Topological creation of vortices and monopoles in spin-1 Bose-Einstein condensates

Mikko Möttönen

QCD group

Department of Applied Physics and Low Temperature Laboratory

Aalto University

PART 0

Background on QCD research

QCD = Quantum Computing and Devices

- Mikko Möttönen, Adj. Prof., group leader
- Jukka Huhtamäki, Dr. Tech., post doctoral researcher
- Paolo Solinas, PhD, post doctoral researcher
- Ville Pietilä, Dr. Sc., post doctoral researcher
- Olli-Pentti Saira, MSc, joint doctoral student with PICO group
- Pekko Kuopanportti, MSc, doctoral student
- Philip Jones, MSc, doctoral student
- Juha-Matti Pirkkalainen, MSc, associate doctoral student
- Juha Salmilehto, MSc student
- Emmi Ruokokoski, MSc student
- Joel Röntynen, summer student 2010
- Juha Kreula, summer student 2010

QCD research topics

- Bose-Einstein condensation (theory)
  - Topological defects (vortices, monopoles, skyrmions)
- Nanoelectronics (theory/experiments)
  - Single-charge pumping[1]
  - Geometric phases[1]
  - Phosphorus donors in silicon[2]
- Quantum information (theory)

[1] in close collaboration with Prof. Jukka Pekola, LTL

[2] in close collaboration with Centre for Quantum Computer Technology, Sydney

Single-charge pumping

Cooper pair paumping

PRL 100, 177201 (2008) PHYSICAL REVIEW LETTERS week ending 2 MAY 2008

Experimental Determination of the Berry Phase in a Superconducting Charge Pump

Mikko Möttönen,[1,2,*] Juha J. Vartiainen,[1] and Jukka P. Pekola[1]

Quasiparticle = single-electron pumping

LETTERS

Hybrid single-electron transistor as a source of quantized electric current

JUKKA P. PEKOLA[1]*, JUHA J. VARTIAINEN[1], MIKKO MÖTTÖNEN[1,2], OLLI-PENTTI SAIRA[1], MATTHIAS MESCHKE[1] AND DMITRI V. AVERIN[3]

Published online: 9 December 2007; doi:10.1038/nphys808

120 nature physics | VOL 4 | FEBRUARY 2008

Phosphorus donors in silicon

NANO LETTERS

The single-atom transistor work

pubs.acs.org/NanoLett

Transport Spectroscopy of Single Phosphorus Donors in a Silicon Nanoscale Transistor

Kuan Yen Tan,[*,†] Kok Wai Chan,[†] Mikko Möttönen,[†,‡,§] Andrea Morello,[†] Changyi Yang,[||] Jessica van Donkelaar,[||] Andrew Alves,[||] Juha-Matti Pirkkalainen,[†,‡] David N. Jamieson,[||] Robert G. Clark,[†] and Andrew S. Dzurak[†]

11

DOI: 10.1021/nl901635j | *Nano Lett.* **2010,** *10,* 11-15

Accepted yesterday!!!

Single-shot readout of an electron spin in silicon

Andrea Morello[1*], Jarryd J. Pla[1], Floris A. Zwanenburg[1], Kok W. Chan[1], Hans Huebl[1†], Mikko Möttönen[1,3,4], Christopher D. Nugroho[1‡], Changyi Yang[2], Jessica A. van Donkelaar[2], Andrew D. C. Alves[2], David N. Jamieson[2], Christopher C. Escott[1], Lloyd C. L. Hollenberg[2], Robert G. Clark[1], and Andrew S. Dzurak[1]

On general quantum theory

PRL **105,** 030401 (2010) PHYSICAL REVIEW LETTERS week ending 16 JULY 2010

Decoherence in Adiabatic Quantum Evolution: Application to Cooper Pair Pumping

J. P. Pekola,[1] V. Brosco,[2,3,4] M. Möttönen,[1,5,6] P. Solinas,[5] and A. Shnirman[7,8]

$$\frac{d\tilde{\rho}_I(t)}{dt} = i[\tilde{\rho}_I(t), w_I(t)] - \frac{1}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'[[\tilde{\rho}_I(t)\otimes\rho_E, \tilde{V}_I(t')], \tilde{V}_I(t)]\right\} + \frac{i}{\hbar^2}\mathrm{Tr}_E\left\{\int_0^t dt'\int_0^{t'} dt''[[\tilde{\rho}_I(t)\otimes\rho_E, [w_I(t'), \tilde{V}_I(t'')]], \tilde{V}_I(t)]\right\}$$

$$\begin{aligned}\dot{\rho}_{gg} &= -2\Im\mathrm{m}(w_{ge}^*\rho_{ge}) - (\Gamma_{ge}+\Gamma_{eg})\rho_{gg} + \Gamma_{eg} + \tilde{\Gamma}_0\Re\mathrm{e}(\rho_{ge}) + \frac{\Re\mathrm{e}(w_{ge})}{\omega_0}[(2\tilde{\Gamma}_+ - \tilde{\Gamma}_0)(1-\rho_{gg}) - (2\tilde{\Gamma}_- - \tilde{\Gamma}_0)\rho_{gg}]\\ &\quad + 2\frac{\Re\mathrm{e}(w_{ge})\Re\mathrm{e}(\rho_{ge})}{\omega_0}(\Gamma_{ge}+\Gamma_{eg}-\Gamma_0),\end{aligned}$$

$$\begin{aligned}\dot{\rho}_{ge} &= iw_{ge}(2\rho_{gg}-1) + i(w_{ee}-w_{gg})\rho_{ge} + i\omega_0\rho_{ge} - i(\Gamma_{ge}+\Gamma_{eg})\Im\mathrm{m}(\rho_{ge}) - \Gamma_\varphi\rho_{ge} + (\tilde{\Gamma}_+ + \tilde{\Gamma}_-)\rho_{gg} - \tilde{\Gamma}_+\\ &\quad + \left[\frac{w_{ge}}{\omega_0}(2\Gamma_- - \Gamma_\varphi) - i\frac{\Im\mathrm{m}(w_{ge})}{\omega_0}(\Gamma_{eg}-\Gamma_{ge})\right]\rho_{gg} - \left[\frac{w_{ge}}{\omega_0}(2\Gamma_+ - \Gamma_\varphi) + i\frac{\Im\mathrm{m}(w_{ge})}{\omega_0}(\Gamma_{eg}-\Gamma_{ge})\right](1-\rho_{gg})\\ &\quad + 2\left[\frac{w_{ge}^*}{\omega_0}\Re\mathrm{e}(\rho_{ge}) + 2i\frac{\Re\mathrm{e}(w_{ge})}{\omega_0}\Im\mathrm{m}(\rho_{ge})\right](\tilde{\Gamma}_0 - \tilde{\Gamma}_+ - \tilde{\Gamma}_-).\end{aligned}$$

PART 1

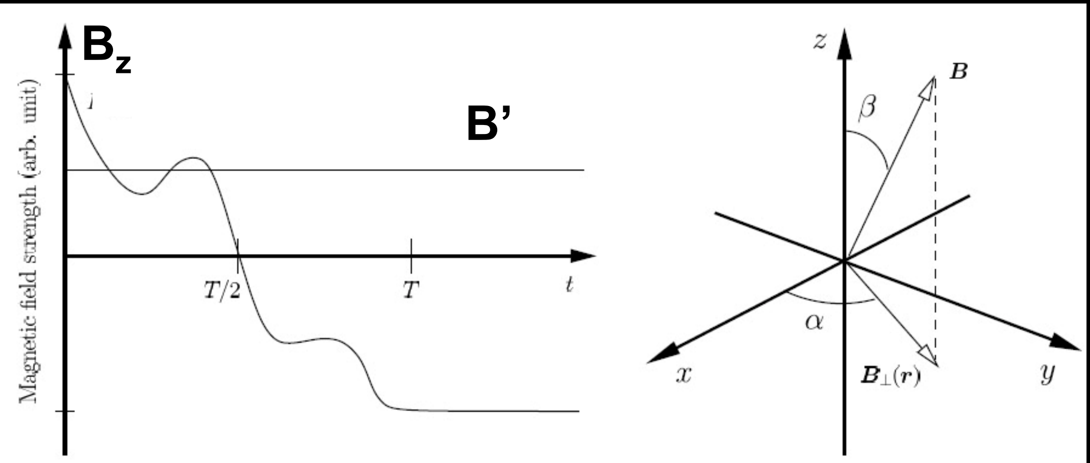
Topological vortex formation (TVF)

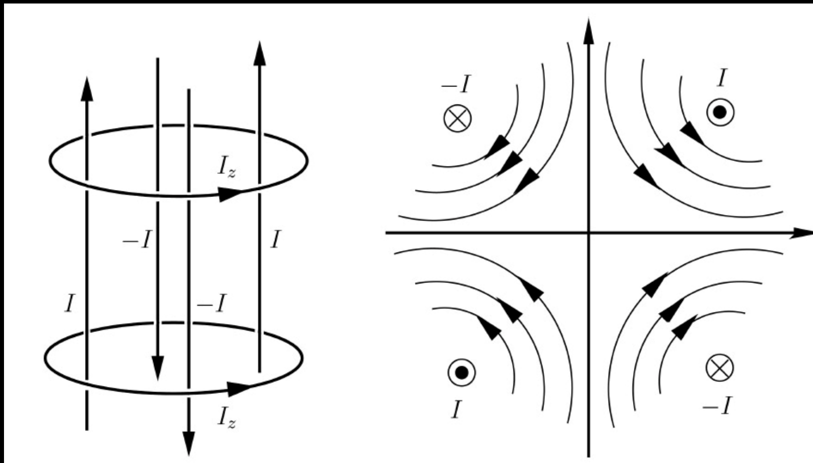
This is about creating multiply quantized vortices adiabatically and deterministically

Physical picture

- Condensate trapped in the WFSS of a Ioffe-Pritchard magnetic trap
- Bias field B_z is reversed adiabatically

