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Defect formation

Arttu Rajantie

Nordita, 1 July 2009

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Topological Defects

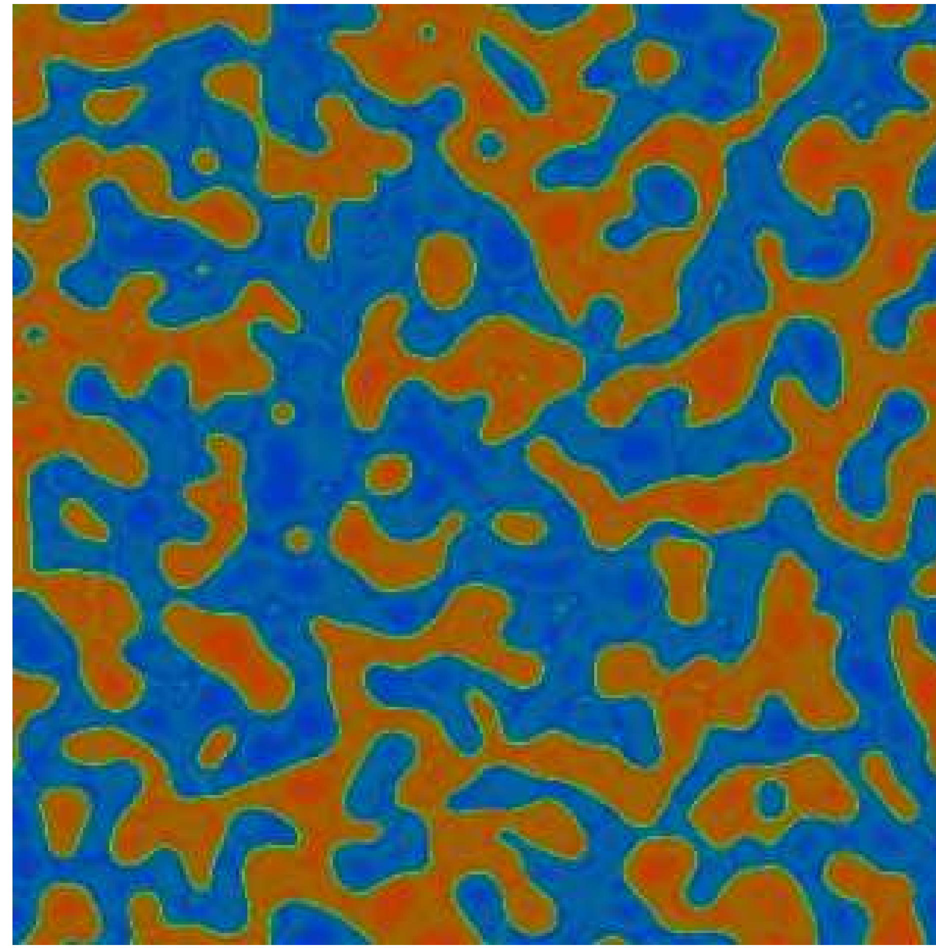
- Localised, topologically stable objects: Different vacuum in different directions
- Exist in systems with
  - spontaneously broken symmetry
  - topologically non-trivial vacuum manifold $\mathcal{M}$:

    $\pi_0(\mathcal{M}) \neq 1$: Domain walls

    $\pi_1(\mathcal{M}) \neq 1$: Vortices/strings

    $\pi_2(\mathcal{M}) \neq 1$: Monopoles

    $\pi_3(\mathcal{M}) \neq 1$: Textures



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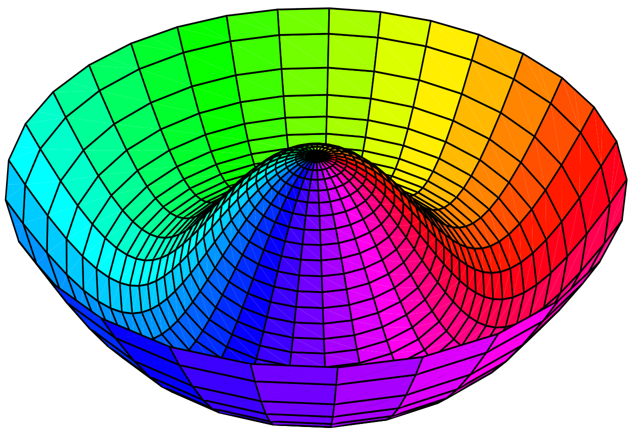
Strings



$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi - \lambda(\phi^* \phi)^2$$

- Continuous U(1) symmetry $\phi \rightarrow e^{i\alpha}\phi$ in
- When $m^2 < 0$, spontaneously broken:

  Circle of degenerate vacua $\langle\phi\rangle = ve^{i\theta}$

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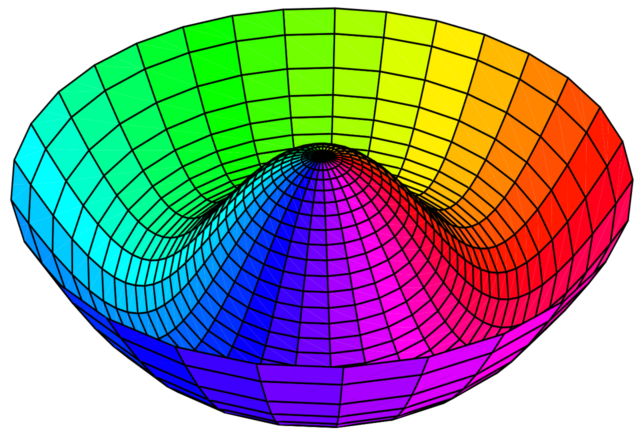
Strings



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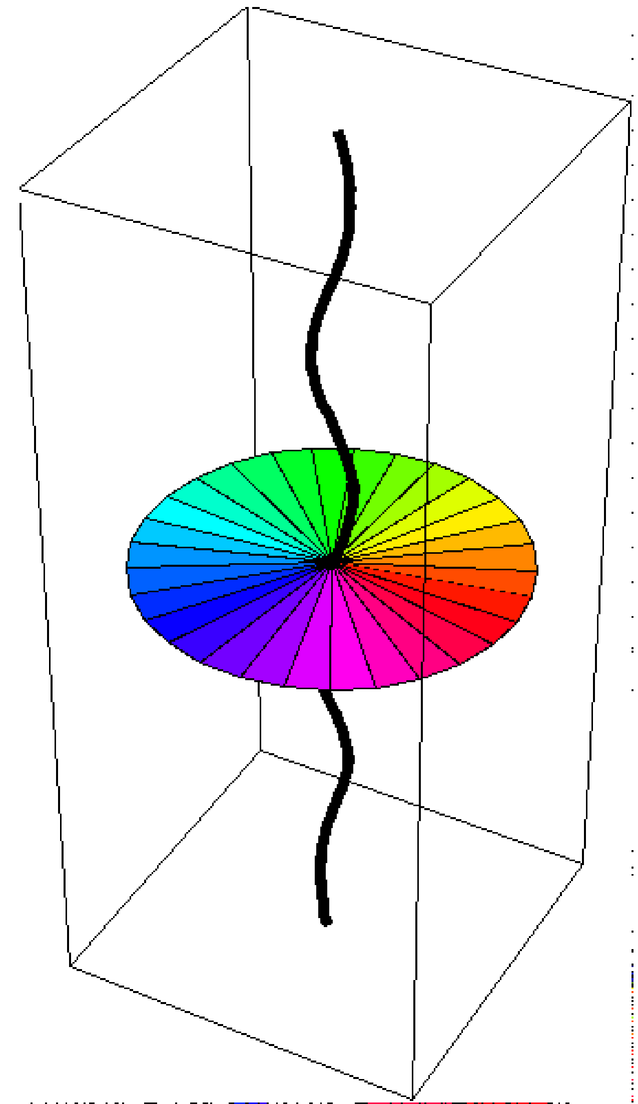
- Continuous U(1) symmetry $\phi \to e^{i\alpha}\phi$ in
- When $m^2 < 0$, spontaneously broken:

  Circle of degenerate vacua $\langle \phi \rangle = ve^{i\theta}$
- Nielsen-Olesen string:
  - Different vacuum in different directions
  - Phase angle θ changes by 2π around the string



Cosmic Strings (Kibble 1976)

- Stable, massive, line-like objects
- Tension μ = energy/length
- Formed in the early universe?
- Predicted by many GUTs:

  $G\mu \sim (M_{\mathrm{GUT}}/M_{\mathrm{Pl}})^2 \sim 10^{-6}$

Cosmic Strings (Kibble 1976)

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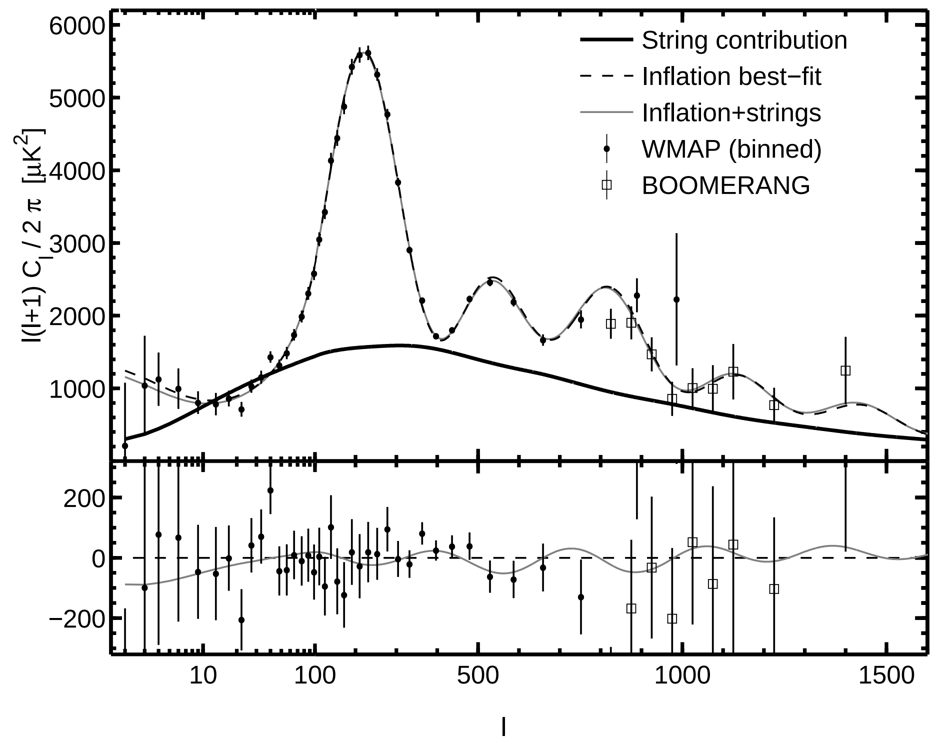
- Formed in the early universe?
- Predicted by many GUTs:
  $G\mu \sim (M_{\mathrm{GUT}}/M_{\mathrm{Pl}})^2 \sim 10^{-6}$
- Cosmic superstrings (Copeland et al 2004):
  $G\mu \sim 10^{-12} \ldots 10^{-6}$

Observing Cosmic Strings?

