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Fractional vortices and Bean-Livingstone barrier in two-component superconductors

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arXiv:1008.1194

Outline

- INTRODUCTION: Multicomponent superconductors. Ginzburg-Landau and London theory. Fractional vortices.

- RESULTS:

1) Gibbs energy of fractional vortices near the surface of superconductor. Stable fractional vortices.
2) Surface barrier for vortex entry.
3) Density of fractional vortices and magnetization curve.

Introduction

Ginzburg-Landau theory

1. Two order parameters

$$\Psi_{A,B} = \left|\Psi_{A,B}\right| \exp\left(i\theta_{A,B}\right)$$

2. Free energy density

$$F = \sum_{j=A,B} \frac{1}{2m_j}\left|(\nabla + ieA)\Psi_j\right|^2 - b_j |\Psi_j|^2 + c_j |\Psi_j|^4$$

$$+\eta\left[\Psi_A^*\Psi_B + c.c.\right] + \frac{\mathbf{H}^2}{8\pi}$$

Two gap superconductivity is realized in

MgB_2,
Liquid metallic hydrogen,
Ferropnictides,
Heavy fermion compounds
...

Liu, Mazin, Kortus, PRL (2001)
Ishida, Nakai, Hosono, J. Phys. Soc. Jpn. (2009)
Jourdan, et al. PRL (2004)
Babaev, Sudbo, Ashcroft, Nature (2004)

Introduction

Ginzburg-Landau theory

Characteristic length scales

Josephson length $$\lambda_J^2 = (\eta\, m_A m_B)^{-1}$$

We will consider the limit of vanishing Josephson coupling

$$\lambda_J = \infty$$

Introduction

Ginzburg-Landau theory

Characteristic length scales

Josephson length $$\lambda_J^2 = \left(\eta\, m_A m_B\right)^{-1} \to \infty$$

Coherence length $$\xi_j^2 = \left(2 m_j b_j\right)^{-1}$$

Magnetic field penetration length $$\lambda^2 = \frac{m_A m_B}{e^2}\left[m_B \left|\Psi_A\right|^2 + m_A \left|\Psi_B\right|^2\right]^{-1}$$

Introduction

London theory

Approximations

1. Neglect Josephson coupling $\lambda_J \gg \lambda \,,\, \xi_{A,B}$

2. Neglect variation of the order parameter modulus $\lambda \gg \xi_{A,B}$

Free energy density

$$F = \frac{1}{8\pi}\left[\mathbf{H}^2 + \lambda_A^{-2}\left(\mathbf{A} - \frac{\phi_0}{2\pi}\nabla\theta_A\right)^2 + \lambda_B^{-2}\left(\mathbf{A} - \frac{\phi_0}{2\pi}\nabla\theta_B\right)^2\right]$$

$$\lambda_{A,B}^2 = \frac{e^2\left|\Psi_{A,B}\right|^2}{2m_{A,B}}$$

Introduction

London theory

The London free energy consists of the

1. Energy of magnetic field and charged current

$$F_m = \frac{1}{8\pi} \int \left[\mathbf{H}^2 + \lambda^2 (\boldsymbol{\nabla} \times \mathbf{H})^2\right] d^2 r.$$

+ 2. Energy of neutral current

$$F_{rel} = \frac{2}{(4\pi)^3} \left(\frac{\Phi_0 \lambda}{\lambda_A \lambda_B}\right)^2 \int (\nabla \varphi_{rel})^2 \, d^2 r$$

Introduction

Fractional vortices

In general two superconducting condensates can have different windings

$$\Psi_A \sim \exp(iL_A\theta)$$

$$\Psi_B \sim \exp(iL_B\theta)$$

$L_A \neq L_B$

We will consider only singly quantized vortices

"A" vortex

$$\Delta\theta_A = \pm 2\pi\delta(\vec{r})$$

$$\Delta\theta_B = 0$$

"B" vortex

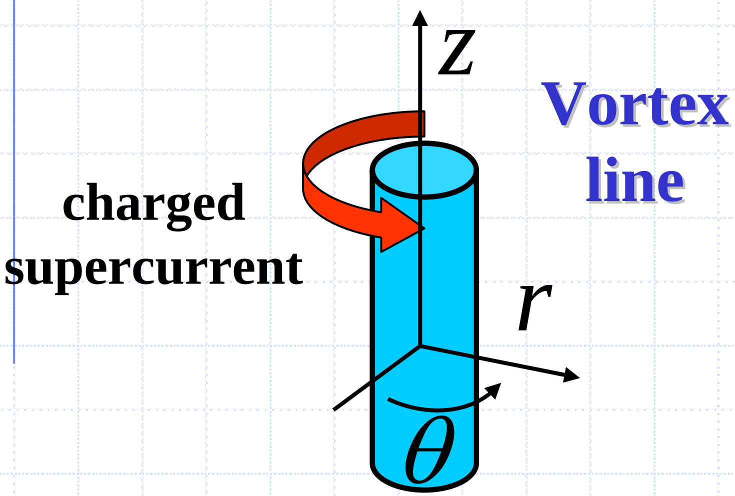
$$\Delta\theta_A = 0$$

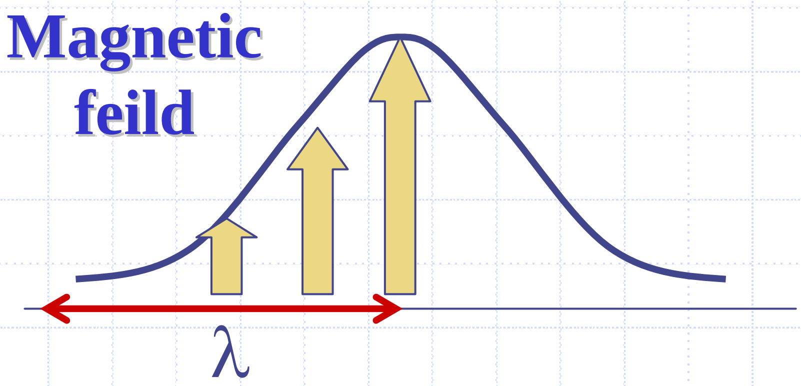
$$\Delta\theta_B = \pm 2\pi\delta(\vec{r})$$

Introduction

Fractional vortices

Magnetic properties (London theory)





