Superfluidity in Quantum Hall Bilayers

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Outline:

I. Introduction I: Quantum Hall Ferromagnets and Textures

II. Introduction II: Bilayer Superfluidity, Merons

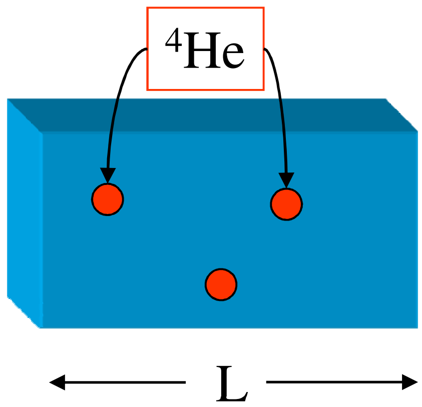
III. Disorder and a “Coherence Network” for the Quantum Hall Bilayer

IV. Effect of Bias: Dissipation in Drag Geometry

V. Summary

I. Introduction (I)

Classic Superfluid: $^4He \Rightarrow$ Bose-Einstein Condensation
Interacting Bose condensates exhibit *phase stiffness*:

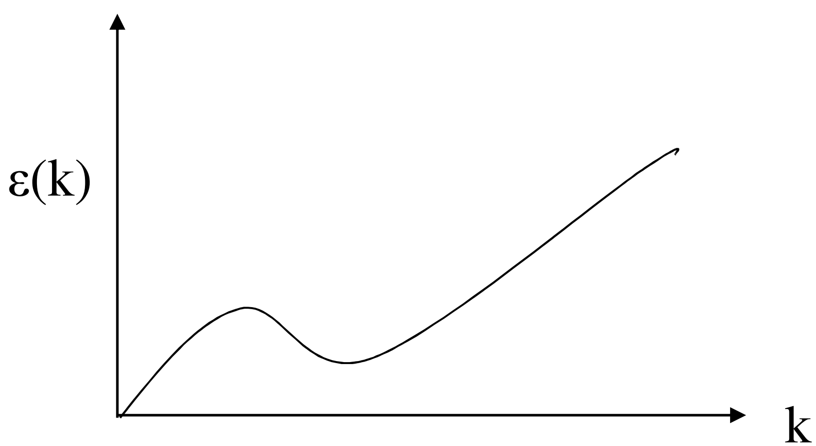


For $\psi(\vec{r} + L\hat{x}) = e^{i\theta}\psi(\vec{r})$,

$$E(\theta) = E_0 + \frac{1}{2}\Upsilon\left[\frac{\theta}{L}\right]^2 L^D$$

D=dimension

“Generalized elasticity”, due to phase of condensate wavefunction.



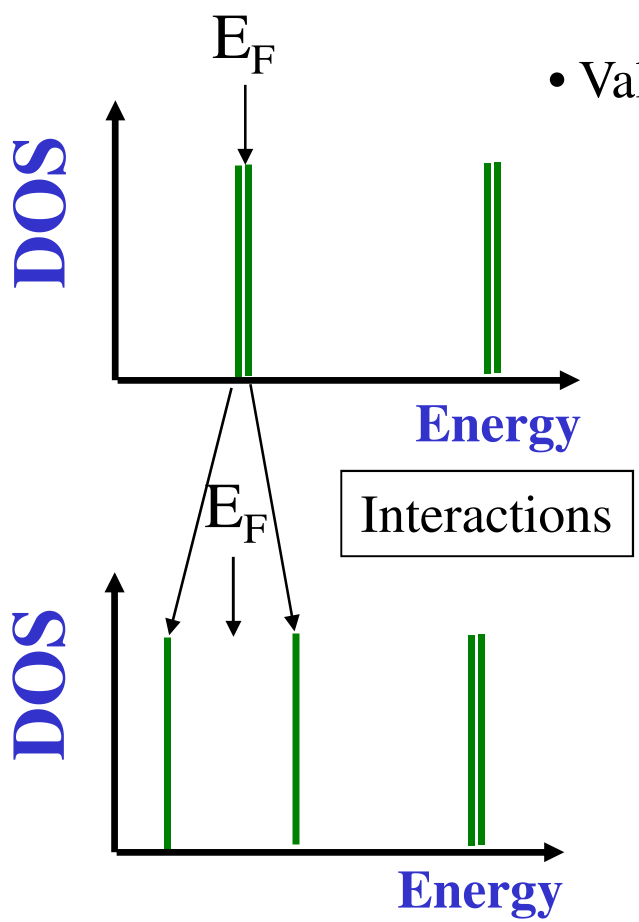
Sound-like excitations:

$\boxed{\varepsilon(k) \sim ck}$ (small k)

Some quantum Hall systems have a similar property:
Quantum Hall Ferromagnetism

Discrete degrees of freedom increase number of states in lowest Landau level:

- Spin ($g\mu_B B \sim 1K$, $e^2/\varepsilon l_0 \sim 100K$)
- Layer
- Valley (Si, AlAs, Graphene)

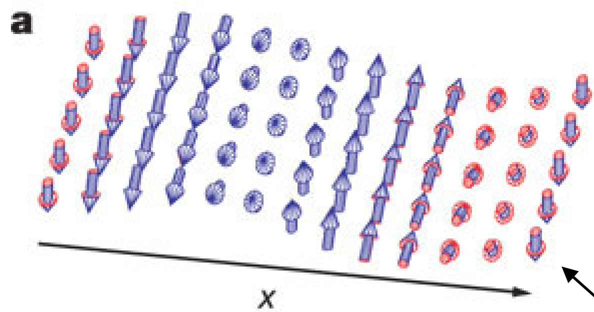


- Filling factor $\nu = 1$
- Exchange tends to force electrons into the same level even when bare splitting between levels is small or absent

⇒System is <u>ferromagnetic</u>
⇒Exchange "spin stiffness"

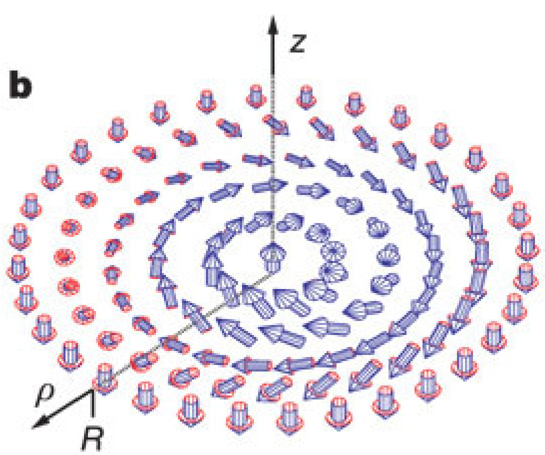
- Renormalizes gap to much larger value than expected from non-interacting theory (even if bare gap is zero!)

Lessons from spin ferromagnetism in QH system:



Effective $O(3)$ degree of freedom in single LL

Spin waves: QH ferromagnets support gapless or nearly gapless collective excitations, even while Hall conductance is quantized



Skyrmions: Topological excitations with non-trivial winding number

[Figures from Rossler et al., Nature **442**, 797 (2006)]

Spin-Charge Coupling

- Constraining electron state to single Landau level induces a connection between "spin" and density modulations. For two component system,

$$\boxed{\delta\rho(\vec{r}) = \varepsilon_{ij}\hat{n}\cdot\left(\partial_i\hat{n}\times\partial_j\hat{n}\right)/8\pi}$$

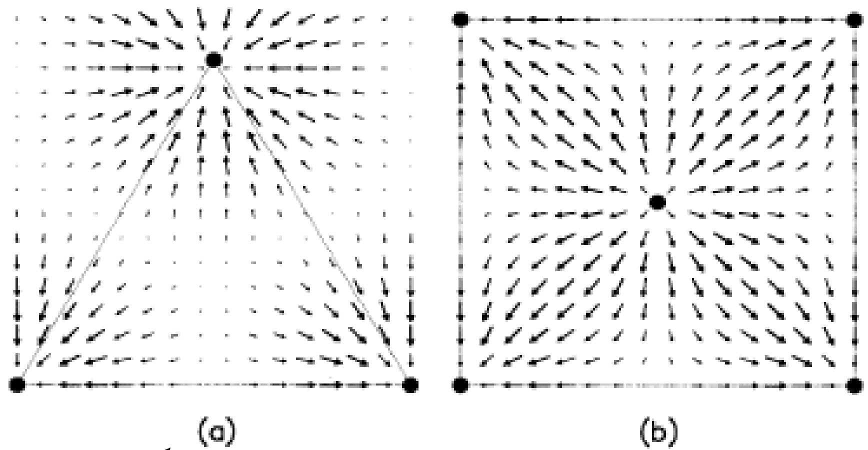
(cf. Lee and Kane,, PRL 1990; Sondhi et al., PRB 1993)

Effective $O(3)$ spin degree of freedom

- For skyrmion state, $\int d^2r\,\delta\rho\left(\vec{r}\right)=\pm 1$ counts number of times n covers the sphere ("Pontryagin index"). In quantum Hall system, **skyrmions carry charge.**

- Skyrmions can be injected into the groundstate by doping away from ν=1.

