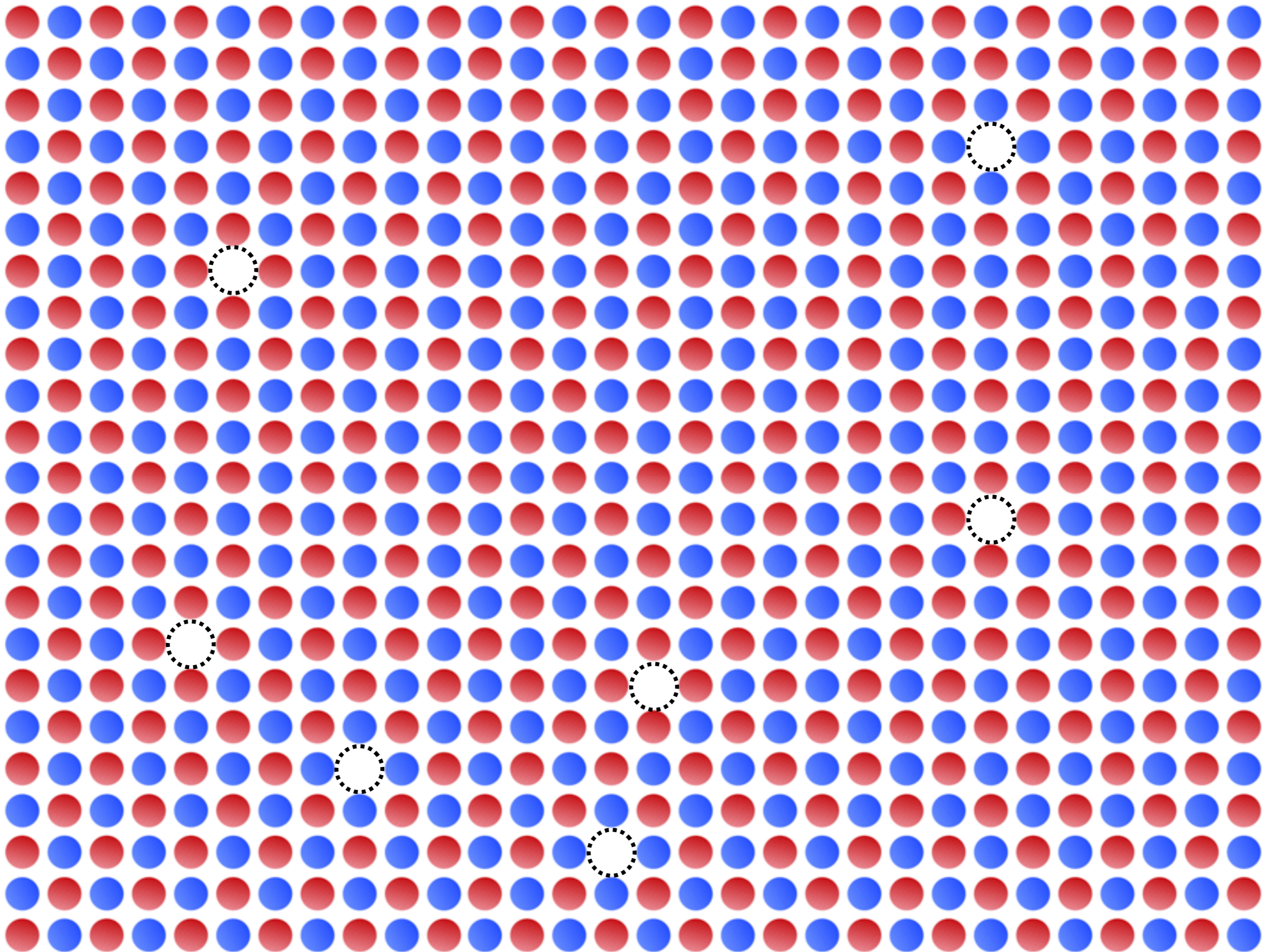


**#2: Spinon = Quantum Domain Walls
in Squeezed Space**

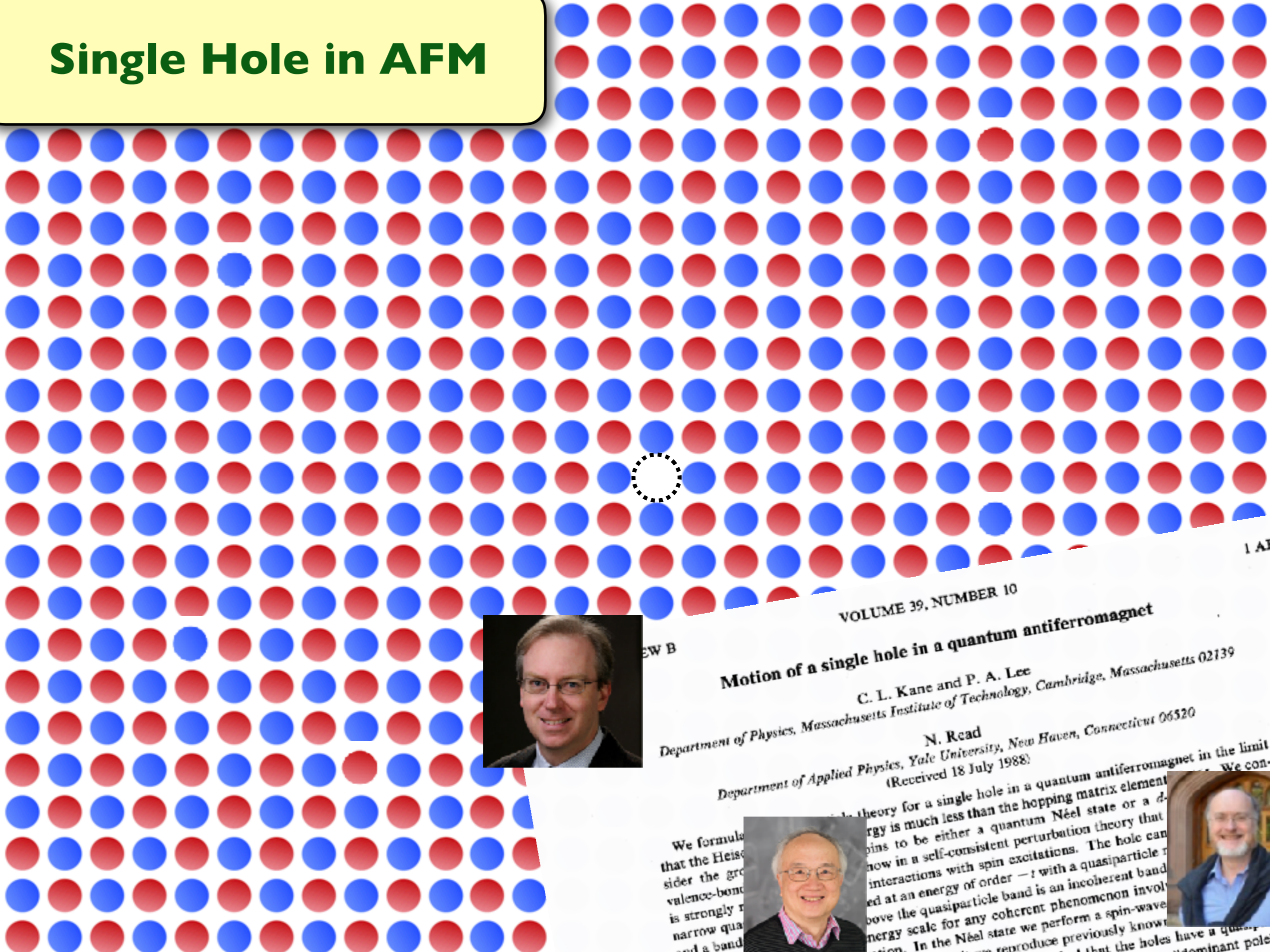


Quantum Domain Walls

Charge Impurities in 2D



Single Hole in AFM



EW B

Motion of a single hole in a quantum antiferromagnet

C. L. Kane and P. A. Lee

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

N. Read

Department of Applied Physics, Yale University, New Haven, Connecticut 06520
(Received 18 July 1988)

We formulate a theory for a single hole in a quantum antiferromagnet in the limit that the Heisenberg energy is much less than the hopping matrix element. We consider the ground state to be either a quantum Néel state or a dimerized state. Our results show that the hole can move in a self-consistent perturbation theory that takes into account interactions with spin excitations. The hole can move above the quasiparticle band at an energy of order δ with a quasiparticle energy scale for any coherent phenomenon involving the Néel state we perform a spin-wave expansion. In the Ising limit we reproduce previously known results. In the Heisenberg limit we employ a "dominant pole" approximation. We conclude that the holes have a quasiparticle character in the Heisenberg limit. The relevance of our results to the motion of a hole in a quantum antiferromagnet is discussed.



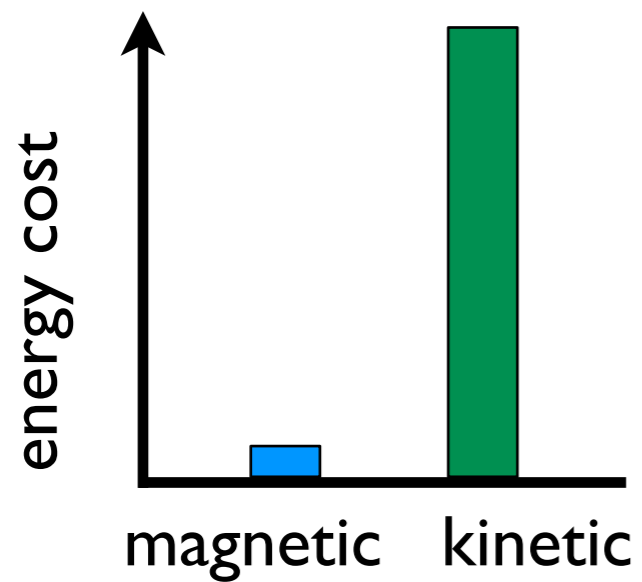
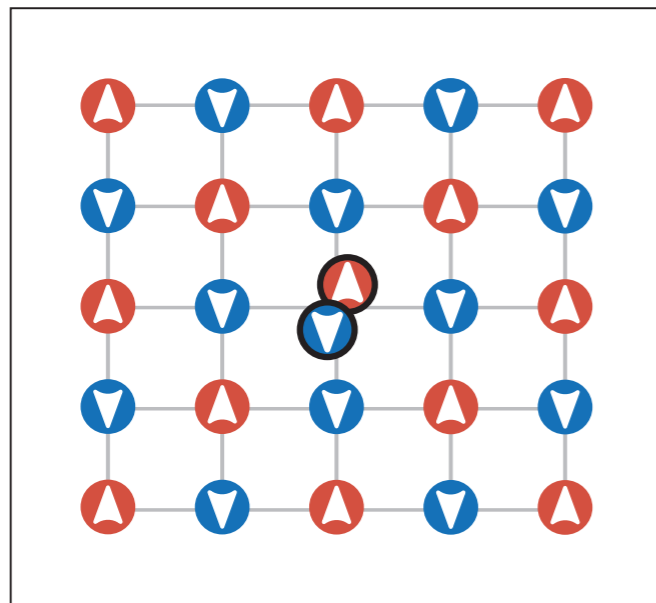
1 APRIL 1989

VOLUME 39, NUMBER 10

Competing Energy Costs: Kinetic vs Magnetic

$$t \gg J$$

0



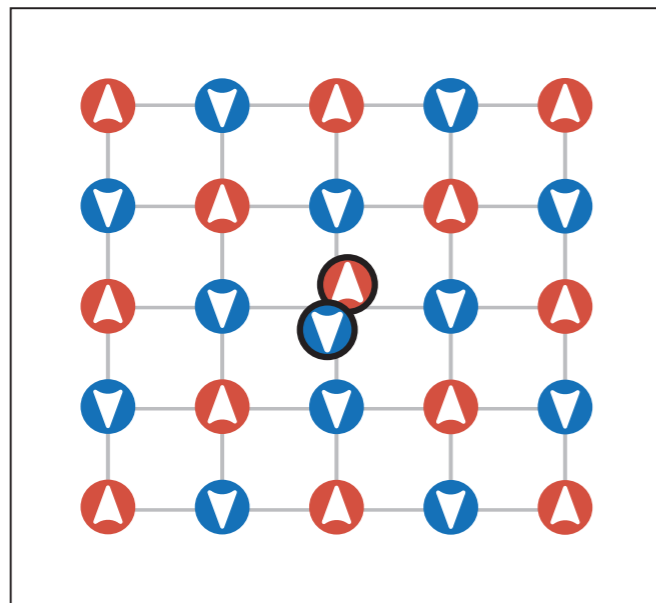
Competing Energy Costs: Kinetic vs Magnetic

$$t \gg J$$

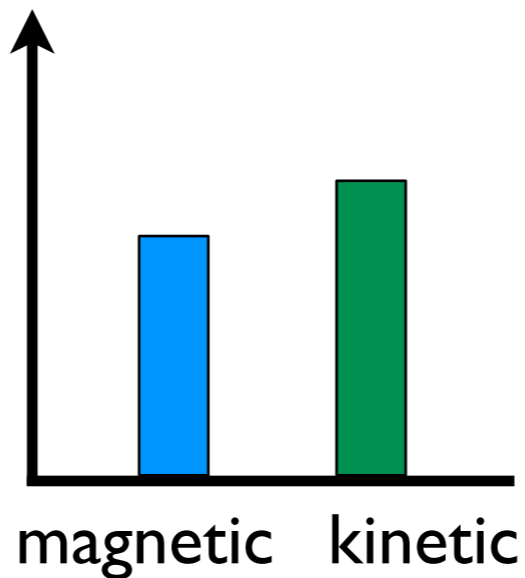
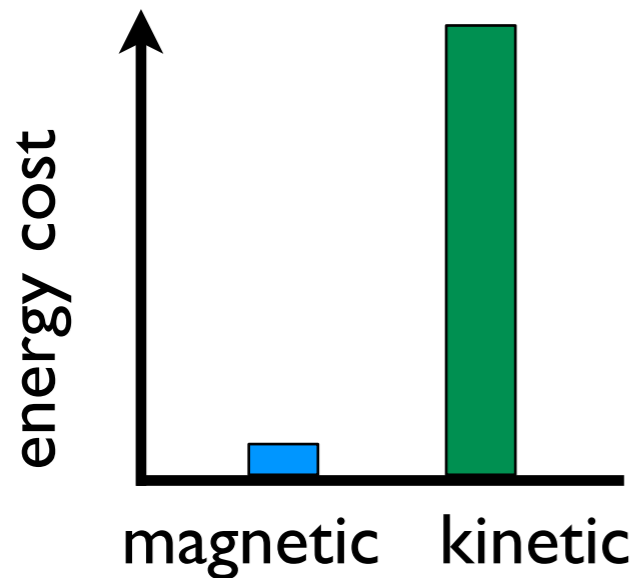
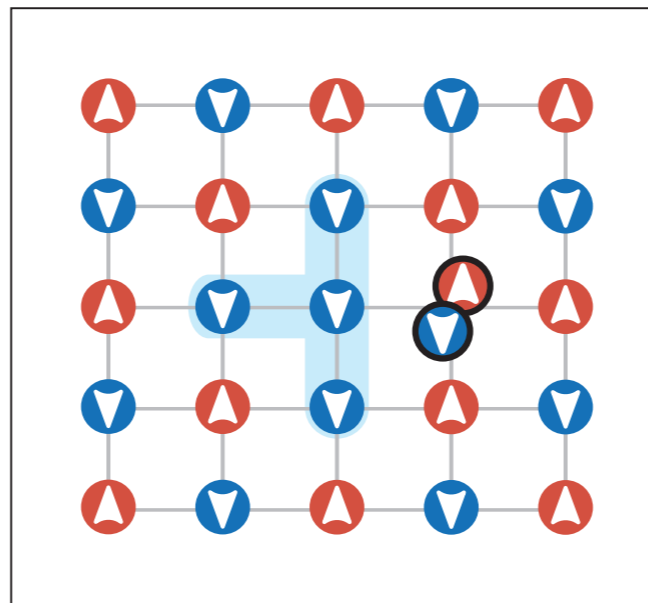
Hopping events



0



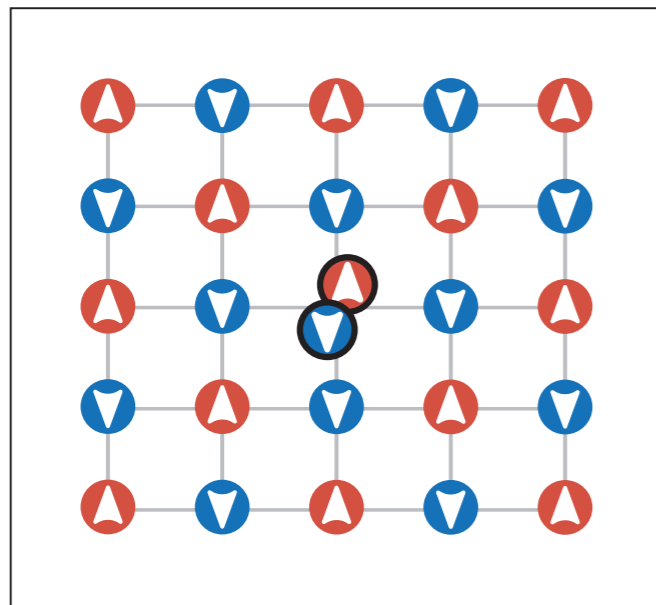
1



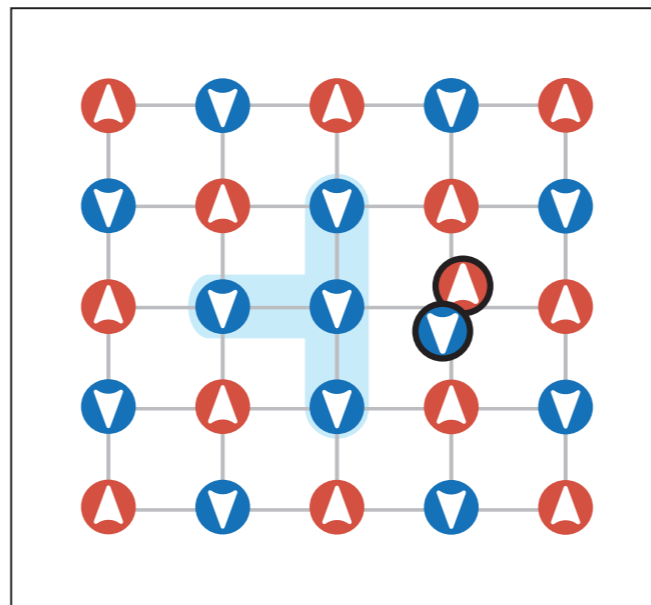
Competing Energy Costs: Kinetic vs Magnetic

