"Quantum solids, liquids, and gases"
Stockholm, Sweden
2 August, 2010

Superfluid unitary Fermi gases in a 1D optical lattice — thermodynamic properties and critical velocity

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(APCTP, RIKEN)

GW, Orso, Dalfovo, Pitaevskii & Stringari, PRA **78**, 063619 (2008).
GW, Dalfovo, Piazza, Pitaevskii & Stringari, PRA **80**, 053602 (2009).

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- Introduction
- Thermodynamic properties
- Critical velocity of superflow
- Loop structures in energy band of Fermi superfluids
- Summary & conclusion

Introduction

Historical overview

Study of superfluidity: More than 70 years of history

1938: superfluidity in liquid 4-He
(Kapitza@Moscow, Allen & Misener@Cambridge)

1938: superfluid 4-He as a BEC (London)

1941: Landau's 2-fluid hydrodynamics

1949: Onsager-Feynman quantization of circulation

1972: superfluidity in liquid 3-He (Osheroff@Stanford)

1995: BEC of alkali atom gases
(Ketterle@MIT, Cornell & Weimann@JILA)

2004: BCS-BEC crossover in 2-component Fermi gases
(JILA, MIT)

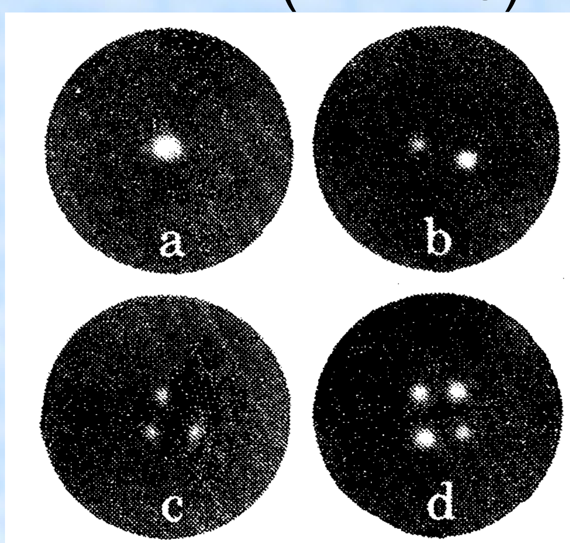
Historical overview

Study of superfluidity: More than 70 years of history

Cold atoms makes seeing is believing!

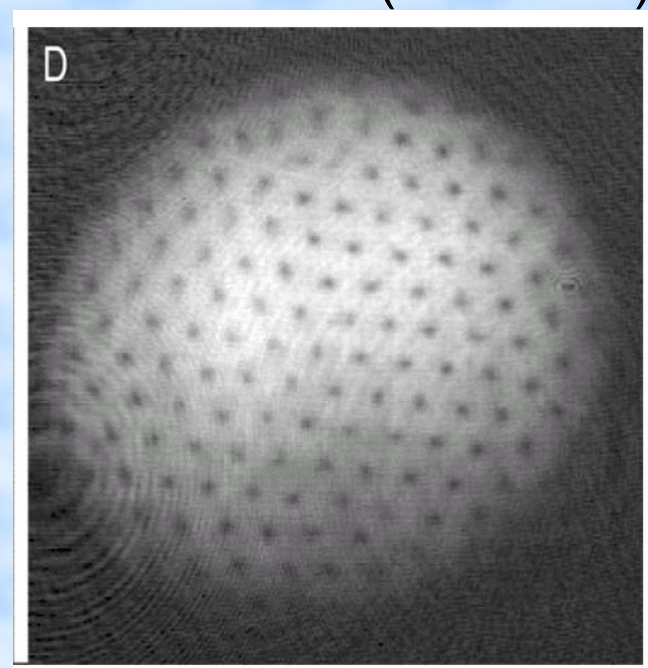
idge)

He II (Nv < 25)



Yamchuck et al. (1979)

BEC of Na (Nv < 150)



Abo-Shaeer et al. (2001)

rd)

ases

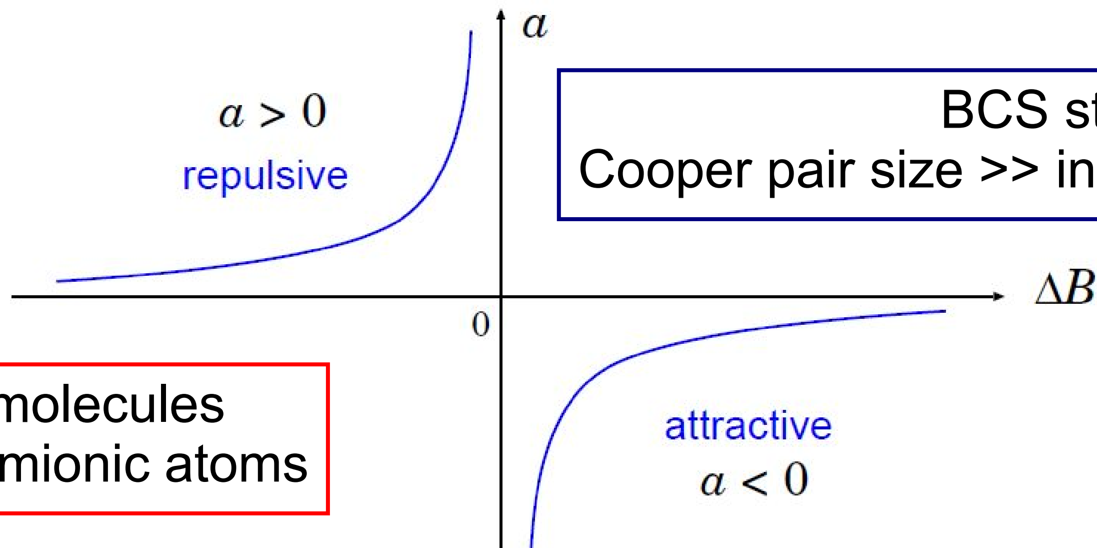
BCS-BEC crossover (1)

Cold atom gases: high controllability

2-component Fermi gases   BCS-BEC crossover by a Feshbach res.

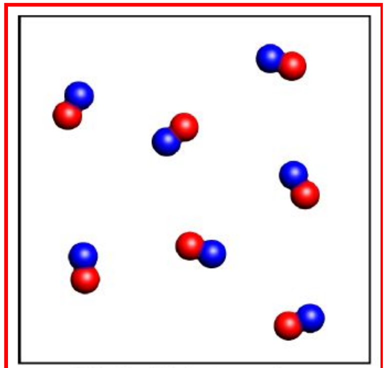
unitary regime: scattering length $a_s \to \pm\infty$ ⇒ **strongly int. Fermi superfluid**

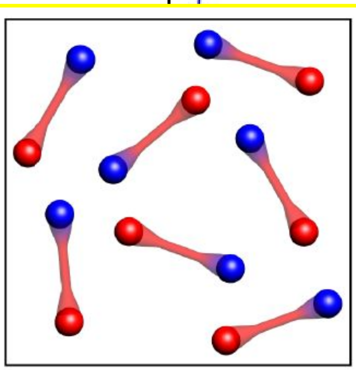
BEC ⟷ Unitarity ⟷ BCS

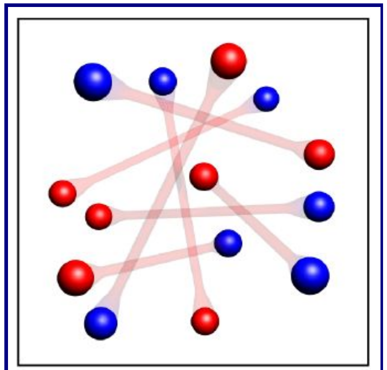


BCS state
Cooper pair size >> interparticle distance

BEC of bosonic molecules
made of 2 bound fermionic atoms


BEC of Molecules


Crossover Superfluid


BCS state

BCS-BEC crossover (2)

At unitarity: $a_s \to \pm\infty$

Scattering length drops from the problem.

Only relevant length scales are
- inverse of the Fermi wave number
- thermal wavelength

All thermodynamic quantities are determined by E_F and T/T_F.

At T=0: $\mu \propto E_F$

$$E_F = \hbar^2 k_F^2 / 2m$$
$$k_F \equiv (3\pi^2 n)^{1/3}$$

Coefficient: $(1+\beta)$ = 0.42 (QMC) Carlson+ (2003)
0.59 (MF) Astrakharchik+ (2004)

Motivation: Experiment @ MIT (1)

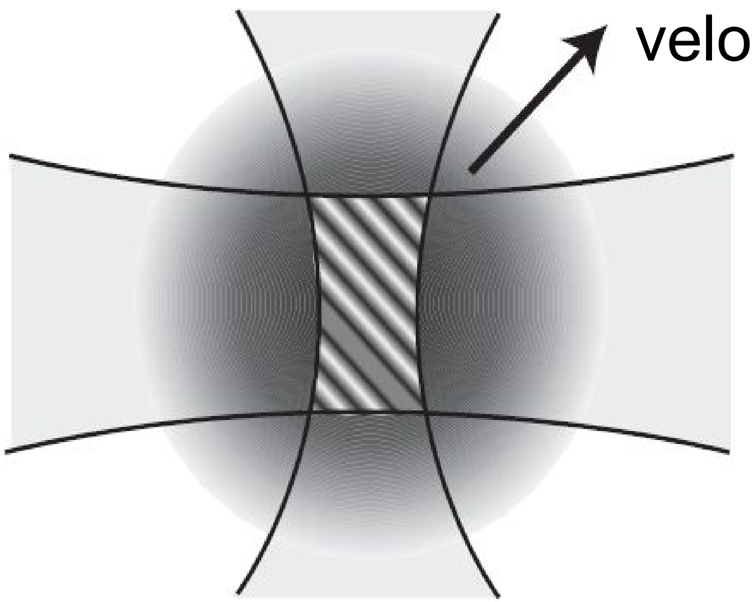
Miller et al., PRL **99**, 070402 (2007)

Instability of superfluids of Fermi gases in a 1D optical lattice

$$V_{\rm lat}(z) = sE_{\rm R}\sin^2 q_{\rm B} z$$

$0.01 \lesssim s \lesssim 2$: weak lattice

recoil energy $E_{\rm R} \equiv \frac{\hbar^2 q_{\rm B}^2}{2m}$; Bragg mom. $q_{\rm B} \equiv \frac{\pi}{d}$



velocity of lattice $v = \lambda \Delta \nu$

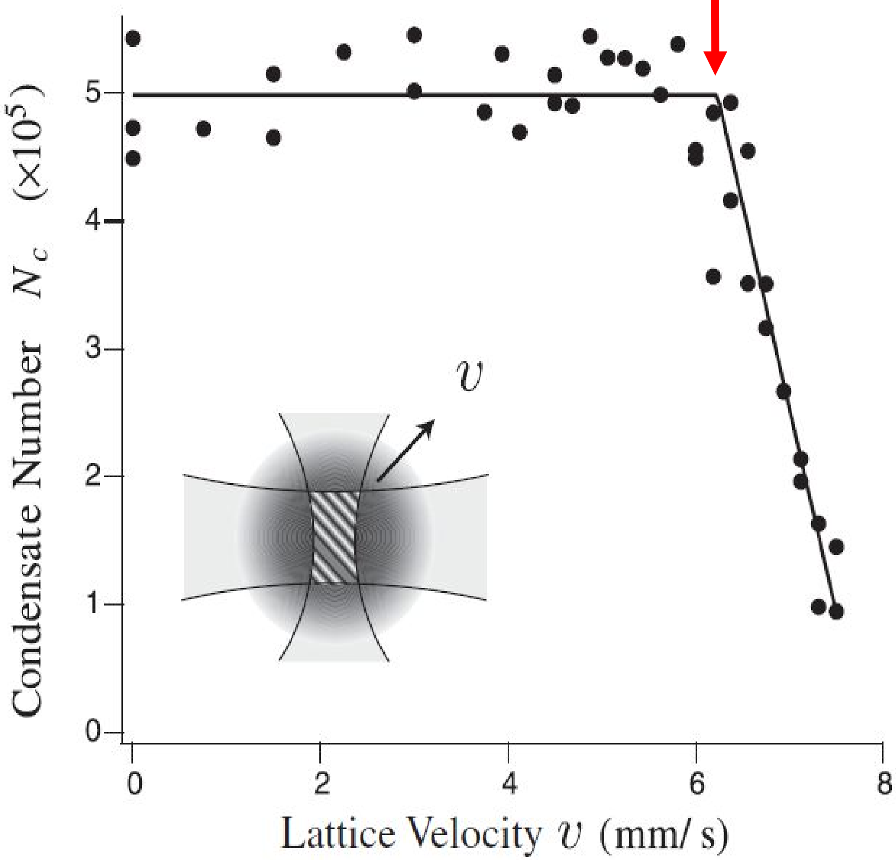
Δv: freq. difference of the 2 lasers

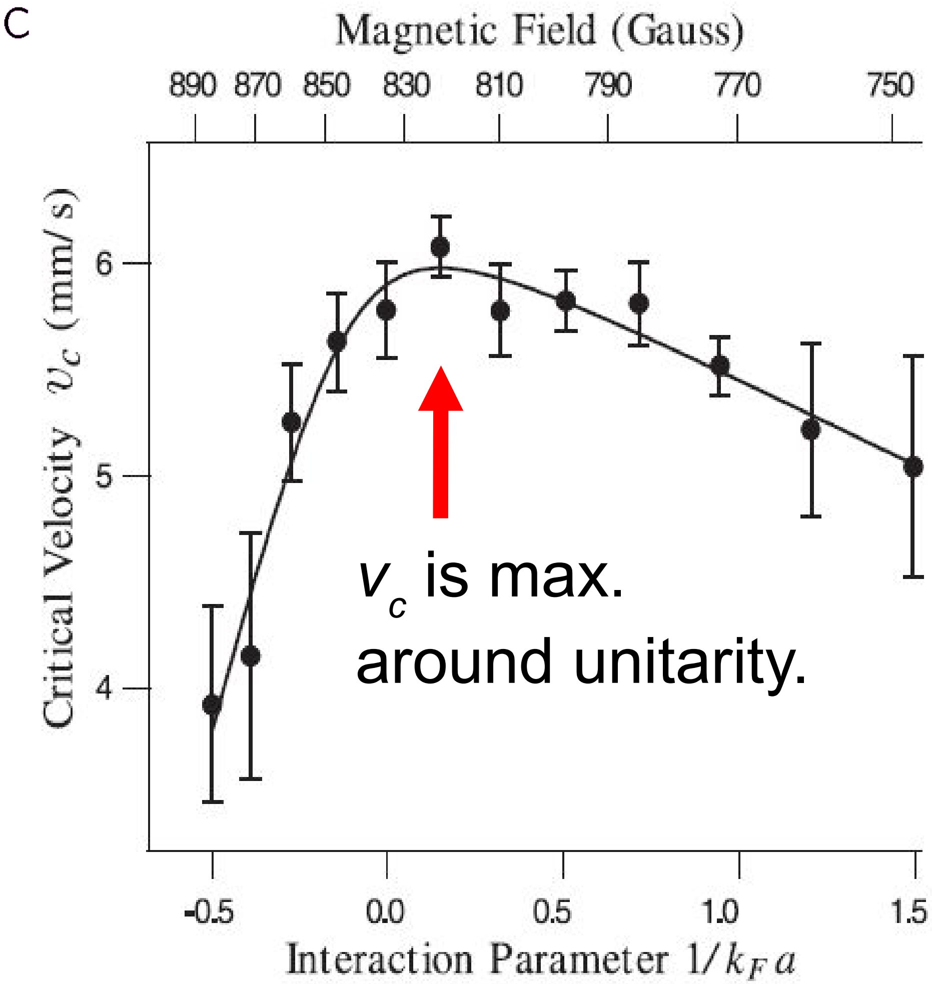
^{6}Li; Feshbach res. @ B=834G

Number of atoms N=10^6

Motivation: Experiment @ MIT (2)

critical velocity v_c



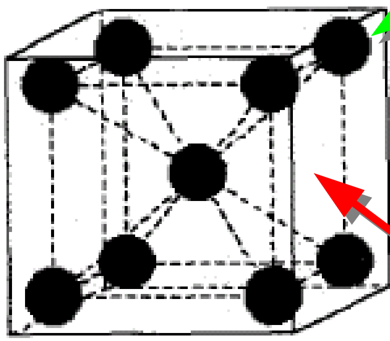


Superfluidity is robust in the unitary regime

Motivation: Simulating neutron star matter

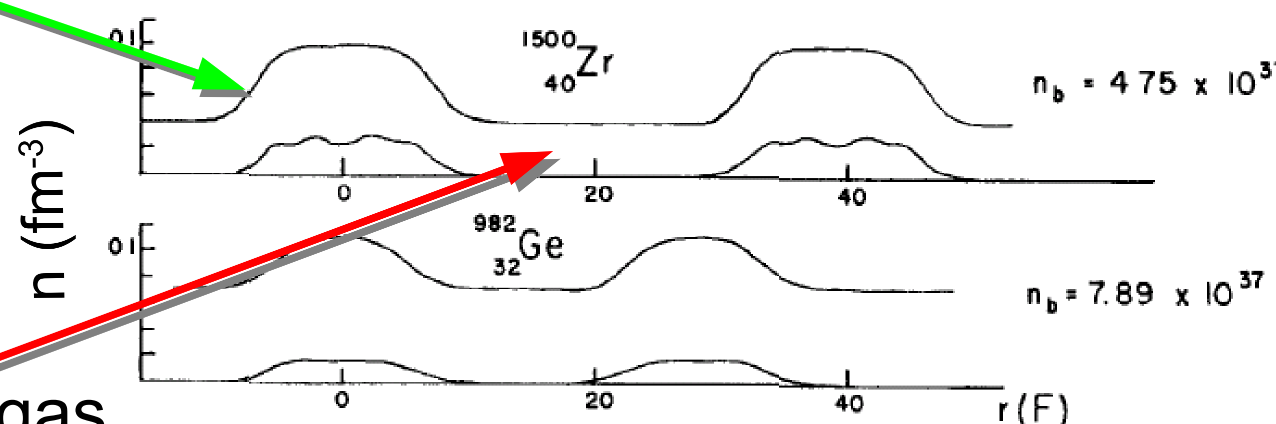
In neutron star crusts: n-rich nuclei are immersed in superfluid neutron gas

n-rich nucleus



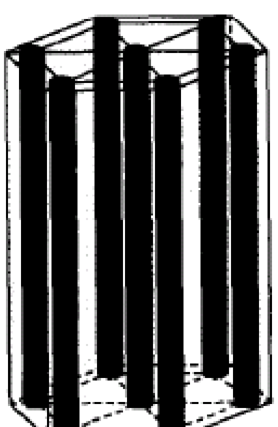
Oyamatsu (1993)

s.f. n-gas

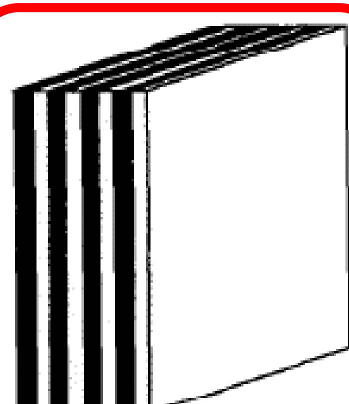


Negele & Vautherin (1973)

Nuclear "pasta": rod-like and slab-like nuclei



spaghetti



lasagna

s.f. n-gas in lasagna phase

Slab-like nuclei (normal nucleons)

⇨ Periodic pot. for s.f. neutrons.

Resembles superfluid Fermi gas in a 1D periodic potential.

Thermodynamic properties in a 1D lattice

GW, Orso, Dalfovo, Pitaevskii & Stringari
PRA **78**, 063619 (2008).

Purpose of our study

BCS-BEC crossover of a 2-component Fermi gas in a 1D optical lattice

$$V_{\mathrm{lat}}(z) = sE_{\mathrm{R}} \sin^2 q_{\mathrm{B}} z \qquad (s \leq 5)$$

Focus on the unitarity limit (T=0)

Effects of the lattice on the basic thermodyn. quantities

chemical potential

$$\mu = \frac{\partial e(n,P)}{\partial n} \ ;$$

incompressibility

$$\kappa^{-1} = n\frac{\partial^2 e(n,P)}{\partial n^2} \ ;$$

effective mass

$$\frac{1}{m^*} = \frac{1}{n}\frac{\partial^2 e(n,P)}{\partial P^2}$$

e: energy density, P: quasimom. of the whole superfulid

EOS

Collective osc.

Sound velocity

Tunneling rate btw wells

Band width

BdG equations

Exp. @ MIT : weak lattice, superfluid phase

Bogoliubov – de Gennes (BdG) eqs.

