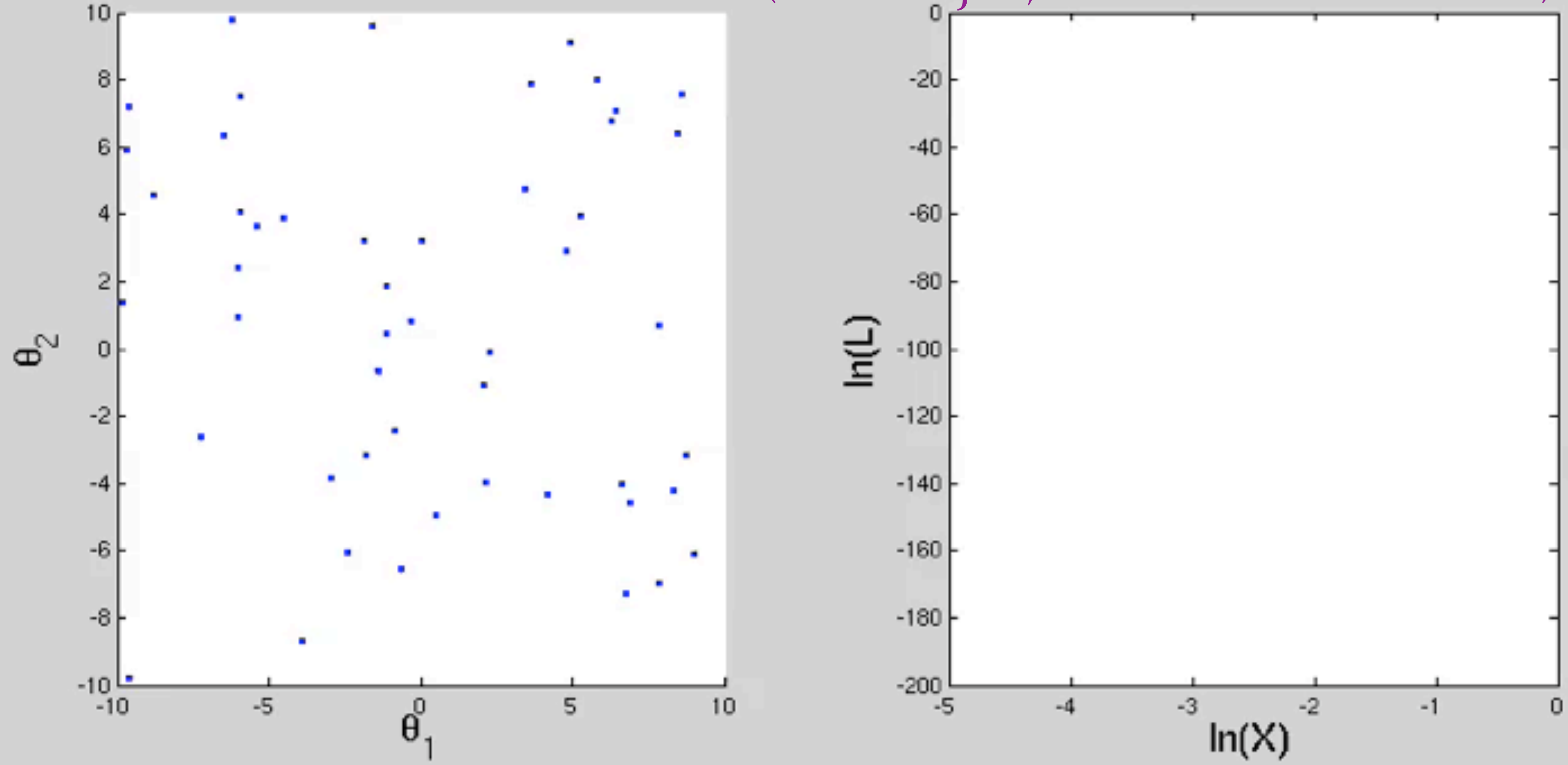
Optimizing dark energy surveys

University of Sussex

Andrew Liddle

March 2011

CosmoNest code in action (Mukherjee, Parkinson and Liddle)



Microsoft-free presentation

Talk theme:
Use of Bayesian inference for design and interpretation of cosmological surveys.

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Terminology: a **model** is a choice of the ***set*** of parameters to be varied in a fit to a dataset.

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- Eventually, sufficiently good data will overturn incorrect choice of prior.
- If you don't know enough to set a prior, why did you bother getting the data?

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Parameter Estimation

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I can do this by combining the parameter likelihoods using Bayesian Model Averaging, adding them together weighted by the model probabilities.

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Prior ranges for those parameters

Choose datasets: WMAP currently the most powerful but need others for `cosmic complementarity'.

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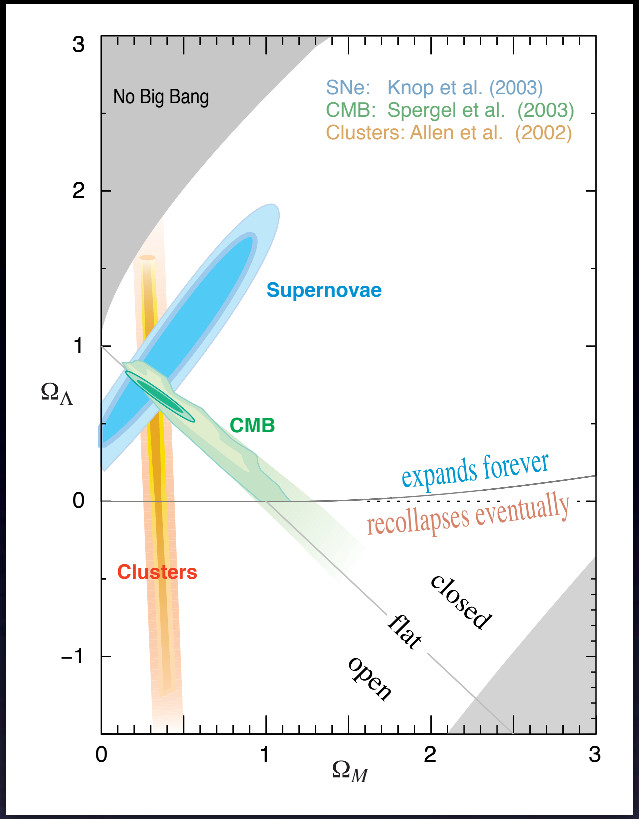

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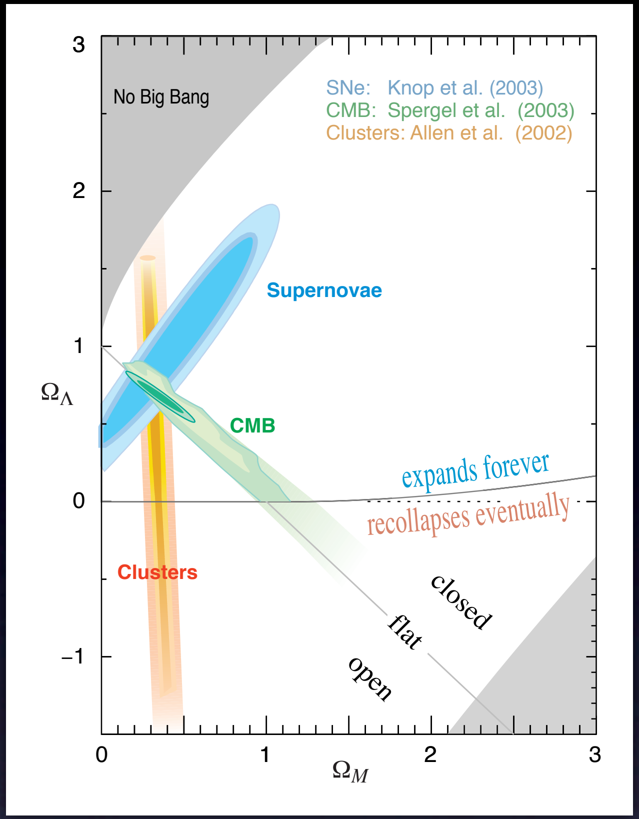
Compute likelihood function

Obtain posterior parameter distribution



Choos

Choos
pow
com

Compu

Obtain

WMAP5, Dunkley et al

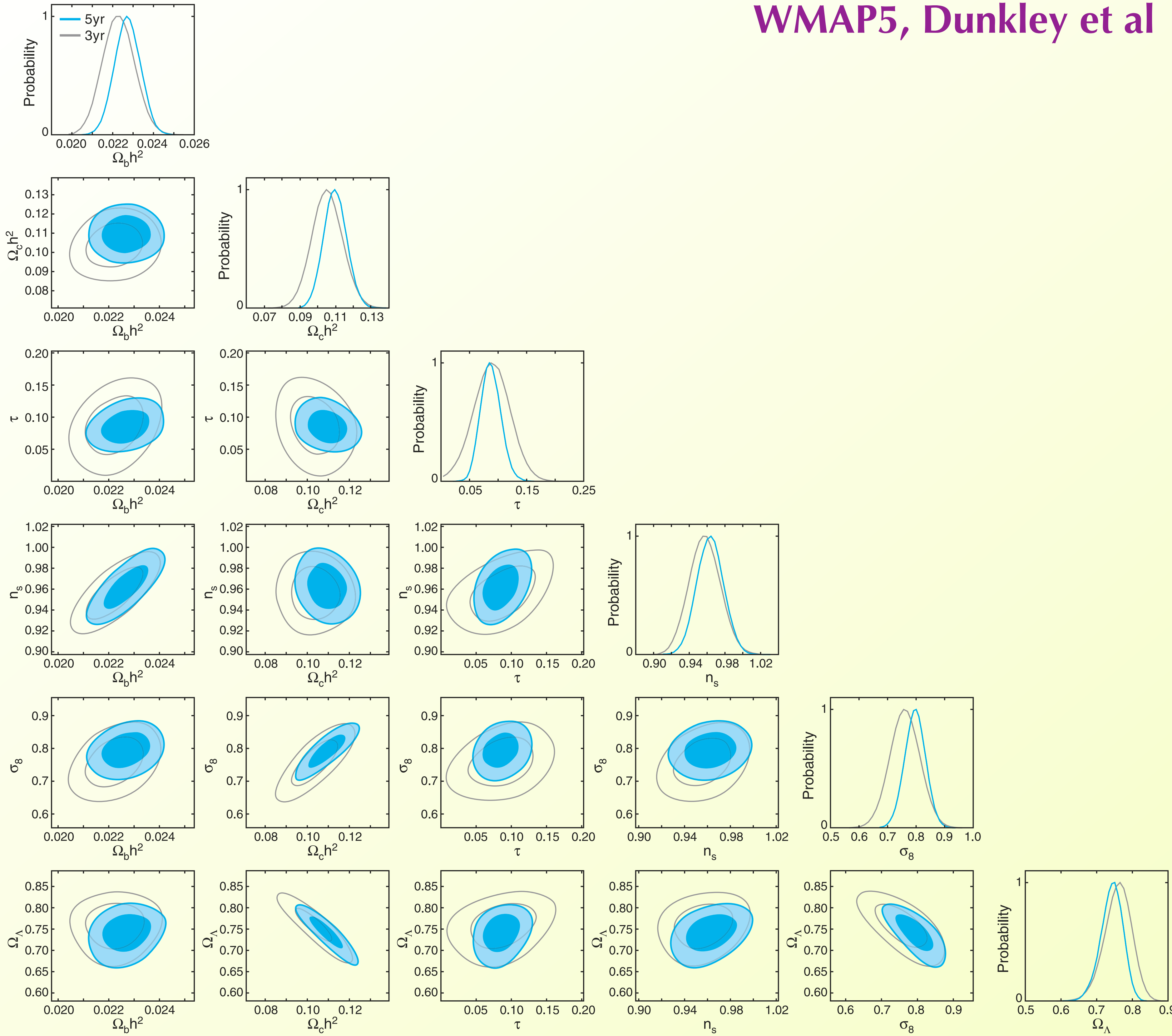
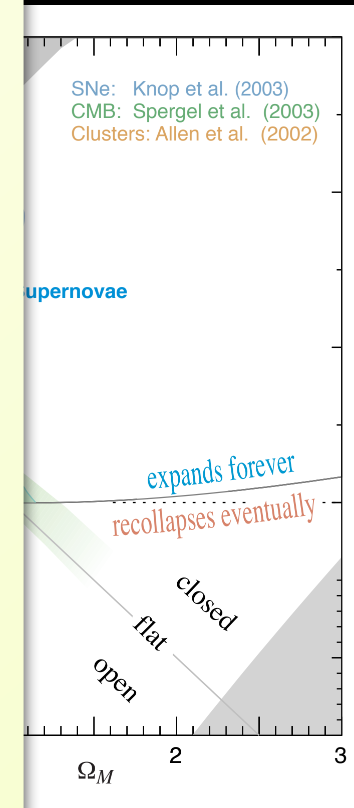