1) Magnetic field distribution

$$H_{v1} = \frac{\phi_A}{2\pi\lambda^2} K_0\left(\frac{|\mathbf{r}-\mathbf{R_A}|}{\lambda}\right)$$

2) Total flux

$$\Phi = \phi_A = \phi_0 \frac{m_A|\Psi_A|^2}{m_B|\Psi_A|^2 + m_A|\Psi_B|^2}$$

is a fraction of flux quantum

3) Magnetic energy

$$F_m = \frac{\phi_A}{8\pi}(\mathbf{H}\cdot\mathbf{z_0})(\mathbf{r})$$

Introduction

Fractional vortices

BUT

The energy of the neutral current is divergent in the infinite superconductors

$$\theta_A = \theta$$

$$\theta_B = const$$

$$\Rightarrow$$

$$\varphi_{rel} = \theta$$

$$(\nabla \varphi_{rel})^2 \sim \frac{1}{r^2}$$

Then the energy of neutral current:

$$F_{rel} = \frac{2}{(4\pi)^3}\left(\frac{\Phi_0 \lambda}{\lambda_A \lambda_B}\right)^2 \int (\nabla \varphi_{rel})^2 \, d^2 r \sim \ln(L)$$

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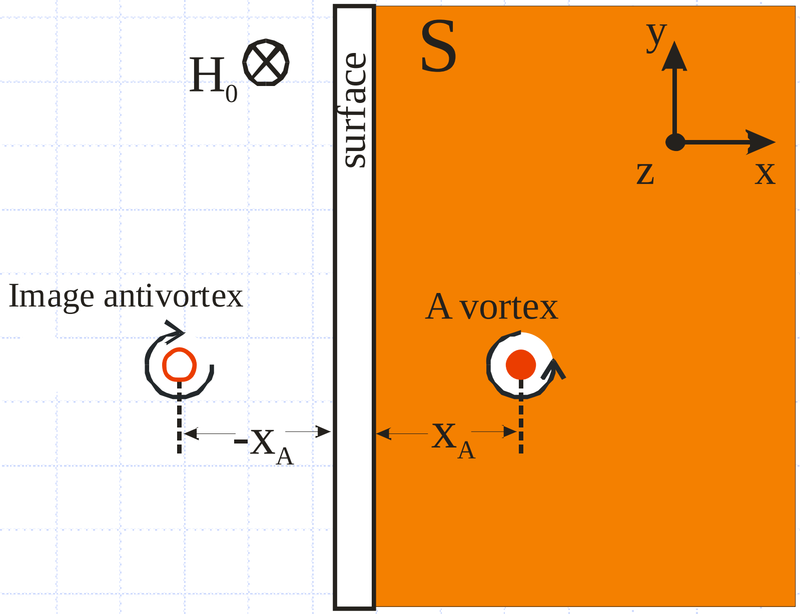
Results

Basic idea

Fractional vortex near the surface of superconductor



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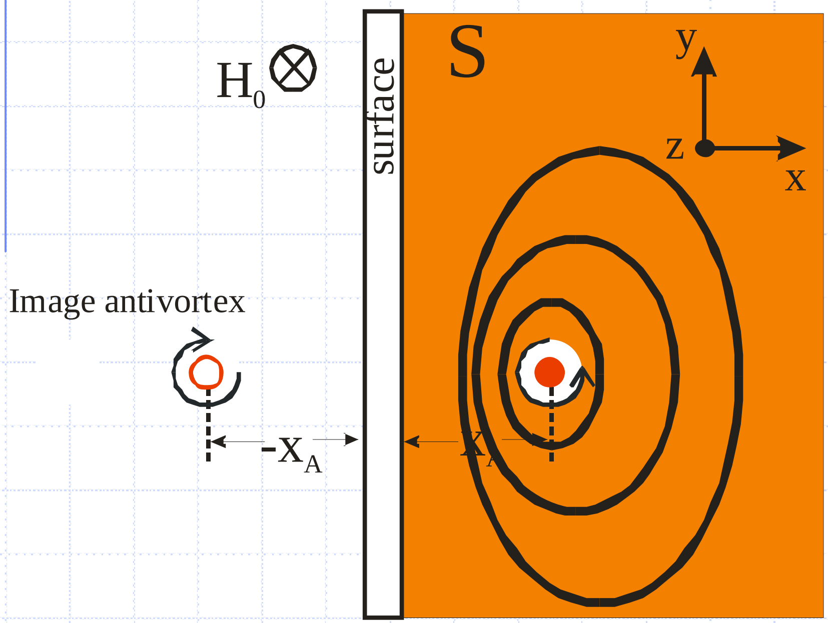
Results

Basic idea

Fractional vortex near the surface of superconductor



Modification of the neutral current distribution due to the image antivortex

$$(\nabla \varphi_{rel})^2 \sim \frac{x_A}{r^3}$$

Then the energy of neutral current:

$$F_{rel} \sim \ln\left(\frac{x_A}{\xi_A}\right)$$

is finite!

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Results

Stable fractional vortex

Vortex stability is determined by the **Gibbs** energy:

$$F_G = F - \frac{\mathbf{H}\cdot\mathbf{H}_0}{4\pi}$$

$$F_G = F_m + F_{rel} + W$$

Interaction with image antivortex (F_m)

Energy of neutral current (F_{rel})

Interaction with Meissner current (W)

