Introduction to Quantum Transport

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Outline

Sakharov conditions

Baryogenesis is one of the cornerstones of the Cosmological Standard Model and tries to explain the observed baryon asymmetry of the Universe (BAU)

$$\eta = \frac{n_B - n_{\bar{B}}}{s} = 0.9 \times 10^{-10}.$$

The celebrated Sakharov conditions state the necessary ingredients for baryogenesis:

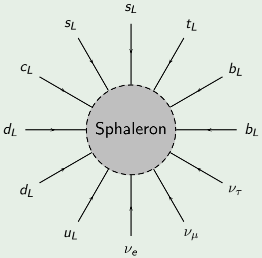
Sakharov conditions

- Baryon number violation
- Charge (C) and charge parity conjugation (CP) are no symmetries
- non-equilibrium

C- and B-violation: The sphaleron process

In the hot universe B- and C-violation is present due to sphaleron processes ('temperature induced instanton').

The effective sphaleron vertex



- $\Delta B = 3,\ \Delta L = 3,\ \Delta N_{CS} = 1$
- $B - L$ conserving
- $B + L$ violating
- Exponentially suppressed after the phase transition (m_W)
- Topological effect of the SU(2) gauge sector

HAII

EWPT is last chance of baryogenesis ($\phi < T$).

First-order electroweak phase transition

Comments on bubble nucleation

- Order parameter of the phase transition is the Higgs vev $\langle h \rangle$
- During the phase transition the particles change their masses
- This is a violent process ($v \sim c$)
- For EWBG, the PT has to be strong, $\phi \gtrsim T$
- In the SM, there is only a cross-over ($m_{Higgs} > 60$ GeV)

KAJANTIE, LAINE, RUMMUKAINEN, SHAPOSHNIKOV ('96)
A first-order electroweak phase transition requires BSM physics.

CP Violation in Mass Matrices

CP violation: Chargino masses in the MSSM and bMSSM

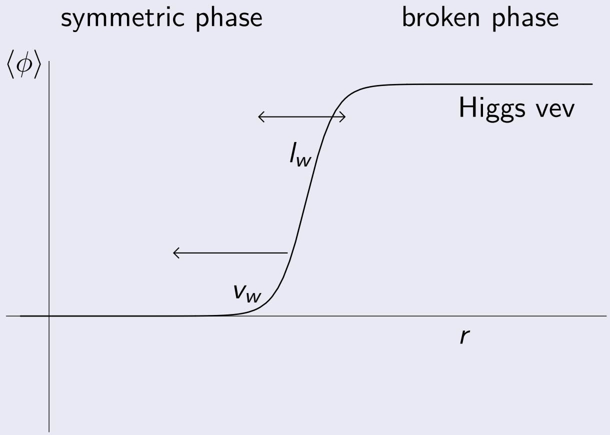
In the MSSM case the mass matrix of the charged higgsinos/winos is:

$$m = \begin{pmatrix} M_2 & g\, h_2(x^\mu) \\ g\, h_1(x^\mu) & \mu_c \end{pmatrix}$$

with M_2 and μ_c containing a CP-odd complex phase. The Higgs fields are during the phase transition space-time dependent.

Picture of Electroweak Baryogenesis

Shaposhnikov ('87)



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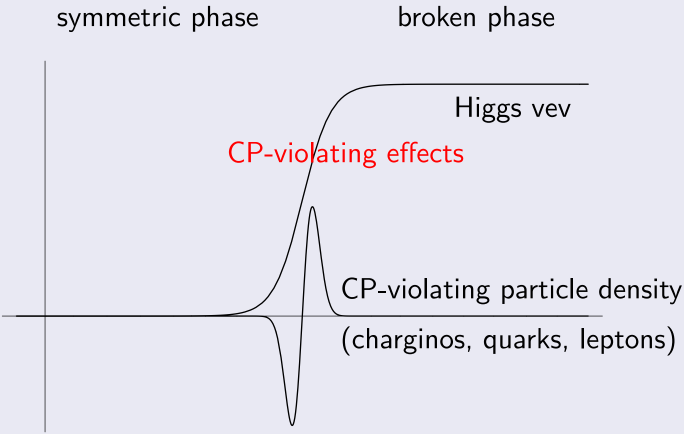