- Observational constraints:
  - Cosmic microwave background
    - Temperature $G\mu \lesssim 10^{-6}$
    - Polarisation $\Rightarrow G\mu \lesssim 10^{-7}$?
      (Bevis et al 2007)



(Bevis et al 2008)

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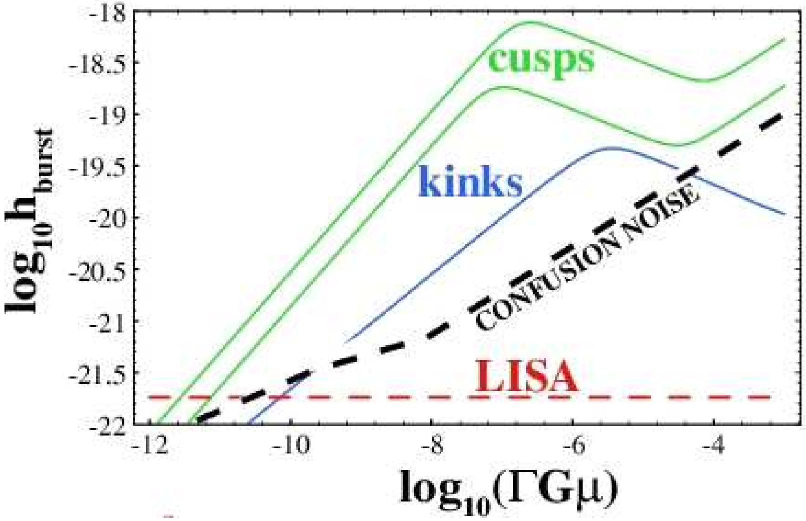
Observing Cosmic Strings?

- Observational constraints:
  - Cosmic microwave background
    - Temperature $G\mu \lesssim 10^{-6}$
    - Polarisation $\Rightarrow G\mu \lesssim 10^{-7}$? (Bevis et al 2007)
  - Gravitational waves
    - Pulsar timing $G\mu \lesssim 10^{-6}$ (Jenet et al 2006)
    - Gravity wave experiments $\Rightarrow G\mu \lesssim 10^{-10}$? (Damour&Vilenkin 2001)



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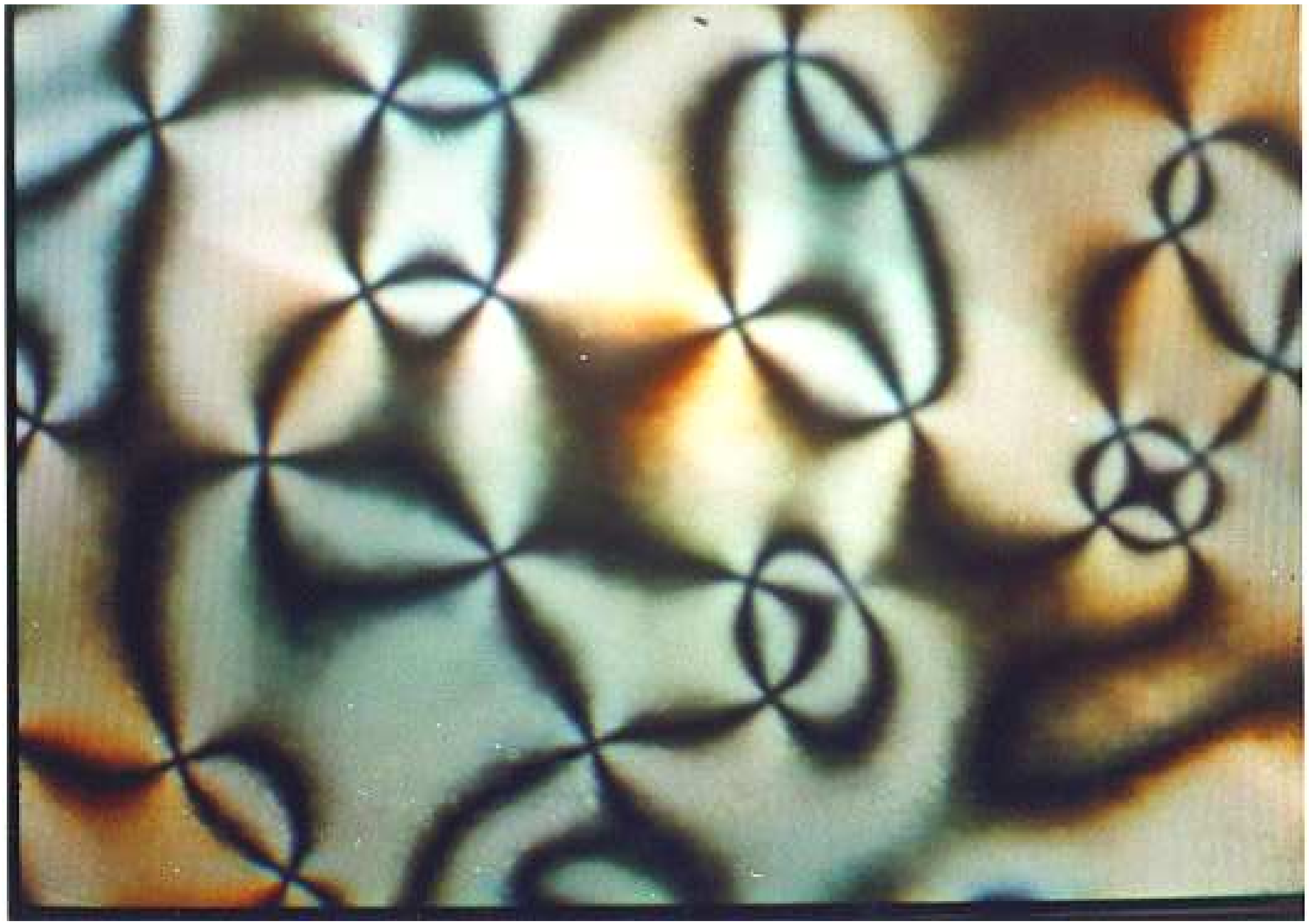
Liquid Crystal Vortex



- Orientation of molecules (Digal et al 1999)

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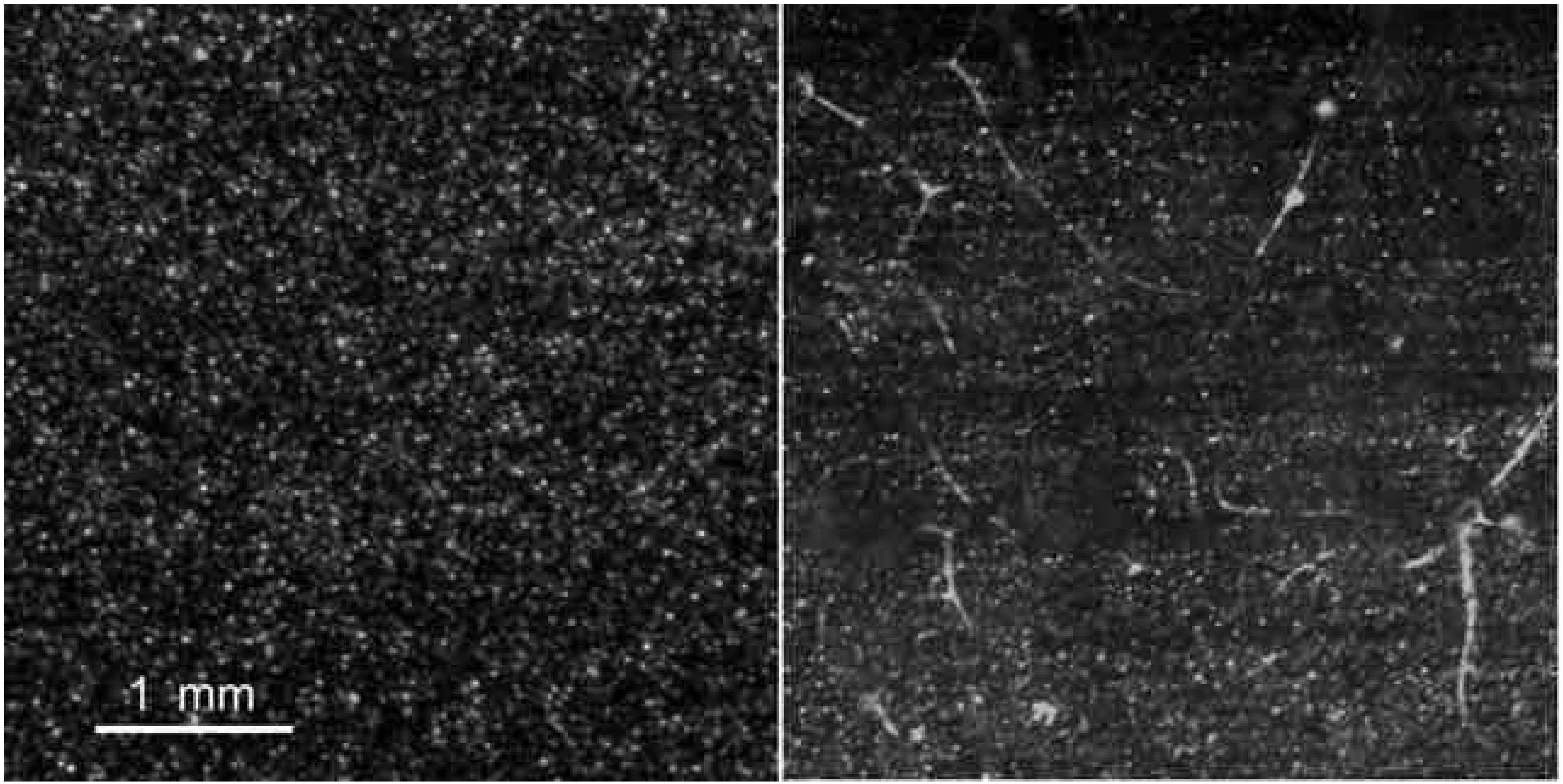
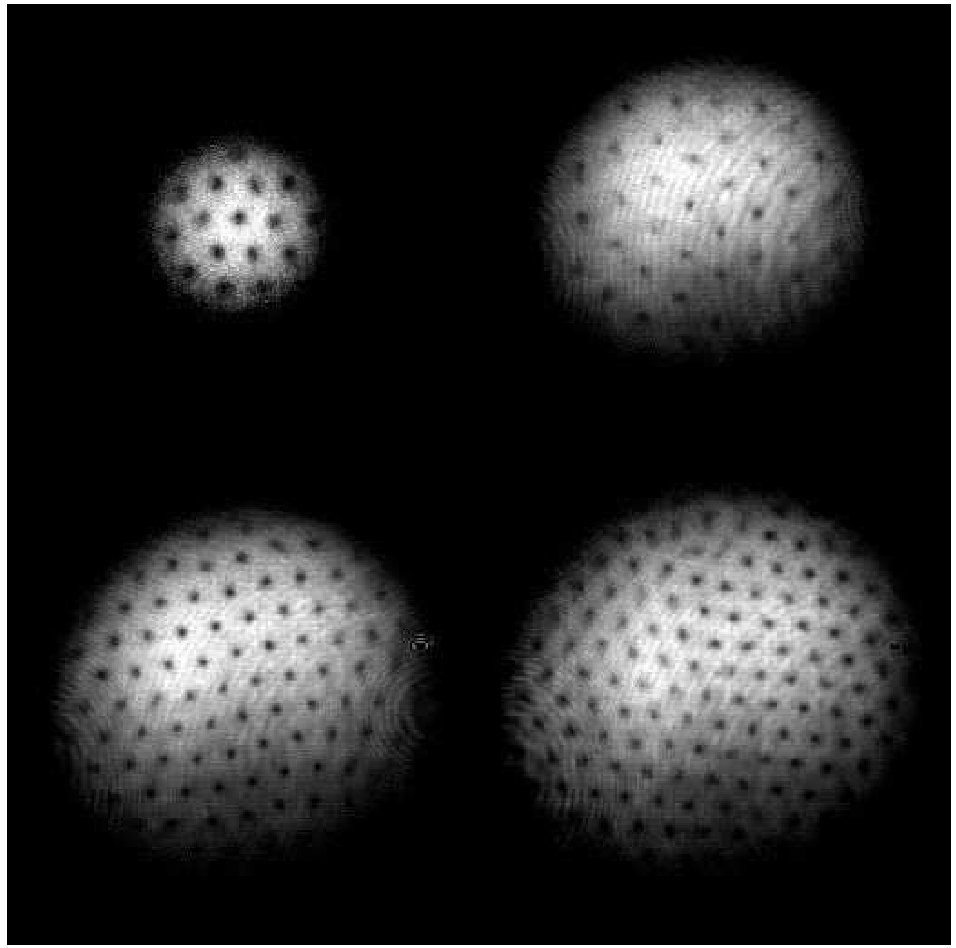
Superfluid Vortex