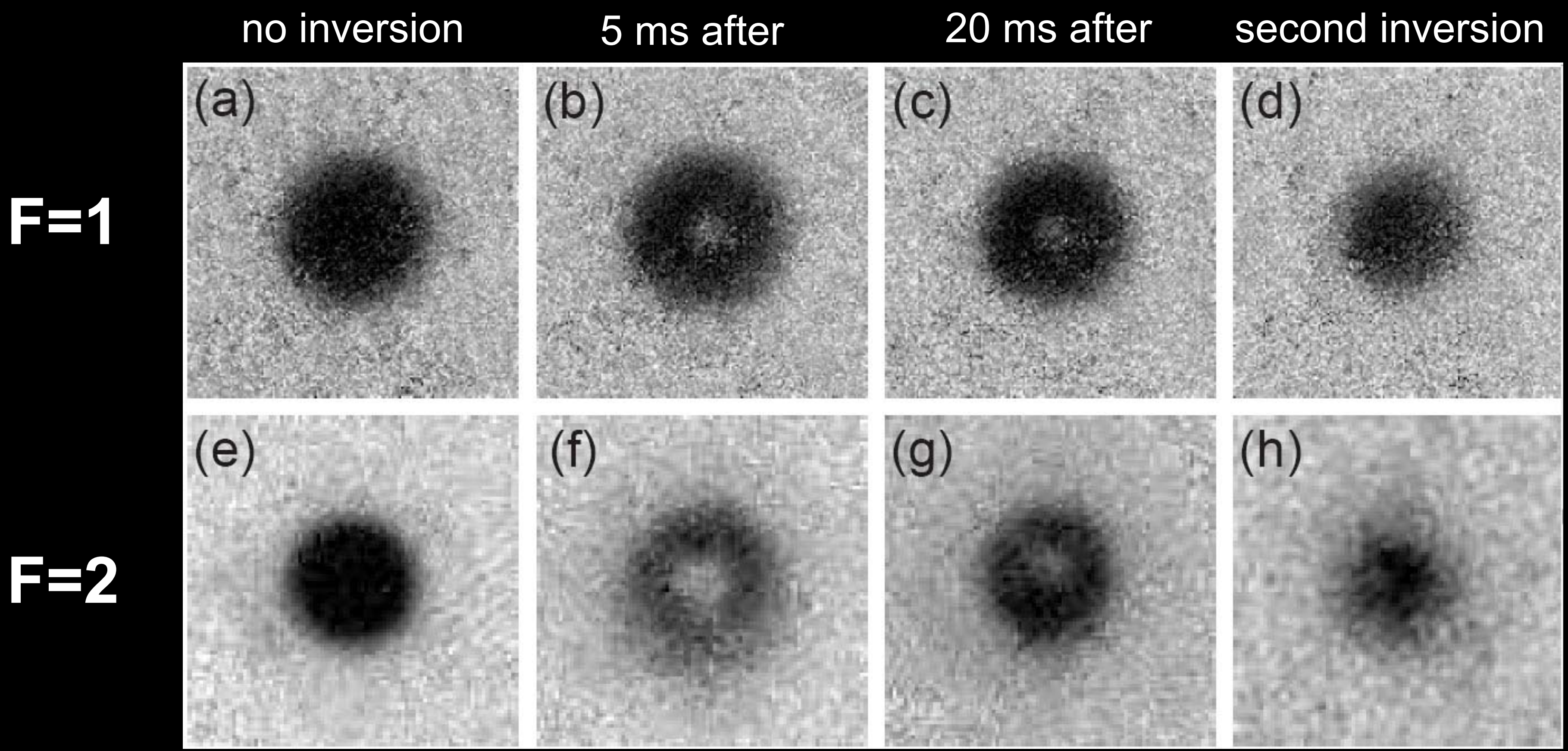
$$B_q(r,\varphi,t) = B'_q(t)r[\cos(\varphi)\hat{x} - \sin(\varphi)\hat{y}]$$



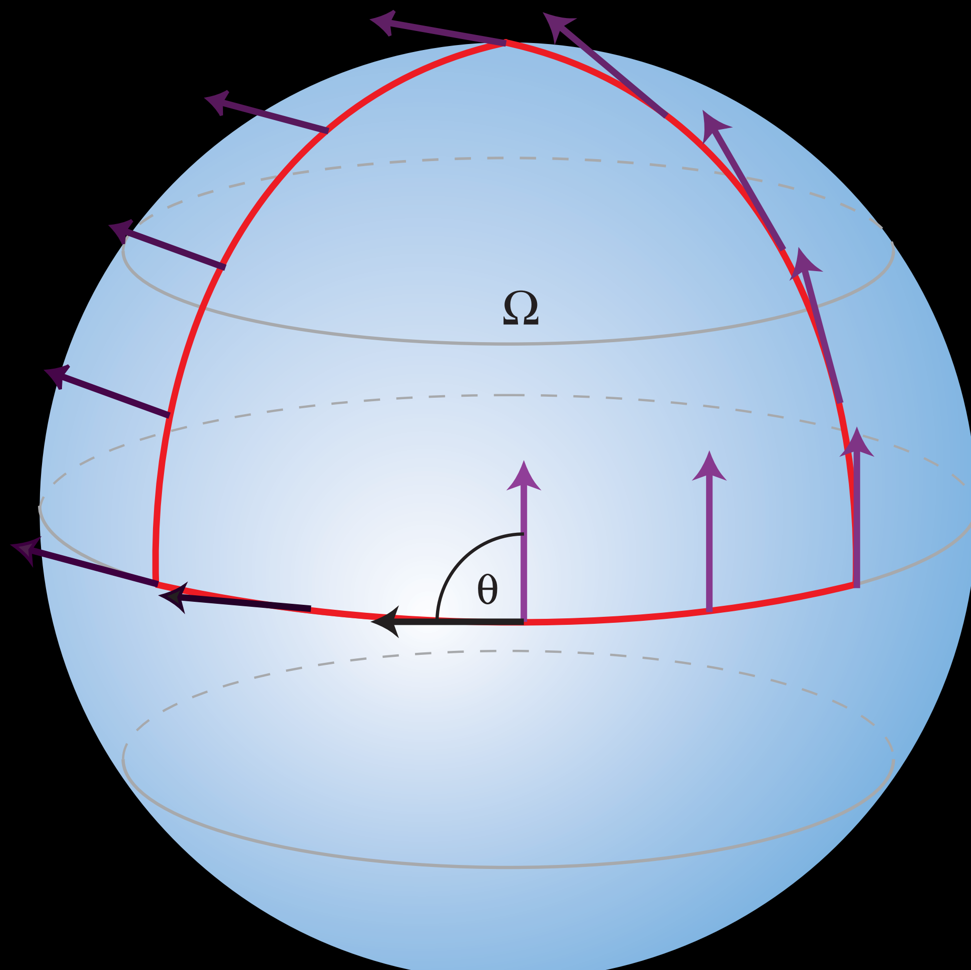


Results of the field reversal

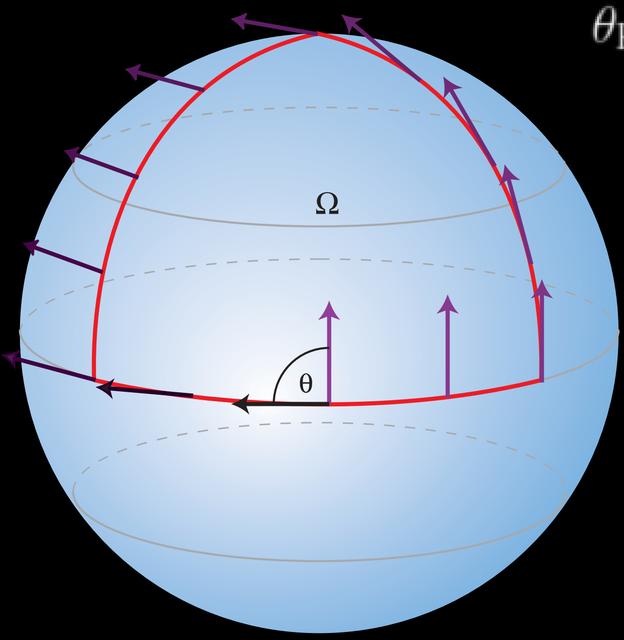
[A. E. Leanhardt et al., PRL 89, 190403 (2002)]



Geometric phases



Berry phase



$$\theta_{\text{Berry}} = i \oint_\gamma \langle 0; \mathbf{q} | \frac{\partial}{\partial \mathbf{q}_i} | 0; \mathbf{q} \rangle d\mathbf{q}^i = i \oint_\gamma \langle 0; \mathbf{q}(t) | \nabla_{\mathbf{q}} | 0; \mathbf{q}(t) \rangle \cdot d\mathbf{q}$$