Results

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Stable fractional vortex

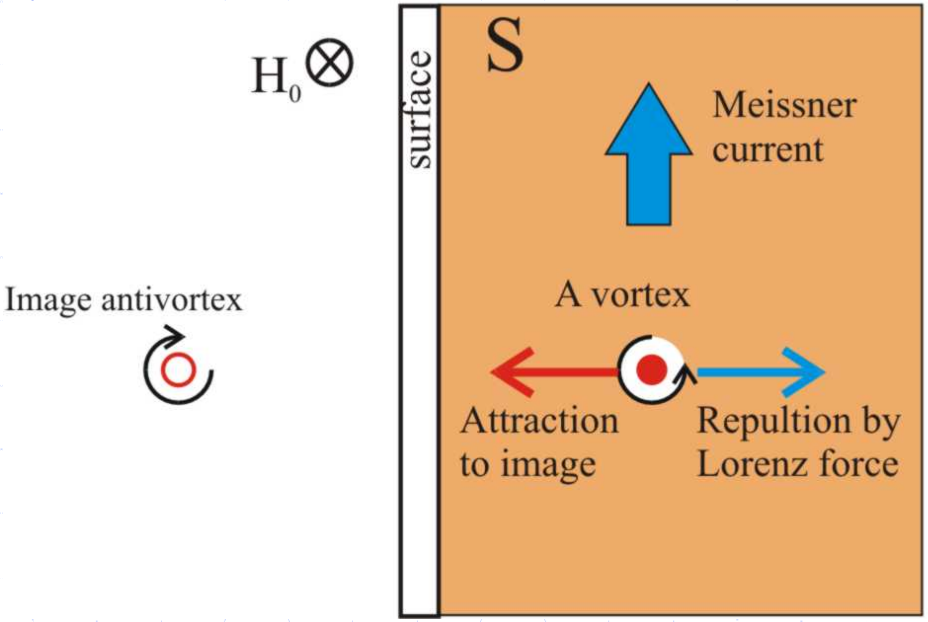
Vortex stability is determined by the **Gibbs** energy:

$$F_G = F - \frac{\mathbf{H} \cdot \mathbf{H_0}}{4\pi}$$

$$F_G = F_m + F_{rel} + W$$

Attraction to surface

Repulsion from surface



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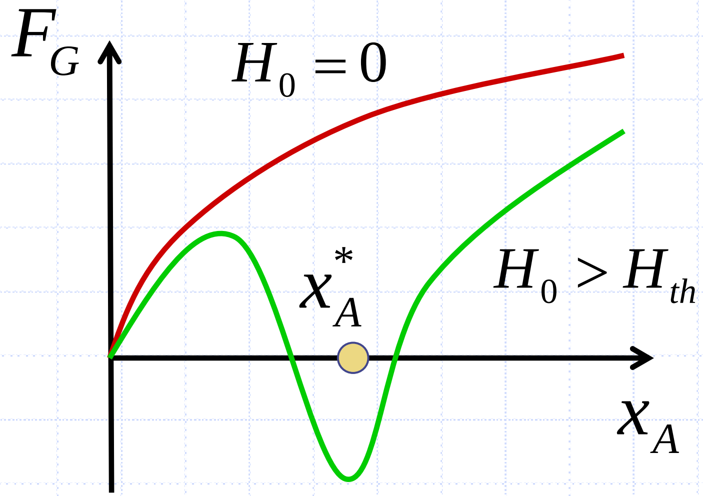
Results

Stable fractional vortex

$$F_G = F_m + F_{rel} + W$$

Attraction to surface (F_m, F_{rel})

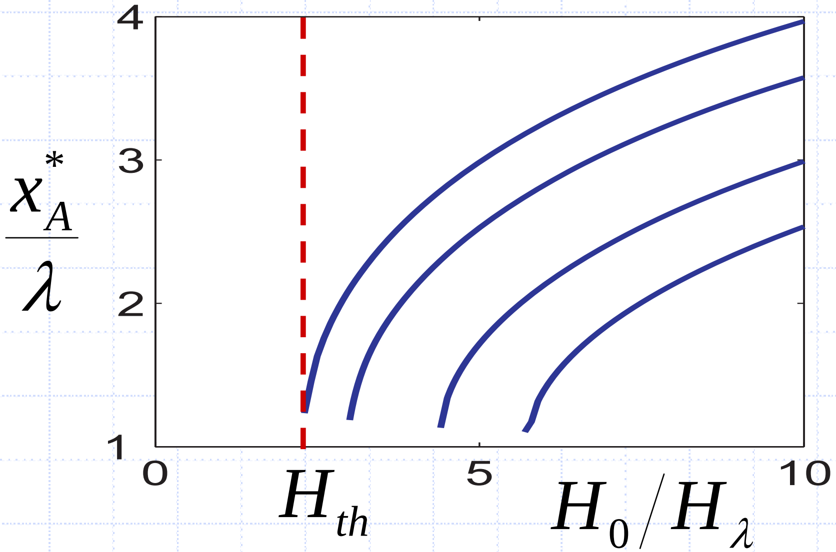
Repulsion from surface (W)



