From Ultracold Atoms to Realization of Strongly Correlated Condensed Matter Models

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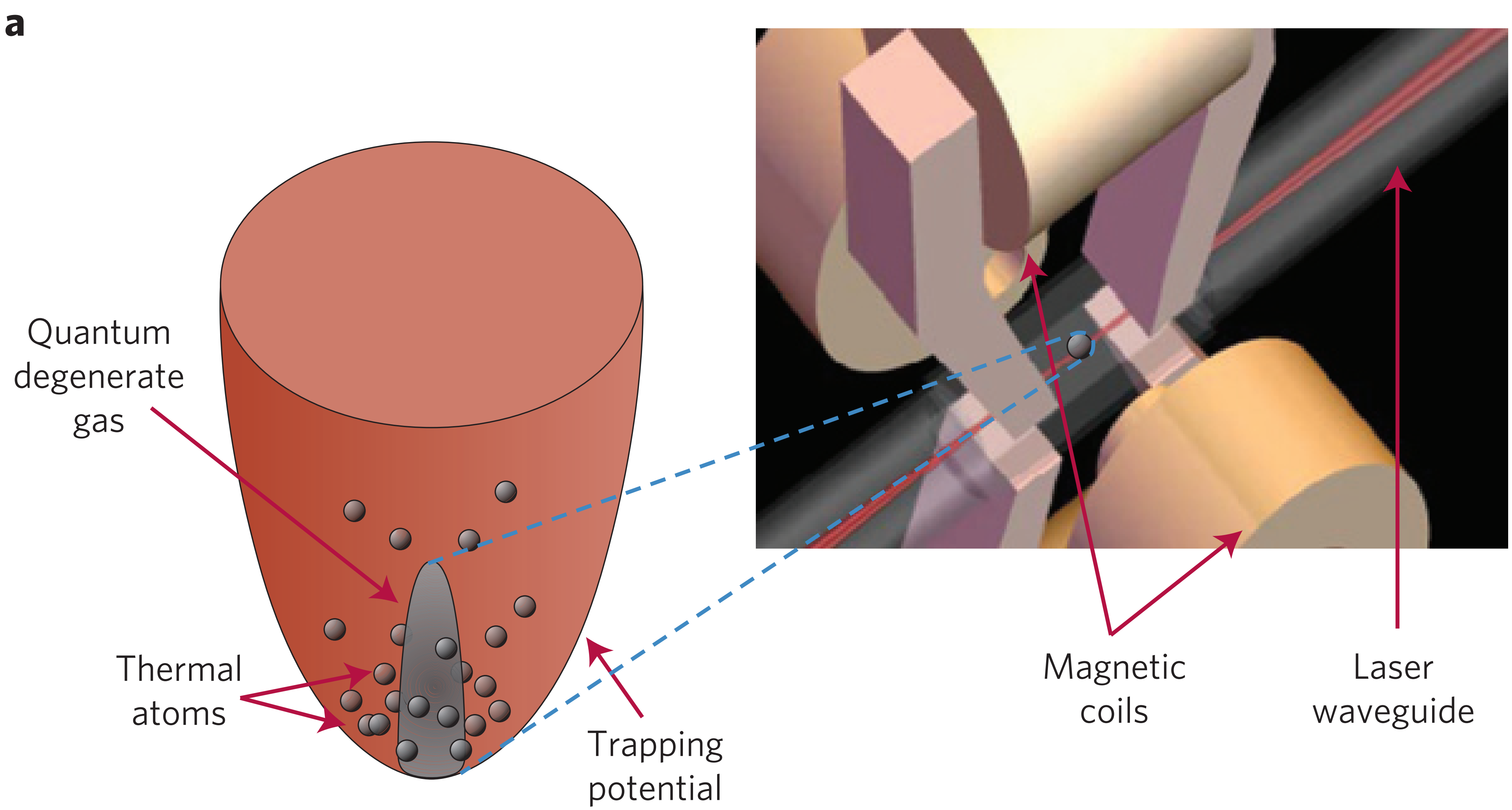
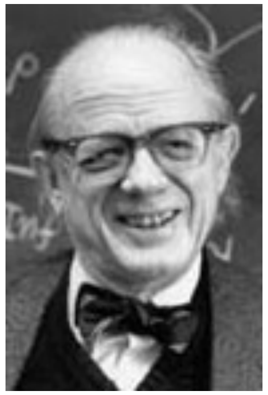
Outline

- Ultracold Atoms Realization of Condensed Matter Models;
- Strongly Correlated States - Realization and Detection;
- Repulsive Interactions - Exotic Strongly Correlated States;
  - ✳ Luttinger Liquids;
- Attractive Interactions - Pairing States and Spin Imbalance;
  - ✳ Analog of the Fulde-Ferrell-Larkin-Ovchinnikov State;
- Interplay of Disorder, Interactions and Confinement;
  - ✳ Dissipative Dipole Oscillations of a Trapped BEC;