$t \gg J$ Hopping events →

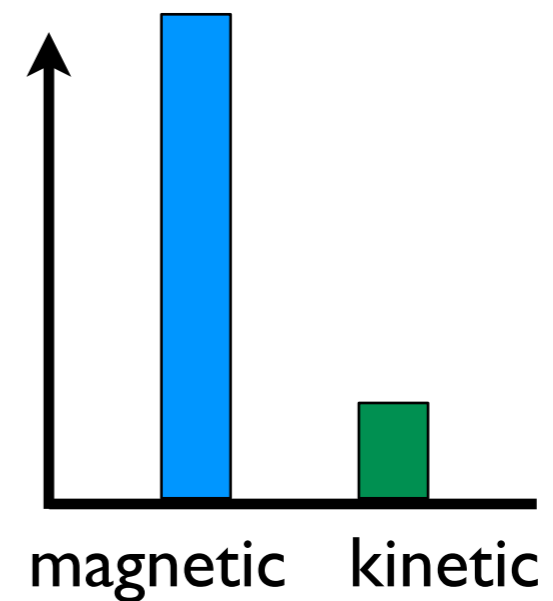
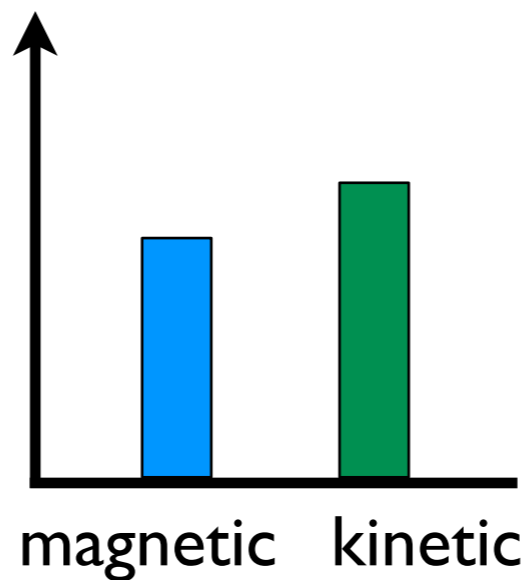
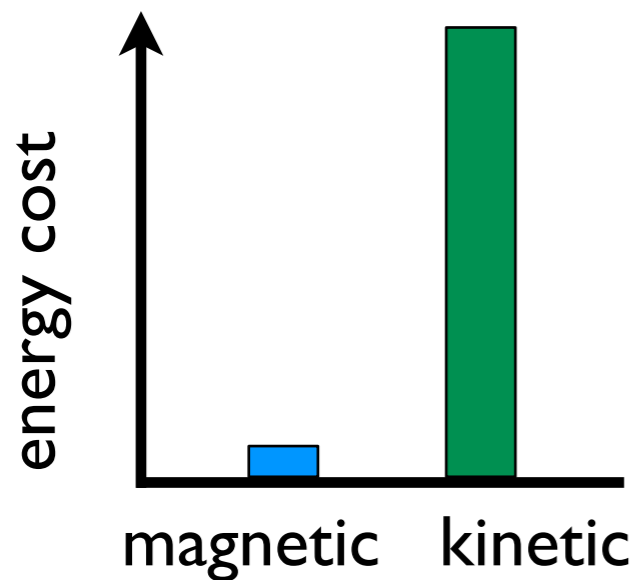
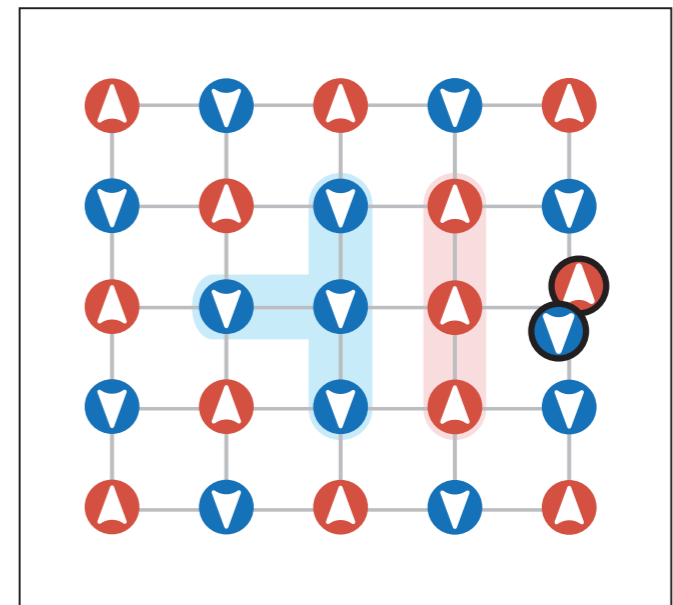
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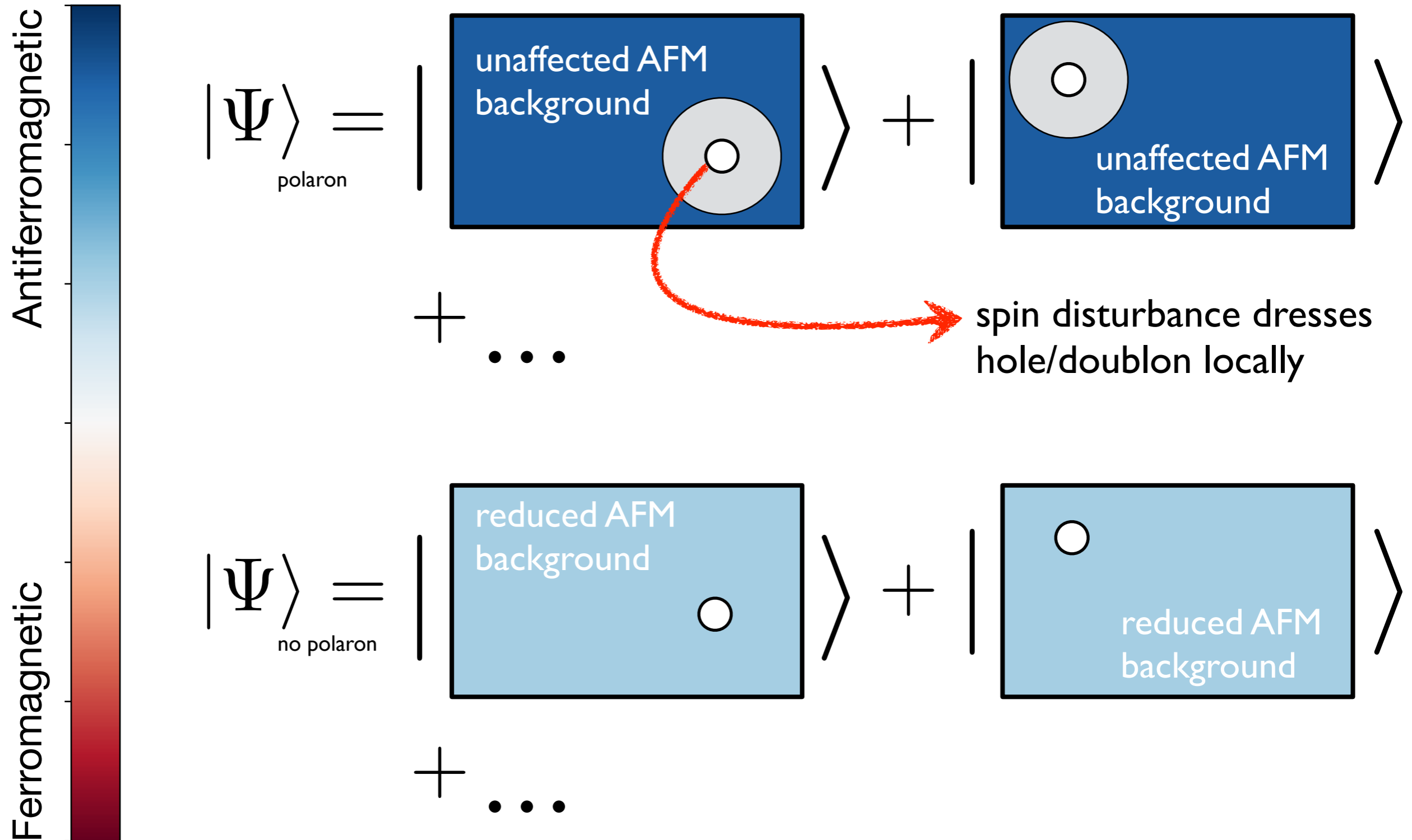
1



2

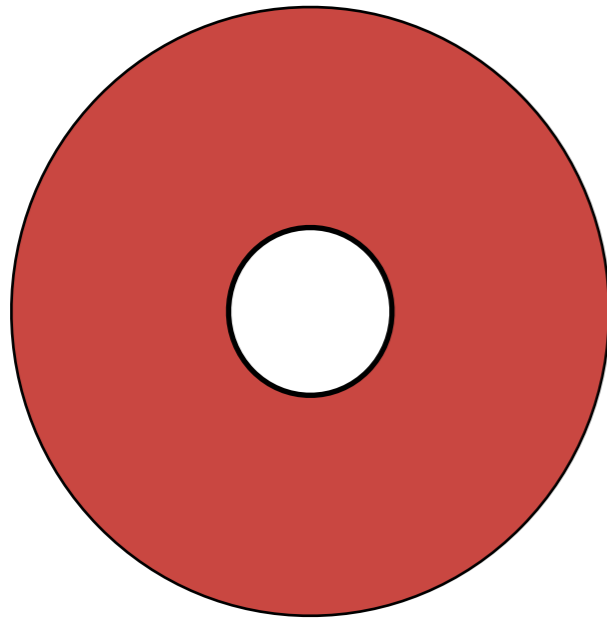


Ground State?



Doping reduces AFM spin order:
[Mazurenko *et al.*, Nature 545 2017]

Polarons in the FHM



▶ **Semiclassical considerations**

$$R \simeq \left(\frac{t}{J} \right)^{\frac{1}{4}} \quad \text{Auerbach, Springer 1994}$$

▶ **Nagaoka ferromagnetism**

Large t/J (large U): dressing becomes ferromagnetic

Nagaoka, Phys. Rev. 147 1966

▶ **Quasiparticle** with bandwidth $W=2J$:

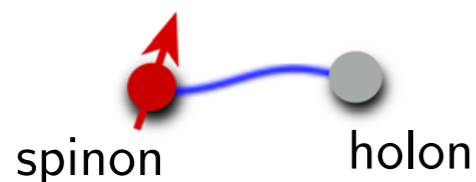
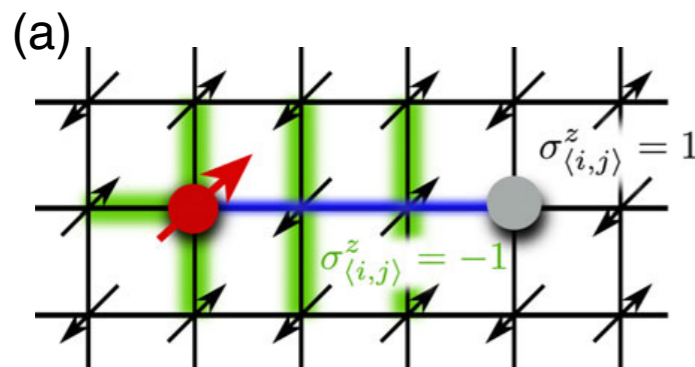
Chernyshev and Wood, arXiv:cond-mat/0208541, 2002

▶ **Attraction between polarons**

→ superconductivity, stripes?

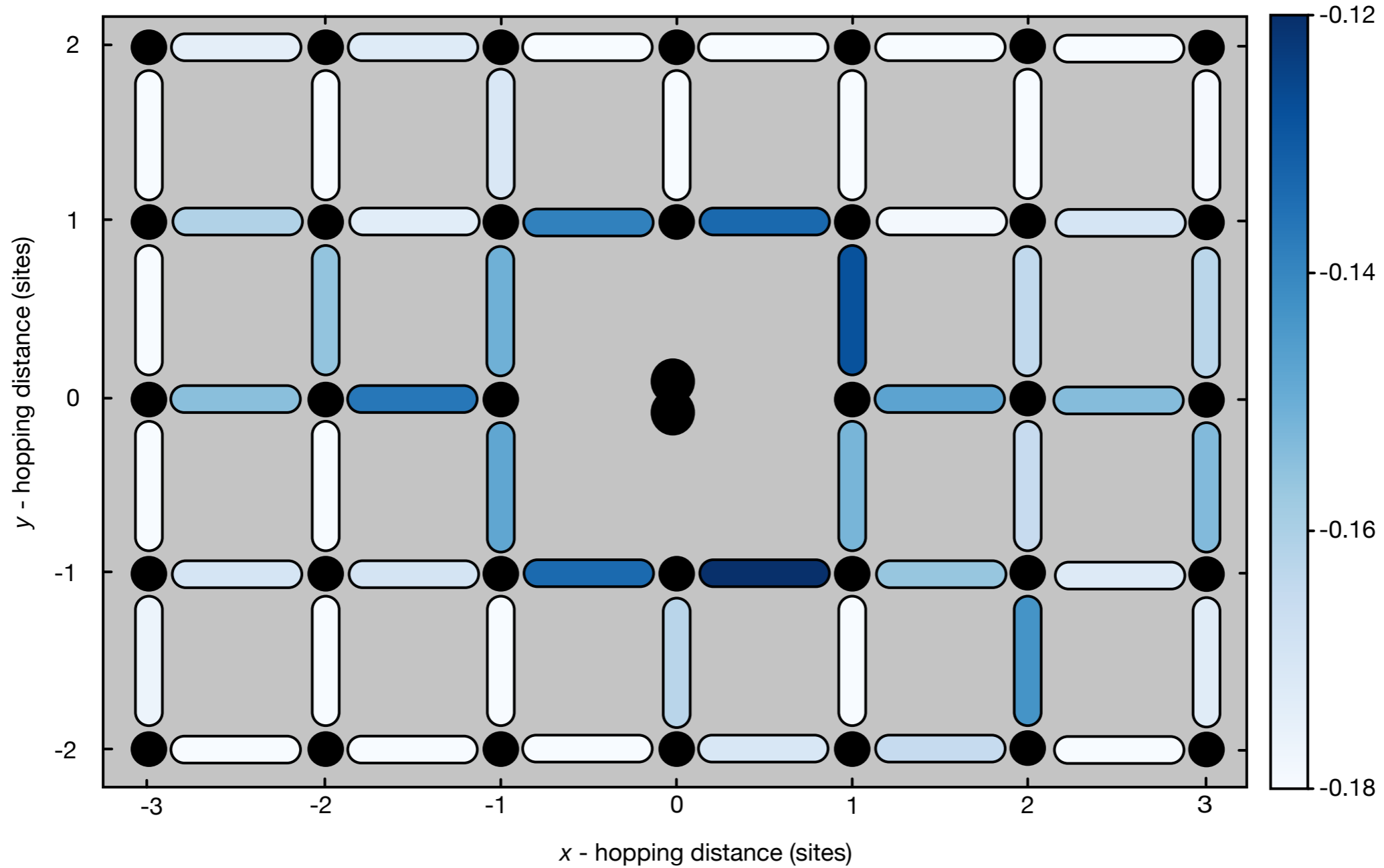
▶ **String picture:** spinon-holon binding string length

$$L \simeq \left(\frac{t}{J} \right)^{\frac{1}{3}}$$



Grusdt *et al.*, PRX 8 2018

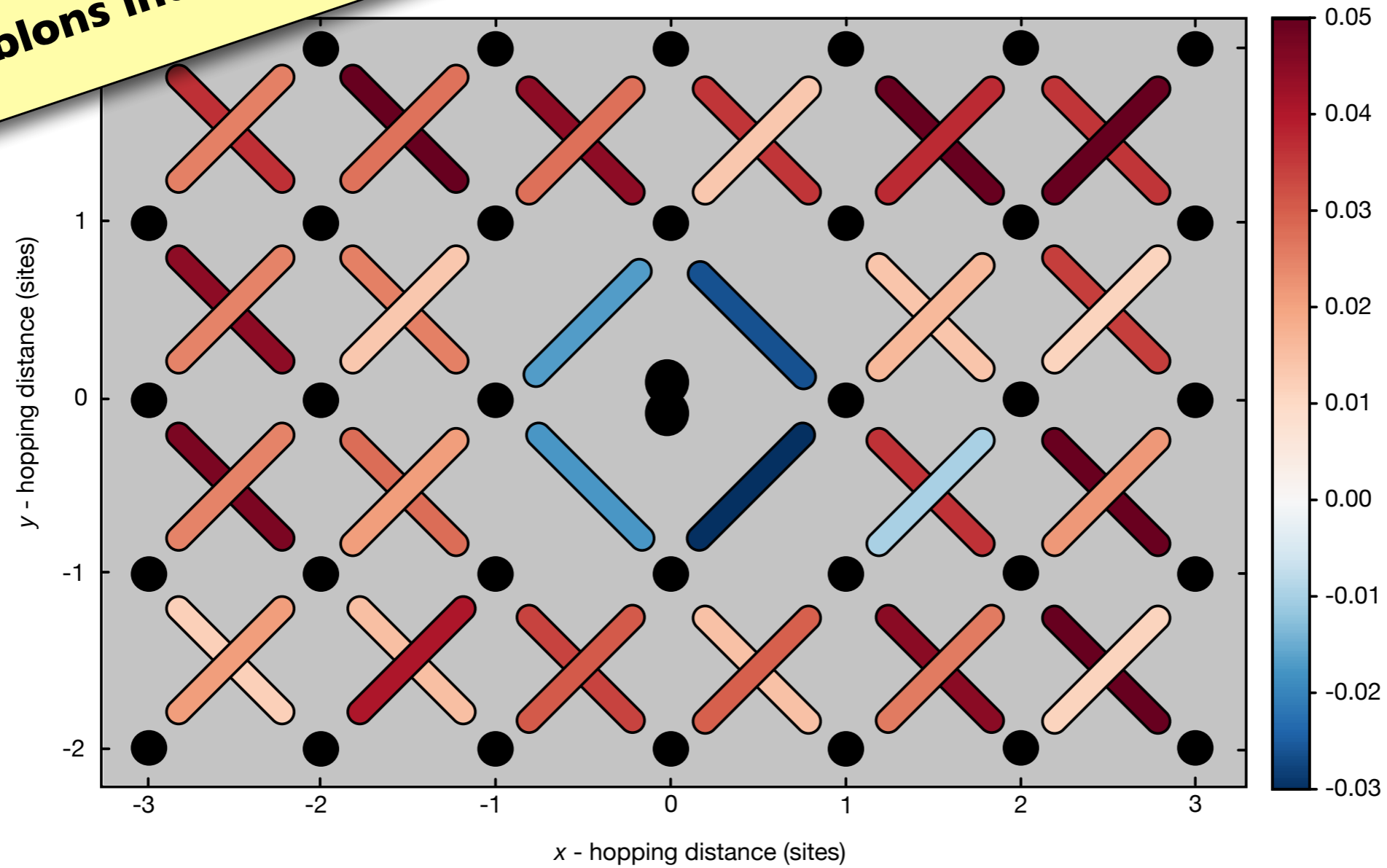
C(1) Local Spin Correlations Around Doublon