From Brey et al, 1995

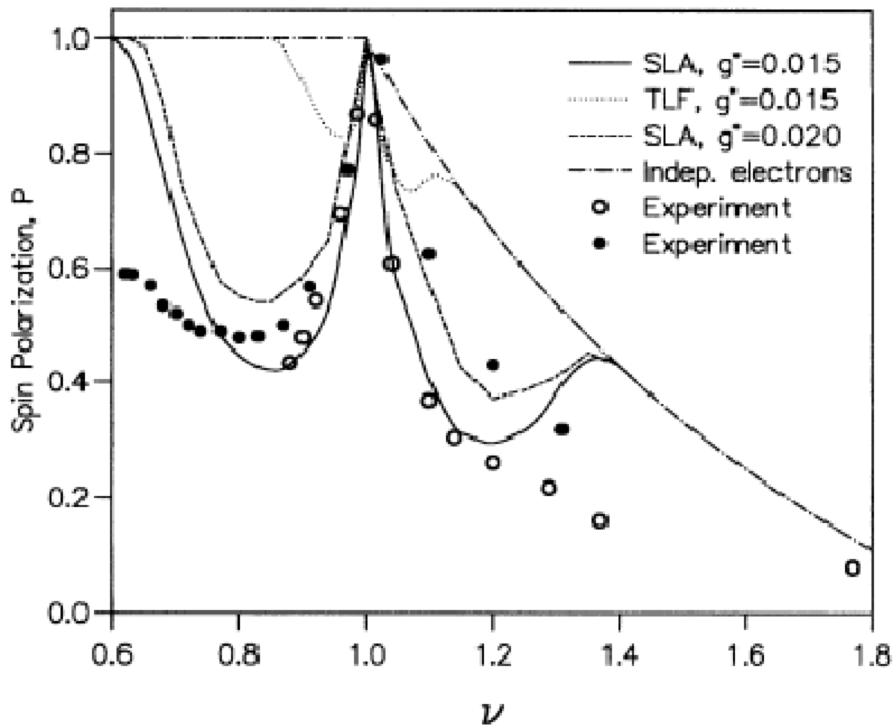


ν=1.2

Triangular lattice of skyrmions

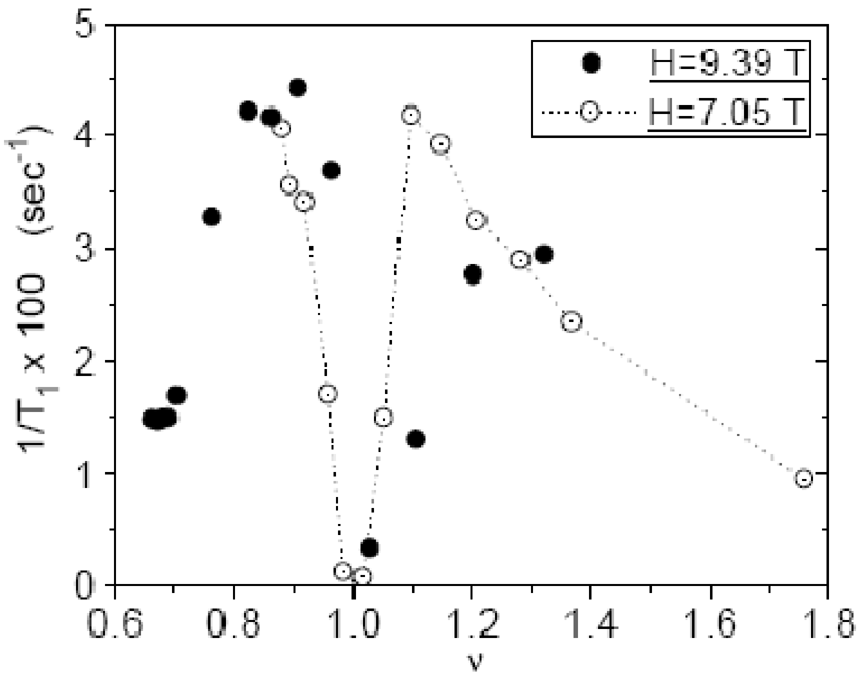
Square lattice of skyrmions (two per unit cell)

Broken symmetry: in-plane spins may be rotated globally at no cost in energy.



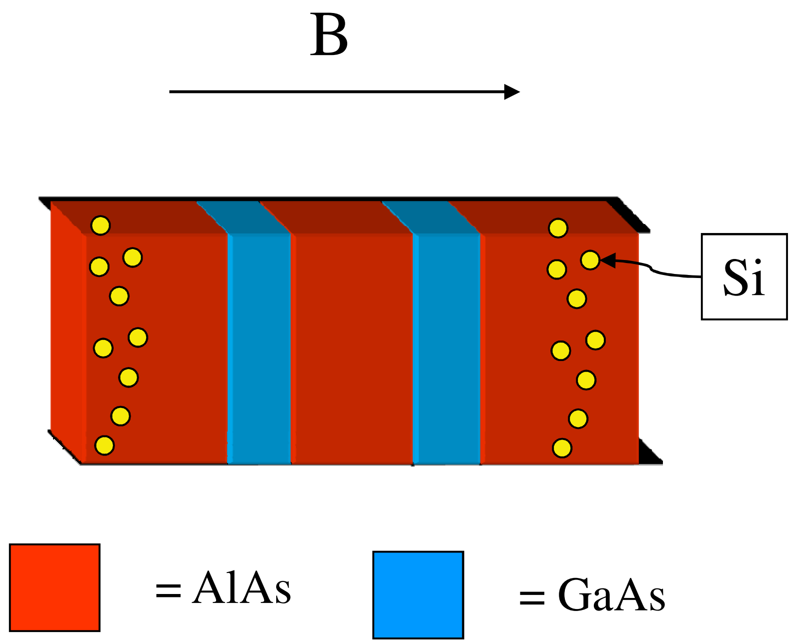
- Spin polarization as measured in NMR experiments degrades rapidly away from ν=1. (Barrett et al., 1995)
- Results can be understood quantitatively with Hartree-Fock theory for skyrme lattice.

• NMR $1/T_1$ relaxation rate greatly enhanced away from $\nu=1$: Evidence of gapless collective mode associated with broken U(1) symmetry in skyrme lattice state

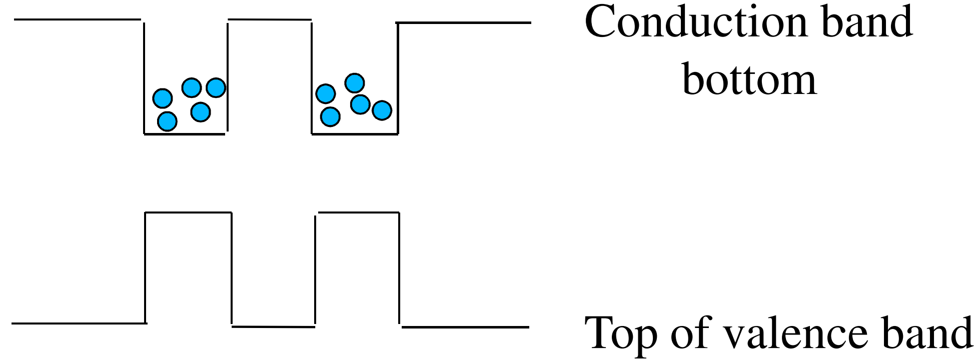


From Tycko et al., Science 1995

Introduction (II): The Quantum Hall Bilayer



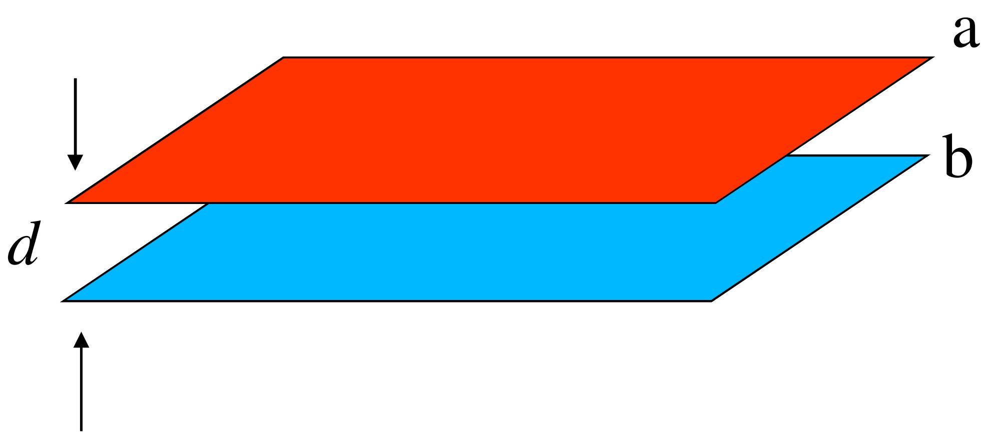
- Expect analogous phenomenology



- Layer index plays role of spin direction: $a_{X,\uparrow}$ annihilates left layer electron, etc.
  $\Rightarrow$ pseudospin

- Charging energy favors equal populations of layers (unless there is a bias.)
  $\Rightarrow$ Easy plane pseudospin ferromagnet

Alternative interpretation:



Particle-hole transformation
on *one* layer

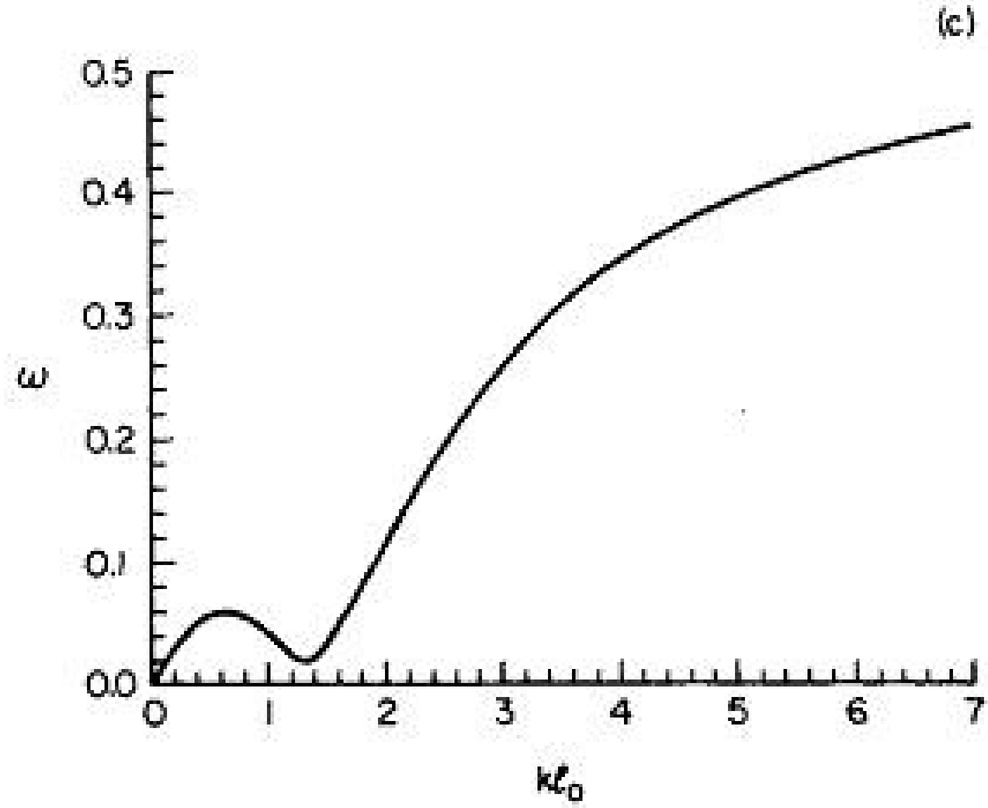
Hole in blue layer

$$\left|\Psi\right\rangle = \prod_X \left[u_X + v_X e^{i\theta} a_X^+ b_X^+\right] \left| all\, blue \right\rangle$$

Electron in red layer

- θ equivalent to in-plane angle of easy-plane ferromagnet
- BCS state of electron-hole pairs $\rightarrow$ counterflow superfluid

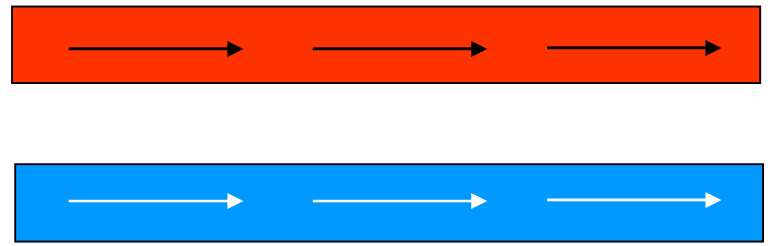
Phase Stiffness for Bilayer quantum Hall system



Roton minimum touches zero for large enough *d*.
Associated with destruction of quantum Hall effect.

[Derived using generalized RPA.
See H.A. Fertig, PRB **40**, 1087 (1989)]

Electron flow

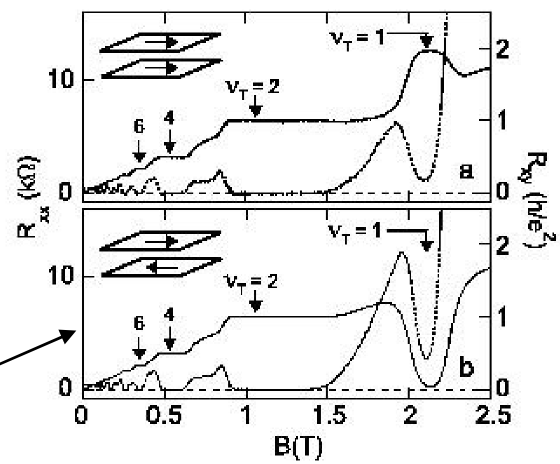


Hole flow

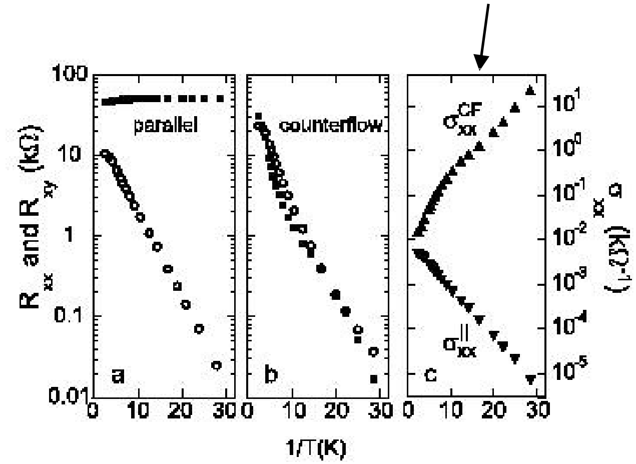
- Because condensation involves electron-hole pairs, expect dissipationless response in counterflow

Kellogg et al., PRL 2004:

Diverging conductivity!



Counterflow resistance



Superfluidity appears to emerge at T=0!

Imperfect superfluidity

Dissipation suggests vortices play a role.
In quantum Hall bilayers, vortices are
really <u>**merons**</u>.

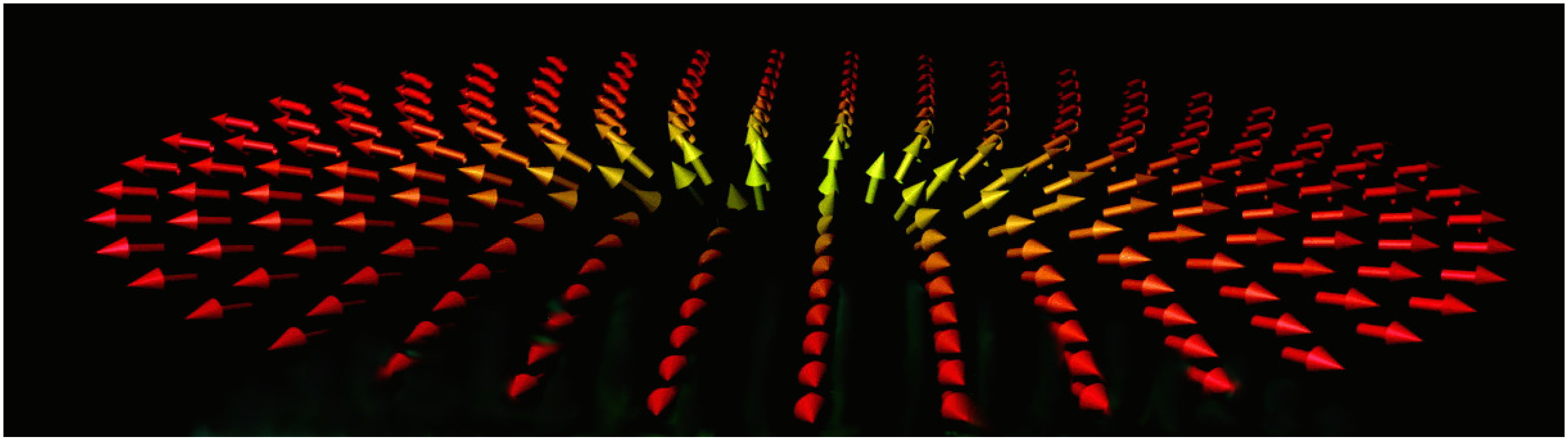
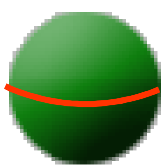


Image from Senthil et al., Science 2004.

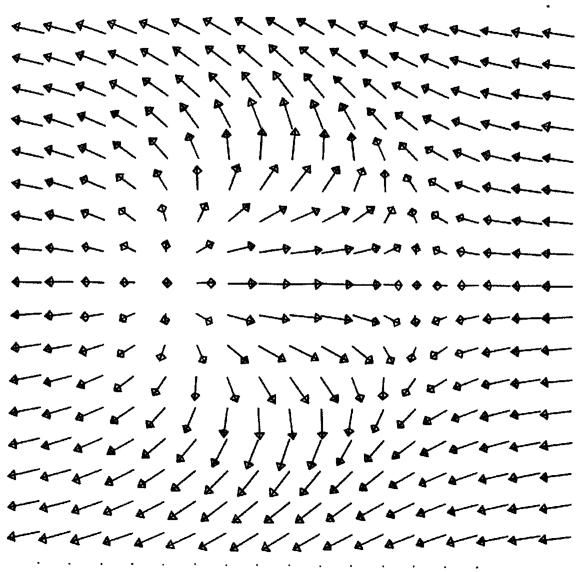
Charge $\pm e/2$



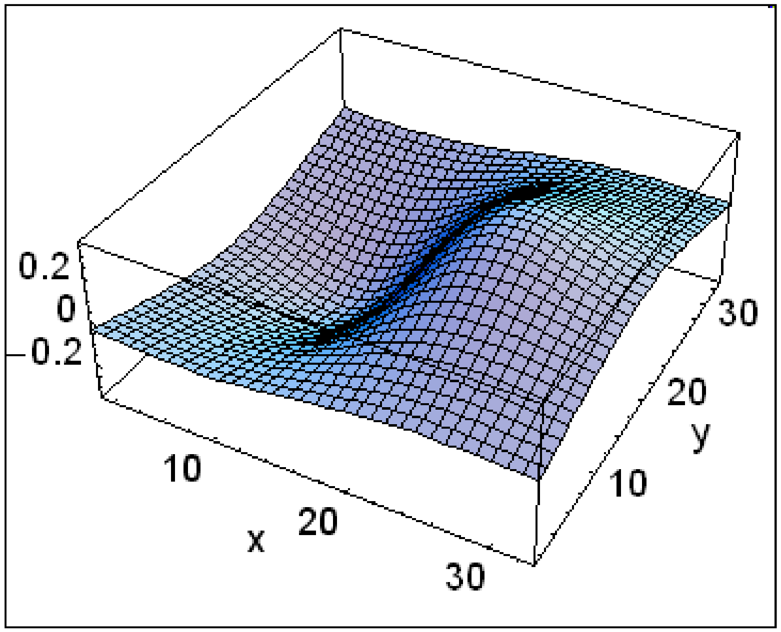
Spin arrows cover only half-sphere

<u>Electric dipole moment</u>:
Charge near core center
localized in a single layer
("Out-of-plane vortex")

Quasiparticles of clean bilayer system are **bimerons**.