P : Qausimom. of the superfluid

$$u_i(\mathbf{r}) = \tilde{u}_i(z) e^{iPz} e^{i\mathbf{k}\cdot\mathbf{r}}$$
$$v_i(\mathbf{r}) = \tilde{v}_i(z) e^{-iPz} e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\Delta(\mathbf{r}) = \boxed{\tilde{\Delta}(z)} e^{2iPz}$$

period of the lattce const. d

$$\begin{pmatrix} \tilde{H}'_P(z) & \tilde{\Delta}(z) \\ \tilde{\Delta}^*(z) & -\tilde{H}'_P(z) \end{pmatrix} \begin{pmatrix} \tilde{u}_i(z) \\ \tilde{v}_i(z) \end{pmatrix} = \epsilon_i \begin{pmatrix} \tilde{u}_i(z) \\ \tilde{v}_i(z) \end{pmatrix}$$

$$\tilde{H}'_P(z) \equiv \frac{1}{2m}\left[k_x^2 + k_y^2 + (-i\partial_z + P + k_z)^2\right] + V_{\mathrm{lat}}(z) - \mu$$

$$V_{\mathrm{lat}}(z) = sE_{\mathrm{R}} \sin^2 q_{\mathrm{B}} z$$

self-consistency relation

$$\tilde{\Delta}(z) = -g \sum_i \tilde{u}_i(z) \tilde{v}_i^*(z) \qquad n = \frac{2}{V} \int d\mathbf{r} \sum_i |v_i(\mathbf{r})|^2$$

Hydrodynamic theory for unitary Fermi gases

Based on LDA: valid when

(length scale of density chage) >> (healing length ξ)

Around P=0: $$\mathcal{E}_0(n,P) \simeq \frac{3}{5}(1+\beta)E_{\rm F} n + \frac{P^2}{2m} n$$

$$\bar{\mathcal{E}} \equiv \frac{1}{d}\int_{-d/2}^{d/2} dz\,[\mathcal{E}_0(n,P) + Vn]$$

Cont. eq. $\cancel{\partial_t n} + \partial_z \frac{\partial \mathcal{E}_0}{\partial P} = 0$

Euler eq. $\cancel{\partial_t P} + \partial_z \left[\frac{\partial \mathcal{E}_0}{\partial n} + V\right] = 0$

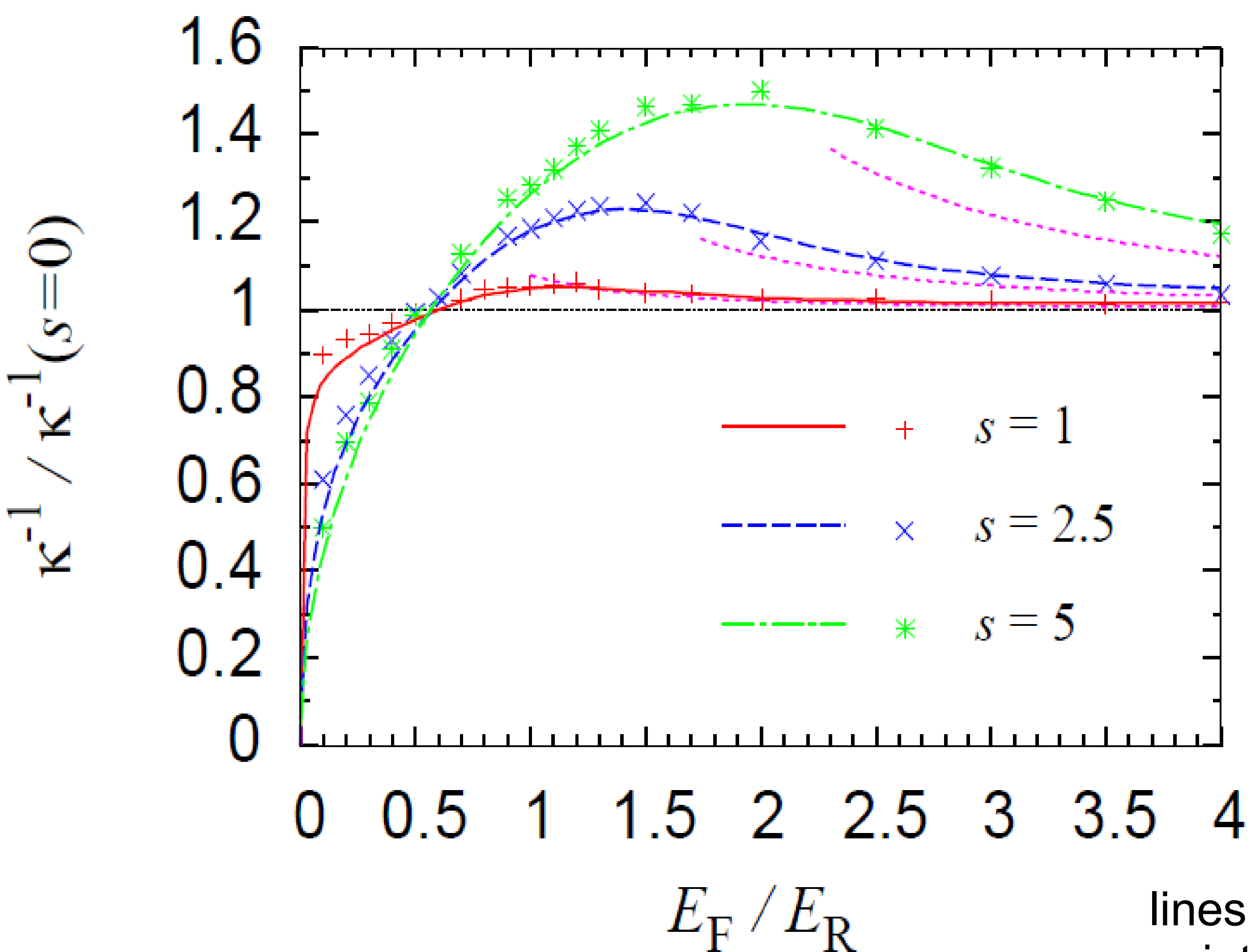
$V/E_{\rm F} \ll 1$

$$\Rightarrow \begin{cases} n(z) \simeq n_0\left[1 - \frac{V}{m}A^{-1}\right] \\ P(z) \simeq P_0\left[1 + \frac{V}{m}A^{-1}\right] \end{cases} \Rightarrow \bar{\mathcal{E}} \Rightarrow \kappa^{-1},\ m^*$$

$$A \equiv \frac{\partial^2 \mathcal{E}_0}{\partial n^2}\frac{\partial^2 \mathcal{E}_0}{\partial k^2} - \left(\frac{\partial^2 \mathcal{E}_0}{\partial n \partial k}\right)^2 = \frac{2}{3}\alpha\frac{n_0^{2/3}}{m} - \frac{k_0^2}{m^2}$$

$$\alpha \equiv (1+\beta)\frac{\hbar^2}{2m}(3\pi^2)^{2/3}$$

Results: incompressibility κ^{-1}

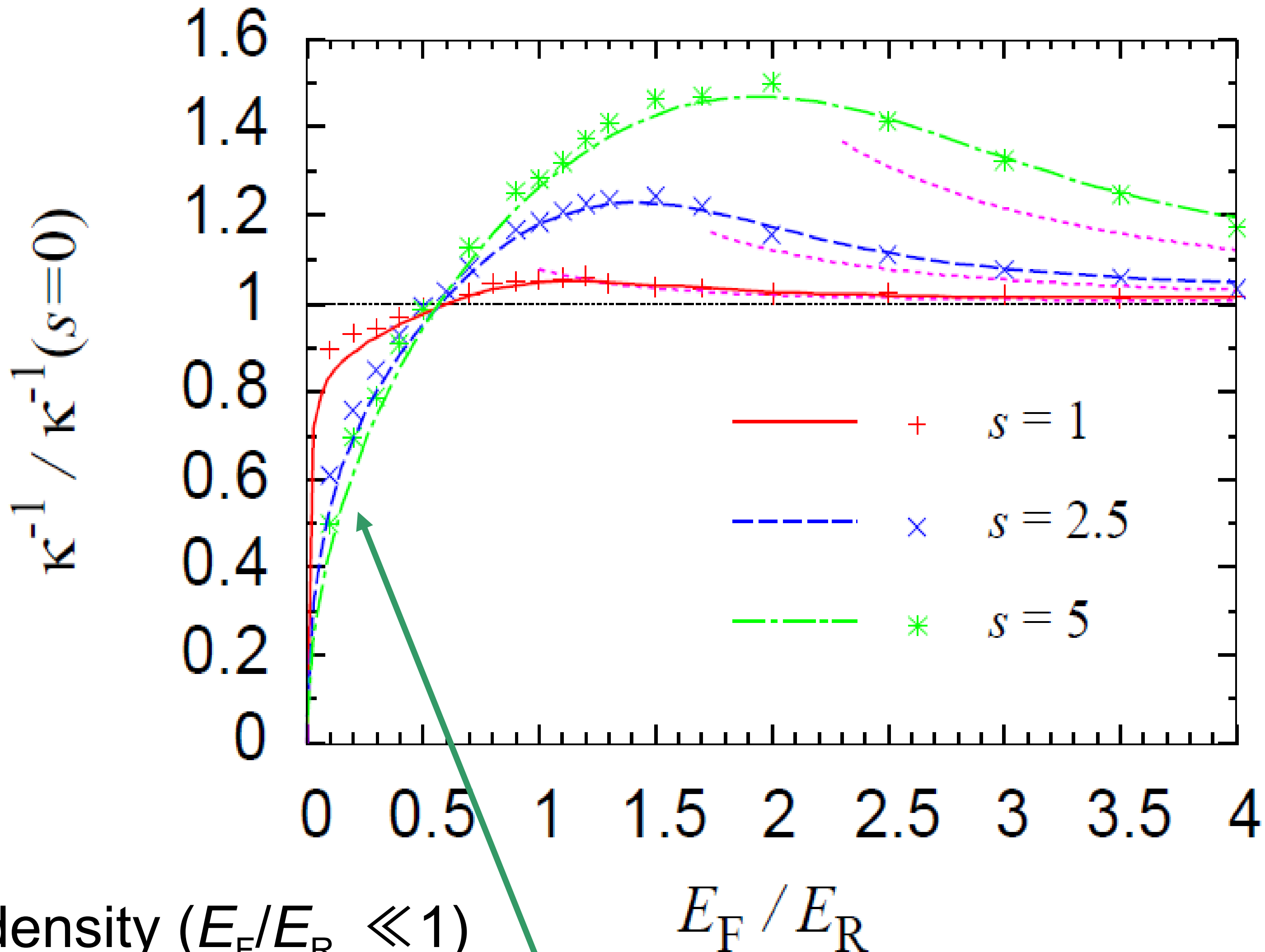


lines: Bulgac & Yu
points: standard

$\kappa^{-1}/\kappa^{-1}(s=0)$ is max. at $E_F/E_R \sim 1$

cf. bosons： sys. becomes uniform and $\kappa^{-1}/\kappa^{-1}(s=0)$ decreases with $gn/E_R\uparrow$.

Results: incompressibility κ^{-1}

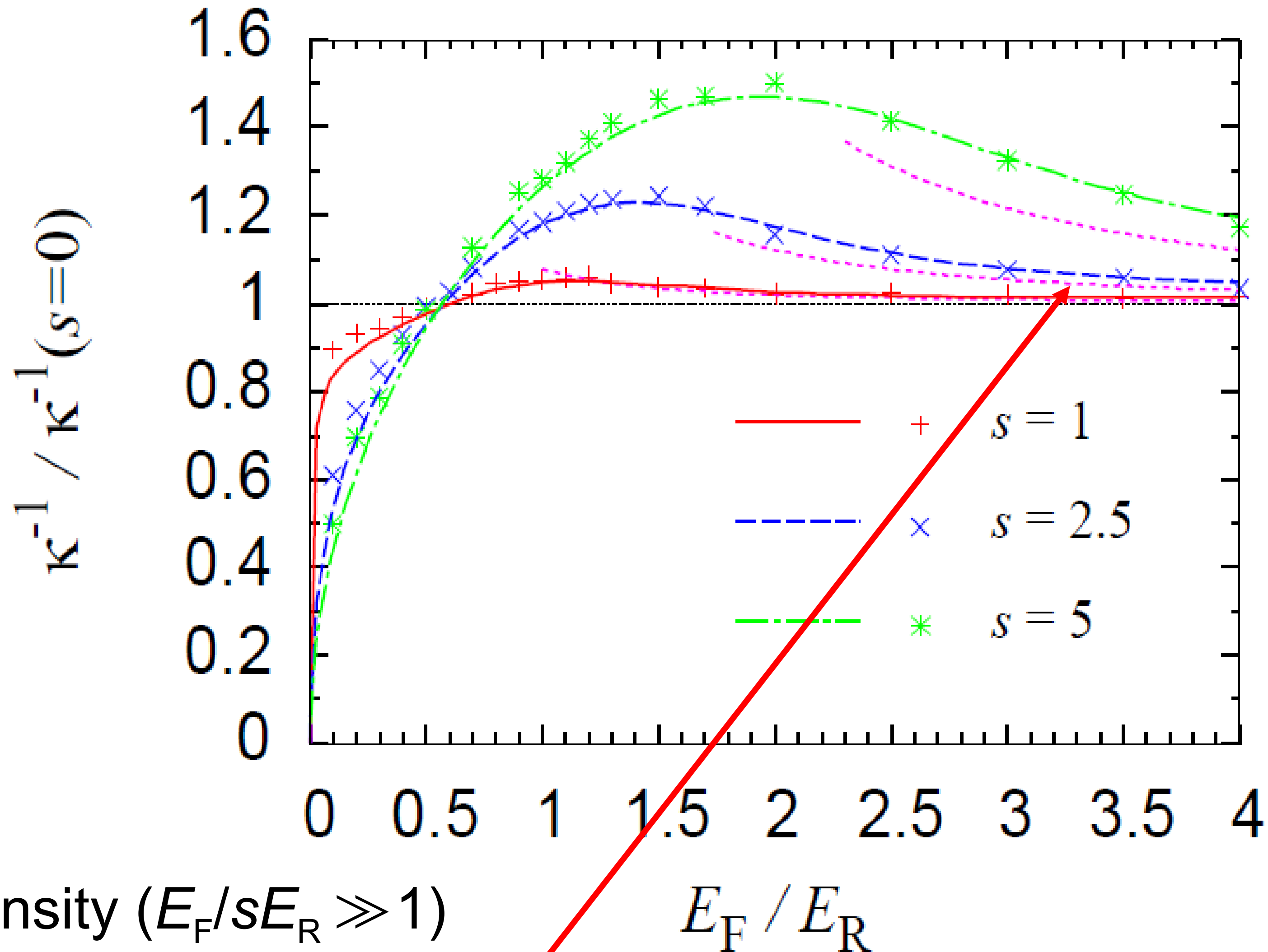


Low density ($E_F/E_R \ll 1$)

Form a bound st. by lattice and behave as bosons ($\kappa^{-1} \propto n$)

$$\kappa^{-1}/\kappa^{-1}(s=0) \propto n/n^{2/3} \propto n^{1/3} \to 0$$

Results: incompressibility κ^{-1}

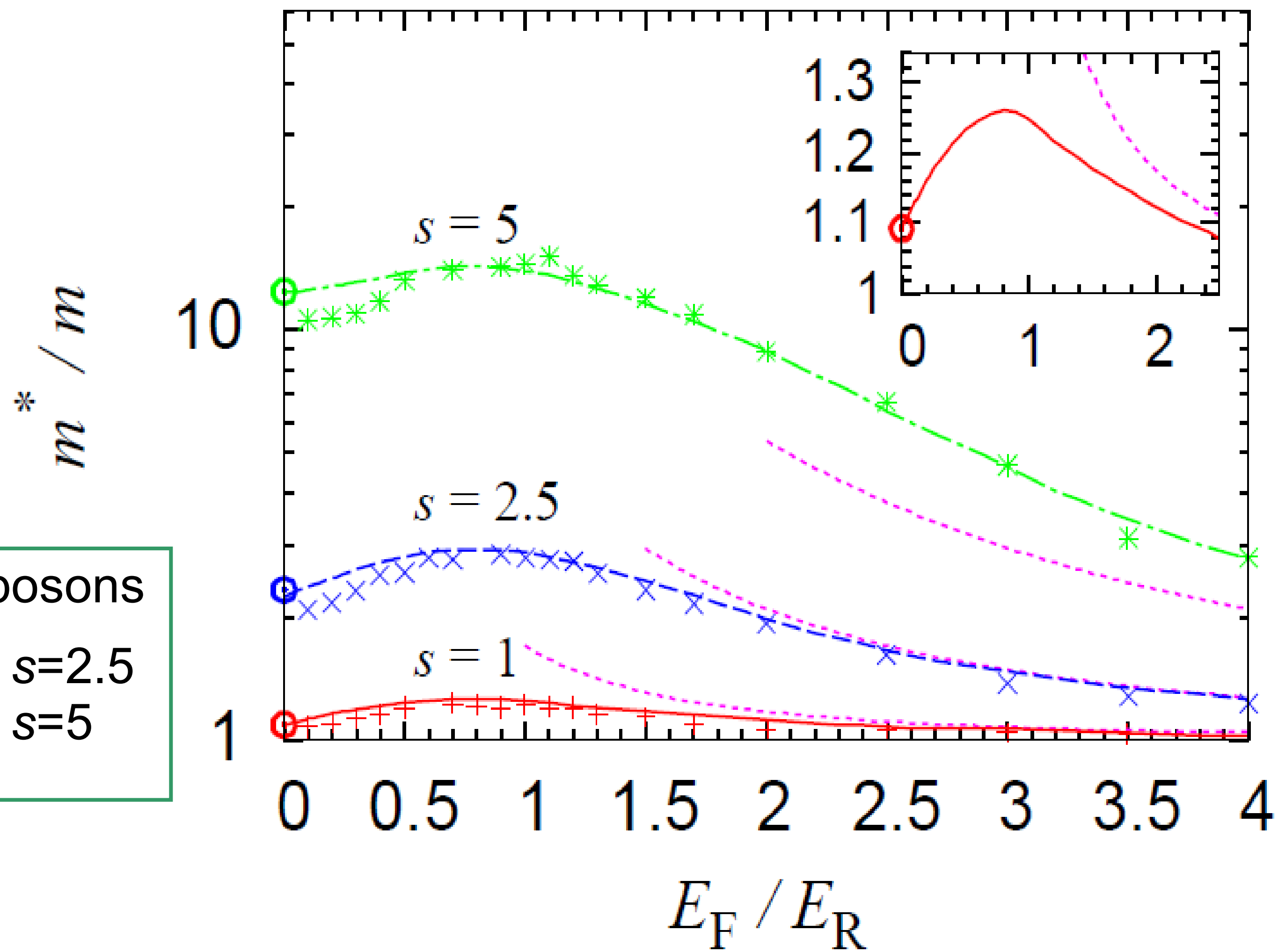


High density ($E_F/sE_R \gg 1$)

System becomes uniform. Lattice is a perturbation.

Hydro. theory ➡ $\kappa^{-1}/\kappa^{-1}(s=0) \simeq 1+\frac{1}{32}\frac{1}{(1+\beta)^2}\left(\frac{sE_R}{E_F}\right)^2+O\left[(sE_R/E_F)^4\right]$