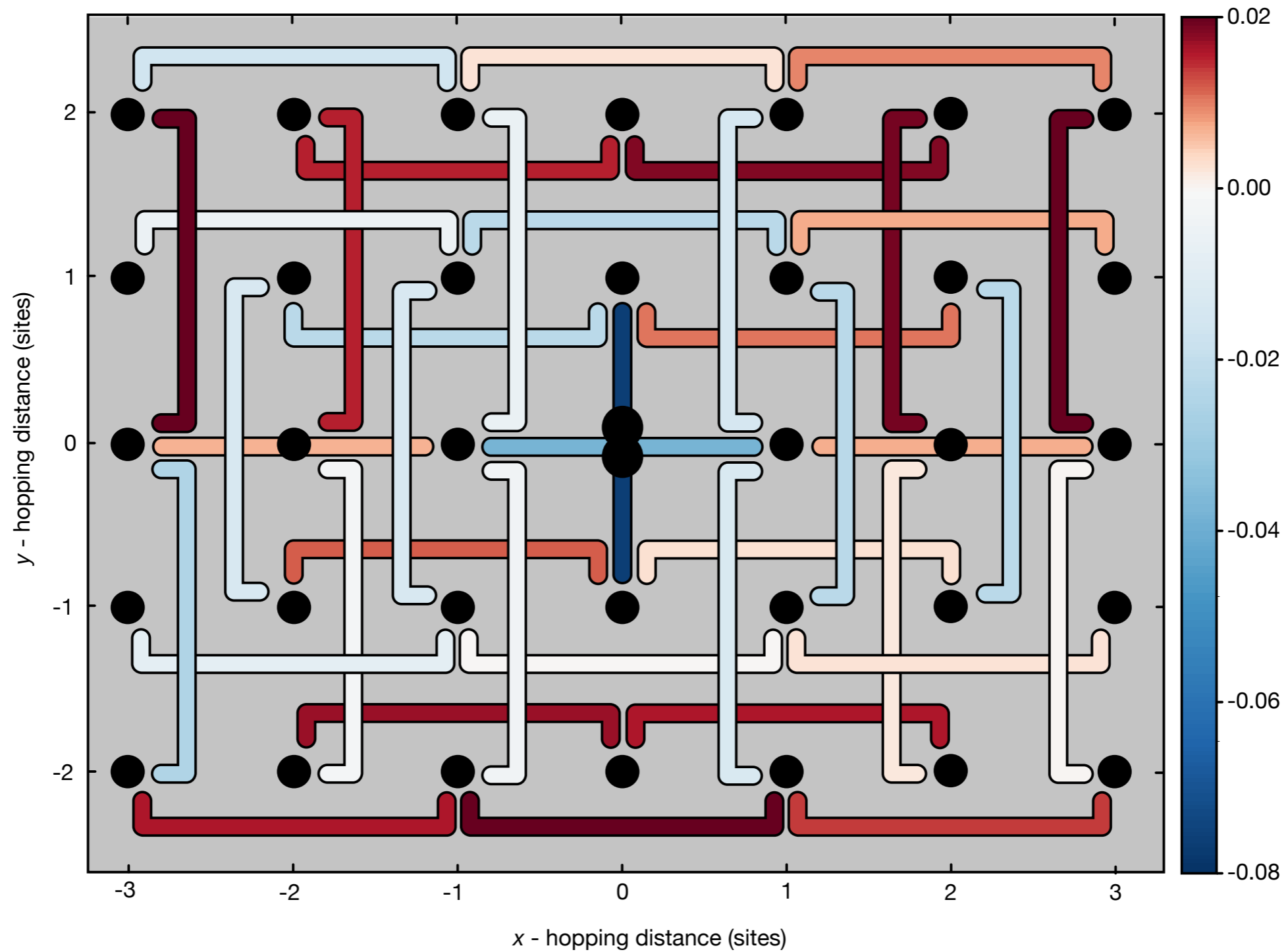
Conditioned C(1) Correlator

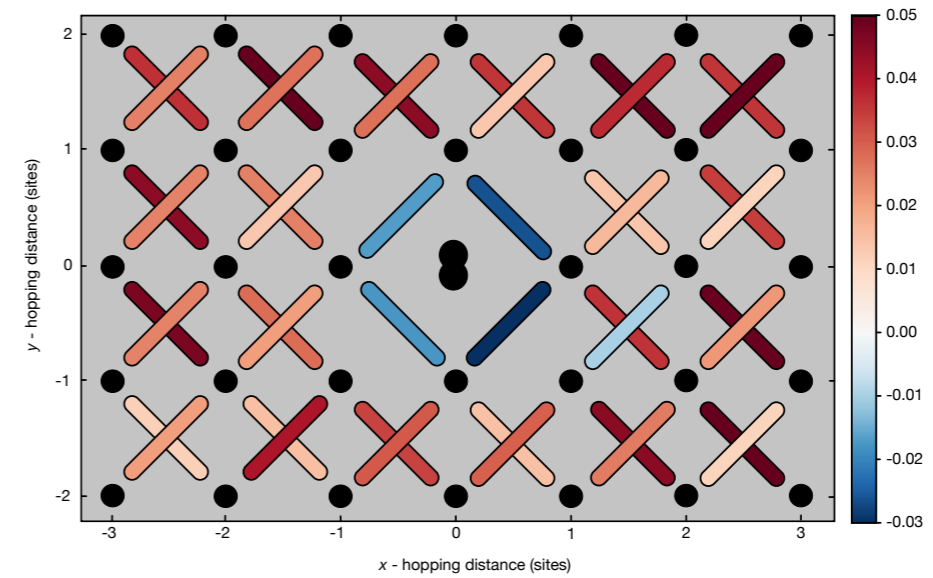
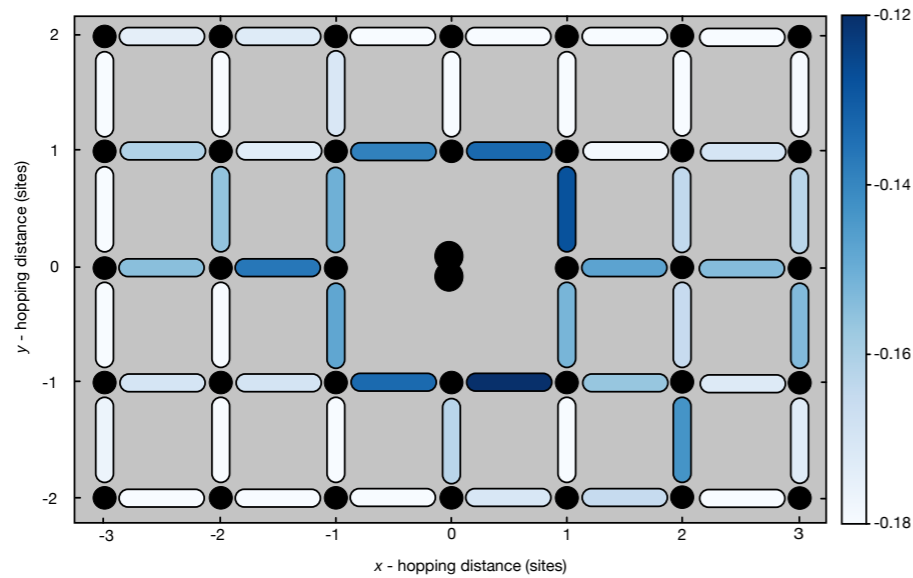
C(1,1) Local Spin Correlations around Doublon

Doublons induce sign flip !

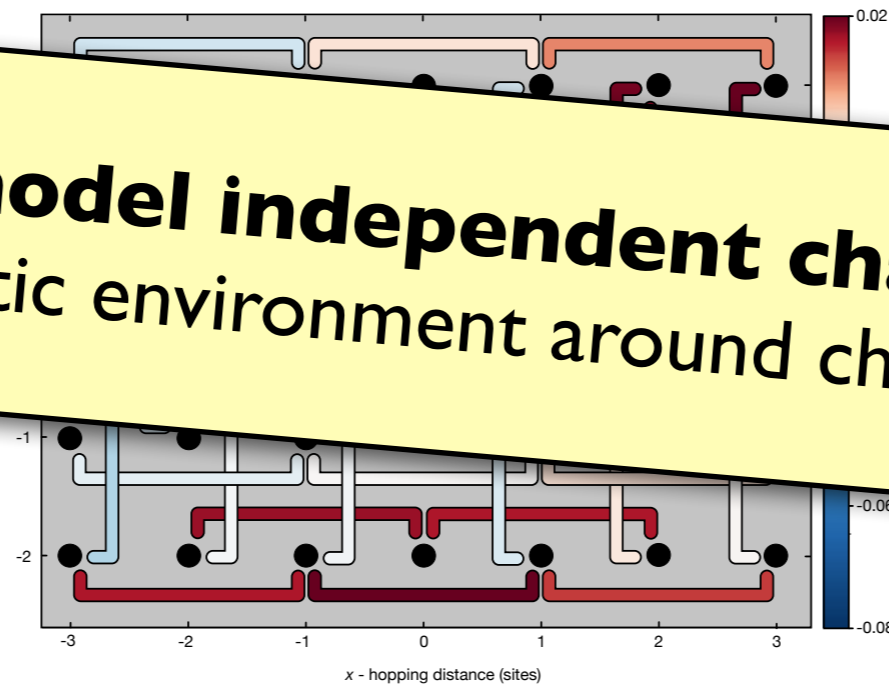


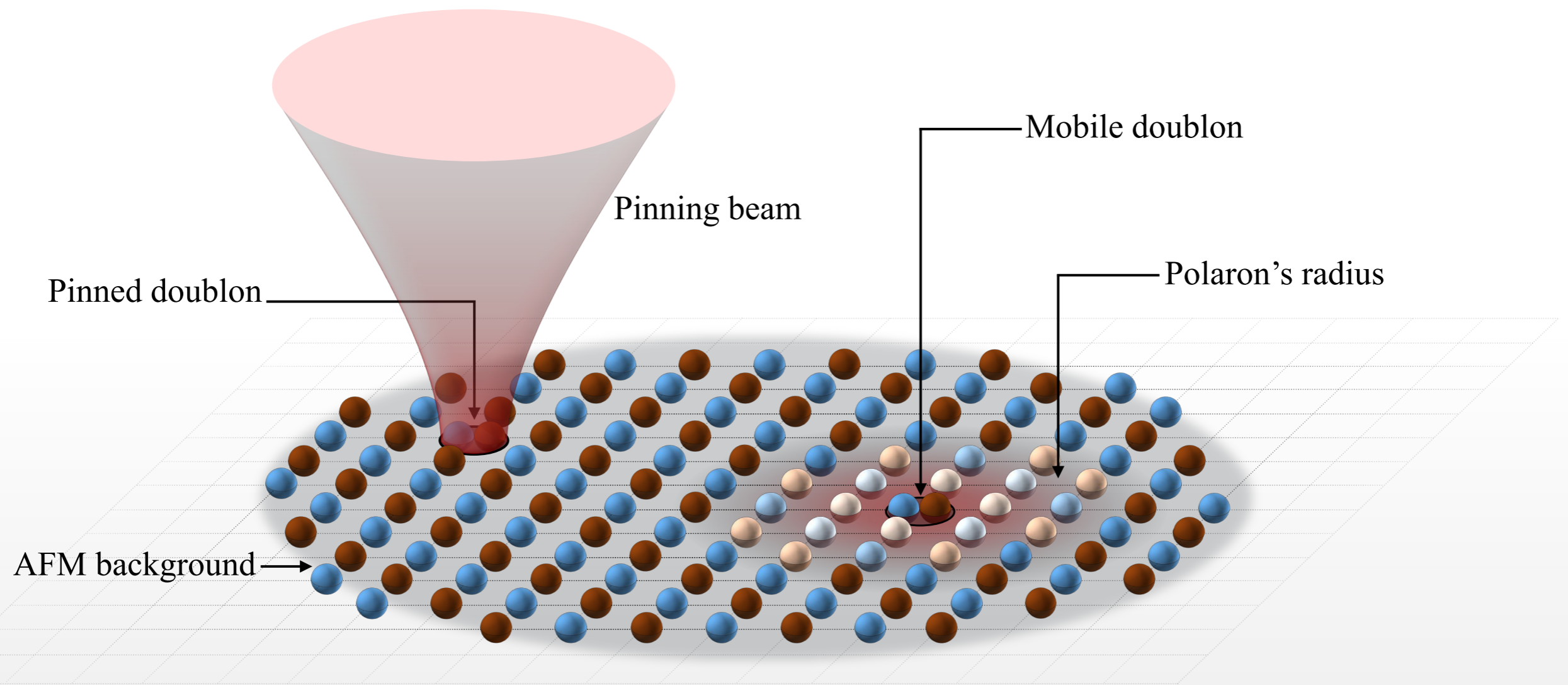
C(2) Local Spin Correlations around Doublon



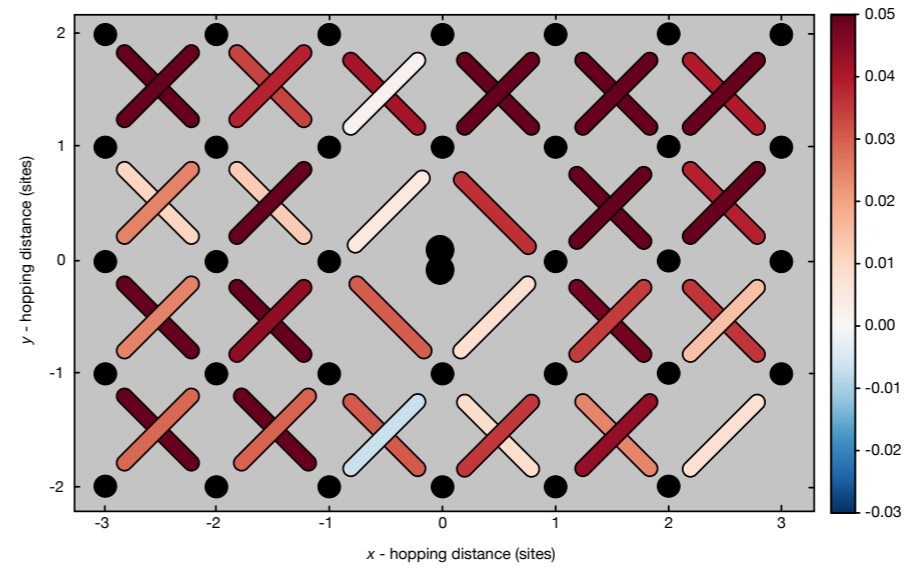
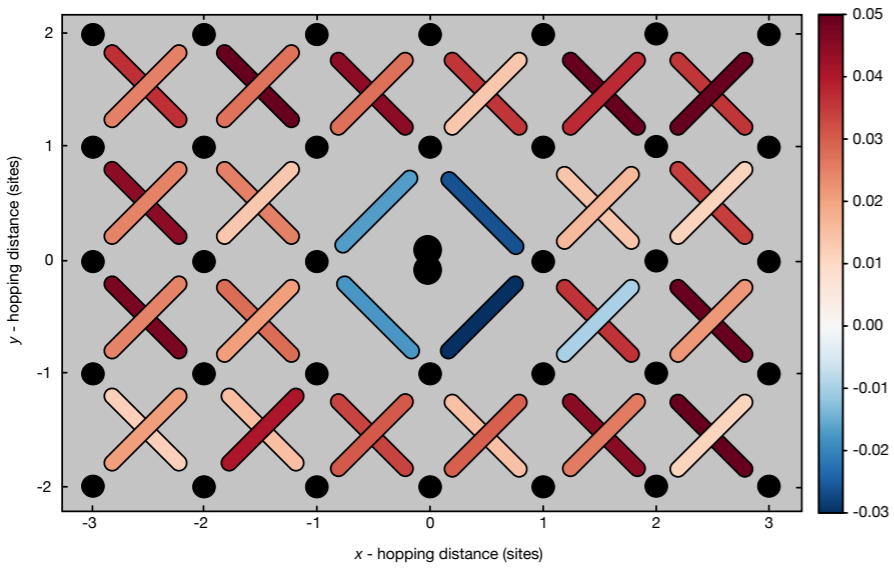
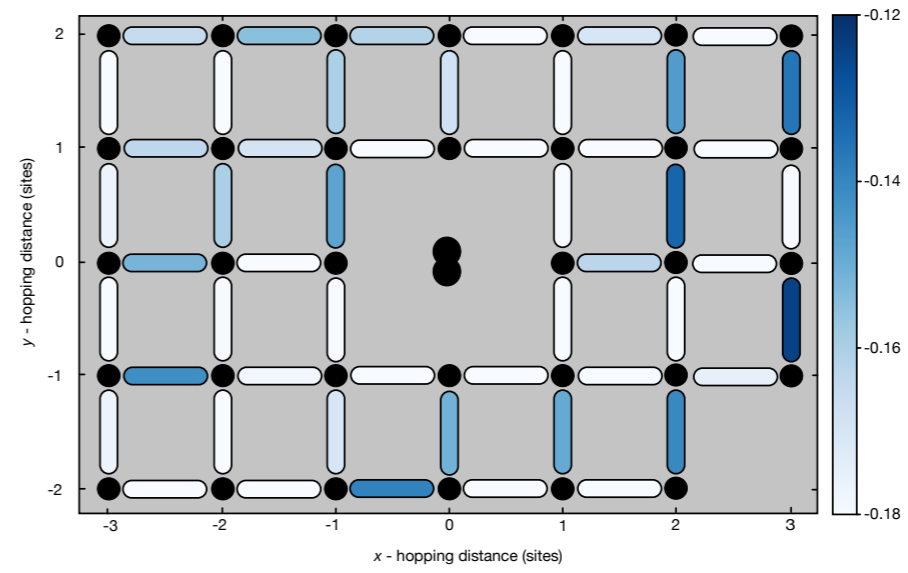
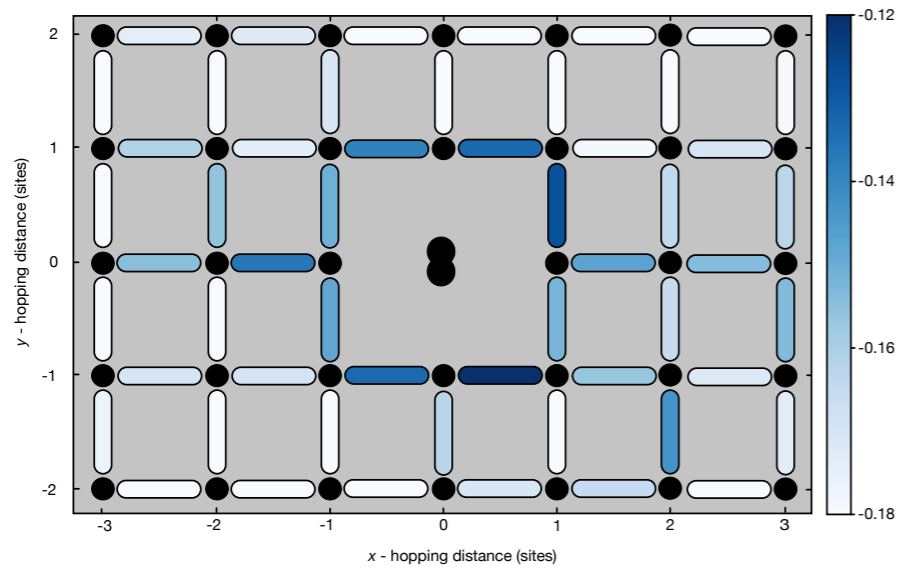


Complete model independent characterisation
of magnetic environment around charge impurity





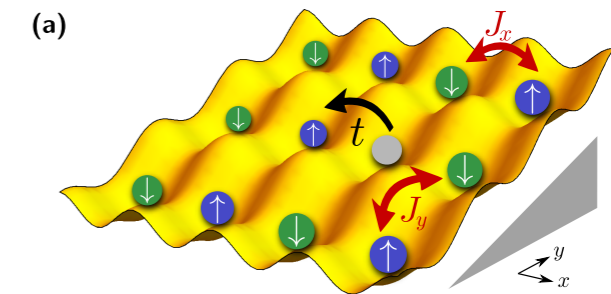
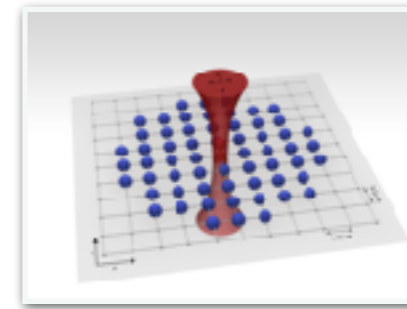
Spin Correlations Around Doublon