Equilibrium position of fractional vortex for

$$\phi_A / \phi_B = \frac{m_A |\Psi_A|^2}{m_B |\Psi_B|^2} = 1, 2, 3, 4$$

$$H_\lambda = \phi_A / \lambda^2$$



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Results

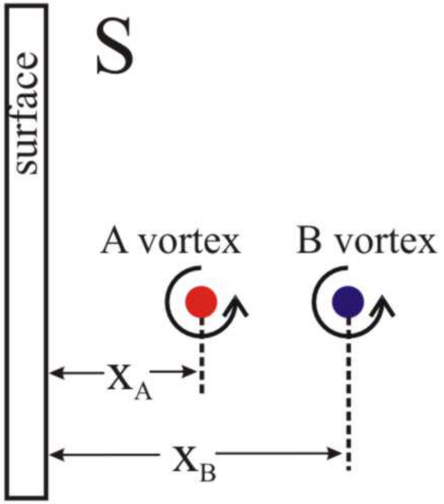
Bean – Livingston barrier

Vortex penetration condition in conventional superconductor

$$\frac{dF_G}{dx_v}\left(x_v=\xi\right)<0$$

Generalization for two-gap superconductor

$$\frac{dF_G}{dx_{A,B}}\left(x_{A,B}=\xi_{A,B}\right)<0$$



Results

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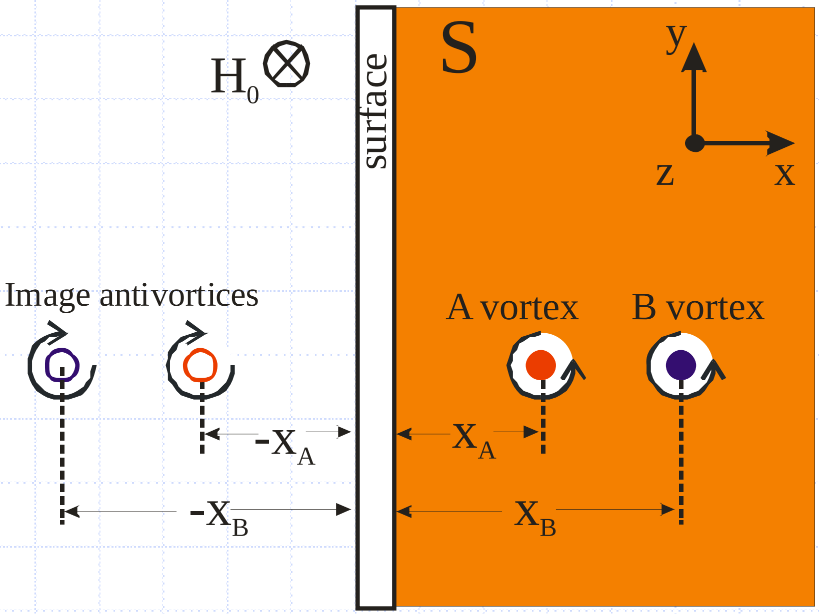
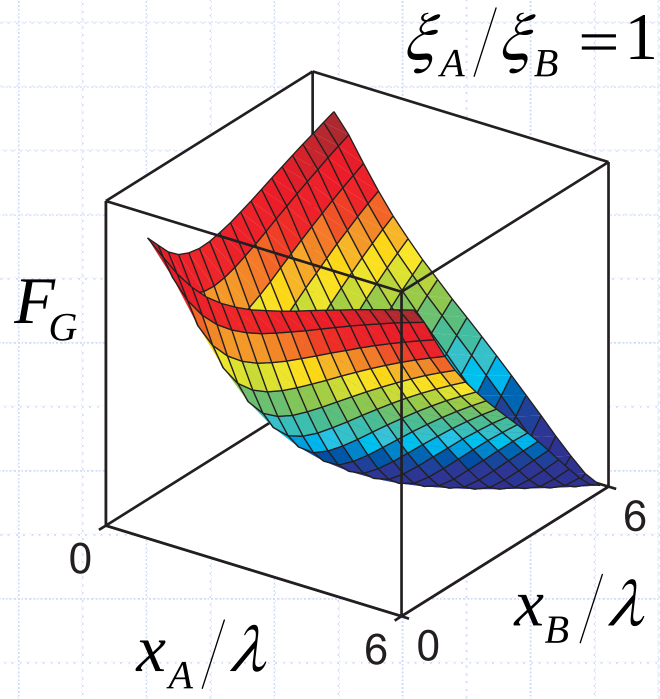
Bean – Livingston barrier

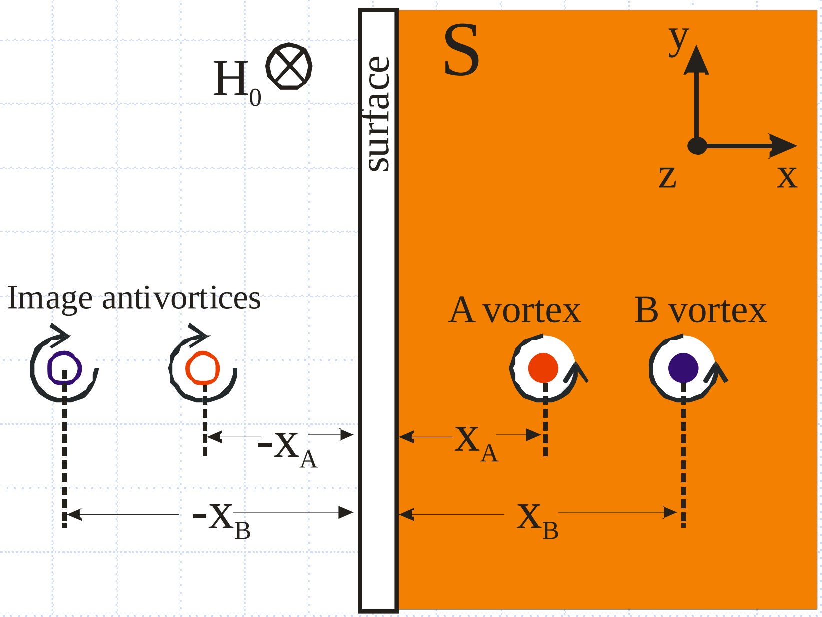


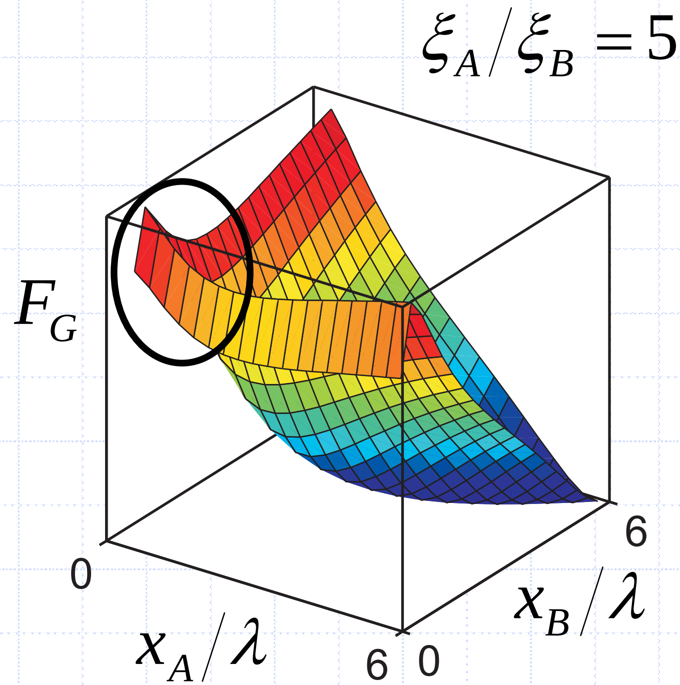


Simultaneous entrance of both "A" and "B" vortices

Results

Bean – Livingston barrier





Field of surface barrier suppression

$$H_{s1} = \min(H_{sA}, H_{sB})$$

Vortex with the largest ξ enters at first

$$H_{sA,B} = \phi_0 / (4\pi\lambda\xi_{A,B})$$

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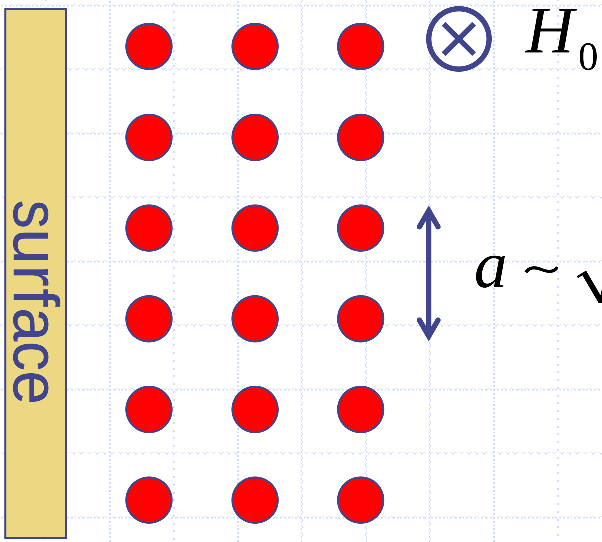
Results

