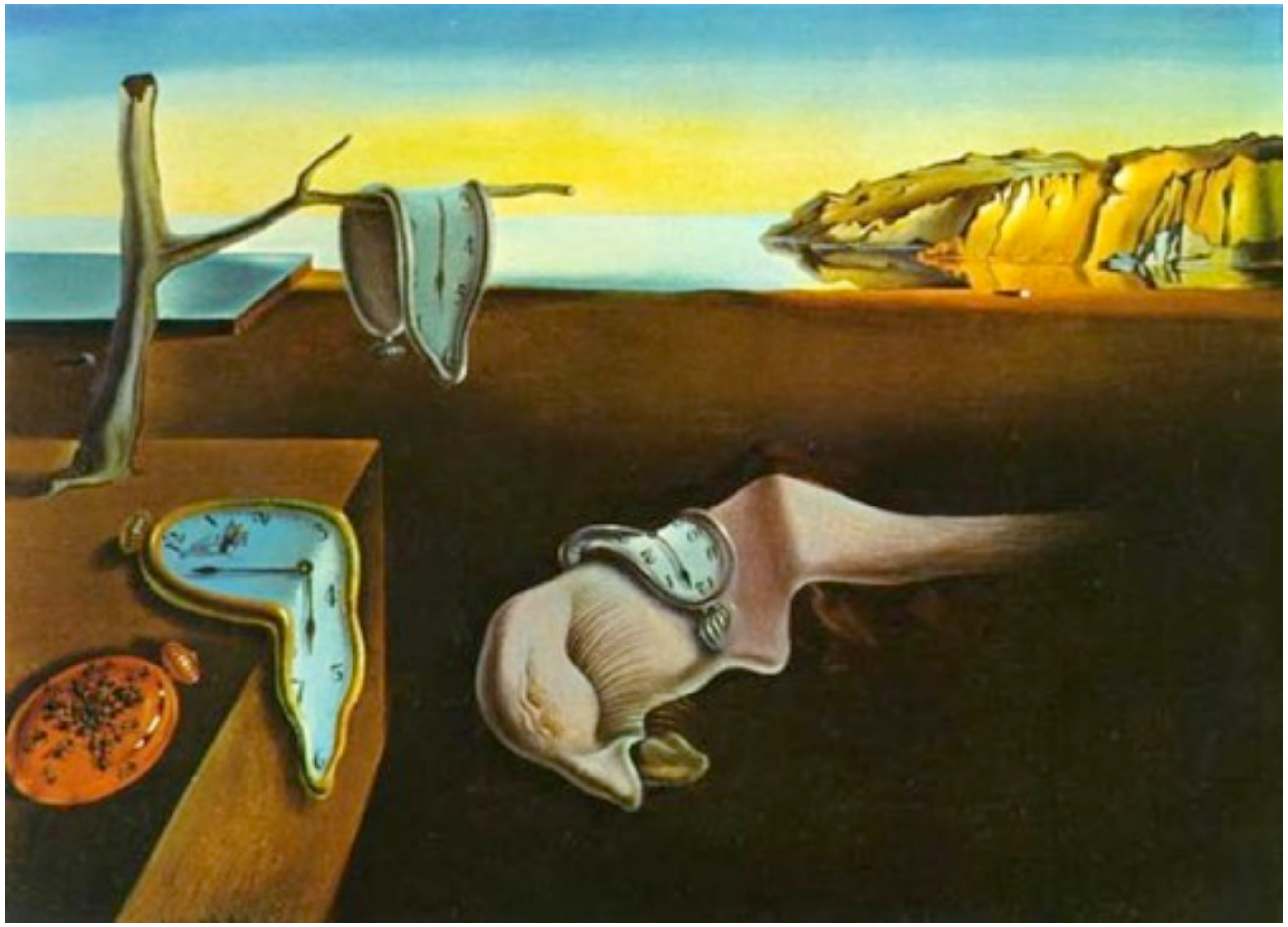
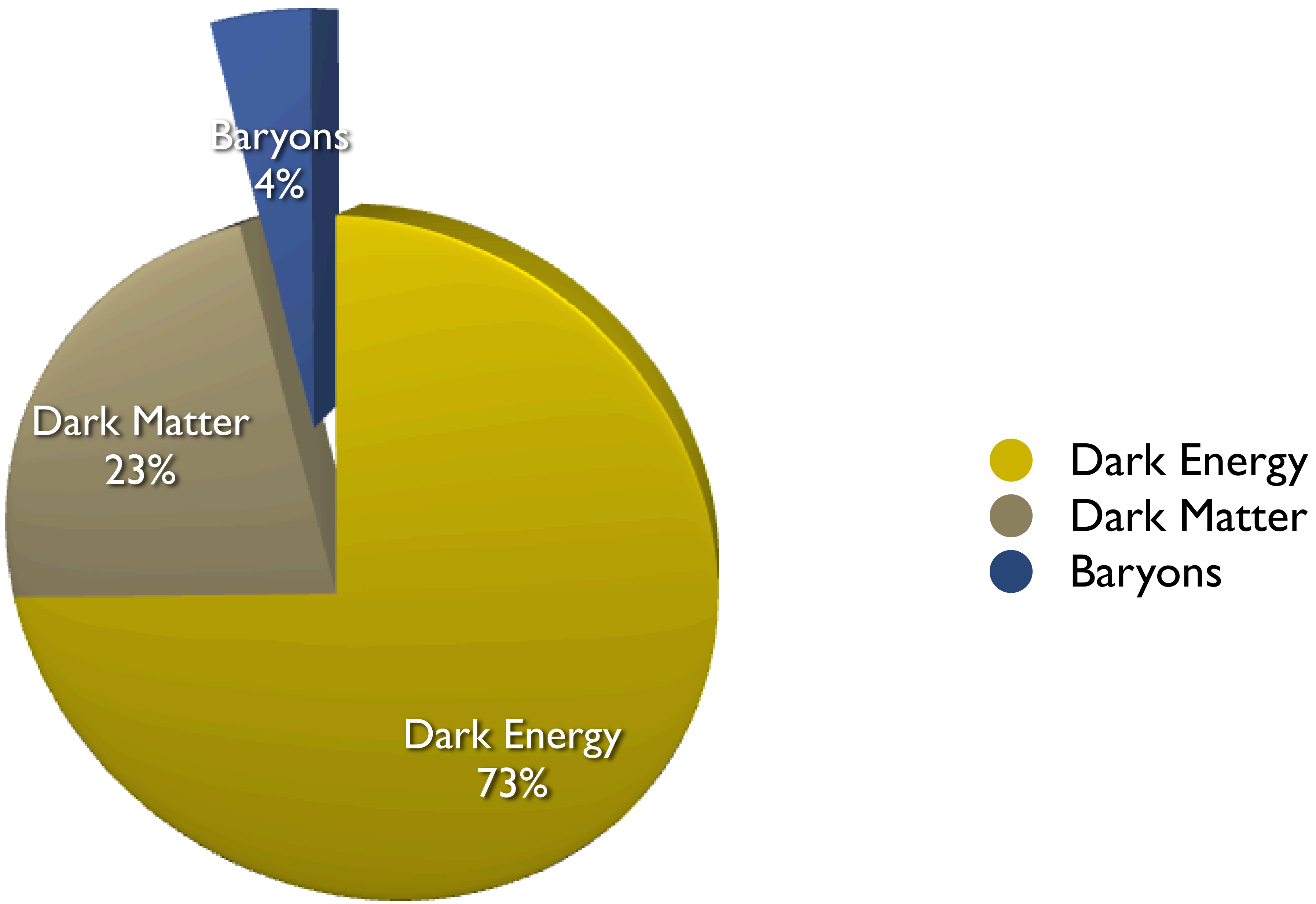
Cosmological tests of gravity