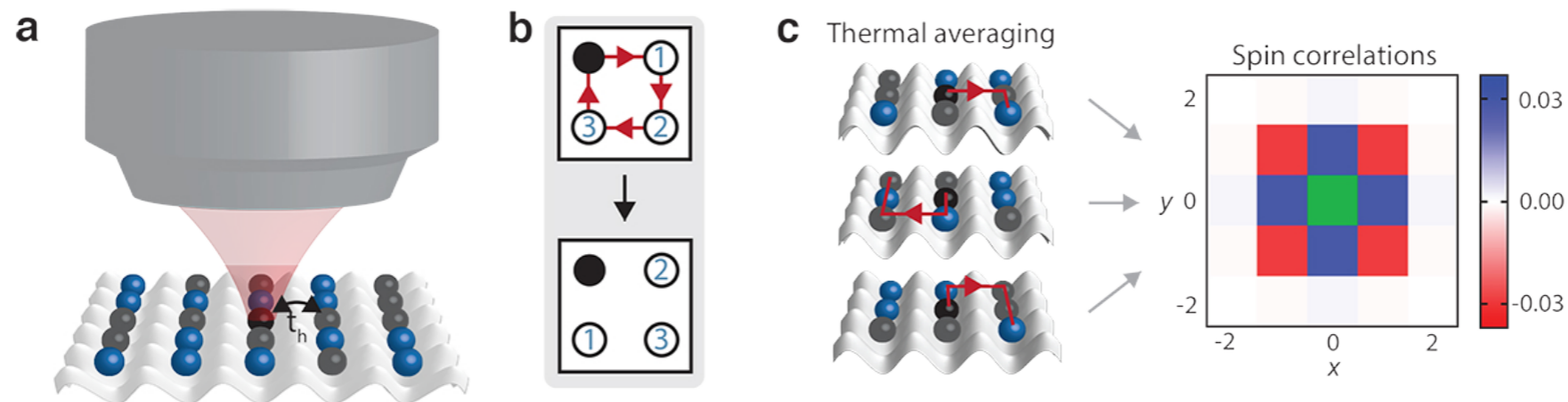
Mobile

Pinned

- ▶ **Polaron-Polaron** correlations
- ▶ **Timescale** of polaron formation
- ▶ **Mixed dimensions**
- ▶ **Dynamical Nagaoka effect**



Grusdt et al, arXiv:1806.04426 2018

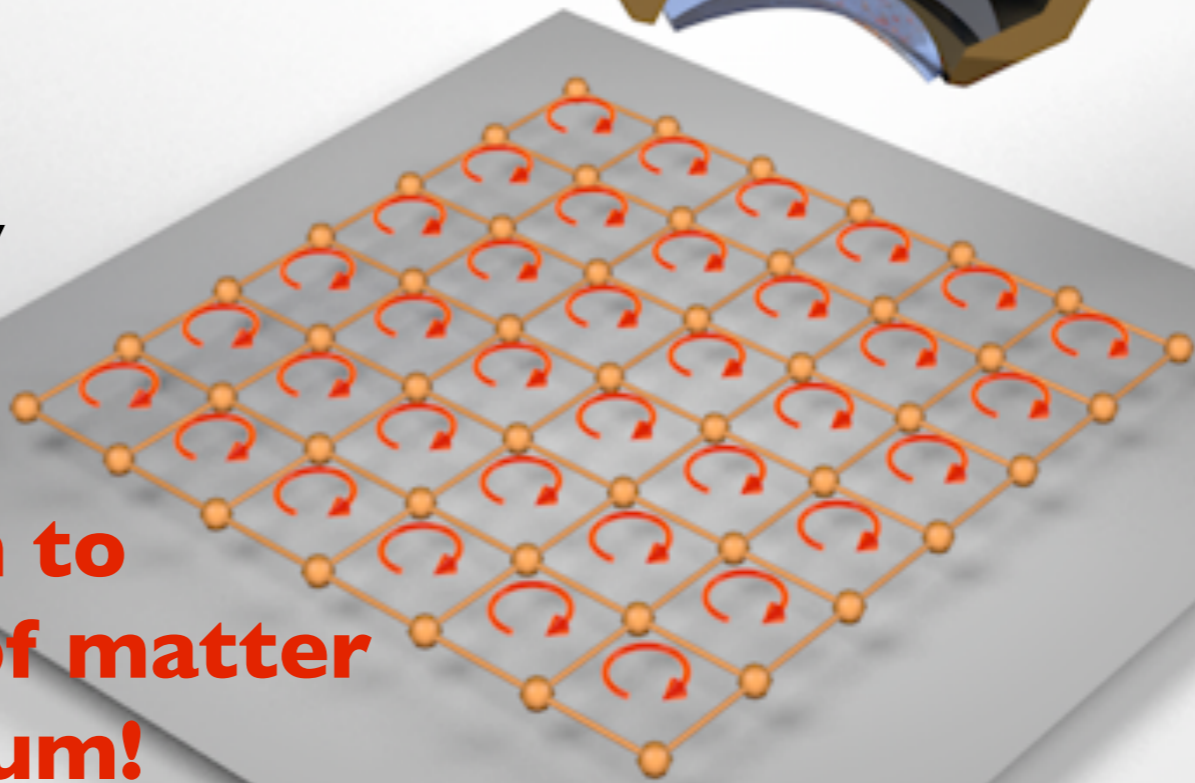
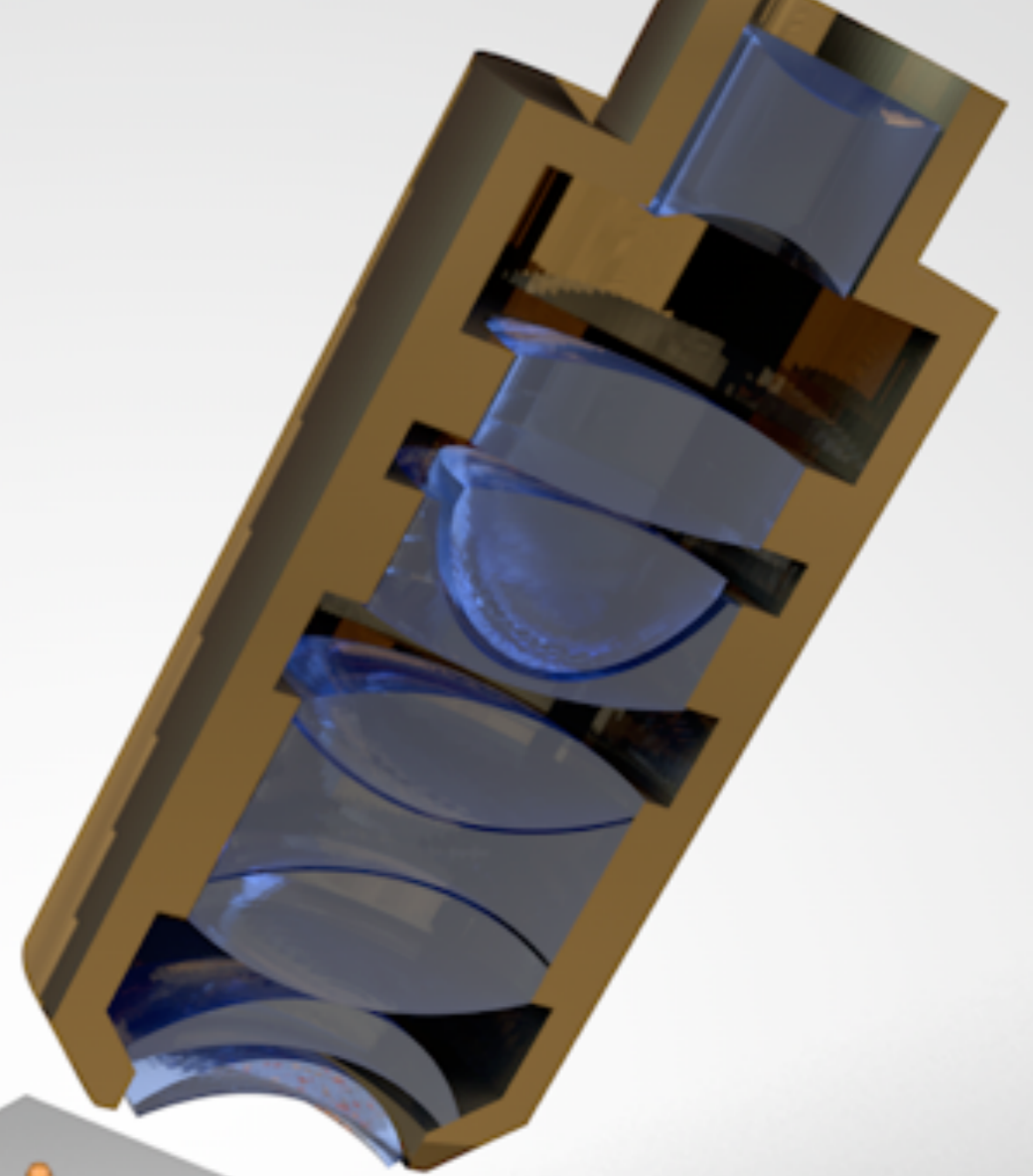


Kanász-Nagy et al, PRB 96 2017

Outlook

- Search for New Phases of Matter
- Extremely Strong Magnetic Field Physics
- Novel Quantum Magnets
- Controlled Quasiparticle Manipulations
- Non-Equilibrium Dynamics (Universality?)
- Thermalization in Isolated Quantum Systems
- Entanglement Measures in Dynamics
- Supersolids
- Cosmology - Black Hole Models?
- High Energy Physics/String Theory
- New clocks/Navigation

**Quantitative testbeds
for theory and system to
discover new phases of matter
in and out of equilibrium!**



Rydberg atoms

- hydrogen-like wave function
 - quantum defect

$$E_{nlj} = - \frac{Ry}{[n - \delta_{lj}(n)]^2}$$

- Strong switchable interactions

$^{87}\text{Rb } 43\text{S}_{1/2}$

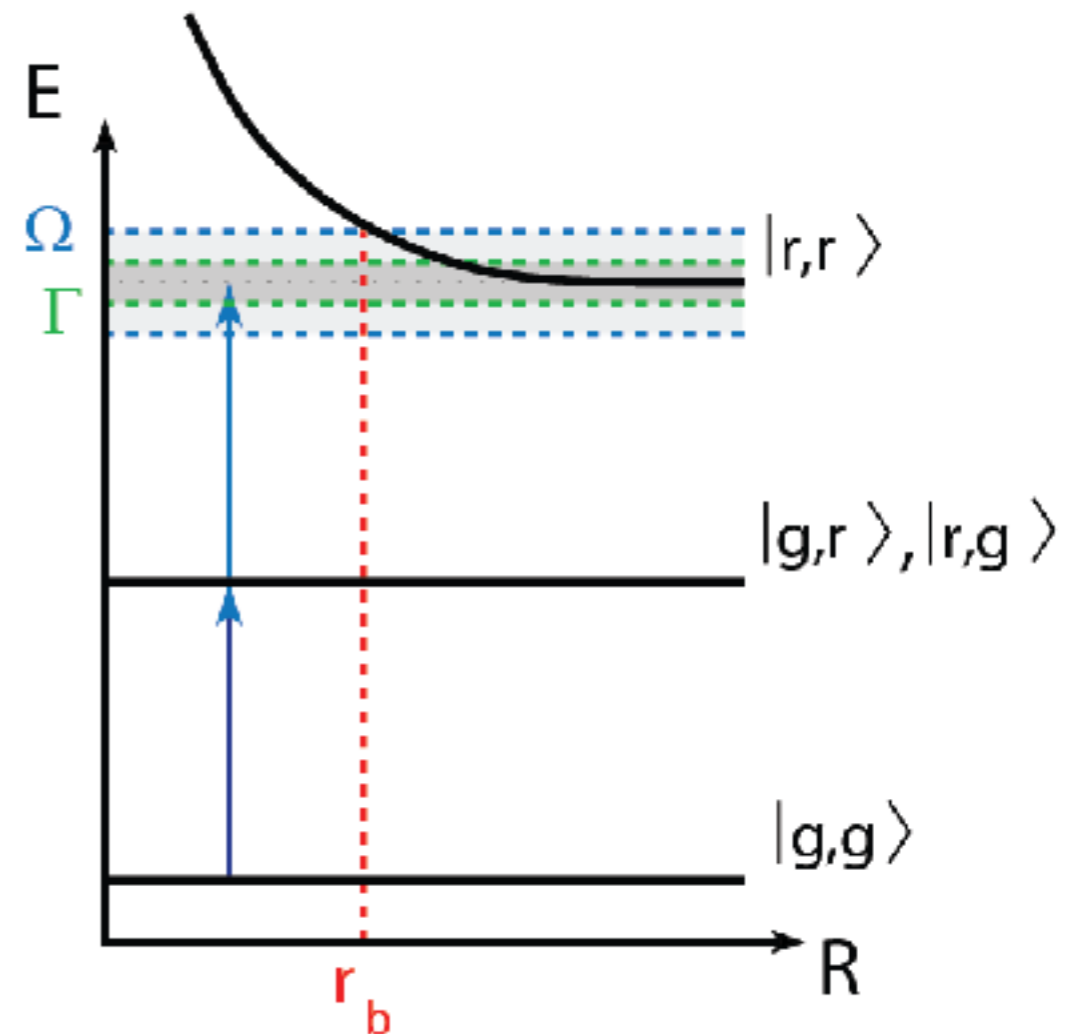
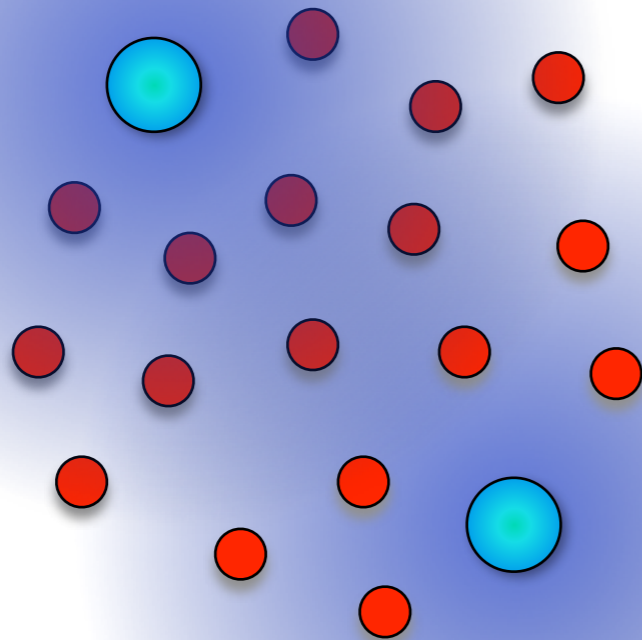
$^{87}\text{Rb } 5\text{S}_{1/2}$



Ø 0.5nm

Ø 250 nm

Property	Scaling	$^{87}\text{Rb } 43\text{S}$
Radius	$(n^*)^2$	$2400 a_0 = 127\text{nm}$
Lifetime (dominated by black body radiation for large n)	$(n^*)^2$	$45 \mu\text{s @ } 20^\circ\text{C}$
van der Waals coefficient	$(n^*)^{11}$	$C_6 = -1.7 \times 10^{19} \text{ a.u.}$
Blockade radius ($\Omega=2\pi$ 200 kHz)	$(n^*)^2$	$\sim 5 \mu\text{m}$



Blockade condition

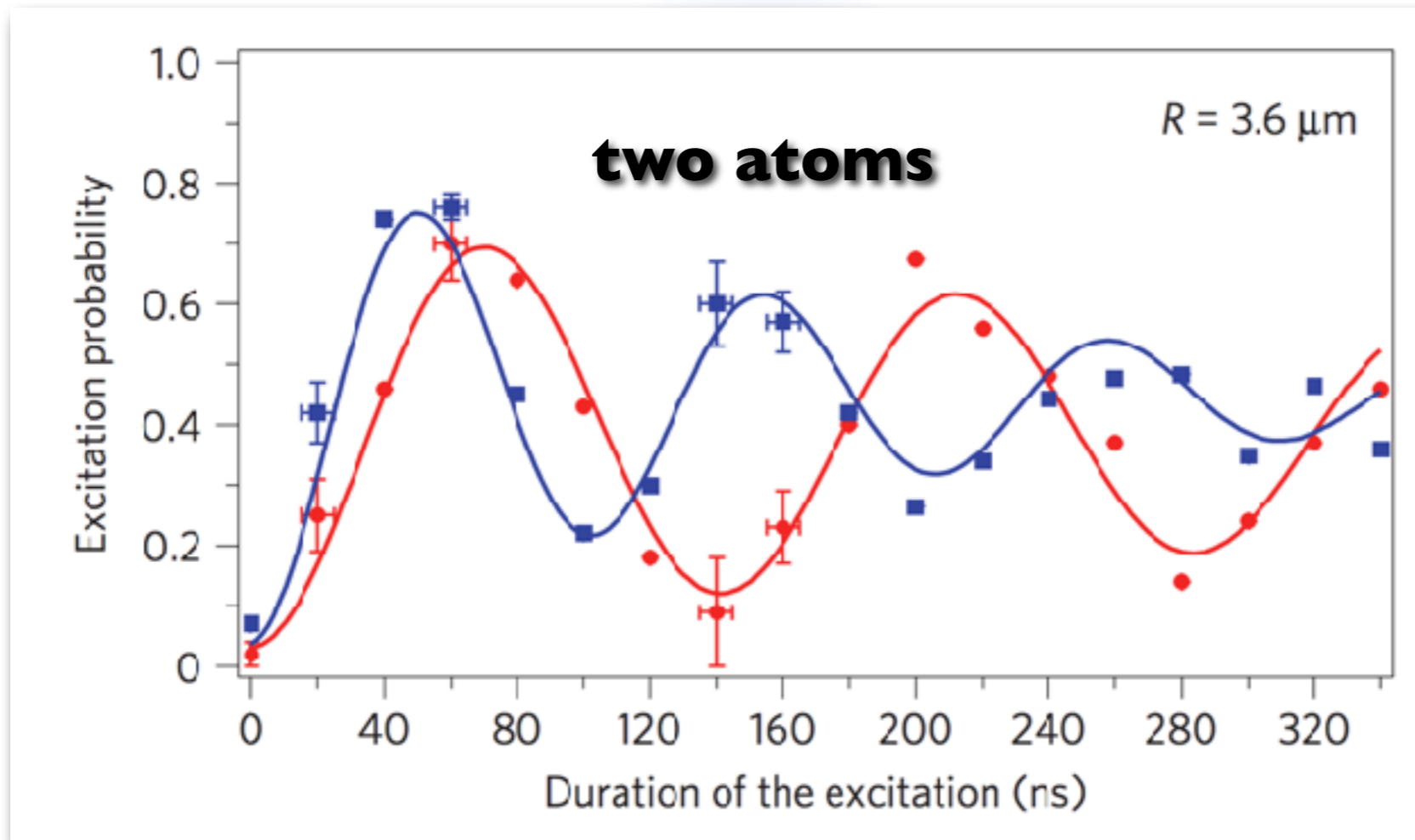
$$\mathcal{V}_{\text{vdW}} = \frac{C_6}{r^6} > \hbar \max(\Gamma, \Omega)$$

$$r_b \equiv \sqrt[6]{\frac{C_6}{\hbar\Omega}}$$

Each superatom:

$$\frac{1}{\sqrt{N}} (|r, 0, 0, 0, \dots\rangle + |0, r, 0, 0, \dots\rangle + |0, 0, 0, \dots, r\rangle)$$

M. Lukin et al. PRL **87**, 037901 (2001)



blockade radius
larger than cloud size!

$\sqrt{N}\Omega_1$ Rabi Oscillations speed up!

Each superatom:

$$\frac{1}{\sqrt{N}} (|r, 0, 0, 0, \dots\rangle + |0, r, 0, 0, \dots\rangle + |0, 0, 0, \dots, r\rangle)$$

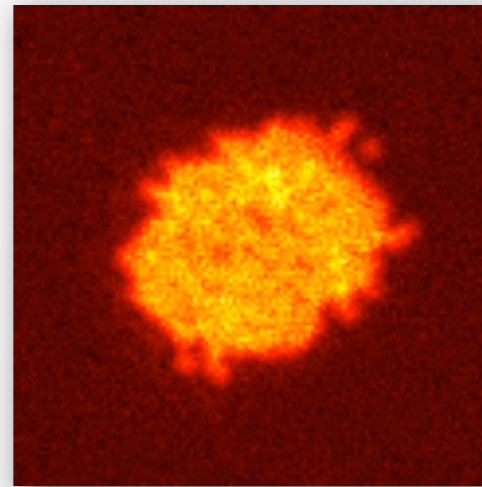
M. Lukin et al. PRL **87**, 037901 (2001)

see work by A. Browaeys & Ph. Grangier, M. Saffman, A. Kuzmich, T. Pfau...