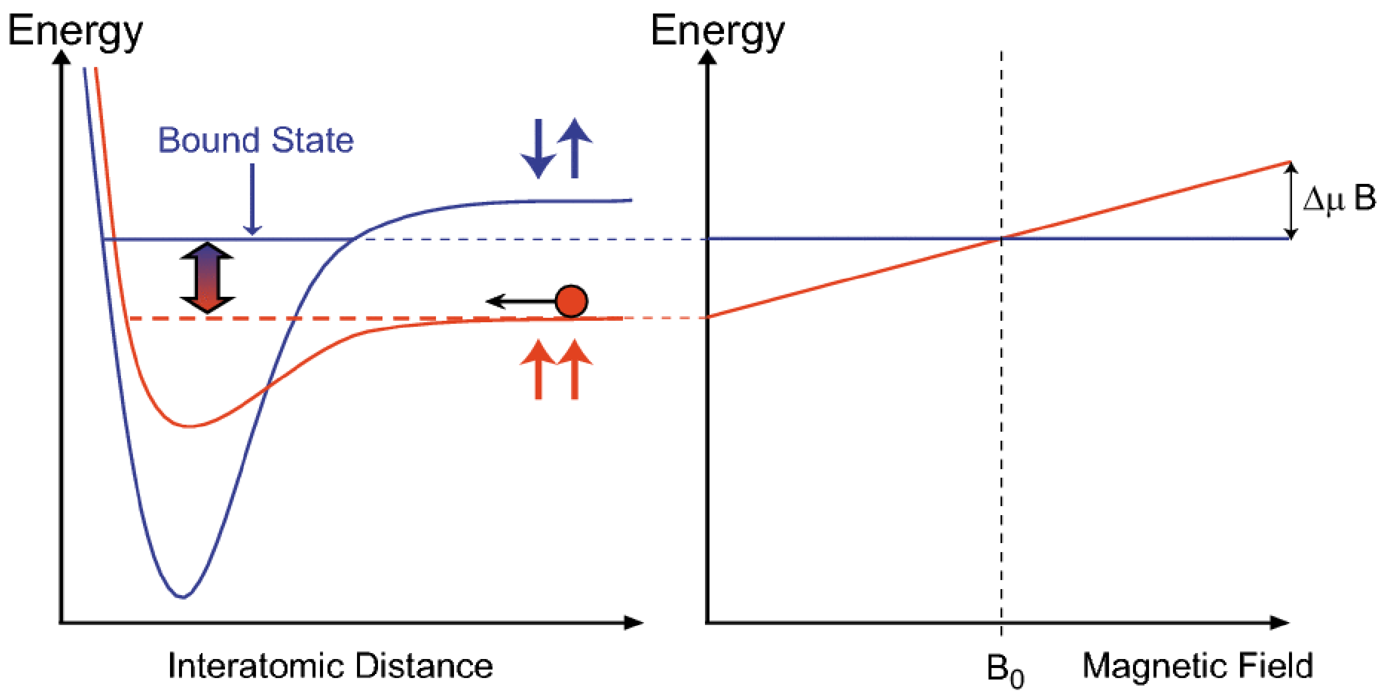
Confinement



L. Sanchez-Palencia and M. Lewenstein, Nat. Phys. **6**, 87 (2010)



Interaction - Feshbach Resonance



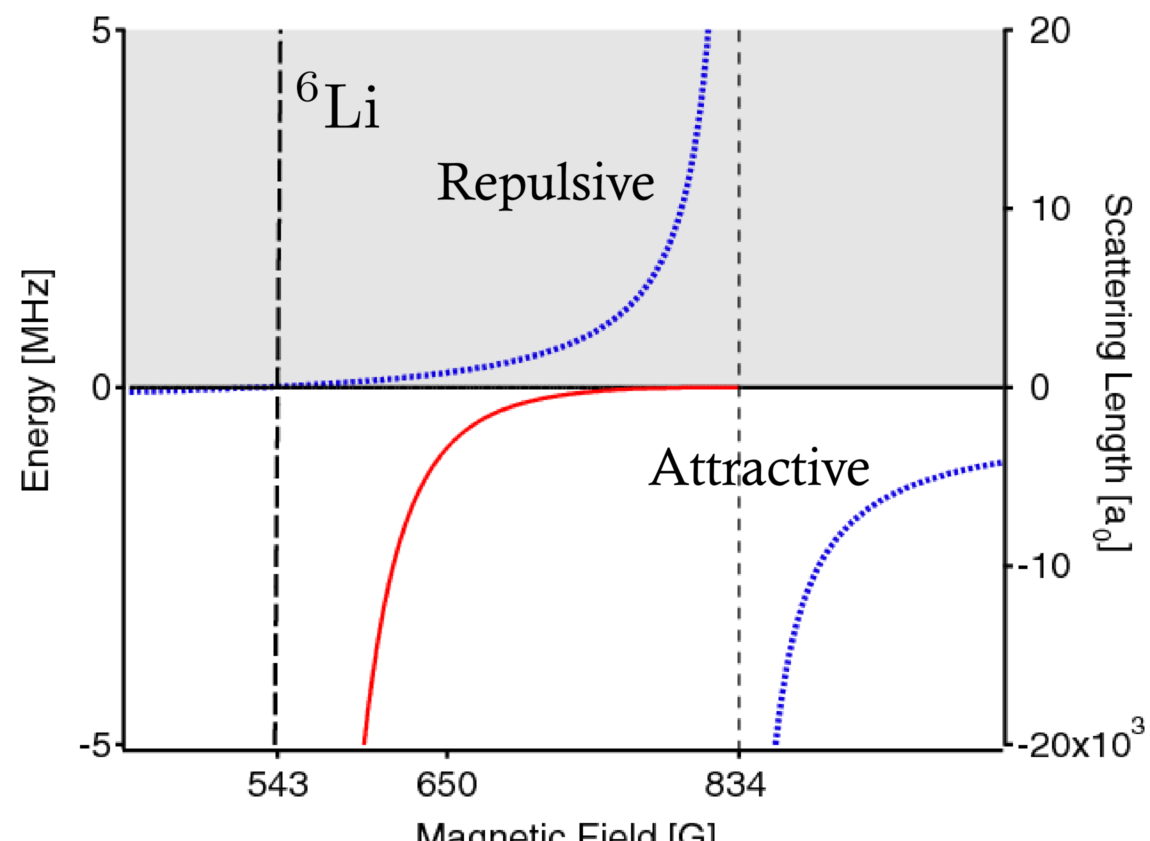
Ketterle, Zwierlein, arXiv:0801.2500

H. Feshbach - late 50s
Unified theory of nuclear scattering

Feshbach resonance occurs as a bound state in the interatomic potential is tuned into resonance with energy of two colliding atoms.

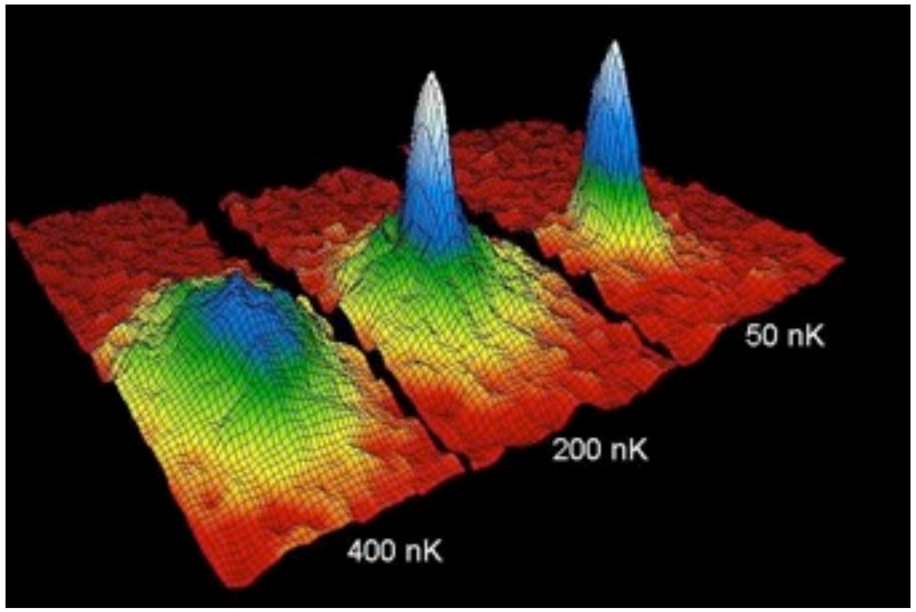
Tuning is achieved by an applied magnetic field and the scattering length and thus the interaction can be changed in the broad range.

$$V(\vec{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\vec{r})$$



Ketterle, Zwierlein, arXiv:0801.2500

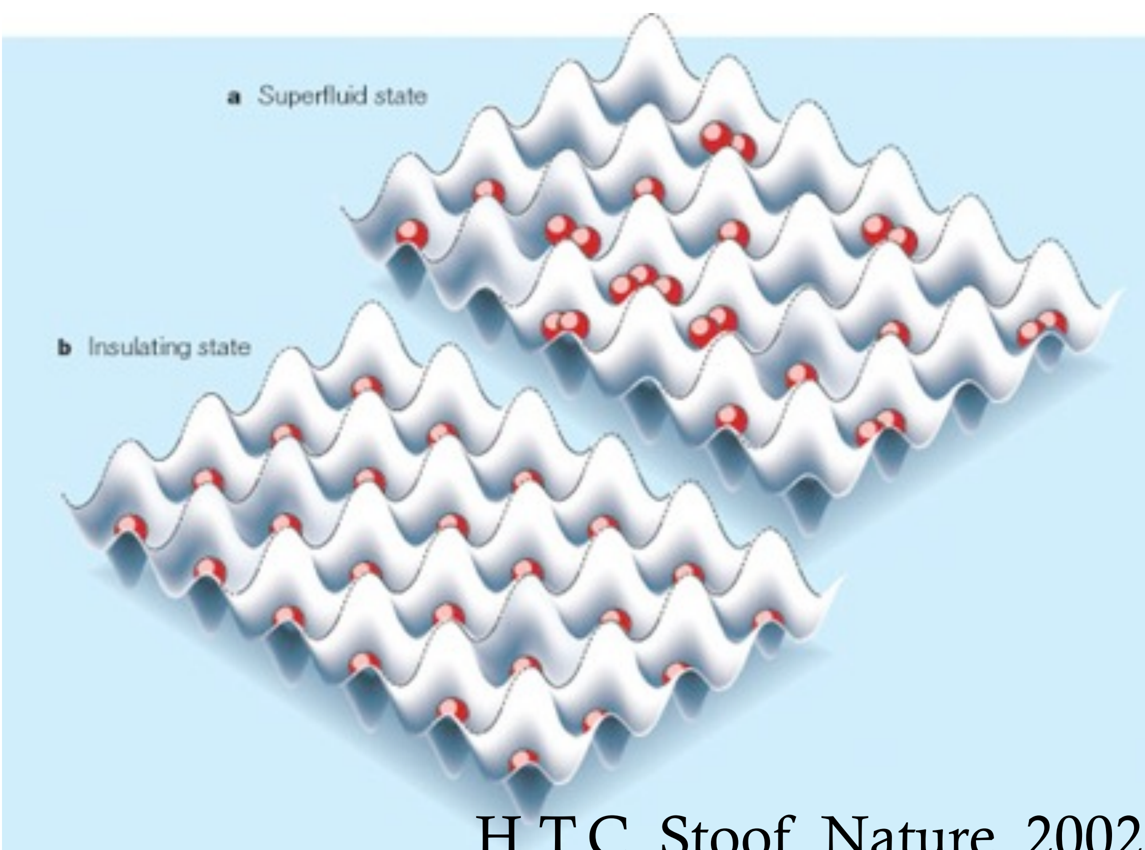
From Atomic BEC to Condensed Matter Physics