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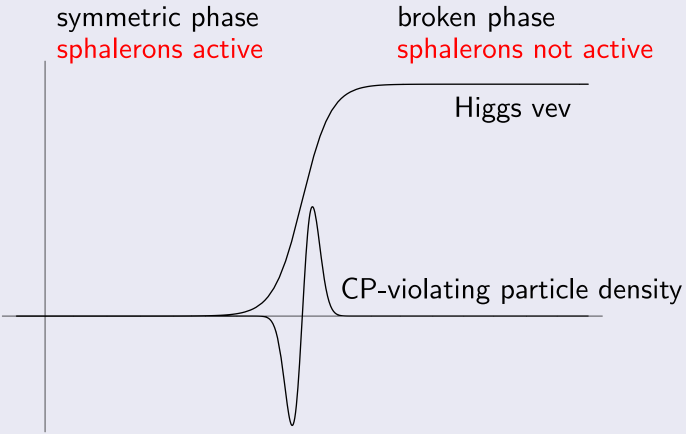
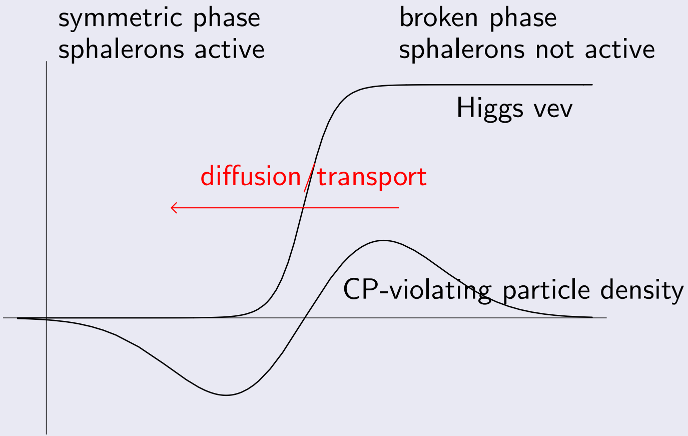


Picture of Electroweak Baryogenesis

Cohen, Kaplan, Nelson ('90)



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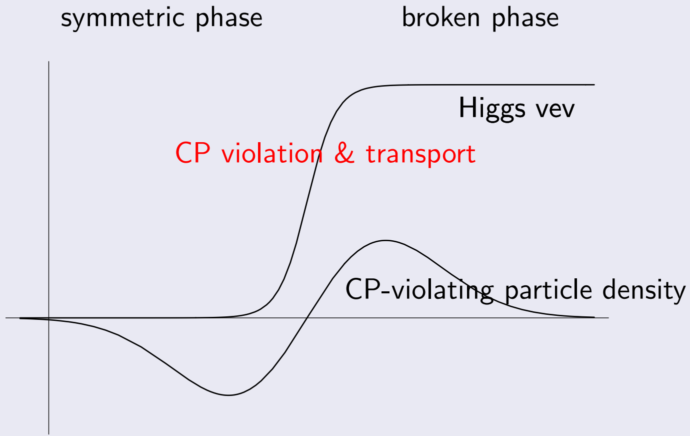
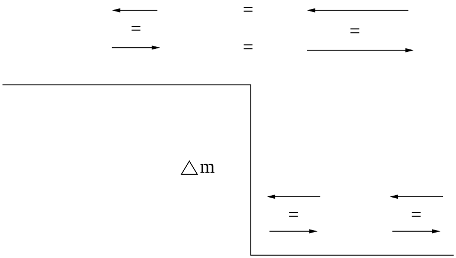


Outline

Particle reflection, $v = 0$

Equilibrium with same temperatures is a steady-state solution. Clasically, particles climb the wall if possible and just replace each other, distribution functions in the wall frame depend on $u_\mu p^\mu = E$.