Results: Effective mass m*



m^*/m for bosons

$\simeq$ 1.2 for s=2.5

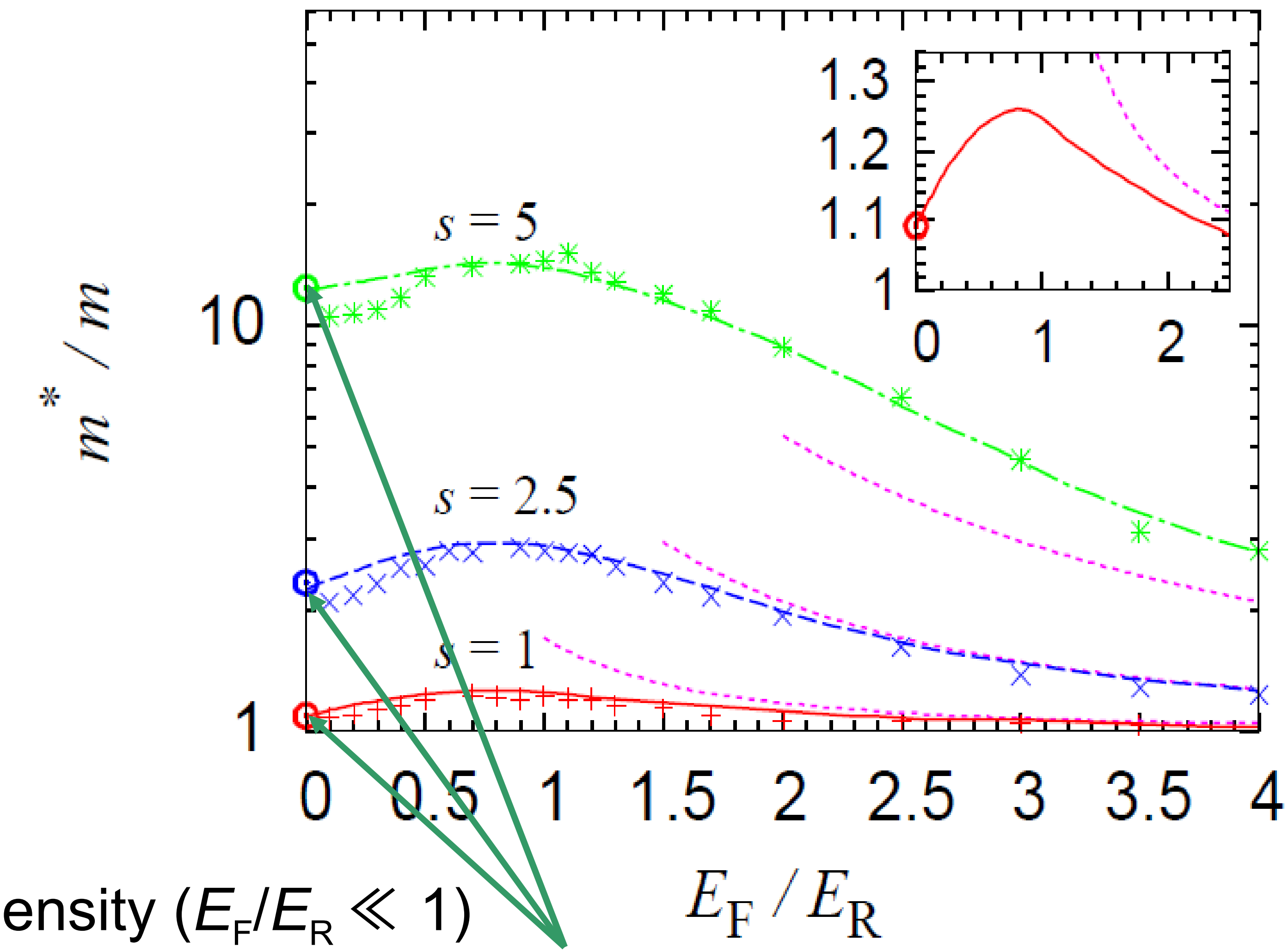
$\simeq$ 1.9 for s=5

Increase of m^* by lattice : more drastic than boson case ⬅ form. of molecules

m^*/m is max. at $E_F/E_R \sim 1$ ⬅ bandgap effect

boson case : wave func. spreads as gn/E_R↑ and tunnel. favored ➡ m^*/m decreases

Results: Effective mass m*

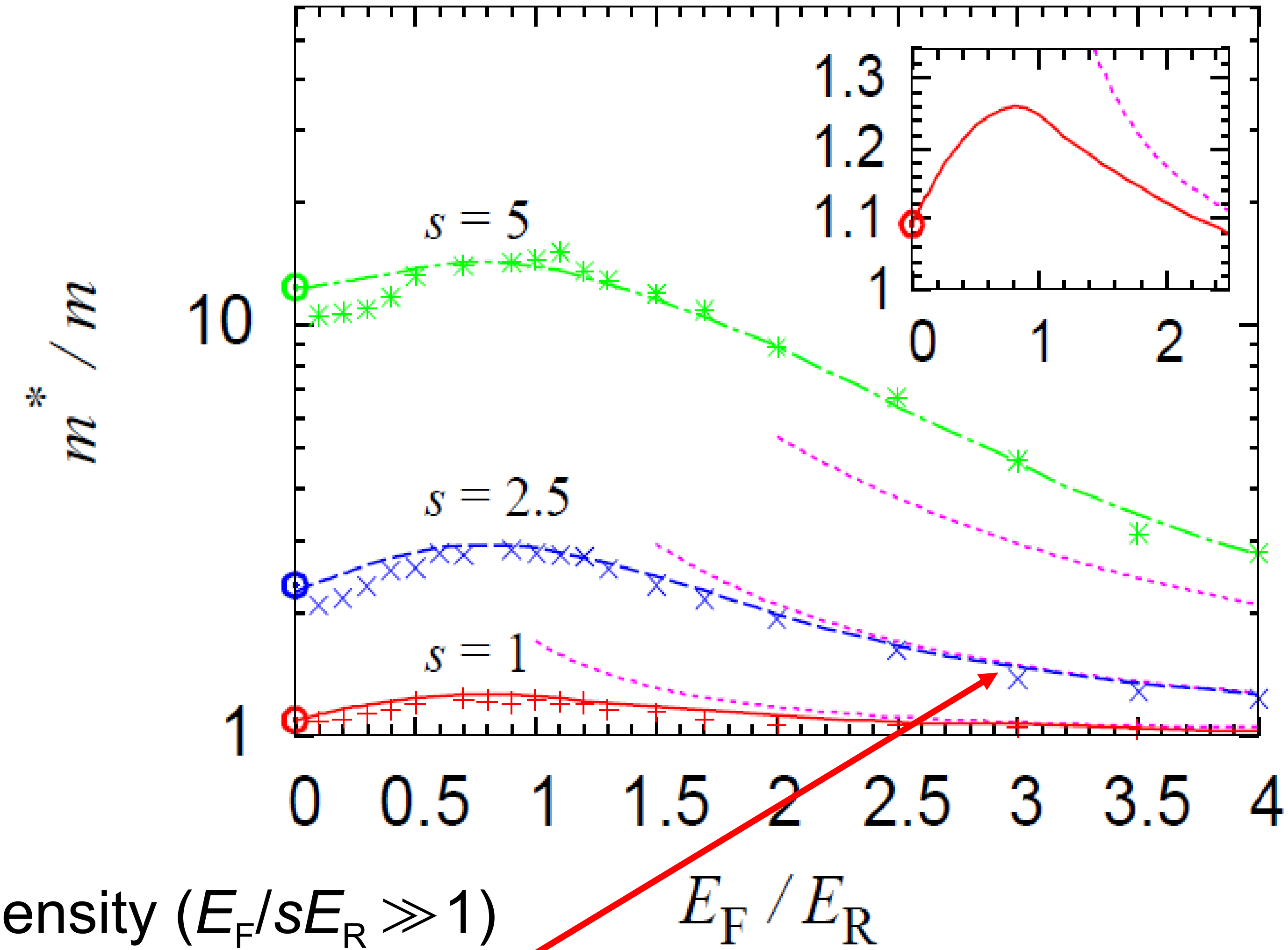


Low density ($E_F/E_R \ll 1$)

Converges to a 2-body result [Orso et al. (2005)].

（BdG eqs. correctly describe intramolecular correlations.）

Results: Effective mass m*



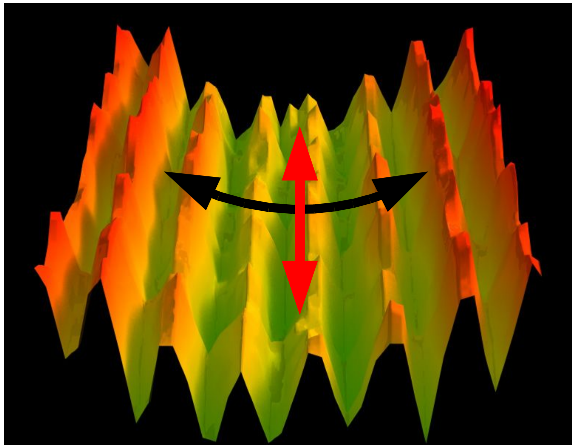
High density ($E_F/sE_R \gg 1$)

hydro. theory ➡ $m^*/m \simeq 1 + \frac{9}{32}\frac{1}{(1+\beta)^2}\left(\frac{sE_R}{E_F}\right)^2 + O\left[(sE_R/E_F)^4\right]$

Consequences

Sound velocity $c_s = \sqrt{\kappa^{-1}/m^*}$

Collective osc. in lattice + trap system



(MIT Group)

Axial dipole mode: probe of m^*

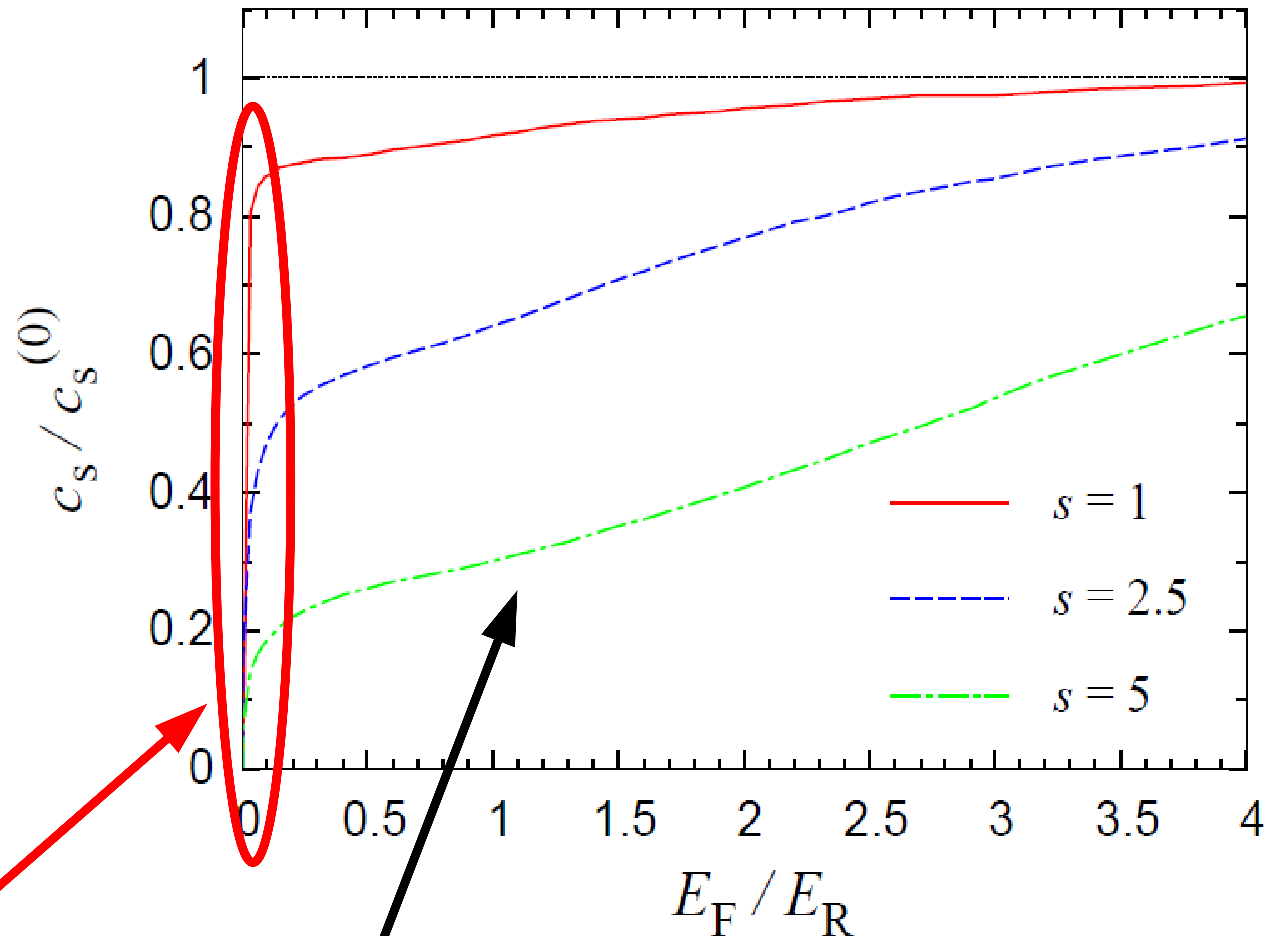
Radial breathing mode: probe of κ^{-1}

Coarse-grained density profile

EOS $\mu(n)$

Sound velocity c_s

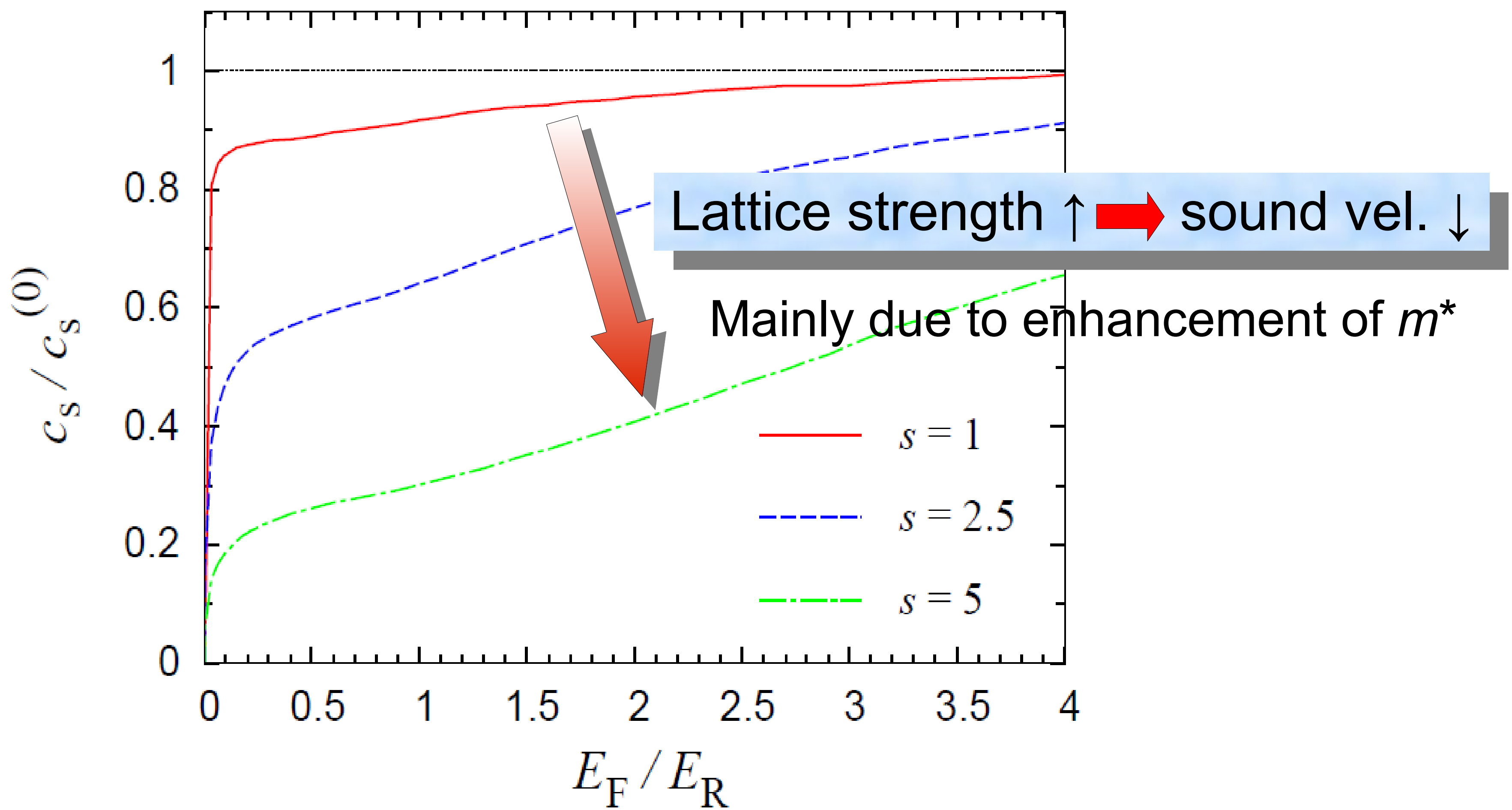
$$c_s = \sqrt{\kappa^{-1}/m^*}$$



- Significant reduction of the sound vel. by lattice ⬅ Due to κ^{-1}
- Smooth density dep. except for the low dens. limit ➡ Propagation of sound wave

Sound velocity c_s

$$c_s = \sqrt{\kappa^{-1}/m^*}$$



- Significant reduction of the sound vel. by lattice ⬅ Due to κ^{-1}
- Smooth density dep. except for the low dens. limit ➡ Propagation of sound wave

Collective modes (1)

harmonic trap + lattice system

$$V(\mathbf{r}) = V_{\rm ho}(\mathbf{r}) + V_{\rm lat}(z); \quad V_{\rm ho}(\mathbf{r}) = \frac{m}{2}(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2)$$

Dipole mode in z-direction : probe of m*

Sum rule → $$\omega_D = \omega_z \sqrt{\frac{m}{\overline{m^*}}}$$

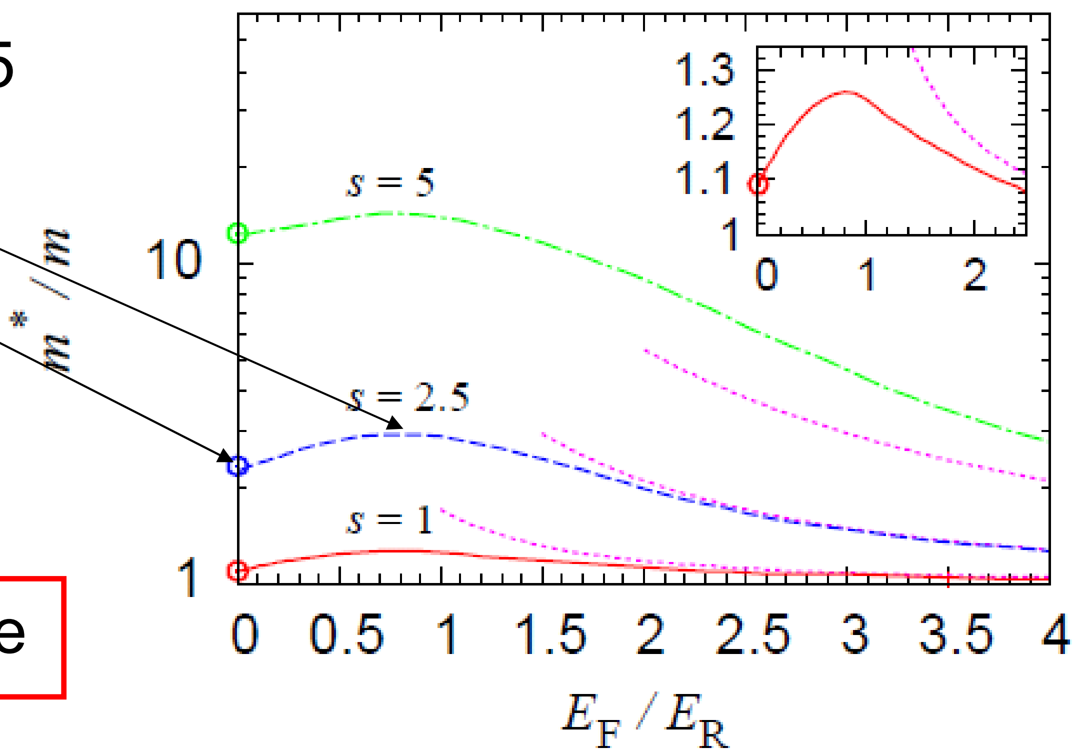
$$\frac{1}{\overline{m^*}} = \frac{1}{N}\int d\mathbf{r}\ \frac{n(\mathbf{r})}{m^*[n(\mathbf{r})]}$$

For central density of $E_F/E_R \lesssim 1.5$

lower bound of ω_D

upper bound of ω_D

$$\omega_D/\omega_z \simeq \begin{cases} 0.89 - 0.96 & (s = 1) \\ 0.59 - 0.66 & (s = 2.5) \\ 0.26 - 0.29 & (s = 5) \end{cases}$$



ω_D decreases drastically by lattice

Collective modes (2)

radial breathing mode： probe of κ^{-1}

cigar shape trap: $\omega_z \ll \omega_\perp$

Without lattice

$$\omega = \sqrt{10/3}\,\omega_\perp \simeq 1.83\omega_\perp$$

lattice strength $s \nearrow$

Linear density dep. of μ (boson-like)

$$\omega = 2\omega_\perp$$

Boson-like behaviors can be observed by radial breathing mode

Density profile (1)

Trap the sys. by a harmonic trap $V_{\text{ho}}(\mathbf{r})$

$$V(\mathbf{r}) = V_{\text{ho}}(\mathbf{r}) + V_{\text{lat}}(z); \quad V_{\text{ho}}(\mathbf{r}) = \frac{m}{2}(\omega_\perp^2 r_\perp^2 + \omega_z^2 z^2)$$

Using the local chemical pot. μ(n) by the BdG eqs.

LDA: $\mu_0 = \mu[n(\mathbf{r})] + V_{\text{ext}}(\mathbf{r})$ with $N = \int d^3r \; n(\mathbf{r})$

⇨ Calculate the coarse-grained density profile $n(\mathbf{r})$

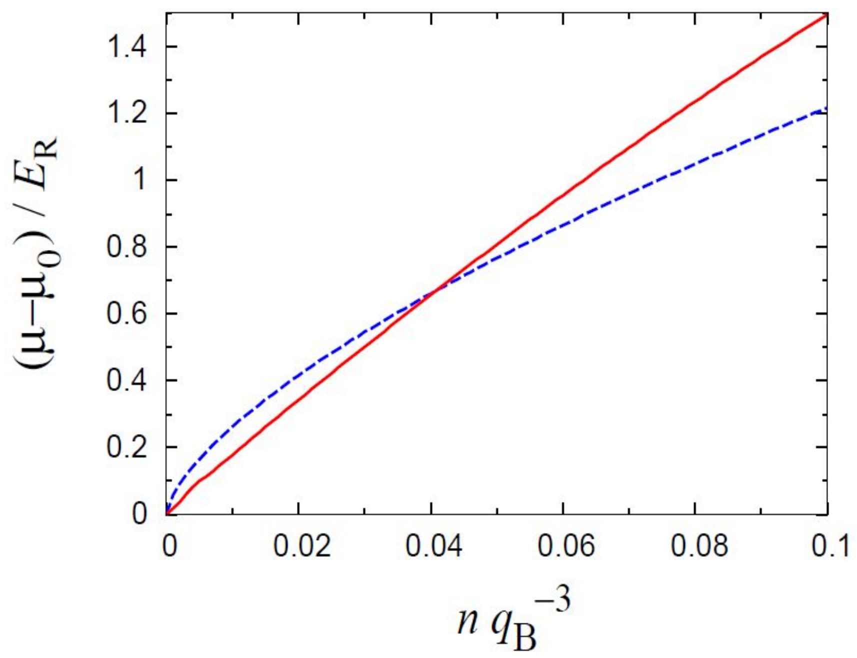
Parameters in the MIT exp.

$$N = 10^6, \; \omega_\perp = \omega_z, \; \omega_z / E_{\text{R}} = 0.01$$

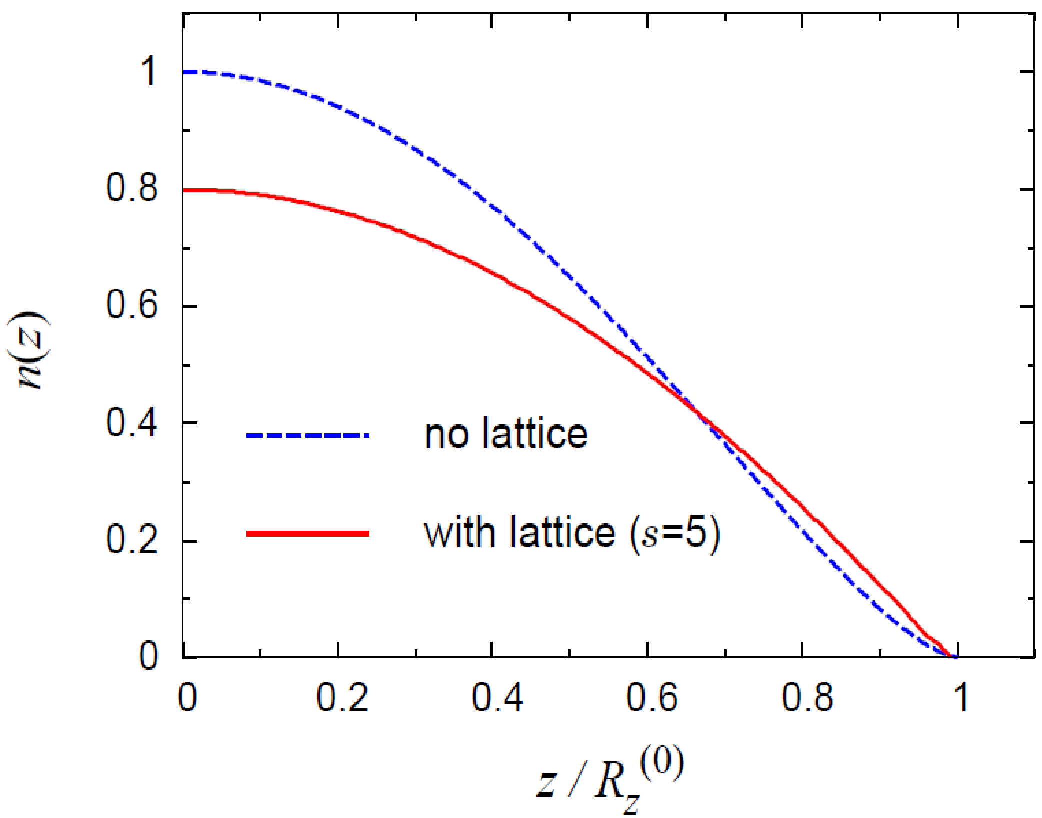
Density profile (2)

For s=5

Chemical potential



Density profile



Linear density dep. of the chemical pot.
due to the formation of molecules and 2D-like behavior by lattice,

Fermionic TF profile

$$n \propto \left(1 - r^2/R^2\right)^{3/2}$$

Inverted parabola

$$n \propto \left(1 - r^2/R^2\right)$$

Take-home messages

Unitary Fermi gas in a1D optical lattice (T=0)

$$V_{\text{lat}}(z) = sE_{\text{R}} \sin^2 q_{\text{B}} z$$

Effects of lattice in low density regime: $E_F \ll E_R$

Formation of molecules (bosons)

(Quasi)-2 dimensionality

→ Linear density dep. of the chemical pot. $\mu \propto n$

- Reduction of the incompressibility κ⁻¹

$$\kappa^{-1} \propto n^{2/3} \quad \Rightarrow \quad \kappa^{-1} \propto n$$

- Drastic increase of the effective mass *m**
- These effects can be probed by collective modes.
- Drastic change of the coarse-grained density profile

fermionic TF profile → inverted parabola

$$n \propto \left(1 - r^2/R^2\right)^{3/2} \quad \Rightarrow \quad n \propto \left(1 - r^2/R^2\right)$$

Critical velocity of superflow in a 1D lattice

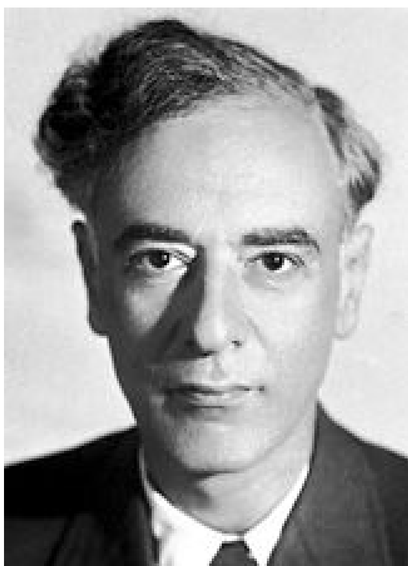
GW, Dalfovo, Piazza, Pitaevskii & Stringari
PRA **80**, 053602 (2009).