Constantinos Skordis (Nottingham)
"Testing General Relativity with cosmology",
Royal Society 28/2/11

Our universe today

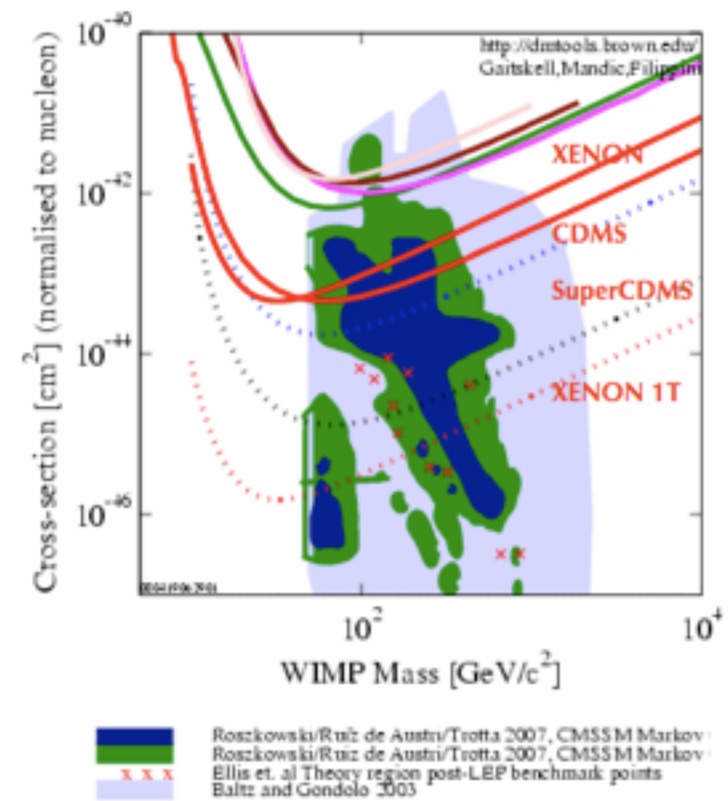


or is it?

Problems with CDM

- Missing satellites
- Cuspy halos?
- Tully-Fisher unexplained
- a_0 unexplained
- Baryon-CDM coincidence
- Voids too empty (Peebles)
- Bullet cluster collision velocity too fast (Lee & Komatsu)
- Tidal dwarf galaxies

Not detected yet



Problems with Λ

1. fine tuning problem : $\rho_{\text{vac,obs}} \sim 10^{-122} M_p^4$

$$\rho_{vac} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3} E_p$$

A low energy physics problem

$$\Lambda = \Lambda_0 + c_\nu m_\nu^4$$

threshold ↓

$$\Lambda = \Lambda_1 + c_e m_e^4 + c_\nu m_\nu^4$$

Λ_0 must cancel Λ_1 to 32 decimal places

2. Coincidence problem :

$\rho_\Lambda \sim \rho_{CDM} \sim \rho_b$ today

"Nothing yet. ...How about you, Newton?"

Rewriting Einstein equations:

$$f(g, \partial g, \partial^2 g, \phi^A, \partial \phi^A, \partial^2 \phi, T^{mat}_{cd})_{ab} = 0$$

$$\Downarrow$$

$$G_{ab}[g] = 8\pi G T_{ab} + U_{ab}[g, \phi^A, mat, \ldots]$$

$$\Downarrow$$

effective dark matter/dark energy

$$P_{eff} = w_{eff} \rho_{eff}$$

Simple parameterizations

1. $\Phi - \Psi = \alpha(t,k)\Phi$ (Bertschinger)

2. $-k^2\Phi = 4\pi G_{eff} a^2 \rho_M \left(\delta_M + 3\mathcal{H}\theta_M\right)$

comoving density perturbation

Two new functions : $\alpha(t,k)$

$G_{eff}(t,k)$

Caveats:
- Depends on initial conditions
- Blind to field content
- May be testing outside theory space
- Does not distinguish gravity from fluids