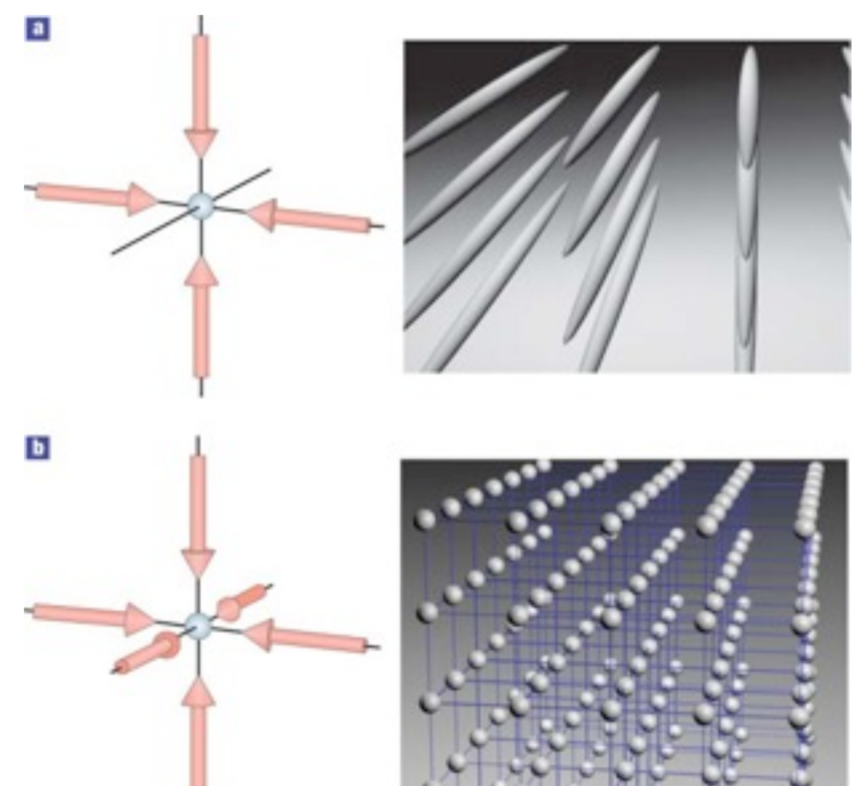
E. Cornell's group

- BEC to BCS crossover;
- Superfluid to Mott insulator transition;
- Berezinskii-Kosterlitz-Thouless physics;
- Tonks-Girardeau gas;
- Anderson Localization and phase transition;
- Spin-imbalance Physics and 3D and 1D;
- FFLO Physics?
- Luttinger liquid?
- Disorder?

Reduction of dimensionality using optical lattices

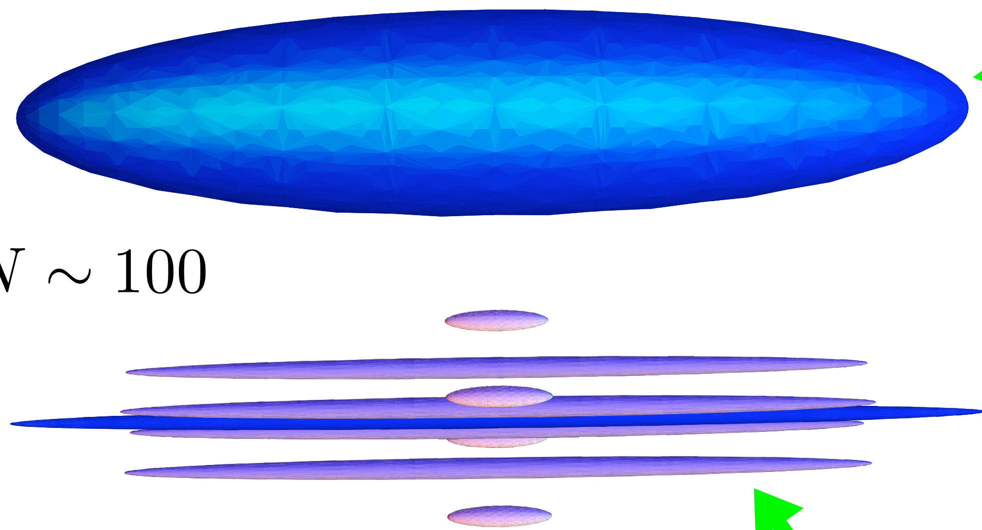


H.T.C. Stoof, Nature, 2002



I. Bloch, Nature Physics, 2005

Reduction to One Dimension



Hamiltonian for a 3D cloud

$$H_{3\mathrm{D}} = \sum_{\sigma=\uparrow,\downarrow} \int d^3x \psi^\dagger_\sigma(\vec{x}) \left[\frac{p^2}{2m} + V(\vec{x})\right] \psi_\sigma(\vec{x})$$
$$+ \; U_{3\mathrm{D}} \int d^3x \psi^\dagger_\uparrow(\vec{x}) \psi^\dagger_\downarrow(\vec{x}) \psi_\downarrow(\vec{x}) \psi_\uparrow(\vec{x})$$

$$V(\vec{x}) = m\omega_\perp^2(x^2+y^2)/2 + m\omega_z^2 z^2/2$$
$$+V_o \left[\sin^2(2\pi x/\lambda) + \sin^2(2\pi y/\lambda)\right]$$

Hamiltonian for a 1D tube $\quad \hbar\omega_z \ll E \ll \hbar\omega_O$

$$H_{1\mathrm{D}} = \sum_{\sigma=\uparrow,\downarrow} \int dz \psi^\dagger_\sigma(z) \left[\frac{p_z^2}{2m} + \frac{m\omega_z^2 z^2}{2}\right] \psi_\sigma(z)$$
$$+ \; U_{1\mathrm{D}} \int dz \psi^\dagger_\uparrow(z) \psi^\dagger_\downarrow(z) \psi_\downarrow(z) \psi_\uparrow(z)$$

$$U_{1\mathrm{D}} = \frac{2\hbar^2 a_{3\mathrm{D}}}{m a_\perp^2} \frac{1}{1 - 1.03 a_{3\mathrm{D}}/a_\perp}$$

Can be readily tuned using **Feshbach resonance** and can become effectively attractive (<0) or **repulsive** (>0).

Realizing 1D Physics - Fermions

Repulsive Interactions

Paradigm of 1D metallic systems - Luttinger Liquid.

Attractive Interactions

Properties of paired states.

Strong Correlations

Different regimes of the Luttinger Liquid and crossover between them.

Spin-coherent and -incoherent Luttinger Liquids

1D analog of the Fulde-Ferrell-Larkin-Ovchinnikov state for spin-imbalanced systems.