S_x, S_y



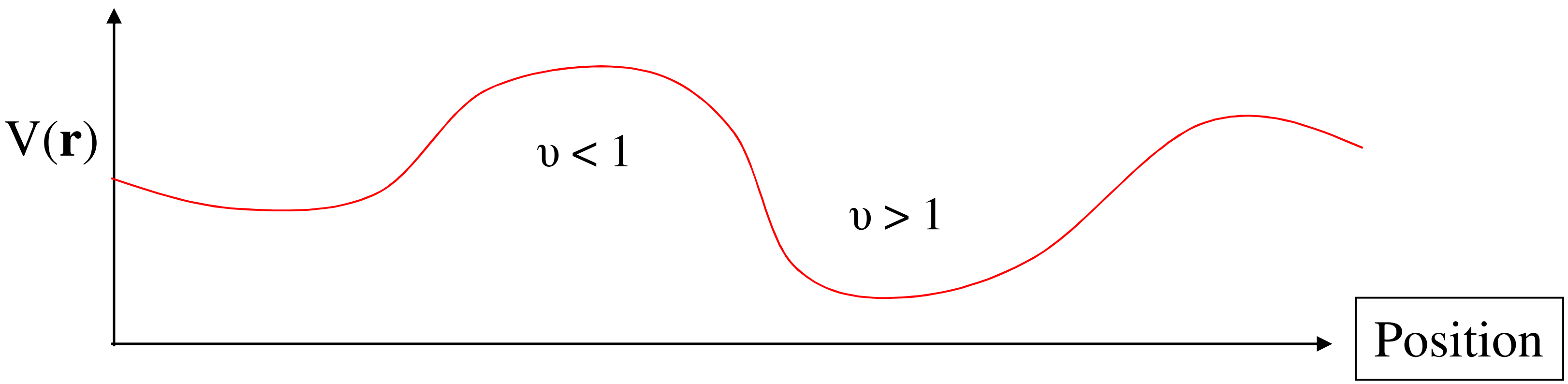
S_z

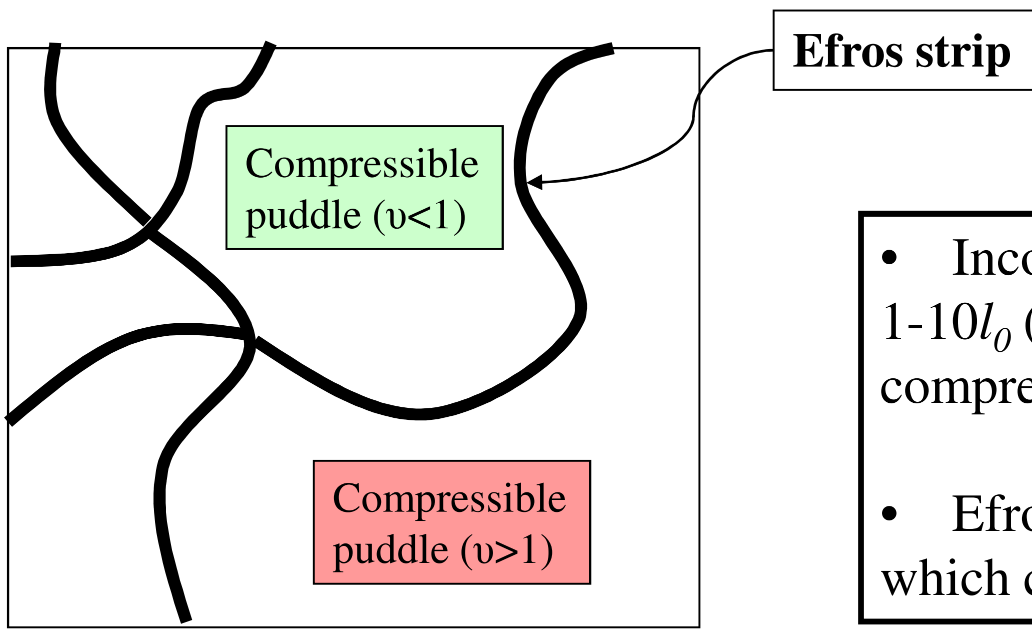
• Note: Bimerons are topologically equivalent to skyrmions. They look different only because they are represented in a rotated spin basis.

III. Disorder-Induced Deconfinement: A "Coherence Network"

[HAF and G. Murthy, PRL (2005)]

Efros model: Potential fluctuations ⇒ nonlinear screening





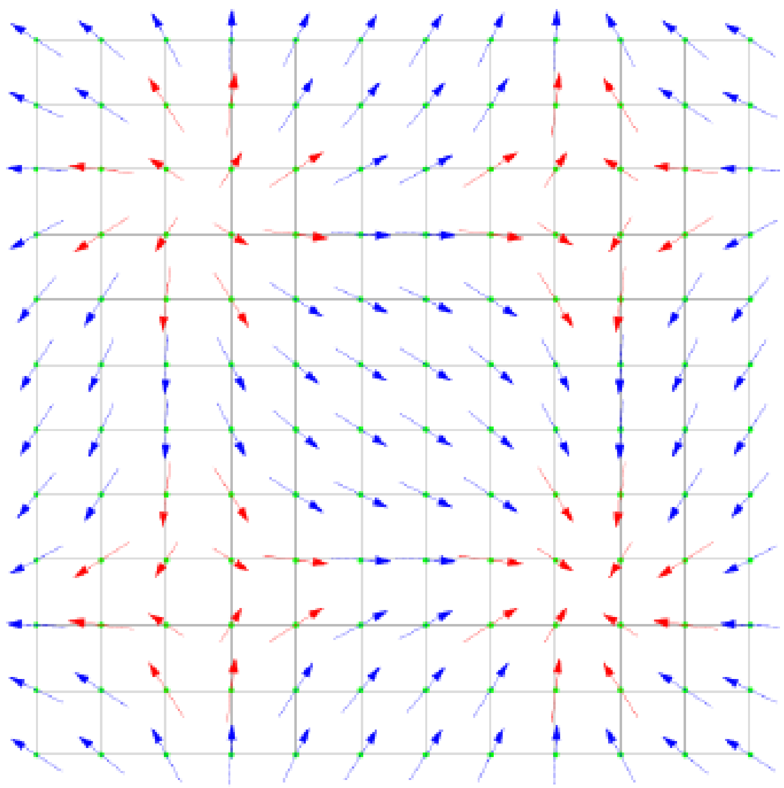
- Incompressible strip of width 1-10l_0 ("Efros strip") surrounds compressible quasiparticle-rich puddles
- Efros strip supports extended state which carries Hall current

Simulation results for periodic system:

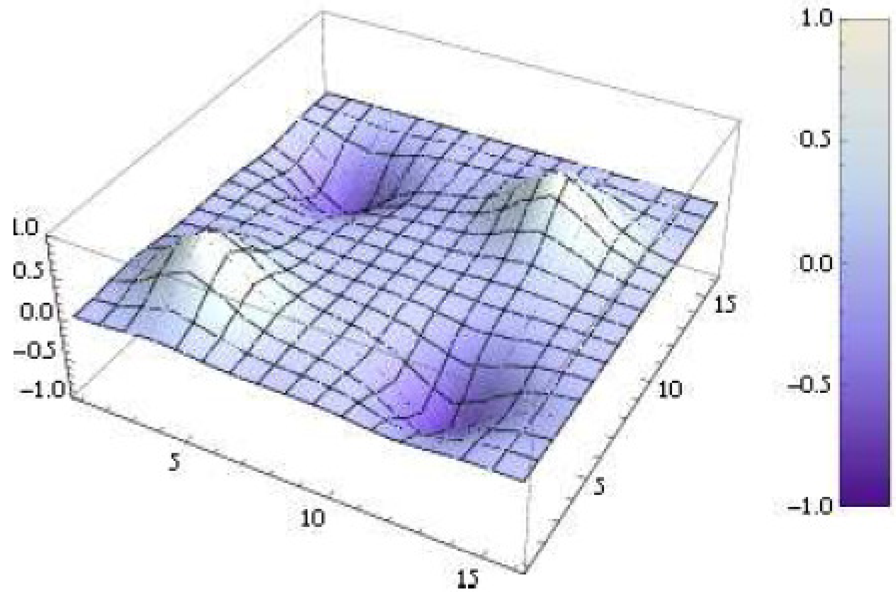
- Lattice model of classical easy-plane ferromagnet, with periodic potential

$$V_{ext}(x) = V \sin(2\pi x / L) \sin(2\pi y / L)$$

For a 12×12 unit cell, with $V = 3.0$ and $U = 8.0$ there is one vortex/antivortex per puddle.