- Complex phase of atom wave function (Bewley et al 2006)

BEC Vortex



- Complex phase of atom wave function (Abo-Shaeer et al 2001)

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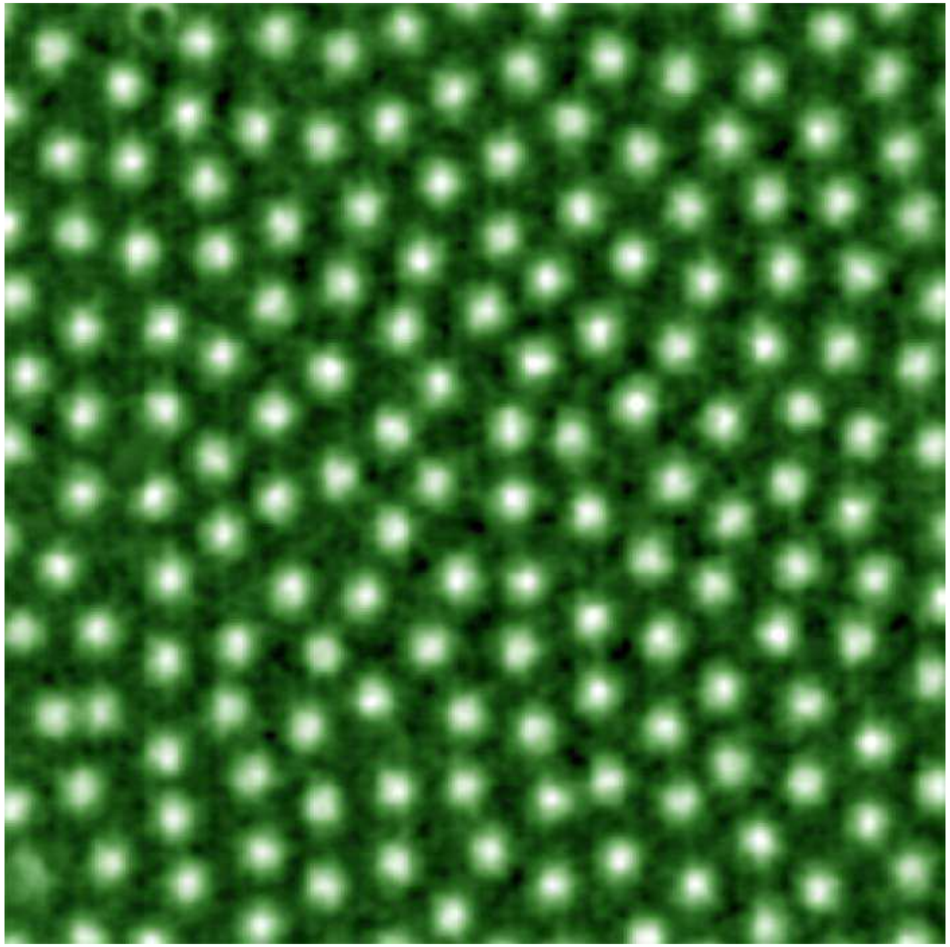
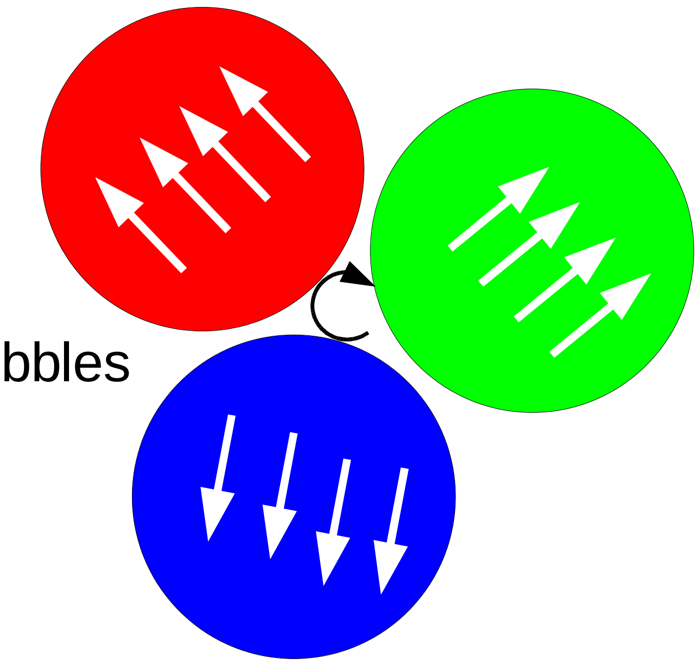
Superconductor Vortex



- Complex phase of Cooper pair wave function (Goa et al)

Kibble Mechanism in First Order Transitions (Kibble 1976)

- Spontaneous symmetry breaking:
  Field has to “choose” a direction
- Typical bubble size R
  - Order parameter uncorrelated between bubbles
  - Finite probability to form a string whenever three bubbles meet (Kibble 1976)
  - Number density per cross-sectional area: $n \sim R^{-2}$



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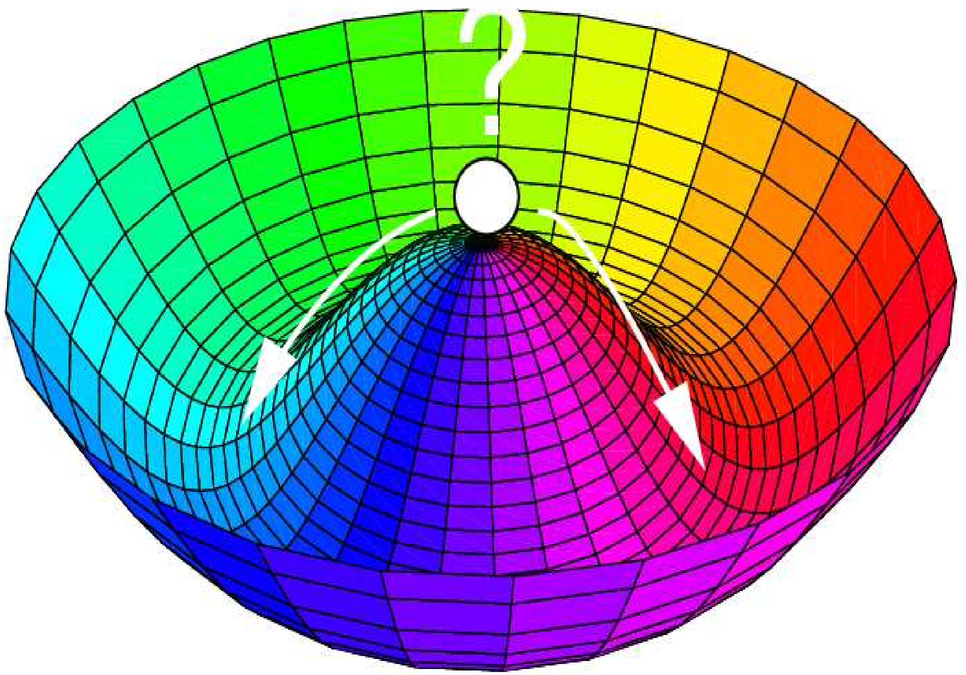
Second Order Phase Transition

$$V(\phi) = m^2\phi^*\phi + \lambda(\phi^*\phi)^2$$

- Gradual change from $m^2 > 0$ to $m^2 < 0$: Transition from symmetric to broken phase
- Caused by decreasing temperature, end of inflation etc
- Choose a direction
- Uncorrelated at distances longer than the correlation length $\xi < 1/H$ $\Rightarrow$ Kibble mechanism
- Number density: $n \sim \xi^{-2}$



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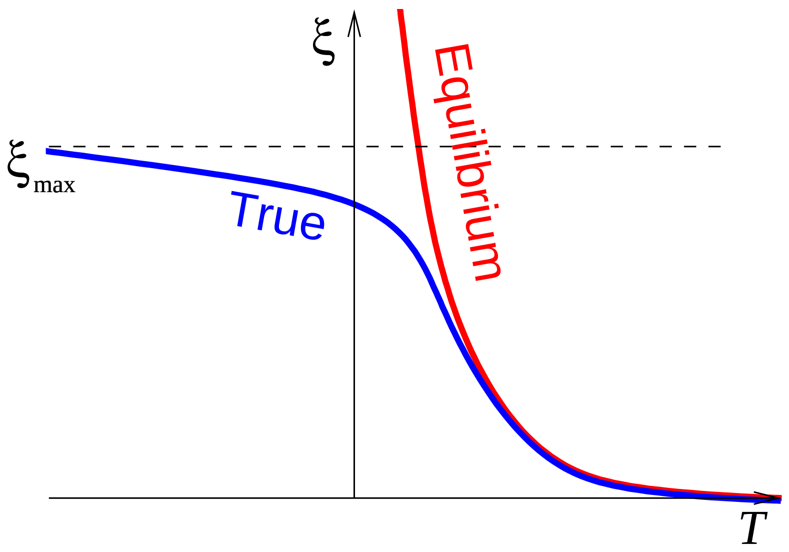
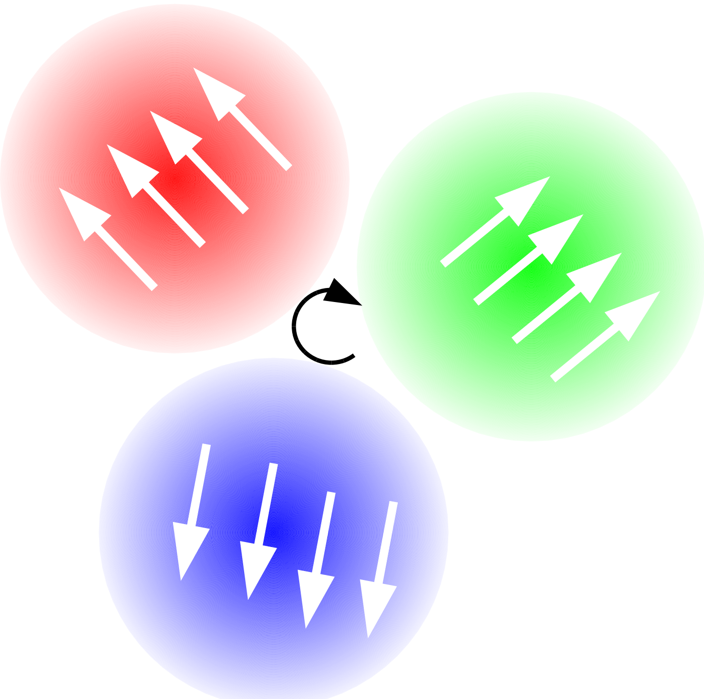
Kibble-Zurek Mechanism (Zurek 1985)