Landau instability (1)

Superflow in a vessel

L. D. Landau (1941)

High velocity ⇒ Kin. energy of flow goes to excitations



Kin. energy of superflow can be dissipated via creation of excitations at $v > v_c$

Energetic instability of superflow

Galilei transf.: $E' = E_0 + \epsilon(p) - \boldsymbol{p}\cdot\boldsymbol{v} + \frac{1}{2}Mv^2$

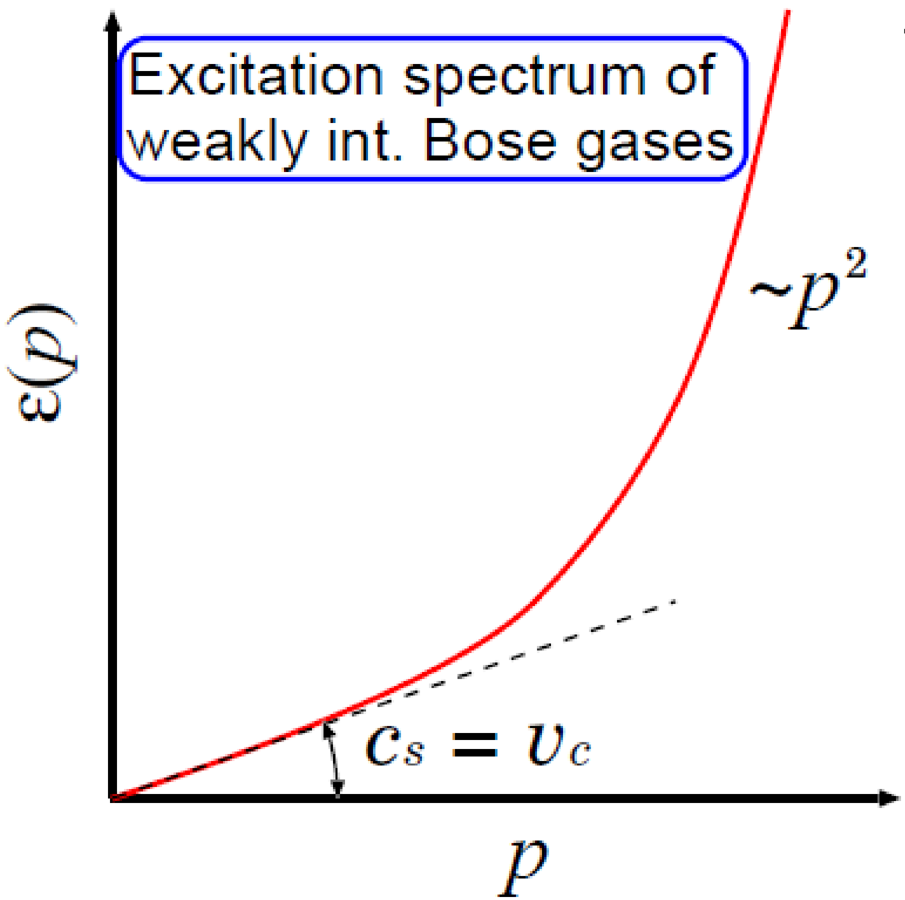
Landau criterion for uniform sys.

critical velocity $$v_c = \min_{\mathbf{p}} \frac{\epsilon(\mathbf{p})}{p}$$

$\varepsilon(\boldsymbol{p})$: energy of excitations

Landau instability (2)

Landau criterion: $$v_c = \min_{\mathbf{p}} \frac{\epsilon(\mathbf{p})}{p}$$



Weakly int. Bose gas

Bogoliubov spectrum

$$\epsilon(p) = \sqrt{\frac{\hbar^2 p^2}{2m}\left(\frac{\hbar^2 p^2}{2m} + 2gn\right)}$$

$$v_c \Rightarrow \text{sound vel. } c_s$$

$$c_s = \sqrt{gn/m}$$

Motivation: Theoretical prediction

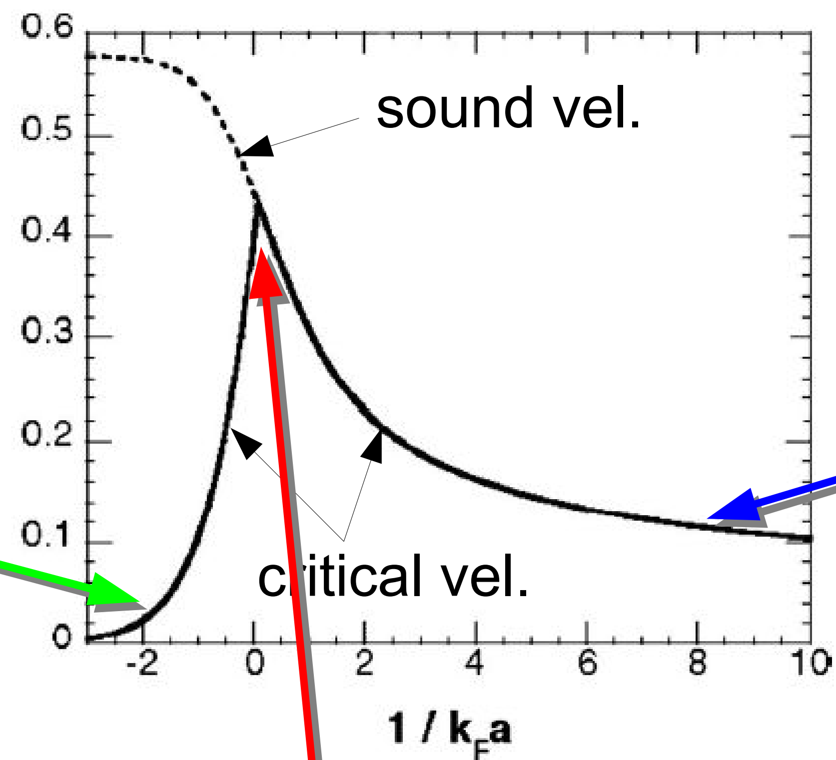
Combescot, Kagan & Stringari, PRA **74**, 042717 (2006).

Critical velocity of uniform Fermi superfluids.

BCS limit

Fermi gas with infinitesimally small gap

v_c is determined by pair breaking.



BEC limit

Weakly int. bosonic mol.

v_c is given by sound vel.

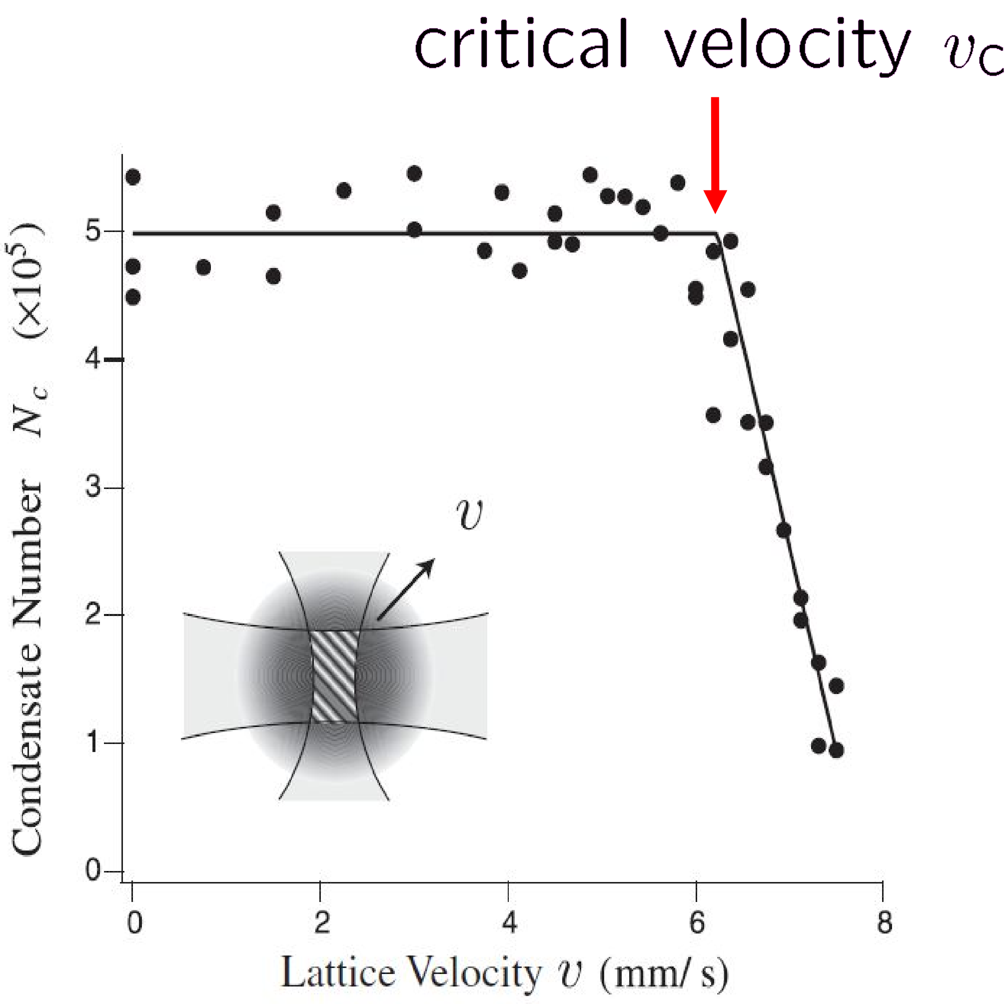
Unitary regime

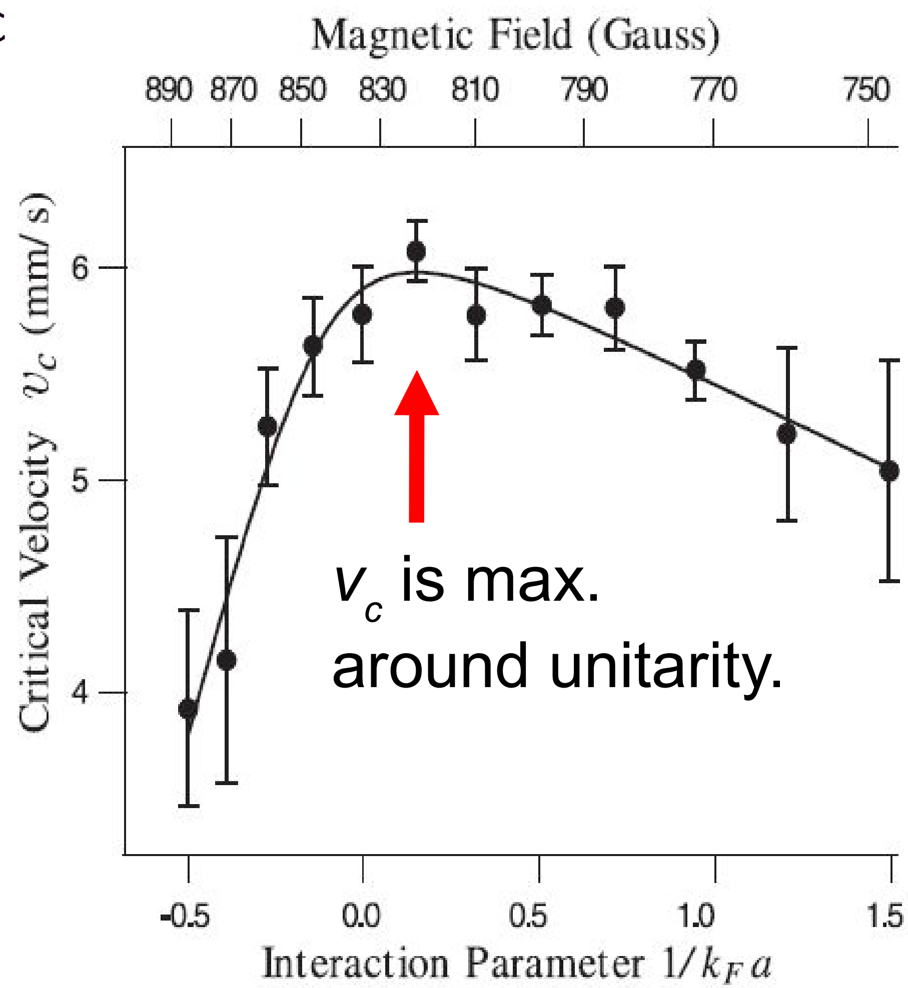
v_c shows maximum.

Motivation: Experiment @ MIT

Miller et al., PRL **99**, 070402 (2007)

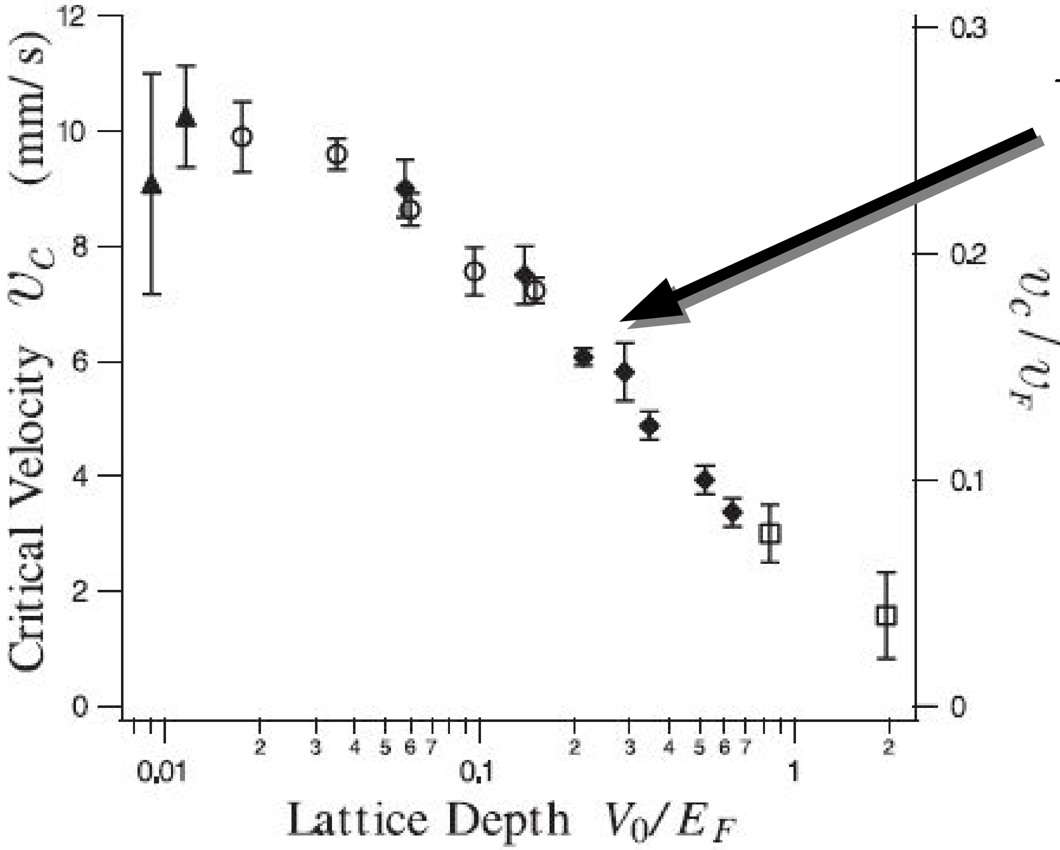
Critical velocity of Fermi gases in a lattice





Motivation: Experiment @ MIT

At unitarity



$V_0 \lesssim E_F$

v_c reduces sensitively with V_0

$$c_s^{(0)} = \frac{(1+\beta)^{1/2}}{\sqrt{3}} v_F \simeq 0.37 v_F$$

$V_0 \rightarrow 0$ limit

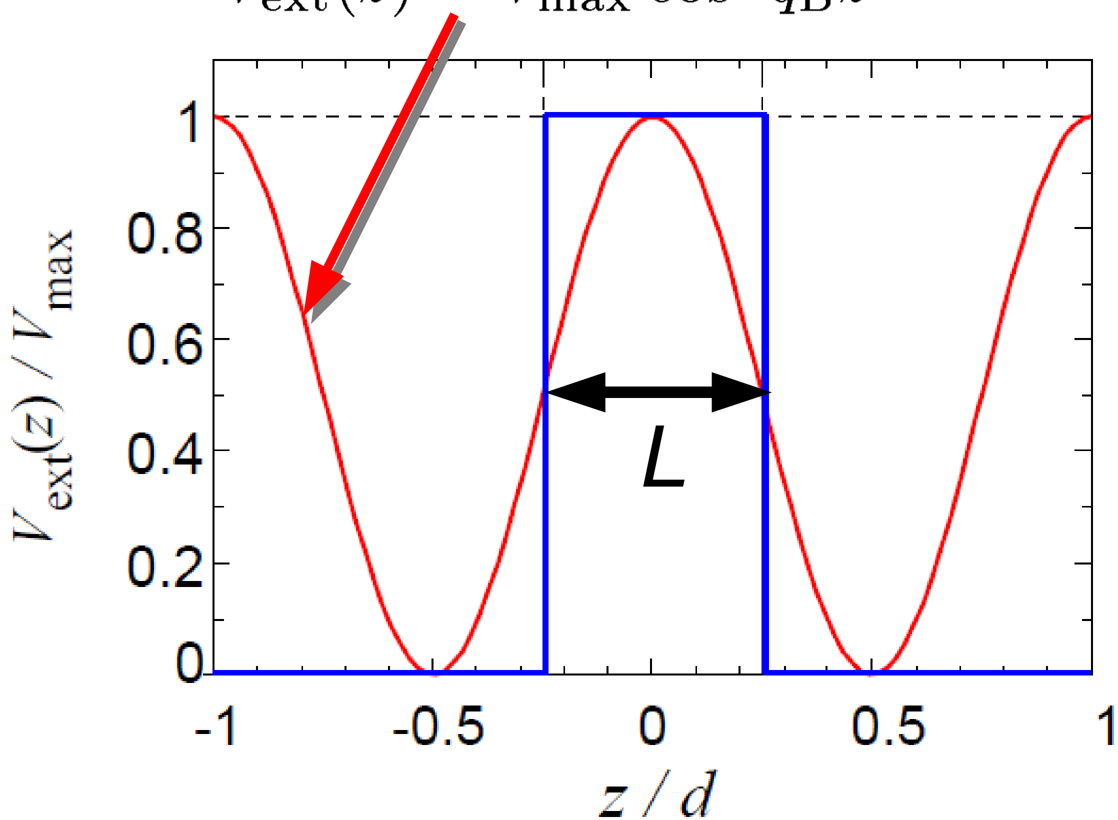
Reproduces the Landau criterion within ~20-30% uncertainty.

Setups

Superfluids at T=0

4 cases:
- Statistics: bosons / fermions
  (BEC) (unitary Fermi gas)
- Potential: periodic / single barrier

$$V_{\rm ext}(z) = V_{\rm max}\cos^2 q_{\rm B} z$$



BEC in a single barrier
Mamaladze & Cheishvili (1966);
Hakim (1997); Leboeuf et al. (2003), etc.

BEC in a lattice
Wu & Niu (2001); Machholm et al. (2003);
Modugno et al. (2004), etc.

Unitary Fermi gas in a single barrier
Spuntarelli, Pieri & Strinati (2007).

Fermi gases in the tight-binding regime
Burkov & Paramekanti (2008).

Length scale of the barrier width *L*:

- Width (single barreir)
- FWHM (periodic pot.)

Critical velocity in the LDA hydrodynamics

Energy density within LDA: $$e(n,P) = \frac{P^2}{2m}n + e(n,0)$$

EOS: $$\mu(n) = \frac{\partial}{\partial n}e(n,0) = \alpha n^\gamma$$

For unitary fermions

$\gamma = 2/3$

$\alpha = (1+\beta)(3\pi^2)^{2/3}\hbar^2/2m$

Local sound vel.: $$mc_s^2(z) = n\frac{\partial}{\partial n}\mu(n) = \gamma\mu(n)$$

Current density $j = n(z)\, v(z)$: const.