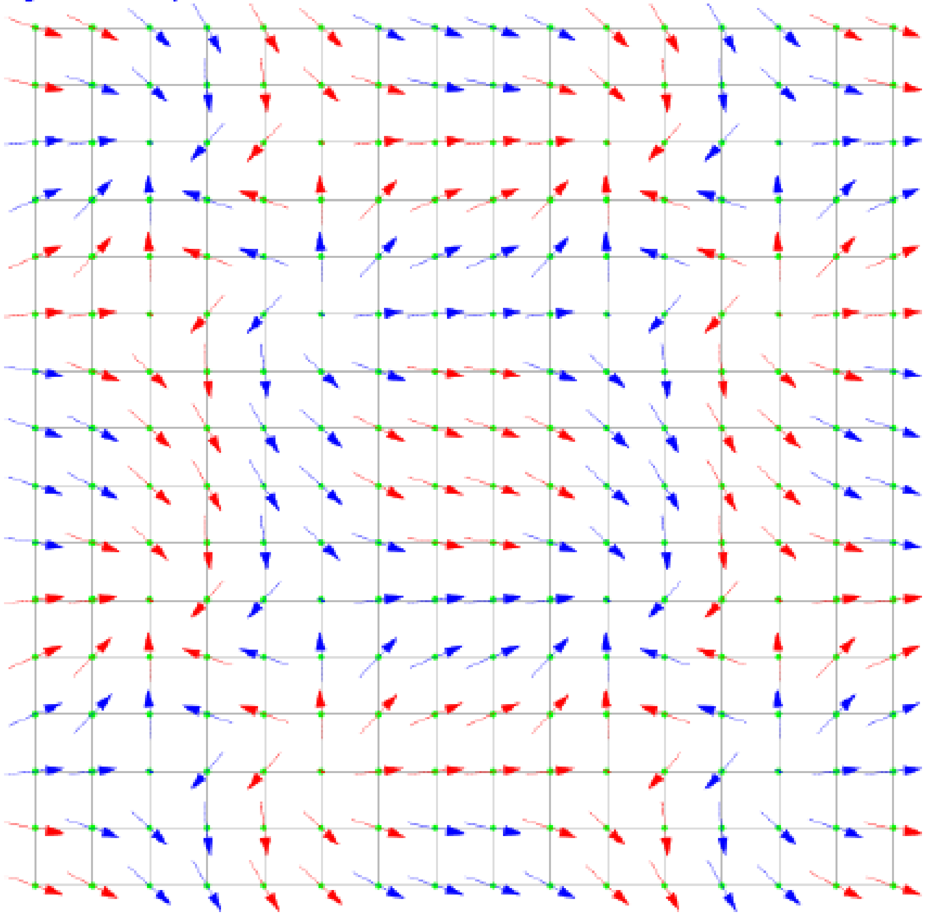


Topological density



Larger V

For $V = 7.0$ at $U = 8.0$ there are four topological objects per puddle. Generically, the phase transitions are first-order. For large U the very first phase transition is second-order.

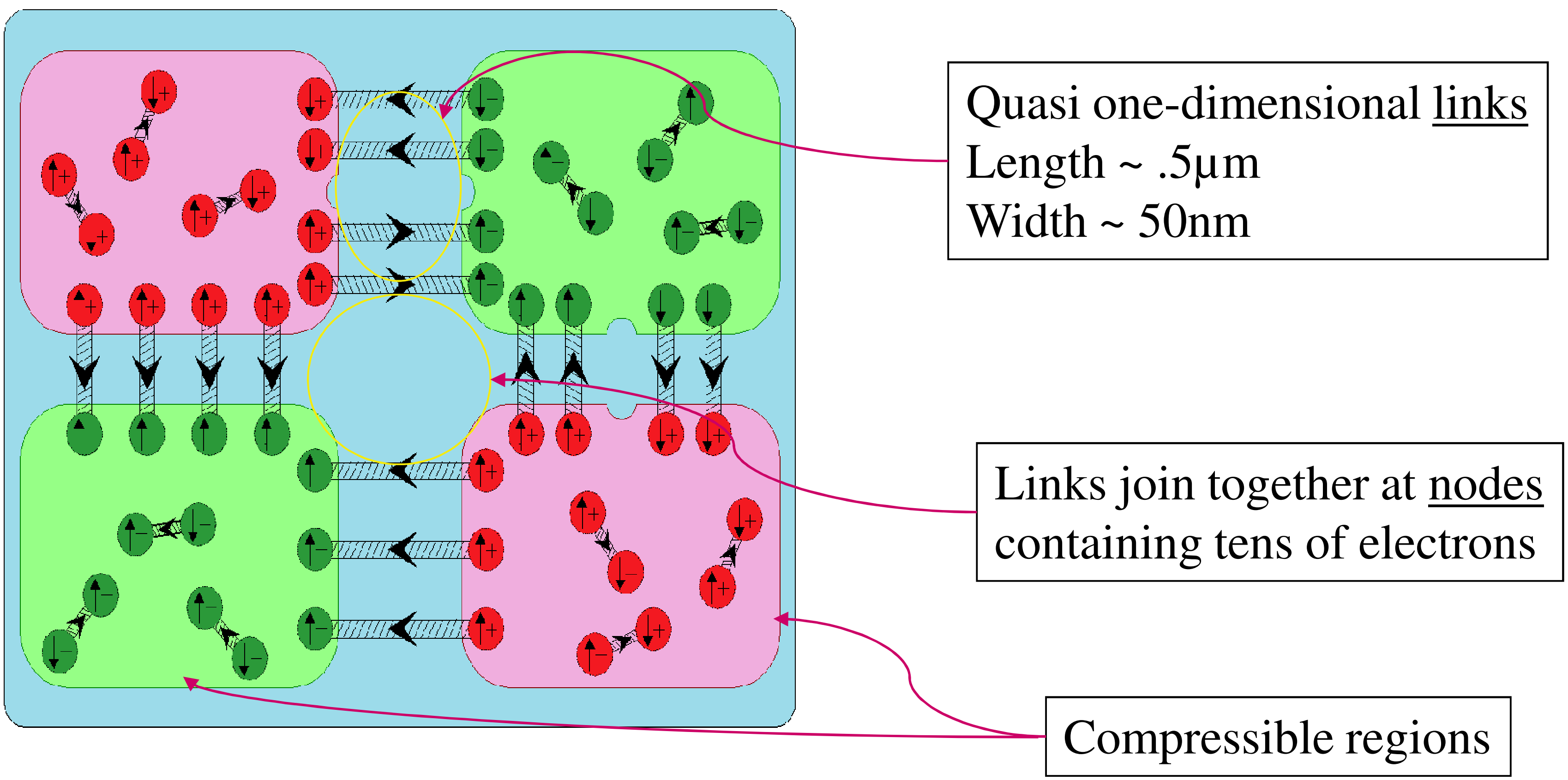


Some observations about ordered potential:

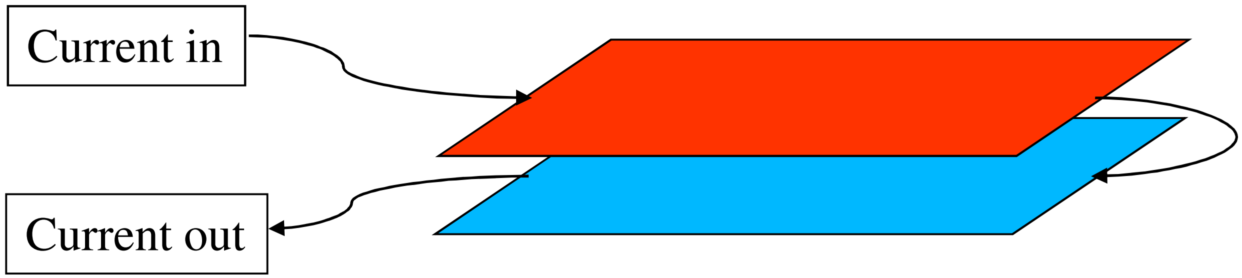
1. $T_{\mathrm{KT}} \longrightarrow 0$ at a transition between different groundstates
2. New low energy excitations near transitions: relevance to dissipation?