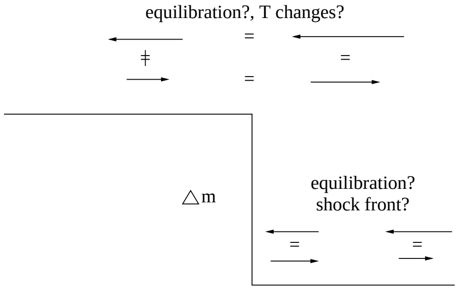
$E,\ \sqrt{p_z^2 - \Delta m^2}$ $\qquad\qquad$ $E,\ p_z$



Particle reflection, $v \ll 1$

For small velocities, a steady-state solution requires interactions (equilibration). The particle distribution functions in equilibrium depend an the wall frame on $u_\mu p^\mu = \gamma(E - v p_z)$.

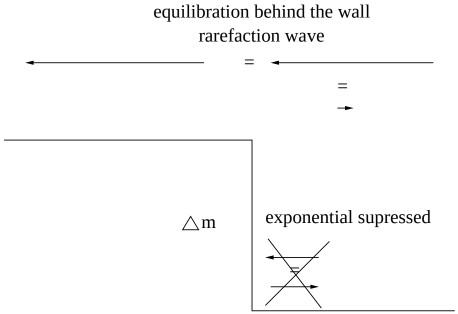
$$E, \sqrt{p_z^2 - \Delta m^2} \qquad\qquad E, p_z$$



Particle reflection, $v \approx 1$

For large velocities, equilibration takes place behind the wall.

$$E, \sqrt{p_z^2 - \Delta m^2} \qquad\qquad E, p_z$$



Bodeker, Moore ('09)

Particle reflection

A moving Higgs wall drives the plasma out of equilibrium

- in front of the wall due to reflections
- behind the wall due to a change in the particle momenta

The picture of particle reflection is only valid for thin walls, $l_w \ll 1/g^2 T$.

Outline

Challenge in EWBG

Connection between macroscopic and microscopic scales

- In electroweak baryogenesis, one is rather interested in the case of thick walls which can be hardly treated in the pseudo-particle reflection picture due to multiple scatterings
- CP violation is a microscopic/quantum effect produced by the interaction of single quanta in the plasma with the wall.
- Transport is a macroscopic/classical effect based on statistical physics and particle densities.

Classical Boltzmann Equations

The system is described by the particle distribution function $n(\mathbf{x}, \mathbf{v}, t)$ on classical phase space.

The Boltzmann equations are based on a particle picture. If on the particles in the plasma acts an external force F, one obtains

$$n(\mathbf{x} + \mathbf{v}\delta t, \mathbf{v} + \mathbf{F}\delta t, t + \delta t) - n(\mathbf{x}, \mathbf{v}, t) = \text{collisions/interactions},$$

and thus

$$(\partial_t + \mathbf{v} \cdot \nabla + \mathbf{F} \cdot \partial_{\mathbf{v}}) n(\mathbf{x}, \mathbf{v}, t) = \text{collisions/interactions}.$$

How can CP violation be incorporated in this classical picture?

In which semi-classical limit can one obtain a phase-space from a quantum theory?

Summary of Approaches to Transport

Semi-classical force / WKB approach

Joyce, Prokopec, Turok, hep-ph/9401352, hep-ph/9408339

Joyce, Cline, Kainulainen, hep-ph/9708393

Huber, Schmidt, hep-ph/0003122

Perturbative mixing / mass insertion approach

Carena, Moreno, Quiros, Seco, Wagner, hep-ph/0011055

Carena, Quiros, Seco, Wagner, hep-ph/0208043

Cirigliano, Profumo, Ramsey-Musolf, hep-ph/0603246

Kadanoff-Baym approach

Kainulainen, Prokopec, Schmidt, Weinstock, hep-ph/0105295

Konstandin, Prokopec, Schmidt, hep-ph/0410135

Konstandin, Prokopec, Schmidt, Seco, hep-ph/0505103

Huber, Konstandin, Prokopec, Schmidt, hep-ph/0606298

Summary of former Approaches

Approach	Cline/Joyce Kainulainen	Carena/Moreno/Quiros Seco/Wagner
CP-violation	dispersion relation WKB	local source term perturbation theory
basis	mass eigenbasis	flavour eigenbasis
quasi-particles	charginos	higgsinos/winos
transport	classical Boltzmann type	classical diffusion
mixing	not included	in the source not in the diffusion
$\hbar$ **order**	second order	first order
comment	momentum?	basis?, finite?