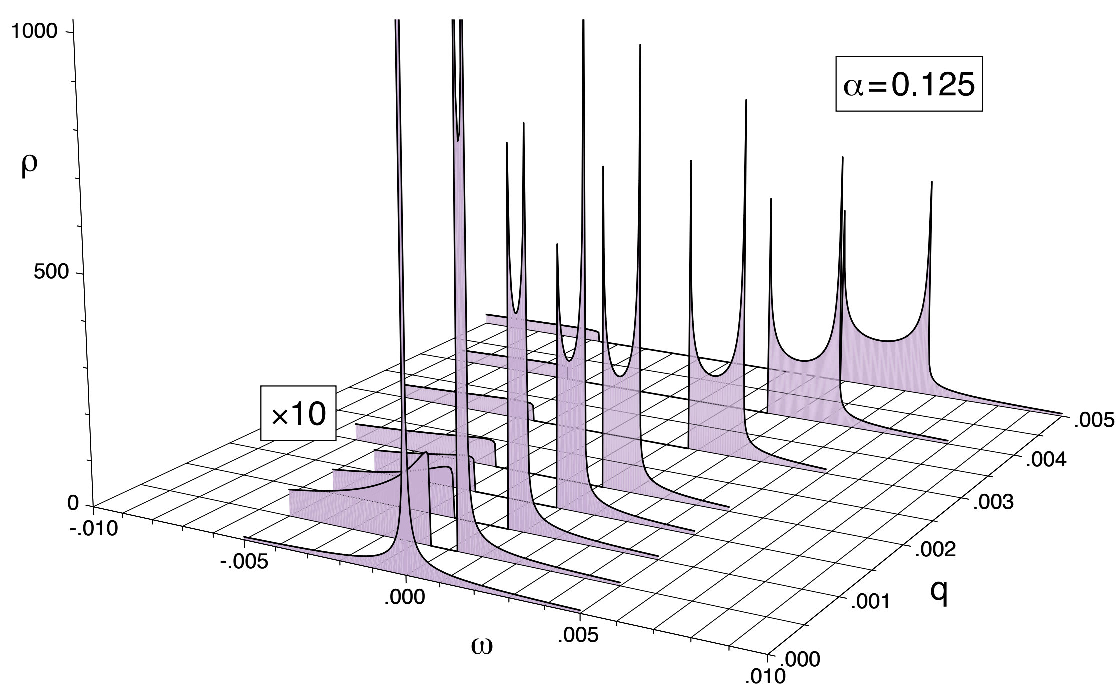
Phase Diagram of a uniform system

(Spin-Coherent)Luttinger Liquid

Breakdown of Fermi liquid theory:

$$G_{\mathrm{coh}}(\mathbf{k},\omega) = \frac{Z_{\mathbf{k}}}{\omega - \tilde{\varepsilon}_{\mathbf{k}} + \mathrm{i}/\tau_{\mathbf{k}}} \qquad \boxed{Z_{\mathbf{k}} \to 0}$$

- No quasiparticles but collective excitations;
- Power law behavior of correlators with interaction dependent exponents;
- Spin-charge separation;
- Suppression of the density of states at the Fermi level;



J. Voit, Rep. Prog. Phys. 58, 977 (1995)

Bosonization:

$$\Psi_\sigma(x) = e^{ik_F x}\psi_{R,\sigma}(x) + e^{-ik_F x}\psi_{L,\sigma}(x)$$

$$\psi_{r,\sigma} = \frac{\eta_{r,\sigma}}{\sqrt{2\pi\alpha}}\exp(ir\sqrt{4\pi}\phi_{r,\sigma})$$

$$J_{r,c/s} = \frac{1}{\sqrt{4\pi}}(\partial_z\varphi_{c/s} - r\Pi_{c/s})$$

$$H_{TL} = \sum_{\alpha=c,s}\frac{v_\alpha}{2}\int dx\left[K_\alpha\tilde{\Pi}_\alpha^2 + \frac{1}{K_\alpha}(\partial_z\tilde{\varphi}_\alpha)^2\right]$$

$$v_c = v_F\sqrt{\left(1+\frac{g_{4,c}}{\pi}\right)^2 - \left(\frac{g_{2,c}}{\pi}\right)^2}$$

$$v_s = v_F\left(1+\frac{g_{4,s}}{\pi}\right)$$

$$K_c = \sqrt{\frac{\pi - g_{2,c} + g_{4,c}}{\pi + g_{2,c} + g_{4,c}}} \qquad K_s = 1$$

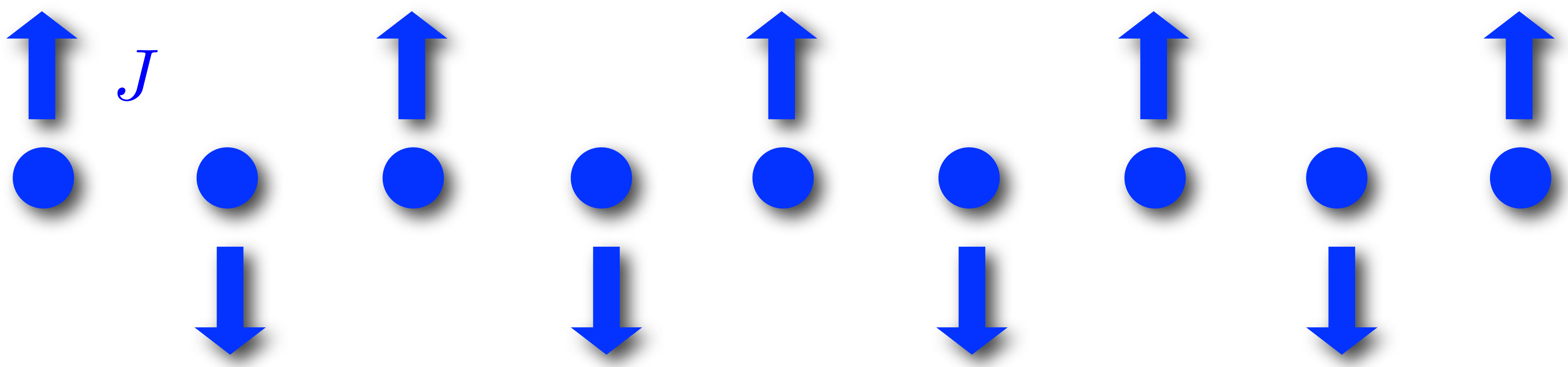
Spin-coherent Vs Spin-incoherent Luttinger Liquid