Bilayer System

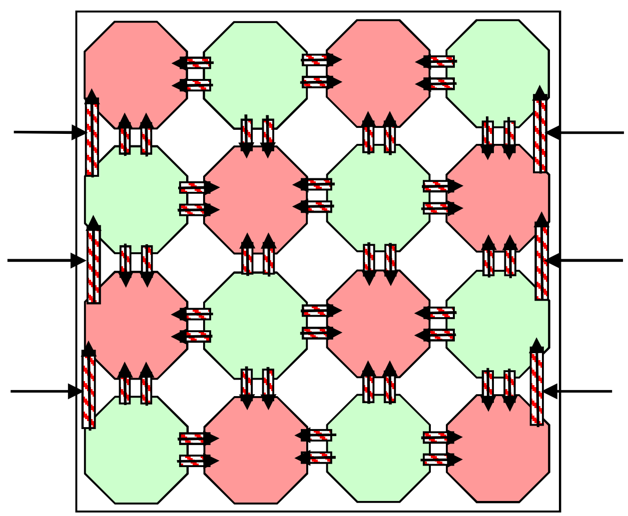
- In bilayer system, Efros strip should support *interlayer coherence*.
- Key assumption: Disorder unbinds vortices (RG calculations, simulations)
- Model system as collection of rotors, Josephson coupled to links



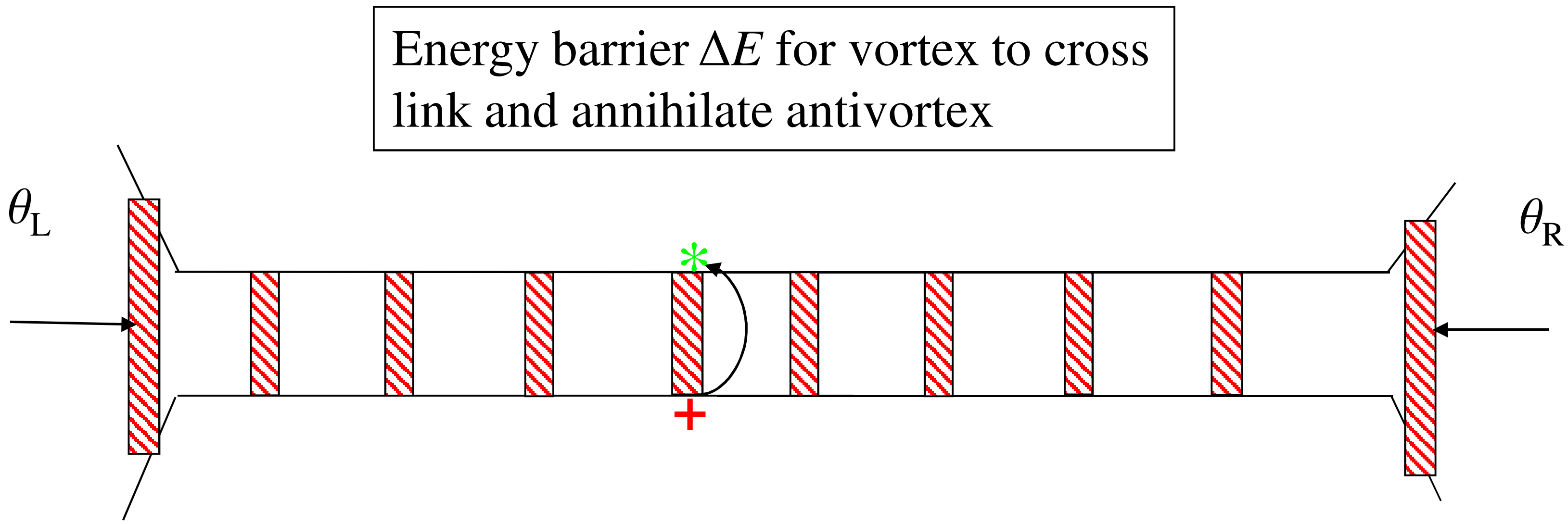
Dissipation in the Counterflow Geometry



Description in coherence network model:



- CF orientation the same at both edges
- Compression of kinks in links
- If compressed state can be maintained statically, get CF superfluidity (*phase stiffness*)
- Can "restoring force" be relaxed?



- For $T > 0$, thermally activated generation/annihilation of vortex-antivortex pairs (solitons) allows new solitons to be injected from edges
- So average phase θ_L can rotate relative to θ_R as solitons created/destroyed.
- Rate of overturn for given I_{CF} at edges

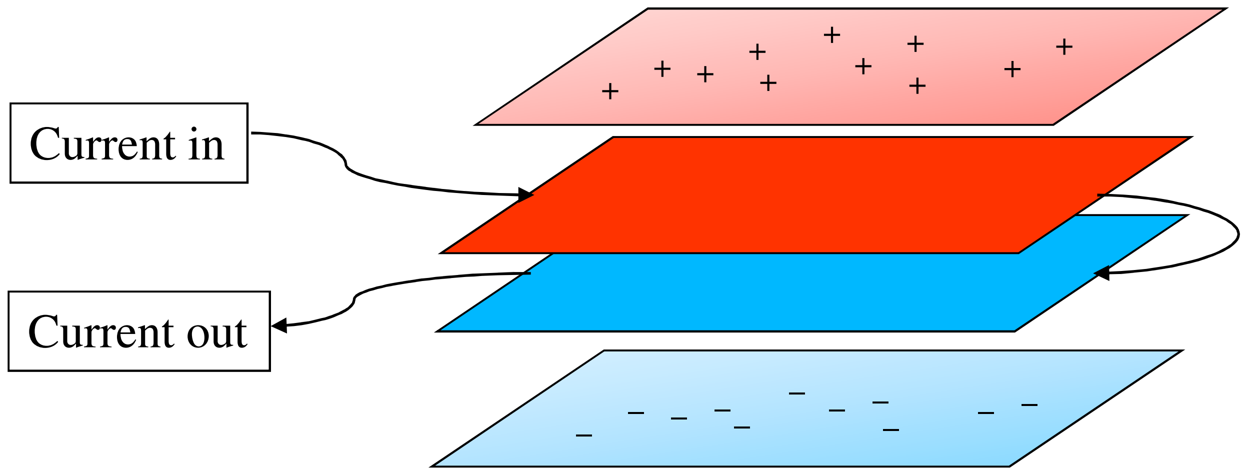
$$d\theta_L/dt = -d\theta_R/dt \sim V_L = -V_R \sim e^{-\Delta E/k_B T}$$

$$\Rightarrow R_{CF} \sim e^{-\Delta E/k_B T} \quad \text{as seen in experiment!}$$

- Note this pumps vortices across sample, so they must be in liquid state.

IV. Effect of Interlayer Bias

- What happens when layers are imbalanced ($\nu_T \neq \nu_B$)?



- System can still be (nearly) coherent – evidence that imbalance *increases* coherence (Eisenstein and coworkers; Joglekar and MacDonald)
- Ordered state in ferromagnet description has $n_z \neq 0$.

PRL 93, 266805 (2004) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2004

Activated Transport in the Separate Layers that Form the $\nu_T = 1$ Exciton Condensate

R. D. Wiersma, J. G. S. Lok, S. Kraus, W. Dietsche, and K. von Klitzing
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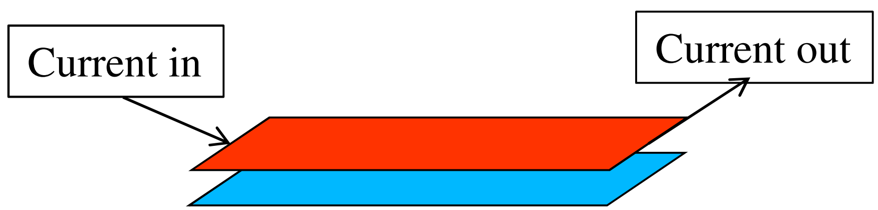
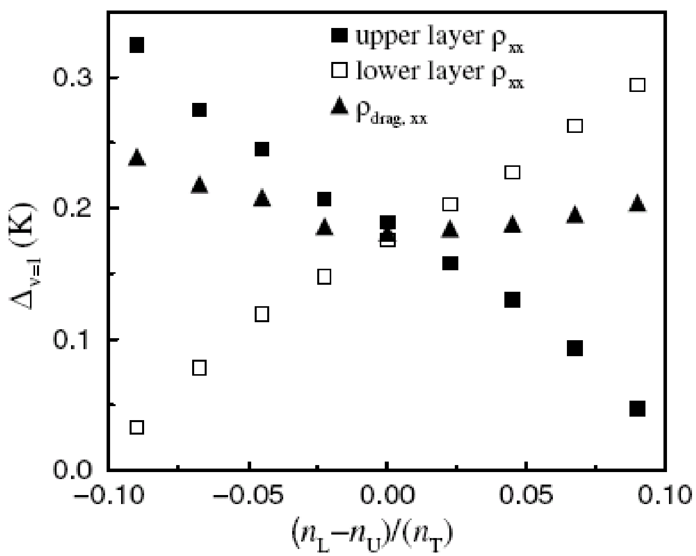
H.-P. Tranitz and W. Wegscheider



Drag measurement: current in only one layer.
Voltage drop measured in either drag or drive layer.

Results:

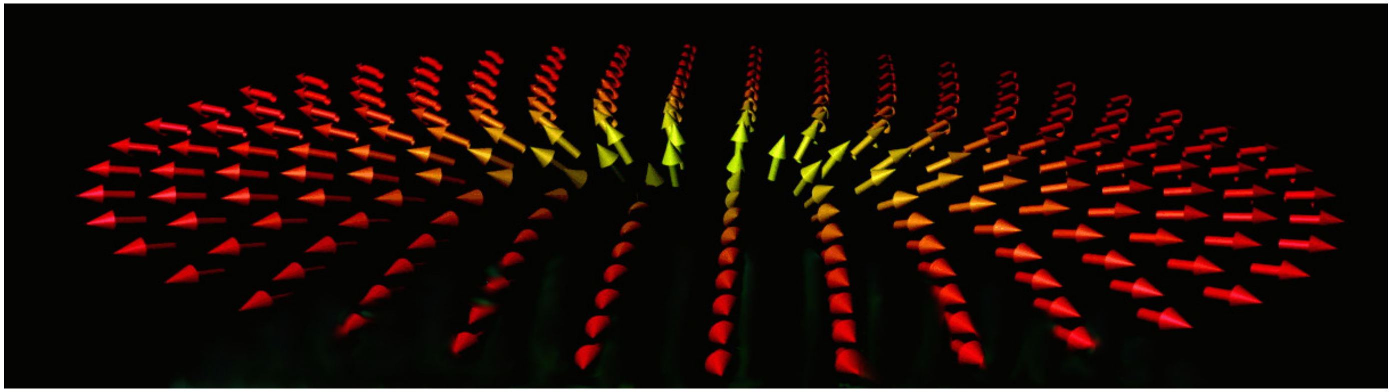
1. Scale of activation energies similar for balanced and imbalanced cases.
2. Different activation energies for drag and drive layers!
3. Activation energy for drive layer *asymmetric* around balance point.
4. Activation energy for drag layer *symmetric* around balance point.