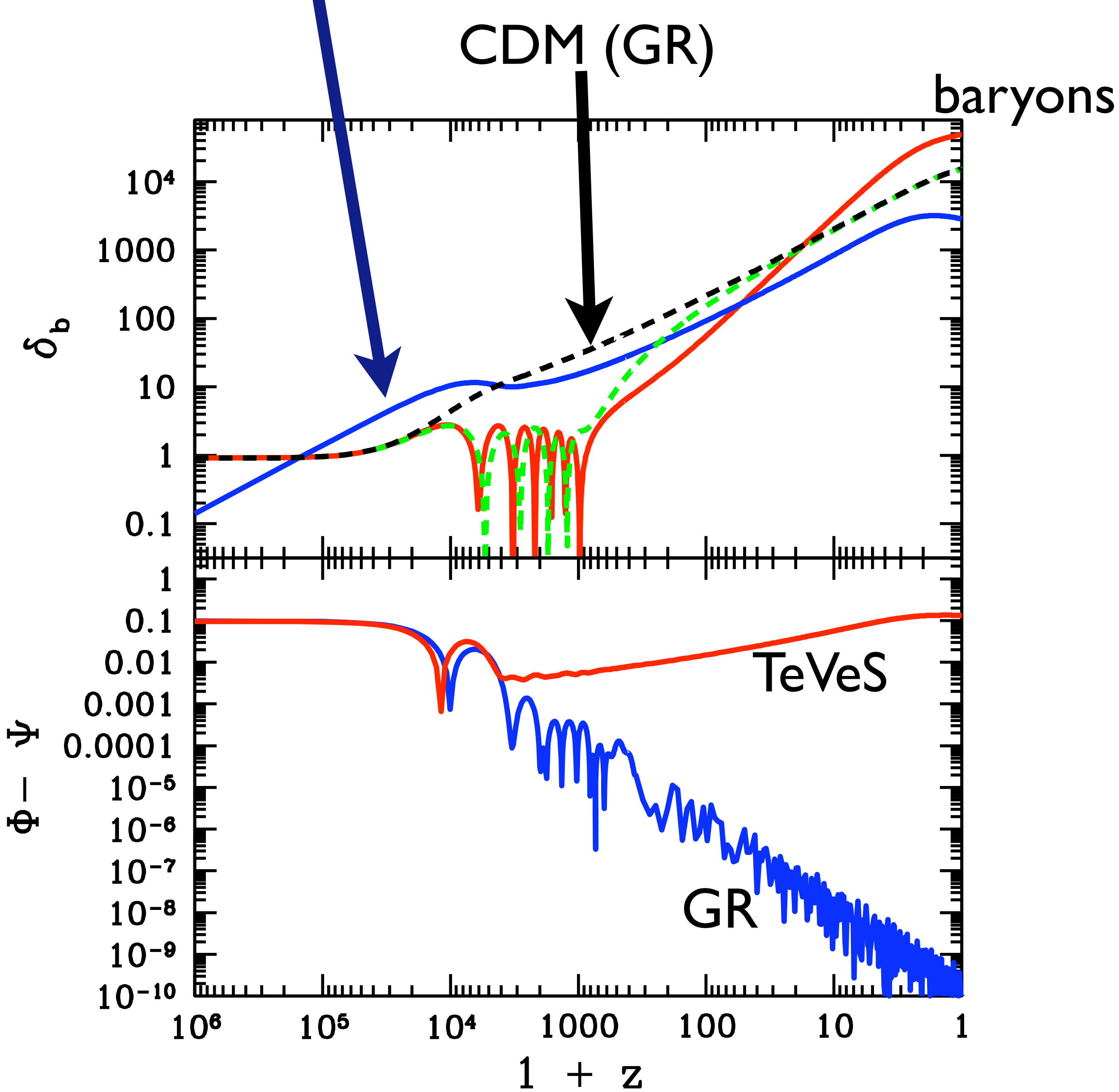
An example: TeVeS

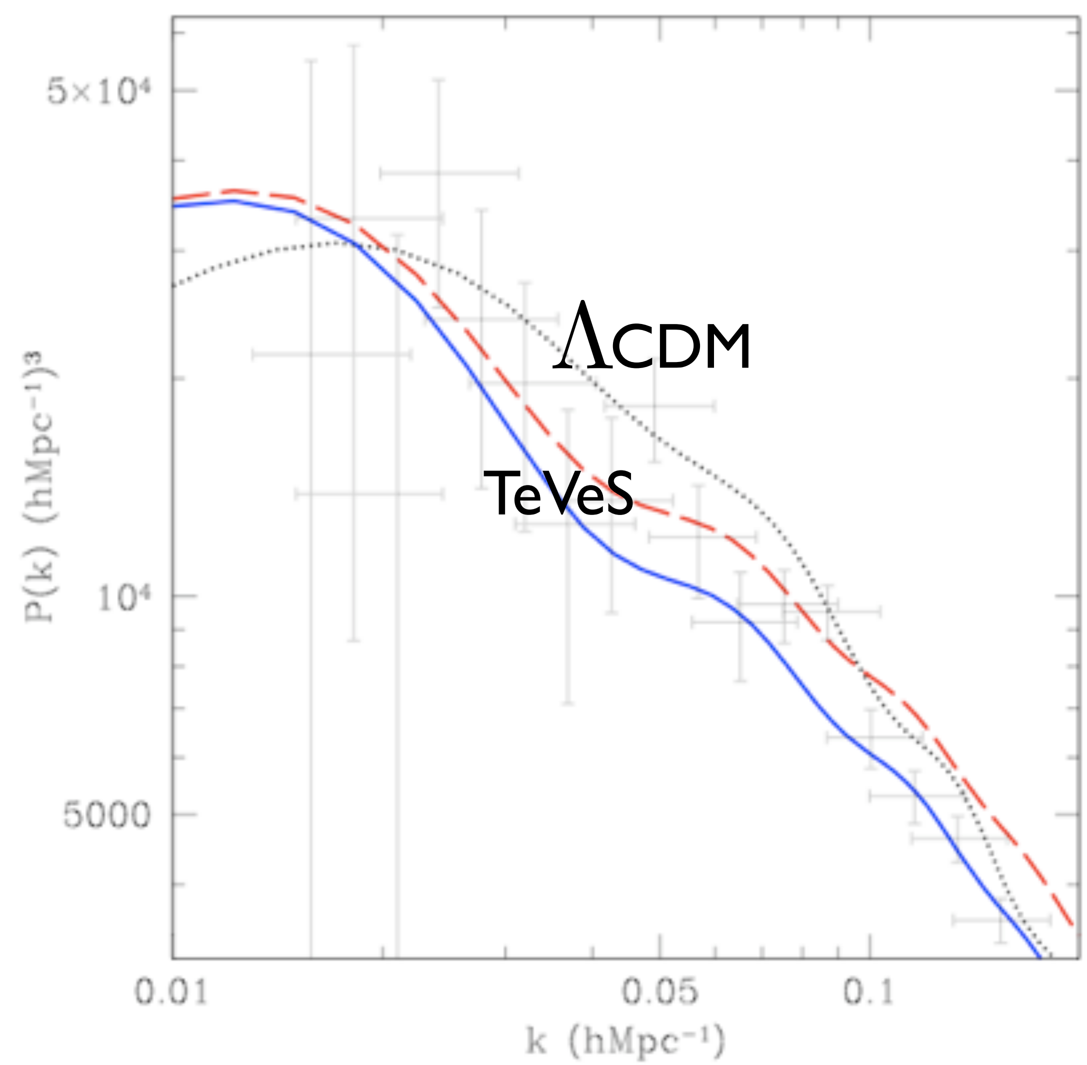
Bekenstein (2004)

two metrics $$g_{\mu\nu} = e^{-2\phi}\left(\tilde{g}_{\mu\nu} + A_\mu A_\nu\right) - e^{2\phi} A_\mu A_\nu$$