Fig. 4.— Constraints from the five-year *WMAP* data on ΛCDM parameters (blue), showing marginalized one-dimensional distributions and two-dimensional 68% and 95% limits. Parameters are consistent with the three-year limits (grey) from Spergel et al. (2007), and are now better constrained.



Parameters of the standard cosmological model

	WMAP5 alone	WMAP5 + BAO + SN
$\Omega_{\rm b} h^2$	0.0227 ± 0.0006	0.0227 ± 0.0006
$\Omega_{\rm cdm} h^2$	0.110 ± 0.006	0.113 ± 0.003
Ω_Λ	0.74 ± 0.03	0.726 ± 0.015
n	$0.963^{+0.014}_{-0.015}$	0.960 ± 0.013
τ	0.087 ± 0.017	0.084 ± 0.016
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The currently-favoured cosmology is a ΛCDM model, in a spatially-flat Universe, with initial conditions of the form expected from simple inflation models.

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It is easy to find over 20 such candidate parameters that have already been discussed in the literature.

Table 2. Candidate parameters: those which might be relevant for cosmological observations, but for which there is presently no convincing evidence requiring them. They are listed so as to take the value zero in the base cosmological model. Those above the line are parameters of the background homogeneous cosmology, and those below describe the perturbations.

Ω_k	spatial curvature
$N_\nu - 3.04$	effective number of neutrino species (CMBFAST definition)
m_{ν_i}	neutrino mass for species 'i'
	[or more complex neutrino properties]
$m_{\rm dm}$	(warm) dark matter mass
$w + 1$	dark energy equation of state
dw/dz	redshift dependence of w
	[or more complex parametrization of dark energy evolution]
$c_{\rm S}^2 - 1$	effects of dark energy sound speed
$1/r_{\rm top}$	topological identification scale
	[or more complex parametrization of non-trivial topology]
$d\alpha/dz$	redshift dependence of the fine structure constant
dG/dz	redshift dependence of the gravitational constant
$n - 1$	scalar spectral index
$dn/d\ln k$	running of the scalar spectral index
r	tensor-to-scalar ratio
$r + 8n_{\rm T}$	violation of the inflationary consistency equation
$dn_{\rm T}/d\ln k$	running of the tensor spectral index
$k_{\rm cut}$	large-scale cut-off in the spectrum
$A_{\rm feature}$	amplitude of spectral feature (peak, dip or step) ...
$k_{\rm feature}$	... and its scale
	[or adiabatic power spectrum amplitude parametrized in N bins]
$f_{\rm NL}$	quadratic contribution to primordial non-gaussianity
	[or more complex parametrization of non-gaussianity]
$\mathcal{P}_S$	CDM isocurvature perturbation ...
n_S	... and its spectral index ...
$\mathcal{P}_{S\mathcal{R}}$	... and its correlation with adiabatic perturbations ...
$n_{S\mathcal{R}} - n_S$	... and the spectral index of that correlation
	[or more complicated multi-component isocurvature perturbation]
$G\mu$	cosmic string component of perturbations

From Liddle 2004

Bayesian model selection

Choose dataset

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Compute model likelihoods, known as the Bayesian evidence
Update prior model probabilities to posterior ones
[option: multi-model inference by Bayesian model averaging]
Interpret

The Bayesian evidence

Bayes theorem again

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Bayes theorem again, but conditioned on a model.

$$P(\theta|D) = \frac{P(D|\theta)P(\theta)}{P(D)} \qquad \Rightarrow \qquad P(\theta|D,M) = \frac{P(D|\theta,M)P(\theta|M)}{P(D|M)}$$

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- If only one model survives, proceed to standard parameter estimation.
- If several models survive, use multi-model inference, e.g. Bayesian model averaging.

Interpretational scale

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$\Delta \ln E < 1$	Not worth more than a bare mention
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The most useful divisions are 2.5 (odds ratio of 12:1) and 5 (odds ratio of 150:1).

(Almost) current dark energy data

Liddle, Mukherjee, Parkinson, and Wang, PRD, astro-ph/0610126

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CMB shift+BAO(SDSS)+SN

data used	Model			
WMAP+SDSS+	$\Delta \ln E$	H	$\chi^2_{\rm min}$	parameter constraints
	Model I: Λ			
Riess04	0.0	5.7	30.5	$\Omega_{\rm m} = 0.26 \pm 0.03,\ H_0 = 65.5 \pm 1.0$
Astier05	0.0	6.5	94.5	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.3 \pm 1.0$
	Model II: constant w, flat prior $-1 \leq w \leq -0.33$			
Riess04	-0.1 ± 0.1	6.4	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.4,\ w < -0.81, -0.70^a$
Astier05	-1.3 ± 0.1	8.0	93.3	$\Omega_{\rm m} = 0.24 \pm 0.03,\ H_0 = 69.8 \pm 1.0,\ w < -0.90, -0.83^a$
	Model III: constant w, flat prior $-2 \leq w \leq -0.33$			
Riess04	-1.0 ± 0.1	7.3	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.5,\ w = -0.87 \pm 0.1$
Astier05	-1.8 ± 0.1	8.2	93.3	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.0 \pm 1.0,\ w = -0.96 \pm 0.08$
	Model IV: w_0–w_a, flat prior $-2 \leq w_0 \leq -0.33,\ -1.33 \leq w_a \leq 1.33$			
Riess04	-1.1 ± 0.1	7.2	28.5	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.1 \pm 1.5,\ w_0 = -0.83 \pm 0.20,\ w_a = --^b$
Astier05	-2.0 ± 0.1	8.2	93.3	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.0 \pm 1.0,\ w_0 = -0.97 \pm 0.18,\ w_a = --^b$
	Model V: w_0–w_a, $-1 \leq w(a) \leq 1$ for $0 \leq z \leq 2$			
Riess04	-2.4 ± 0.1	9.1	28.5	$\Omega_{\rm m} = 0.28 \pm 0.04,\ H_0 = 63.6 \pm 1.3,\ w_0 < -0.78, -0.60^a,\ w_a = -0.07 \pm 0.34$
Astier05	-4.1 ± 0.1	11.1	93.3	$\Omega_{\rm m} = 0.24 \pm 0.03,\ H_0 = 69.5 \pm 1.0,\ w_0 < -0.90, -0.80^a,\ w_a = 0.12 \pm 0.22$

LambdaCDM

Constant W {

W_0-W_a {

(Almost) current dark energy data

Liddle, Mukherjee, Parkinson, and Wang, PRD, astro-ph/0610126

CMB shift+BAO(SDSS)+SN

	data used		Model		
	WMAP+SDSS+	$\Delta \ln E$	H	$\chi^2_{\rm min}$	parameter constraints
					Model I: Λ
LambdaCDM	Riess04	0.0	5.7	30.5	$\Omega_{\rm m} = 0.26 \pm 0.03,\ H_0 = 65.5 \pm 1.0$
	Astier05	0.0	6.5	94.5	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.3 \pm 1.0$
					Model II: constant w, flat prior $-1 \le w \le -0.33$
Constant W	Riess04	-0.1 ± 0.1	6.4	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.4,\ w < -0.81, -0.70^a$
	Astier05	-1.3 ± 0.1	8.0	93.3	$\Omega_{\rm m} = 0.24 \pm 0.03,\ H_0 = 69.8 \pm 1.0,\ w < -0.90, -0.83^a$
					Model III: constant w, flat prior $-2 \le w \le -0.33$
	Riess04	-1.0 ± 0.1	7.3	28.6	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.0 \pm 1.5,\ w = -0.87 \pm 0.1$
	Astier05	-1.8 ± 0.1	8.2	93.3	$\Omega_{\rm m} = 0.25 \pm 0.03,\ H_0 = 70.0 \pm 1.0,\ w = -0.96 \pm 0.08$
					Model IV: w_0–w_a, flat prior $-2 \le w_0 \le -0.33,\ -1.33 \le w_a \le 1.33$
W_0-W_a	Riess04	-1.1 ± 0.1	7.2	28.5	$\Omega_{\rm m} = 0.27 \pm 0.04,\ H_0 = 64.1 \pm 1.5,\ w_0 = -0.83 \pm 0.20,\ w_a = --^b$
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Conclusion: LambdaCDM currently favoured but all models still alive

That’s where we are at present.