Uniform system

$T \ll J \ll E_F$ $\quad$ $k_F = \dfrac{\pi n}{2}$

Spins are correlated

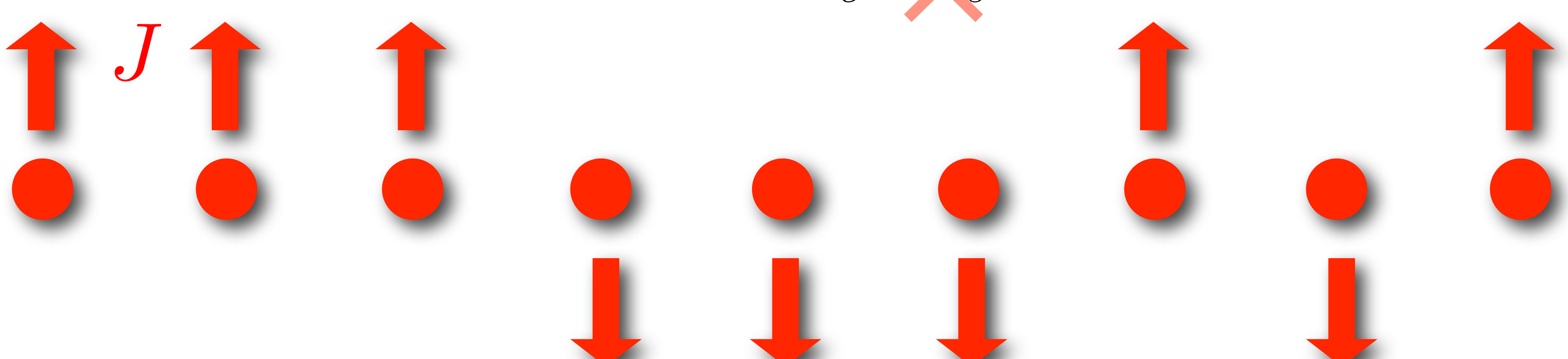
$$H = H_c + H_s$$



$J \ll T \ll E_F$ $\quad$ $k_F = \pi n$

Spins are uncorrelated

$$H = H_c + \cancel{H_s}$$



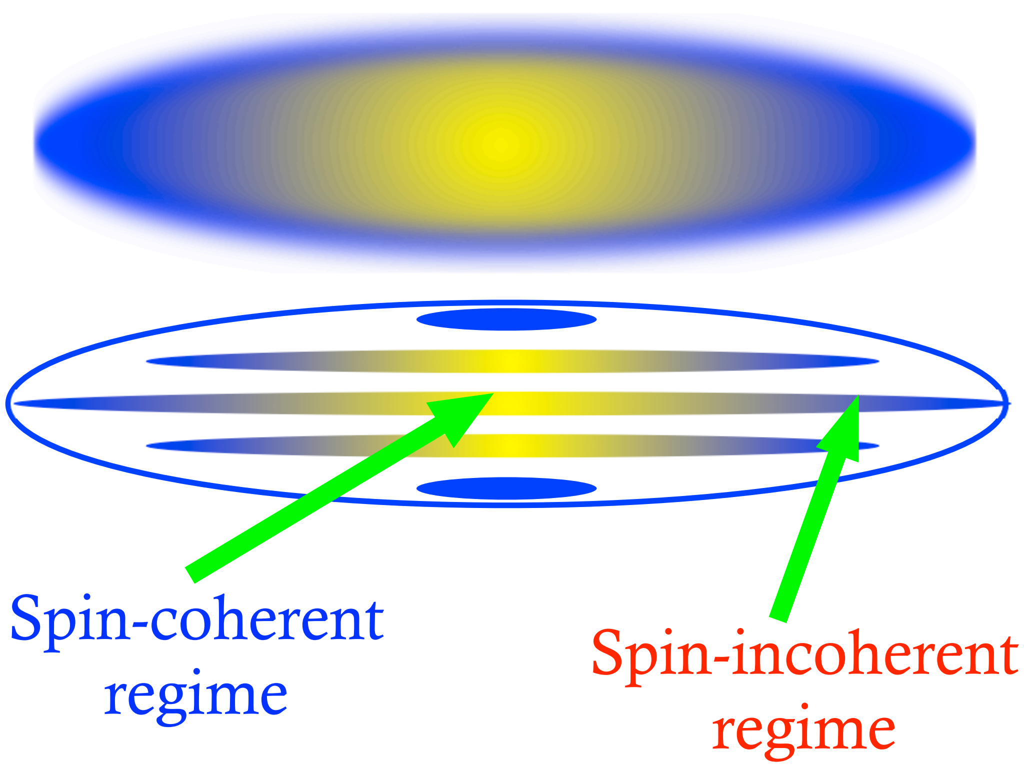
Spin-coherent Vs Spin-incoherent Luttinger Liquid

- Low-energy excitations are collective and separately carry spin and charge (spin-charge separation);
- Single-particle Green function decays as power law and the exponent depend on the LL interaction parameters in spin and charge sector;
- Low-energy excitations consist of only charge excitations, spin excitations are non-propagating (non-unitary spin-charge separation) [Cheianov and Zvonarev, PRL, 2004, Fiete and Balents, PRL, 2004];
- Single-particle Green function decays as power law in the charge sector with a non-unitary exponent; decay in the spin sector has an exponential nature [Cheianov and Zvonarev, PRL, 2004, Fiete and Balents, PRL, 2004];

Single-Particle Green Function

$$G(x,x',\tau) \sim \frac{e^{-(\ln 2/\pi)\tilde{k}_F|r|}}{((\alpha\ \mathrm{sign}\tau + v_c\tau)^2 + r^2)^{\Delta}} \left(\frac{e^{-i\tilde{k}_F r} e^{i\zeta}}{(\alpha\ \mathrm{sign}\tau + v_c\tau + ir)} + c.c. \right)$$

Spin-coherent and Spin-incoherent Luttinger Liquids in a Trap



Local Density (Thomas-Fermi) approximation

$$\frac{dE[\langle\rho\rangle]}{d\langle\rho\rangle} = \mu - V_{\text{trap}}(z)$$

$$\langle\rho\rangle(z) = \langle\rho_0\rangle\sqrt{1-\frac{z^2}{R^2}}$$

The spin-coherent regime at the center of the cloud crosses over to the spin-incoherent counterpart at the edges.

$$H_{\text{SC}} = \sum_{\alpha=c,s}\int dz \frac{v_\alpha(z)}{2}\left[K_\alpha(z)\Pi_\alpha^2 + \frac{1}{K_\alpha(z)}(\partial_z\varphi_\alpha)^2\right]$$

$K_c(z) \approx K_c(0)$ approximation

$K_s(z) = 1$

$$H_{\text{SI}} = \int dz \frac{v_c(z)}{2}\left[K_c\Pi_c^2 + \frac{1}{K_c}(\partial_z\varphi_c)^2\right]$$

$K_c(z) = 1/2$

Spin-coherent and Spin-incoherent Luttinger Liquids in a Trap

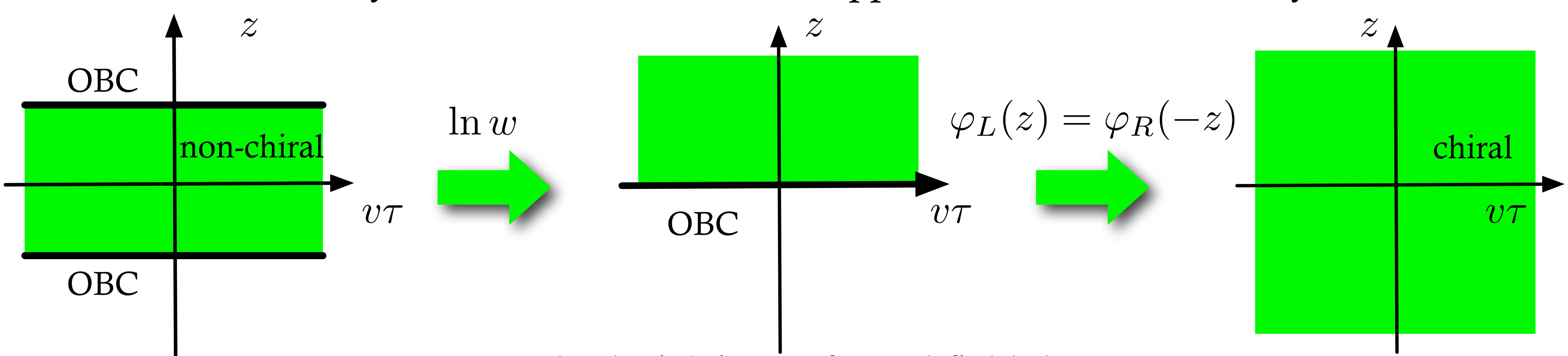
Using nonstandard transformations, we map a system in a trap onto a **finite uniform system with open boundary conditions (OBCs)** at the the edges.

$$d\tilde{z}_\alpha = dz/\tilde{v}_\alpha(z) \qquad v_\alpha(z) = v_\alpha(0)\tilde{v}_\alpha(z) = v_{\alpha,0}\tilde{v}_\alpha(z)$$

$$\tilde{\varphi}_\alpha(\tilde{z}_\alpha) = \varphi_\alpha(z(\tilde{z}_\alpha)) \qquad \tilde{\Pi}_\alpha(\tilde{z}_\alpha) = \tilde{v}_\alpha(z(\tilde{z}_\alpha))\Pi_\alpha(z(\tilde{z}_\alpha))$$

$$H_{\mathrm{SC}} = \sum_{\alpha=c,s} \frac{v_{\alpha,0}}{2} \int_{-\tilde{z}_\alpha(R)}^{\tilde{z}_\alpha(R)} d\tilde{z}_\alpha \left[K_{\alpha,0}\tilde{\Pi}_\alpha^2 + \frac{1}{K_{\alpha,0}} (\partial_{\tilde{z}_\alpha}\tilde{\varphi}_\alpha)^2 \right] \qquad H_{\mathrm{SI}} = \frac{v_{c,0}}{2} \int_{-\tilde{z}_c(R)}^{\tilde{z}_c(R)} d\tilde{z}_c \left[K_c\tilde{\Pi}_c^2 + \frac{1}{K_c} (\partial_{\tilde{z}_c}\tilde{\varphi}_c)^2 \right]$$

Performing a conformal transformation and an analytic continuation, a finite non-chiral system with OBCs can be mapped to an infinite chiral system.



standard trick in conformal field theory

How to Observe These Regimes?
Density-Density Correlator

Detect spin-charge separation (Recati et al., PRL, 2003; Kecke et al., PRL 2005)

Our proposal - Density-Density Correlator (Kakashvili, Bhongale, Pu and Bolech, PRA, 2008; see also Altman, Demler, and Lukin, PRA, 2004)

$$G_{\uparrow\downarrow}(z,z') = \langle \delta\rho_\uparrow(z)\delta\rho_\downarrow(z')\rangle$$

