Minimal Technicolor on the lattice

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Electroweak phase transition, NORDITA 2009

Background:

- **The Higgs field is special in the Standard Model:**
  - it has not been found
  - it is scalar
  - $\Rightarrow$ hierarchy problem, vacuum stability, unitarity bound ...
- **Can we do without a scalar?**
- Consider the EW symmetry breaking and χSB in QCD:

	EWSB	χSB
condensate (Breaks EW):	Higgs vev v	$\bar{\psi}\psi$ chiral condensate
goldstone bosons:	eaten by W,Z (gauged)	π-mesons
radial excitation:	Higgs particle	scalar meson

Technicolor

Technicolor (TC): Electroweak symmetry breaking $\longrightarrow$ chiral symmetry breaking of "Techni-QCD", (technigauge + techniquarks Q), with $\Lambda_{\rm TC} \approx \Lambda_{\rm EW}$.

- Quarks have technicolor and EW charge
- After chiral symmetry breaking:
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson: Higgs
  - $\Rightarrow$ Pseudoscalars $\rightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons

- Works well for the Higgs-gauge sector
- However, the Yukawa sector is messy in the SM $\rightarrow$ messy solutions in Technicolor

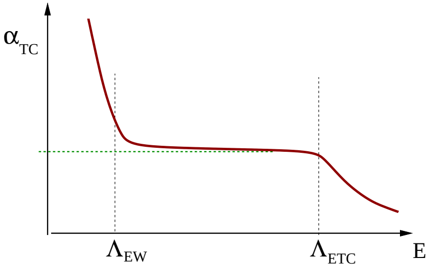
Yukawa couplings to SM quarks?

$\Rightarrow$ **Extended technicolor (ETC):** new gauge interaction coupling normal quarks and techniquarks.

- At low energy, these give $\mathcal{L}_{\rm Yukawa} \sim \frac{1}{\Lambda_{\rm ETC}} \langle \bar{Q} Q \rangle \, \bar{q} \, q$
- **Experimental constraint:** $\Lambda_{\rm ETC} \gg \Lambda_{\rm EW} = \Lambda_{TC}$, due to FCNC's.
- Typically must require $\Lambda_{ETC} \sim 100\text{–}1000 \times \Lambda_{EW}$

Walking coupling

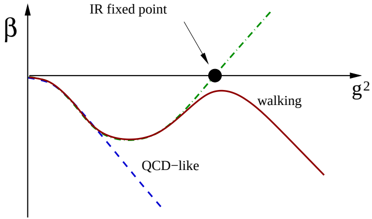
To make this work in practice, the coupling constant of the theory should evolve very slowly over a wide range of energy:



β-function

$$\beta = -\mu \frac{dg}{d\mu}$$

is almost zero at moderately strong coupling



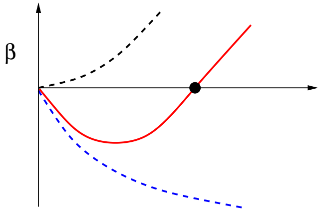
Symmetric representation

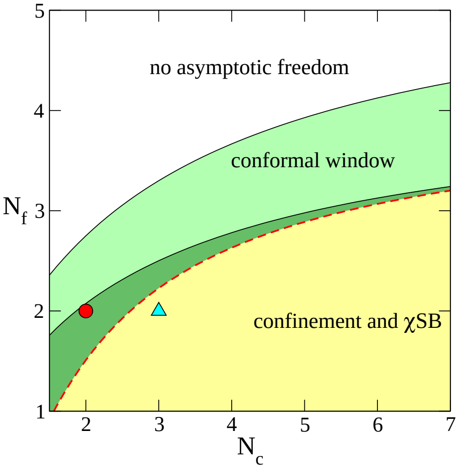
- The required behaviour is difficult to satisfy using fundamental rep. quarks. (Need large $N_f \to$ difficult to avoid FCNC's.)
- However, for SU(N) with 2-index symmetric representation quarks ($\square\square$) $N_f \leq 5$ is sufficient to reach conformal behaviour for any N_c (at least in perturbative analysis).
- There $\exists$ "conformal window" with IR fixed point

  [Dietrich, Sannino, Tuominen; Dietrich, Sannino]

Conformal window

Within the conformal window there ∃ IR fixed point





Lattice simulations have been done at $N_c = 2$, $N_f = 2$ and $N_c = 3$, $N_f = 2$

Why lattice simulations?