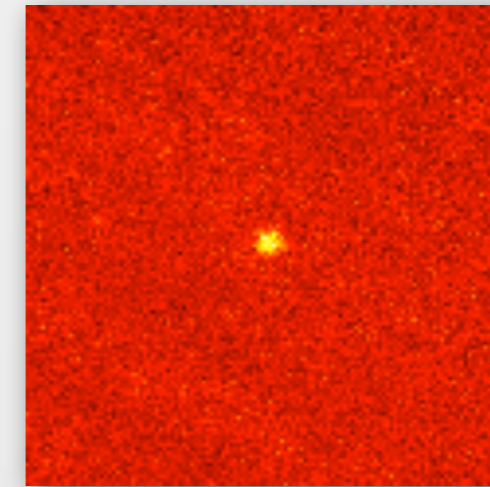




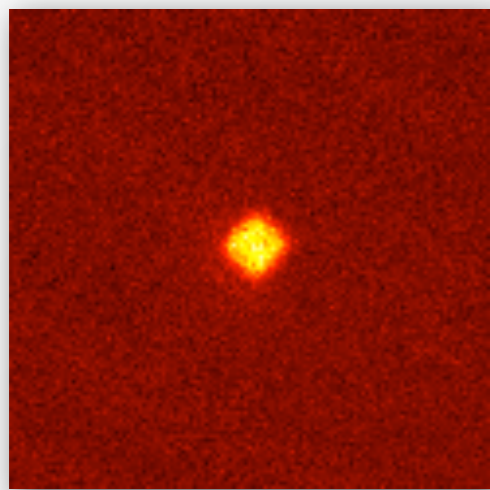
Digital Mirror
(Size Control)



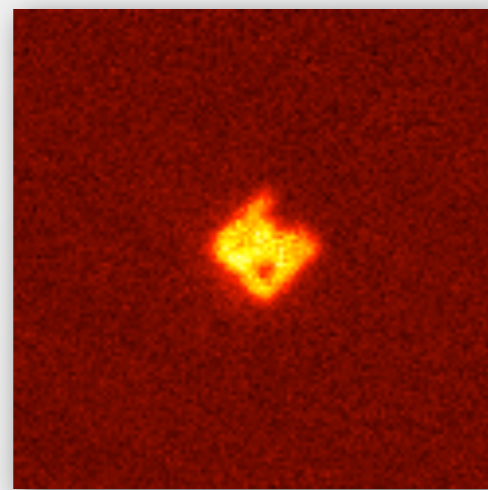
Initial MI



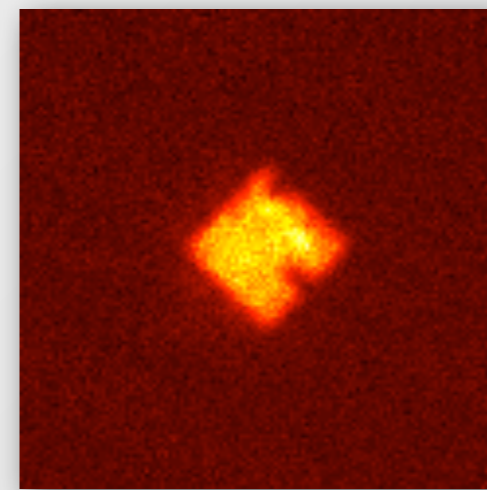
Single Atom



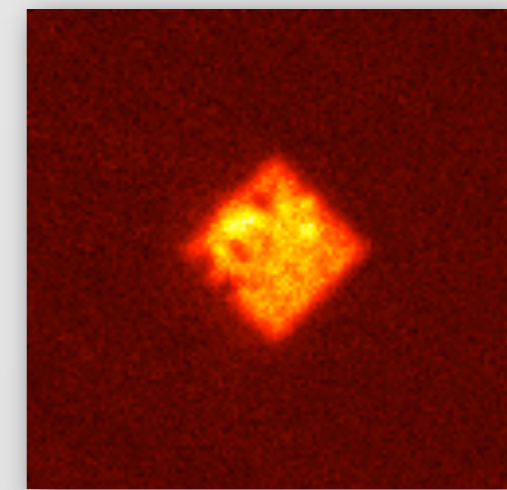
3x3



5x5



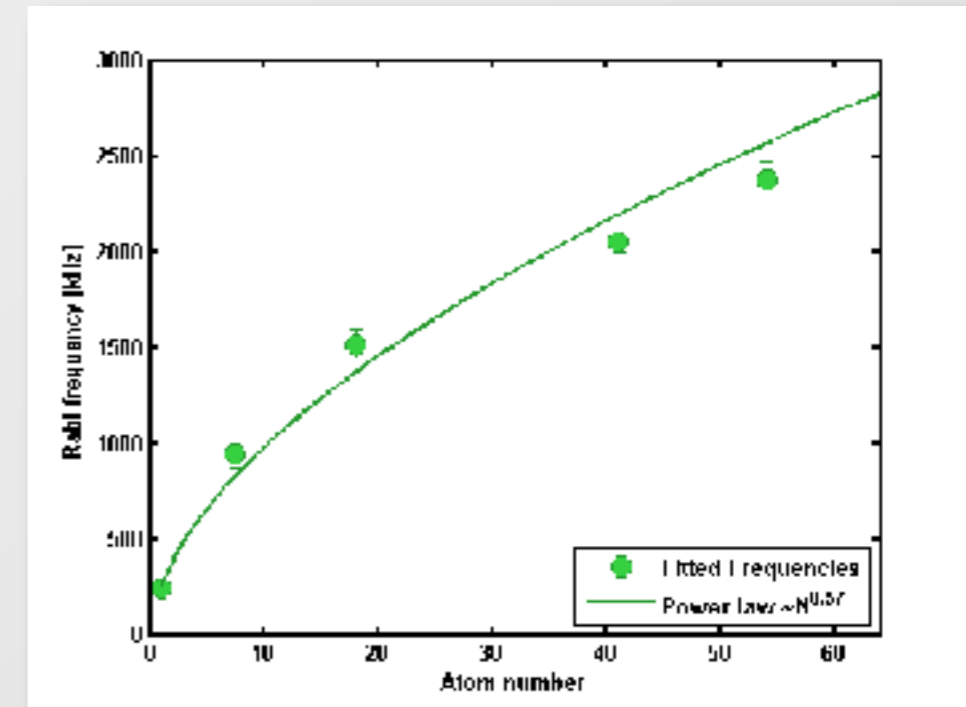
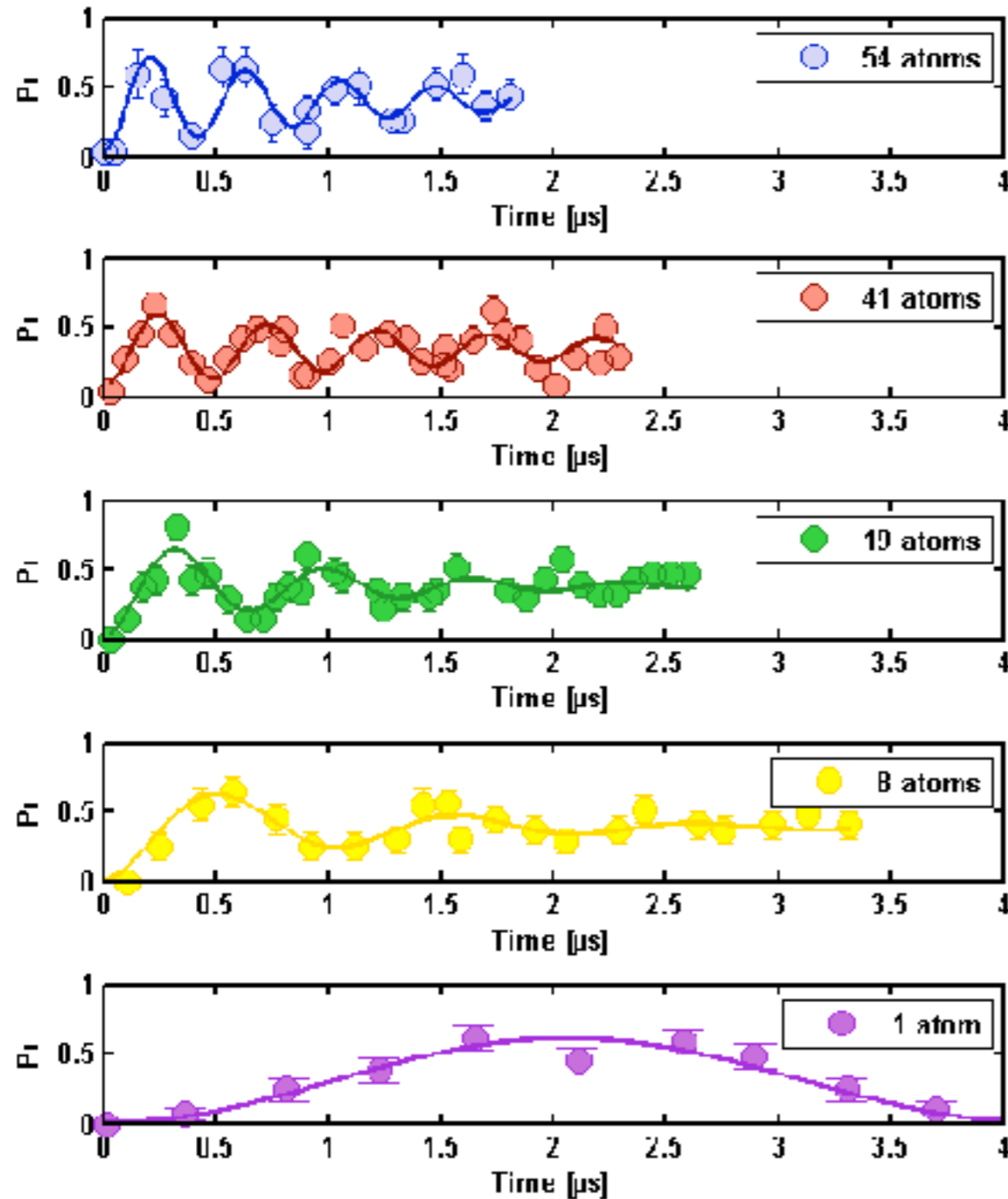
7x7



8x8

atoms

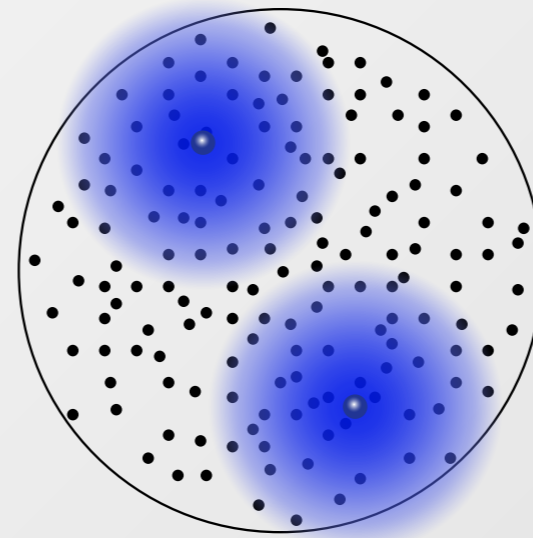
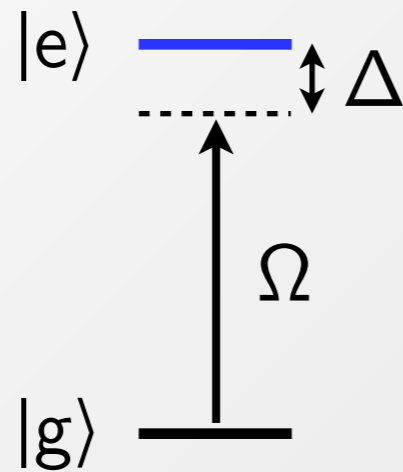




Single atom non-linearity controls dynamics of >50 atoms!

J. Zeiher et al., Phys. Rev. X **5**, 031015 (2015)





no mechanical motion on the timescale of the internal dynamics

$$H = \frac{\hbar\Omega}{2} \sum_i \left(\sigma_{eg}^{(i)} + \sigma_{ge}^{(i)} \right) + \sum_{i \neq j} \frac{V_{ij}}{2} \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} - \Delta \sum_i \sigma_{ee}^{(i)}$$

coherent coupling

interaction between Rydberg atoms

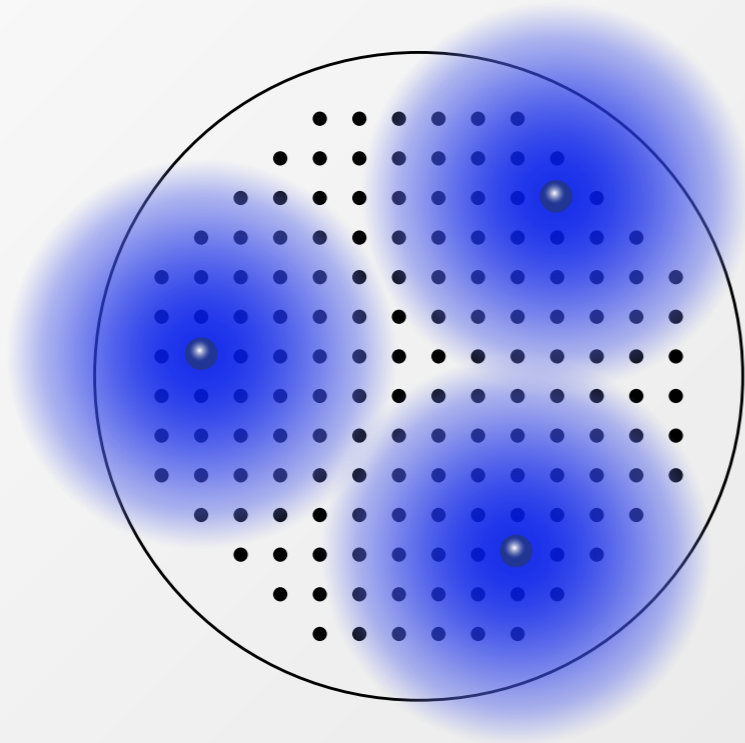
"chemical potential"

$$V_{ij} = C_\alpha |r_i - r_j|^{-\alpha}$$

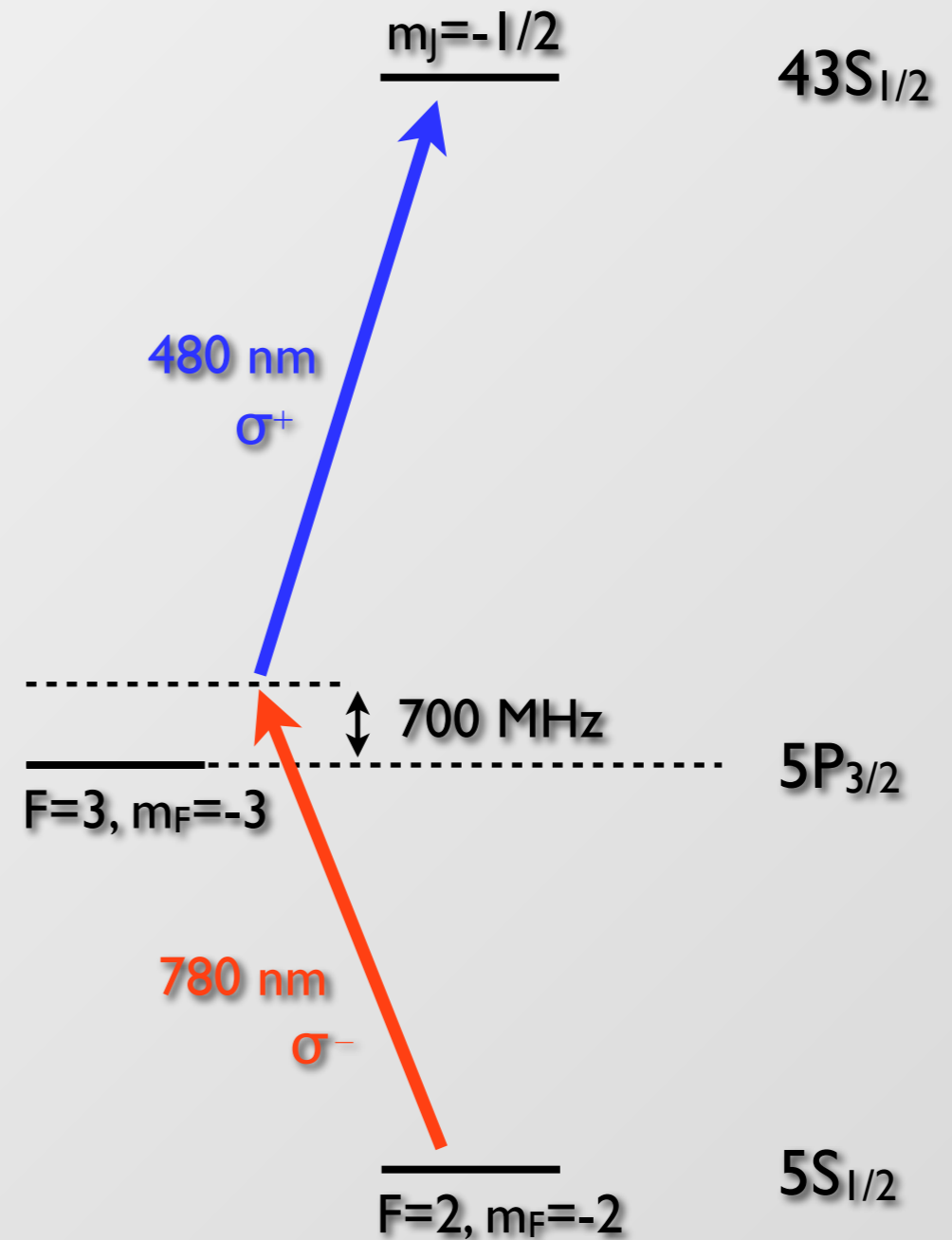
This work: $\alpha=6$, repulsive



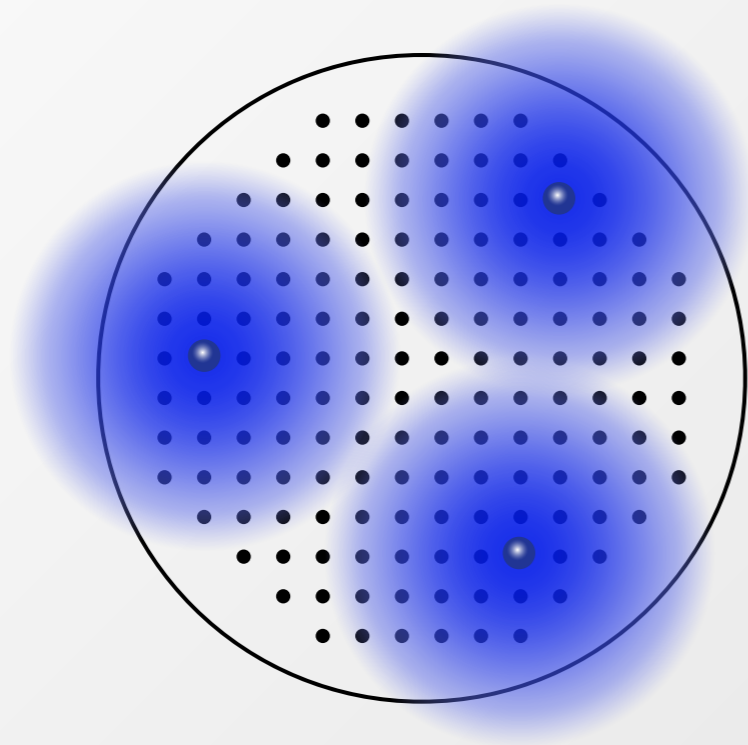
Excitation and detection of the Rydberg atoms