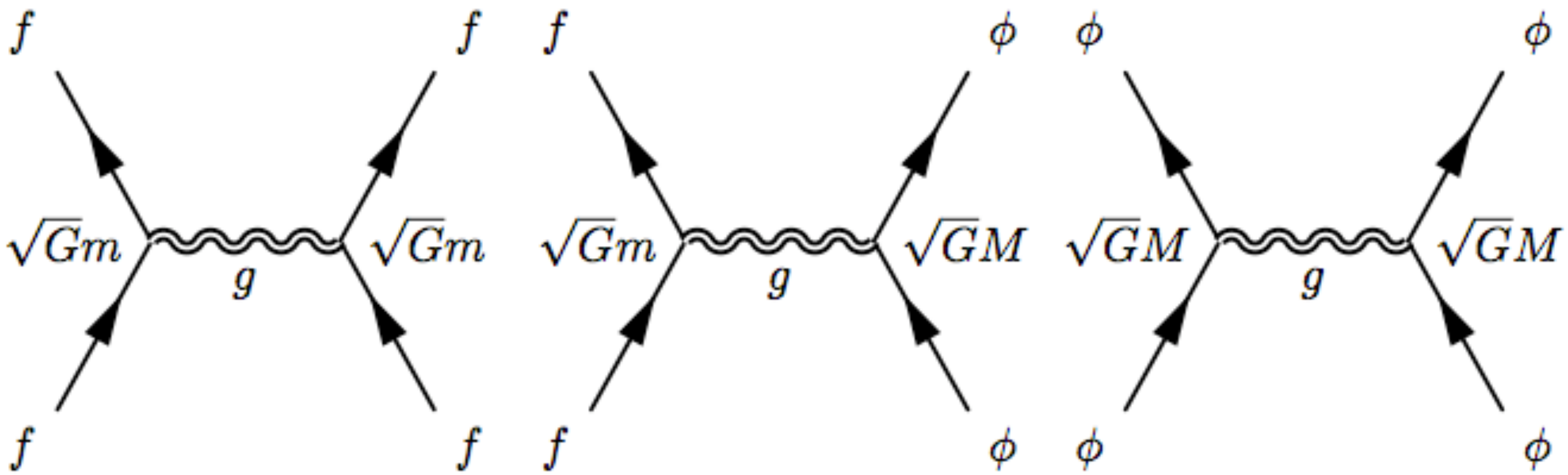
$$A^\mu A_\mu = -1$$

$A_i = \vec{\nabla}_i \alpha$

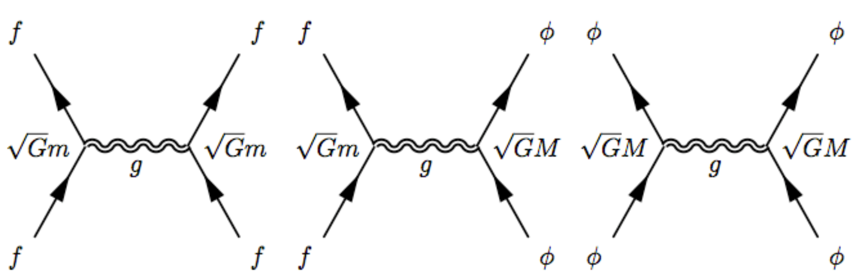


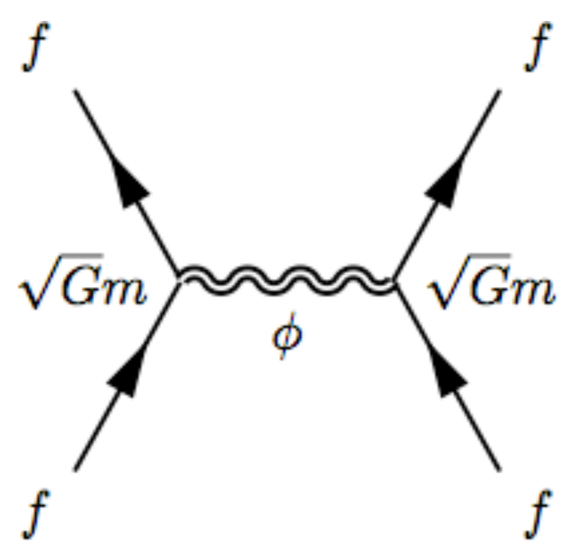


Dark fluids or gravity?



ϕ is fluid





ϕ mediates gravity

A classification

Type 0: no additional dof

Type 1: two additional dof, two additional field equations
e.g. Brans-Dicke, quintessence

Type 2: two additional dof, no additional field equations
e.g. f(R)

Type 3: more than 2 additional dof, more than 2 field equations
e.g. TeVeS, DGP

Consistent framework

C. S. (2009)

- Specification of the FRW background
- Consistent modification of linearized cosmological equations
  - Gauge form-invariance
  - Bianchi identity and energy conservation
- Field content
  - Number of time derivatives
  - Dynamical or non-dynamical fields
- Parameters
  - Expansion in known functions
- Associate theories with parameters

Assumption I

SM (+CDM?) matter couples to g_{ab} → $S_m[g_{ab}, \chi^A]$

↓

$$\nabla_a T^a{}_b = 0$$

$$G_{ab}[g] = 8\pi G T_{ab}[\chi^A, g] + U_{ab}[g, \phi^A]$$

↓

$$\nabla_a U^a{}_b = 0$$

Nb: coupling

$$U_{ab}[g, \phi^A, \chi^A] \qquad \nabla_a U^a{}_b = J_b = -\nabla_a T^a{}_b$$

Example 1: Brans-Dicke

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\nabla\phi)^2\right] + S_m[g]$$

$$G_{ab} = 8\pi G T_{ab} + U_{ab}$$

where

$$U_{ab} = (1-\phi)G_{ab} + \nabla_a\nabla_b\phi - \nabla^2\phi g_{ab} + \frac{\omega}{\phi}\left[\nabla_a\phi\nabla_b\phi - \frac{1}{2}(\nabla\phi)^2 g_{ab}\right]$$

$$\nabla_a U^a{}_b = 0$$

$$\frac{2\omega}{\phi}\nabla^2\phi + R + \left[\frac{d\omega}{d\phi} - \frac{\omega}{\phi}\right]\frac{(\nabla\phi)^2}{\phi} = 0$$

Example 2: $f(R)$

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[R + f(R)] + S_m$$

↓

$$G_{ab} = 8\pi G T_{ab} + U_{ab}$$

where

$$U_{ab} = \nabla_a\nabla_b f_R - f_R R_{ab} + \left(\frac{1}{2}f - \nabla^2 f_R\right) g_{ab}$$

↓

$\nabla_a U^a{}_b = 0$ for any $f(R)$

Assumption 2

FRW + linear perturbations is a good approximation on large scales

$$ds^2 = -dt^2 + a^2 d\ell_K^2$$

$$G_{ab}[g] = 8\pi G T_{ab}[\chi^A, g] + U_{ab}[g, \phi^A]$$

$${G^0}_0: \qquad 3H^2 + \frac{3K}{a^2} = 8\pi G \sum_i \rho_i + X$$

$${G^i}_i: \qquad -2\frac{\ddot{a}}{a} - H^2 = 8\pi G \sum_i P_i + Y$$

Bianchi identity gives $\dot{X} + 3H(X+Y) = 0$

Linearized equations

C. S. (2009)

4 scalar components in g_{ab} :

$$\delta G_{00} = -2a^2\Psi \qquad \delta g_{0i} = -a^2\vec{\nabla}_i\zeta \qquad \delta g_{ij} = a^2\left[-2\Phi\delta_{ij} + D_{ij}\nu\right]$$

Gauge form-invariance

$$2(\vec{\nabla}^2 + 3K)(\Phi - \vec{\nabla}^2\nu) - 6\tfrac{\dot a}{a}(\dot\Phi - \tfrac{1}{3}\vec{\nabla}^2\zeta) - 6\tfrac{\dot a^2}{a^2}\Psi = 8\,\pi G a^2\rho\delta$$

gauge ↓ transform

$$2(\vec{\nabla}^2 + 3K)(\Phi' - \vec{\nabla}^2\nu') - 6\tfrac{\dot a}{a}(\dot\Phi' - \tfrac{1}{3}\vec{\nabla}^2\zeta') - 6\tfrac{\dot a^2}{a^2}\Psi' = 8\pi G a^2\rho\delta' + [FRW\,eq.]\xi$$

choice

$$-2\vec{\nabla}^2\eta - \frac{\dot a}{a}\dot h = 8\pi G a^2\delta\rho \qquad\qquad -2\vec{\nabla}^2\Phi + 6\,\frac{\dot a}{a}\left(\dot\Phi + \frac{\dot a}{a}\Psi\right) = 8\pi G a^2\delta\rho$$

Example 1:

C. S. (2009)

$$2\left(\vec{\nabla}^2+3K\right)\left(\Phi-\vec{\nabla}^2\nu\right)-6\mathcal{H}\left(\dot{\Phi}-\frac{1}{3}\vec{\nabla}^2\zeta\right)-6\mathcal{H}^2\Psi=8\pi G a^2\mathcal{O}(t,\vec{\nabla}^2)\delta\rho$$

gauge transform

↓

$$\mathcal{O}=1$$

General invariance fixes ALL gauge-variant terms with the FRW background

Parameterization freedom is ONLY contained in gauge-invariant terms

Example 2

Background : ΛCDM
No higher than 2 time derivatives
No additional fields

C. S. (2009)

$$\delta G_{ab} = 8\pi G \delta T_{ab}^{\Lambda CDM} + \delta U_{ab}$$

$\delta U^a{}_b$ must be gauge-invariant

$$\Psi_{GI} = \Psi - \ddot{\nu} - \dot{\zeta} - \frac{\dot{a}}{a}(\dot{\nu} + \zeta)$$ contains 2nd derivatives

$$\Phi_{GI} = \Phi + \frac{1}{3}\vec{\nabla}^2\nu + \frac{\dot{a}}{a}(\dot{\nu} + \zeta)$$ contains 1st derivatives