Now for the future!

What are we trying to achieve?

Goal: to define the key model tests to be carried out and, where possible, to optimize survey strategies to achieve them. First we need to figure out which are the interesting models.

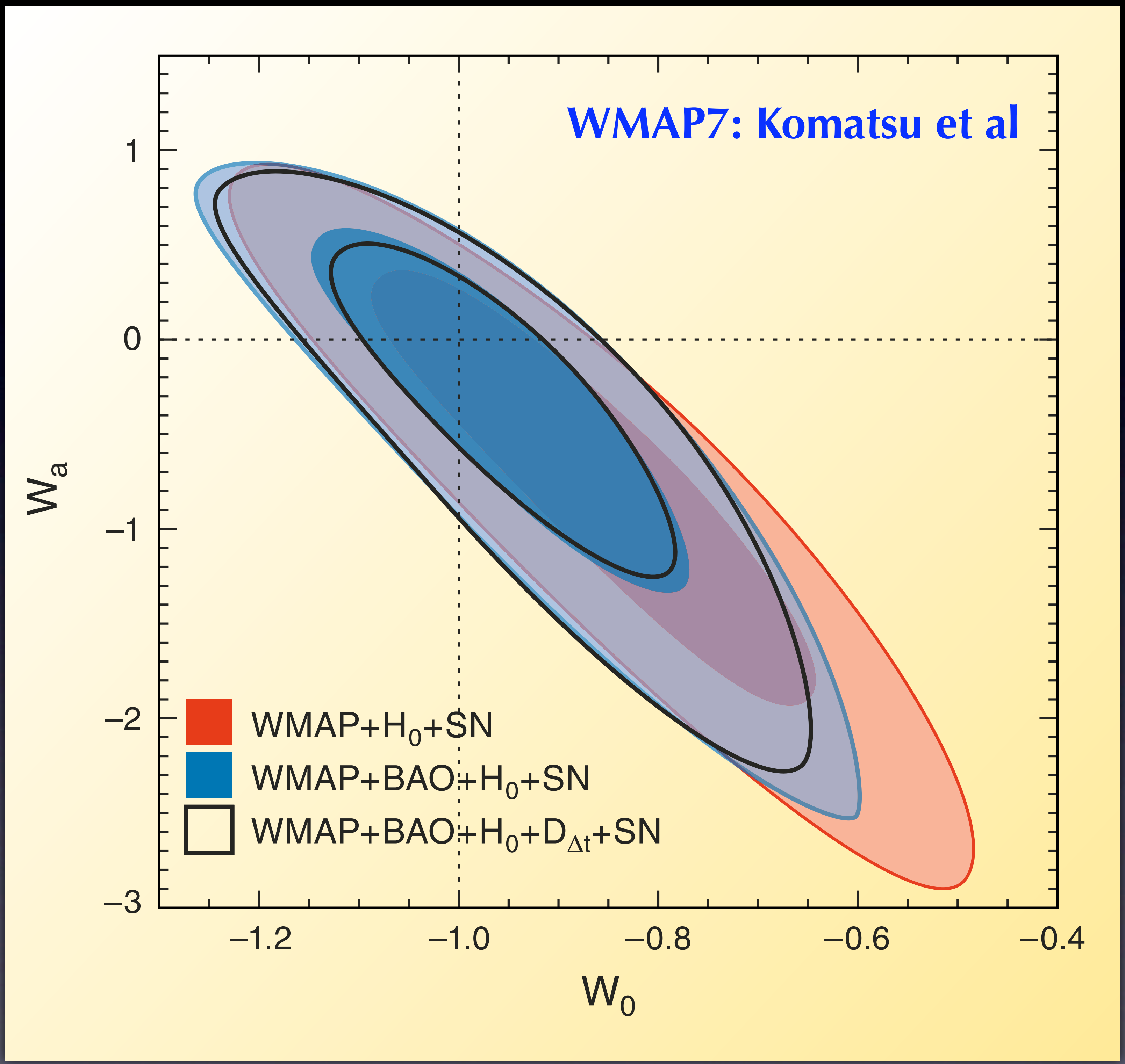
- **ΛCDM:** The current baseline cosmological model.
- **Phenomenological dark energy models, eg CPL $w = w_0 + (1-a)\, w_a$:**

  The most common candidate alternative for dark energy studies.
- **Fundamental physics dark energy models, eg inverse power-laws, Albrecht-Skordis, etc:**

  Many candidate models in the literature though many fail to fit current data. Not clear which are best motivated.
- **Modified gravity models:**

  Determination of best candidate modified gravity models required.
- **Inhomogeneous Universe models:** Not clear if there are any.

Parameter estimation tests



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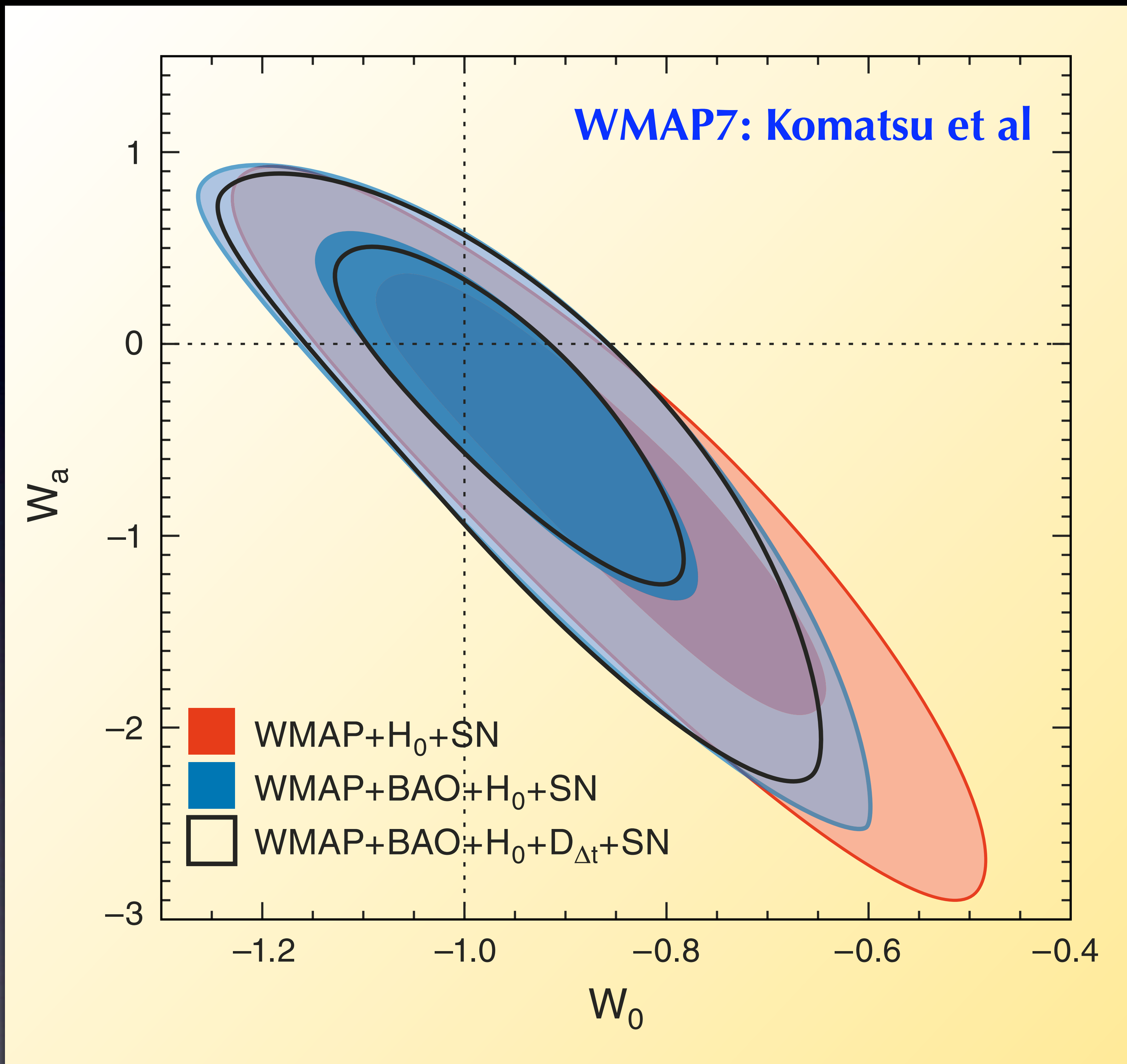

This graph answers the following question:

If we assume that the w_0-w_a dark energy model is correct, how good are our constraints on those parameters?

Model tests

However that wasn't the question we wanted to answer, which was:

Between the ΛCDM model and the dark energy model, which is the better description of the data?
[I.e., can one of them be ruled out with respect to the other?]

This question can only be answered with a model-level analysis.

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Another way of expressing this: do you think that the ***prior*** probabilities of w_0 = -1 and of w_0 = -0.9 are equal?

I would argue that they are not just different in magnitude, but that the former is finite while the latter is infinitesimal.

