INTRODUCTION TO:

MASSIVE GRAVITY

STEFAN SJÖRS

LECTURE NOTES FROM PREP-SCHOOL
AT "THE RETURN OF de SITTER"
FEB 28 - MARCH 4, 2011.

References

* Boulware + Deser : "Can Gravitation Have a Finite Range?" Phys. Rev. D 6, 3368–3382 (1972)

  Discusses Newtonian Potentials and light bending; constraints and gauge invariance; and uses the Hamiltonian approach when discussing degrees of freedom and ghosts.

* van Dam + Veltman : "Massive and Massless Yang-Mills and Gravitational Fields". Nuclear Physics B, volume 22, Issue 2, 15 September 1970, Pages 397–411

  Discusses the VDVZ discontinuity using a Feynman diagrammatic approach giving explicit propagators.

* Fierz + Pauli : Proc. R. Soc. Lond. A 1939 173, 211–232

  original paper...

* Vainshtein : "To the Problem of non-vanishing Gravitation mass" Physics Letters B, Volume 39, Issue 3, 1 May 1972, Pages 393–394

  Short, Russian style...

* Arkani-Hamed + Georgi + Schwartz : "Effective Field Theory for Massive Gravitons and Gravity in Theory Space" hep-th/0210184

  EFT approach to massive gravity

* Hinterbichler : "Massive Gravity" (online lecture notes)
  very good lecture notes that covers basically what is covered in these notes and more.

* Maggiore : "Gravitational Waves" (book)
  Good introduction to linearized gravity and massive gravity, pedagogical!

* Rubakov + Tinyakov : "Infrared-modified gravities and massive gravitons" hep-th arXiv: 0802.4379
  3+1 decomposition of massive gravity à la cosmological perturbation theory where all helicities become apparent.

* Zee : "Quantum Field Theory In a Nutshell" (book)
  Full of good disussions on propagators, potentials, and all that...

* Feynman : "Feynman Lectures on Gravitation" (book)
  Nice introduction to gravity from a QFT point of view.

* Wu-Ki Tung : "Group Theory in Physics" (book)
  Poincaré algebra and all that...

* Wald : "General Relativity" (book)
  If you want formulas ...

* Misner + Thorne + Wheeler : "Gravitation"
  Nice physical pictures and formulas ...

Introduction

The weak field limit of General Relativity, i.e. linearized Einstein's theory, reproduces Newtonian gravity for non-relativistic sources. The potential energy between two soures m, M is given by

$$V(r) = -G\frac{Mm}{r}, \qquad G = \text{Newton's constant}$$

This gives rise to the familiar attractive $1/r^2$ force law, which has been tested from very small scales $\simeq 0.1$ mm up to scales of clusters of galaxies $\lambda_c \simeq$ Mpc, which we know are held together by gravity.

As we will derive in Lecture 1, when the graviton is appended with a mass m_g, the potential energy takes the characteristic Yukawa form

$$V(r; m_g) = -G\frac{Mm}{r}e^{-m_g r}$$

At scales $r \ll \lambda_g \equiv m_g^{-1}$, far below the Compton wave-length of the graviton, the Yukawa potential reproduces the Newtonian one

$$V(r; m_g) = -G\frac{Mm}{r}\left(1 + \mathcal{O}\left(\frac{r}{\lambda_g}\right)\right),$$

but at scales $r \simeq \lambda_g$ the graviton mass kicks in and gravity becomes exponentially weak.

If we demand that $\lambda_g \gtrsim \lambda_c \simeq$ Mpc then we arrive at the incredibly small bound on the graviton mass

$$m_g \lesssim 10^{-30} \, eV .$$

This is not the full story though and as we will see in the end of Lecture 1, and discuss in more detail in Lecture 2, there is a finite difference in the prediction for light bending in the linearized massive case and the linearized massless case, even if we are well inside λ_g. We will indeed show that the light bending around the sun differs by a factor of 4/3

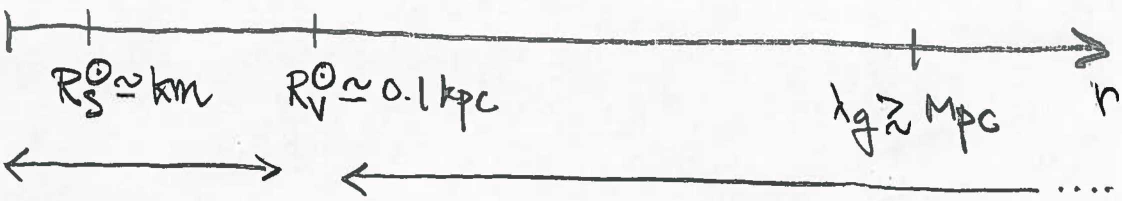
$$\Delta\theta_{\odot}^{massless} = \frac{4}{3} \Delta\theta_{\odot}^{massive} , \quad predictions .$$

This is the famous van Dam-Veltman-Zakharov discontinuity (VDVZ) and since $\delta_{exp}(\Delta\theta_{\odot}) \simeq 10^{-4}$ one is lead to conclude that massive gravity is excluded.

But as Vainshtein pointed out, in the presence of a source that curve spacetime, the gravitational field becomes strong and linear theory breaks down. Vainshtein showed that at distances $r \gg R_V$ away from the source linear theory is valid, while at $r \ll R_V$ non-linear effects become important and a non-linear completion could recover general relativity.

Vainshtein showed that $R_V = (R_S \lambda_g^4)^{1/5}$, were R_S is the Schwarzschild radius of the source. For the sun $R_S^{\odot} \simeq$ km, $\lambda_g \gtrsim$ Mpc which implies $R_V \gtrsim 0.1$ kpc which is well outside the solar system and linearized massive gravity makes no predictions for light bending.

We arrive at a picture for the <u>sun</u>:



Non-linear completion that reproduces GR:

Nothing new for observers to calculate but new physics for theoreticians to invent

Linear massive gravity:

New physics to observe but nothing new to invent.

In Lecture 3 we will explicitly verify the Vainshtein recovery of GR in the so called Decoupling limit of massive gravity in a non-linear completion suggested by de Rham and Gabadadze.