Density of fractional vortices



$$a \sim \sqrt{\xi\lambda} \ll \lambda$$

"A" or "B" vortices

We consider the range of magnetic fields

$$H_{s2} > H_0 > H_{s1}$$

$$H_{s1} = \min(H_{sA}, H_{sB})$$

$$H_{s2} = \max(H_{sA}, H_{sB})$$

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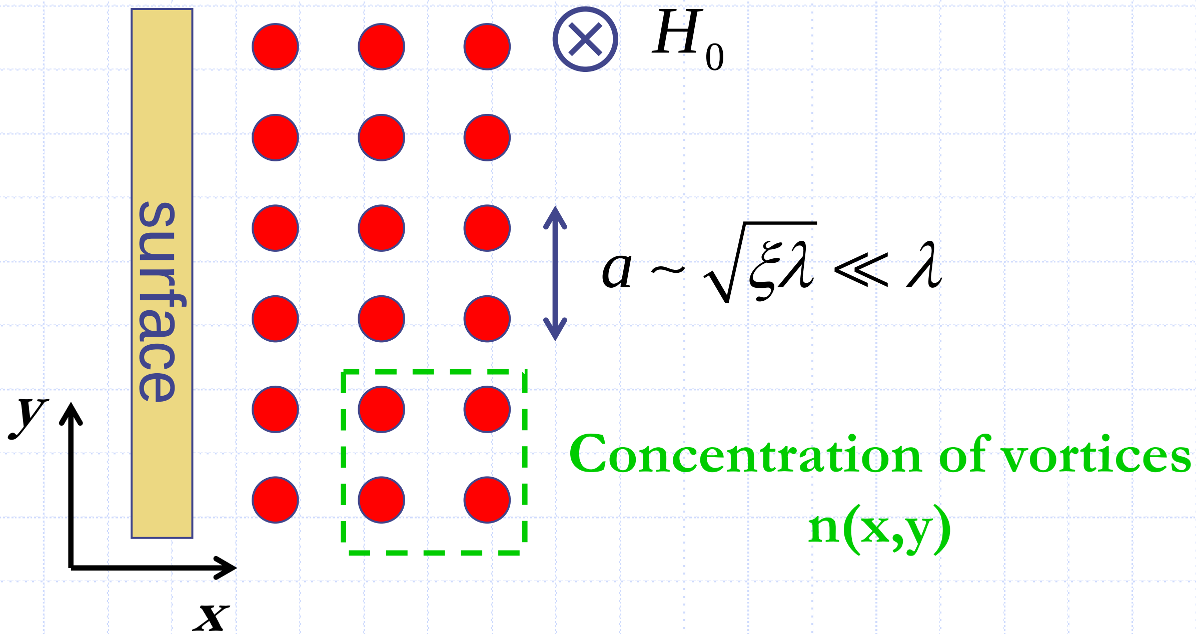
Results

Density of fractional vortices



Results

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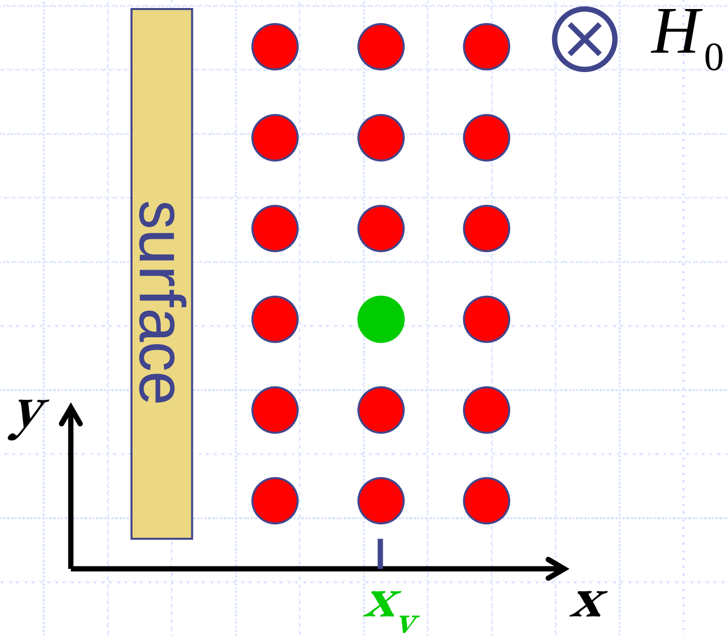
Density of fractional vortices



To find the equilibrium concentration n(x) let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G(x_v) = 0$$

Results

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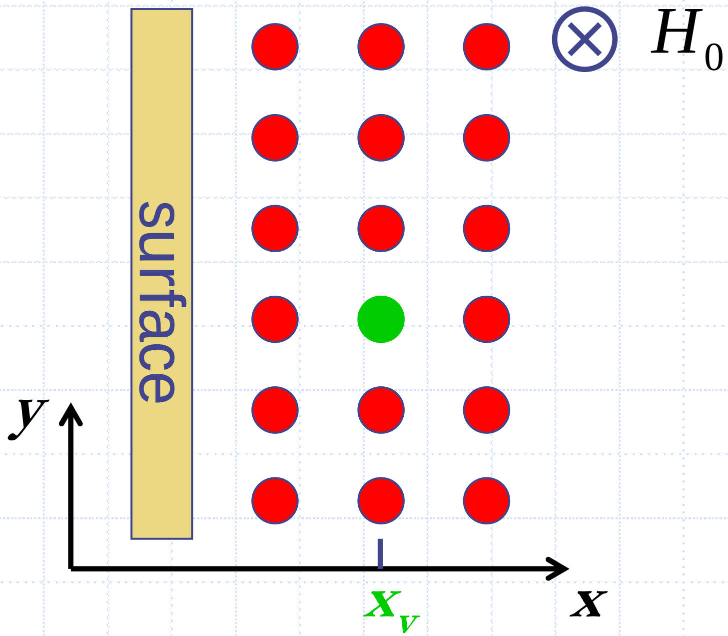
Density of fractional vortices