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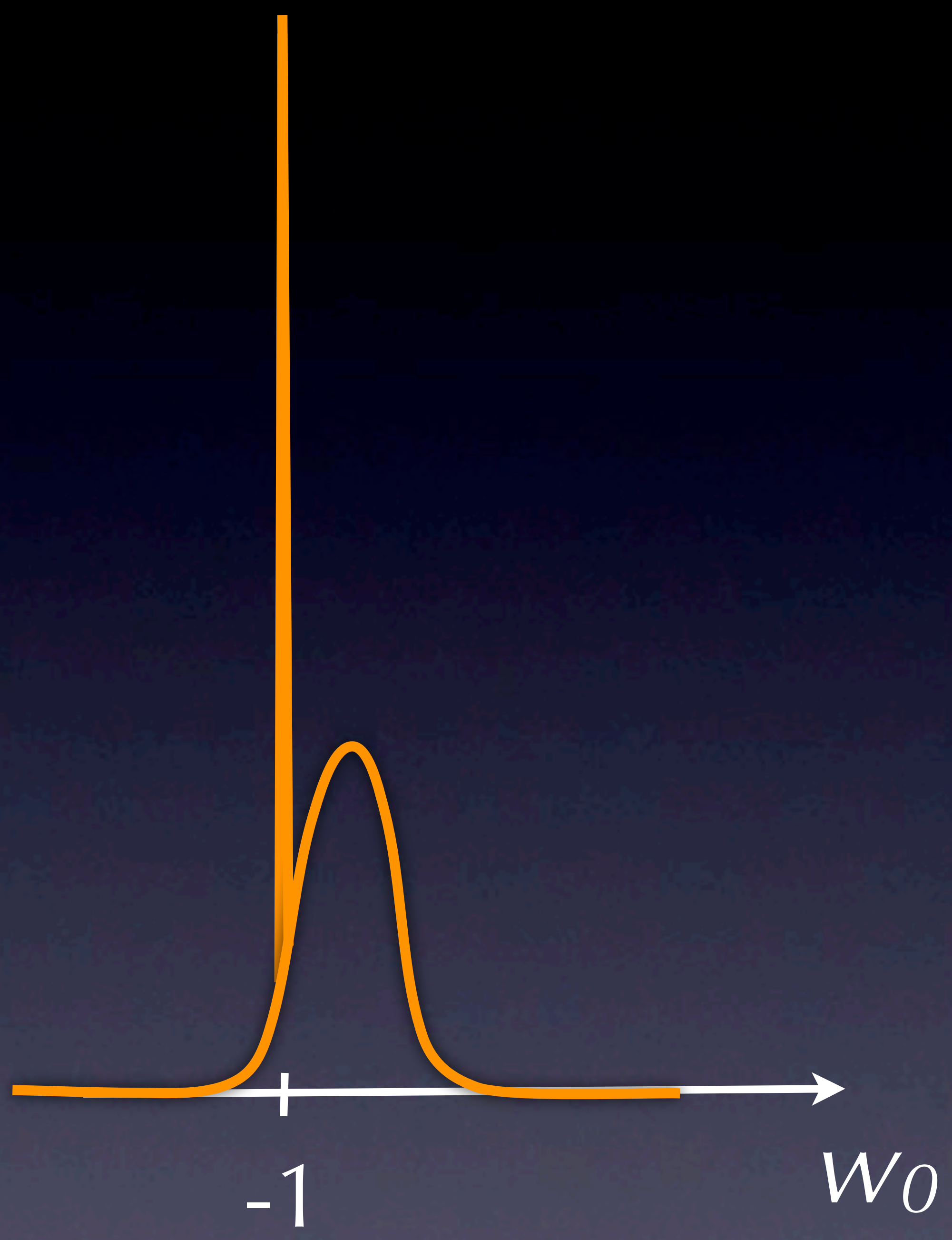
Within each of these models we also have a probability distribution for the parameters.

Likelihood of w_0 given all models



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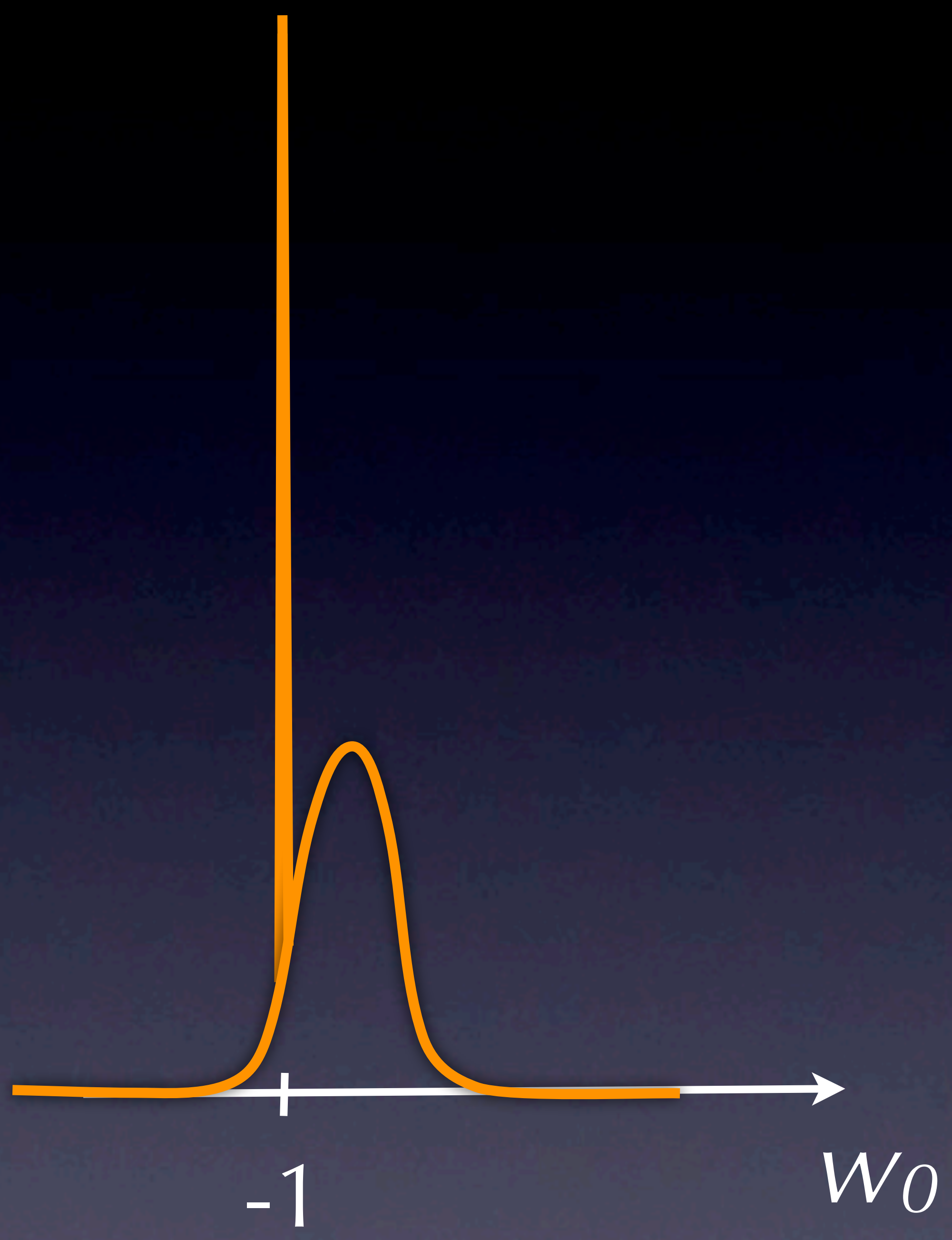

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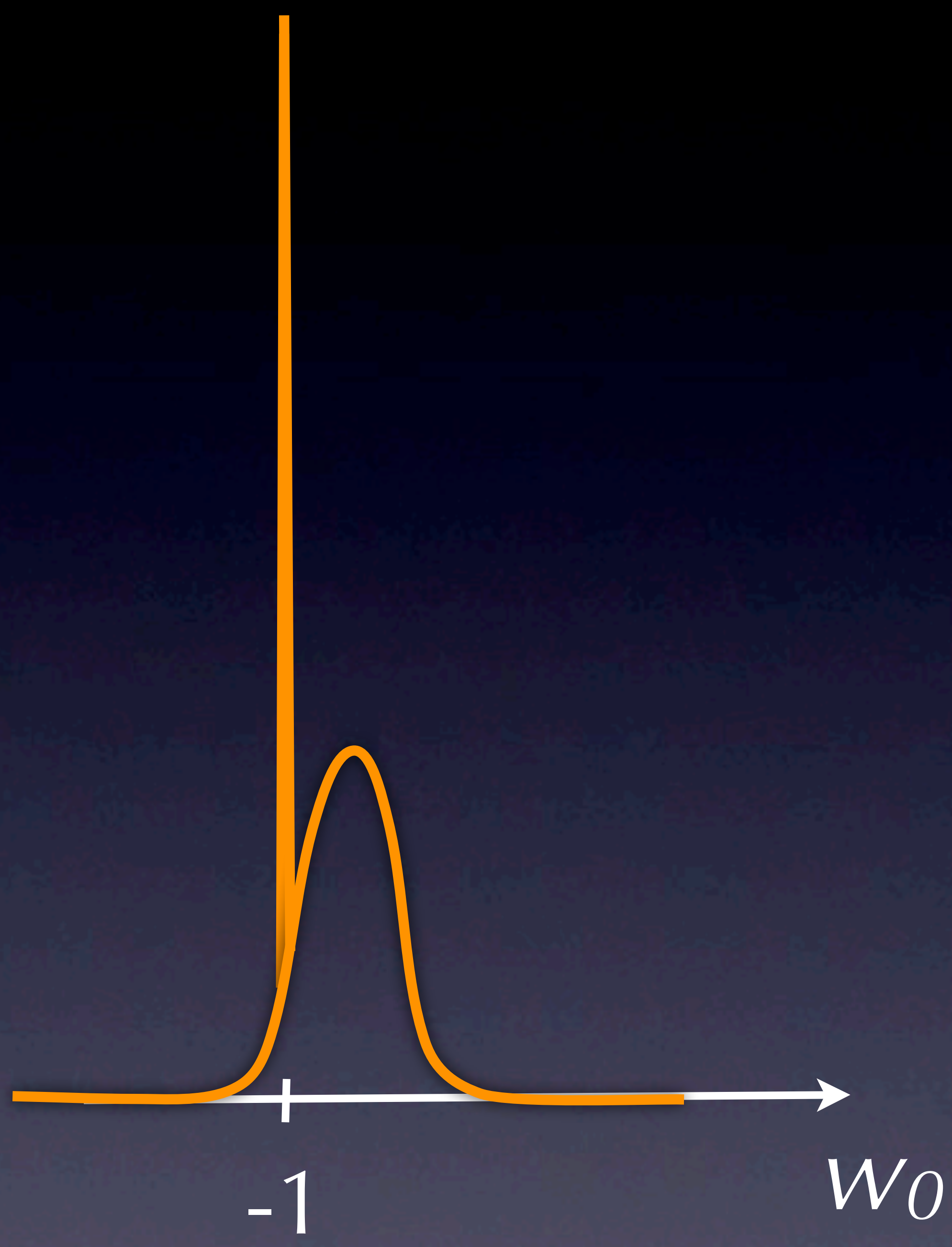