Within LDA: $n(z)$ is smallest at pot. maxima z_0.

⇒ $v(z)$ is largest at z_0.

$$v(z_0) = c_s(z_0)$$ ⇒ Critical velocity v_c

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}}\left[\frac{2\mu_{j_c}}{2+\gamma}\left(1-\frac{V_{\max}}{\mu_{j_c}}\right)\right]^{(2/\gamma)+1}$$

⇐ Determined only by V_{max}

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For unitary fermions
$\gamma = 2/3$
$\alpha = (1+\beta)(3\pi^2)^{2/3}\hbar^2/2m$

Applies for fermions & bosons in *any* potential.

Within LDA: $n(z)$ is smallest at pot. maxima z_0.

Current density $j = n(z)\, v(z)$: const.

→ $v(z)$ is largest at z_0.

$$v(z_0) = c_s(z_0)$$ → Critical velocity v_c

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Hydrodynamic analysis for excitations

Energetic (Landau) inst. : long-wavelength excitations give smallest v_c

Hydro. analysis for excitations is appropriate.

Dispersion rel. of excitations (long-wavelength phonons)

$$\omega = \frac{\partial^2 e}{\partial n_0 \partial P} q + \sqrt{\frac{\partial^2 e}{\partial n_0^2} \frac{\partial^2 e}{\partial P^2}} |q|$$

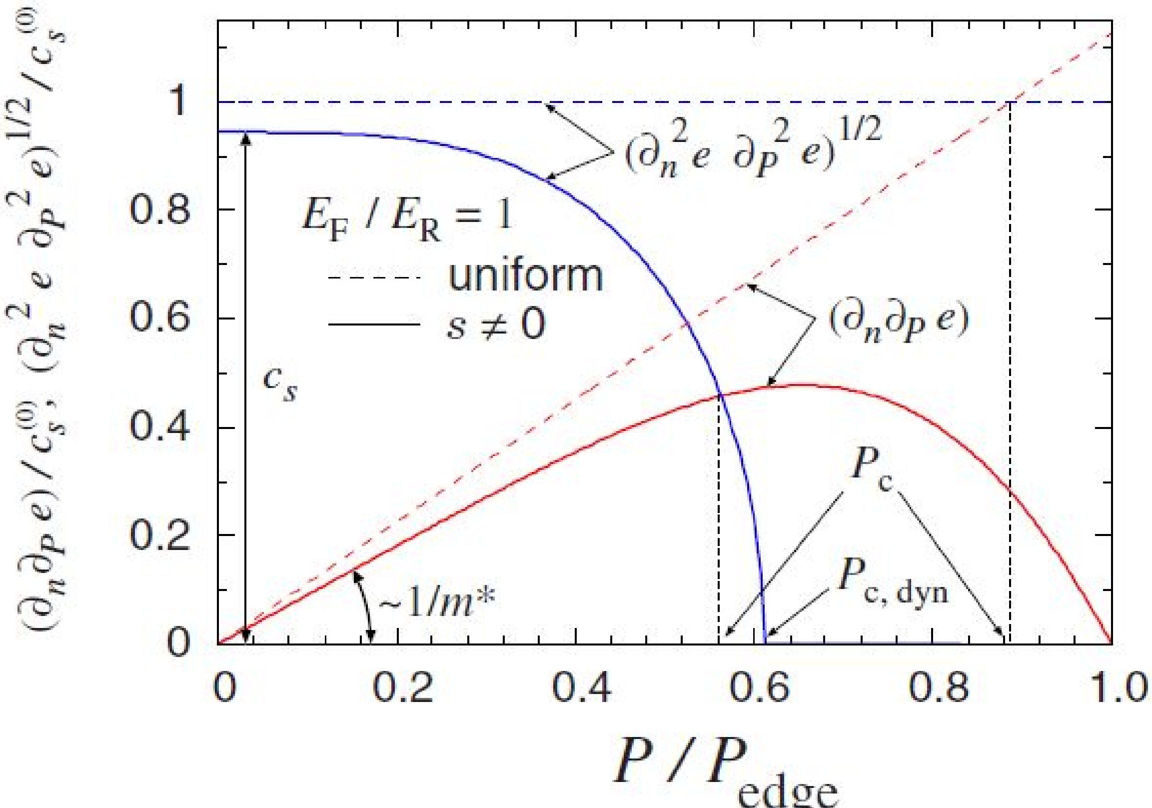
(n_0 : avr. density)

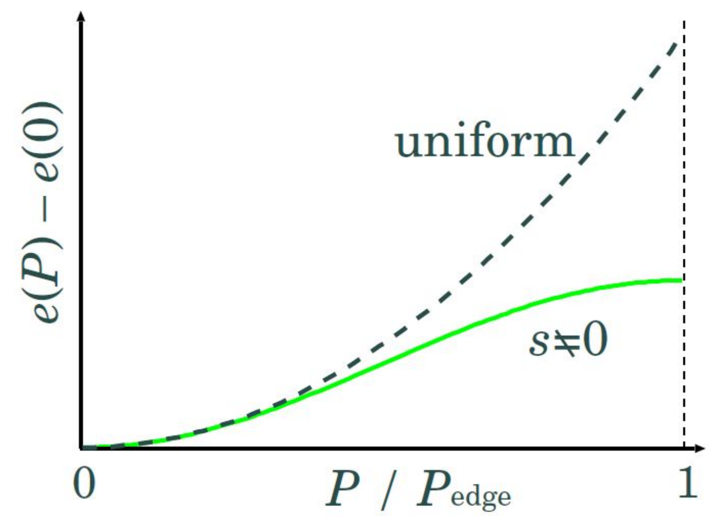
$$\frac{\partial^2 e}{\partial n_0 \partial P} = \sqrt{\frac{\partial^2 e}{\partial n_0^2} \frac{\partial^2 e}{\partial P^2}}$$ → Energetic inst.

[$\partial_n^2 e < 0$ or $\partial_P^2 e < 0$ → Dynamical inst.]

Critical velocity for the energetic instability (1)

$$\frac{\partial^2 e}{\partial n_0 \partial P} = \sqrt{\frac{\partial^2 e}{\partial n_0^2}\frac{\partial^2 e}{\partial P^2}}$$ → Energetic inst.





Brillouin zone edge
$P_{edge} = 0.5 q_B$

$s \to 0$: $P_c/m = v_c = c_s^{(0)}$: Landau criterion

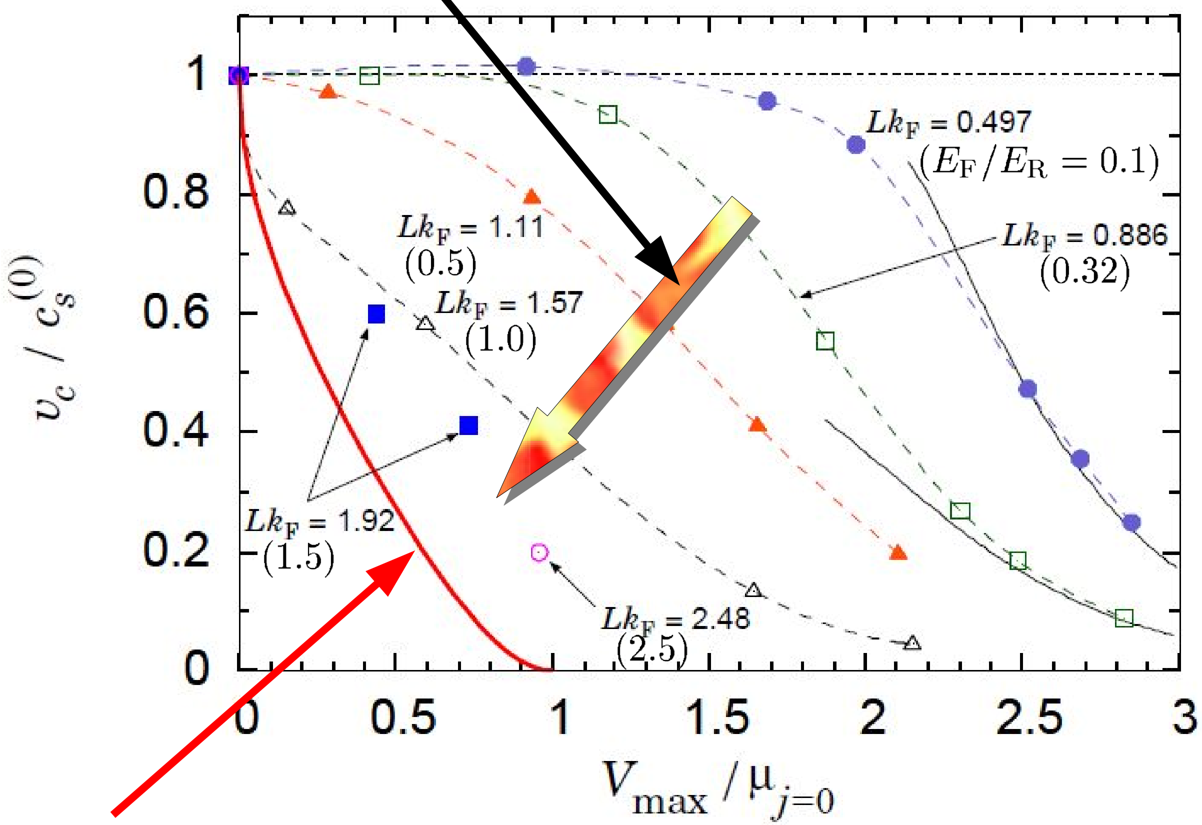
$s \to \infty$: $c_s \propto 1/\sqrt{m^*}, 1/\sqrt{m^*} \to 0$ → $P_c = P_{c,\ dyn} = \hbar q_B/2$

$e(n,P)$: cos-dep. on P

Critical velocity for the energetic instability (2)

$$k_F d \nearrow \quad \Rightarrow \quad v_c/c_s^{(0)} \searrow$$

sys. becomes more sensitive to the lattice.



s.f. hydro. theory within LDA (valid for $d/\xi \sim k_F d$ »1)

flow vel. = local sound vel. at pot. max.

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}} \left[\frac{2\mu_{j_c}}{2+\gamma} \left(1 - \frac{V_{max}}{\mu_{j_c}} \right) \right]^{\frac{2}{\gamma}+1}$$

$$\mu(n) = (1+\beta)E_F \equiv \alpha n^\gamma$$

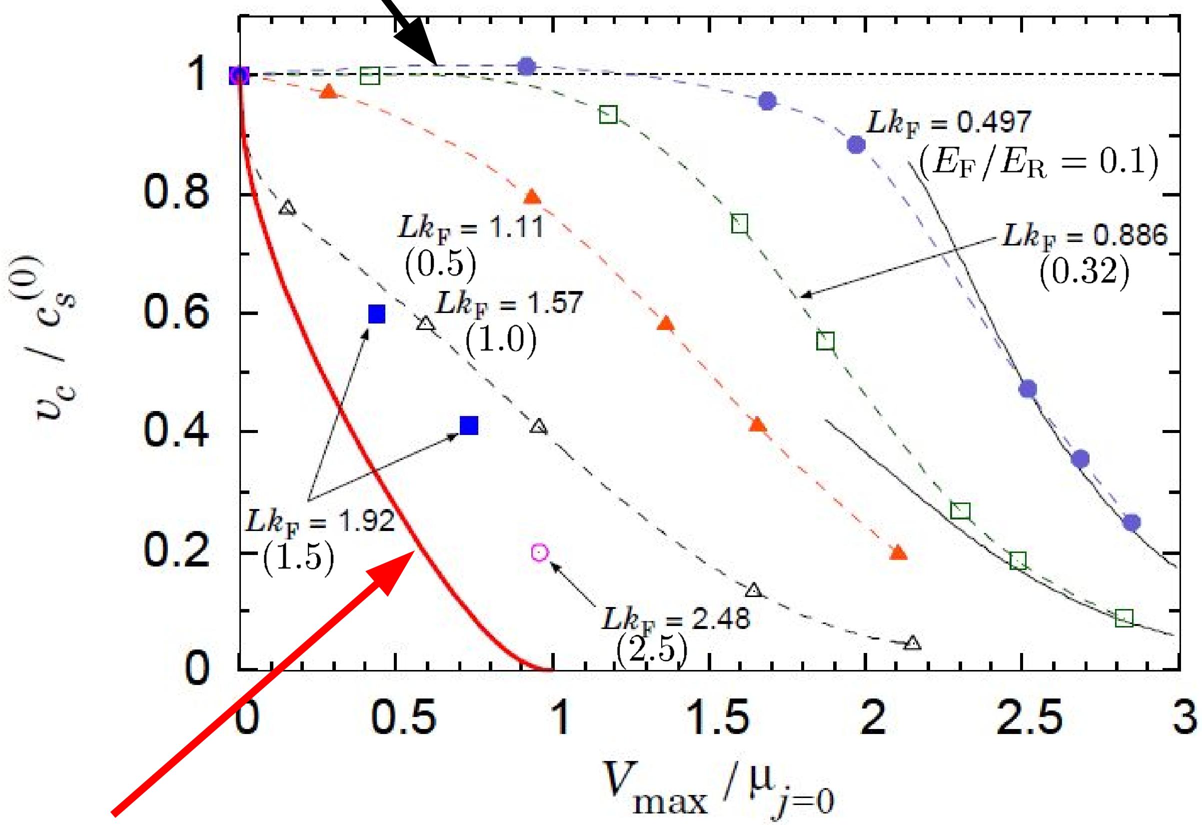
(universal for lattice and single barrier)

Critical velocity for the energetic instability (3)

Insensitive to V_{max} : δn is forced to be periodic with d.

➡ Hard to excite if $k_F d \ll 1$.

(this insensitivity is absent in single barrier case)



s.f. hydro. theory within LDA (valid for $d/\xi \sim k_F d \gg 1$)

flow vel. = local sound vel. at pot. max.

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}} \left[\frac{2\mu_{j_c}}{2+\gamma} \left(1 - \frac{V_{max}}{\mu_{j_c}} \right) \right]^{\frac{2}{\gamma}+1}$$

$$\mu(n) = (1+\beta)E_F \equiv \alpha n^\gamma$$

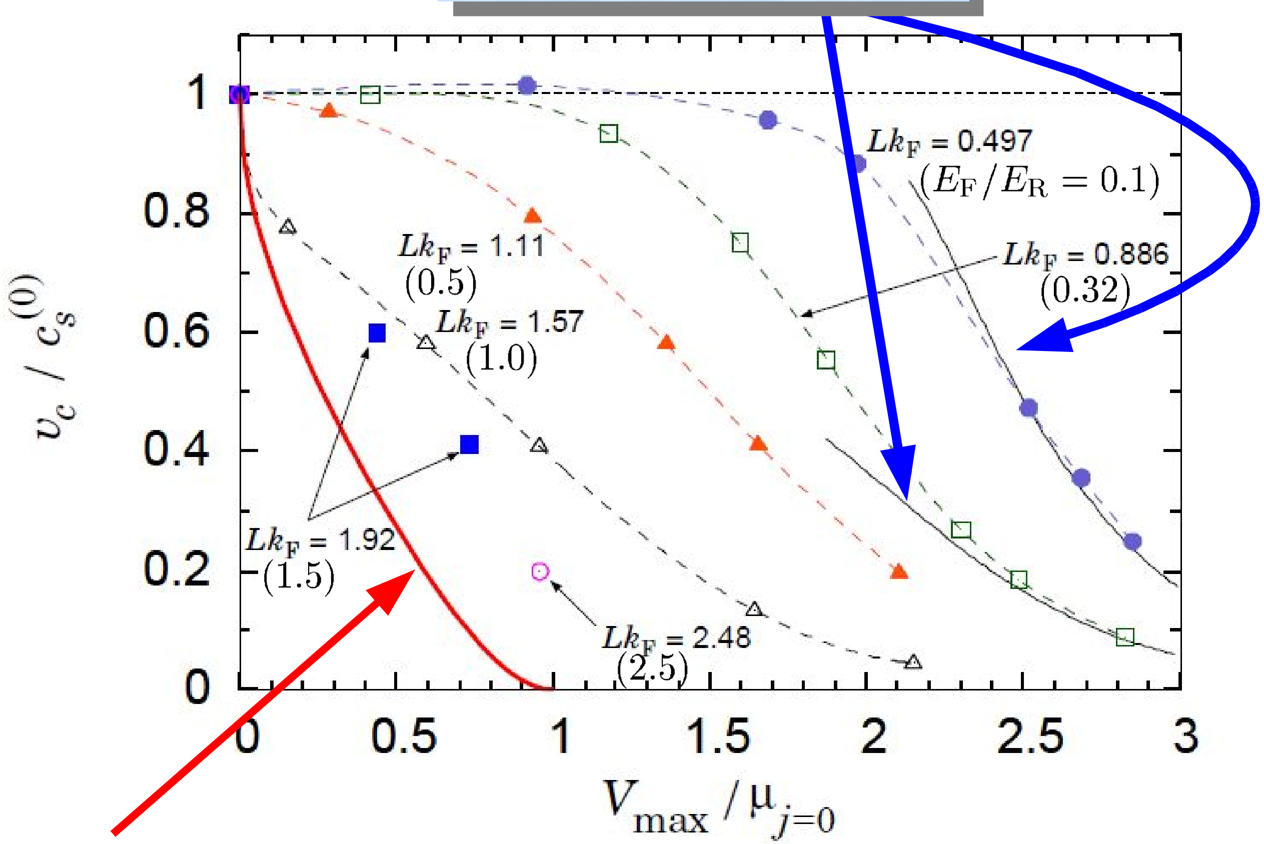
(universal for lattice and single barrier)

Critical velocity for the energetic instability (4)

For large *s* : cos band $e(n_0, P) = e(n_0, 0) + \delta_J[1 - \cos(\pi P/P_{\rm edge})]$

Band width δ_J is determined by *m** : $\delta_J = n_0 P_{\rm edge}^2/\pi^2 m^*$

$$v_c = P_{\rm edge}/\pi m^*$$



s.f. hydro. theory within LDA (valid for *d/ξ* ~ $k_F d$ »1)

flow vel. = local sound vel. at pot. max.

$$j_c^2 = \frac{\gamma}{m\alpha^{2/\gamma}}\left[\frac{2\mu_{j_c}}{2+\gamma}\left(1 - \frac{V_{\rm max}}{\mu_{j_c}}\right)\right]^{\frac{2}{\gamma}+1}$$

$\mu(n) = (1+\beta)E_F \equiv \alpha n^\gamma$

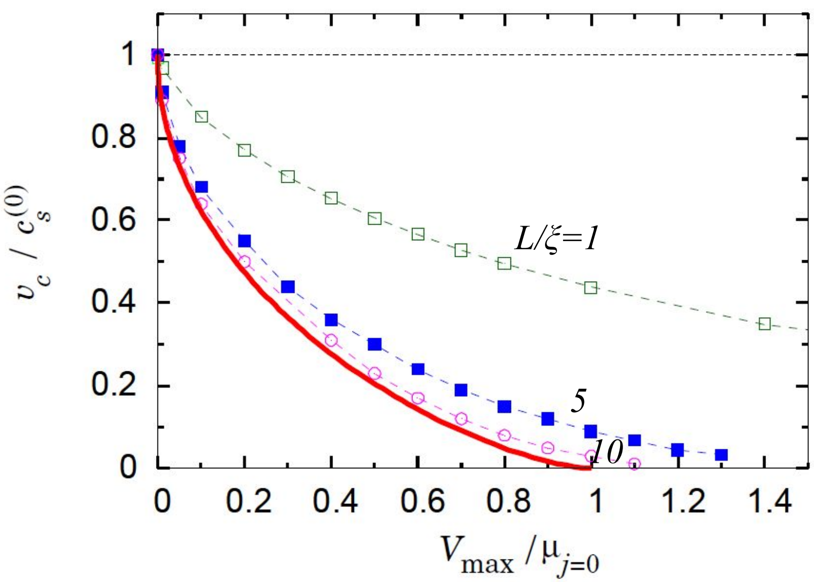
(universal for lattice and single barrier)

Lattice vs single barrier

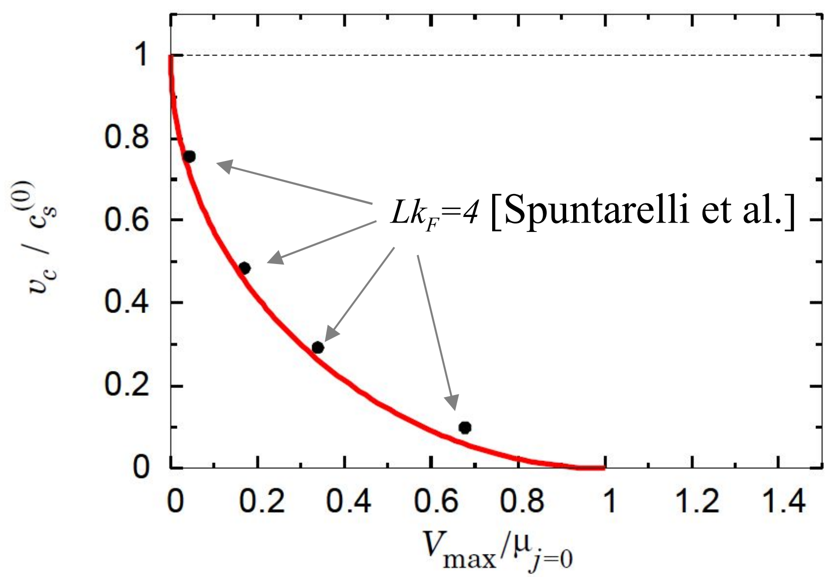
Bosons

Fermions (@ unitarity)