To find the equilibrium concentration n(x) let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G\left(x_v\right) = 0$$

Integral equation to determine the concentration n(x)

$$\alpha \int_0^{\infty} \left[\frac{(x - x_v)}{|x - x_v|} e^{-|x - x_v|} + e^{-|x + x_v|} \right] n(x) dx +$$
$$\beta \int_{x_v}^{\infty} n(x) dx = \gamma e^{-x_v},$$

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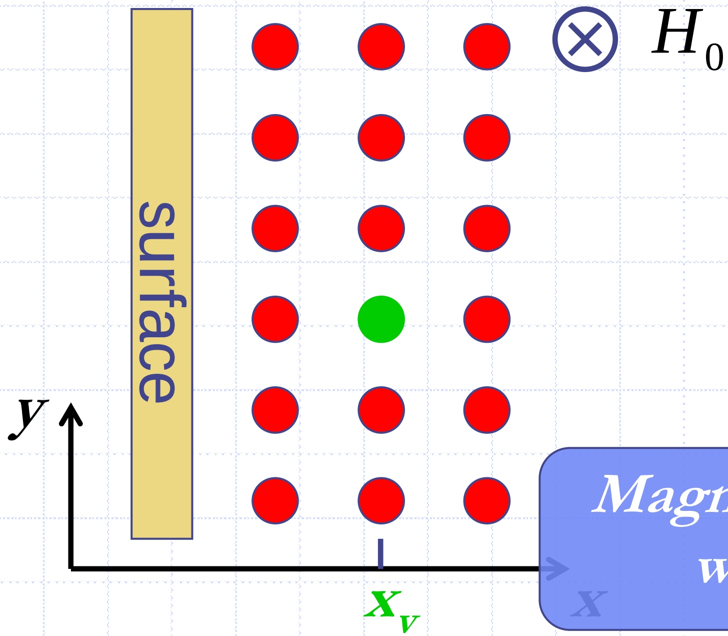
Results

Density of fractional vortices



To find the equilibrium concentration **n(x)** let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G(x_v) = 0$$

Magnetic interaction with vortices

$$\alpha \int_0^{\infty} \left[\frac{(x - x_v)}{|x - x_v|} e^{-|x-x_v|} + e^{-|x+x_v|} \right] n(x) dx +$$

$$\beta \int_{x_v}^{\infty} n(x) dx = \gamma e^{-x_v},$$

Integral equation to determine the concentration n(x)

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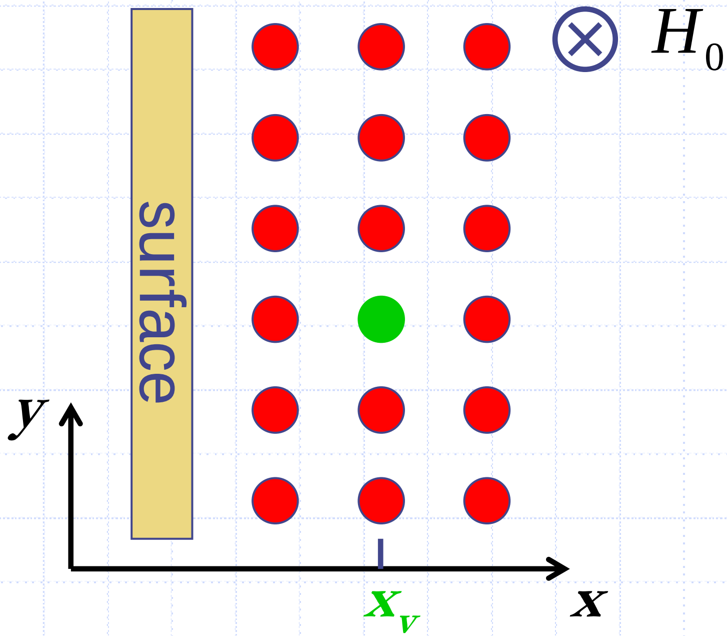
Results

Density of fractional vortices



To find the equilibrium concentration n(x) let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G(x_v) = 0$$

Magnetic interaction with image anti vortices

Integral equation to determine the concentration n(x)

$$\alpha \int_0^{\infty} \left[\frac{(x - x_v)}{|x - x_v|} e^{-|x-x_v|} + e^{-|x+x_v|} \right] n(x) dx +$$

$$\beta \int_{x_v}^{\infty} n(x) dx = \gamma e^{-x_v},$$

Results

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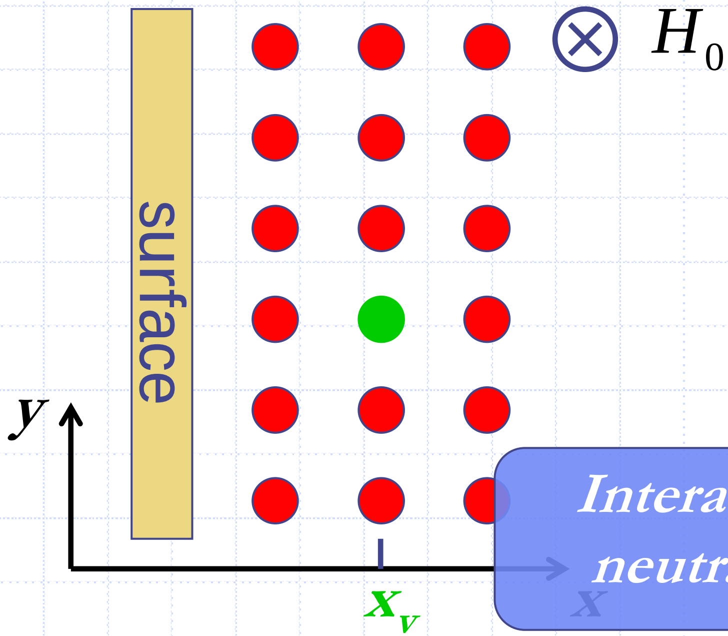
Density of fractional vortices