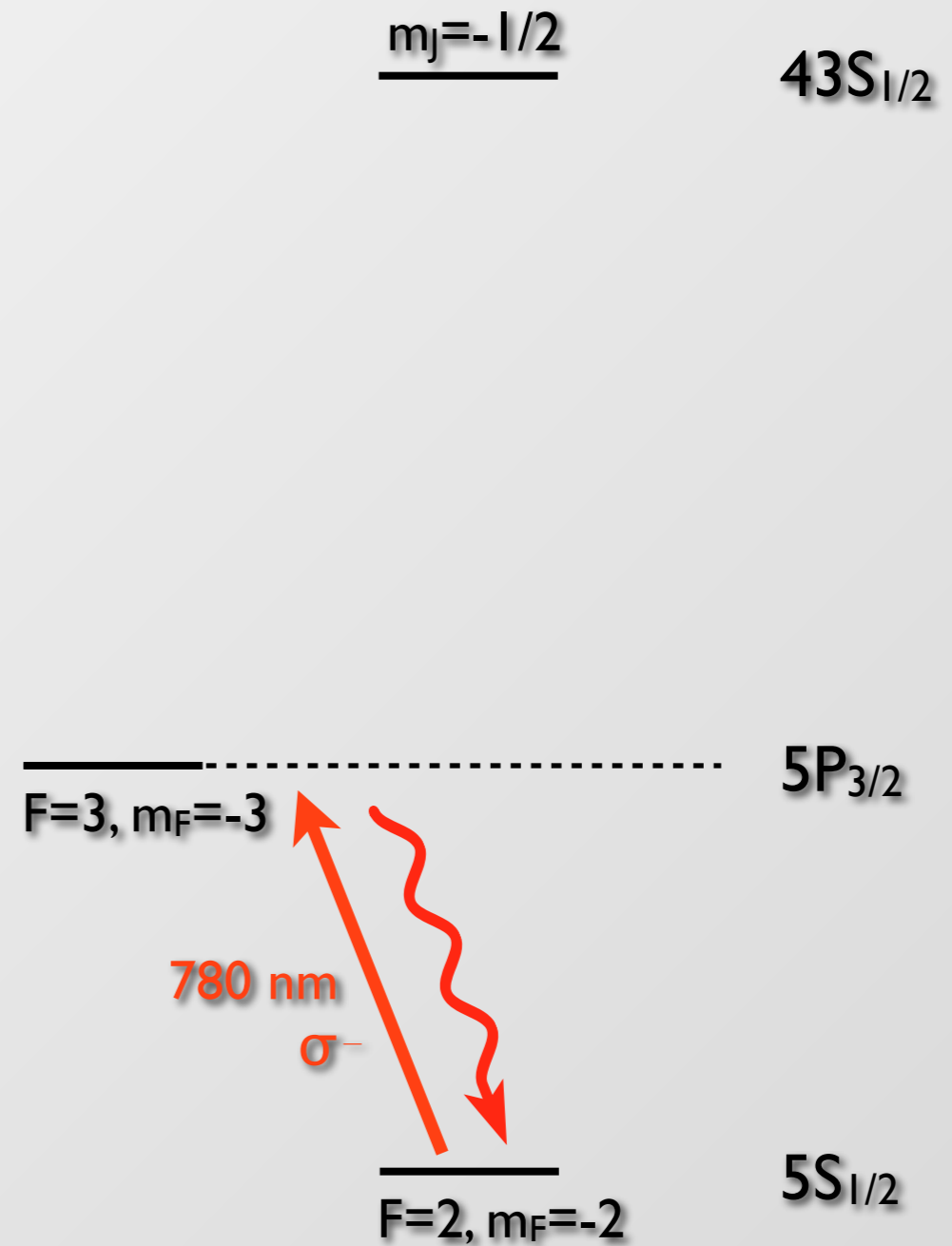
- two-photon Rabi frequency:
 $\Omega/2\pi = 170(20)$ kHz
- resonant excitation:
 $\Delta = 0$
- blockade radius:
 $R_b = 4.9(1)$ μm



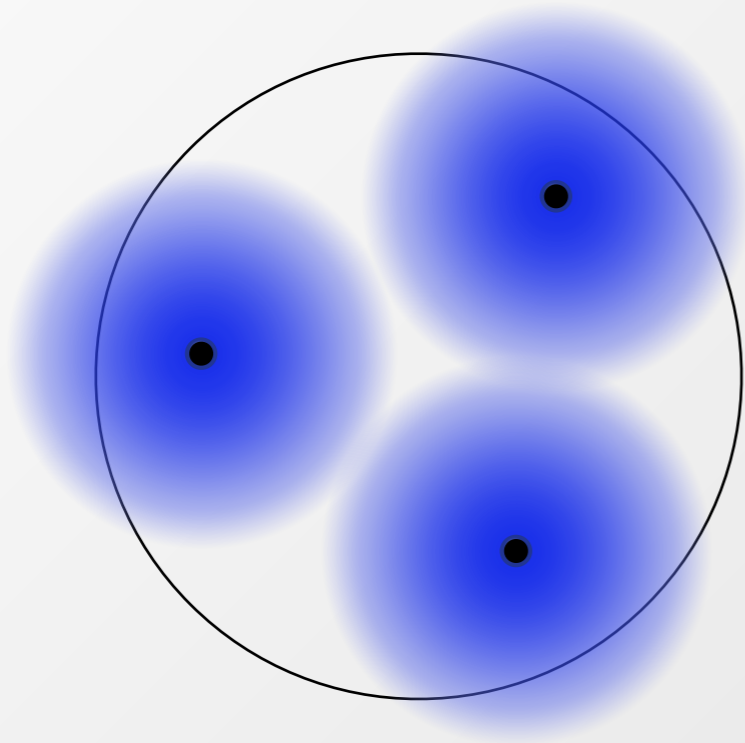
Excitation and detection of the Rydberg atoms



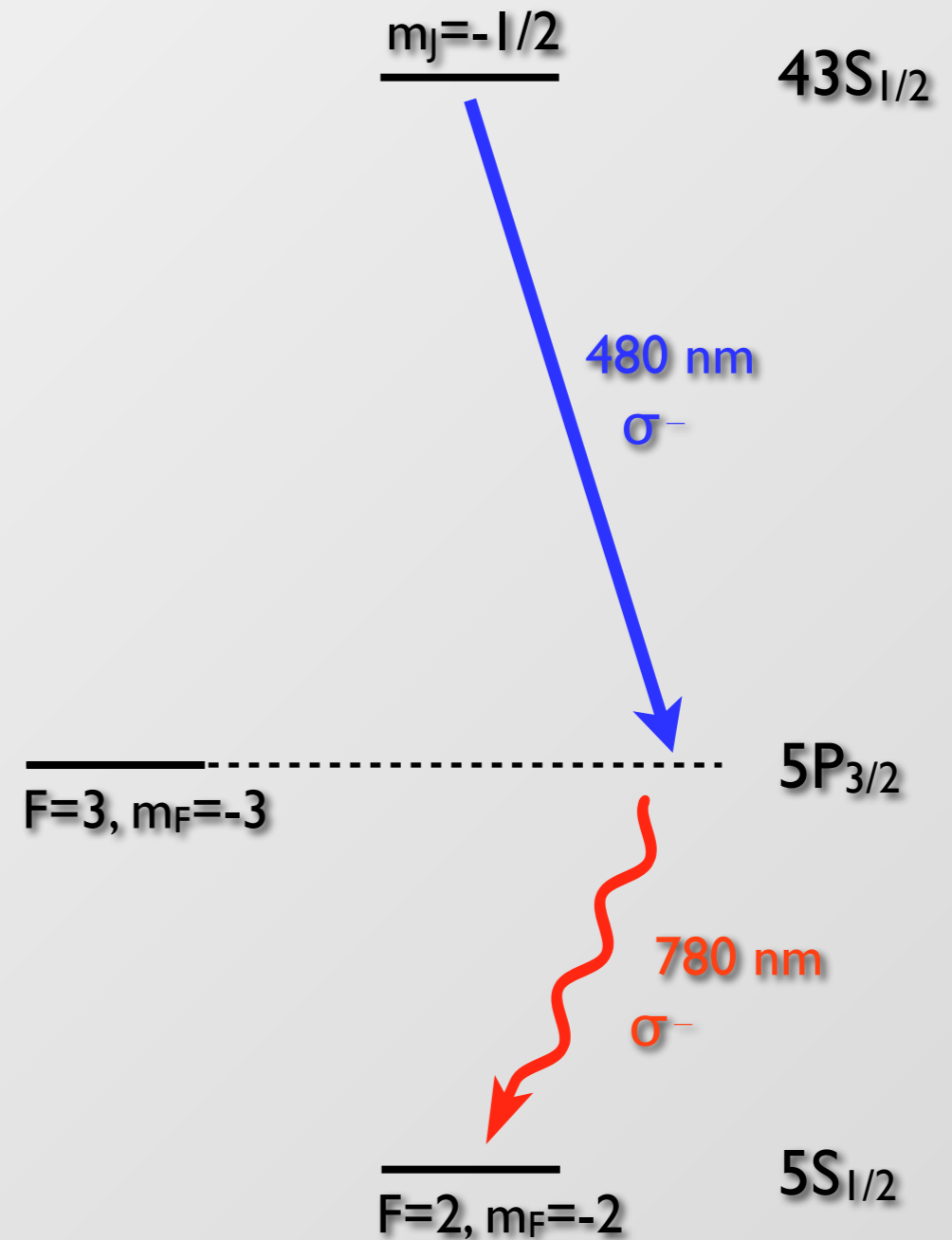
- removal pulse duration: 10 μ s
- survival probability: 0.1 %

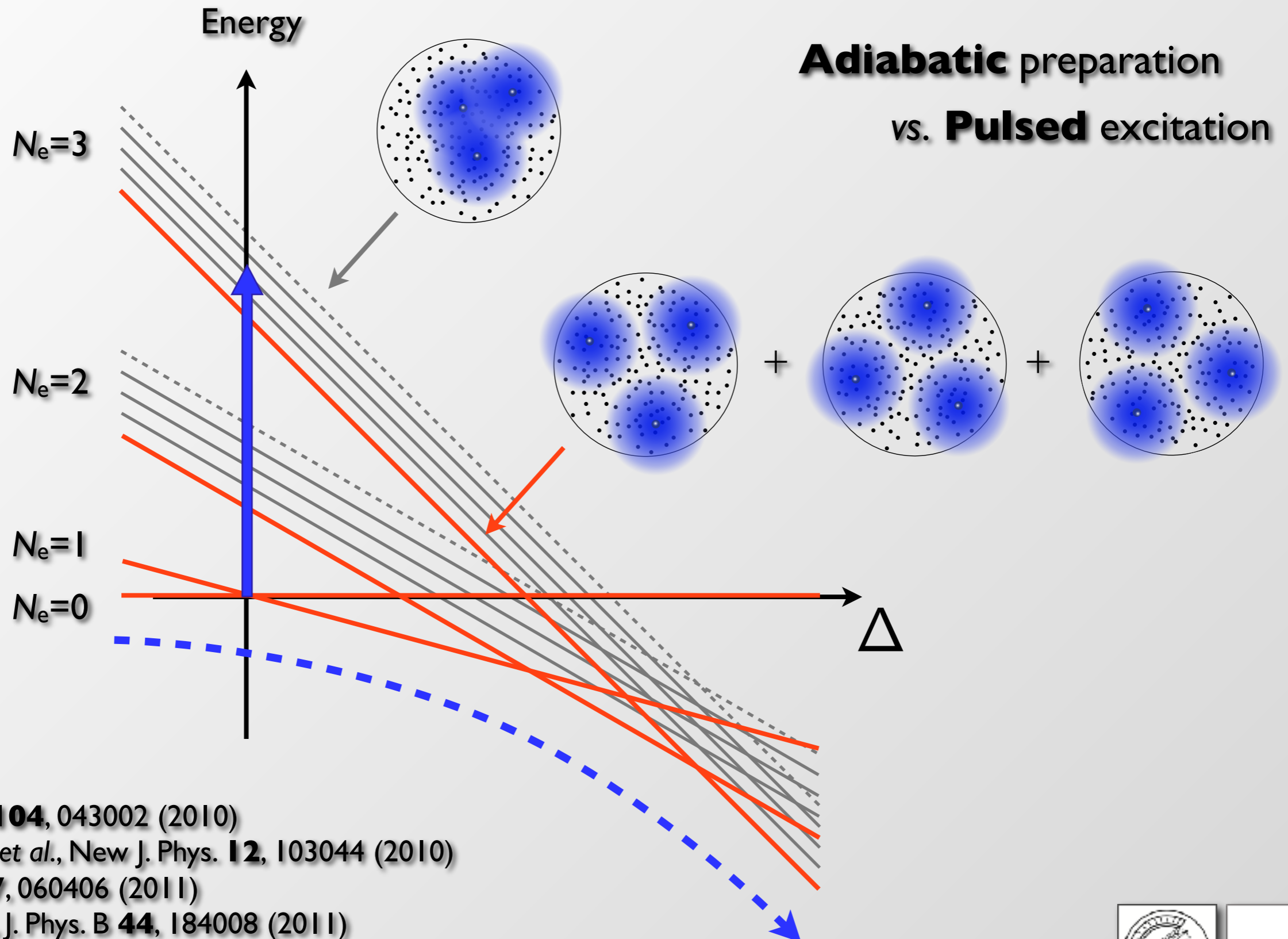


Excitation and detection of the Rydberg atoms



- deexcitation pulse duration: 2 μs
- detection efficiency: 75(10) %
- overall resolution: ~ 500 nm





Pohl *et al.*, PRL **104**, 043002 (2010)

Schachenmayer *et al.*, New J. Phys. **12**, 103044 (2010)

Ji *et al.*, PRL **107**, 060406 (2011)

van Bijnen *et al.*, J. Phys. B **44**, 184008 (2011)

Gärttner *et al.*, arXiv:1203.2884v2 (2012)



Dynamical Crystallization in the Dipole Blockade of Ultracold Atoms

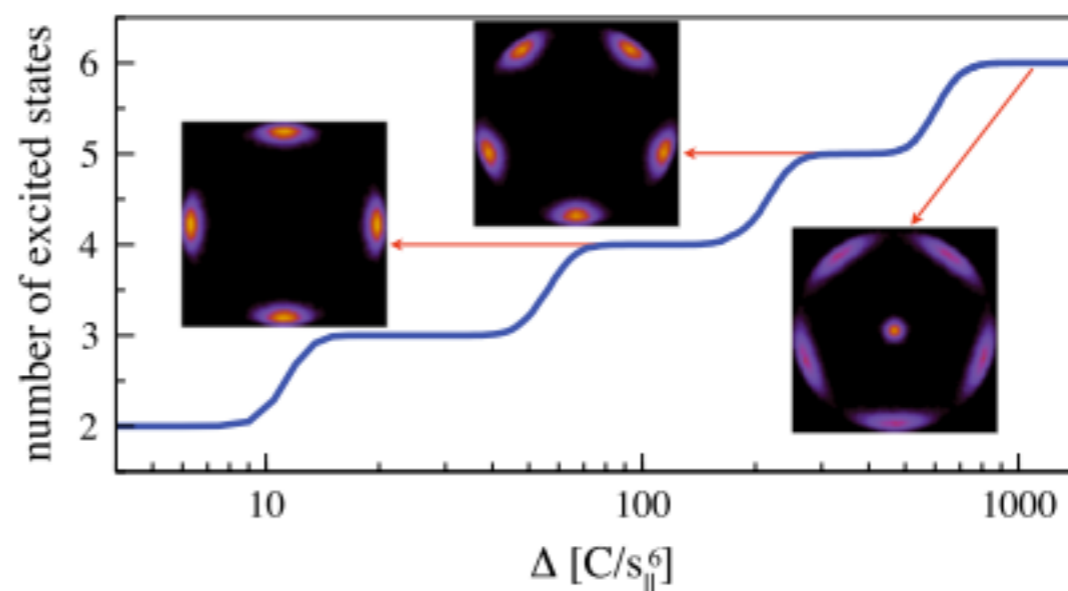
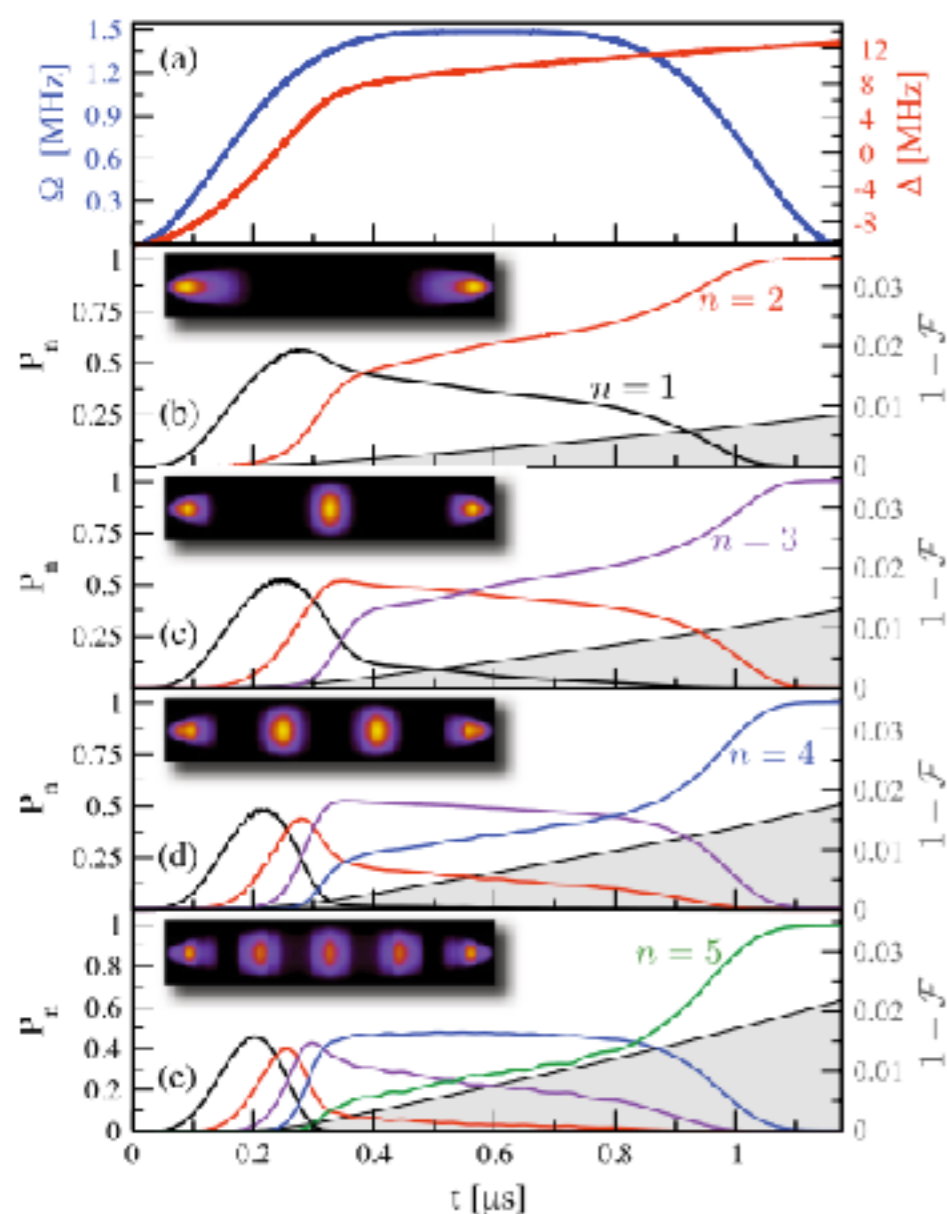
T. Pohl,^{1,2} E. Demler,^{2,3} and M. D. Lukin^{2,3}