Parameterize Einstein equations

C. S. (2009)

Constraints: 1st derivatives

$$\delta U^0{}_0 = \mathcal{A}\Phi_{GI}$$

$$\delta U^0{}_i = \mathcal{B}\vec{\nabla}_i\Phi_{GI}$$

Propagation: 2nd derivatives

$$\delta U^i{}_i = \mathcal{C}_1\Phi_{GI} + \mathcal{C}_2\dot{\Phi}_{GI} + \mathcal{C}_3\Psi_{GI}$$

$$\delta U^i{}_j - \frac{1}{3}U^k{}_k\delta^i{}_j = D^i{}_j\left[\mathcal{D}_1\Phi_{GI} + \mathcal{D}_2\dot{\Phi}_{GI} + \mathcal{D}_3\Psi_{GI}\right]$$

Use Bianchi identity

C. S. (2009)

$$\nabla_a U^a{}_b = 0$$

↓

$$[\ldots]\,\Phi_{GI} + [\ldots]\,\dot{\Phi}_{GI} + [\ldots]\,\Psi_{GI} = 0$$

x2

↓

$$\mathcal{C}_3 = \mathcal{D}_3 = 0 \qquad \mathcal{A} = -\frac{\dot{a}}{a}\mathcal{C}_2 \qquad \mathcal{B} = \frac{1}{3}\mathcal{C}_2 + \frac{2}{3}\left(\vec{\nabla}^2 + 3K\right)\mathcal{D}_2$$

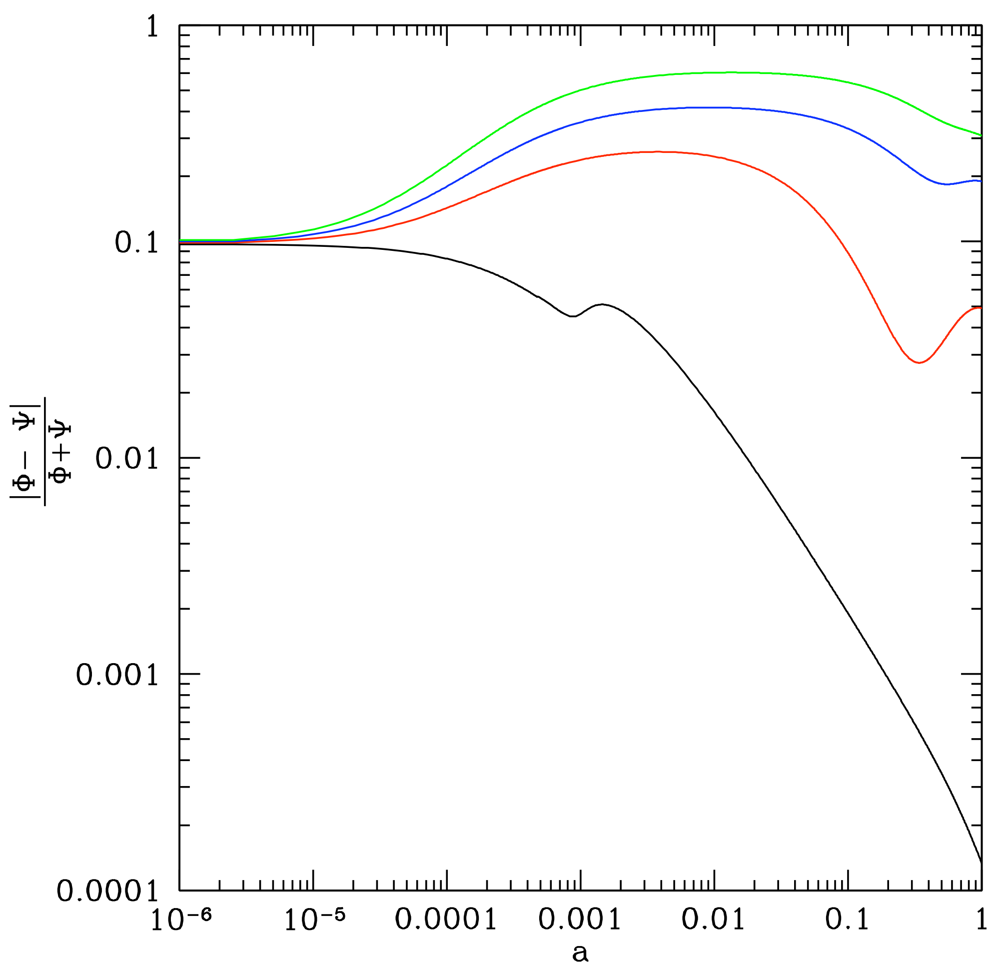
$$\dot{\mathcal{A}} + \frac{\dot{a}}{a}\mathcal{A} - \vec{\nabla}^2\mathcal{B} + \frac{\dot{a}}{a}\mathcal{C}_1 = 0$$

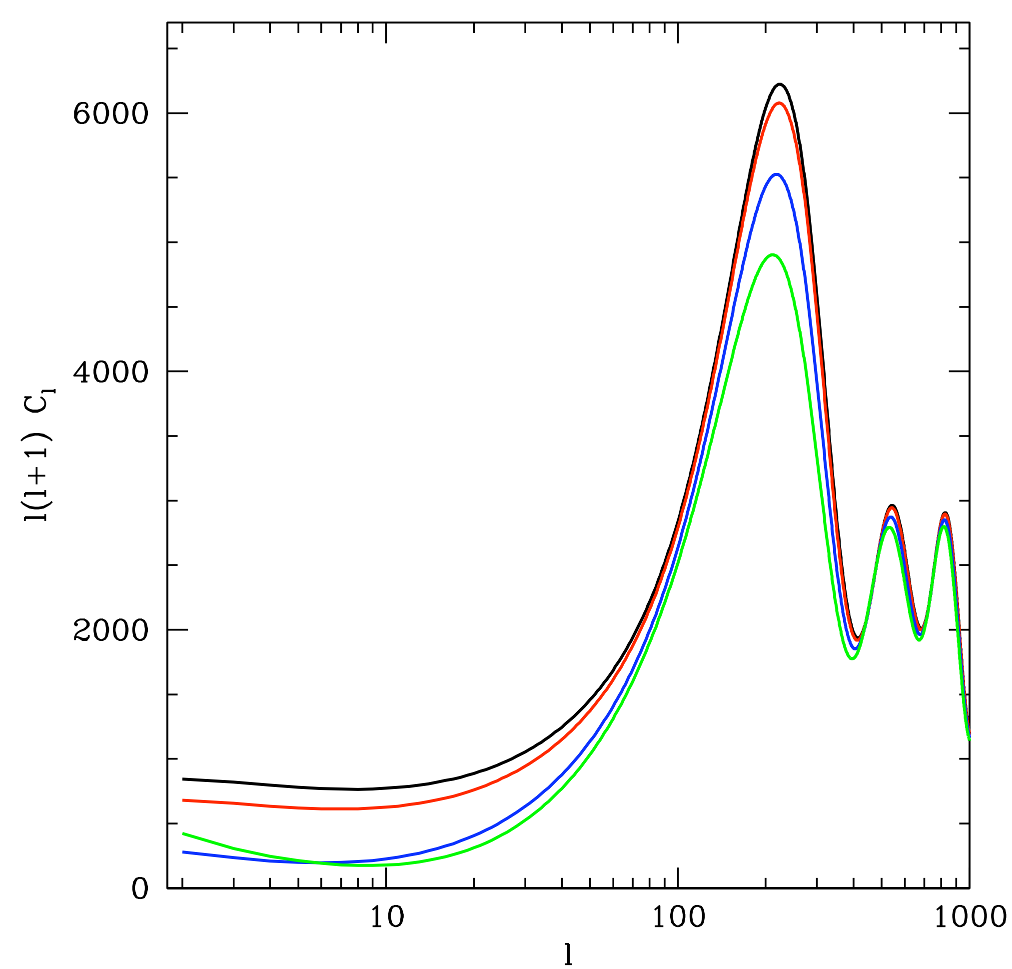
$$\dot{\mathcal{B}} + 2\frac{\dot{a}}{a}\mathcal{B} - \frac{1}{3}\mathcal{C}_1 - \frac{2}{3}\left(\vec{\nabla}^2 + 3K\right)\mathcal{D}_1 = 0$$