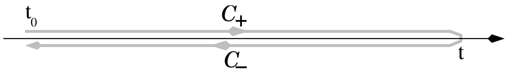
Outline

Transport Theory and EWBG

Starting point of our formalism are the Kadanoff-Baym equations that are the statistical analogue to the Schwinger-Dyson equations:

$$\int d^4z \left(S_0^{-1}(x_\mu, z_\mu) - \Sigma(x_\mu, z_\mu)\right) S(z_\mu, y_\mu) \quad = \quad \mathbb{1}\, \delta(x_\mu - y_\mu)$$

where the self-energy Σ and the Green function S contain an additional 2×2 structure from the *in-in*-formalism



$$\Sigma = \begin{pmatrix} \Sigma^t & \Sigma^> \\ \Sigma^< & \Sigma^{\bar t} \end{pmatrix} = \begin{pmatrix} \Sigma^{++} & \Sigma^{+-} \\ \Sigma^{-+} & \Sigma^{--} \end{pmatrix}, \quad S = \begin{pmatrix} S^t & S^> \\ S^< & S^{\bar t} \end{pmatrix}.$$

Of these four entries only two are independent and $S^<$ encodes the particle distribution function.

Transport Theory and EWBG

In Wigner space we have

$$X_\mu = (x_\mu + z_\mu)/2, \quad k_\mu = FT(x_\mu - z_\mu).$$

Then the Kadanoff-Baym equations read

$$e^{-i\Diamond}\{S_0^{-1} - \Sigma, S\} \;=\; 0.$$

with

$$2\Diamond\{A, B\} := \partial_{X^\mu} A \, \partial_{k_\mu} B - \partial_{k_\mu} A \, \partial_{X^\mu} B$$

and all quantities are functions of X_μ and k_μ. The super-/sub-scripts $<, >, R, \mathcal{A}$ denote the additional 2×2 structure of the *in-in formalism*.

Without interaction

$$e^{-i\Diamond}\{S_0^{-1}(z), S^<\} = 0, \quad e^{-i\Diamond}\{S_0^{-1}(z), S_\mathcal{A}\} = 0$$

Particle densities

Using the KMS condition and the correct normalization one obtains in equilibrium

$$iS^{<} = 2\pi \,\mathrm{sign}(k_0)\, \delta(k^2 - m^2)\; n(k_0), \qquad n(k_0) = \frac{1}{\exp(\beta k_0) - 1}$$

The particle density can (also away from equilibrium) be read off from $S^{<}$

$$\int_{k_0>0} \frac{dk_0}{2\pi}\, 2ik_0\, S^{<} = n(\mathbf{k}, t, \mathbf{x}).$$

Thus the Wigner space yields in the semi-classical limit the usual phase space (however, not necessarily positive).

A Simple Example

Free bosonic theory with one flavour and a **constant** mass:

$$e^{-i\diamondsuit}\{S_0^{-1}, S^<\} = 0, \quad S_0^{-1} = k^2 - m^2.$$

The hermitian/anti-hermitian parts are the so called constraint/kinetic equations:

$$(k^2 - m^2)S^< = 0, \quad k^\mu \partial_{X_\mu} S^< = 0$$

which at low energies $k^\mu = (m, m\mathbf{v})$ is of Boltzmann type

$$m(\partial_t + \mathbf{v} \cdot \nabla)S^< = 0.$$

Gradient Expansion

Consider

$$e^{-i\diamondsuit}\{S_0^{-1}(z), S^<\} = 0, \quad S_0^{-1} = k^2 - m^2(z)$$

Since the background in the MSSM is weakly varying ($l_w \approx 20/T_c$) the Moyal star product can be simplified by the semi-classical approximation

$$\partial_k \partial_X \approx \frac{1}{T_c l_w} \approx \frac{1}{20} \ll 1 \quad \rightarrow \quad e^{-i\diamondsuit} \approx 1 - i\diamondsuit - \frac{1}{2}\diamondsuit^2 \cdots$$