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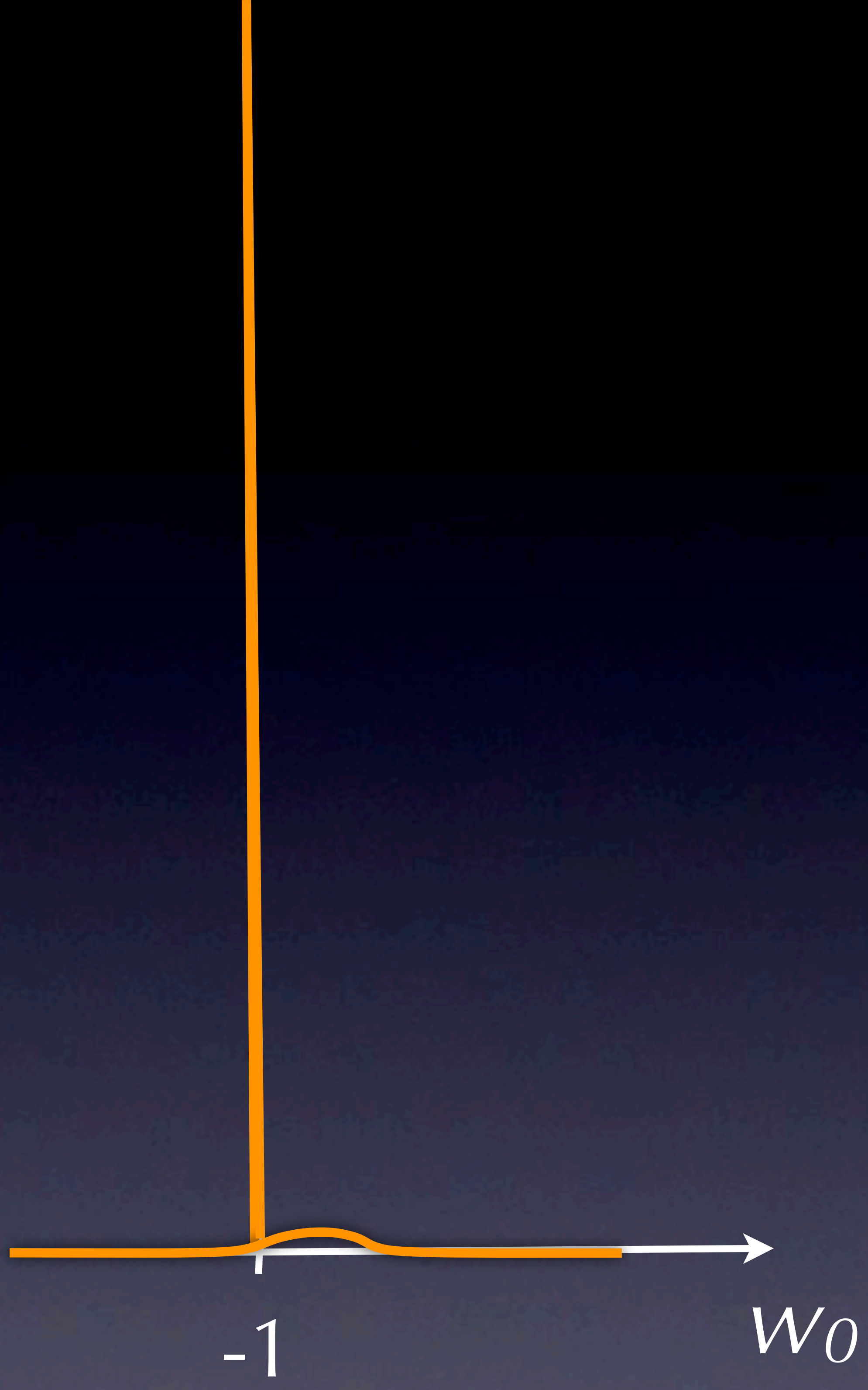

A priori preference for Λ

Model tests/inference

Model-level inference can be used at several levels:

- **Model-level tests**

  Deploy Bayesian model selection tools to compare model classes.

- **Model selection forecasting**

  Evaluate the capability of proposed experiments to answer model selection questions, by defining model selection Figures of Merit (FoMs). Explore outcomes contingent on each model class (including ΛCDM) being correct.

- **Survey optimization**

  Vary survey configurations in order to optimize ability to carry out identified model test priorities.

Forecasts for dark energy

Mukherjee, Parkinson, Corasaniti, Liddle, Kunz, MNRAS, astro-ph/0512484

$$w = w_0 + (1-a)w_a$$

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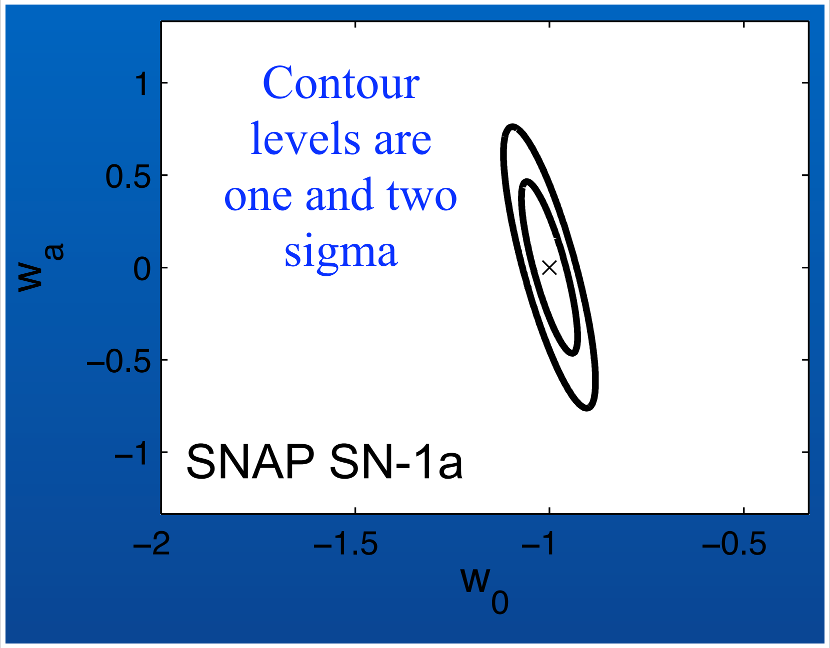

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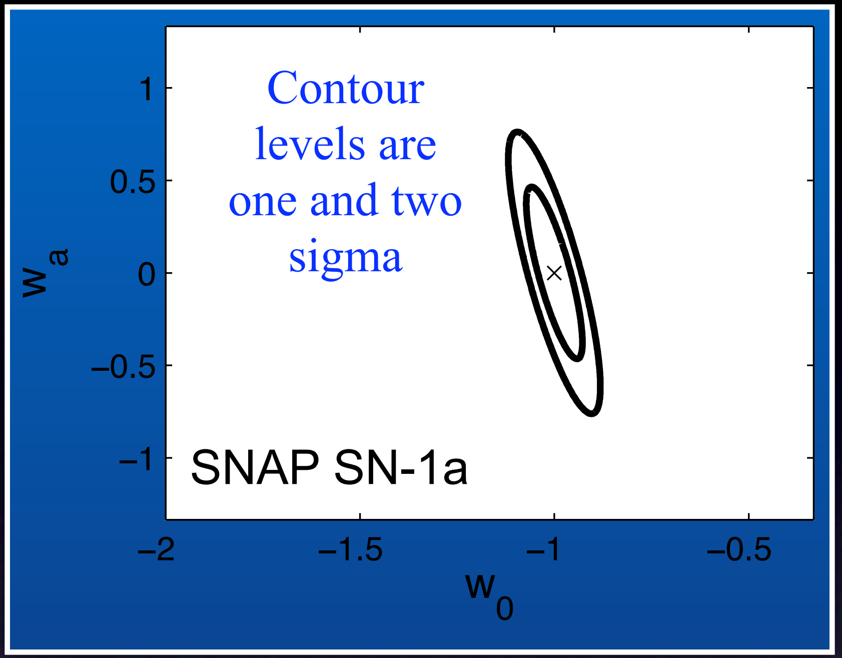

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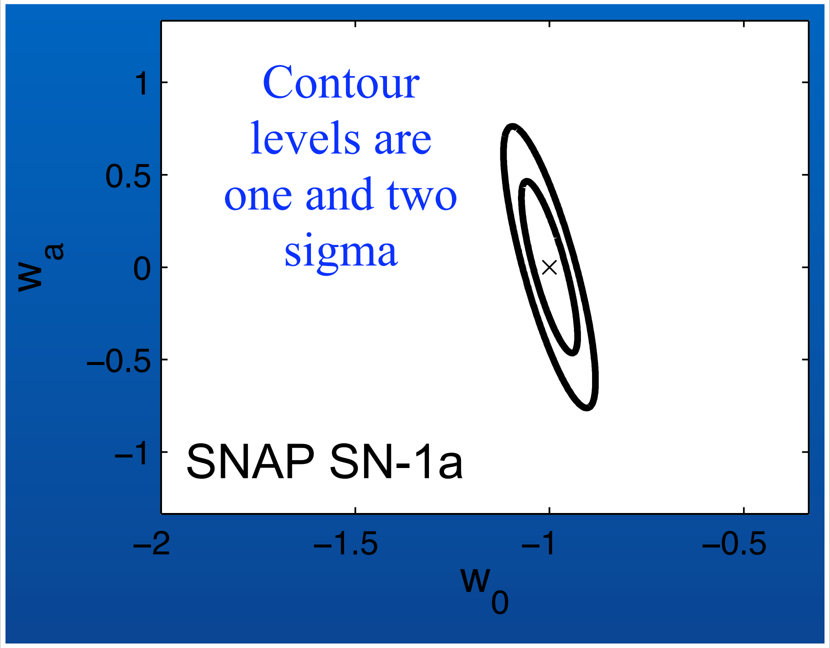

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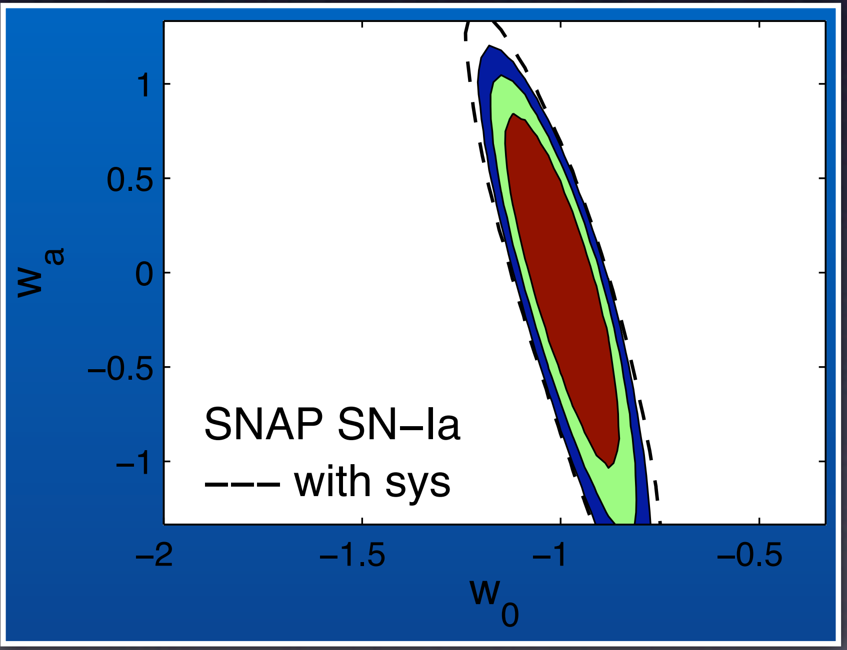