A simple solution (C.S 2009)

new parameter: β

$$\mathcal{B} = \mathcal{C}_1 = \mathcal{D}_1 = 0 \qquad \mathcal{A} = \beta\frac{H_0^2}{a} \qquad \mathcal{C}_2 = -\beta\frac{H_0^2}{\dot{a}} \qquad \mathcal{D}_2 = \beta\frac{H_0^2}{2\dot{a}}\frac{1}{\vec{\nabla}^2}$$





Generalization

(P. Ferreira & C.S 2010)

ΛCDM $\rightarrow$ wCDM + perturbations

$$\delta G_{ab} = 8\pi G(\delta T^M_{ab} + \delta T^E_{ab}) + \delta U_{ab}$$

All operators in U_{ab} solved in terms of

$$\zeta_p \equiv \mathcal{D}_1$$

$$g_p \equiv k\mathcal{D}_2$$

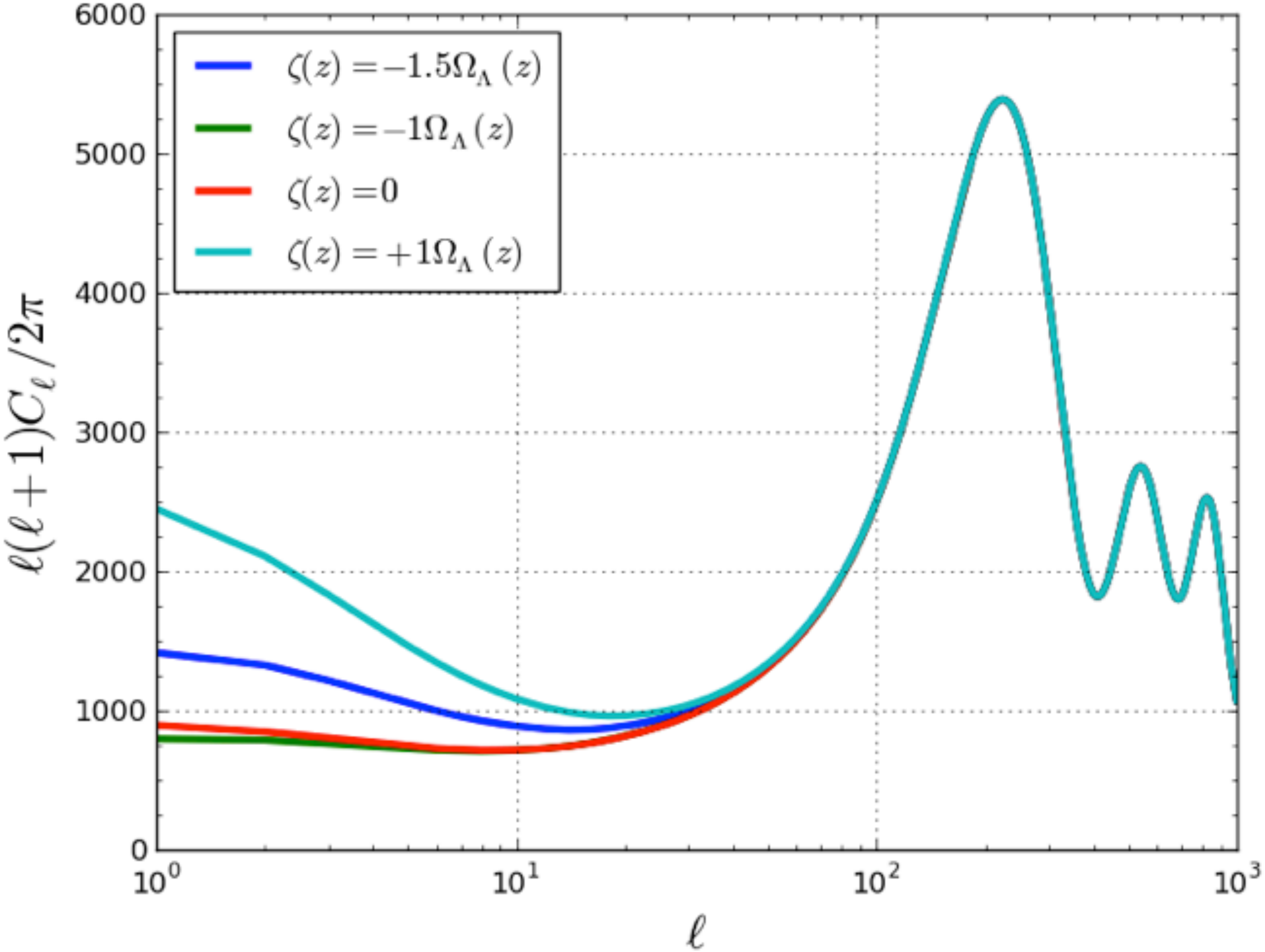
$$-k^2\Phi = 4\,\pi G_{eff} a^2 \rho(\delta + 3\frac{\dot{a}}{a}\theta)$$