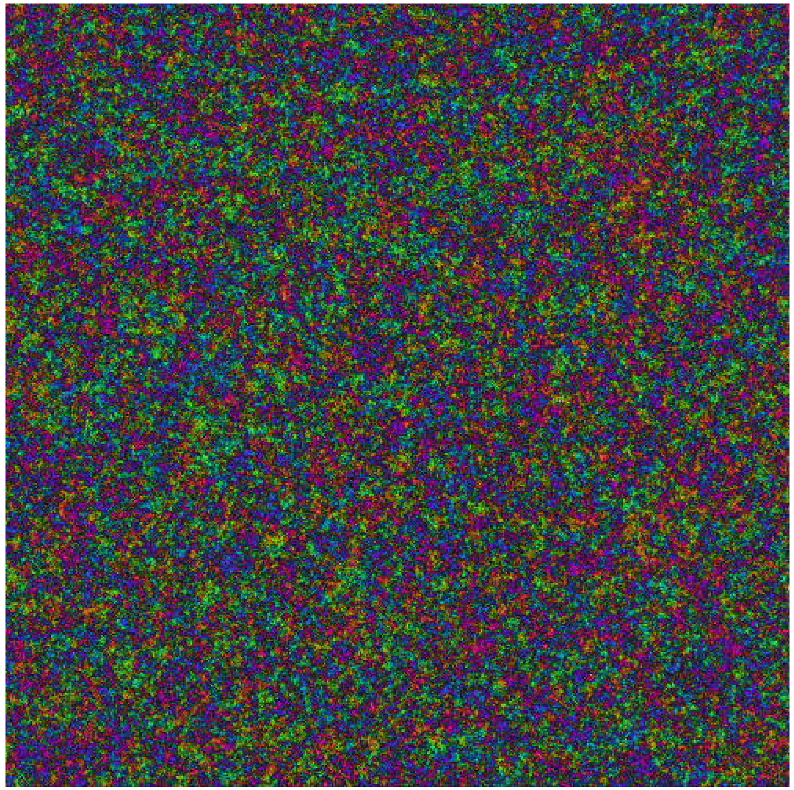




- Correlation length grows as $\xi \sim (T - T_c)^{-\nu}$
- Critical slowing down: $\tau \sim (T - T_c)^{-\mu}$
- ξ freezes out to $\hat{\xi} \propto \tau_Q^{\nu/(1+\mu)}$:

  Sets the string number density

Simulation Tests



- Classical field theory simulations confirm predictions

Gauge Fields: Abelian Higgs Model

$$\epsilon = \frac{1}{2}\vec{B}^2 + |\vec{D}\phi|^2 + m^2\phi^*\phi + \lambda(\phi^*\phi)^2$$

- $\vec{D} = \vec{\nabla} + ie\vec{A}$, $\vec{B} = \vec{\nabla} \times \vec{A}$
- Gauge symmetry $\phi(x) \to e^{i\alpha(x)}\phi(x)$, $\vec{A} \to \vec{A} - (1/e)\vec{\nabla}\alpha$
  - Phase angle not θ physical observable - Kibble argument?
- Higgs phase:
  - Minimize energy: $\vec{D}\phi = 0 \leftrightarrow e\vec{A} \approx \vec{\nabla}\theta$
  - Gauge field cancels gradient energy

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Cold Electroweak Baryogenesis

- Electroweak theory: $\pi_3(\mathcal{M}) = \mathbb{Z}$

  $\Rightarrow$ Textures
- Gauge field compensates: $N_{\rm CS} = N_{\rm w}$
  - Either $N_{\rm CS}$ or $N_{\rm w}$ changes, depending on texture size
  - Biased by CP violation
  - Anomaly $\Delta B = 3\Delta N_{\rm CS}$ leads to baryon asymmetry (Turok&Zadrozny 1990)
- Can take place at the end of inflation: $T = 0$

  (Krauss&Trodden; Copeland,Lyth,AR&Trodden; Tranberg&Smit)

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  (Krauss&Trodden; Copeland,Lyth,AR&Trodden; Tranberg&Smit)
  - In thermal transition dominated by gauge dynamics:

    Thermal sphalerons

Flux Quantisation

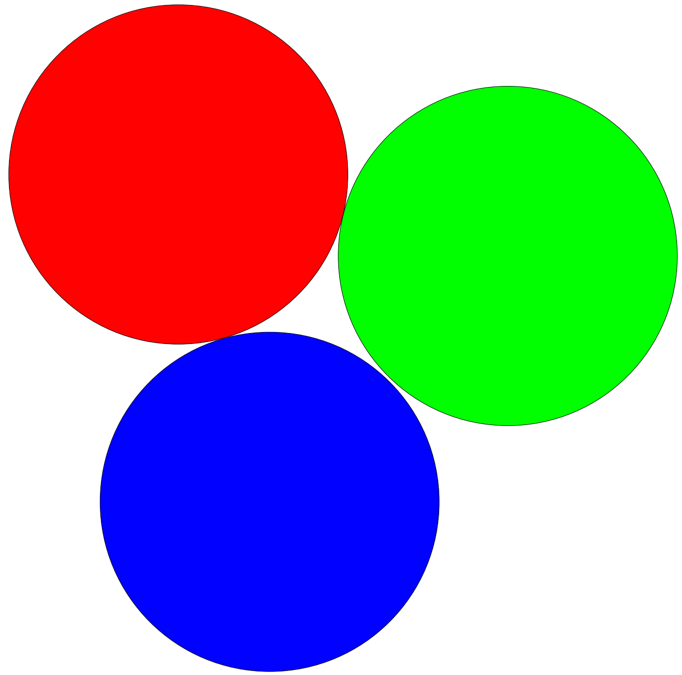
- Minimize energy: $\vec{D}\phi = 0 \leftrightarrow e\vec{A} \approx \vec{\nabla}\theta$
- No magnetic field in the Higgs phase:
  $\vec{\nabla} \times \vec{A} \approx (1/e)\vec{\nabla} \times \vec{\nabla}\theta = 0$
- Confined into strings

$$\Phi = \oint d\vec{x} \cdot \vec{A} = (1/e) \oint d\vec{x} \cdot \vec{\nabla}\theta = N_{\rm w}\Phi_0$$

- Flux quantum $\Phi_0 = 2\pi/e$ per string

First Order Phase Transition

- Bubble radius R
- Hole of area $\sim R^2$ between bubbles



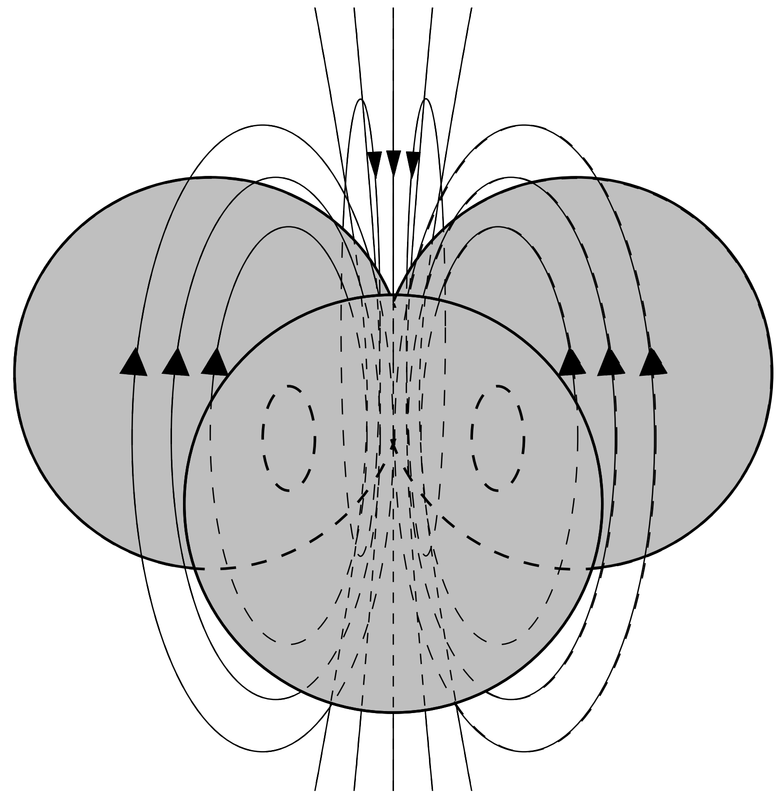
First Order Phase Transition

- Bubble radius R
- Hole of area $\sim R^2$ between bubbles
- Thermal flux through hole: $\Phi \approx \sqrt{RT}$
- Conservation of flux:

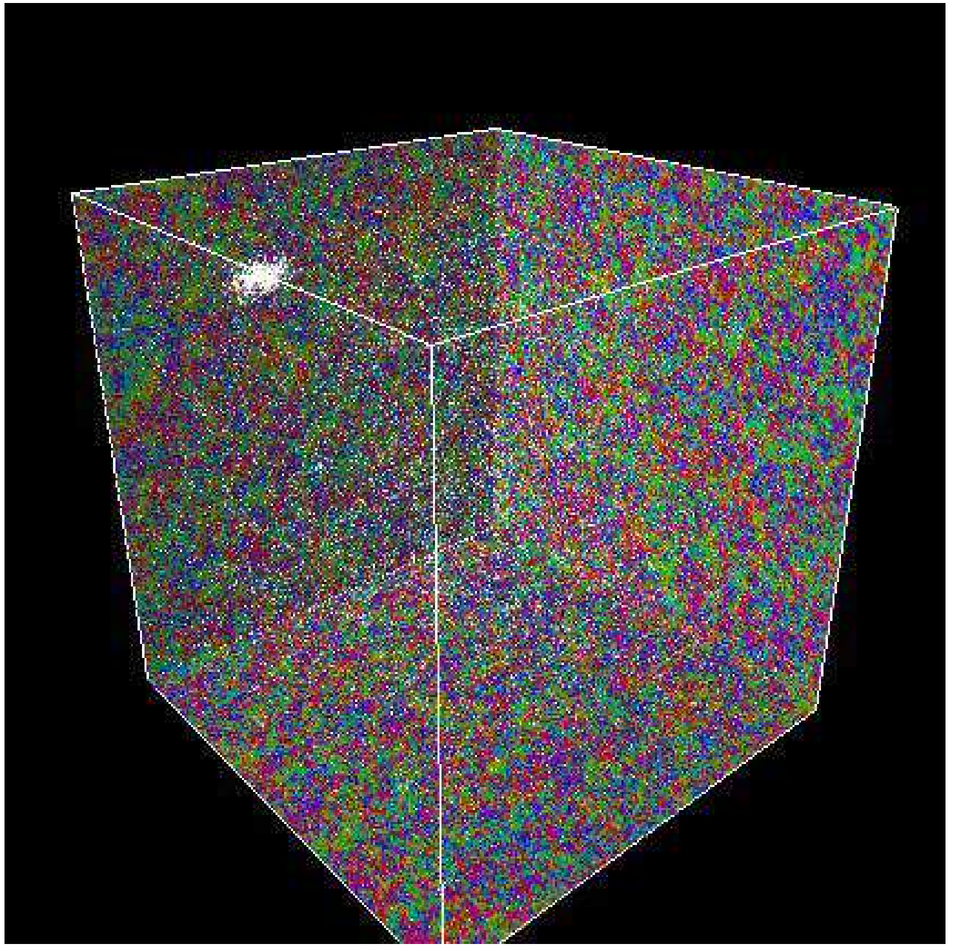
  Must get confined in a string

  with winding $N_{\mathrm{w}} = \Phi/\Phi_0 \approx e\sqrt{RT}$

  (Donaire&AR 2006)