- χSB is essentially non-perturbative: lattice simulations are needed to check whether the scenario works.
- Conformal point is at non-zero coupling.
- We study here $N_c = 2$, $N_f = 2$ -case; the simplest model in this class: "Minimal technicolor".
- Also studied by [Catterall, Sannino; Del Debbio, Patella, Pica].
- $N_c = 3$, $N_f = 2$ has been studied by [DeGrand, Shamir, Svetitsky].

What is studied?

- Particle spectrum: do we observe chiral symmetry breaking (QCD) or do all modes become massless as $m_q \to 0$ (no χSB, possibly conformal)
- Measure the evolution of the coupling directly using "Schrödinger functional" method

Model:

- SU(2) gauge action in fundamental rep.
- massless fermions in symmetric ($\equiv$ adjoint for SU(2)) rep.

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} i \gamma_\mu D_\mu \psi$$

- On the lattice:
  - gauge fields U in the fundamental rep.
  - For the fermion action, these transformed into adjoint rep

    $$V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b]$$

    $a, b = 1, 2, 3.$
  - We use standard Wilson action

Lattice phase diagram

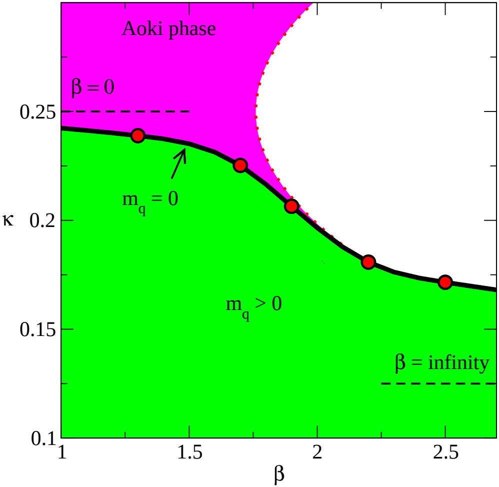
Lattice parameters:

$$\beta = \frac{4}{g^2} \qquad \kappa = \frac{1}{8 + 2m_{q,\mathrm{bare}}}$$

We determine physical quark mass from the axial Ward identity

$$m_q = \lim_{t\to\infty} \frac{1}{2} \frac{\partial_t V_{\mathrm{PS}}(t)}{V_{\mathrm{PP}}(t)}$$

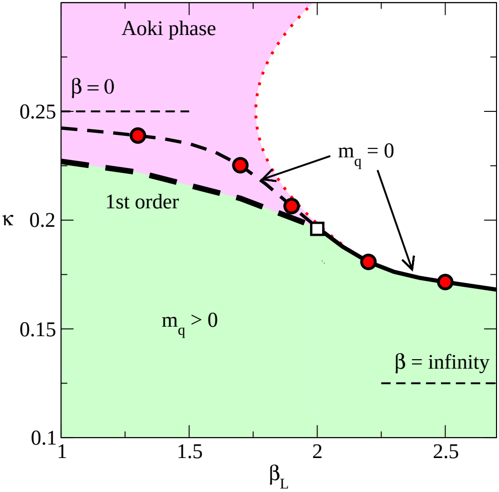
$m_q(\beta, \kappa) = 0$ determines the critical line $\kappa_c(\beta)$



Lattice phase diagram

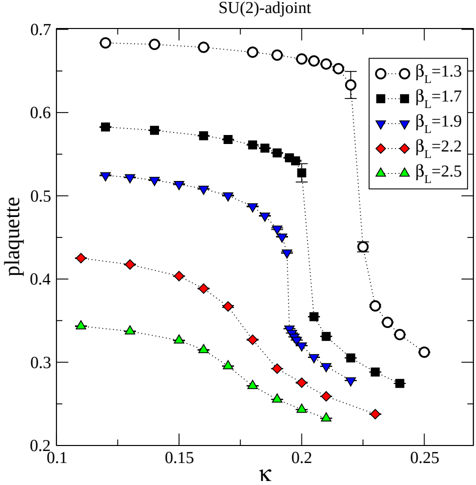
More precisely: at $\beta \lesssim 2$ there appears an unphysical 1st order phase transition at $m_q > 0$, preventing us to go to zero m_q

At $\beta \gtrsim 2$ no sign of a transition, $m_q \to 0$ limit possible. This is the relevant region for us.