The simplest example for a transport equation in a varying background is for one bosonic flavour with real mass (up to first order in $\diamondsuit$)

$$\begin{aligned} (k^2 - m^2)S^< &= 0 \\ (k^\mu \partial_\mu - \frac{1}{2}(\partial_z m^2)\, \partial_{k_z})S^< &= 0. \end{aligned}$$

Fast walls and reflection

For fast walls, most particles in the plasma have high $p_z \gg \Delta m, 1/l_w$ and hence the Kadanoff-Baym approach should agree with the reflection picture

Integration yields that the plasma depends behind the wall on

$$u_\mu p^\mu - u_z \frac{\Delta m^2}{2p_z} + O(\Delta m^4/p_z^4)$$

what for $v \approx 1$ this agrees with the former result from reflections

$$p_z \to \sqrt{p_z^2 - \Delta m^2} \approx p_z - \frac{\Delta m^2}{2p_z}.$$

CP violation

Prokopec, Schmidt, Weinstock ('01)

Consider a fermion with complex mass term

$$S_0^{-1} = \not{k} - P_L\, m(z) - P_R\, m^*(z)$$

and

$$m(z) = |m(z)|\, e^{i\theta(z)}.$$

Then the kinetic equation up to second order in $\Diamond$ reads (s denotes the spin of the particle)

$$\left[k^\mu \partial_\mu - \frac{1}{2}(\partial_z m^2)\, \partial_{k_z} - \frac{s}{2k_0}\partial_z (m^2 \partial_z \theta)\, \partial_{k_z} \right] S_s^<(k^\mu, z) \quad = \quad 0$$

which leads to CP-violating particle densities. This agrees with the findings of Cline/Joyce/Kainulainen in the WKB framework.

Fermionic Systems

After spin projection the fermionic system of equations reads

$$
\begin{aligned}
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right) S_0^s - (2isk_z + s\partial_z)\, S_3^s - 2im_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_1^s - 2im_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_2^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right) S_1^s - (2sk_z - is\partial_z)\, S_2^s - 2im_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_0^s + 2m_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_3^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right) S_2^s + (2sk_z - is\partial_z)\, S_1^s - 2m_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_3^s - 2im_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_0^s &= 0\\
\left(2i\tilde{k}_0 - \frac{k_0\partial_t + \vec{k}_{\|}\cdot\nabla_{\|}}{\tilde{k}_0}\right) S_3^s - (2isk_z + s\partial_z)\, S_0^s + 2m_h e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_2^s - 2m_a e^{\frac{i}{2}\overleftarrow{\partial_z}\overrightarrow{\partial_{k_z}}} S_1^s &= 0\,,
\end{aligned}
$$

where $S_0 \dots S_3$ are 2×2 matrices in flavour space and s denotes the spin.

Transport in the Chargino sector

Konstandin, Prokopec, Schmidt ('04)

The chargino transport equations for the left/right handed densities are of the form

$$k^\mu \partial_\mu S^< + \frac{i}{2}\left[m^2, S^<\right] + sources/forces = collisions$$

Comments

- $S^<$ is a 2×2 flavor matrix
- The term $\frac{i}{2}\left[m^2, S^<\right]$ will lead to an oscillatory behaviour of the off-diagonal particle densities, similar to neutrino oscillations with frequency $\sim (m_1^2 - m_2^2)/k_z$.
- The source contains first order contributions that correspond to the sources in the approach of Carena et al.
- The source contains second order contributions that correspond to the sources in the approach of Cline et al.