Simulation of a First Order Transition



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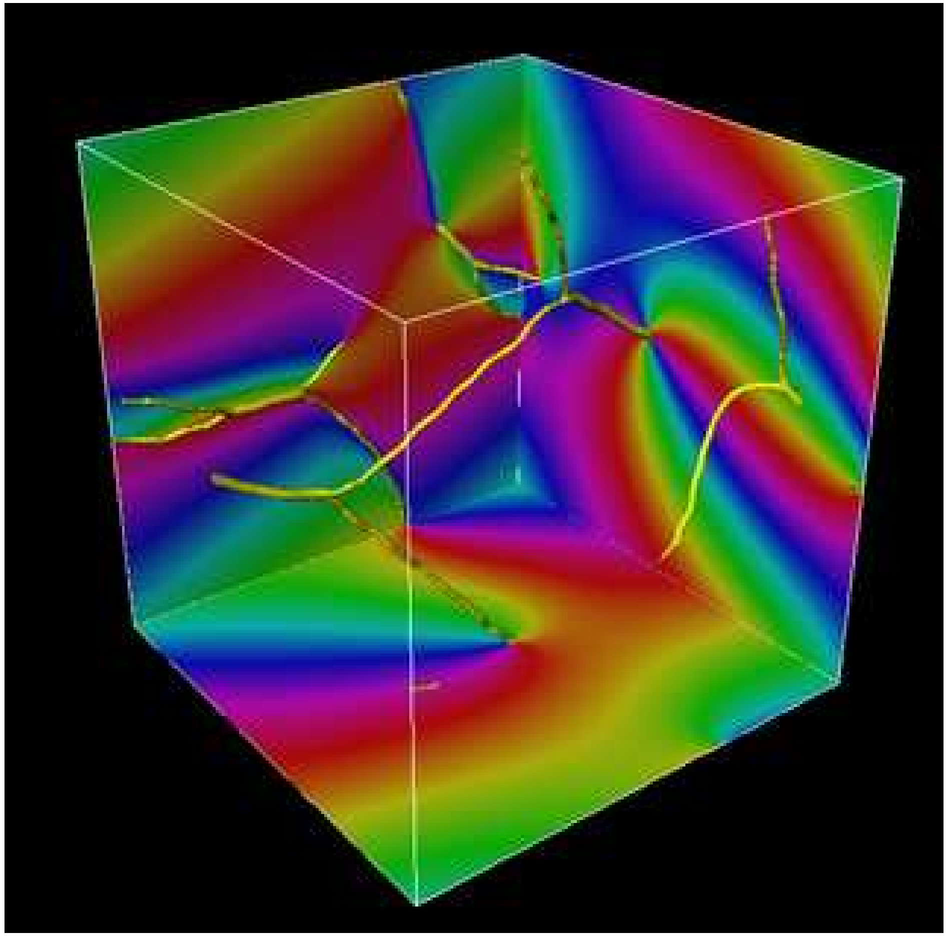
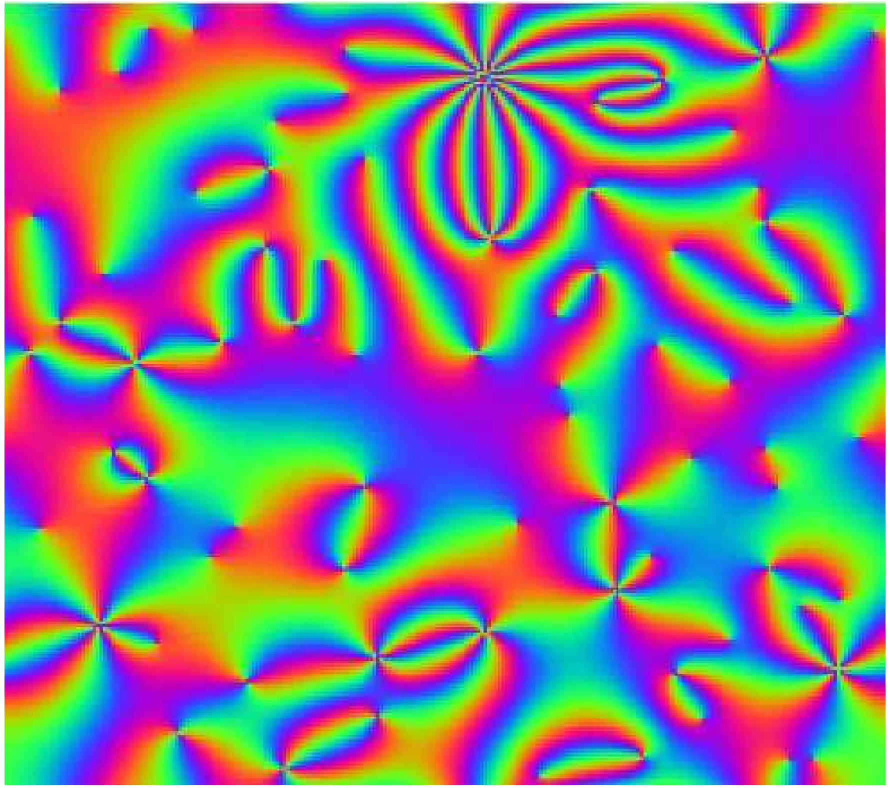
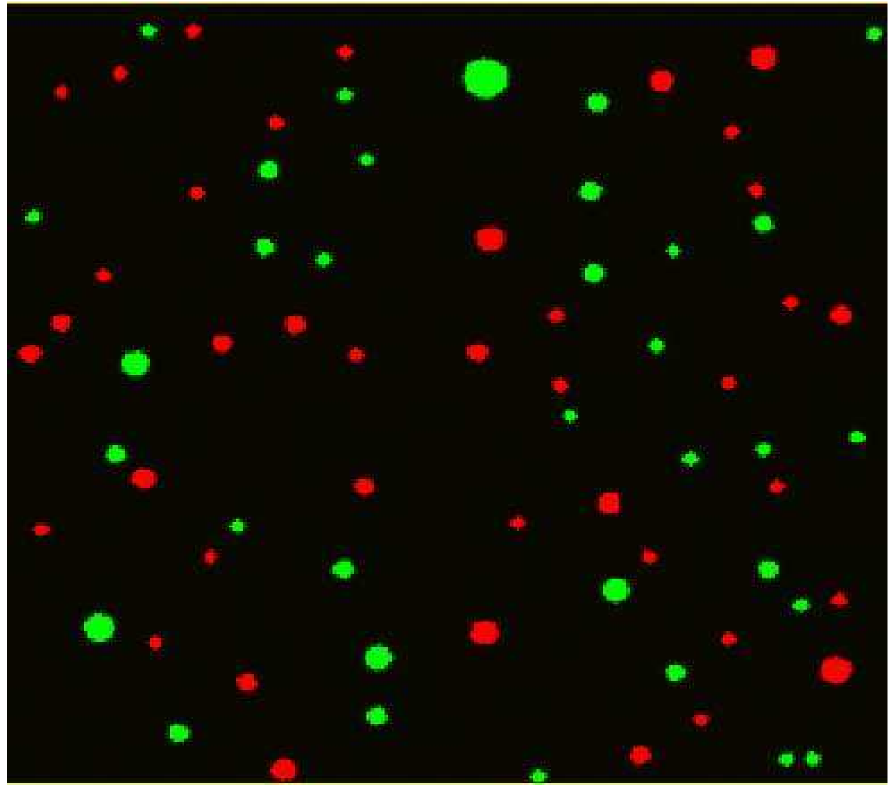


First Order Phase Transition in 2D



Phase angle



Magnetic field

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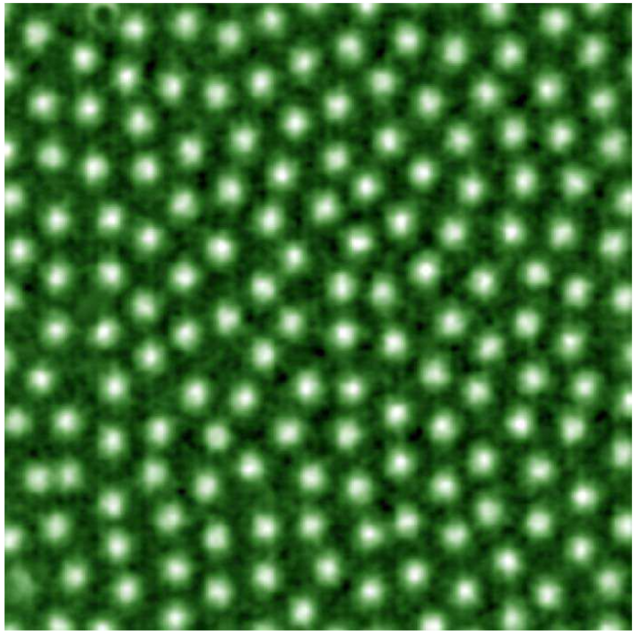
Flux Trapping in a Continuous Transition

- Magnetic flux is conserved

$$\frac{d}{dt}\Phi = -\oint_{\partial C} d\vec{x}\cdot\vec{E}$$

- Uniform initial field: String number density $\vec{\rho} = (e/2\pi)\vec{B}$

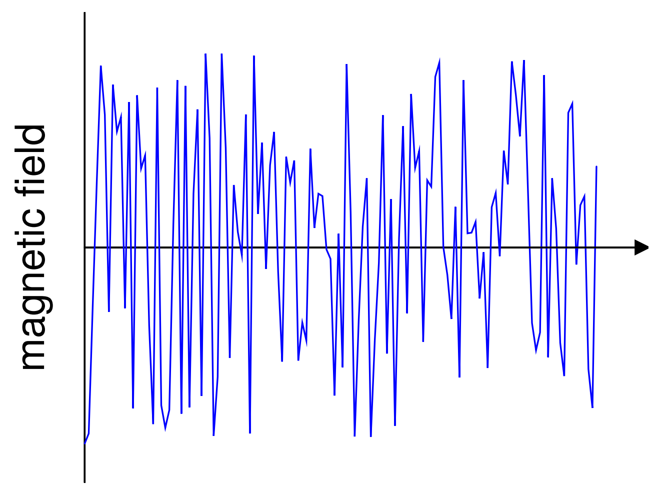


Flux Trapping in a Continuous Transition

- Magnetic flux is conserved

$$\frac{d}{dt}\Phi = -\oint_{\partial C} d\vec{x} \cdot \vec{E}$$

- Non-uniform field: Short-wavelength $\lambda \lesssim \lambda_c$ modes decay



Flux Trapping in a Continuous Transition

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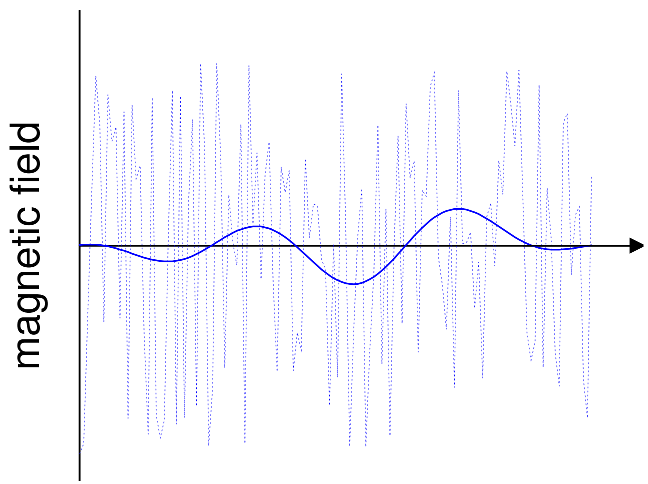
- Long-wavelength modes survive