Bosons through a barrier



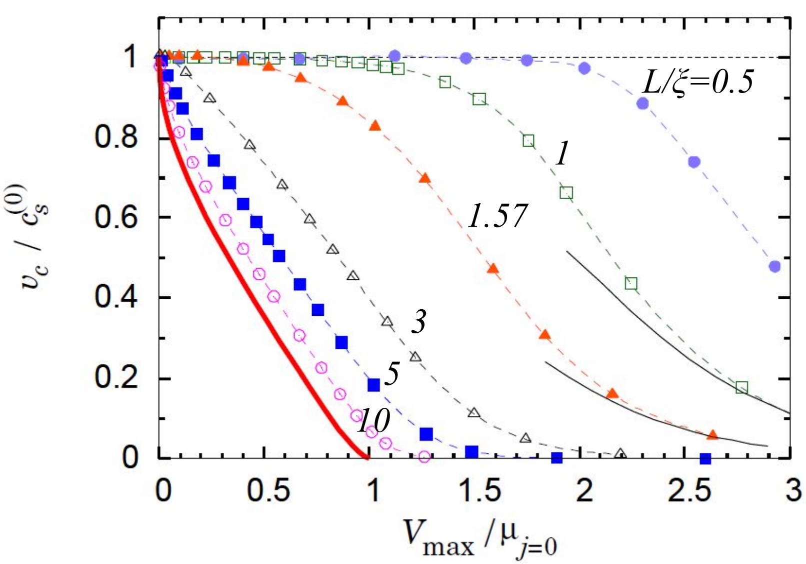
Fermions through a barrier



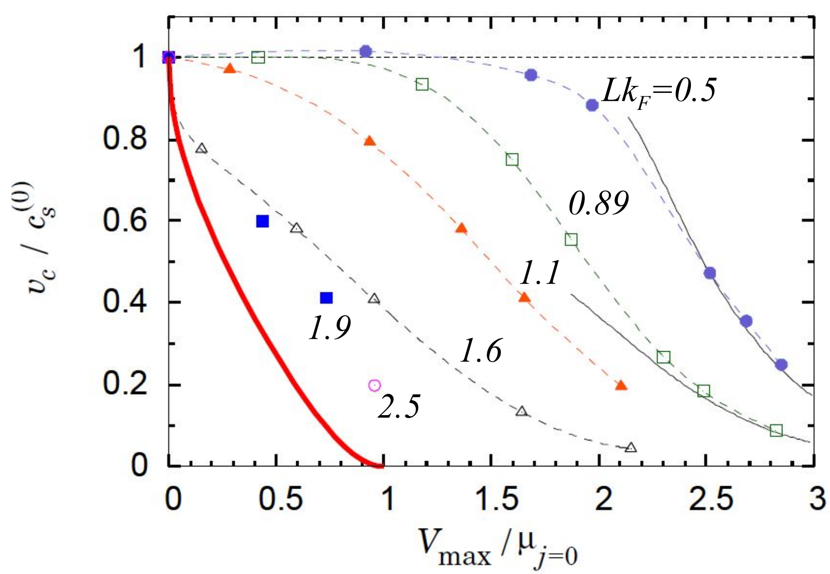
Barrier

L: width of barrier

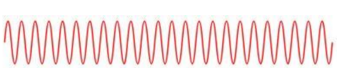
Bosons in a lattice



Fermions in a lattice



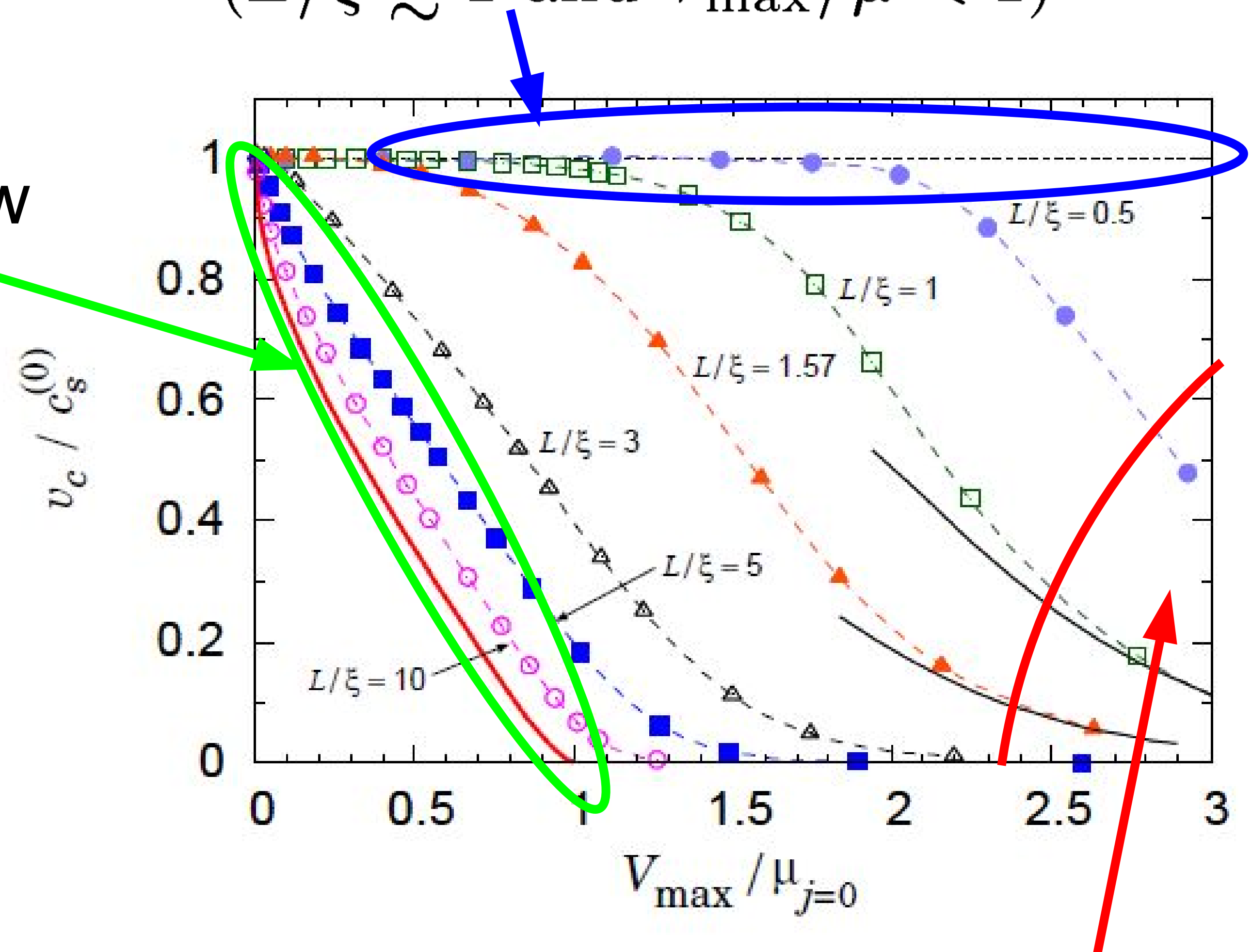
Lattice



Three limiting regimes

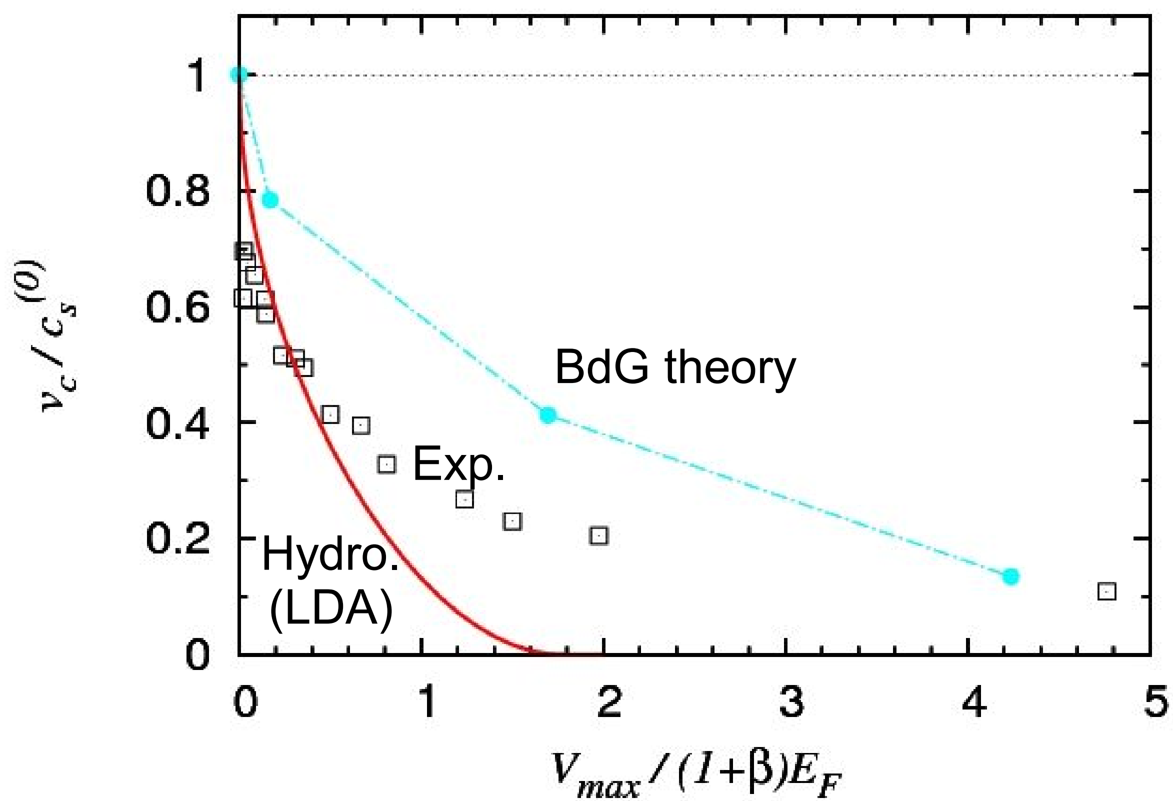
2. Macroscopic flow through thin & weak barriers
$(L/\xi \lesssim 1 \text{ and } V_{\max}/\mu < 1)$

1. Hydrodynamic flow in the LDA
$(L/\xi \gg 1)$



3. Weakly coupled superfluids separated by thin & strong barriers
$(L/\xi \lesssim 1 \text{ and } V_{\max}/\mu \gg 1)$

Comparison with experimental data (1)



If E_F is given by the total number of atoms in the trap:

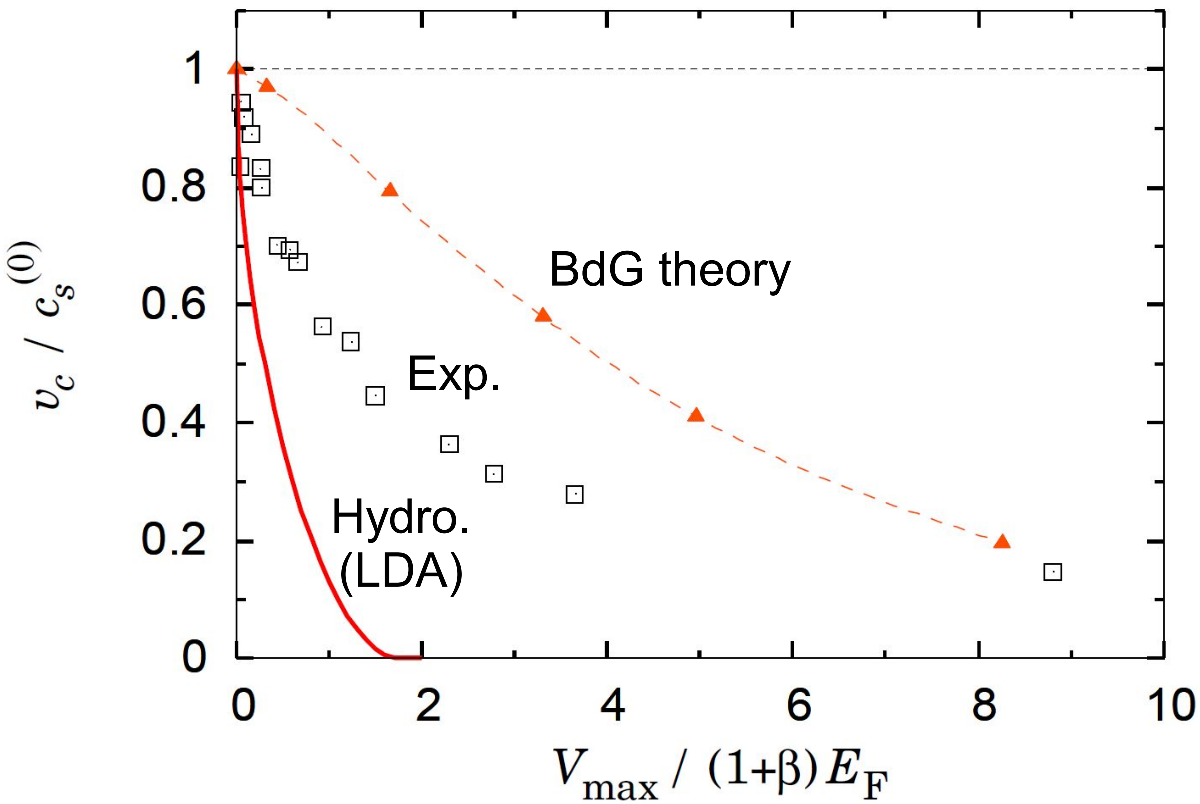
$$E_\mathrm{F}/E_\mathrm{R} \simeq 1 \quad (k_\mathrm{F} d/2 \simeq 1.6)$$

Exp. @ MIT
Miller et al.,
PRL **99**, 070402 (2007)

Exp. data are expected to be far from hydro. (LDA) and close to BdG.

Significant discrepancy even in the uniform limit.

Comparison with experimental data (2)



If E_F is given by the density at e^{-2} beam waist:

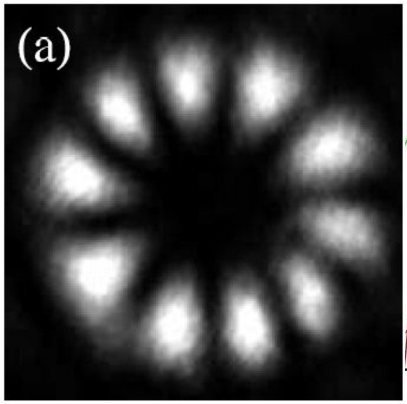
$$E_F / E_R \simeq 0.5 \quad (k_F d/2 \simeq 1.1)$$

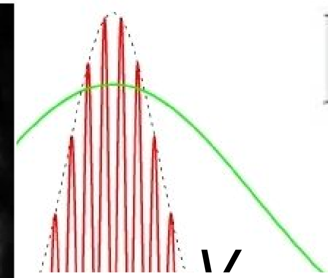
Problem: Which density n_0?
Which V_{max}?

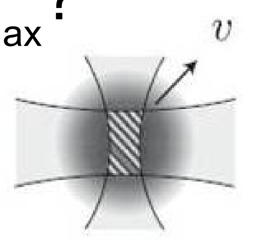
Still significant difference btw exp. data and BdG results.

There is still something to be understood.

Other config.: e.g., annulus trap







Franke-Arnold et al. (2007).

Take-home messages

Critical velocity of the Landau instability for 1D periodic pot. & single barrier

- Expression applicable for any potential in hydro regime.
- v_c/c_s decreases with increasing L/ξ.
- Large difference btw single barrier and periodic pot.

Single barrier: No plateau.

- μ is fixed by the asymptotic density.
- Density variation with >L is allowed.

Periodic pot.: Plateau at smaller L/ξ.

- Pot. extends whole sys.
- μ is influenced by the variation of V_{max}.
- Density variation with period L.

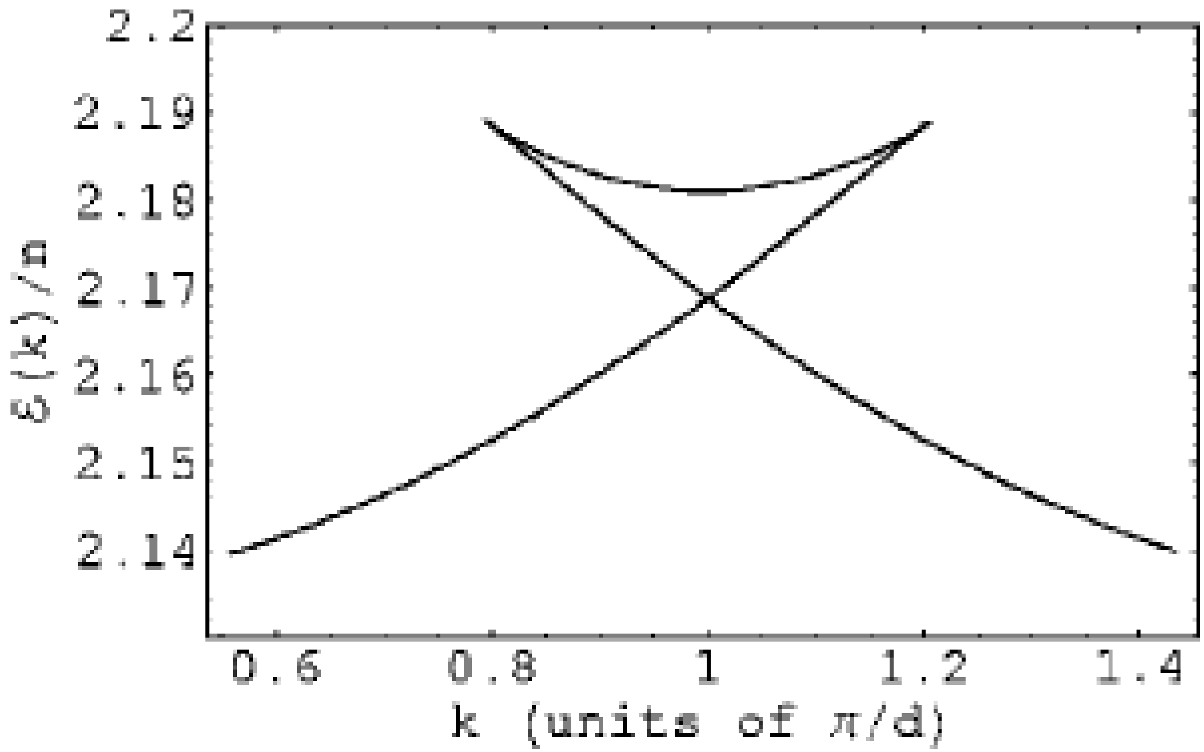
Swallow-tail band structure in BCS-BEC crossover

With F. Dalfovo, R. Scott, G. Orso

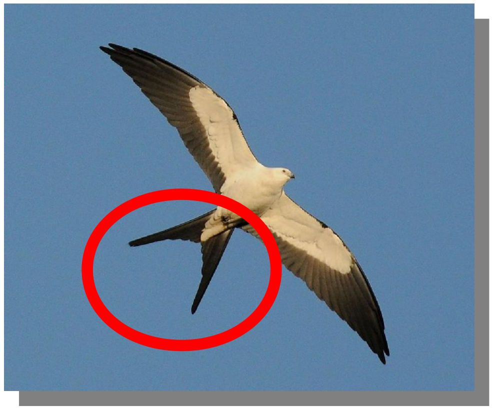
Swallow tail of BEC in a lattice (1)

Nonlinearity & Mode coupling btw different period

Swallow tail band structure



Wu, Diener & Niu (2002)
Diakonov et al. (2002)
Machholm, Pethick & Smith (2003)
Seaman, Carr & Holland (2005)
Danshita & Tsuchiya (2007), etc.



[Diakonov et al. (2002), also Wu et al. (2002)]

Swallow tail of BEC in a lattice (2)

GP eq.: $$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + g|\psi|^2\psi = \mu\psi$$

Int. $g|\psi|^2$: Non-linearlity; favors trans. inv.
favors quadratic band

Lattice $$V(z) = V_{\max}\sin^2 q_{\mathrm{B}} z = \frac{V_{\max}}{2}(1-\cos 2q_{\mathrm{B}} z)$$

Couples the st. with k differing by ±2π/d

Creates periodic band structure; favors sinusoidal

Trial wave func. $$\psi = \sqrt{n}e^{ikz}(\cos\theta + \sin\theta e^{-i2\pi z/d})$$

$$\epsilon = \frac{1}{d}\int_{-d/2}^{d/2} dz \left[\frac{\hbar^2}{2m}|\partial_z\psi|^2 + V(z)|\psi|^2 + \frac{g}{2}|\psi|^4\right]$$

Variatoin w.r.t. θ → $k = k(\theta)$

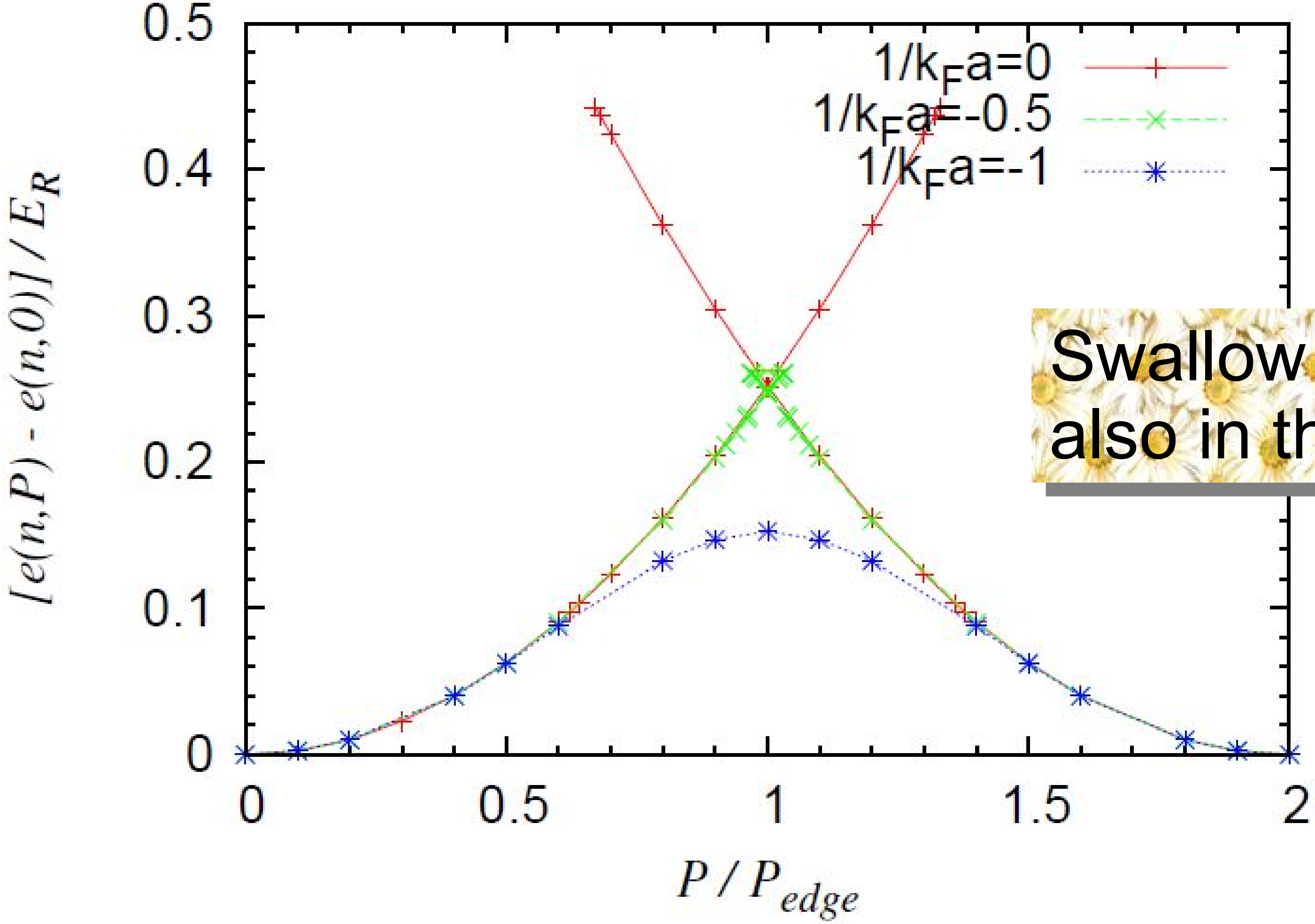
Condition for swallow tail: $gn > V_{\max}/2$

Swallow tail of Fermi superfluids along BCS-BEC crossover

Source of non-linearity: Δ(r) (~ E_F @ unitarity)

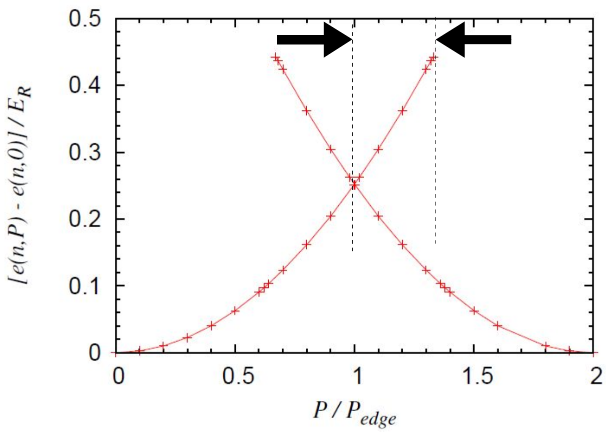
Necessary cond.: $E_{\mathrm{F}} \gtrsim V_{\max}(\equiv sE_{\mathrm{R}})$

Method: BdG s = 0.1, E_F/E_R = 2.5



Swallow tails appear also in the Fermi s.f.