¹Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany

²ITAMP-Harvard-Smithsonian Center for Astrophysics, Cambridge Massachusetts 02138, USA

³Physics Department, Harvard University, Cambridge Massachusetts 02138, USA

(Received 26 July 2009; revised manuscript received 23 October 2009; published 27 January 2010)

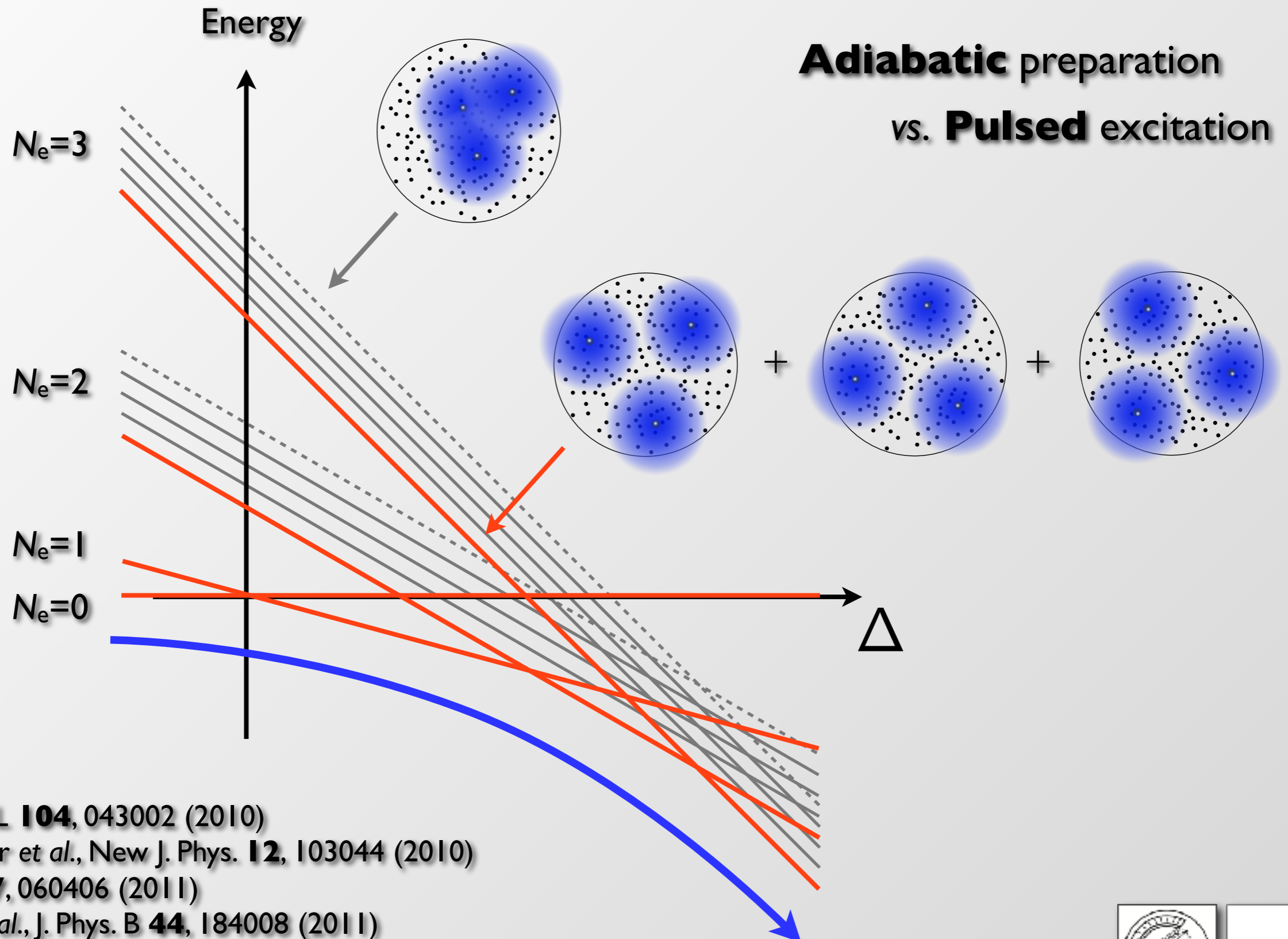


Coherent Control of Many-Body System through Adiabatic Sweeps

Theory see:

T. Pohl et al. PRL 2010; G. Pupillo et al. PRL 2010,
R.M.W van Bijnen et al. J. Phys. B:At. Mol. Opt. Phys. (2011)

see also: H. Weimer et al., PRL 2008



T. Pohl *et al.*, PRL **104**, 043002 (2010)

J. Schachenmayer *et al.*, New J. Phys. **12**, 103044 (2010)

Ji *et al.*, PRL **107**, 060406 (2011)

R. van Bijnen *et al.*, J. Phys. B **44**, 184008 (2011)

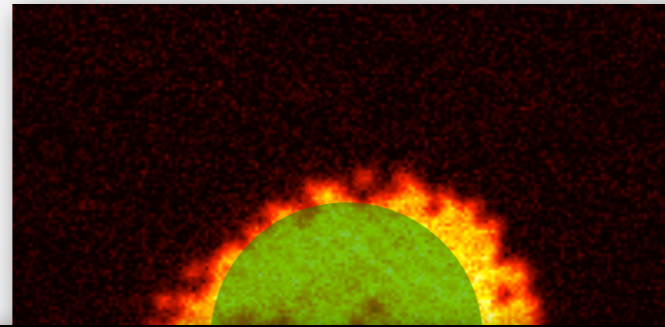
M. Gärttner *et al.*, PRA **88**, 043410 (2012)

D. Petrosyan J. Phys. B **46**, 141001 (2013)



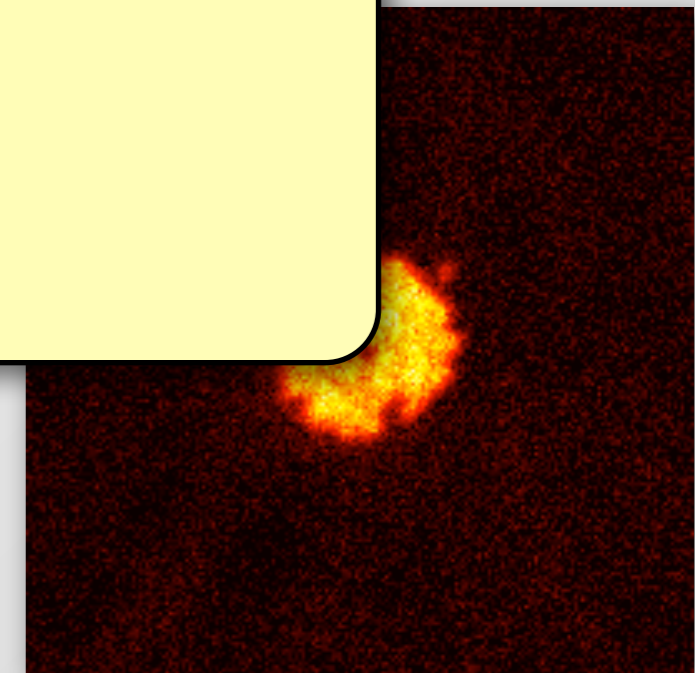
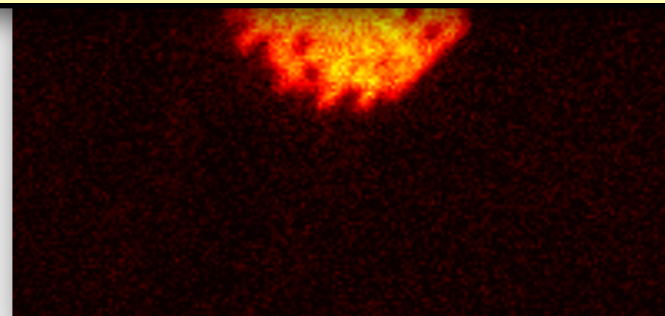
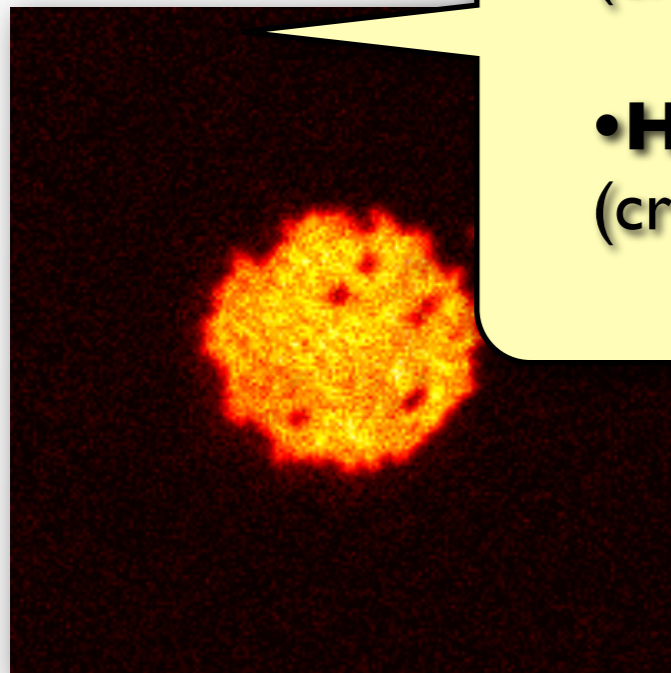


Digital Mirror
(Size Control)



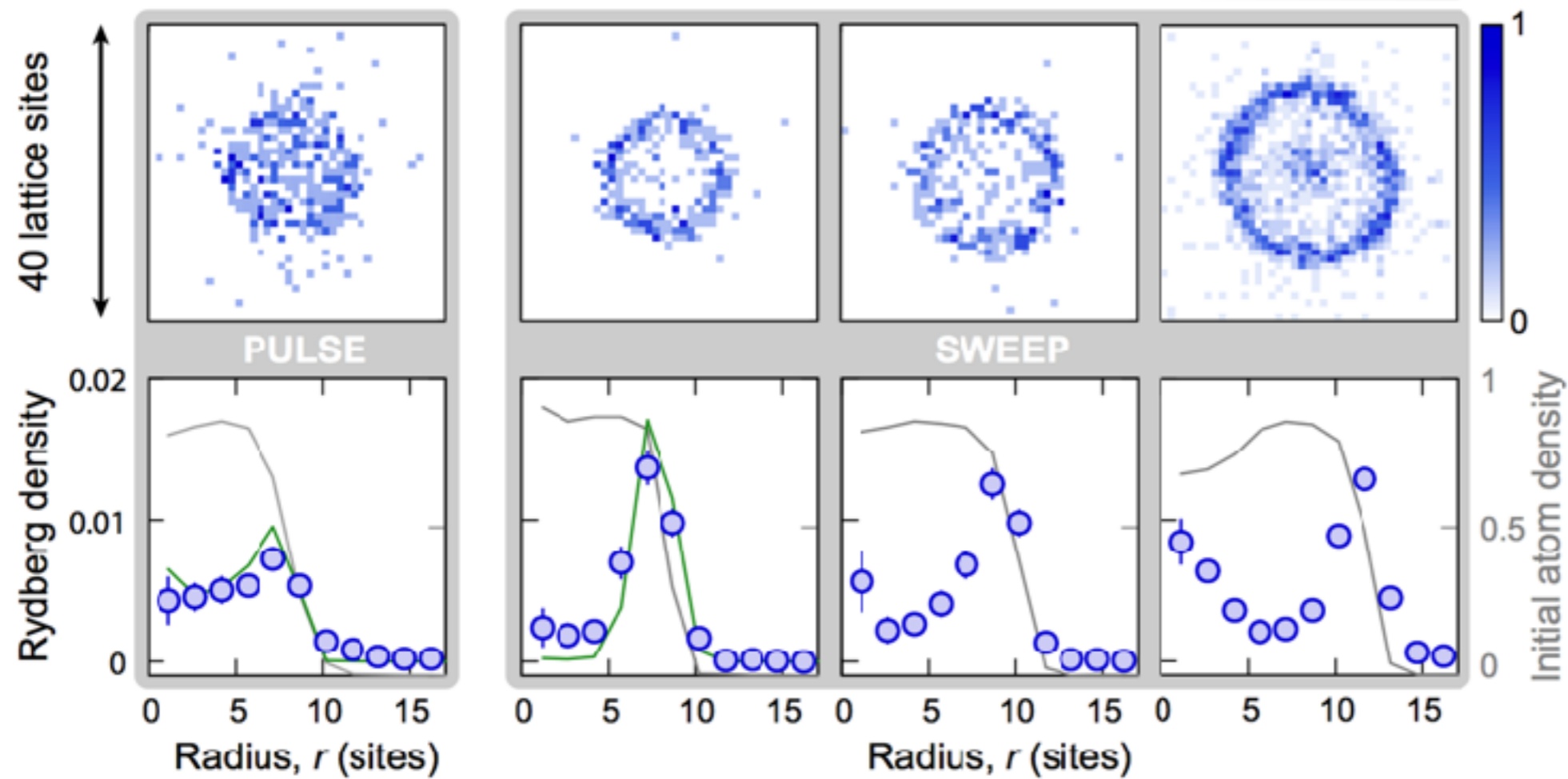
Fluctuating Size and

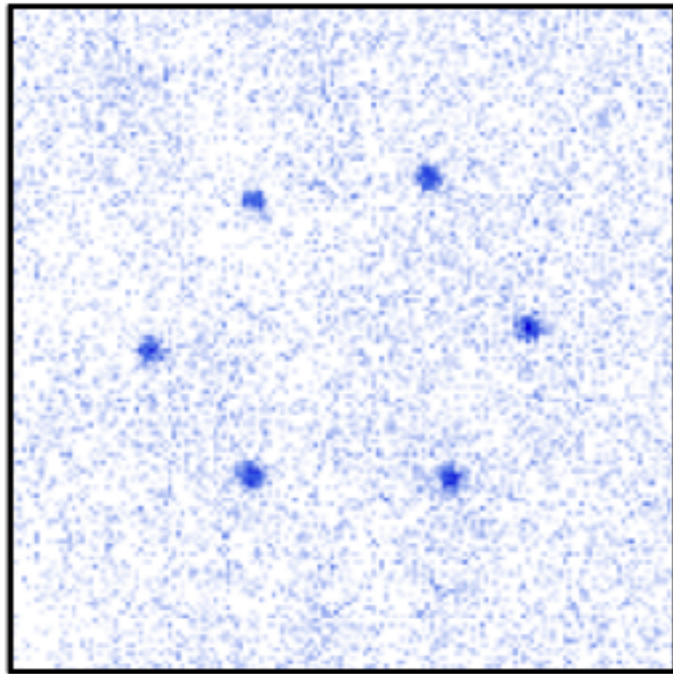
- **Sub Shot Noise Atom Number Preparation**
- **Geometric & atom number control**
(crucial e.g. for quantum criticality)
- **Hard wall potentials realized**
(crucial for edge states)



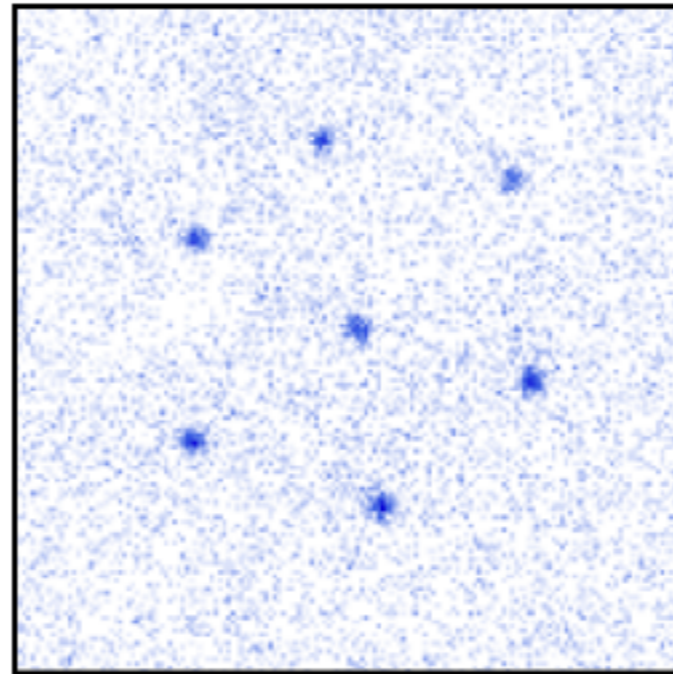
Size & atom number perfectly controlled

Pulsed vs swept excitation - localization of excitations to border of system!