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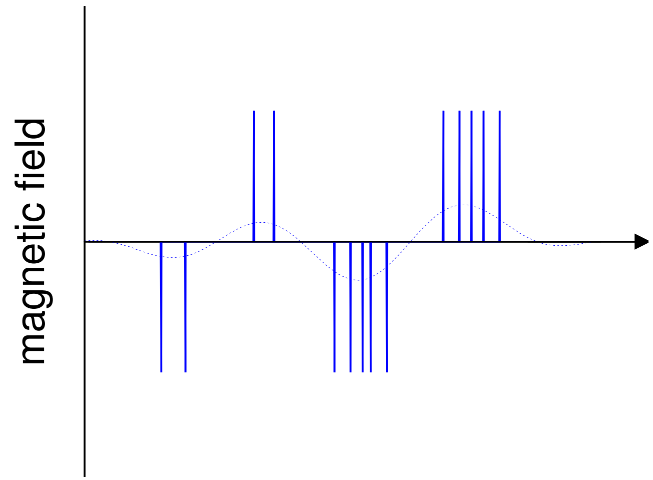

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- Long-wavelength modes form strings $\vec{\rho} = (e/2\pi)\vec{B}$ (Hindmarsh&AR 2000)
- Correlator at long distances $r \gg \lambda_c$:

$$\langle \rho_i(x)\rho_j(y)\rangle = (e/2\pi)^2\langle B_i(x)B_j(y)\rangle$$

- Given by a scalar function $G(k)$:

$$\langle \rho_i(\vec{k})\rho_j(\vec{q})\rangle = (2\pi)^3\delta(\vec{k}+\vec{q})\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)G(k)$$

Long-Range Correlations (AR 2009)

- Initial state:
  - Thermal ($T > 0$):

$$G(k) = \frac{e^2}{4\pi^2} G^B(k) \approx \frac{e^2}{4\pi^2} \frac{k}{2} \coth \frac{k}{2T}$$

$$\langle \rho_i(\vec{x}) \rho_j(\vec{y}) \rangle \sim -\frac{e^2 T}{|\vec{x}-\vec{y}|^3}$$

  - Vacuum ($T = 0$):

$$G(k) = \frac{e^2}{4\pi^2} G^B(k) \approx \frac{e^2}{4\pi^2} \frac{k}{2}$$

$$\langle \rho_i(\vec{x}) \rho_j(\vec{y}) \rangle \sim -\frac{e^2}{|\vec{x}-\vec{y}|^4}$$

3D Simulation

- 256x256x256 lattice, spacing $\delta x = 1$
- Thermal initial conditions with $T_{\rm ini} = 0.5$
- Radiation dominated with $H_{\rm ini} = 0.1$: Evolve until $a = 2$
- Scalar coupling $\lambda = 1$,

  m^2 such that transition takes place at $a = \sqrt{2}$

$$
\begin{aligned}
\partial_\tau^2 \tilde{\phi} &= \vec{D}^2 \tilde{\phi} + (m^2 a^2 + \partial_\tau^2 a/a)\tilde{\phi} - 2\lambda|\tilde{\phi}|^2\tilde{\phi} \\
\partial_\tau \tilde{E}_i &= \partial_j F_{ij} + 2e\mathrm{Im}\tilde{\phi}^* D_i \tilde{\phi}, \\
\partial_i \tilde{E}_i &= 2e\mathrm{Im}\tilde{\phi}^* \partial_\tau \tilde{\phi}
\end{aligned}
$$

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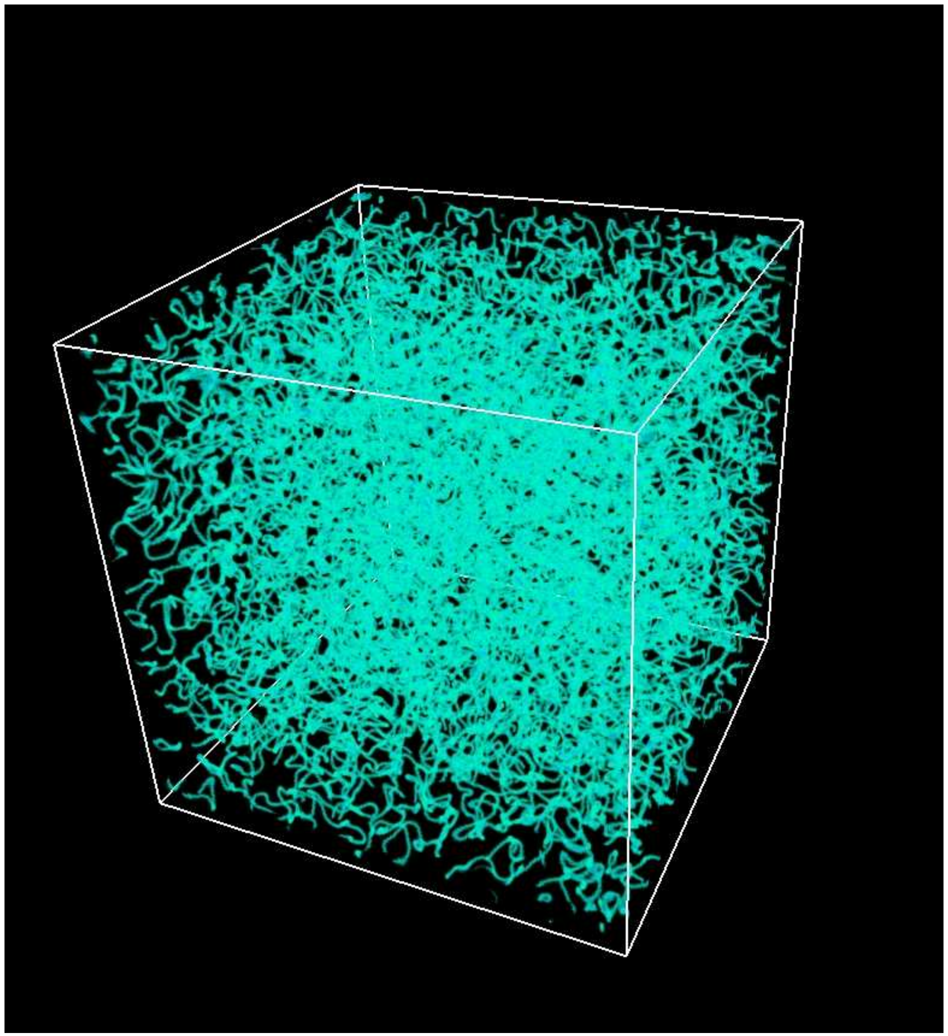
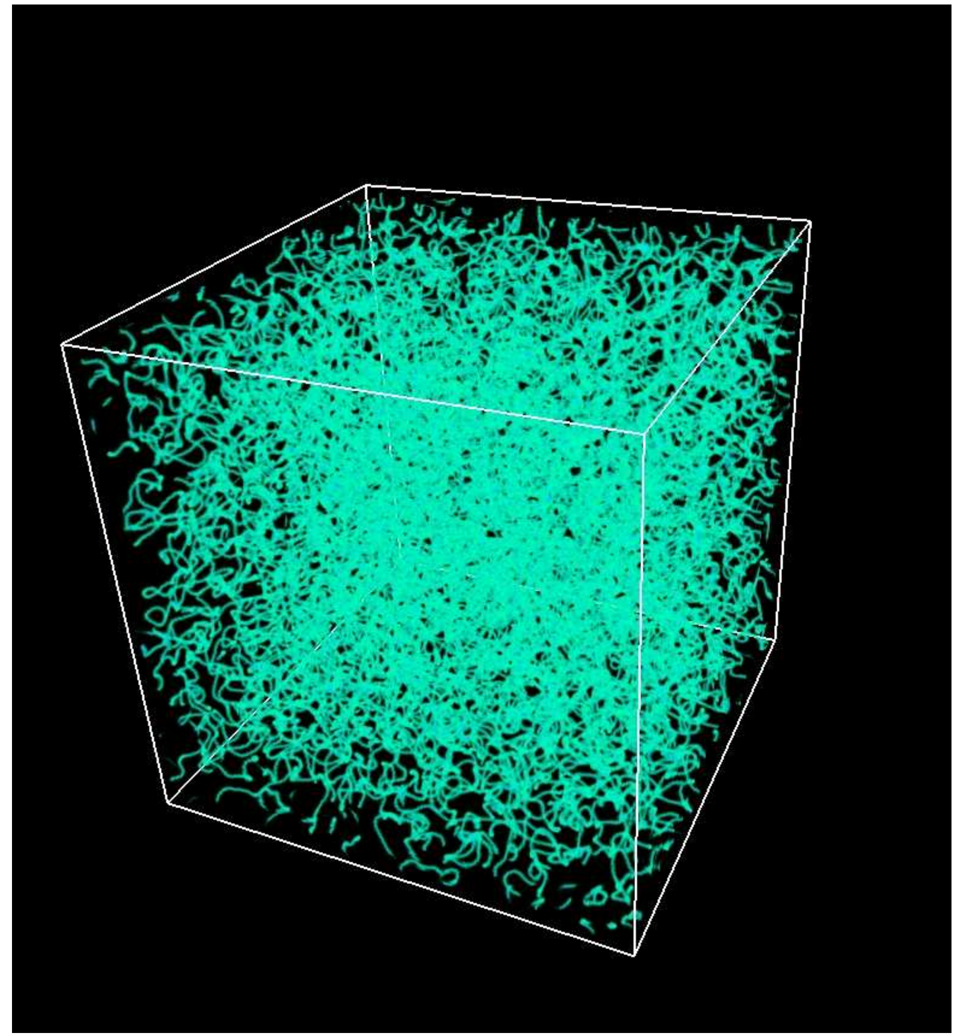
3D Simulation