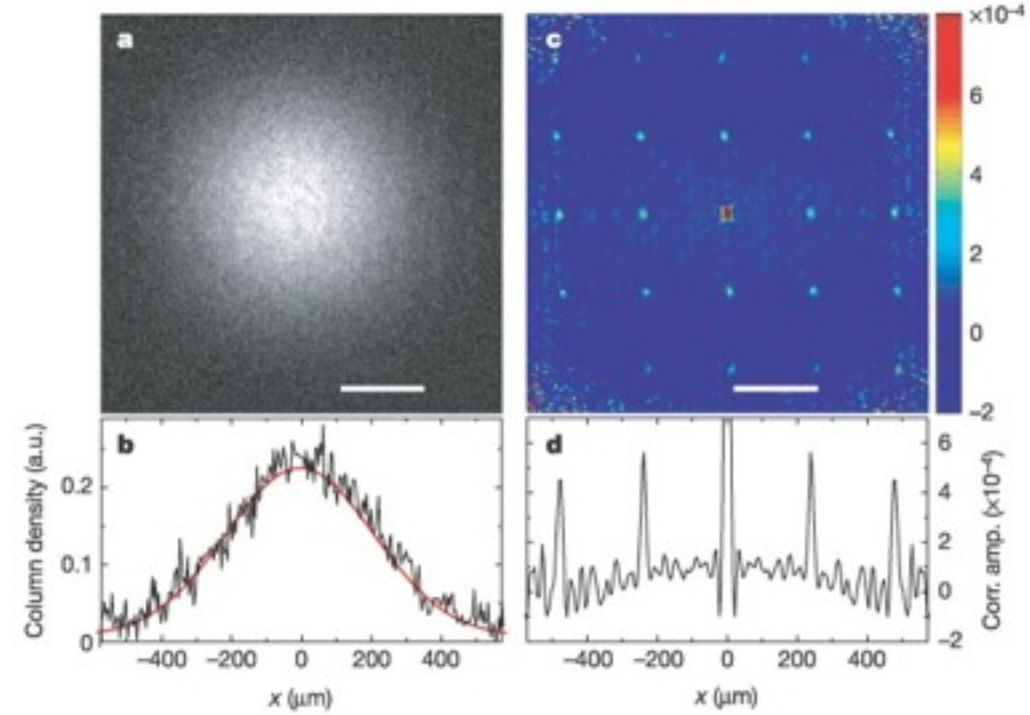
$$G^{\mathrm{SC}}_{\uparrow\downarrow}(z,z') = G_c(z,z') - G_s(z,z')$$

$$G_\alpha(z,z') = -\frac{K_{\alpha,0}}{\tilde{R}^2_\alpha}\left(\frac{1}{\sin^2\Delta^-_\alpha} + \frac{1}{\cos^2\Delta^+_\alpha}\right)$$

$$G^{\mathrm{SI}}_{\uparrow\downarrow}(z,z') = -\frac{1}{2\tilde{R}^2_c}\left(\frac{1}{\sin^2\Delta^-_c} + \frac{1}{\cos^2\Delta^+_c}\right)$$

$$\Delta^\pm_\alpha = \frac{\pi(\tilde{z}_\alpha(z) \pm \tilde{z}_\alpha(z'))}{4\tilde{z}_\alpha(R)}$$

$$\tilde{R}^2_\alpha = 64\tilde{z}^2_\alpha(R)\tilde{v}_\alpha(z)\tilde{v}_\alpha(z')$$