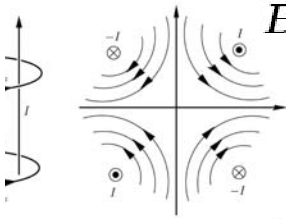
$$\theta_{\text{dyn}} = - \int_{t_0}^{t} \varepsilon_{\mathbf{q}(\tau)} d\tau / \hbar$$

$$\hat{H} = \hat{\mu} \cdot B \Rightarrow \theta_{\text{Berry}} = -m_z \Omega$$

Local Berry phase in TVF

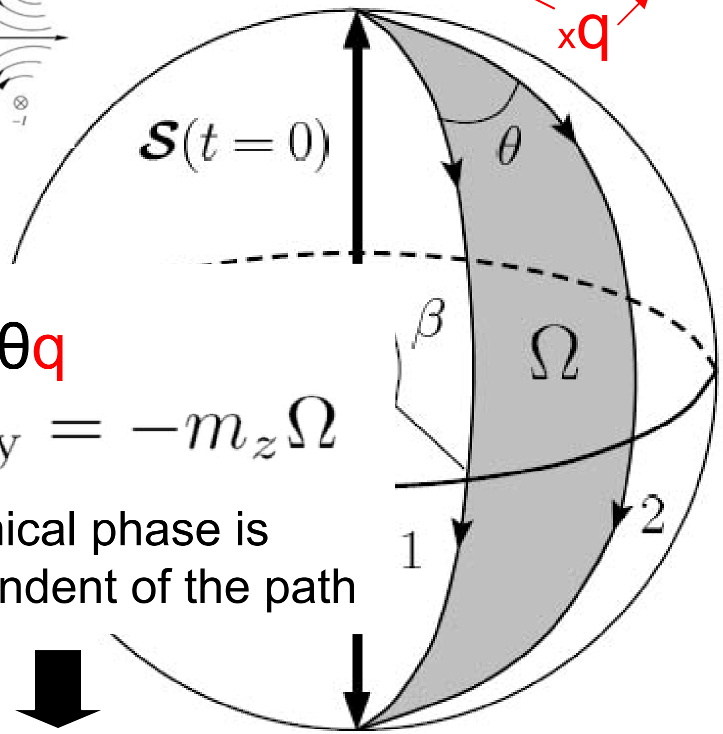
$$\boldsymbol{B}_q(r,\varphi,t) = B'_q(t) r[\cos(\varphi)\hat{\boldsymbol{x}} - \sin(\varphi)\hat{\boldsymbol{y}}]$$

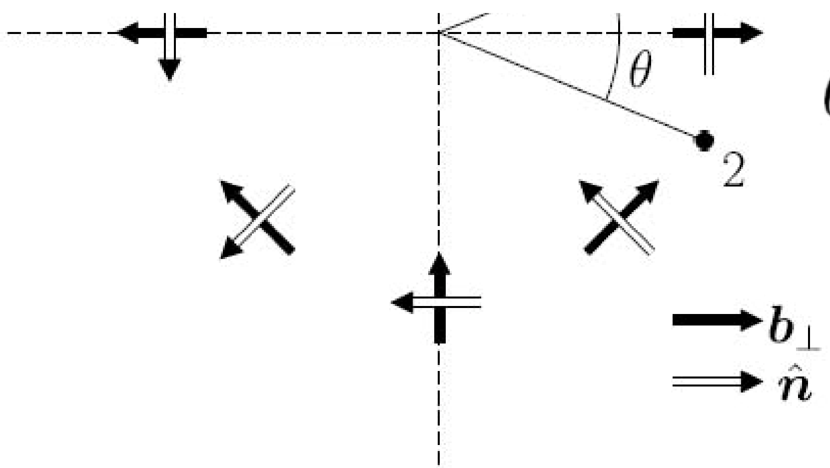
x q



q arises from the topology of the field

→ *topological* VF





Ω=-2θq

$$\theta_{\text{Berry}} = -m_z \Omega$$

Dynamical phase is independent of the path

$2m_z q$-quantum vortex

F=1 GP equation

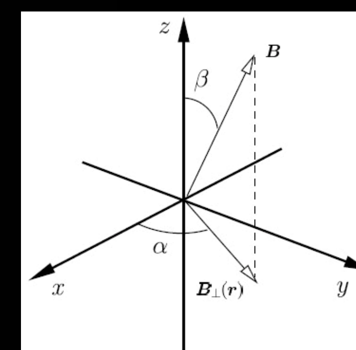
$$i\hbar\partial t\Psi = [\hat{h}_0 + c_0|\Psi|^2 + c_2(\Psi^\dagger \mathcal{F}\Psi)\cdot\mathcal{F}]\Psi$$

$$\hat{h}_0 = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{opt}} \boxed{+ g_F\mu_B \boldsymbol{B}\cdot\mathcal{F}}$$

Weak-field seeking states

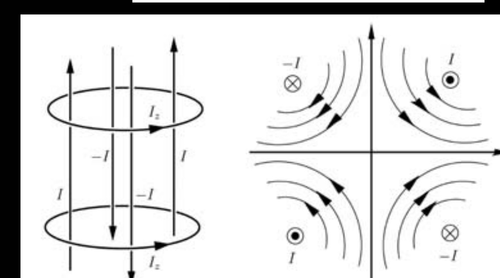
F=1

$$\Phi_1 = \frac{1}{2} f_0 \begin{pmatrix} 1+\cos\beta \\ \sqrt{2}\sin\beta e^{-i\varphi} \\ (1-\cos\beta)e^{-2i\varphi} \end{pmatrix} = f_0\,\zeta(r,\varphi)$$

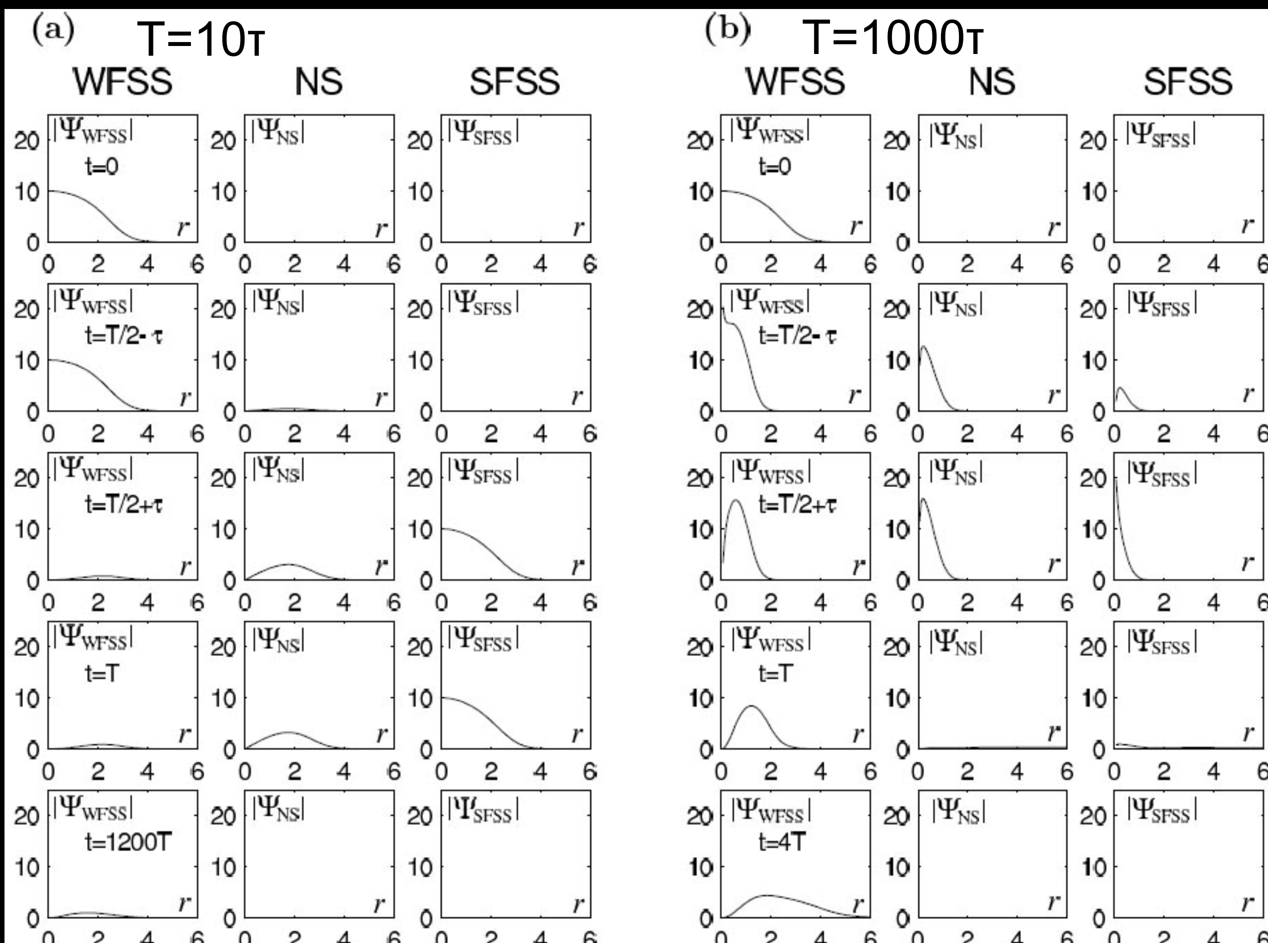


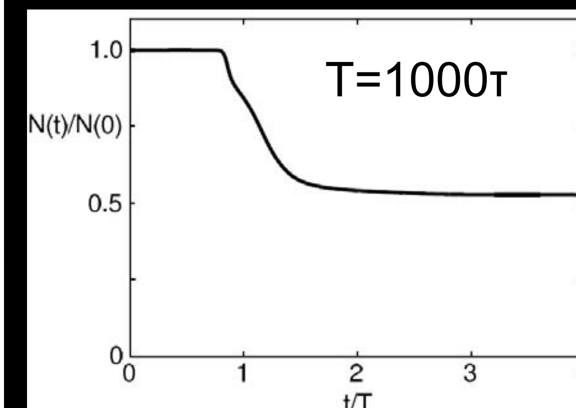
F=2

$$\zeta_2(\mathbf{r}) = \begin{pmatrix} \cos^4\frac{\beta}{2} \\ 2\cos^3\frac{\beta}{2}\sin\frac{\beta}{2}\,e^{i\phi} \\ \sqrt{6}\cos^2\frac{\beta}{2}\sin^2\frac{\beta}{2}e^{2i\phi} \\ 2\cos\frac{\beta}{2}\sin^3\frac{\beta}{2}\,e^{3i\phi} \\ \sin^4\frac{\beta}{2}\,e^{4i\phi} \end{pmatrix}$$