Red: Λ mildly favoured
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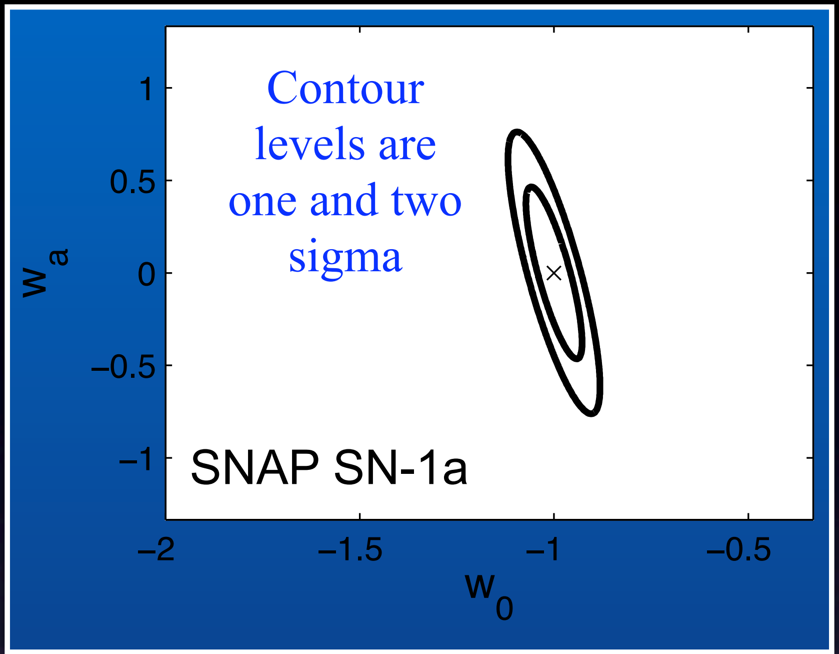

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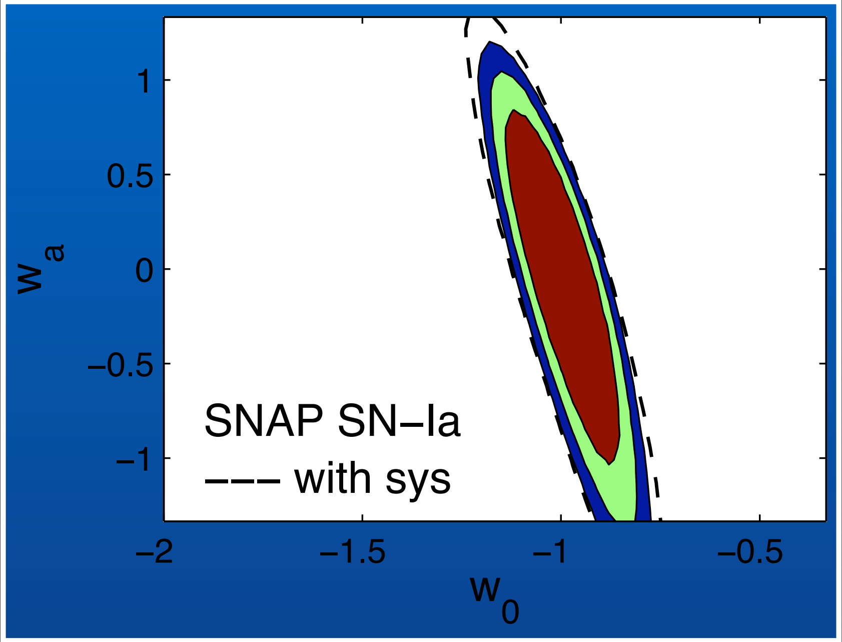

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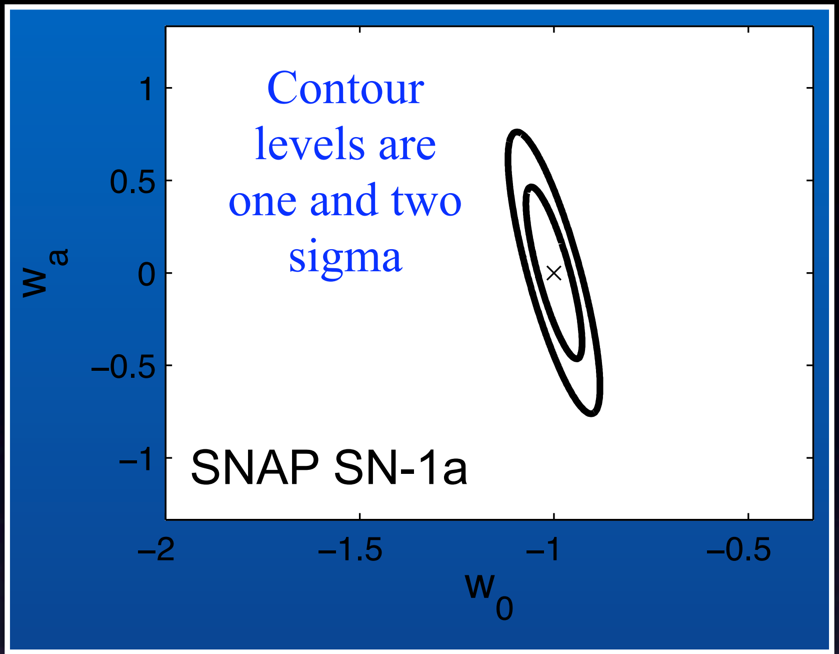

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If the dark energy model is right, will my experiment support it over ΛCDM?

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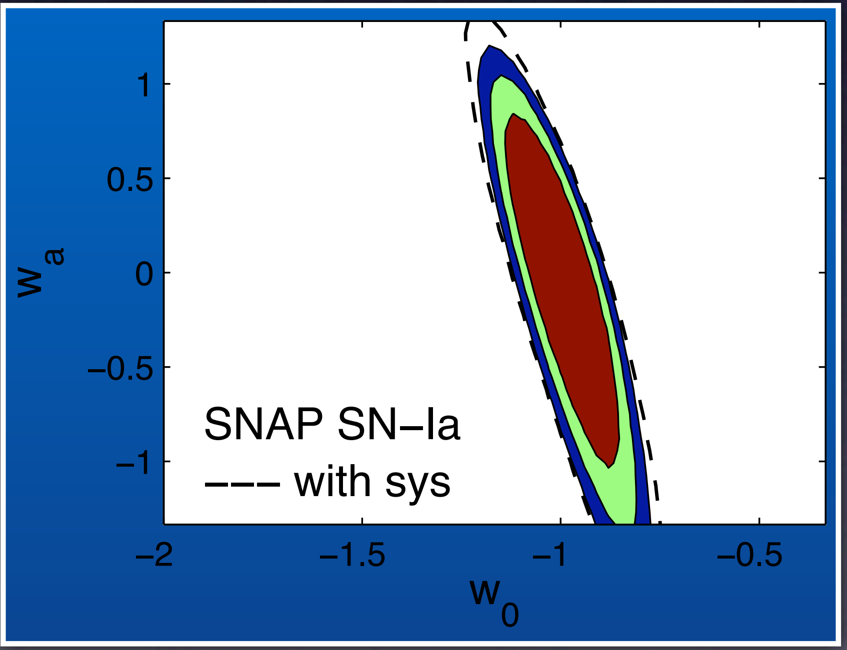
If ΛCDM is excluded, can I distinguish between quintessence and modified gravity models?



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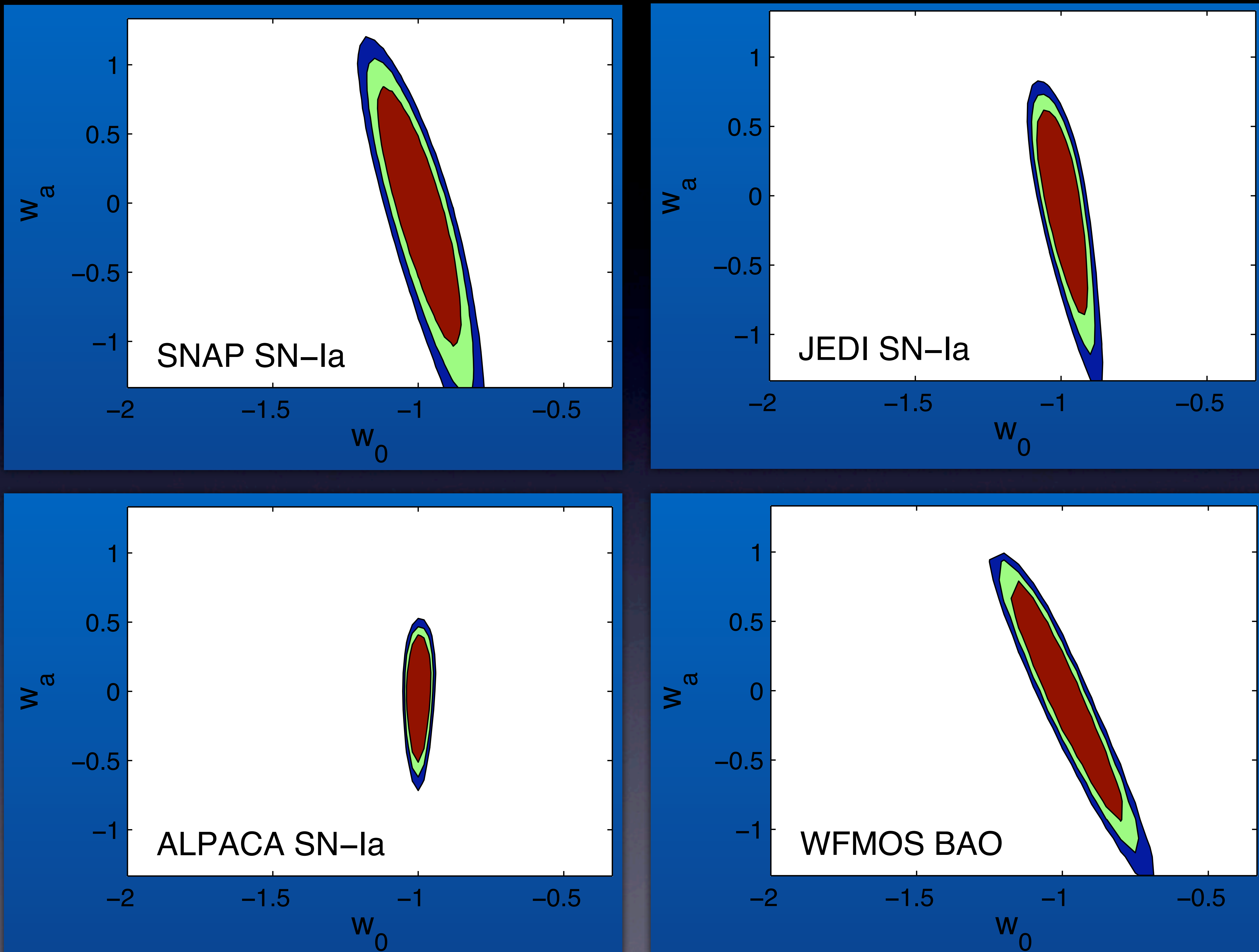

SNAP SN–Ia
JEDI SN–Ia
ALPACA SN–Ia
WFMOS BAO
w0
wa
−2
−1.5
−1
−0.5
1
0.5
0

(Almost) current dark energy data

Liddle, Mukherjee, Parkinson, and Wang, PRD, astro-ph/0610126

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Future forecasts informed by current data

Trotta, astro-ph/0504022; Liddle, Mukherjee, Parkinson, and Wang, astro-ph/0610126

Bayesian philosophy: continual updating of probabilities as new data comes in.