Sources for EWBG

This approach resembles two mechanisms of EWBG from former approaches Joyce, Prokopec, Turok ('96) Cline, Joyce, Kainulainen ('97,'00) Fromme, Huber ('06)

The dispersion shift source from the WKB approach:

$$\mathcal{S}^{(2)} \sim \left\{ m^{\dagger\prime\prime} m - m^{\dagger} m^{\prime\prime}, \partial_{k_z} S^{<} \right\} .$$

Carena, Moreno, Quiros, Seco, Wagner ('00)
Carena, Quiros, Seco, Wagner ('02)
Cirigliano, Profumo, Ramsey-Musolf ('06)
Cirigliano, Ramsey-Musolf, Tulin, Lee ('06)

Sources from flavor mixing effects, e.g.

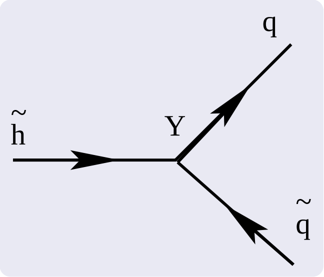
$$\mathcal{S}^{(1)} \sim \left[m^{\dagger\prime} m - m^{\dagger} m^{\prime}, \partial_{k_z} S^{<} \right] .$$

CP violation from mixing appears only on the off-diagonal in the mass eigenbasis.

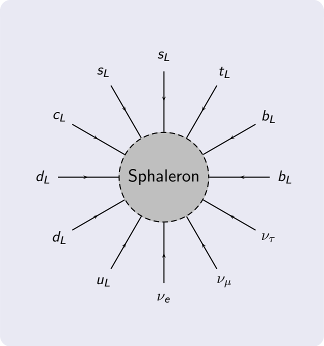
Determination of the BAU

Huet, Nelson ('95)

The missing parts to determine the baryon asymmetry of the universe are:



and



Diffusion and the Sphaleron

The originally used system of diffusion equations is of the form (for a recent treatment see Chung, Garbrecht, Tulin ('08))

$$
\begin{aligned}
v_w\, n_Q' &= D_q\, n_Q'' - \Gamma_Y \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H} \right] - \Gamma_m \left[\frac{n_Q}{k_Q} - \frac{n_T}{k_T} \right] \\
&\quad -6\,\Gamma_{ss} \left[2\,\frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9\,\frac{n_Q + n_T}{k_B} \right] + \text{Sources} \\
&\dots
\end{aligned}
$$

where n_Q, n_T, n_H, n_h denote particle densities, $\Gamma_{ss}, \Gamma_m, \Gamma_Y$ interaction rates, k_Q, k_T, k_H statistical factors and D_q a diffusion constant.

However, these equations are classical and back-reactions on the charginos cannot be taken into account in our approach.

Advantages and Disadvantages

No ambiguities
No divergences
WKB and mixing effects
Flavor oscillations

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No ambiguities
No divergences
WKB and mixing effects
Flavor oscillations

No quantum transport in quark sector
No quantum back-reactions on the charginos

Outline

EWBG in the MSSM

CP violation: Chargino masses in the MSSM

In the MSSM case the mass matrix of the charged higgsinos/winos is:

$$m = \begin{pmatrix} M_2 & g\, h_2(x^\mu) \\ g\, h_1(x^\mu) & \mu_c \end{pmatrix}$$

with M_2 and μ_c containing a CP-odd complex phase. The Higgs fields are during the phase transition space-time dependent.

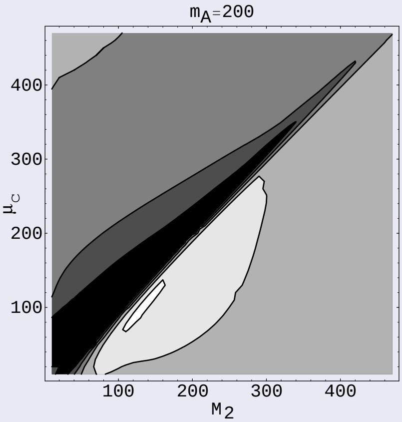
Phase transition in the MSSM

In the MSSM the strength of the phase transition depends mostly on the loop effects of the bosons. A strong phase transition fulfilling the current mass bounds on the Higgs is possible if the stops are relatively light, $m_{top} \sim m_{stop}$.