Solving the F=1 GP equation



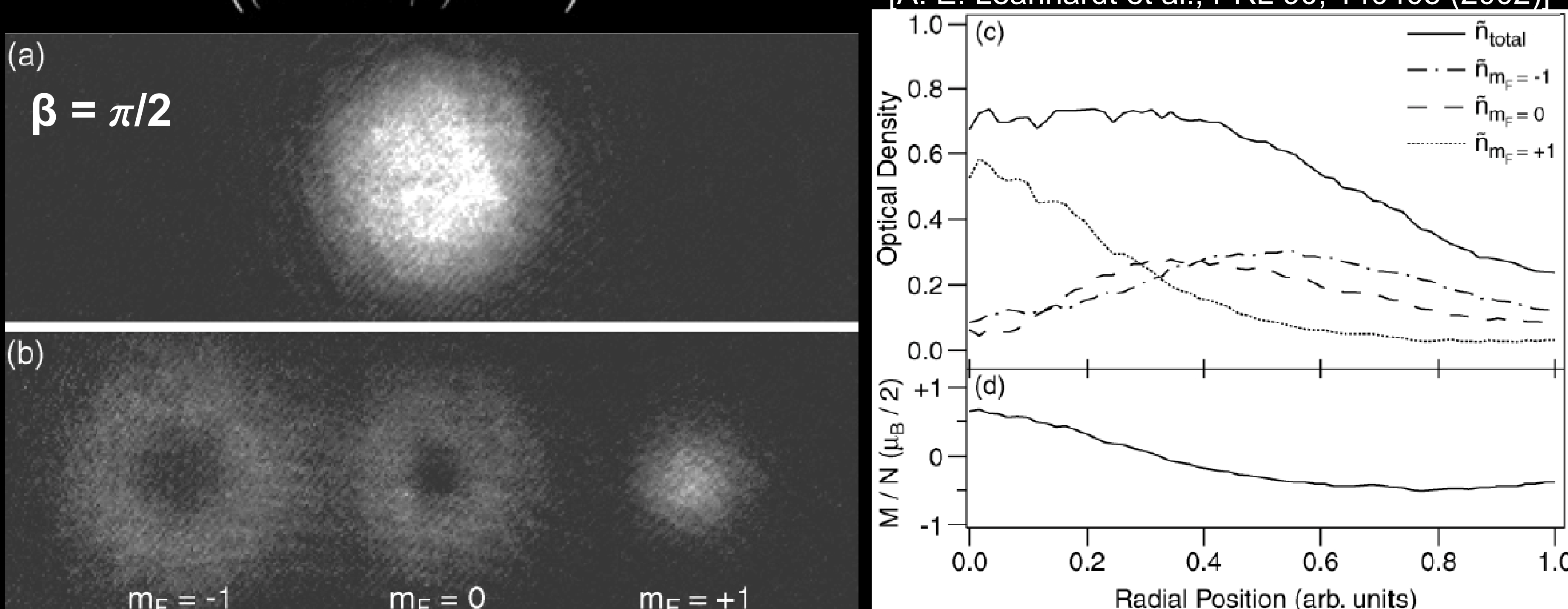


[Ogawa et. Al, Phys. Rev. A **66**, 013617 (2002)]

Coreless vortex formation

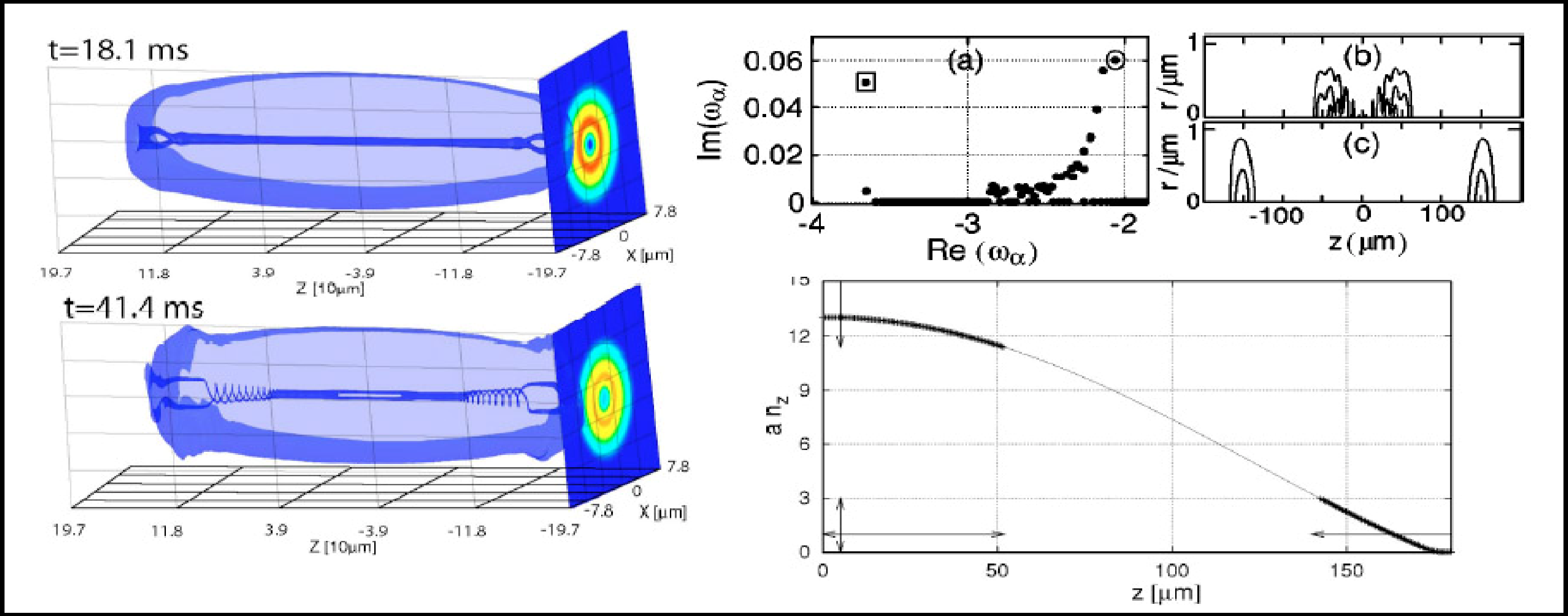
$$\Phi_1 = \frac{1}{2} f_0 \begin{pmatrix} 1+\cos\beta \\ \sqrt{2}\sin\beta e^{-i\varphi} \\ (1-\cos\beta)e^{-2i\varphi} \end{pmatrix} = f_0\,\zeta(r,\varphi)$$

[A. E. Leanhardt et al., PRL 90, 140403 (2002)]



Multi-quantum vortices are dynamically unstable

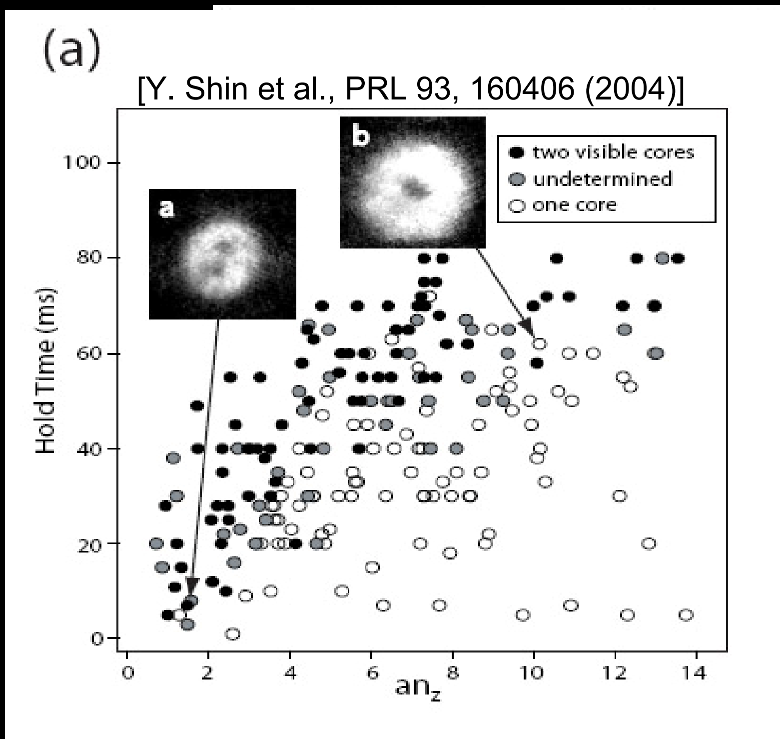
[MM et al., PRA 68, 023611 (2003)]

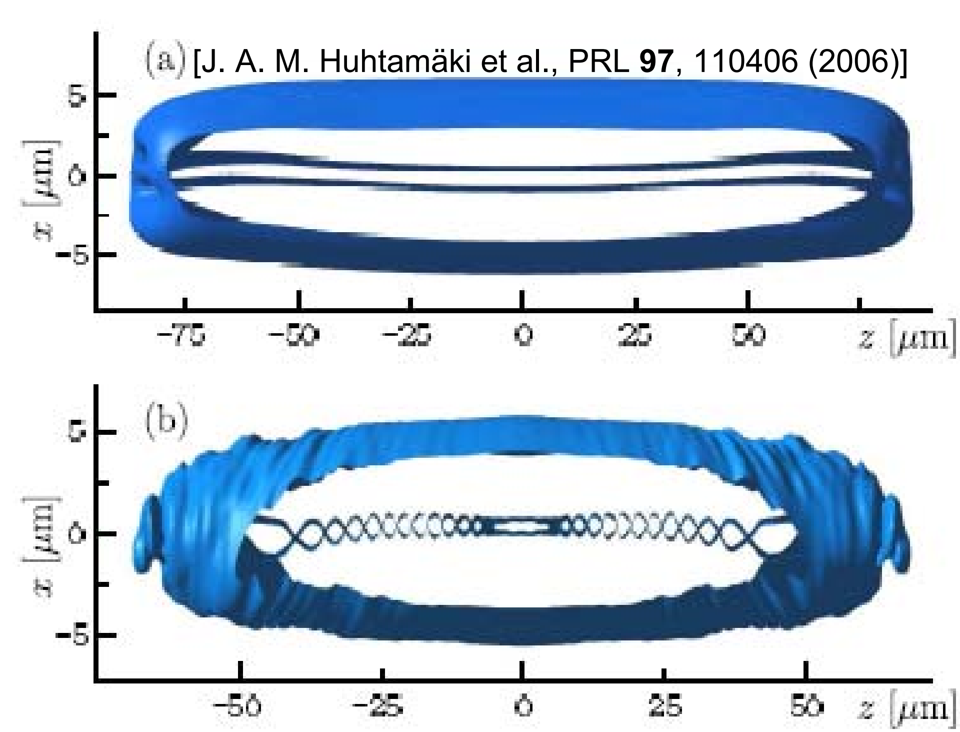


See also H. Nielsen and E. Lundh, PRA 77, 013604 (2008); PRA 74, 063620 (2006)