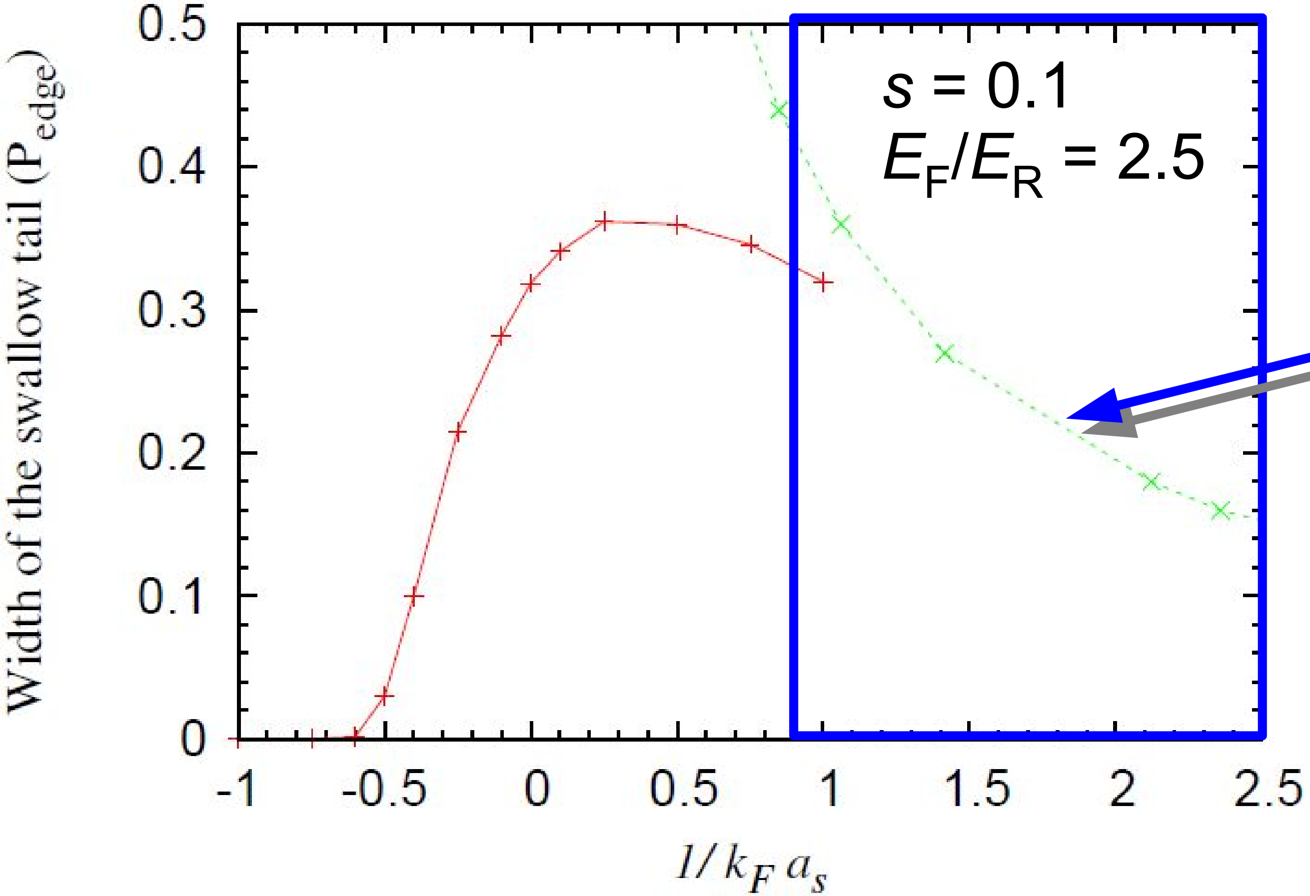
Width of swallow tail in BCS-BEC crossover



Width of the swallow tail $P_{\rm wid}$

= (P @ edge of tail) – (P @ BZ edge)

1. BEC limit → weakly int. bosonic mol.



$1/k_{\rm F}a_s \to \infty$

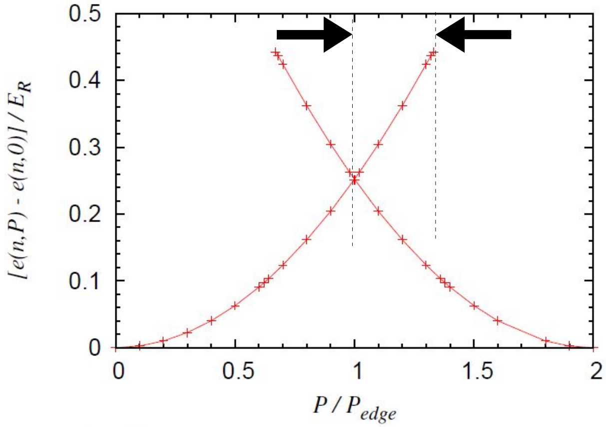
→ $P_{\rm wid} \searrow 0$

$P_{\rm wid}$ obtained by GPE with

$$a_{dd} = 2a_s$$

(cf. exact value $a_{dd} = 0.6a_s$)

Width of swallow tail in BCS-BEC crossover



Width of the swallow tail P_{wid}

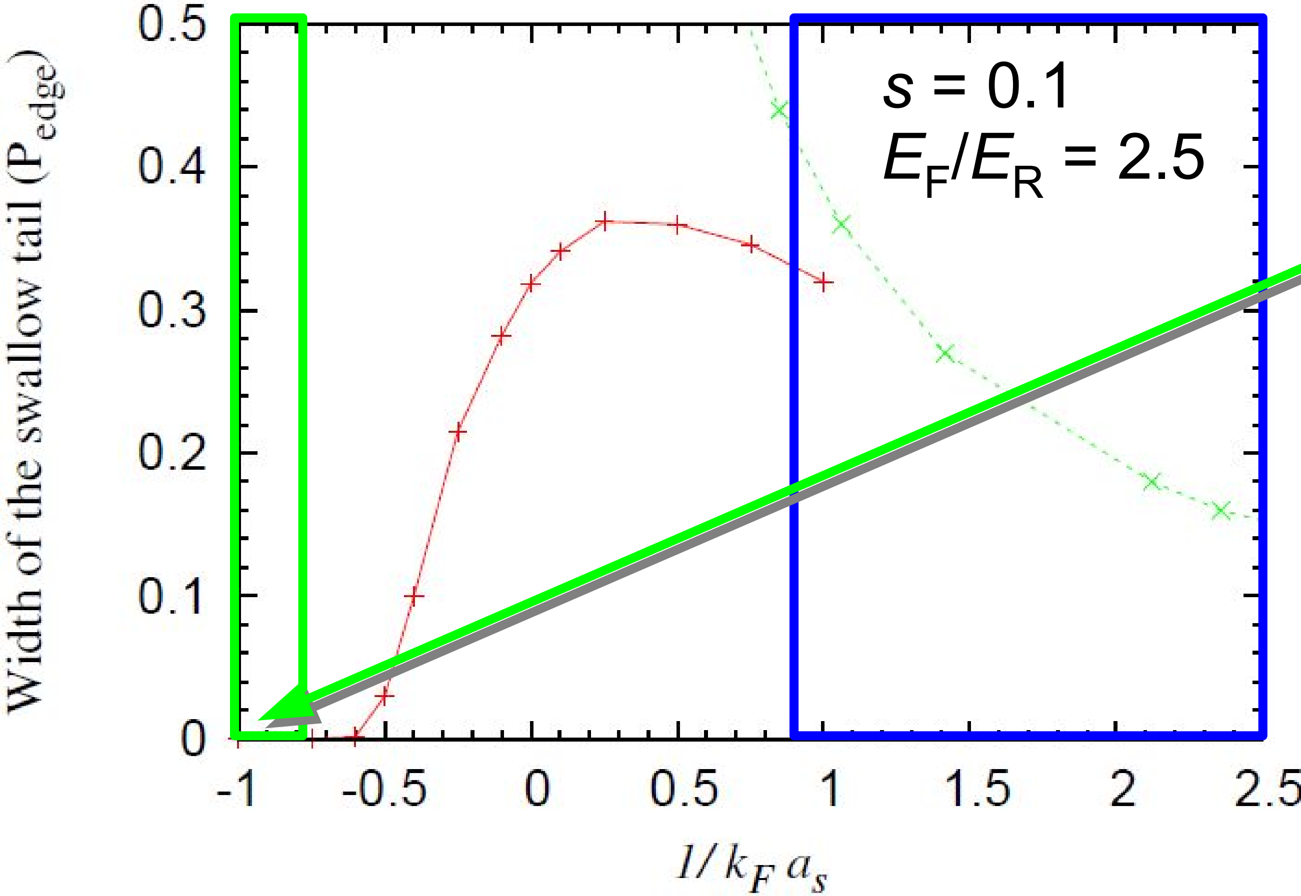
= (P @ edge of tail) – (P @ BZ edge)

2. BCS limit → non-int. fermionic atoms

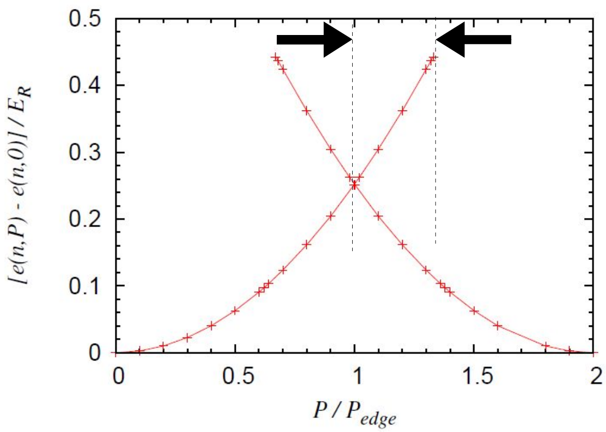
No swallow tails.

$$1/k_{\mathrm{F}}a_s = -\infty$$

→ $P_{\mathrm{wid}} = 0$



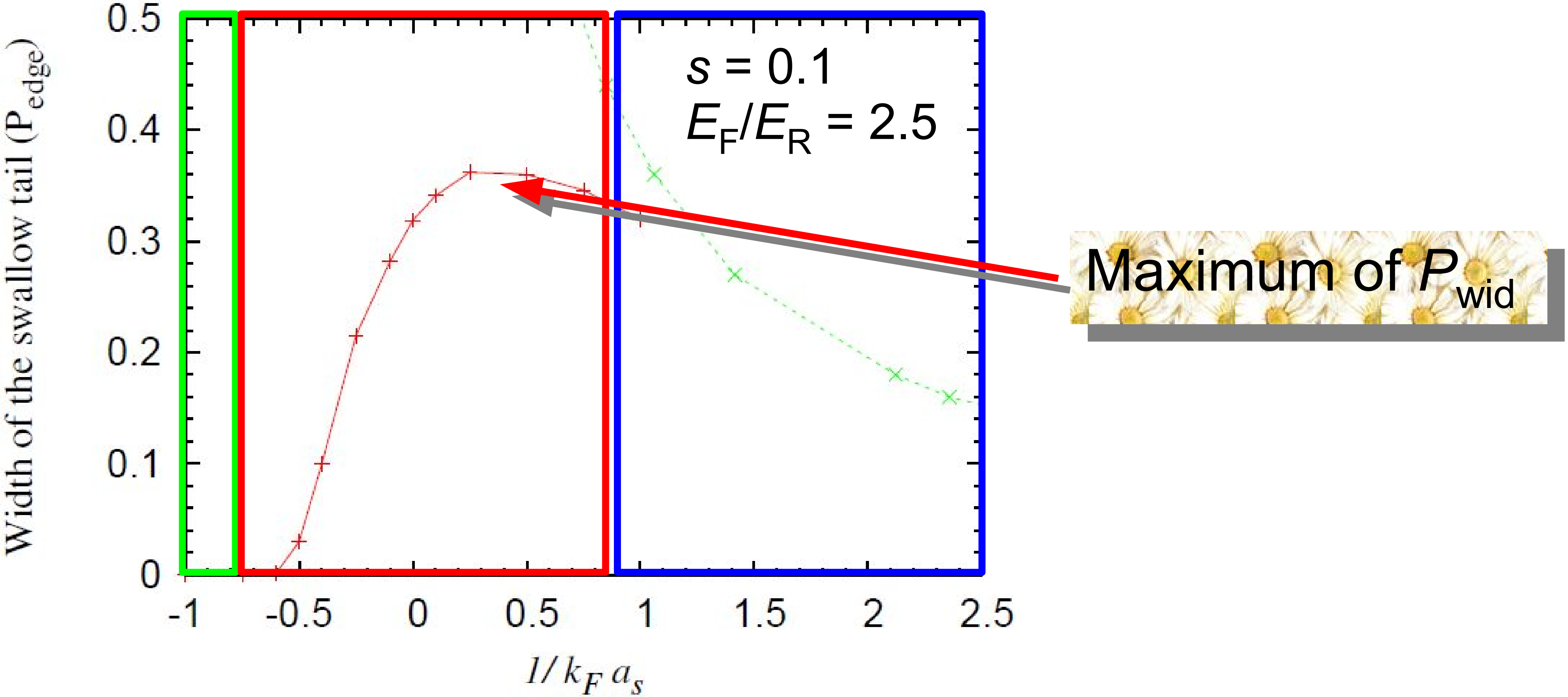
Width of swallow tail in BCS-BEC crossover



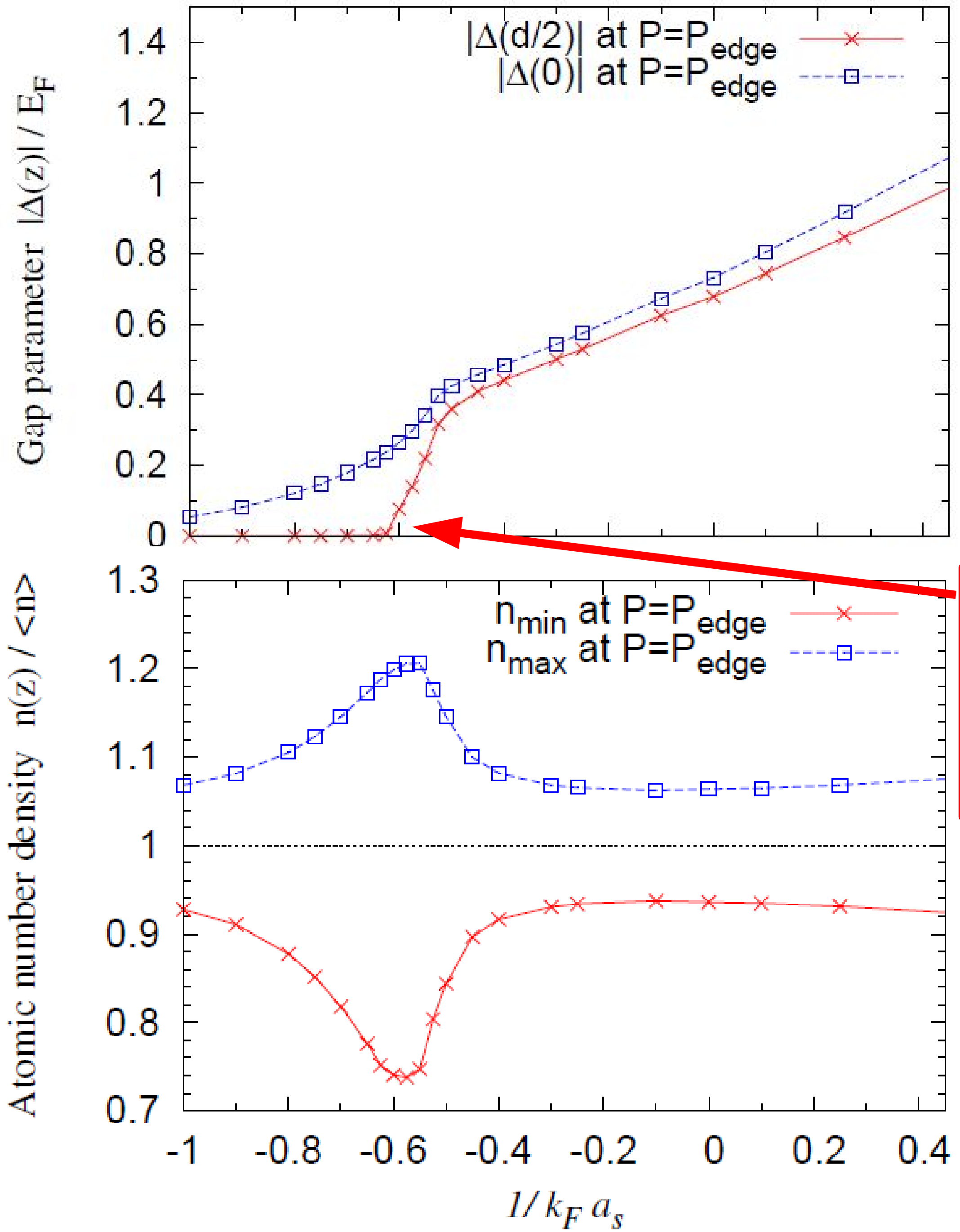
Width of the swallow tail P_{wid}

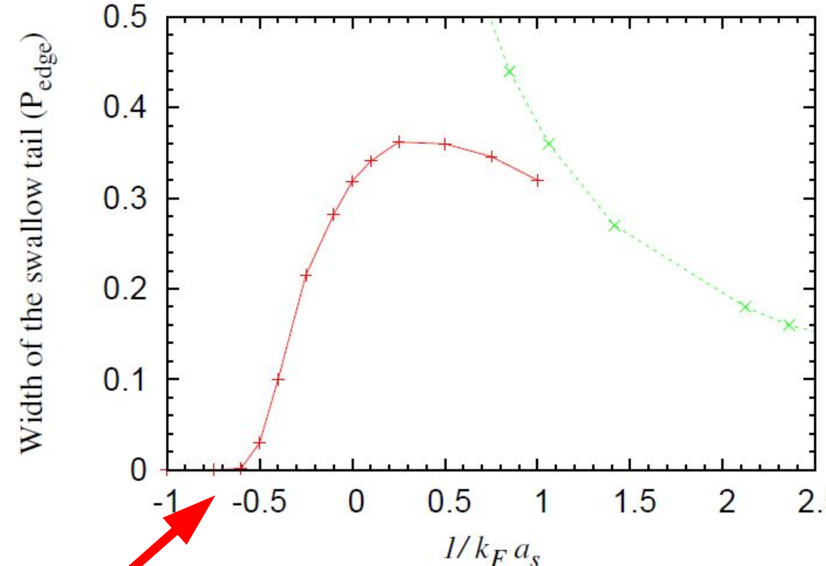
= (P @ edge of tail) – (P @ BZ edge)

3. Unitarity ⇨ Strongly int. fermions



Gap parameter & density at BZ edge





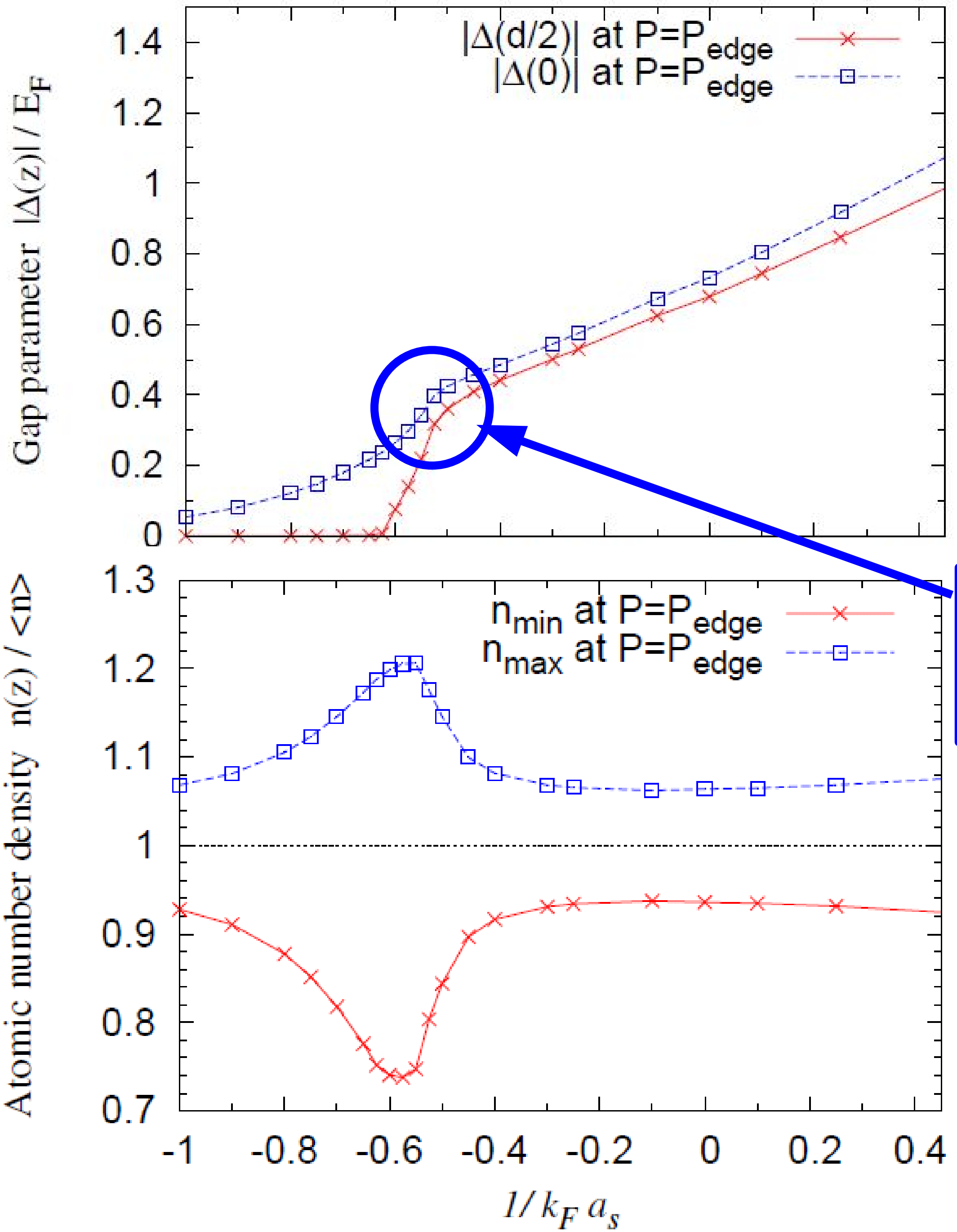
Occurrence of swallow tail relates to sudden change of Δ(d) @ BZ edge.