1st order transition

The 1st order transition is cleary visible in the plaquette ($\propto F^2_{\mu\nu}$) expectation value (on small volumes)

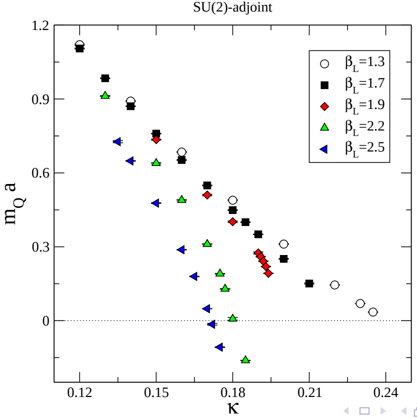


Quark mass

Techniquark mass is determined through the axial Ward identity

$$m_{\rm Q} = \lim_{t\to\infty} \frac{1}{2} \frac{\partial_t V_{\rm AP}(t)}{V_{\rm PP}(t)},$$

where $V_{\rm AP}$ is axial-pseudoscalar and $V_{\rm PP}$ is pseudoscalar-pseudoscalar current



I Particle spectrum

SU(2) + fundamental quarks:

- 2-quark and quark-antiquark states $\bar{q}q$, qq (degenerate except in isoscalar channel)
- glueballs

SU(2) + adjoint quarks:

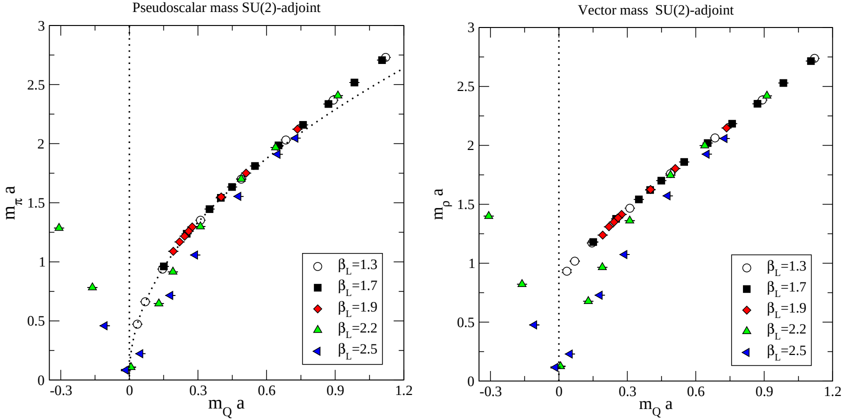
- $\bar{Q}Q$, QQ "mesons" – π, ρ ...
- QQQ "baryons" – "proton"
- Qg quark-gluon state
- glueballs

What to expect for the spectrum:

- If QCD-like χSB: as $m_Q a \to 0$,
  - $m_\pi \propto m_Q^{1/2}$
  - other states have finite mass.
- If IR fixed conformal point: $m_Q a \to 0$, all states become massless.
- If walking behaviour: at high energy $\sim$ conformal, at small χSB.
- On the lattice extrapolation $m_Q a \to 0$ is required. Too large m_Q or too small V can lead to misleading results.

Results

Pseudoscalar ("π") and vector ("ρ") masses

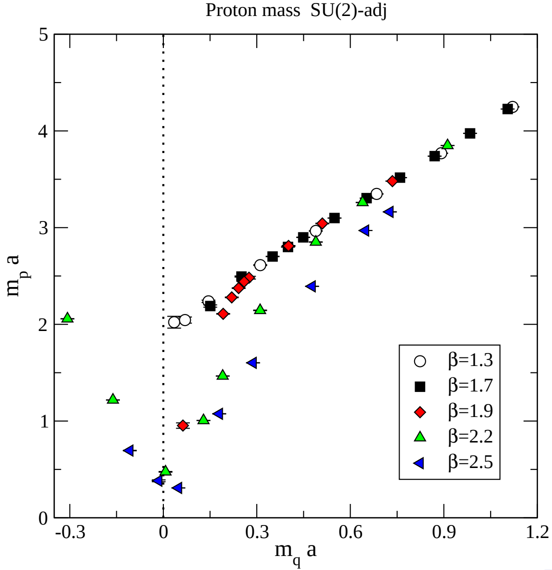


At small β, looks like χSB. However, we cannot go to $m_q \to 0$ because of the 1st order transition.

At large β masses $\to 0$?