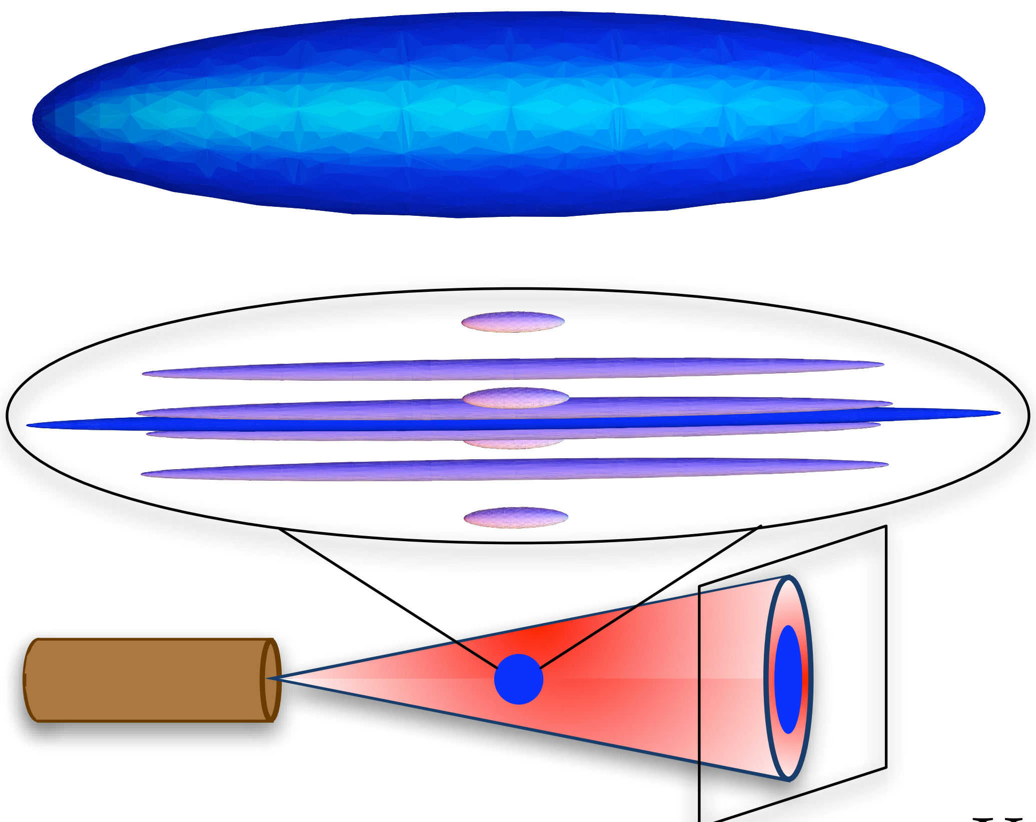
Fölling et al., Nature, 2005

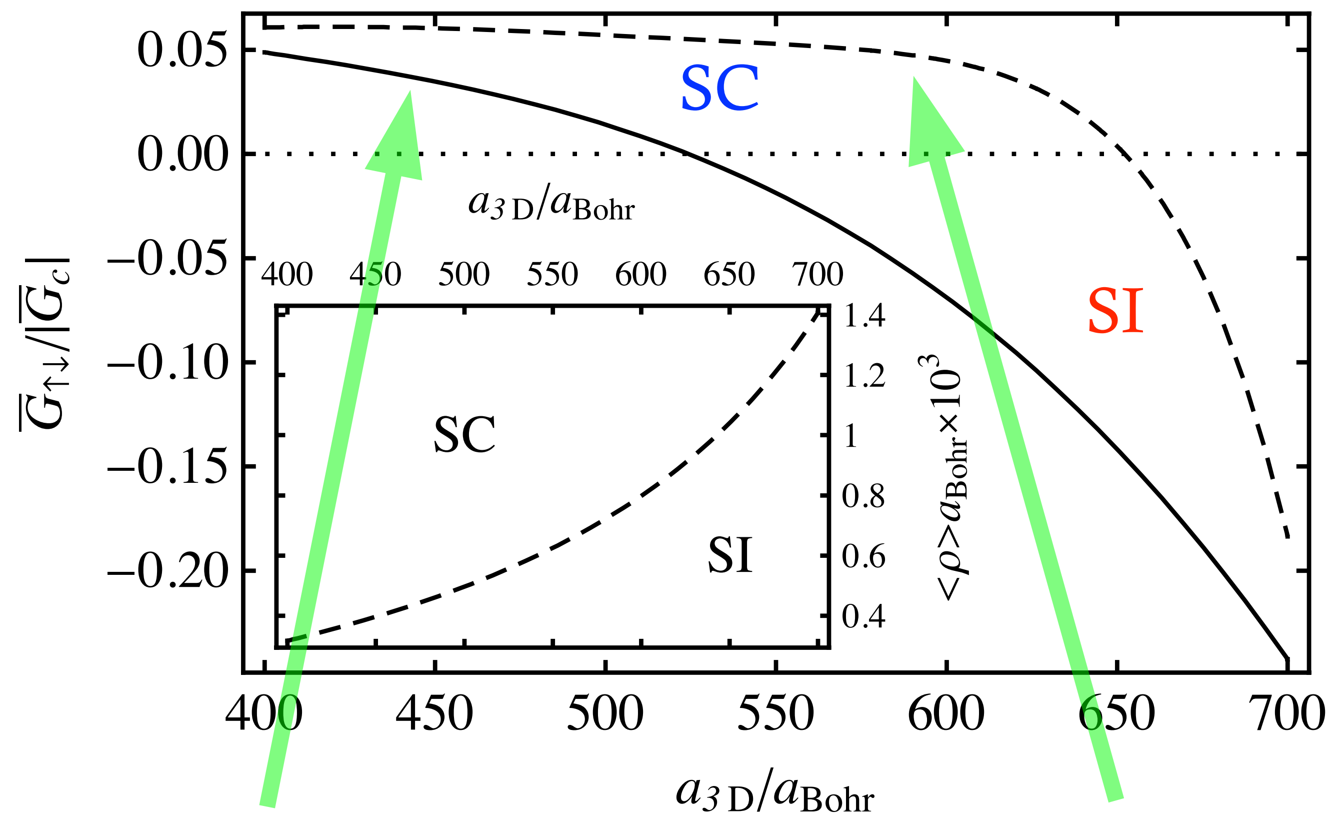
$$G^{\mathrm{SC}}_{\uparrow\downarrow}(z,z') > 0$$

$$G^{\mathrm{SI}}_{\uparrow\downarrow}(z,z') < 0$$

$$G^{\mathrm{NI}}_{\uparrow\downarrow}(z,z') = 0$$

How to Observe These Regimes?





Uncorrelated array of tubes

Central tube

- Robust observable, with high signal-to-noise ratio;
- Natural observable for cold atom experiments;
- Highly unusual for condensed matter realizations.

$$\bar{G}_{\uparrow\downarrow} = \sum_i \int_{-R}^{R} G^{ii}_{\uparrow\downarrow}(z,z')dzdz'$$

P. K., S. G. Bhongale, H. Pu and C. J. Bolech, Phys. Rev. A **78**, 041602(R) (2008)

Realizing 1D Physics

Repulsive Interactions

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Attractive Interactions

Properties of paired states.

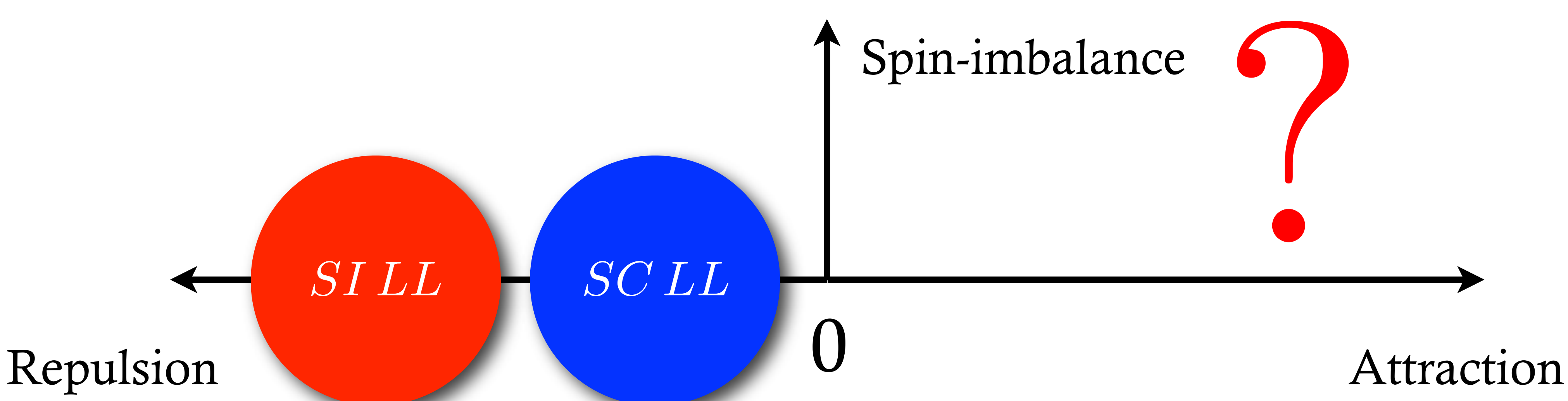
Strong Correlations

Different regimes of the Luttinger Liquid and crossover between them.

Spin-coherent and -incoherent Luttinger Liquids

1D analog of the Fulde-Ferrell-Larkin-Ovchinnikov state for spin-imbalanced systems.