Mesurements on vortex splitting

- TOF absorbtion images from a 30 μm slice in *z* direction

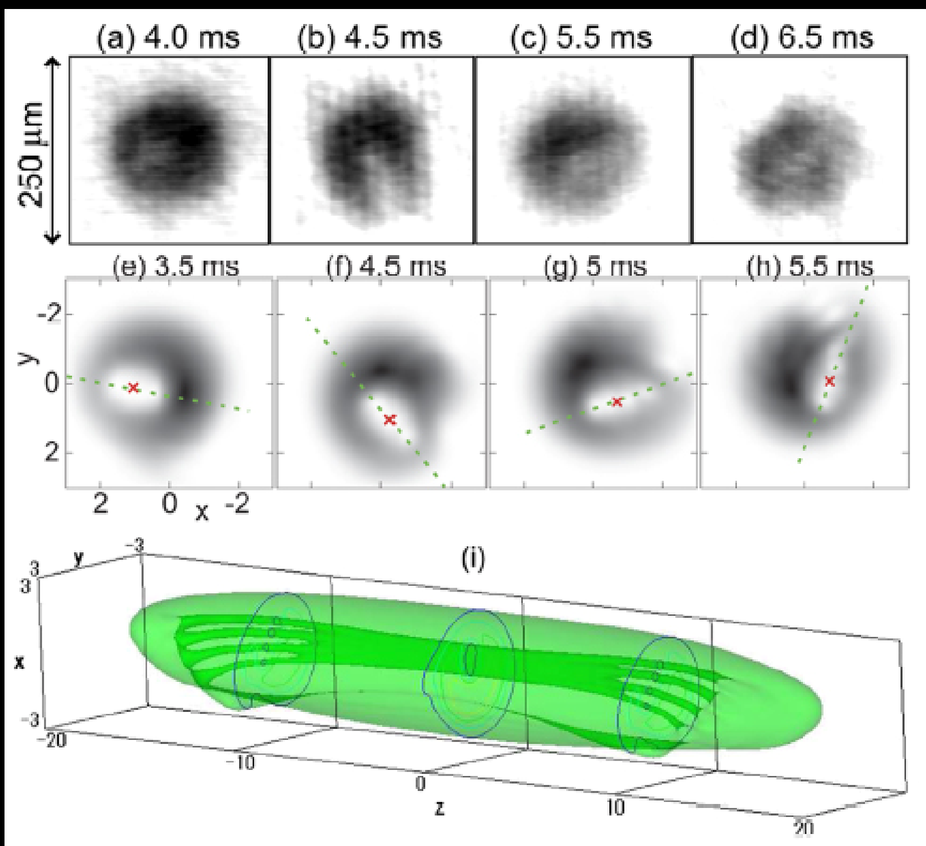




[Y. Shin et al., PRL 93, 160406 (2004)]

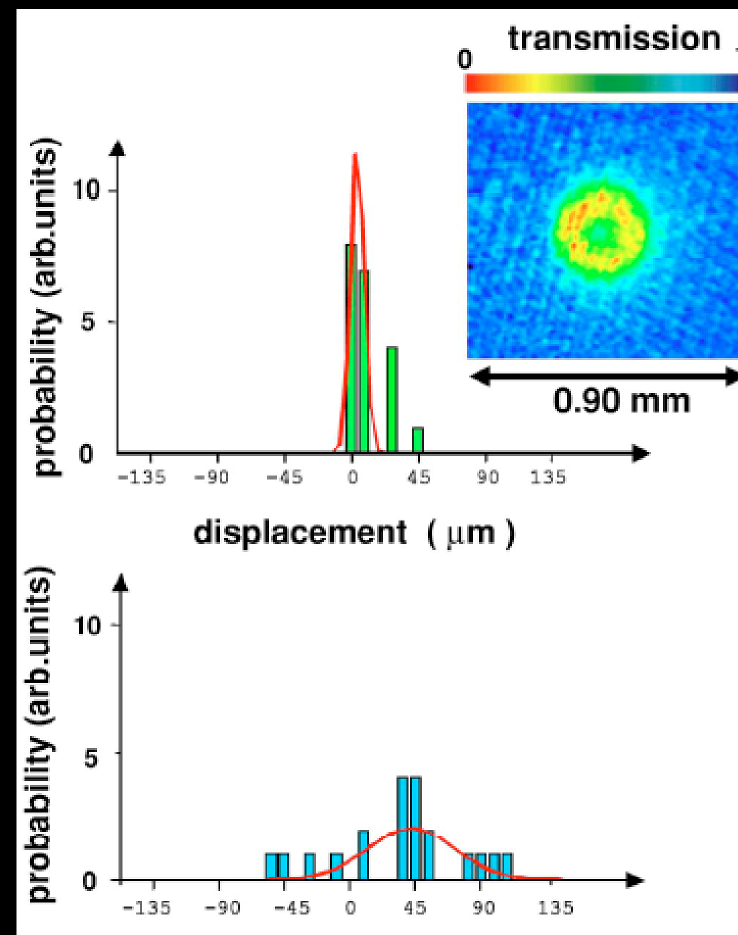
4-quantum vortex splitting

[Isoshima et al., PRL **99**, 200403 (2007), Kyoto group]



Gravity compensation

[Kumakura et al., PRA **73**, 063605 (2006)]



PART 2

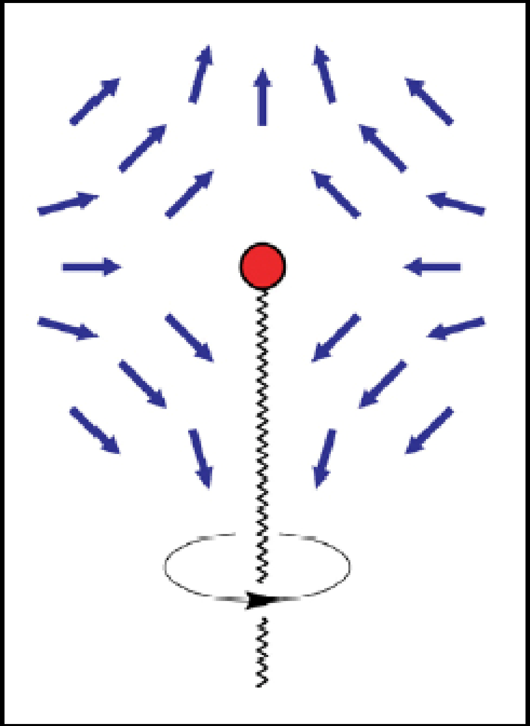
PRL **103,** 030401 (2009) PHYSICAL REVIEW LETTERS week ending 17 JULY 2009

Creation of Dirac Monopoles in Spinor Bose-Einstein Condensates

Ville Pietilä[1,2] and Mikko Möttönen[1,2,3]

Employed magnetic field

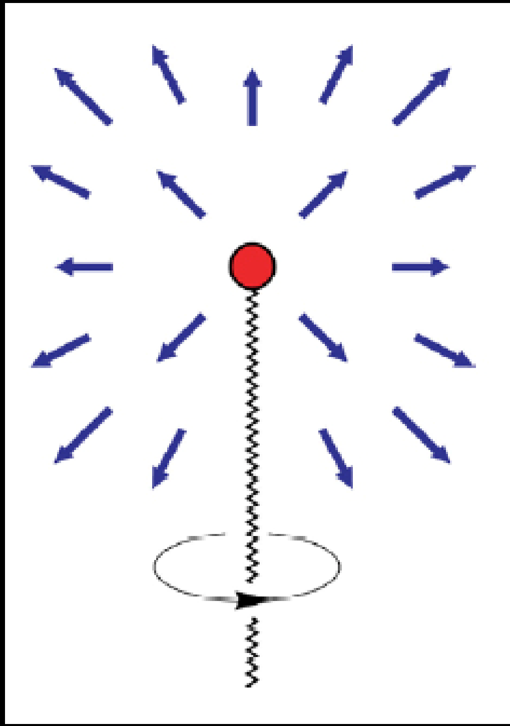
$$B(r,t) = B_1'(x\hat{x} + y\hat{y}) + B_2' z\hat{z} + B_0(t)\hat{b}$$