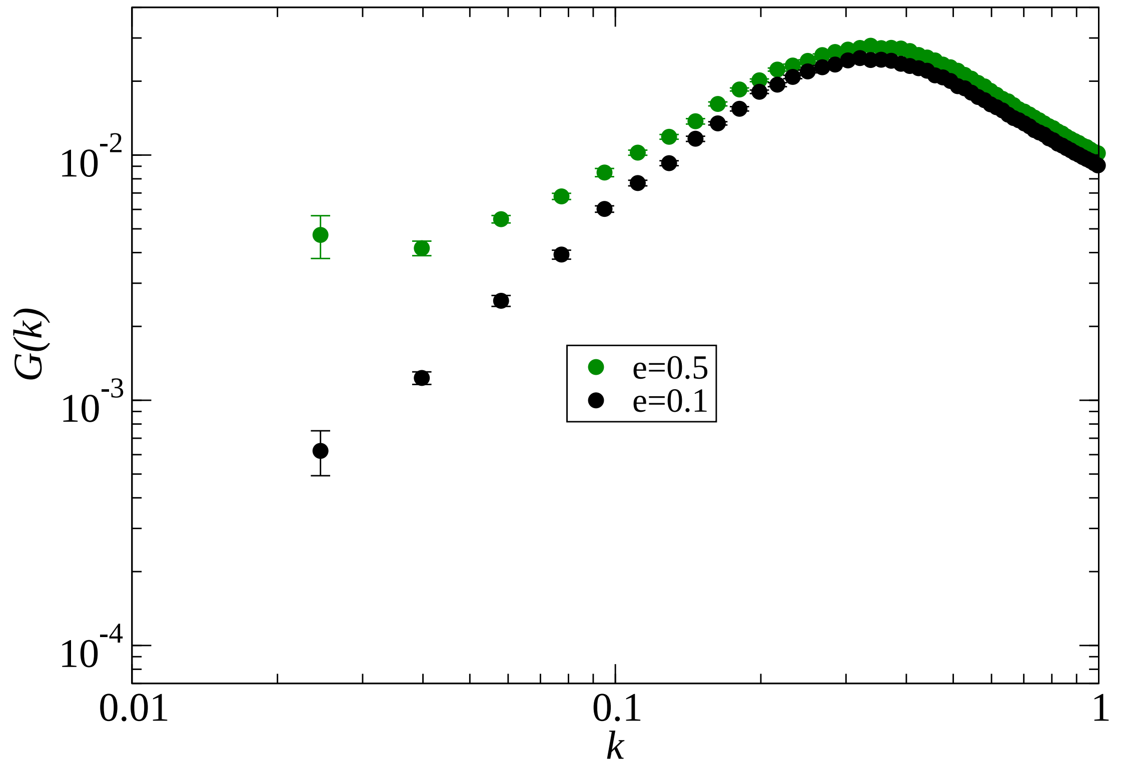


$e = 0.1$



$e = 0.5$

3D Simulation



- Winding number correlator:

$$\langle \rho_i(\vec{k})\rho_j(\vec{q})\rangle = (2\pi)^3\delta(\vec{k}+\vec{q})\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right)G(k)$$

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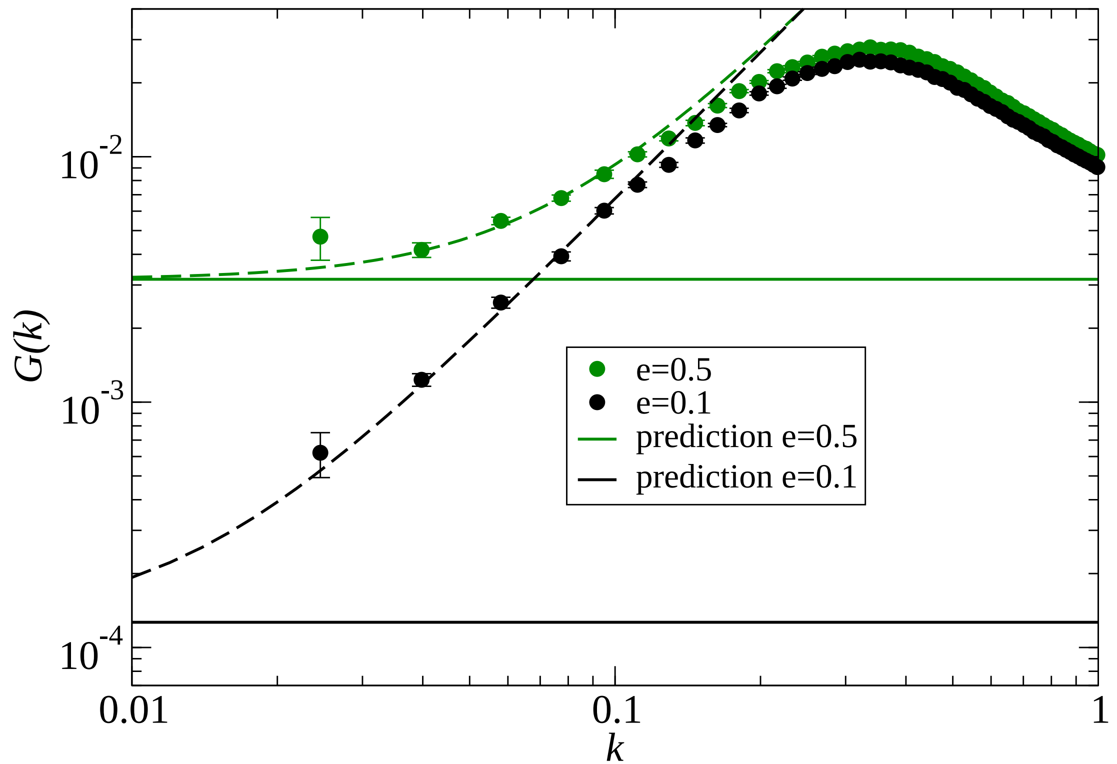
3D Simulation



- Flux trapping prediction:

$$\langle \rho_i(\vec{k})\rho_j(\vec{q})\rangle = \left(\frac{e}{2\pi}\right)^2 \langle B_i(\vec{k})B_j(\vec{q})\rangle_{\rm ini}$$

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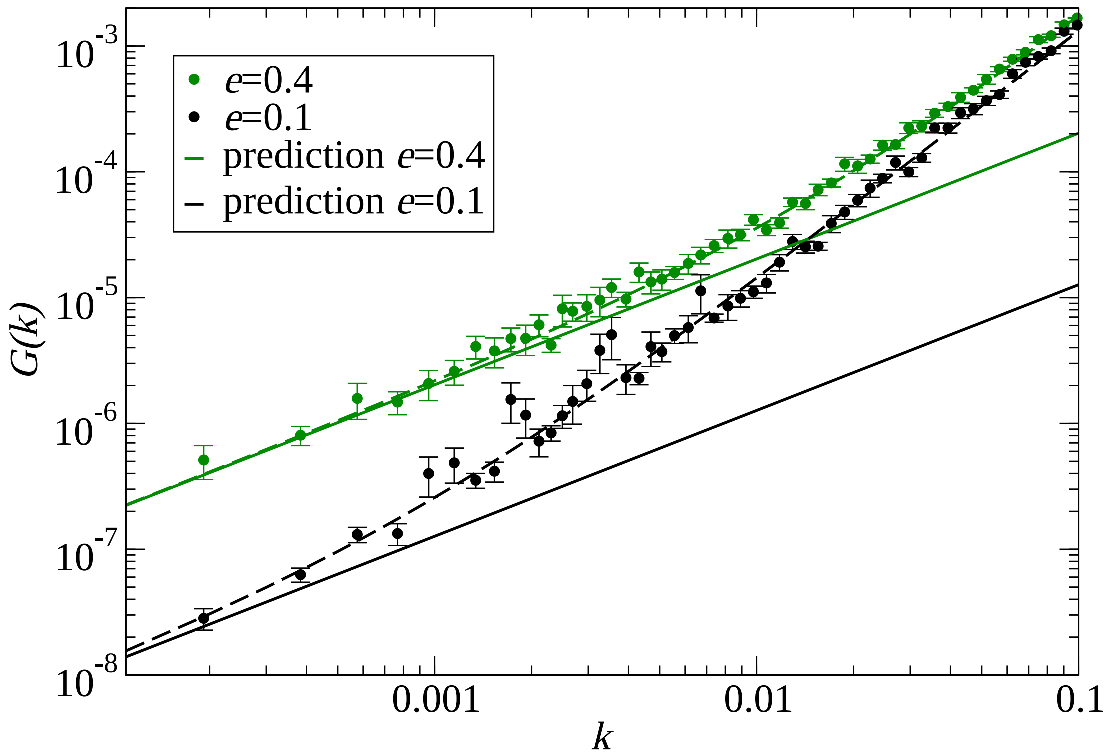
Zero-Temperature Phase Transition



- $128 \times 128 \times 32768$ lattice
- Classical field theory simulation:

  Initial Gaussian fluctuations with the quantum vacuum two-point function

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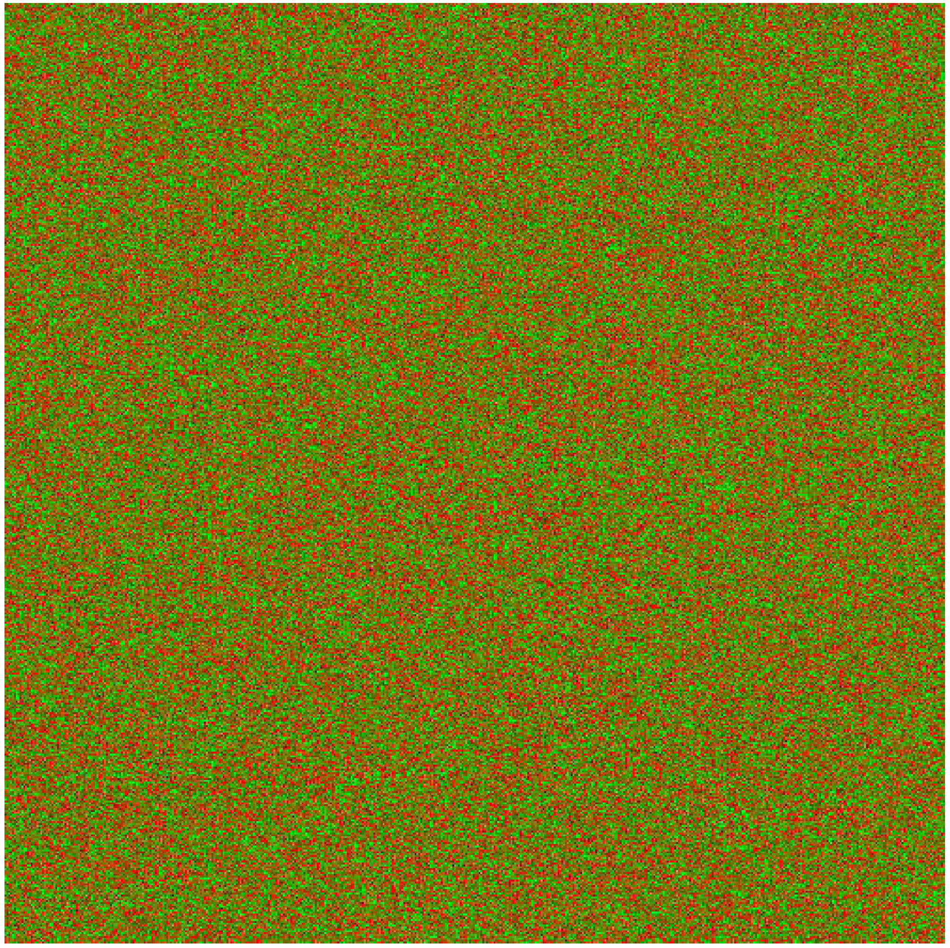
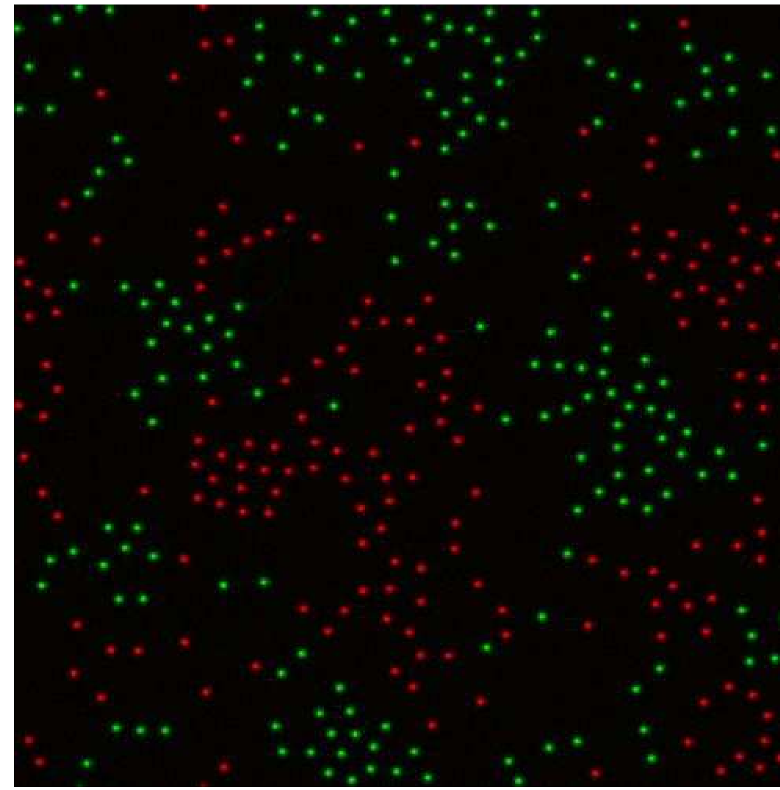
Short-Distance Effects