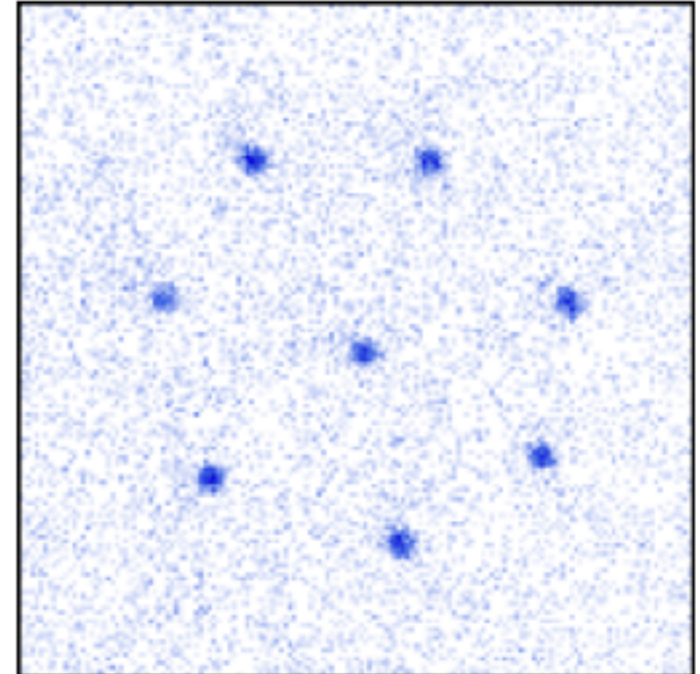




6

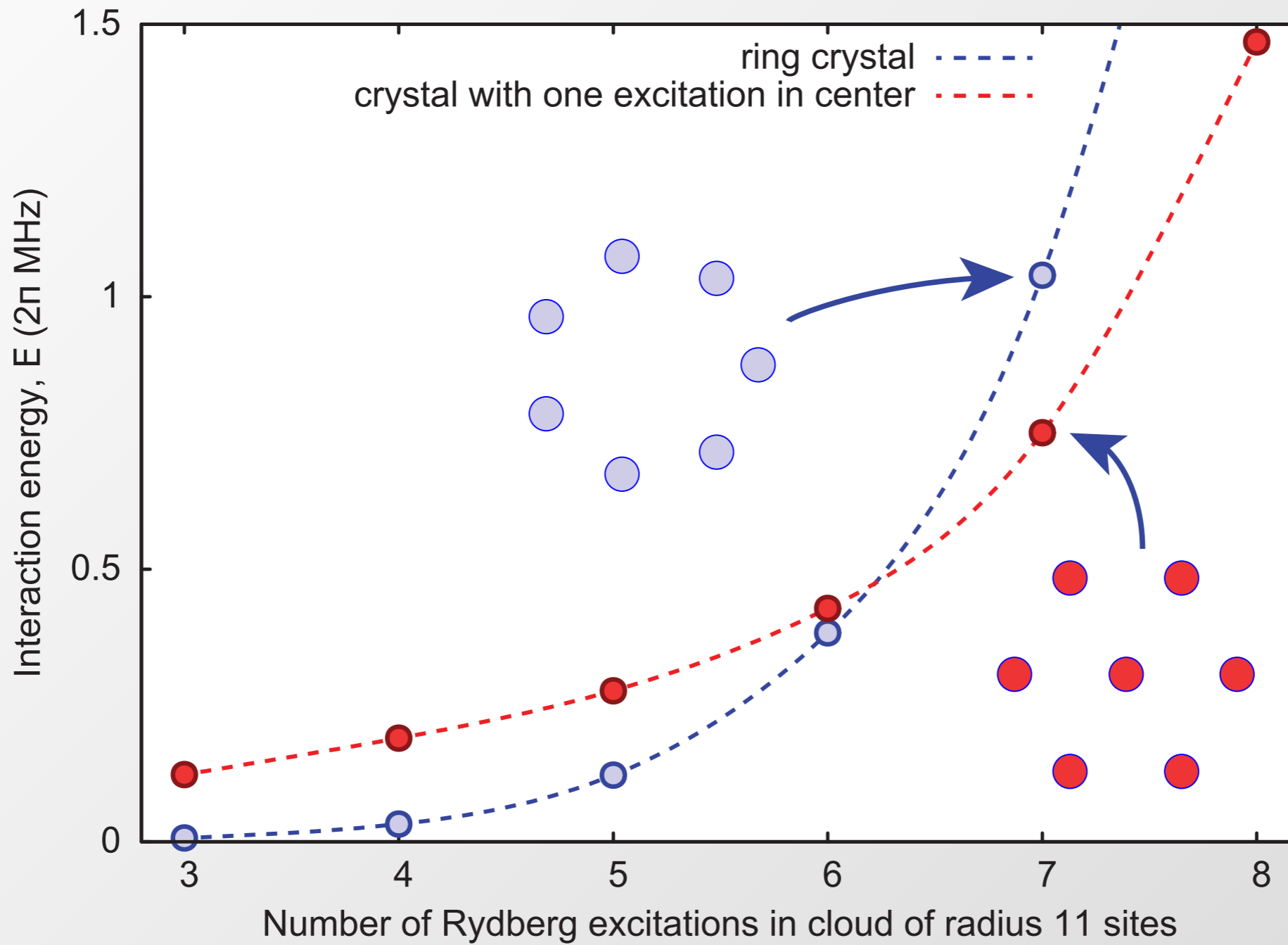


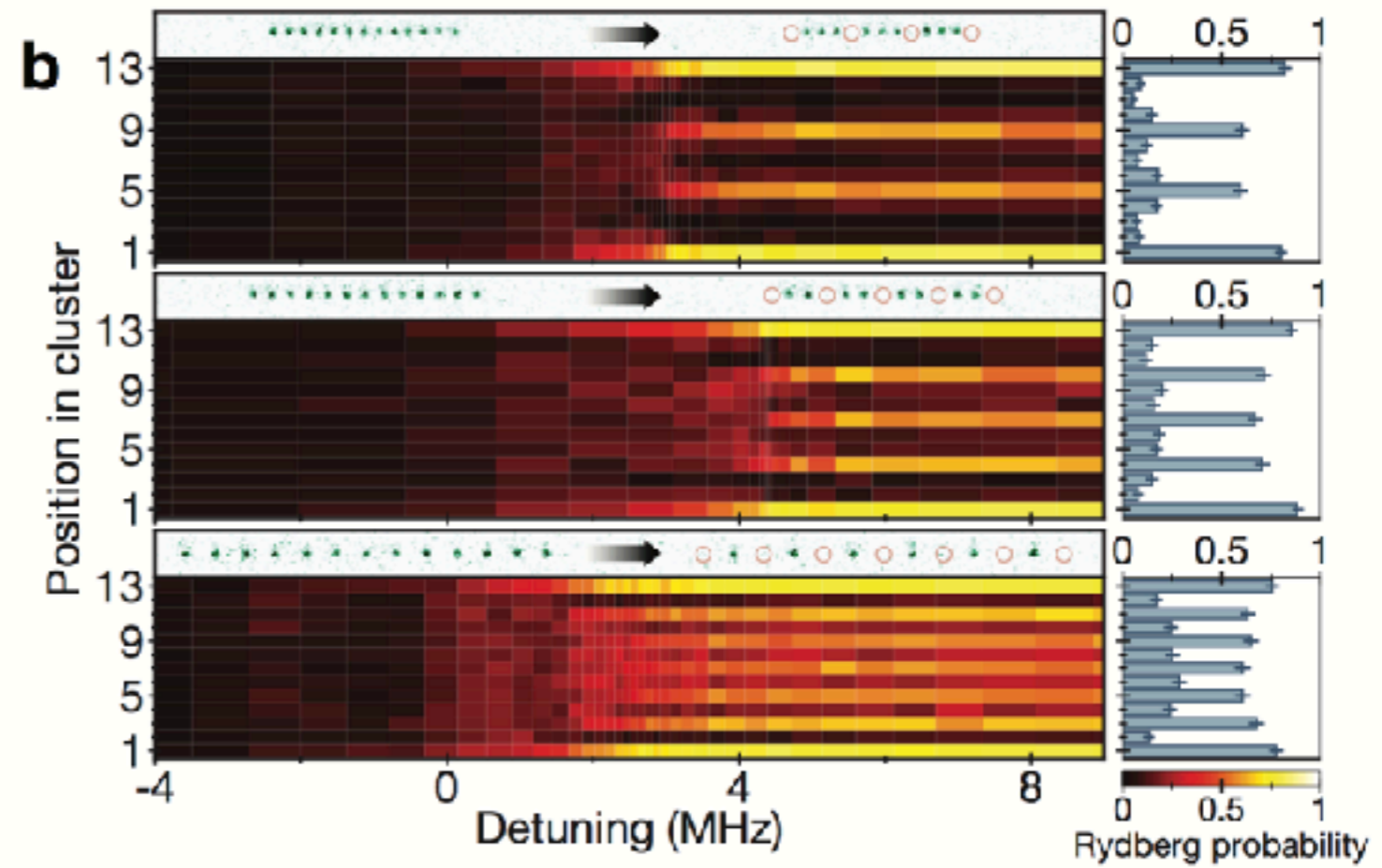
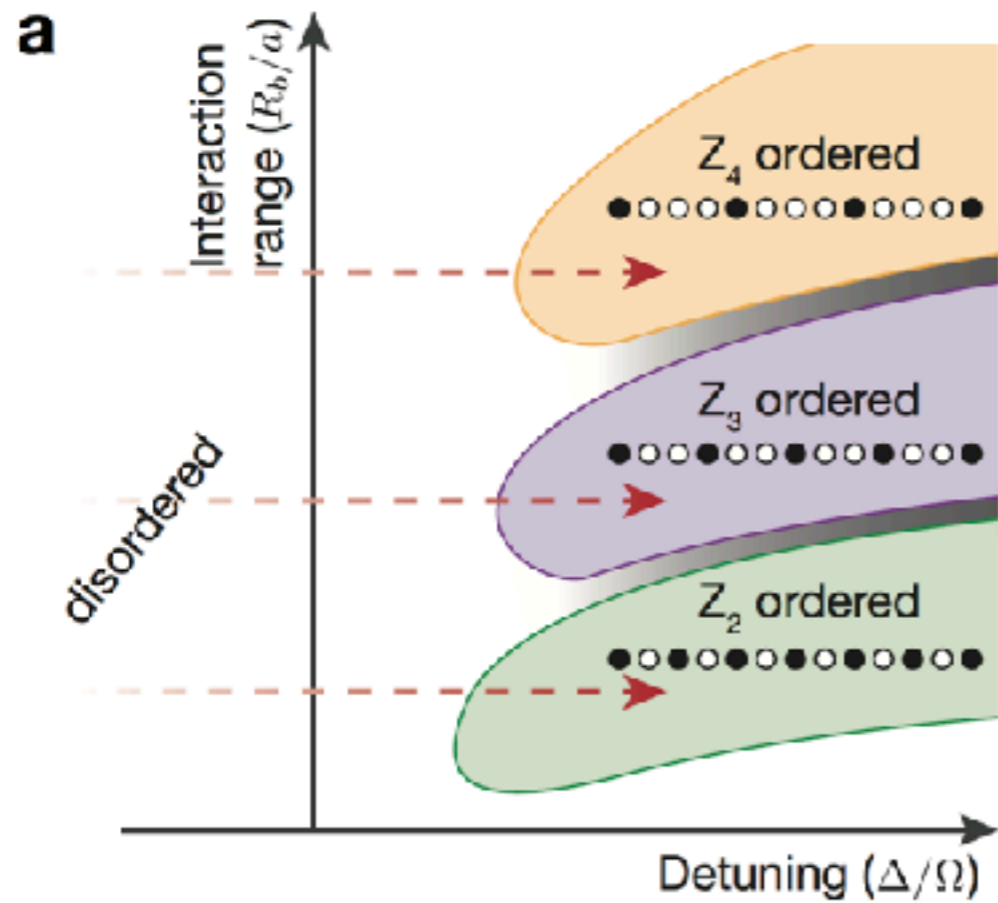
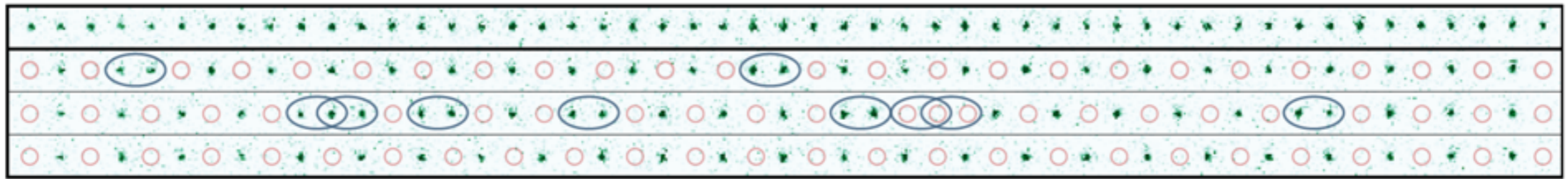
7



8

Rydberg Crystal configurations





Smaller Blockade/Larger Cloud

- ✓ Larger Rydberg Crystals
- ✓ Larger Rydberg Atoms cp. to Lattice Spacing
- ✓ Adiabatic Sweeps to Deterministically Prepare Crystal Structures
- ✓ Show Coherence of Crystalline Superpositions! a **Quantum Crystal?**

T. Pohl et al, (2010), van Bijnen et al. (2011), Gärtner et al. (2012),...

Larger Blockade/Smaller Cloud

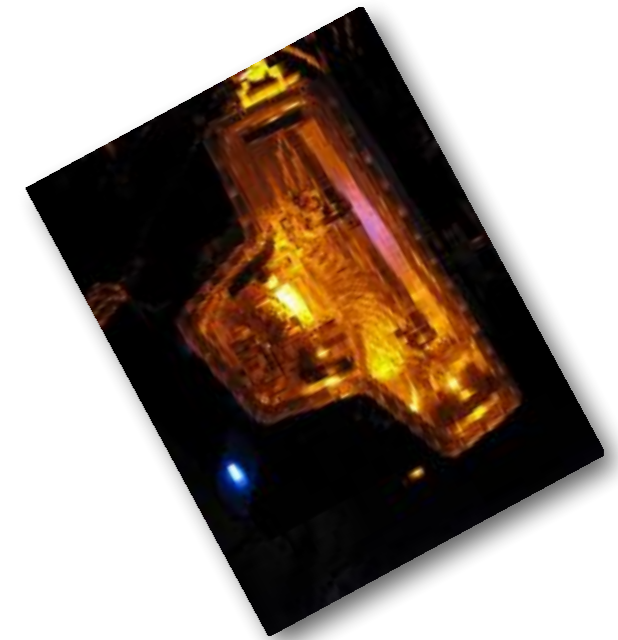
- ✓ Collectively enhanced Rabi oscillations
- ✓ Large Entangled states (e.g. EIT schemes)

M. Lukin et al. (2001), D. Moller et al. (2008), M. Müller et al. (2009), H. Weimer et al. (2009)...

Dressed Rydberg Atom Regime

- ✓ Admix controlled long range interactions

G. Pupillo et al, (2010), Henkel et al. (2010), Schachenmeyer et al. (2010), Honer et al. (2010), Cinti et al. (2010), Johnson & Rolston (2010)...

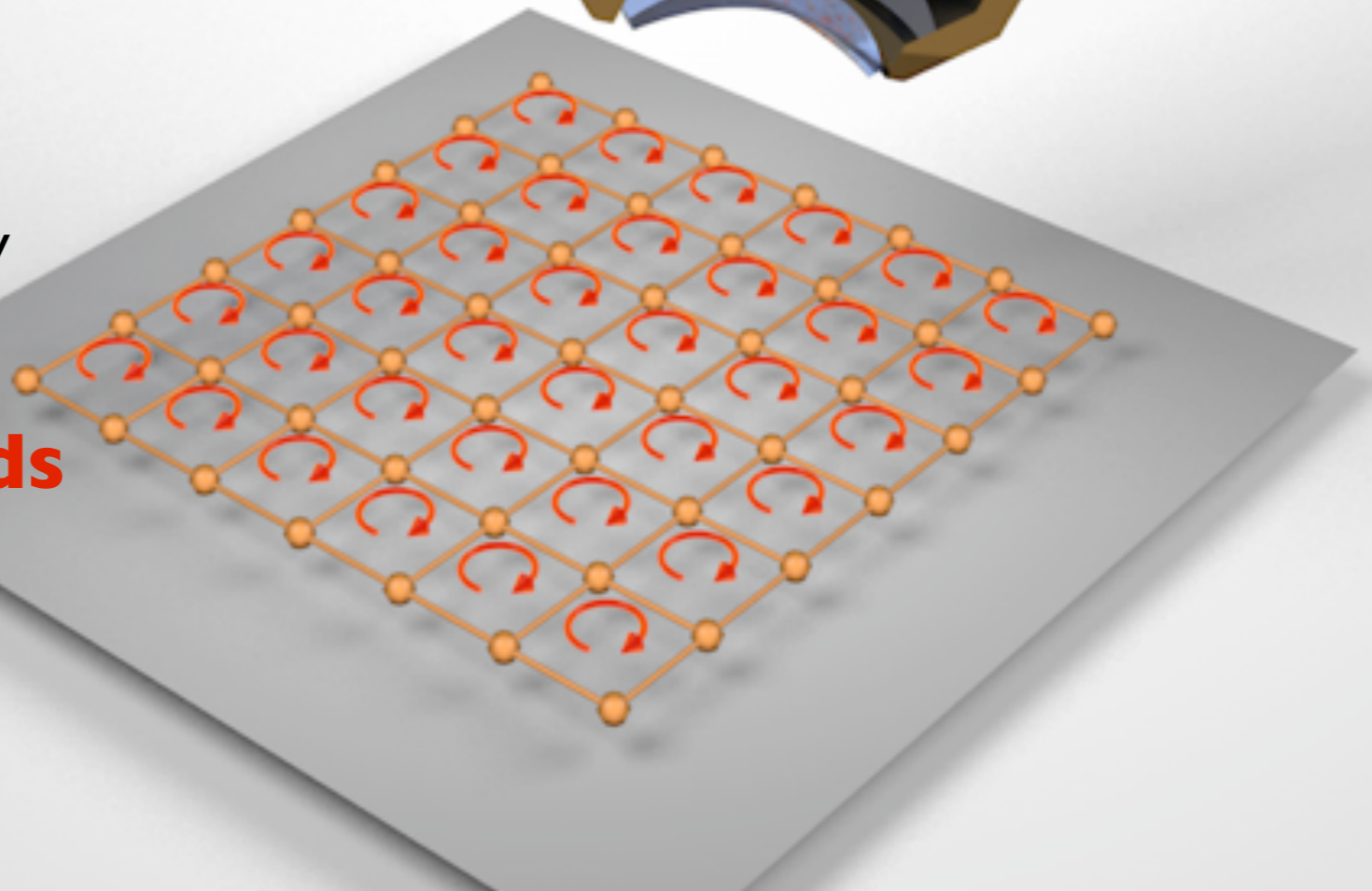
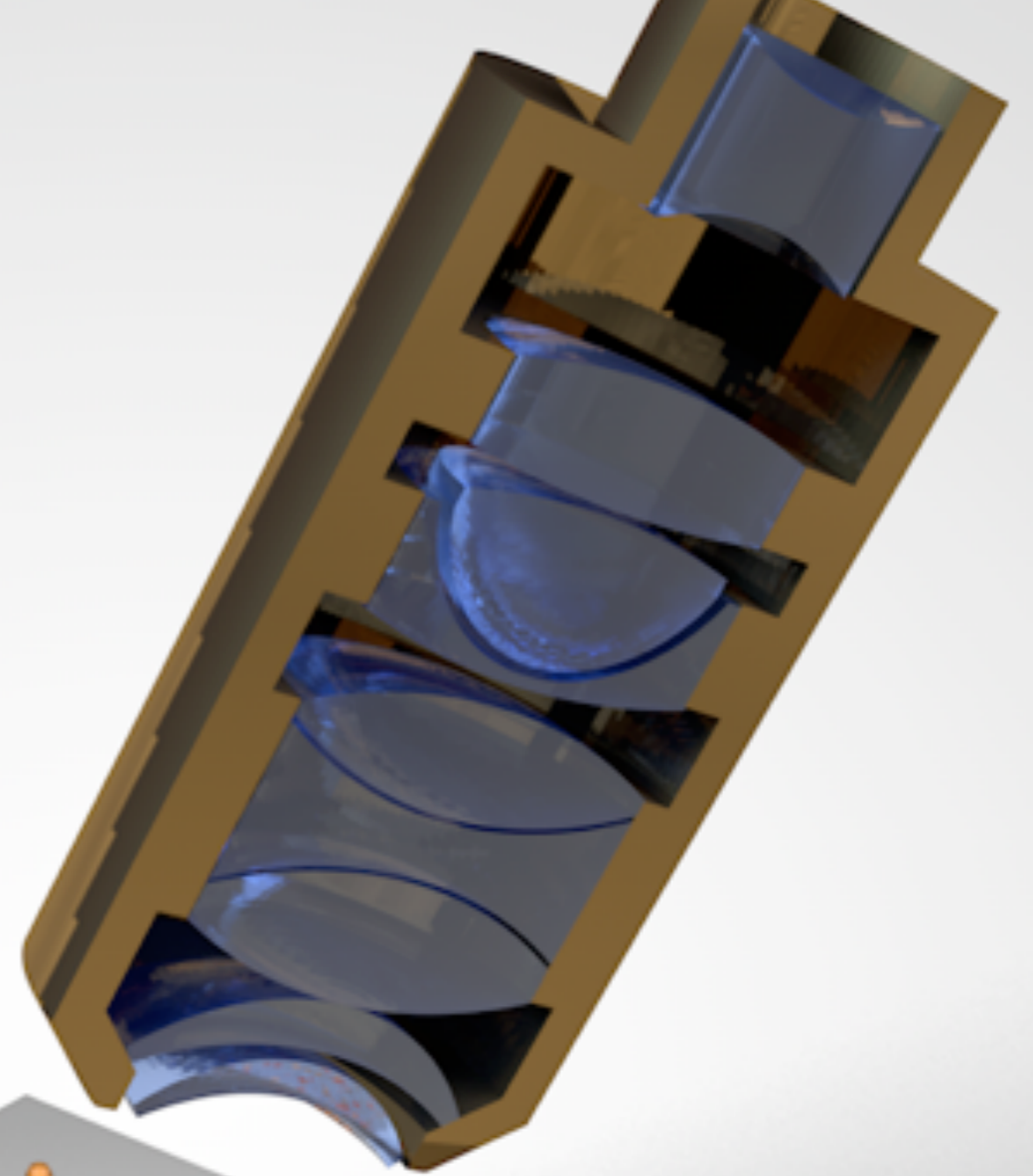


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- Thermalization in Isolated Quantum Systems
- Entanglement Measures in Dynamics
- Supersolids
- Cosmology - Black Hole Models?
- High Energy Physics/String Theory
- New clocks/Navigation

**Quantitative testbeds
for theory!**

⋮





www.quantum-munich.de

Groups of: E. Altman, I. Bloch,
J. Dalibard & P. Zoller