Numerical Results

Konstandin, Prokopec, Schmidt, Seco ('05)

Parameters chosen: $v_w = 0.05$, $l_w = 20/T_c$, CP-phase maximal.



Due to the flavor oscillations, EWBG requires in the MSSM quasi-degenerate chargino masses.

Conclusions in the MSSM

CP violation in the MSSM is based on mixing between different flavors (charginos).

MSSM electroweak baryogenesis is a constrained scenario

- A light stop to acquire a strong first-order phase transition
- The condition $\mu_c \approx M_2 \lesssim 400$ GeV of the *a priori* unrelated parameters M_2 and μ_c
- A large CP-violating phase that is testable by next generation EDM experiments

Why is the nMSSM interesting?

PANAGIOTAKOPOULOS, PILAFTSIS ('02)

The nearly Minimal Supersymmetric Standard Model has the following effective superpotential

$$W_{nMSSM} = \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 - \frac{m_{12}^2}{\lambda} \hat{S} + W_{MSSM},$$

and has the virtues to solve the μ-problem of the MSSM by introducing a dynamical μ-term

$$\mu = -\lambda \langle S \rangle \, .$$

In this model singlet self-couplings are forbidden by a R'-symmetry. The resulting model has neither problems with the stability of the hierarchy nor with domain walls (but λ might develop a Landau pole).

CP violation: Chargino masses in the nMSSM

Huber, Konstandin, Prokopec, Schmidt ('06)

In the nMSSM the μ term contains a z-dependent complex phase

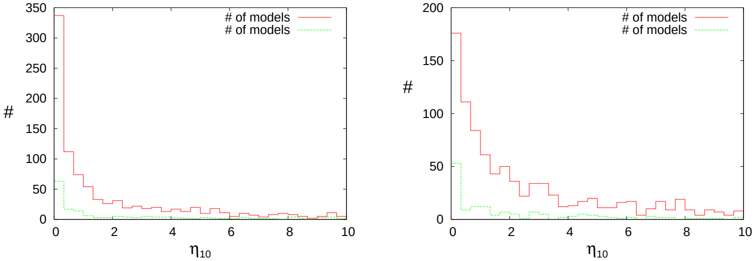
$$\mu(z) = -\lambda \langle S \rangle = -\lambda \phi_s(z) \, e^{i q_s(z)}$$

In the nMSSM second order sources dominate

- The dynamical parameter $\mu = \lambda \langle S \rangle$ leads to a dominating sources of WKB type
- Charginos are generically non-degenerate ($M_2 \gtrsim \mu$)
- Thin wall profiles

Numerical Results

A numerical analysis of the BAU leads to the following result (sets passed LEP constraints and have a first order phase transition)



The left (right) plot shows the generated BAU for $M_2 = 1$ TeV ($M_2 = 200$ GeV). 50% (63%) of the models are in accordance with observation. The lower models fulfill the the EDM bounds with 1 TeV sfermion masses, 4.8 % (6.2 %).

Conclusions in the nMSSM

CP violation in the nMSSM is based on mixing between different chiralities.

EWBG in the nMSSM is very promising

- Strong first order phase transition due to tree-level dynamics
- $\eta_{10} \gtrsim 1$ for most of the parameter space
- EDMs eventually small due to small $Arg(M_2 \mu_c)$
- two loop EDMs relatively small due to $tan(\beta) \sim O(1)$

Outline

Final Remarks on EWBG

The Kadanoff-Baym equations provide a first principle approach to quantum transport.

They unite semi-sclassical force and mixing effects in one framework.

Electroweak baryogenesis is the main application of quantum transport equations so far.

Flavored leptogenesis?