$$G_{eff} = \frac{G}{1 - \mathcal{H}\frac{g_p}{k}}$$

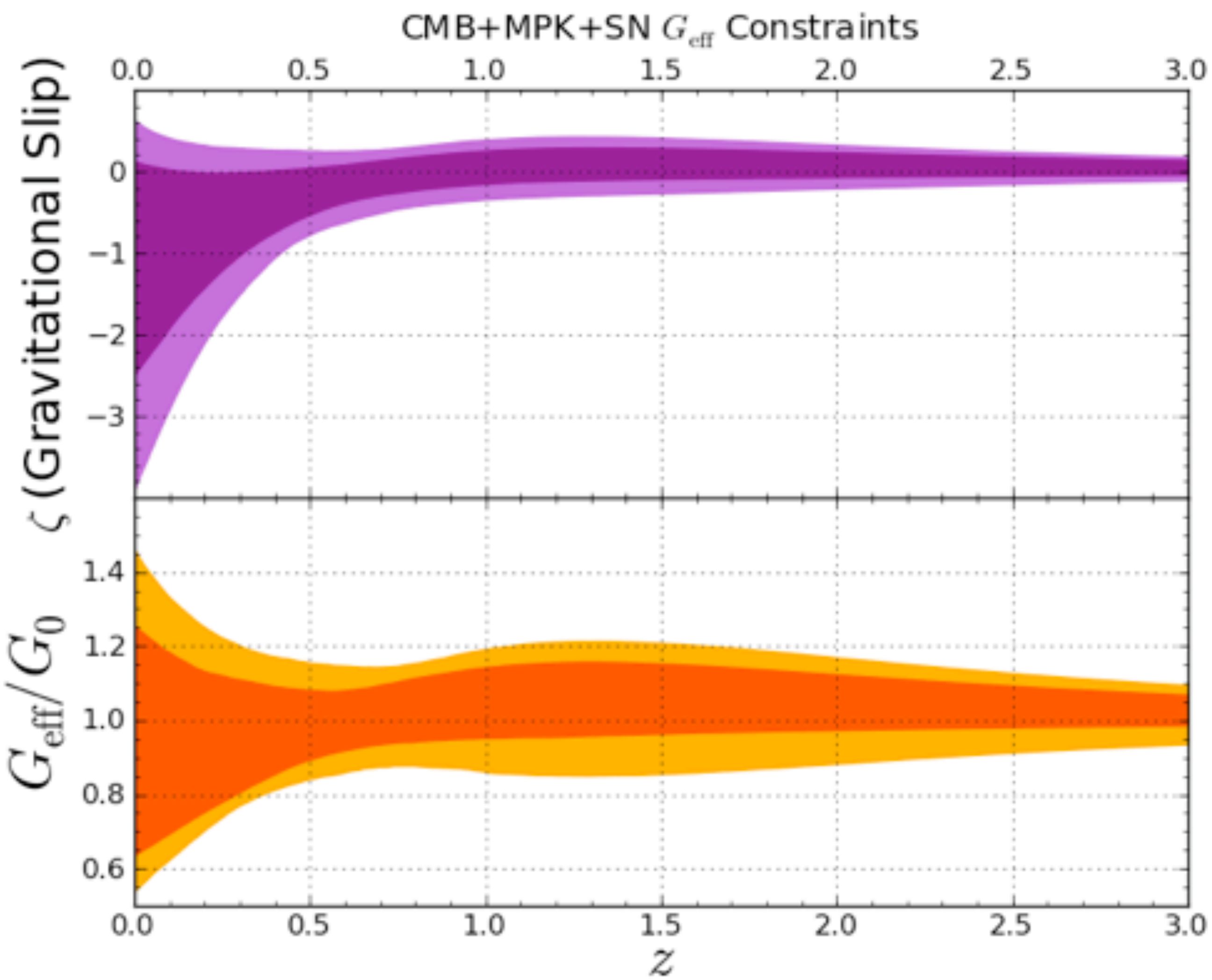
$$\Phi - \Psi = \zeta_p \Phi + \frac{g_p}{k}\dot{\Phi}$$