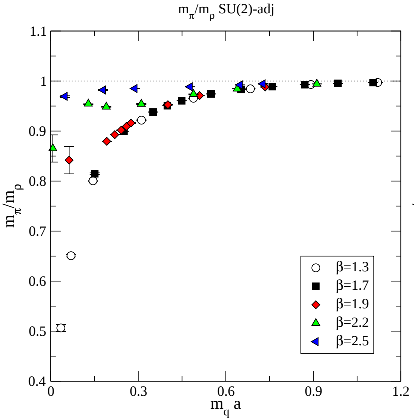
Results

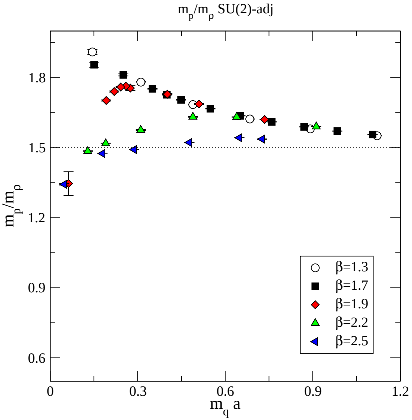
3-quark state ("proton") mass



Results

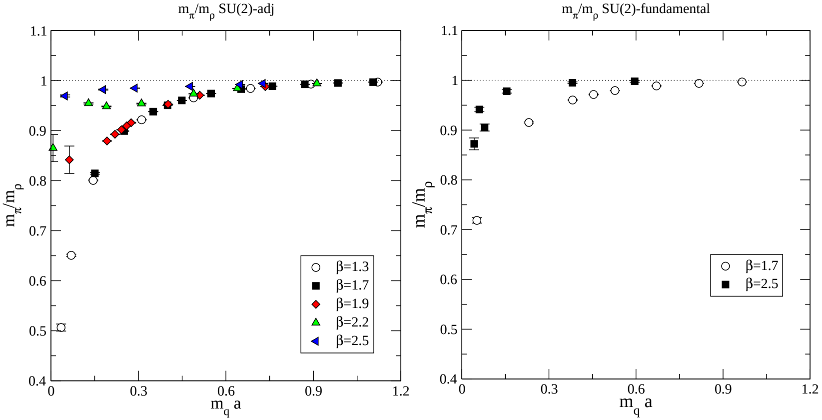
Mass ratios m_π / m_ρ and $m_{\mathrm{Proton}} / m_\rho$





At large β looks like $\sim$ massless, possibly conformal.
However, . . .

Compare with fundamental rep.



Fundamental rep looks also $\sim$ massless at large β!

What does this imply?

In fundamental rep we know what happens:

- There is χSB, observed at small $\beta = 4/g^2$
- At large β lattice spacing is small, and lattice size $L \ll 1/\Lambda \sim$ hadron size. Thus, system looks like $\sim$ conformal.
- This is a finite resolution issue; at large enough volume and high enough numerical accuracy χSB is again observed at any β

In adjoint rep:

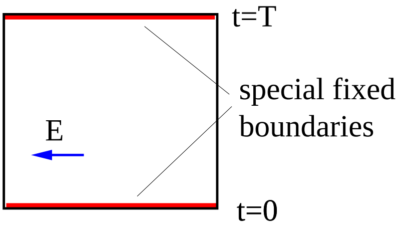
- Theory compatible with conformal at large β (small coupling), but could be also finite volume/resolution issue as in fundamental rep.
- Mass spectrum not sufficient to tell the difference!
- Direct evaluation of β-function required!

II Evolution of the coupling

Schrödinger functional: Generate a *background* chromoelectric field using non-trivial boundary conditions, parametrised by angle η

At the classical level, we have

$$\frac{dS_{\text{class.}}}{d\eta} = \frac{A}{g^2}$$



where $A(\eta)$ is a known constant.
At the quantum level, we measure the coupling through

$$\begin{aligned} \frac{1}{g^2} &= \frac{1}{A}\frac{dS}{d\eta} \\ &= \text{const.} \times \langle(\text{boundary plaq.})\rangle \end{aligned}$$

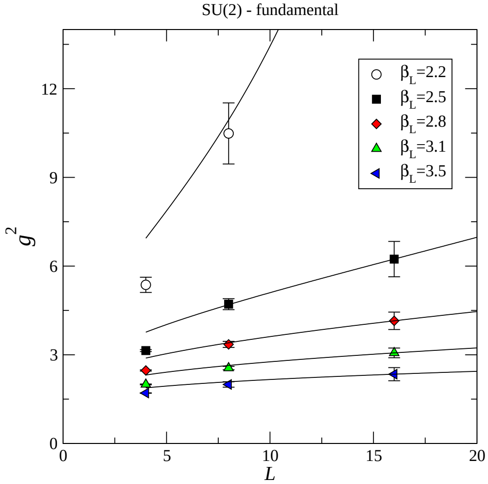
- Evaluates $1/g^2$ at length scale L, the lattice size
- This has been used very succesfully in QCD by Alpha collaboration

Evolution of the coupling

Measure g^2 at different β_L (lattice spacing a) and lattice sizes

Testing with **fundamental representation**:

- L/a grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \rightarrow$ small lattice spacing $\rightarrow$ small volume
- Continuous line: coupling evaluated from 2-loop perturbative β-function (fixed to measurement at $L/a = 16$)

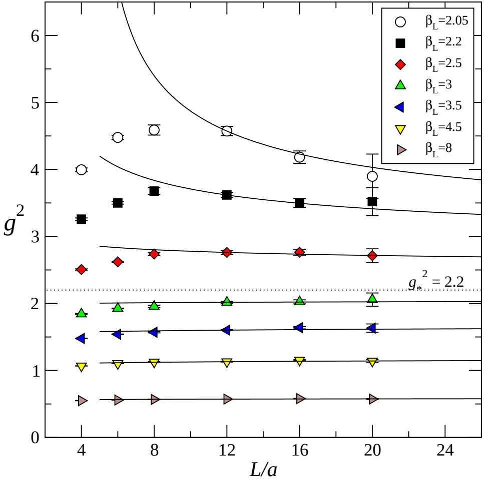


Evolution of the coupling

Measure g^2 at different β_L (lattice spacing) and lattice sizes

In **adjoint representation**:

- At large β_L: $g^2(L)$ is small and increases with L (asymptotic freedom)
- At small β_L: $g^2(L)$ large and *decreases* as L increases
  $\Rightarrow$ β-function positive here!
- As $L/a \to \infty$, in all cases $g^2(L)$ apparently flows towards a fixed point, $g_*^2 \approx 2 \ldots 3$.
  $\Rightarrow$ conformal behaviour!
- Continuous line: coupling evaluated with fitted β-function ansatz (later)

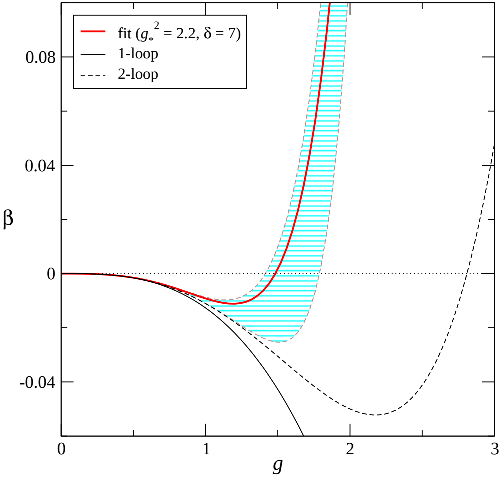


β-function

We can describe β-function with an ansatz

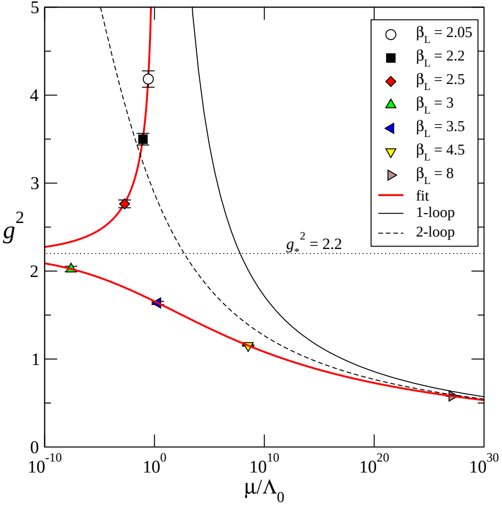
$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

where b_1, b_2 are perturbative constants and b_3 and δ are fit parameters:



Coupling constant

Integrating the β-function we obtain the coupling:
Asymptotically free branch at $g^2 < g_*^2$, non-free branch at $g^2 > g_*^2$. These are disconnected!
Large error bands (not shown)



Conclusions

- SU(2)+2 adjoint fermions appears to have an IR fixed point.
- No χSB, no "walking".
- Can we deform this to walking? Yes, for example by giving small mass to techniquarks:
  – when $\mu \gg m_q$, we are flowing towards the IR fixed point
  – when $\mu \lesssim m_q$, quarks decouple and pure SU(2) gauge theory dominates: confinement, (χSB?)
- Quark mass term against the spirit of technicolor – coupling to Standard Model fields?
- Work to be done: use improved fermions (clover), walking (with m_q), finite T, different groups and reps . . .
- Predictions obtainable from this TC model:
  - Measure $\langle \bar{Q}Q \rangle$, set to v_{Higgs}
  - Measure QQ scalar mass $\rightarrow$ Higgs mass
  - Exotic particle spectrum (ρ: lightest exotic particle)
  - Modified by ETC corrections