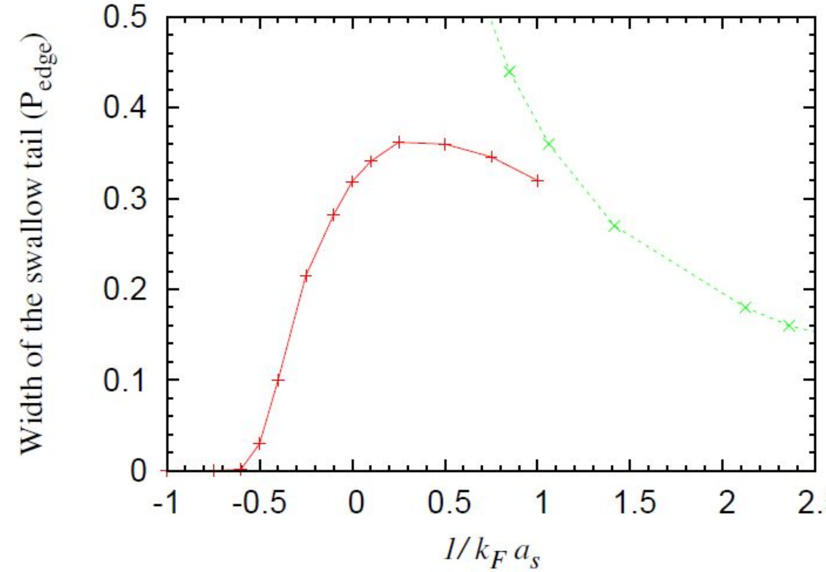
No swallow tail: π-st. @ P_{edge}

Swallow tail: non-zero current @ P_{edge}

➡ Non-zero Δ(d)

Gap parameter & density at BZ edge





Further sudden change of $\Delta(z)$ @ BZ edge.

Unique for Fermion case

Take-home messages

Band structure of Fermi superfluids
along BCS-BEC crossover in a 1D periodic potential

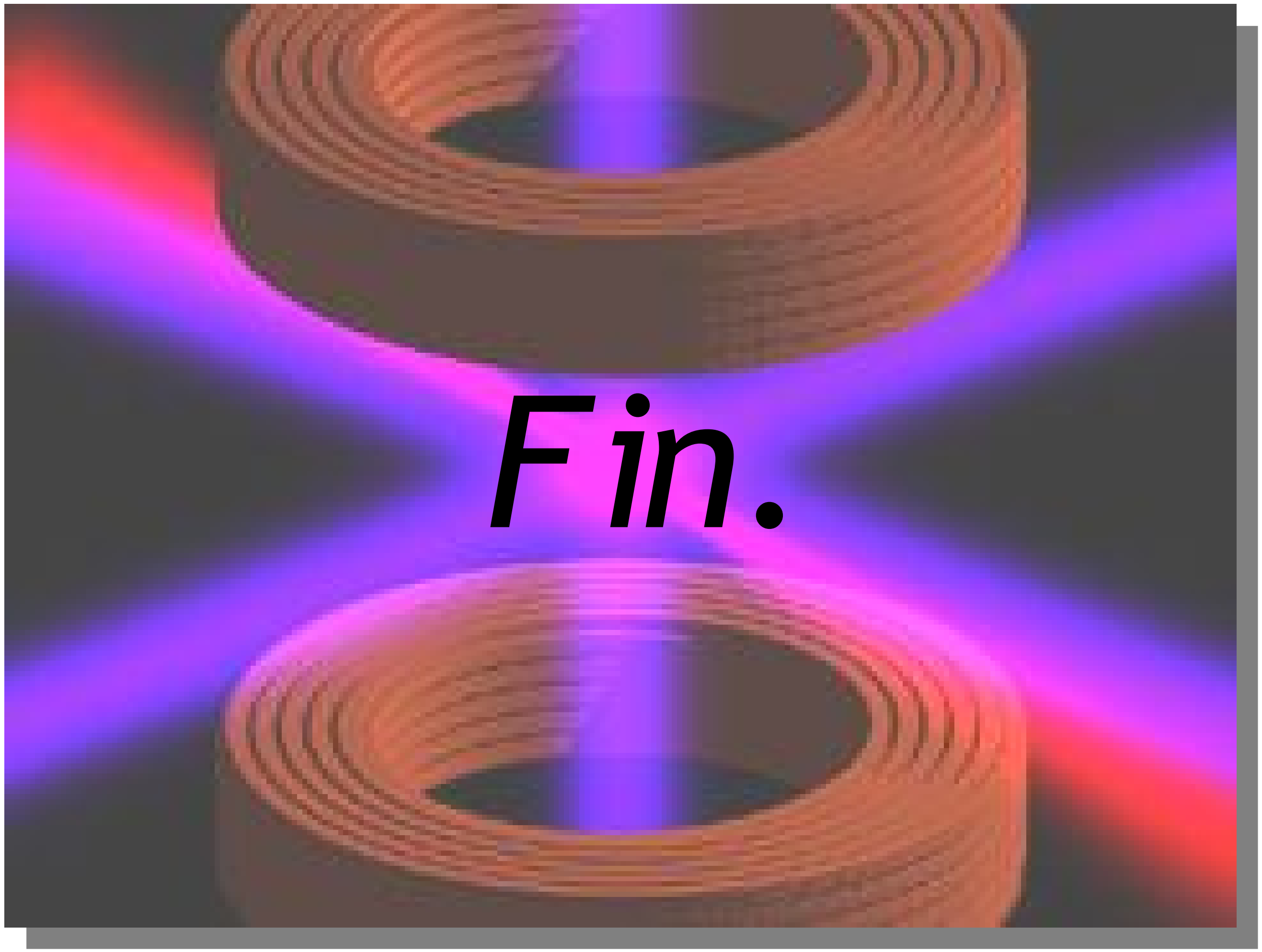
- Swallow tails are observed.

- Width of the tail is largest around unitarity.

- Occurrence of the swallow tail in the BCS side is strongly related to non-trivial behavior of $\Delta(z=d)$ & $n(z=d)$ at BZ edge.

  Effects of the Andreev-like states?

  Different mechanism from the BEC side.



Thank you for your attention.

Hydrodynamic analysis for excitations (2)

Energy density $e(n, P)$

Cont. eq. $$\partial_t n + \partial_z \frac{\partial e}{\partial P} = 0$$

Euler eq. $$\partial_t P + \partial_z \frac{\partial e}{\partial n} = 0$$

Perturbations

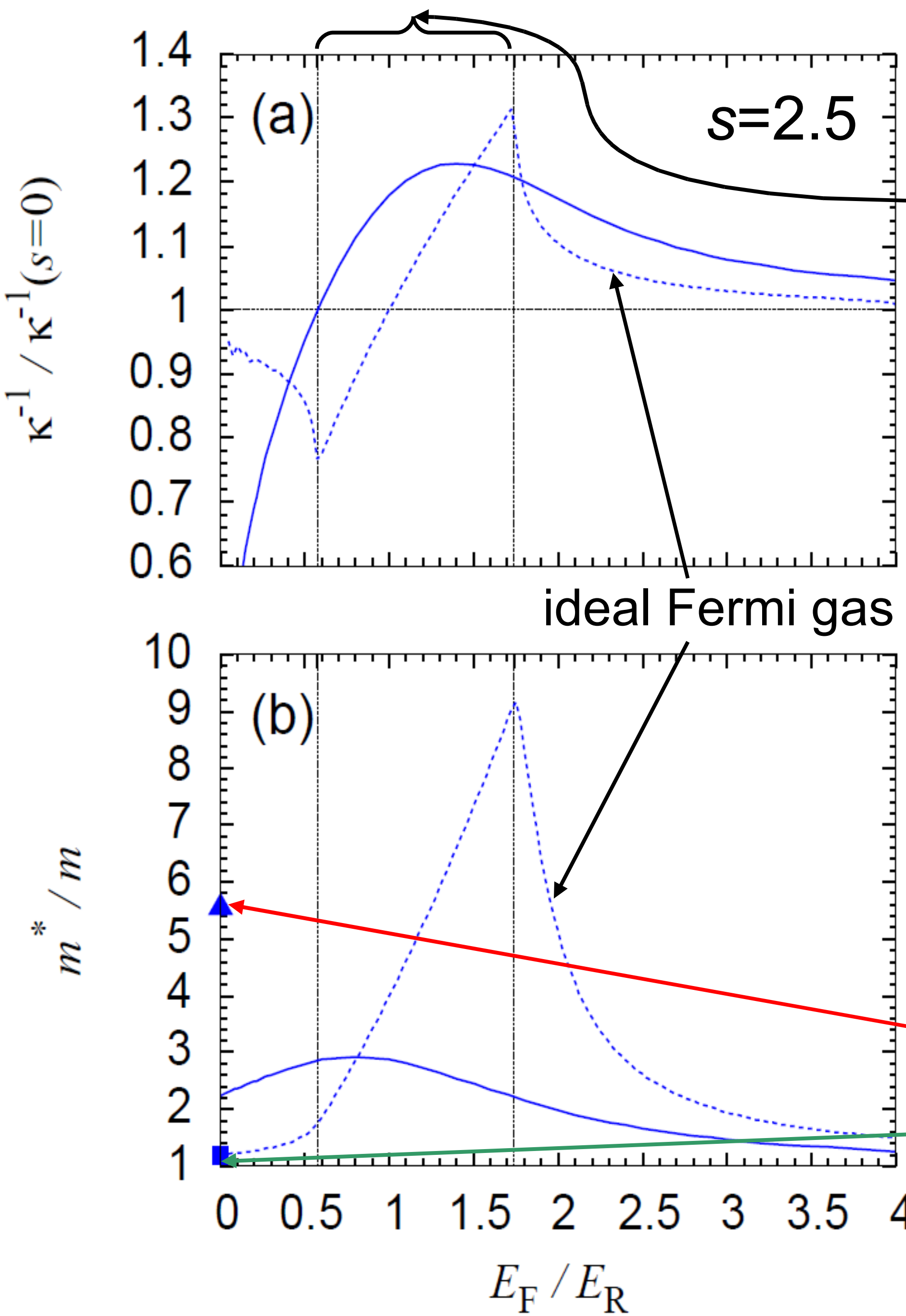
$$n(z) = n_0 + \delta n(z), \quad P(z) = P_0 + \delta P(z)$$

with $\delta n, \delta P \sim \exp\left[i(qz - \omega t)\right]$

Dispersion rel. of excitations (long-wavelength phonons)

$$\omega = \frac{\partial^2 e}{\partial n \partial P}\, q + \sqrt{\frac{\partial^2 e}{\partial n^2}\frac{\partial^2 e}{\partial P^2}}\, |q|$$

Comparison with the ideal Fermi gas



Ideal Fermi gas: clear band effects due to the sharp Fermi surface.

Between cusps: μ is in the bandgap

only trans. modes can be occupied

➡ 2D-like system

$$\kappa^{-1}, m^* \propto n$$

Drastic increase of κ^{-1}, m^*

Unit. Fermi gas: band effects smeared by the broad Fermi surface.

Mol. in unitary Fermi gas: delocalized

point molecules with mass 2*m*

atoms with mass *m*

Unitary Fermi gas is intermediate situation.

Bloch band structure (1)

BEC in a lattice

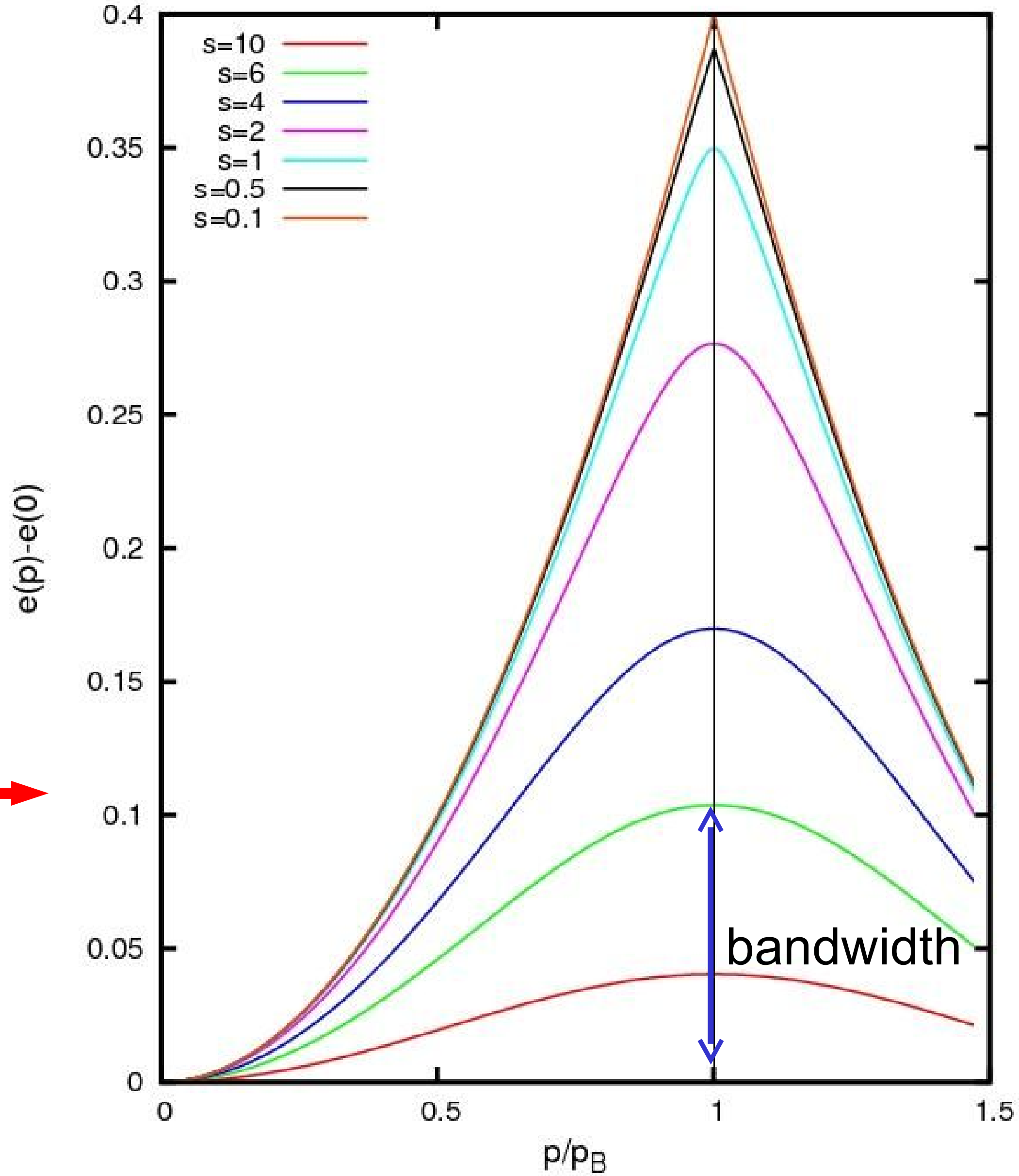
p = quasi-momentum
p_B = Bragg quasi-momentum
$E_R = p^2_B/2m$ = recoil energy
$V_{max} = sE_R$ = lattice strength

Lowest Bloch band for the same *gn=0.4* and different *s*

Small *s*: quadratic

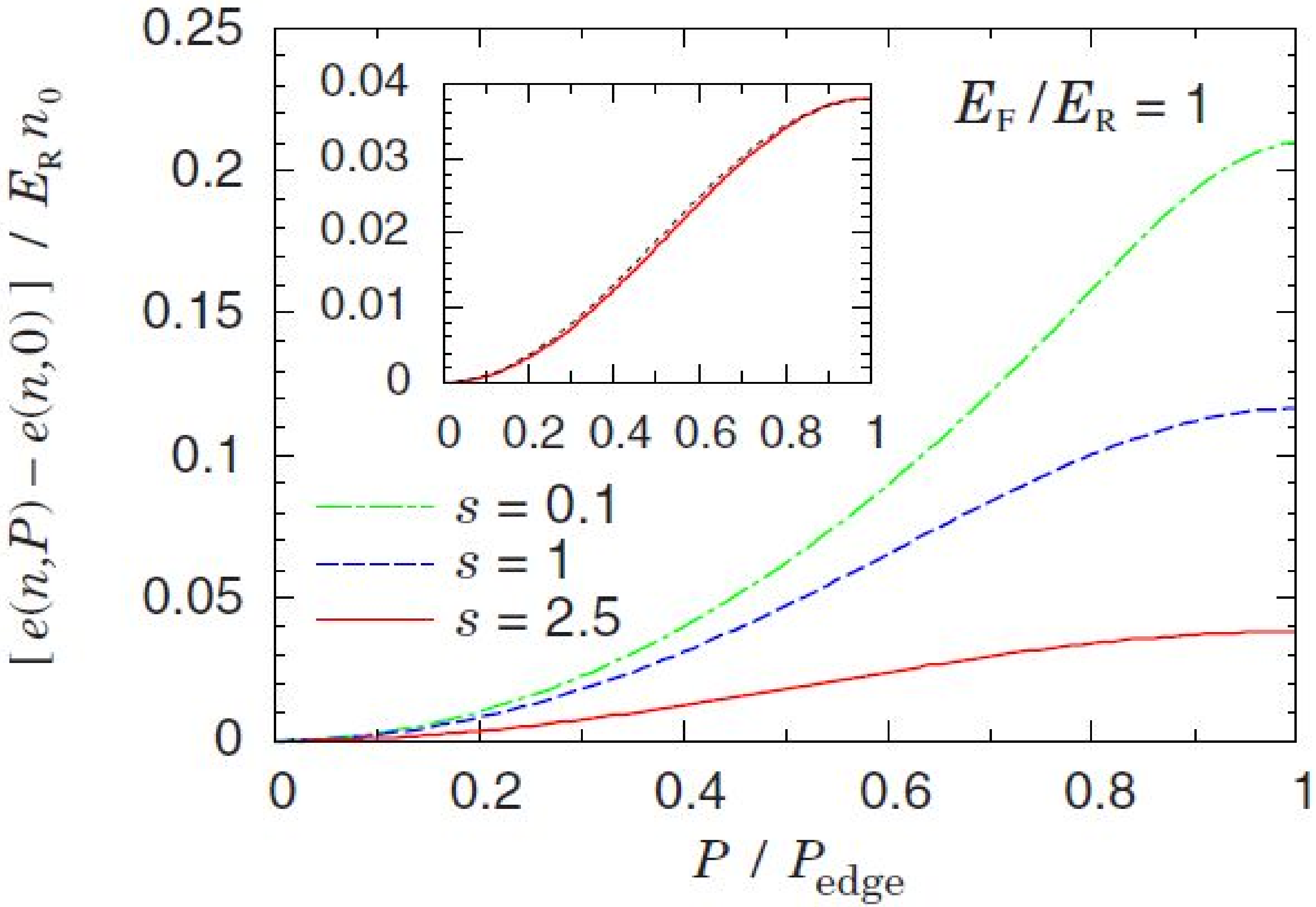
Large *s*: sinusoidal

Energy density vs. quasi-momentum



Bloch band structure (2)

Unitary Fermi gases in a lattice



Small *s*: quadratic

Large *s*: sinusoidal