⇒ Use current probabilities to forecast future experiment outcomes

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- If future experiments are still inconclusive, how tight will be the limits they can impose on dark energy properties? — Tighter than you expect!

Survey optimization

Once we have defined our FoM, we can do better than just forecast how good our experiment will be. We can optimize our experiment to maximize the FoM. However ...

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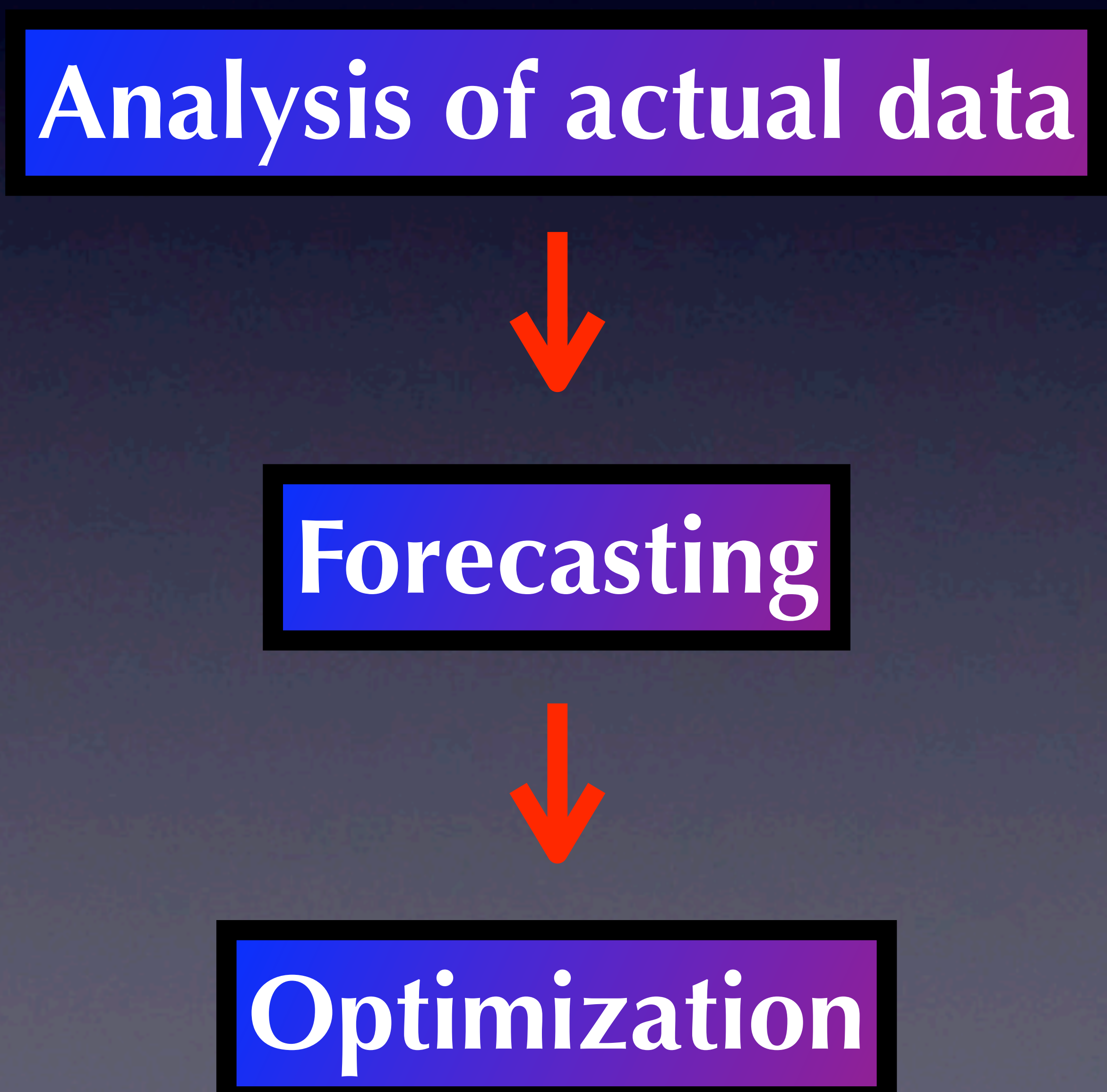

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Analysis of actual data

↓ Requires repetition over representative samples of the expected `true' Universe.

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↓

Optimization

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Forecasting

↓ Requires repetition over possible surveys *and* representative samples of the expected `true' Universe.

Optimization

Optimization of the WFMOS BAO survey

Parkinson et al., arXiv:0905.3410

Example: optimizing the survey parameters of the (now-defunct) proposed WFMOS BAO survey.

Here we use the parameter estimation DETF FoM. Although we varied several survey parameters, only the upper redshift limit proved important.

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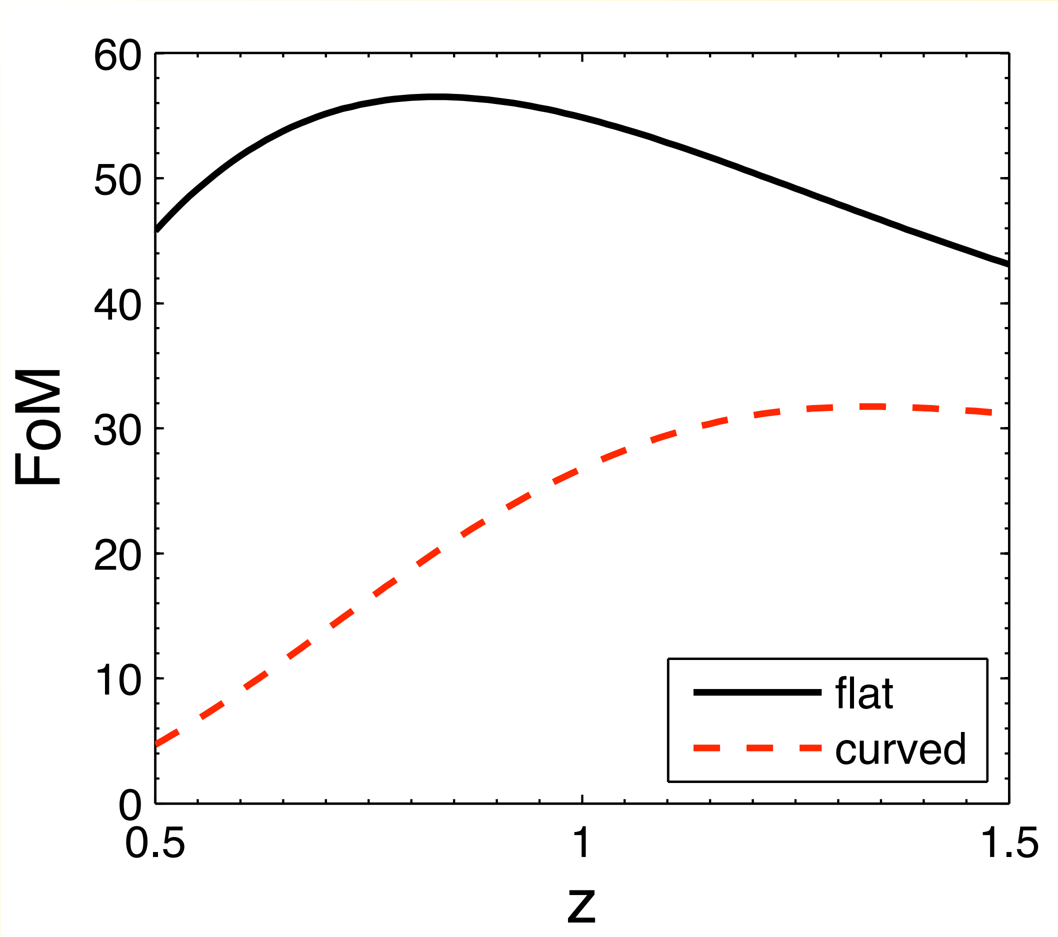

Figure 5. The FoM as a function of the upper redshift limit of the survey, for both the flat case and for the case including curvature. All surveys use $z_{\rm min} = 0.1$ and a minimal exposure time of 15 minutes, as discussed in the text. Measuring the curvature requires targeting a larger redshift range.