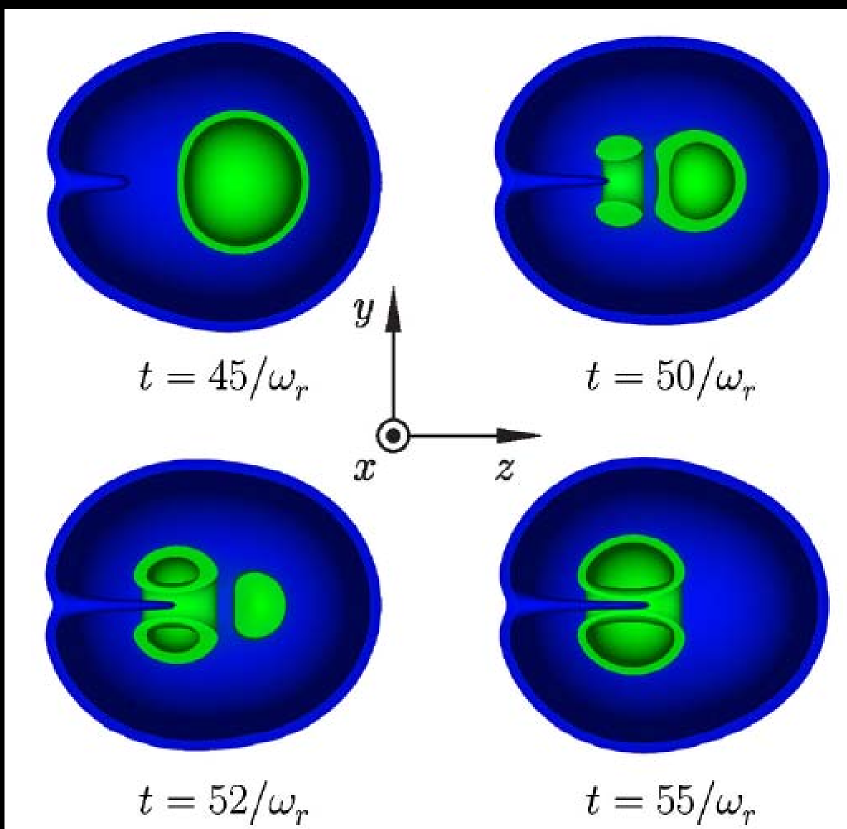
Analytical result:

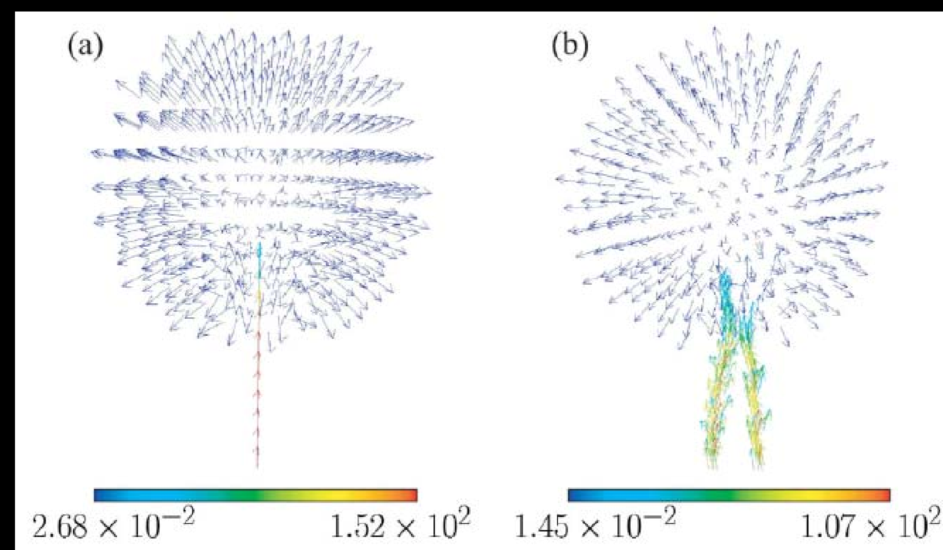
If the spin aligns with the field, the vorticity assumes form:

However, we observe no monopole in the ground state



Bringing in the node of the magnetic field adiabatically



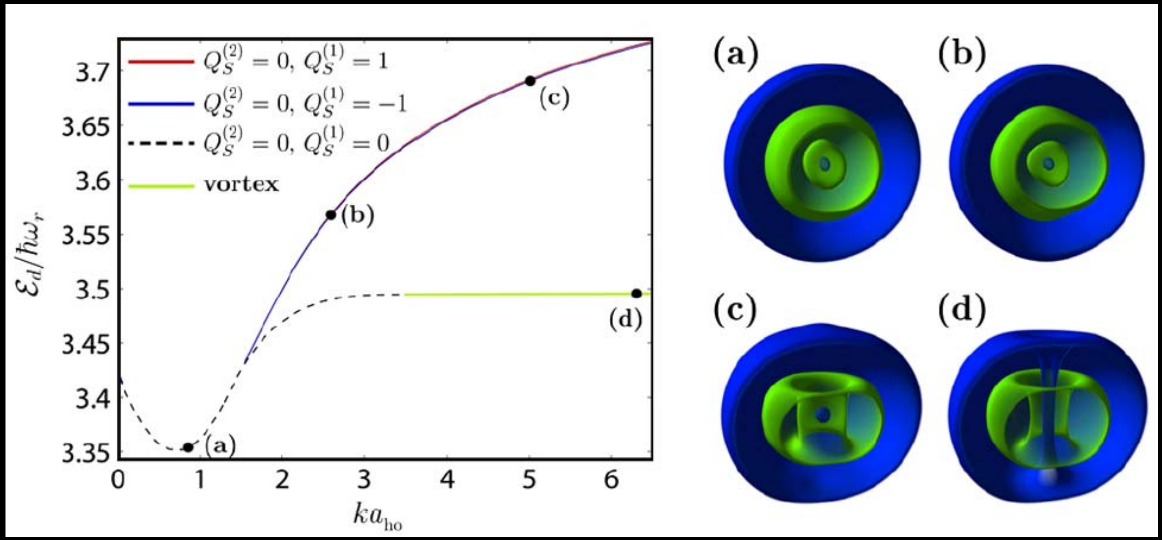


Related study with artificial non-Abelian gauge fields

PRL **102**, 080403 (2009) PHYSICAL REVIEW LETTERS week ending 27 FEBRUARY 2009

Non-Abelian Magnetic Monopole in a Bose-Einstein Condensate

Ville Pietilä[1,2] and Mikko Möttönen[1,2,3]

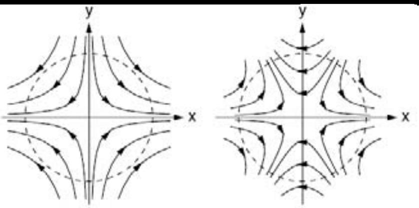


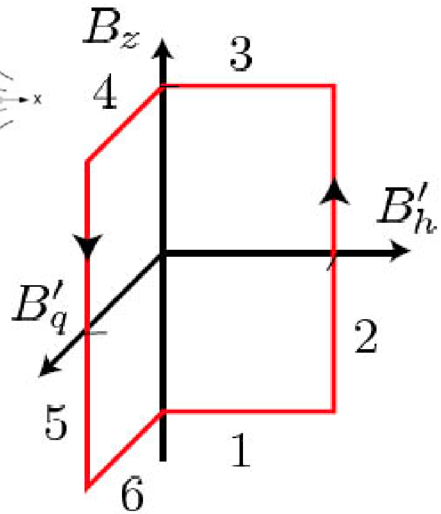
PART 3

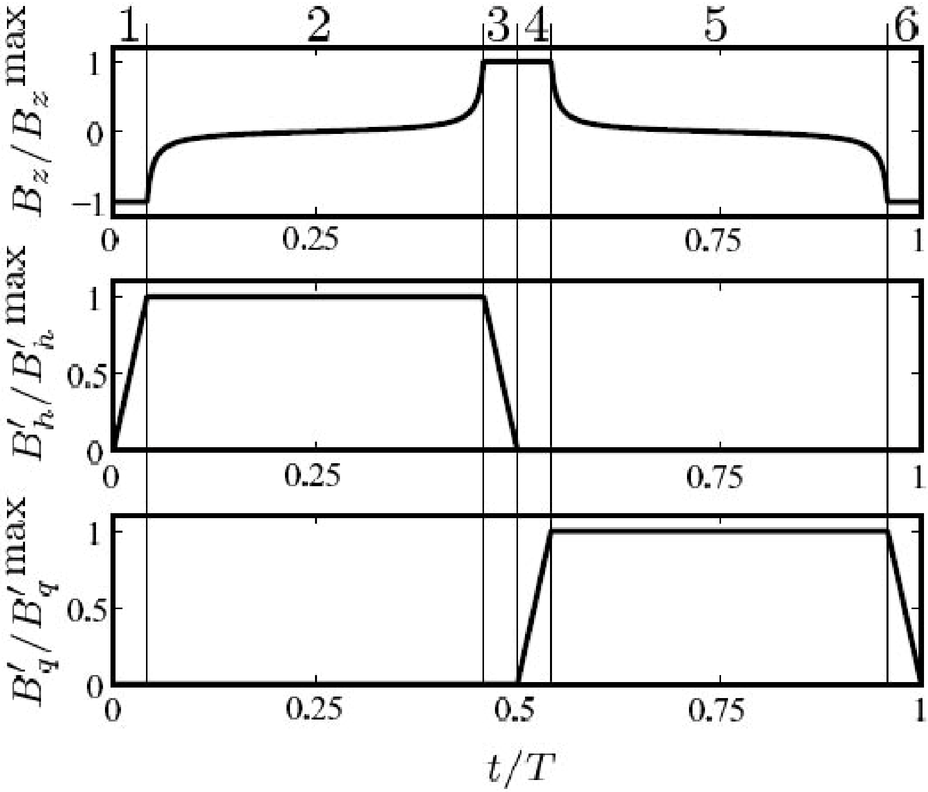
Cyclic vortex formation
THE VORTEX PUMP

Pumping vorticity

[MM et al., PRL **99**, 250406 (2007)]







q=1 → $\mathbf{B}_q(r,\phi,t) = B'_q(t)r[\cos(\phi)\hat{\mathbf{x}} - \sin(\phi)\hat{\mathbf{y}}]$

q=2 → $\mathbf{B}_h(r,\phi,t) = B'_h(t)r[\cos(2\phi)\hat{\mathbf{x}} - \sin(2\phi)\hat{\mathbf{y}}]$