2D Simulation

Short-Distance Effects

- Clusters of equal-sign vortices:
  - $N \approx eT^{1/2}\lambda_c^{2-D/2}$ per cluster
    (Hindmarsh&AR, 2000)
  - Kibble mechanism: No clusters



gauge

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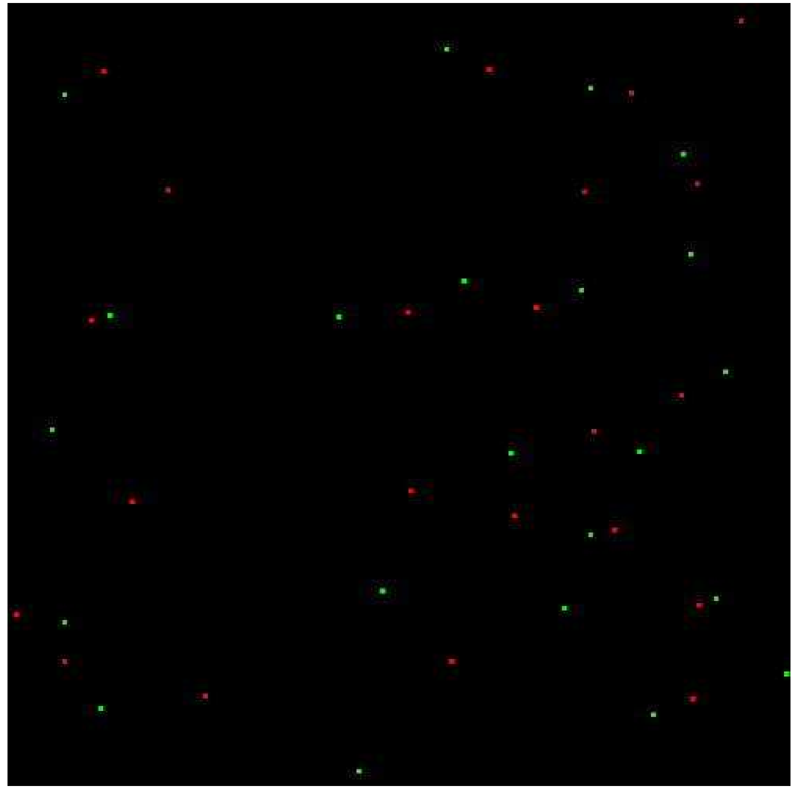

global

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Short-Distance Effects

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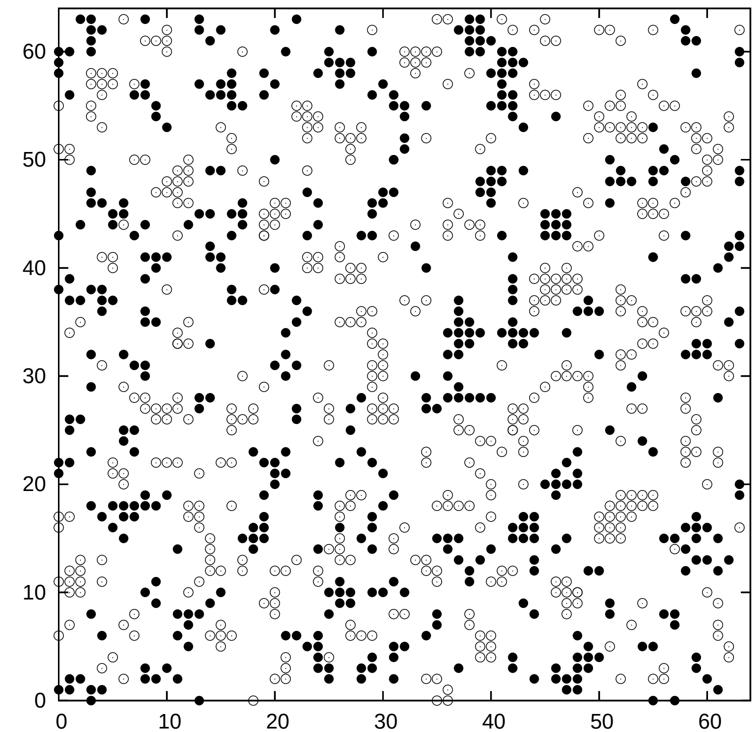
- Also in 3D



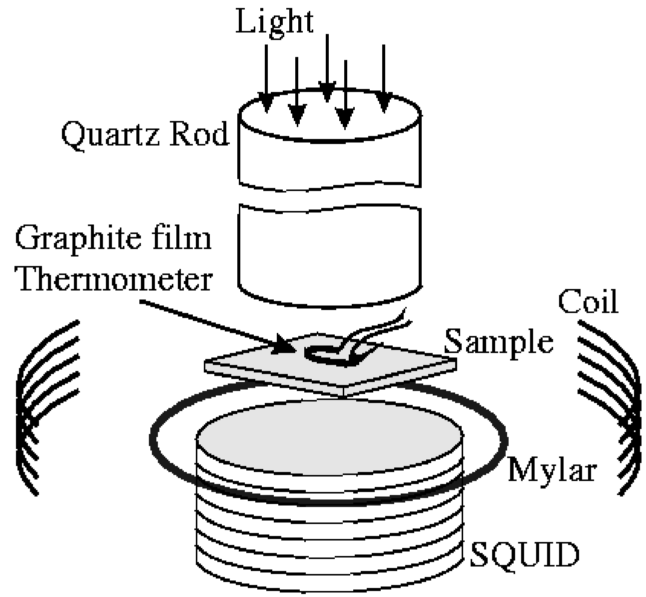
(Blanco-Pillado et al 2007)

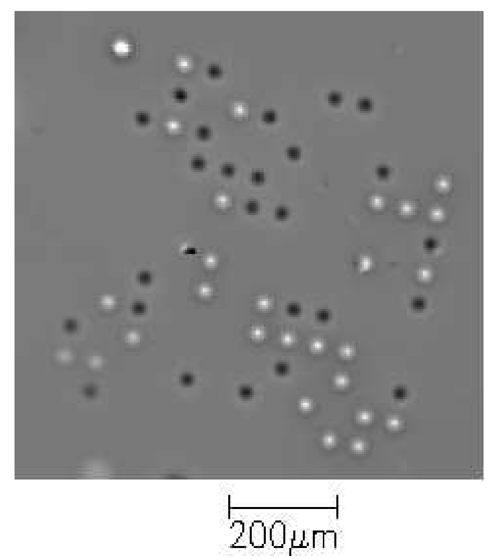
Cosmology

- Short-distance effects washed out quickly
  - Clustering probably not relevant
  - Thick strings ($N_{\rm w} > 1$) can modify string evolution
- Long-range correlations potentially very important
  - Correlations on superhorizon scales:
    Strings pile up as the horizon grows
  - Thermal initial state ruled out: $T_{\rm CMB}/H_0 \sim 10^{14}$ infinite strings
  - Vacuum initial state: $M_{\rm Pl}/g_*^{1/2}T_{\rm inf} \sim 10$ infinite strings
    - with preferred direction
    - correlations from “nothing”
    - are classical arguments valid?

Superconductor Experiments

- Quench from $T > T_c$ to $T < T_c$
  - Fully quantum mechanical
  - Better test than any simulation
- Measurements of net flux
  (Carmi et al, Monaco et al)
- Array of rings (Kirtley et al 2003)
  - Shows clustering
- Magneto-optical imaging:
  - Experiments being planned (Golubchik 2008)
  - Individual vortices: Correlations





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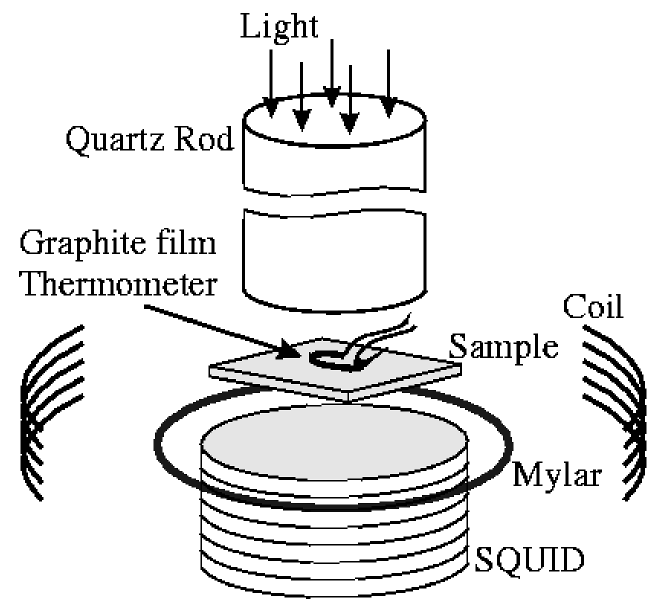

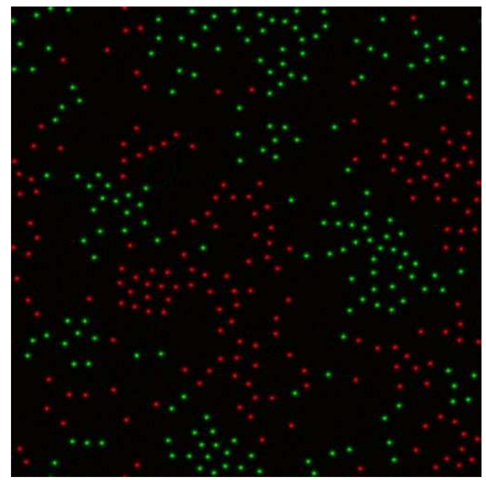

Conclusions

- Global symmetry: Kibble-Zurek mechanism
- Gauge symmetry: Kibble-Zurek + flux trapping
  - Thick strings from first order transitions
  - Dominant mechanism at long distances
  - Gauge field correlations survive in defect distribution:
    Infinite-range correlations from “nothing”?
- Can be tested in superconductor experiments