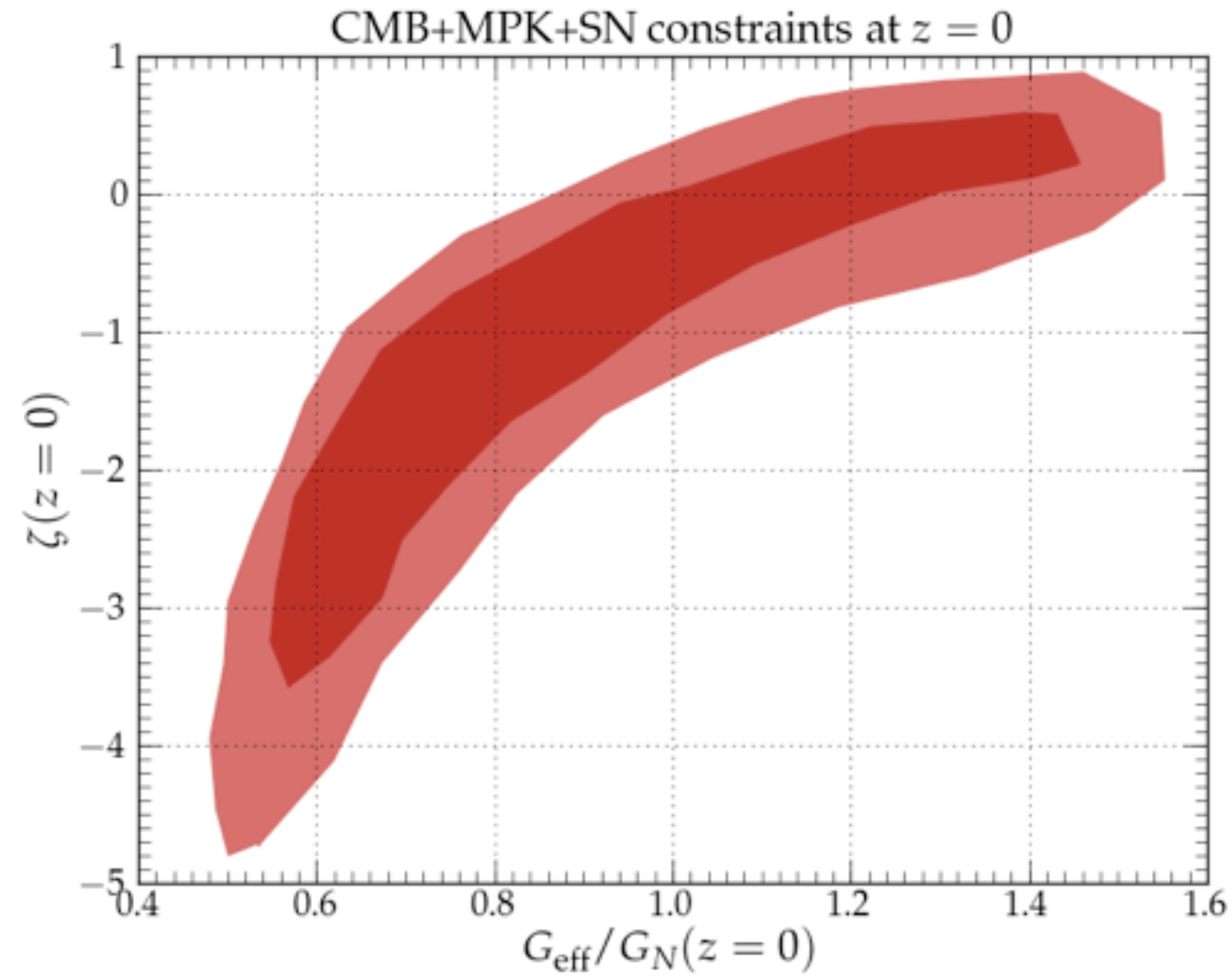
Current constraints

T. Baker, P. Ferreira, C. S., J. Zunz (in preparation)



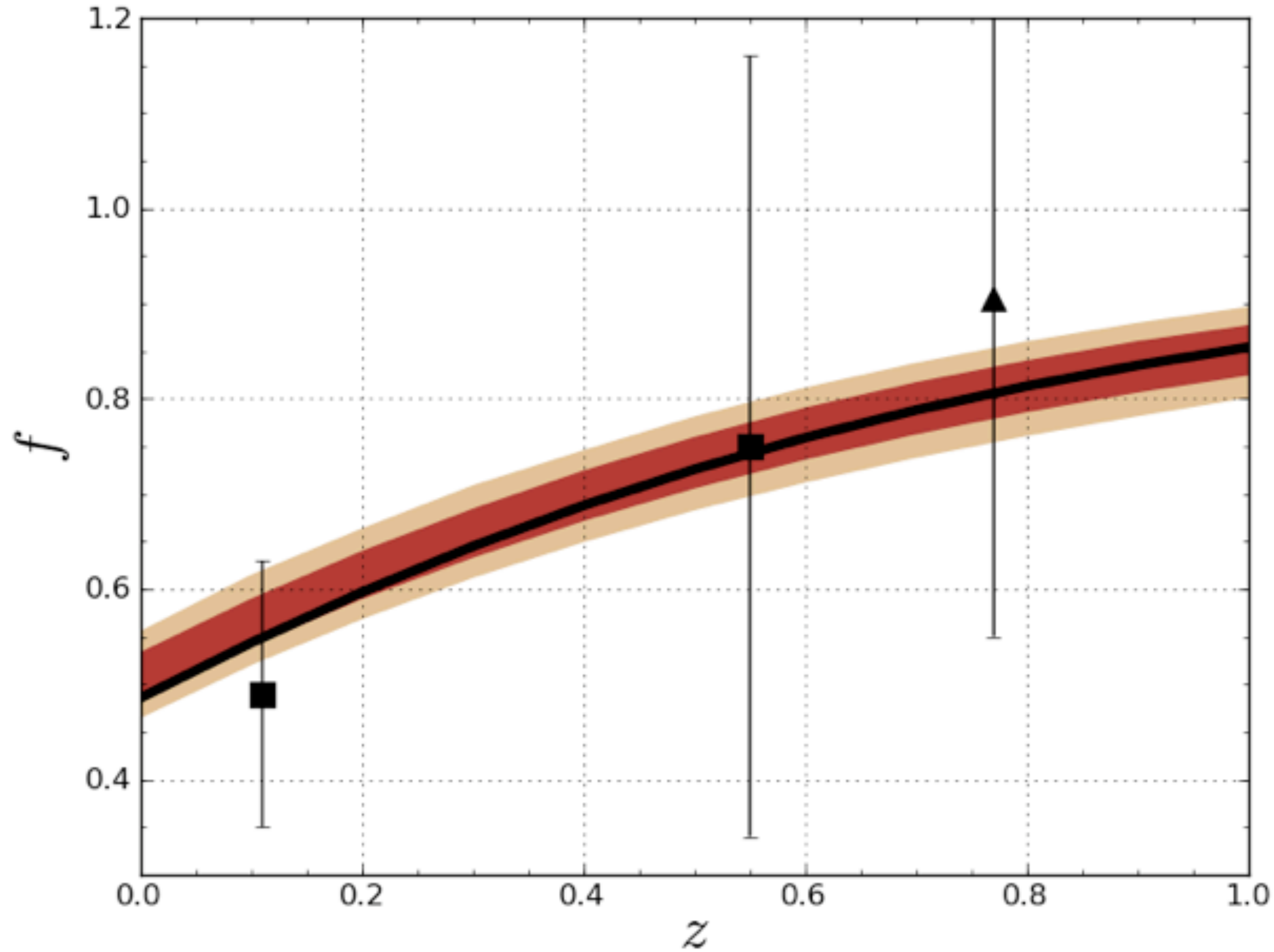
Results: (preliminary)





T. Baker, P. Ferreira, C. S., J. Zunz (in preparation)

Results: other data?



T. Baker, P. Ferreira, C. S., J. Zunz (in preparation)

Dynamical fields

e.g. $A^\mu A_\mu = -1$

(Baker, Ferreira, C.S., Zuntz)

FRW $X = X(a, \dot{a})$

Scalar mode $A_i = \vec{\nabla}_i \alpha$

$$\alpha_{GI} = \alpha - \frac{a}{6\dot{a}}(\chi - k^2\nu) \qquad \beta_{GI} = \dot{\alpha} + \Xi + \frac{1}{6}(\chi - k^2\nu)$$

U_{ab}:

$$U_\Delta = -\frac{1}{2}a^2(X+Y)(\chi - k^2\nu) + \mathcal{A}_1\Phi_{GI} + \mathcal{A}_2\alpha_{GI} + \mathcal{A}_3\beta_{GI}$$

Bianchi leads to field equations

$$\mathcal{O}_1^{(i)}\ddot{\alpha}_{GI} + \mathcal{O}_2^{(i)}\dot{\alpha}_{GI} + \mathcal{O}_3^{(i)}\alpha_{GI} + \mathcal{O}_4^{(i)}\Phi_{GI} + \mathcal{O}_5^{(i)}\dot{\Phi}_{GI} + \mathcal{O}_6^{(i)}\Psi_{GI} = 0$$

$$i = 1 \ldots 2$$

$\mathcal{O}^{(i)} = \mathcal{O}^{(i)}[\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i, \mathcal{D}_i]$ + constraints

N time derivatives

(Baker, Ferreira, C.S., Zuntz)

$$U_\Delta = \sum_{i=0}^{N-2} \mathcal{A}_i^{(\hat{\Phi})}(t)\frac{d^i}{dt^i}\Phi_{GI} + \sum_{i=0}^{N-3} \mathcal{A}_i^{(\hat{\Psi})}(t)\frac{d^i}{dt^i}\Psi_{GI}$$

$$U_\Theta = \sum_{i=0}^{N-2} \mathcal{B}_i^{(\hat{\Phi})}(t)\frac{d^i}{dt^i}\Phi_{GI} + \sum_{i=0}^{N-3} \mathcal{B}_i^{(\hat{\Psi})}(t)\frac{d^i}{dt^i}\Psi_{GI}$$

$$U_P = \sum_{i=0}^{N-1} \mathcal{C}_i^{(\hat{\Phi})}(t)\frac{d^i}{dt^i}\Phi_{GI} + \sum_{i=0}^{N-2} \mathcal{C}_i^{(\hat{\Psi})}(t)\frac{d^i}{dt^i}\Psi_{GI}$$

$$U_\Sigma = \sum_{i=0}^{N-1} \mathcal{D}_i^{(\hat{\Phi})}(t)\frac{d^i}{dt^i}\Phi_{GI} + \sum_{i=0}^{N-2} \mathcal{D}_i^{(\hat{\Psi})}(t)\frac{d^i}{dt^i}\Psi_{GI}$$

e.g. $f(R)$

$$U_\Sigma = f_R(\Psi_{GI} - \Phi_{GI}) - \frac{2}{a^2} f_{RR}\left[3\ddot{\Phi}_{GI} + 3\mathcal{H}(\dot{\Psi}_{GI} + 3\dot{\Phi}_{GI}) + 6\frac{\ddot{a}}{a}\Psi_{GI} - \vec{\nabla}^2(2\Phi_{GI} - \Psi_{GI})\right]$$

Outlook

- GR + visible matter fails to explain cosmological observations
  - Dark matter, Dark energy?
  - or gravity?
- Reconstruct the field content
  - consistent framework: gauge form-invariance
  - test theory space
  - observations
- Extend to non-linear scales