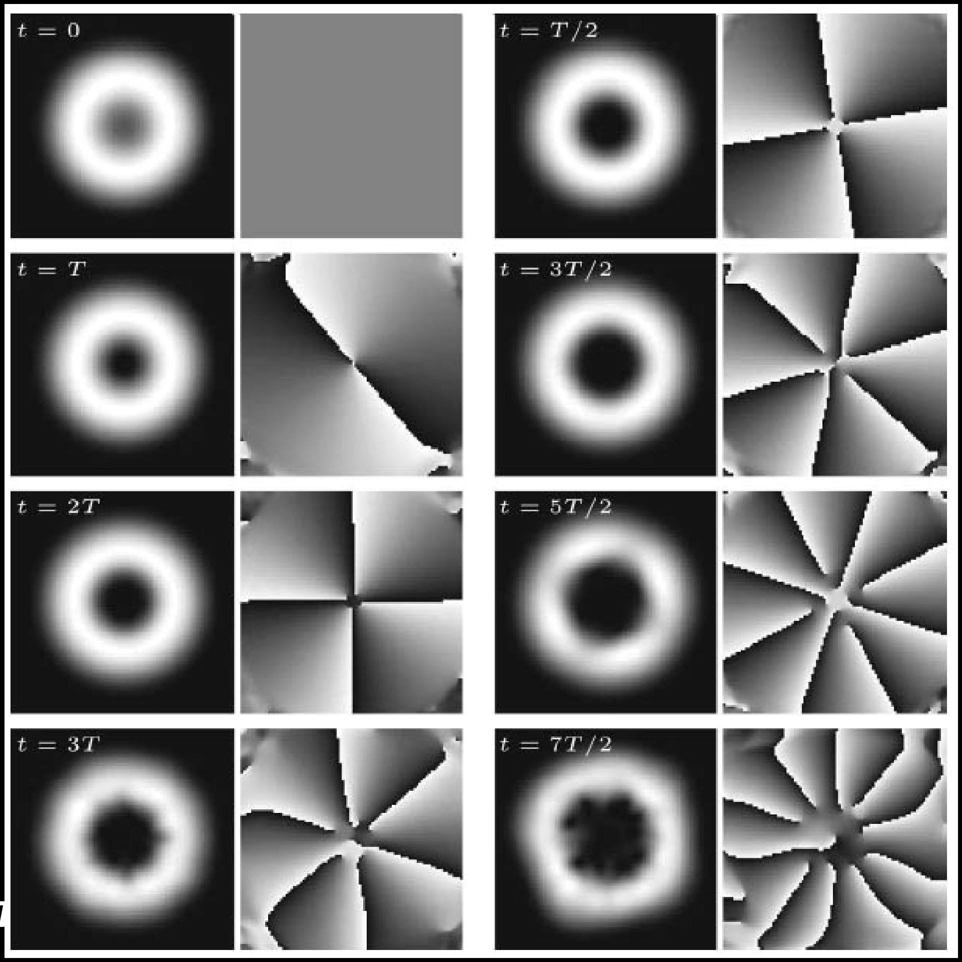
Ω=-2θq

$$\theta_{\mathrm{Berry}} = -m_z\Omega$$

$2m_z q$-quantum vortex

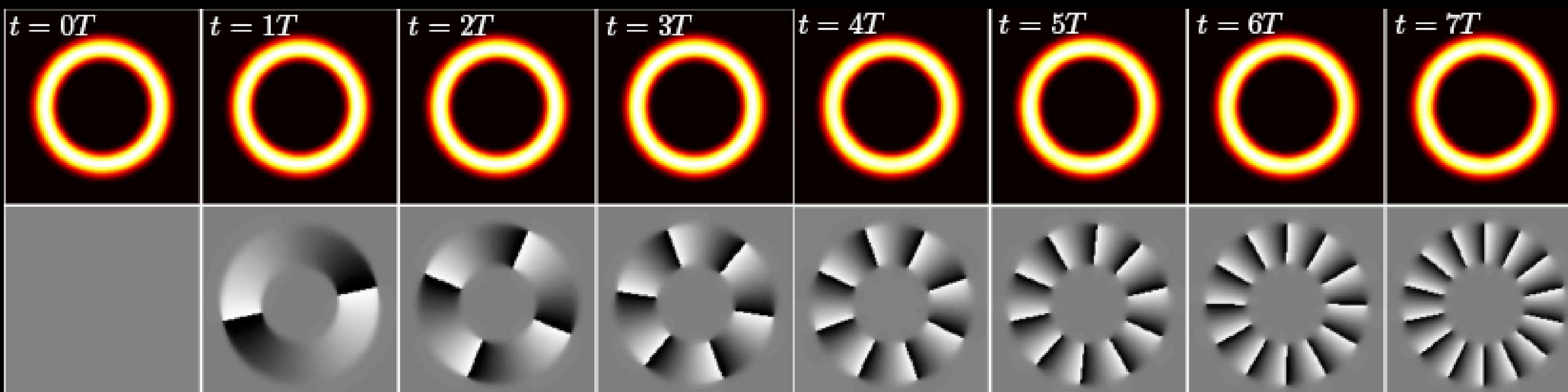
Adiabatic pumping

- Using optical plug along *z* axis and optical harmonic trap in all directions
- Starting from the SFSS



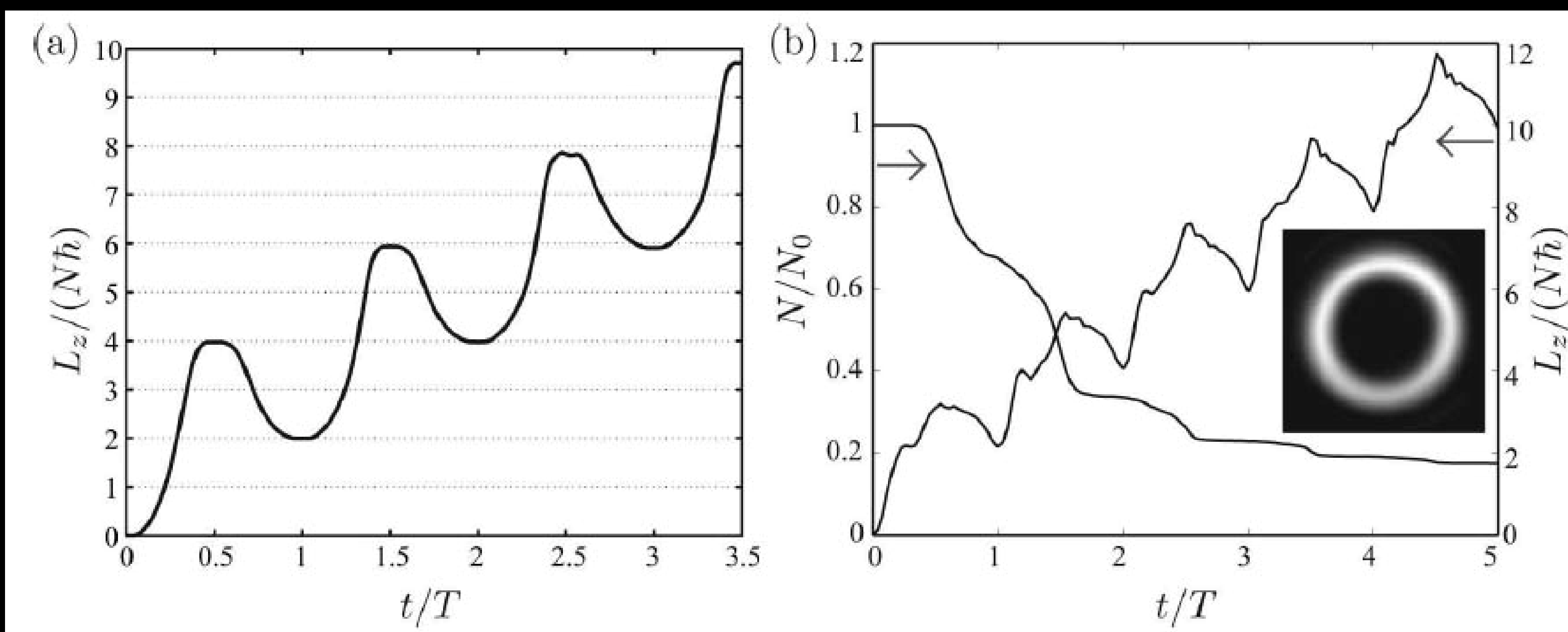
Pumping with a stronger plug

[P. Kupanportti et al., arXiv:1006.0636 (2010)]



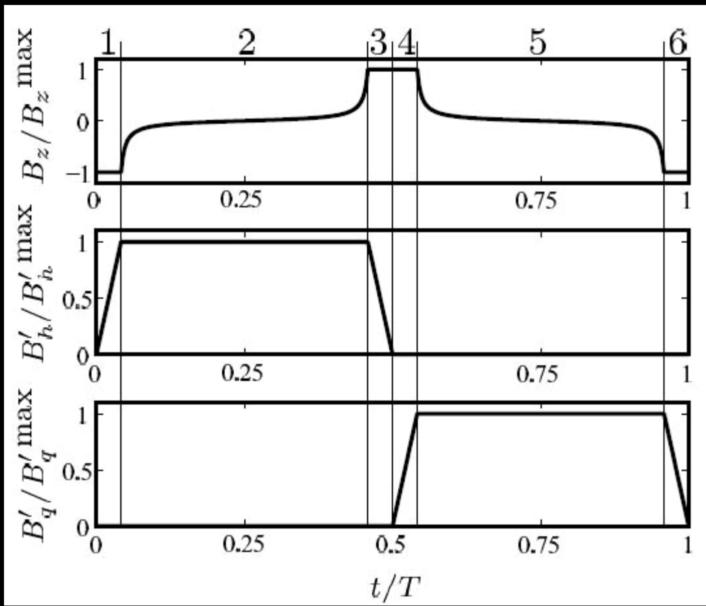
Fast pumping

- No optical potentials → ”easy” to realize in experiments
- Starting from the WFSS (SFSS escapes the trap)