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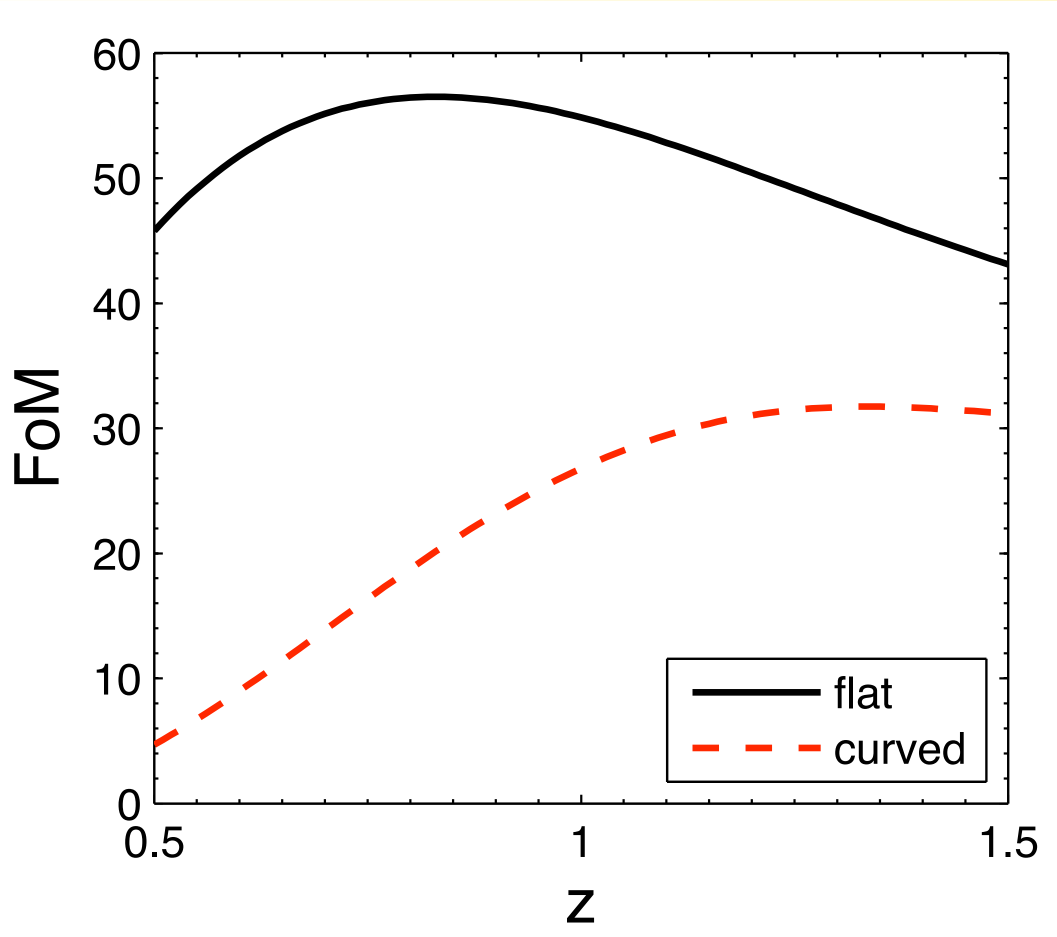
Survey optimization	without Ω_k	with Ω_k
FoM (Ω_k set to zero)	57	48
FoM (Ω_k allowed to vary)	15	32



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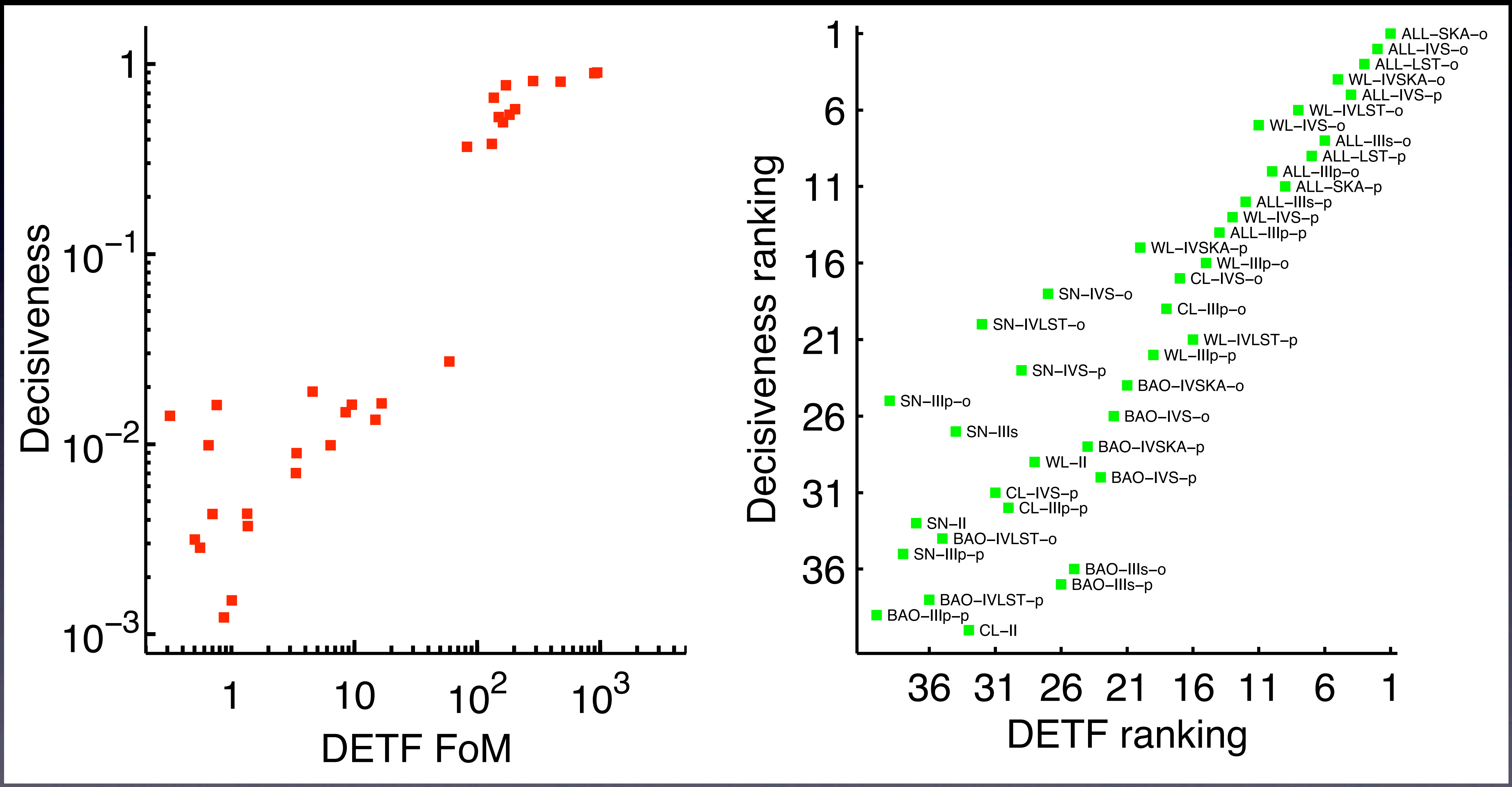
- Expected strength of evidence (Trotta et al. 2010). Measures the average Bayes factor expected, given present knowledge.

A model selection FoM in action

Trotta, Kunz & Liddle, MNRAS, arXiv:1012.3195



Decisiveness (computed in a Gaussian approximation) compared with the DETF FoM for the experiments described in the DETF report.

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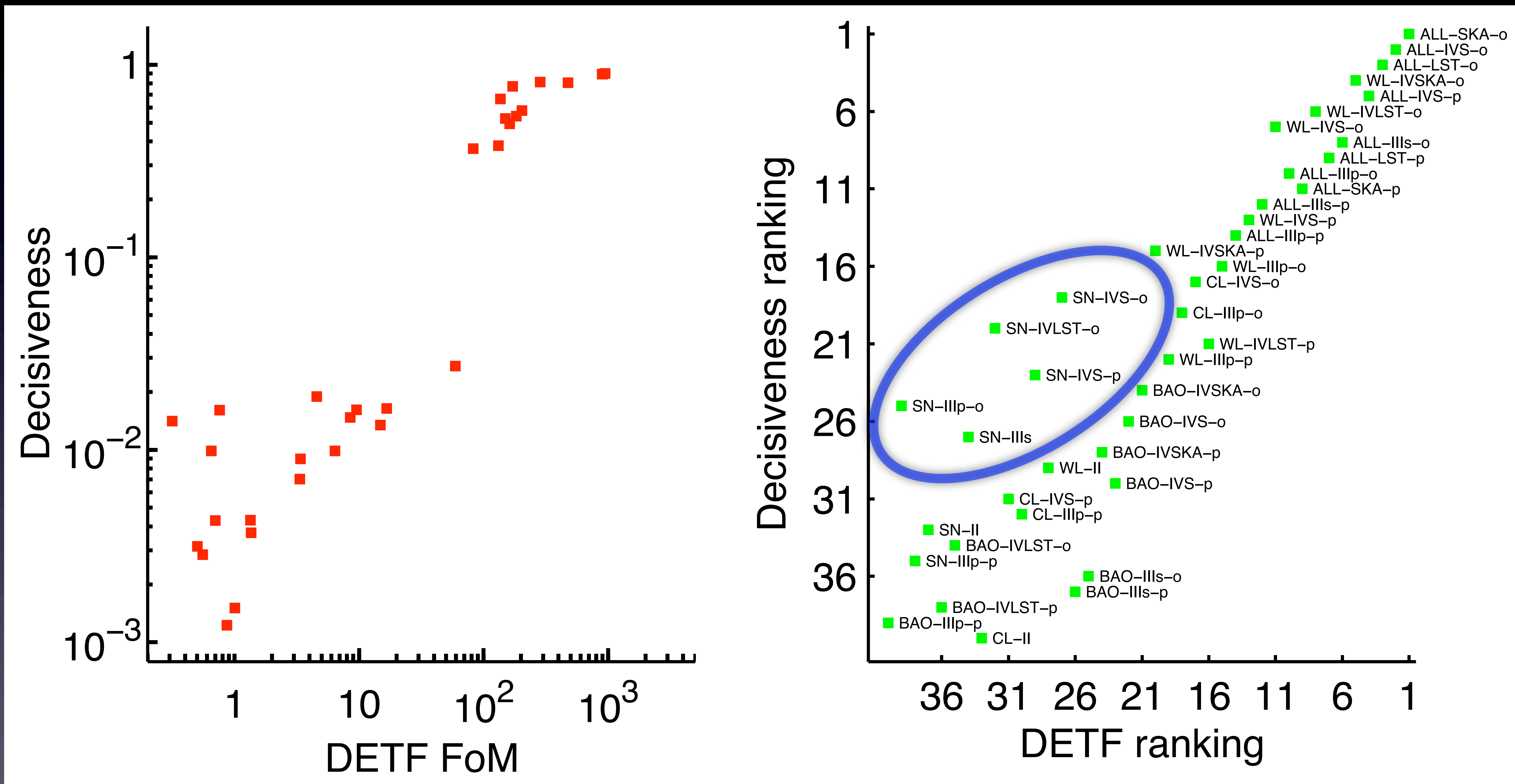

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- Such techniques can positively support simpler models, and set more stringent conditions for inclusion of new parameters.
- Model selection forecasting is a powerful tool for experimental design and comparison, and is readily applied to dark energy and other experiments.