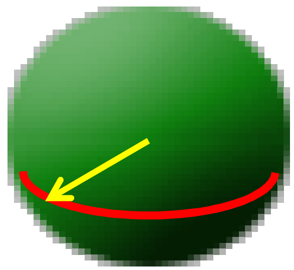
Does this imply different quasiparticles for different layers?

Activation Energies

1. "Spin" for a meron configuration tilts out of plane in the core ("Polarization"). Since bias breaks "which layer" symmetry, merons of different polarization will have different activation energies: polarization may be parallel or antiparallel to bias.

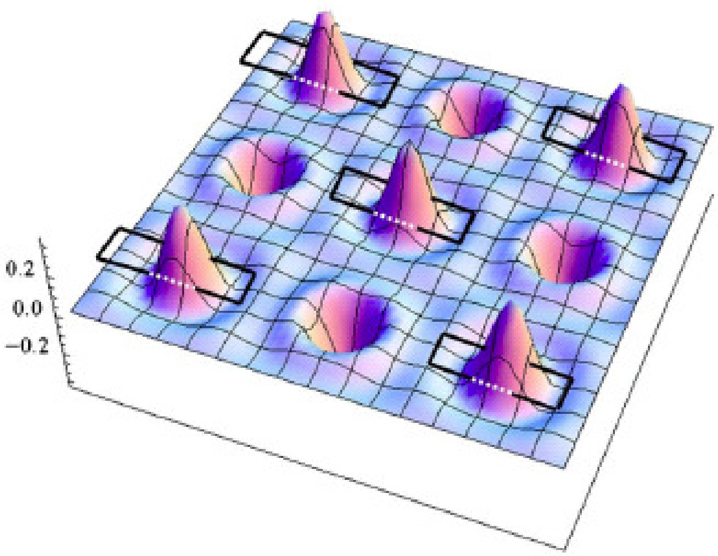


2. Charge of a meron depends on polarization: top or bottom of Bloch sphere covered, depending on polarization.



$q(\uparrow,\text{Vortex}) = -\nu_B$
$q(\uparrow,\text{Antivortex}) = \nu_B$
$q(\downarrow,\text{Vortex}) = \nu_T$
$q(\downarrow,\text{Antivortex}) = -\nu_T$

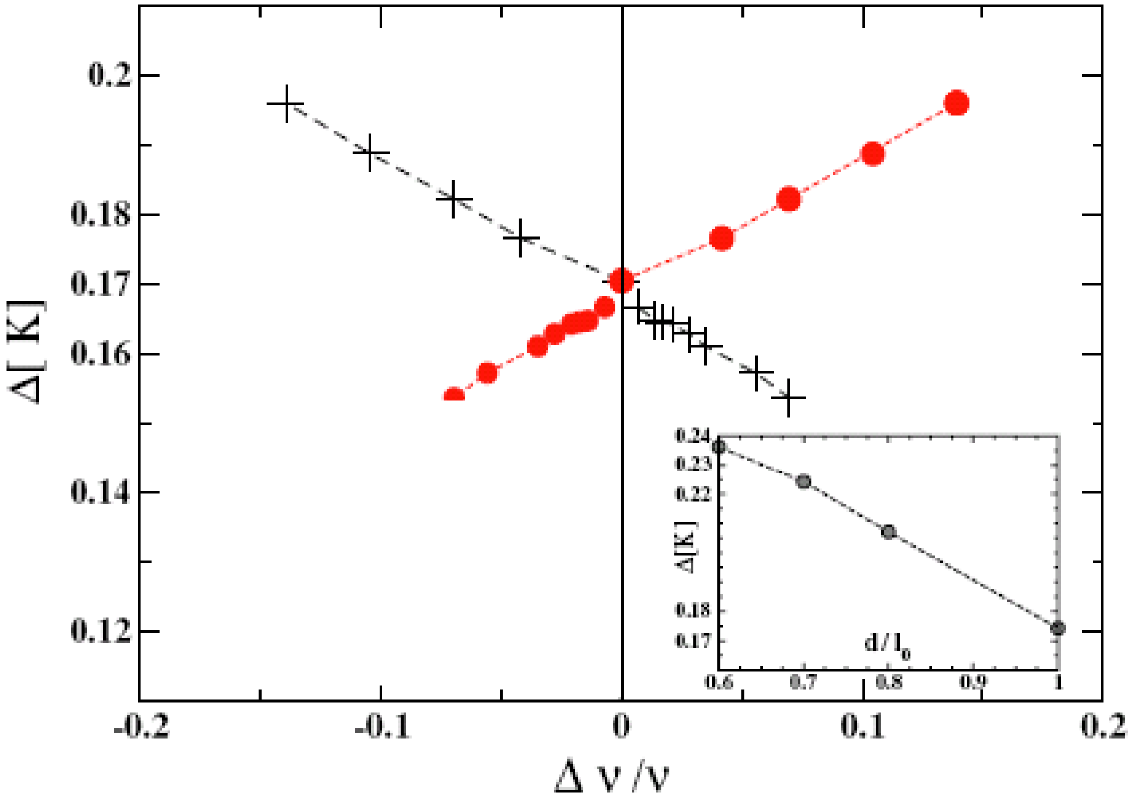
Merons with these different structures must cross incompressible strips
$\Longrightarrow$ different activation energies



Estimate this effect with <u>Hartree-Fock</u>:

• Lattice of barriers
• Meron-antimeron lattice in bias potential
• Activation energy = difference of energy for meron “on-barrier” and “off-barrier”

Result:



- Correct trend, but slope about a factor of 2 too small
- Discrepancy most likely reflects absence of quantum fluctuations in HF

“Which layer” problem for voltage drops

Why don’t we just see the lowest activation energy quasiparticle in both layers?

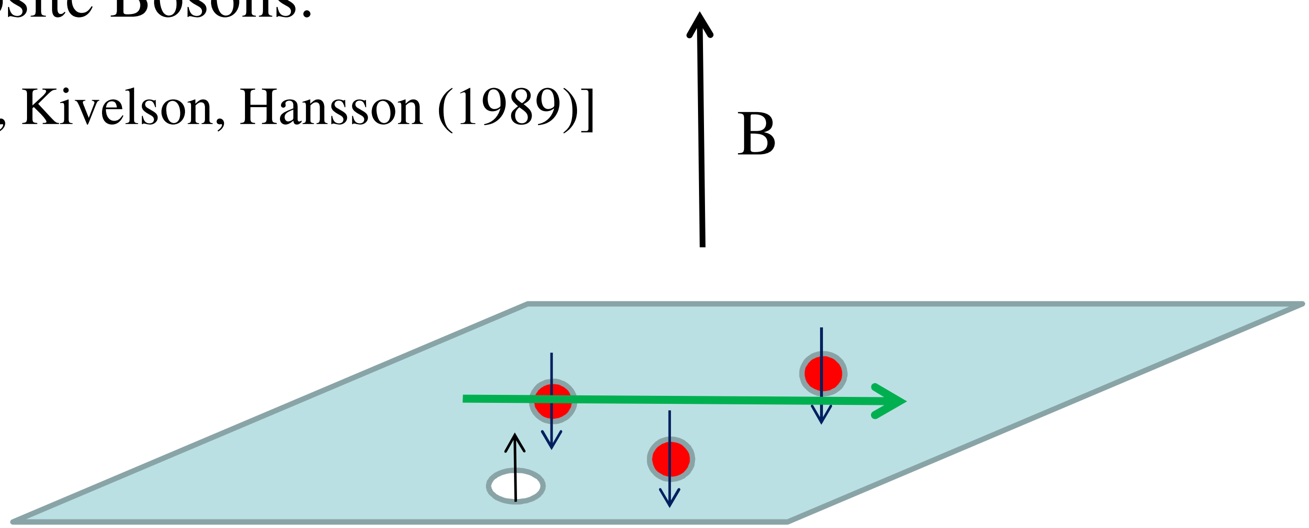
- Need to understand how motion of merons generates voltage drops in each layer

- Josephson relation for interlayer coherence phase angle tells us *interlayer* voltage. For unbiased bilayer can use symmetry to infer single layer voltage drops. What to do for a biased bilayer?

- Answer: Use composite boson description!

Composite Bosons:

[Zhang, Kivelson, Hansson (1989)]



- Describe electrons as a bosons, each carrying a single magnetic flux quantum directed in opposite direction of **B**.
- On average, magnetic field is cancelled. ν=1 quantum Hall groundstate is a Bose condensate of uniform density.
- Charged quasiparticles analogous to magnetic flux quanta in thin-film superconductor.

Current $\Longrightarrow$ Transverse force $\Longrightarrow$ Dissipative voltage drop

• **For bilayer**, since merons carry charge, they also carry magnetic flux quanta proportional to that charge.