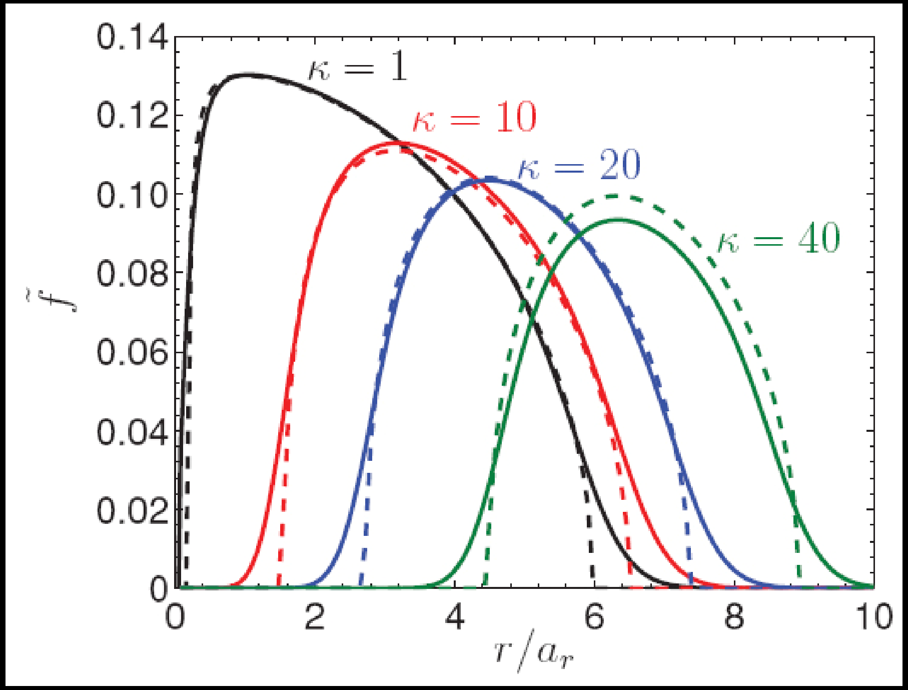


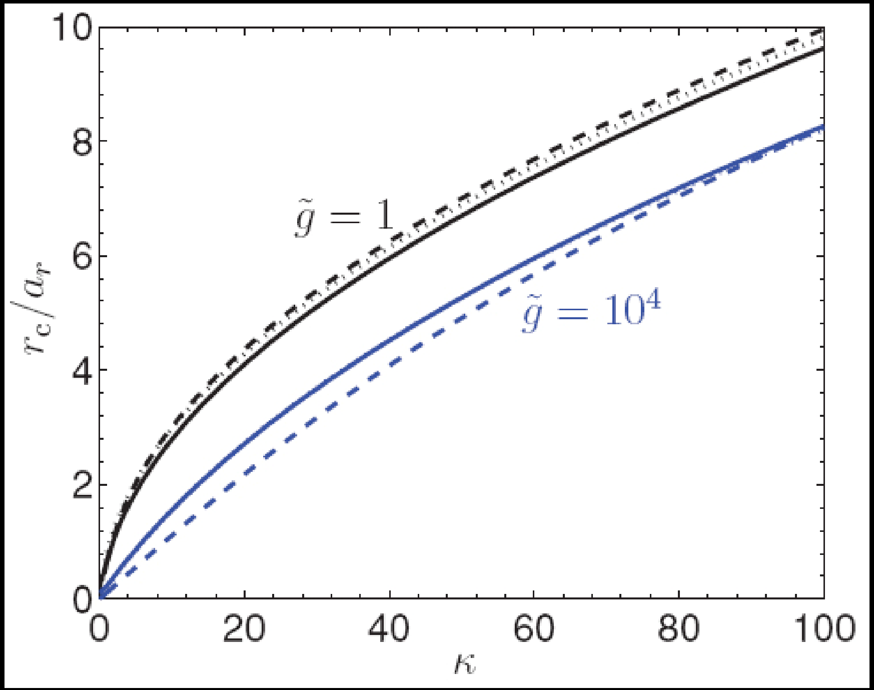
How much can we pump?

- Adiabaticity determined by the transverse field strength (Laudau-Zener tunneling)
- Can we speed up during pumping?



Giant vortex core size



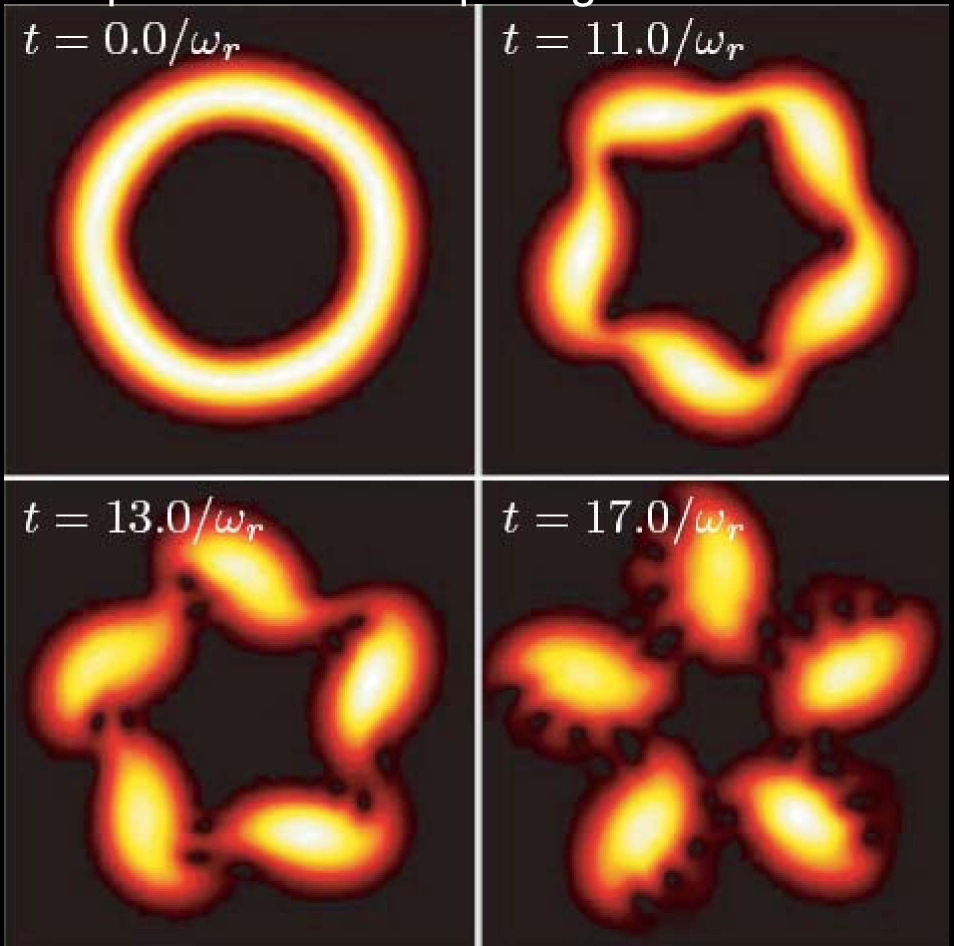


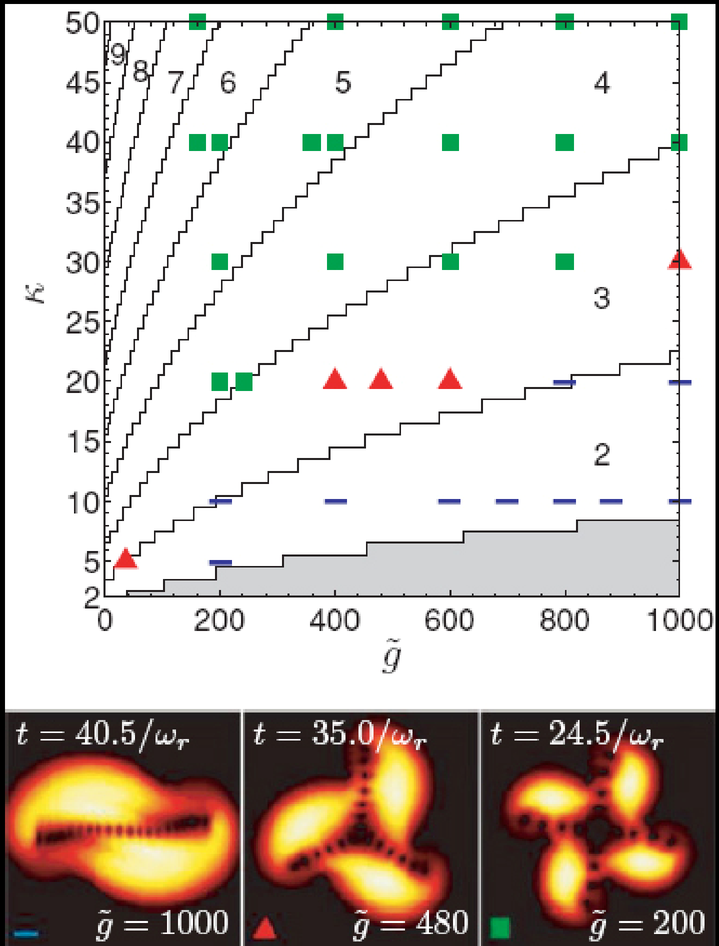
[P. Kupanportti, E. Lundh et al., PRA 81, 023603 (2010)]

What happens to a giant vortex?

[P. Kupanportti and MM, PRA 81, 033627 (2010)]

30 quantum vortex splitting





K=20