To find the equilibrium concentration n(x) let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G(x_v) = 0$$

Interaction with neutral current

Integral equation to determine the concentration n(x)

$$\alpha \int_0^\infty \left[\frac{(x - x_v)}{|x - x_v|} e^{-|x-x_v|} + e^{-|x+x_v|} \right] n(x) dx +$$

$$\beta \int_{x_v}^\infty n(x) dx = \gamma e^{-x_v},$$

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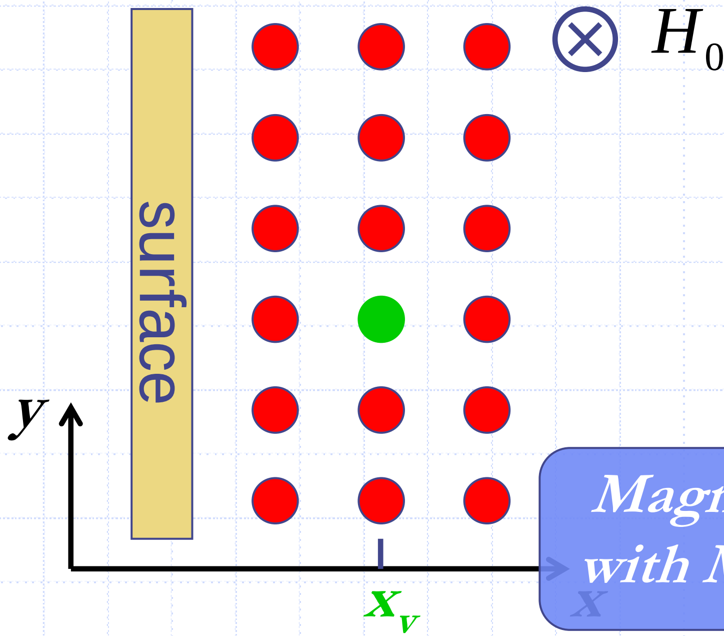
Results

Density of fractional vortices



To find the equilibrium concentration n(x) let us consider the force acting on the probe vortex

$$f_x = \frac{d}{dx_v} F_G(x_v) = 0$$

Magnetic interaction with Meissner current

$$\alpha \int_0^\infty \left[\frac{(x-x_v)}{|x-x_v|} e^{-|x-x_v|} + e^{-|x+x_v|} \right] n(x)dx + \beta \int_{x_v}^\infty n(x)dx = \gamma e^{-x_v},$$

Integral equation to determine the concentration n(x)

Results

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Density of fractional vortices

Integral equation can be solved analytically

$$\alpha \int_0^\infty \left[\frac{(x - x_v)}{|x - x_v|} e^{-|x-x_v|} + e^{-|x+x_v|} \right] n(x)dx + \beta \int_{x_v}^\infty n(x)dx = \gamma e^{-x_v}$$

$$\frac{d^2 n}{dx^2} = \frac{\beta}{\beta + 2\alpha} n.$$

The solution is

$$n(x) = Ae^{-x/L_n}.$$

$$L_n = \lambda \sqrt{\frac{2\alpha + \beta}{\beta}} = \lambda \sqrt{1 + \phi_A/\phi_B}$$

$$A = \frac{H_0}{\phi_A} \frac{(L_n^2 - \lambda^2)}{L_n^2}.$$

Results

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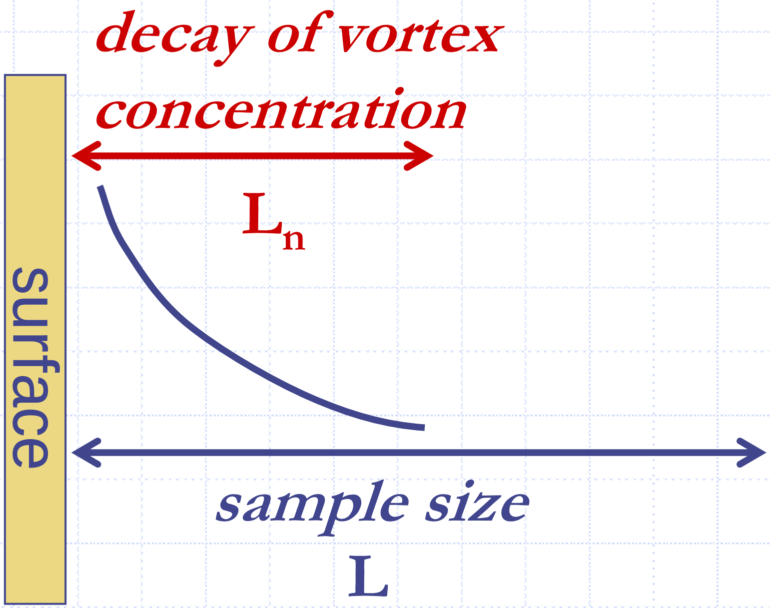
Magnetic moment of fractional vortices



Magnetic flux given by one vortex:

$$\Phi_A = \phi_A \left(1 - e^{-x_A/\lambda}\right)$$

Total magnetic moment:

$$M_f = \frac{1}{4\pi L} \int_0^\infty n(x)\Phi_A(x)dx,$$

Results

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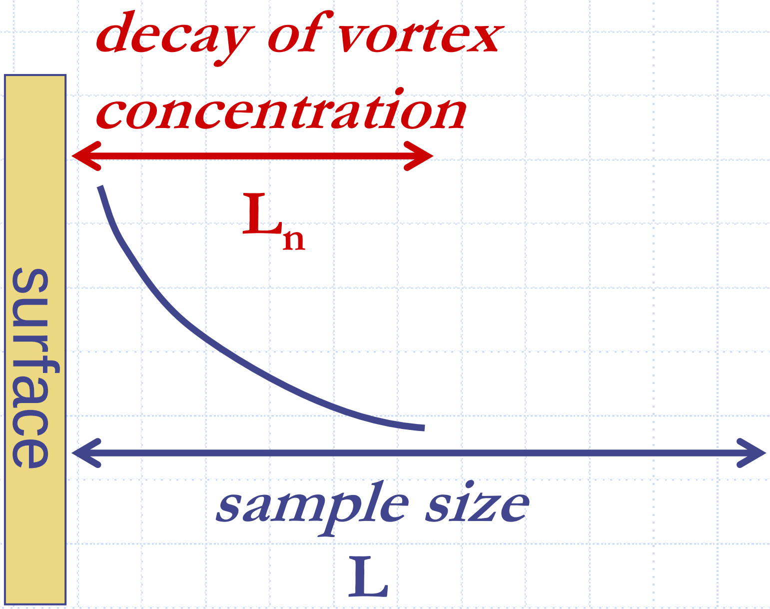
Magnetic moment of fractional vortices