Flux subject to force due to total current (I_{top}+I_{bot}):

$$\vec{F}_{flux} = -\frac{eq\Phi_0}{W}(\vec{I}_{top} + \vec{I}_{bot}) \times \hat{z}$$

W = width, Φ_0 = magnetic flux quantum

Interlayer phase vorticity *s* subject to force due counterflow current, in comoving frame (Stone, 1996):

$$\vec{F}_{CF} = \frac{es\Phi_0}{W}\left[\nu_{top}\vec{I}_{bot} - \nu_{bot}\vec{I}_{top}\right] \times \hat{z}$$

$$\Longrightarrow \boxed{\vec{F}_{Tot} = \frac{e\Phi_0}{W}\left[\left(s\,\nu_{top} - q\right)\vec{I}_{bot} - \left(s\,\nu_{bot} + q\right)\vec{I}_{top}\right]}$$

Suppose current flows *only* in top layer (I_{bot}= 0). Recall:

$q(\uparrow,\text{Vortex}) = -\nu_{\text{bot}}$
$q(\uparrow,\text{Antivortex}) = \nu_{\text{bot}}$
$q(\downarrow,\text{Vortex}) = \nu_{\text{top}}$
$q(\downarrow,\text{Antivortex}) = -\nu_{\text{top}}$

F_{tot}= 0 for ($\uparrow$,s=+1) and ($\uparrow$,s= -1) !!

Only vortices with polarization in bottom layer are driven!
Precise cancellation a direct result of the spin-charge relation.

Vortex (i.e., meron) motion $\Longrightarrow$ voltage drops

1. Motion of interlayer phase angle vorticity $\Longrightarrow$ interlayer voltage drop

$$\Delta V = \Delta V_{top} - \Delta V_{bot} = -\frac{h}{e} y_0 \sum_{s,\sigma} n_{s\sigma} s u_{s\sigma}$$

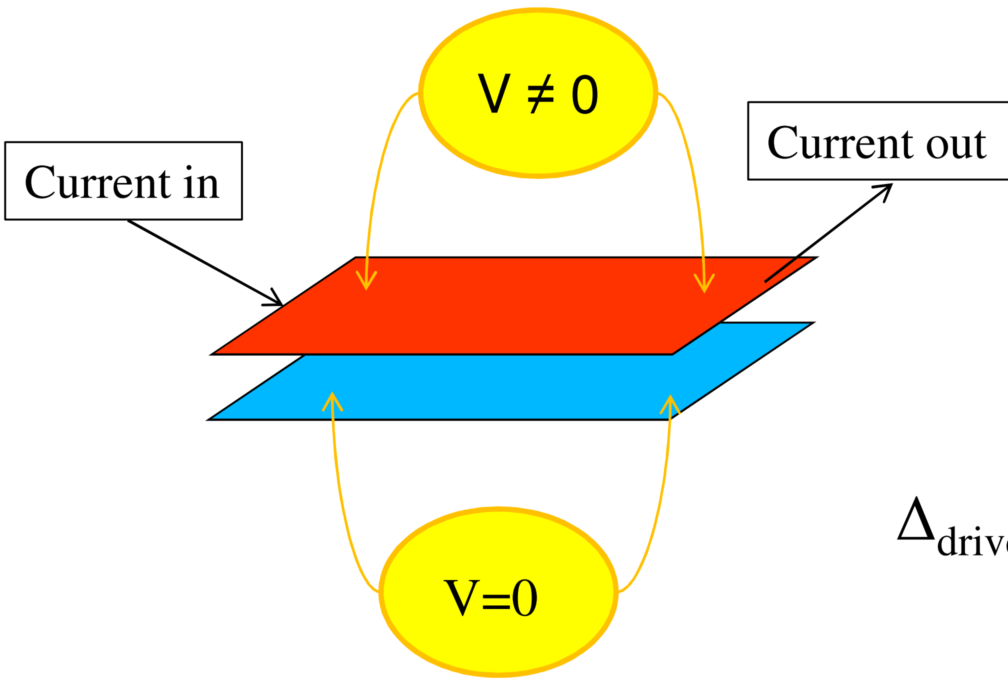
2. Motion of flux $\Longrightarrow$ layer-independent voltage drop

$$(\nu_{top} \Delta V_{top} + \nu_{bot} \Delta V_{bot}) = -\frac{h}{e} y_0 \sum_{s,\sigma} n_{s\sigma} q_{s\sigma} u_{s\sigma}$$

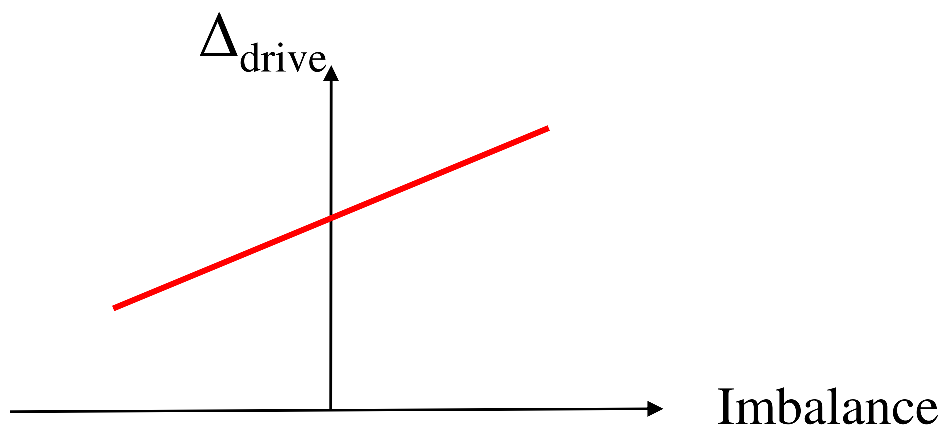
y_0 = distance between voltage probes

$\boxed{\text{For } I_{bot}=0, \text{ find } \Delta V_{bot} = 0!}$

At this level, in a drag measurement all the voltage drop is in the drive layer, and is determined by the activation energy of a single polarization of meron.

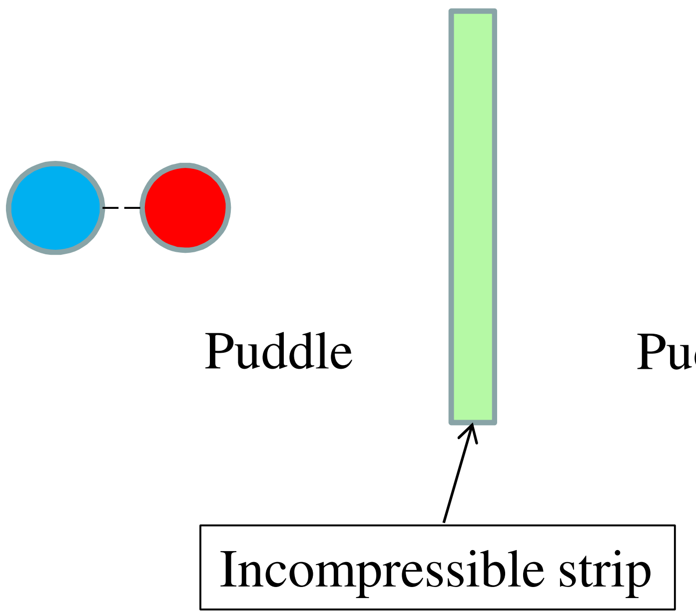


- Agrees with experimental observation that drive layer voltage >> drag layer voltage



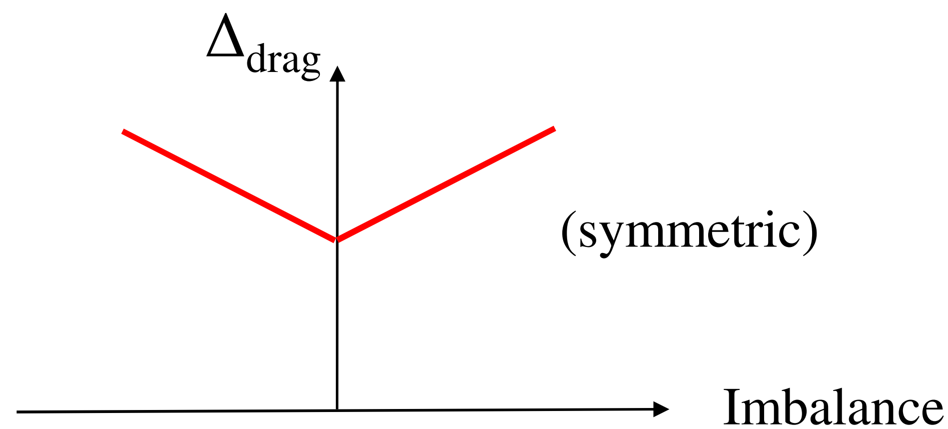
What about observed activation energy in drag layer?

Merons are not always unpaired: undriven merons may be dragged across barriers.



Meron core size > incompressible strip width

$\Longrightarrow$ activation barrier $\approx$ $\mathrm{Max}(\Delta_{\uparrow}, \Delta_{\downarrow})$



Summary

- Describe the bilayer $\nu = 1$ system in terms of an exciton condensate.
- Disorder induces meron-antimerons pairs, spoiling perfect superfluidity.
- Coherence network model explains vanishing dissipation in counterflow in zero temperature limit.
- Biased system displays multiple activation energies in drag experiments.
- Describing the system in terms of composite bosons allows full description of voltage drops, and demonstrates that this behavior is a natural consequence of unpaired merons thermally hopping over barriers between meron-rich regions.

Refs: HAF and G. Murthy, PRL **95,** 156802 (2005).
B. Roostaei, K. Mullen, HAF, S. Simon, PRL **101,** 046804 (2008) .
J. Sun, G. Murthy, HAF, N. Bray-Ali, PRB **81**, 195314 (2010).