Magnetic flux given by one vortex:

$$\Phi_A = \phi_A \left(1 - e^{-x_A/\lambda}\right)$$

Total magnetic moment:

$$M_f = \frac{1}{4\pi L} \int_0^\infty n(x) \Phi_A(x) dx,$$

$$M_f = \frac{H_0}{4\pi} \frac{L_n - \lambda}{L}.$$

Results

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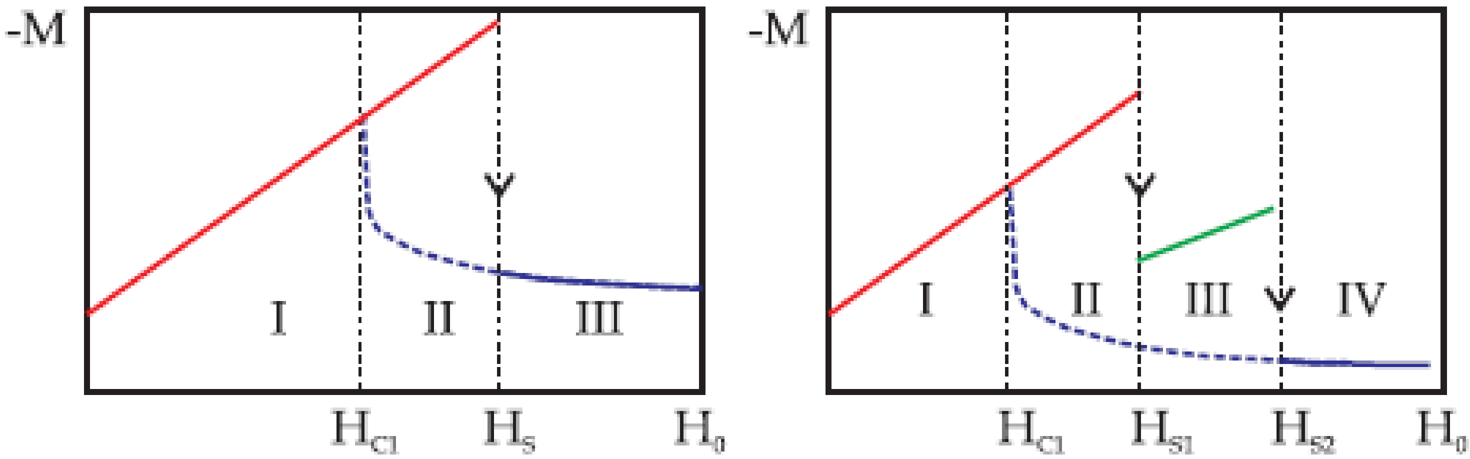
Magnetization curve

Single component superconductor	Two-component superconductor



I : Meissner state, no vortices

II : Overheated Meissner state

III : Vortex state in the bulk

I : Meissner state, no vortices

II : Overheated Meissner state

III : Fractional vortices confined near the surface

IV : Vortex state in the bulk

Results

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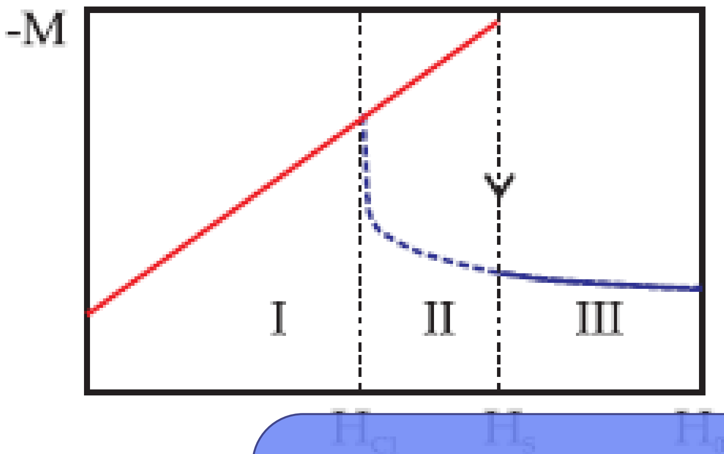
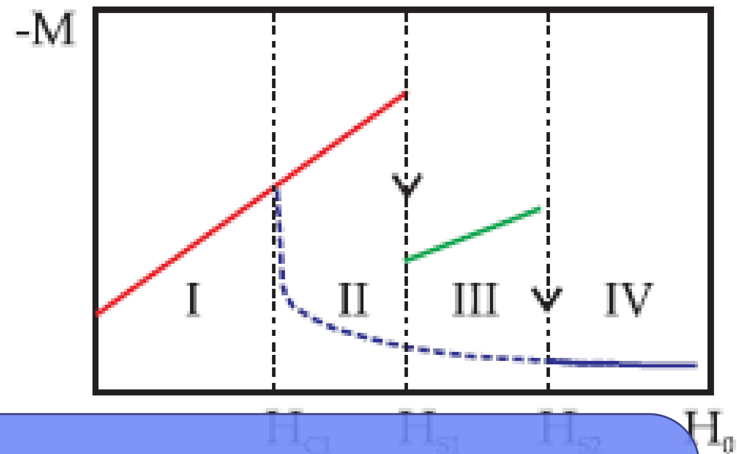
Magnetization curve

Single component superconductor



Two-component superconductor



Magnetization curve features two jumps in contract to single gap superconductors

Single component superconductor:

I : Meissner state, no vortices

II : Overheated Meissner state

III : Vortex state in the bulk

Two-component superconductor:

I : Meissner state, no vortices

II : Overheated Meissner state

III : Fractional vortices confined near the surface

IV : Vortex state in the bulk

Thank you for attention!