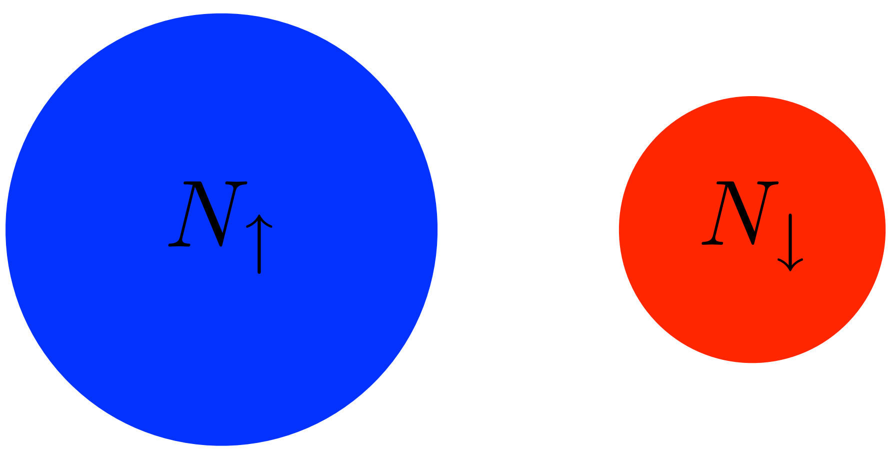
Phase Diagram of a uniform system

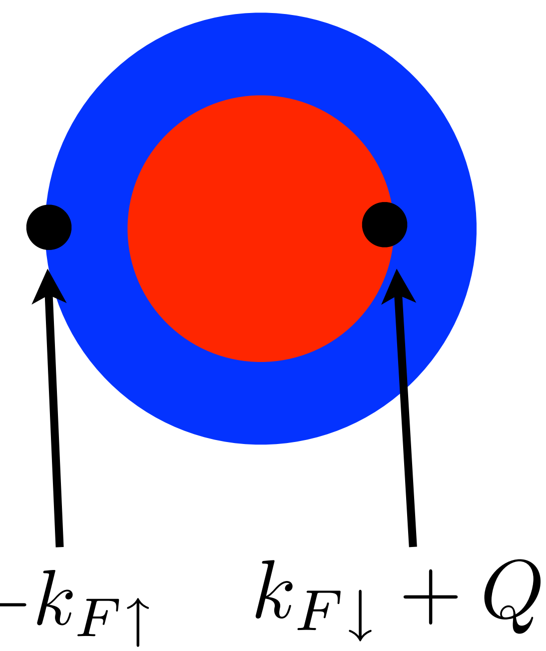


Attractive Interactions - Pairing States for Spin-imbalanced Systems

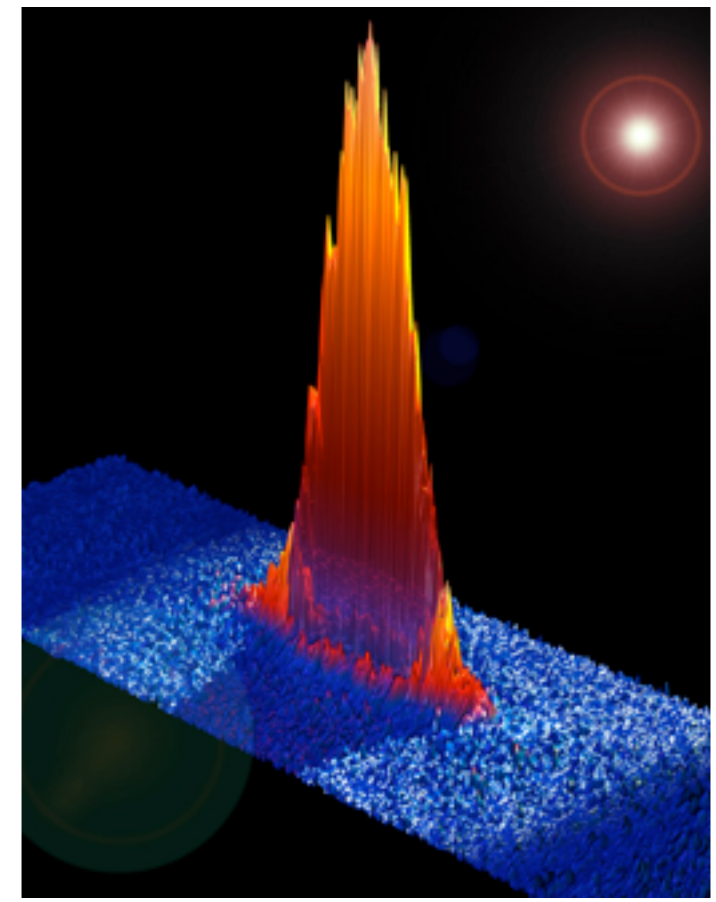
Spin-imbalanced case - Exotic Superconductivity
FFLO (Fulde, Ferrell, Larkin and Ovchinnikov) State;

Quasi-1D organic superconductors, heavy-fermion materials, neutron stars.





$$\Delta(x) \sim e^{iQx}; \quad \sin(Qx)$$



R. Hulet's Group

Experiment:

<u>*R. Hulet*</u> - Experimental realization using 2D optical lattices.

Recent theoretical considerations:

Mean Field - *Huse, Parish*;
QMC - *Casula, Ceperley, Mueller*;
Bogoliubov - De Gennes Equation - *Liu, Hu, Drummond, Baksmaty, Bhongale, Pu*;
Bosonization - *Yang, Liu, Zhao, Liu*;
DMRG - *Rizzi, Polini, Cazalilla, Bakhtiari, Tosi, Fazio*;
Bethe Ansatz - *Orso, Hu, Liu, Drummond, Mueller, Kakashvili, Bolech, Zhao, Liu*;

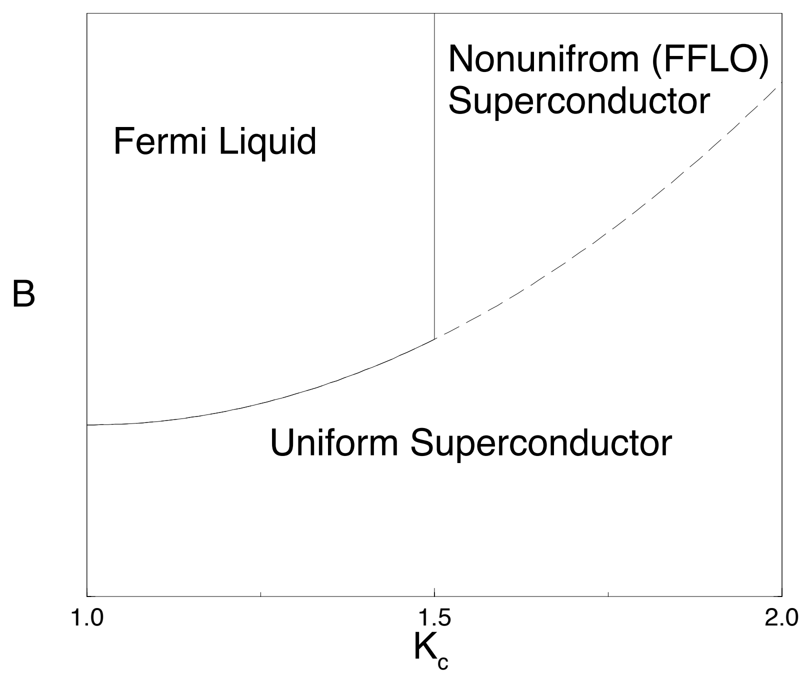
FFLO in One-Dimensional Systems

Zero Temperature

Bosonization

K. Yang, PRB, 2001;

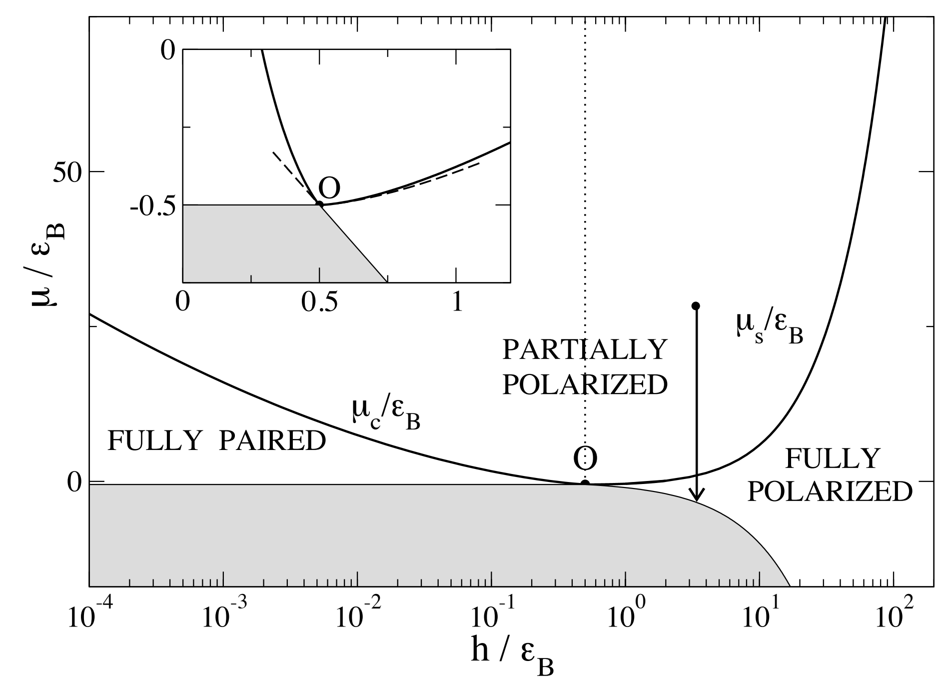
E. Zhao, W. V. Liu, arXiv: PRA, 2008.



Bethe Ansatz

G. Orso, PRL, 2007;

H.Hu, X.-J. Liu, P. Drummond, PRL, 2007.



No long-ranger order;
Power-law decay of the order parameter;

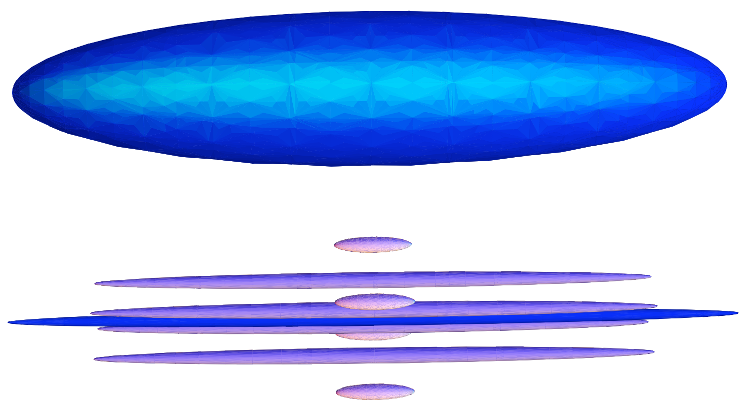
$$< \Delta^\dagger(x)\Delta(x') > \propto e^{iQ(h)(x-x')}|x-x'|^{-(1/K_c+1/2)}$$

$$Q(h) \propto (h-h_c)^{1/2} \qquad p \neq 0$$

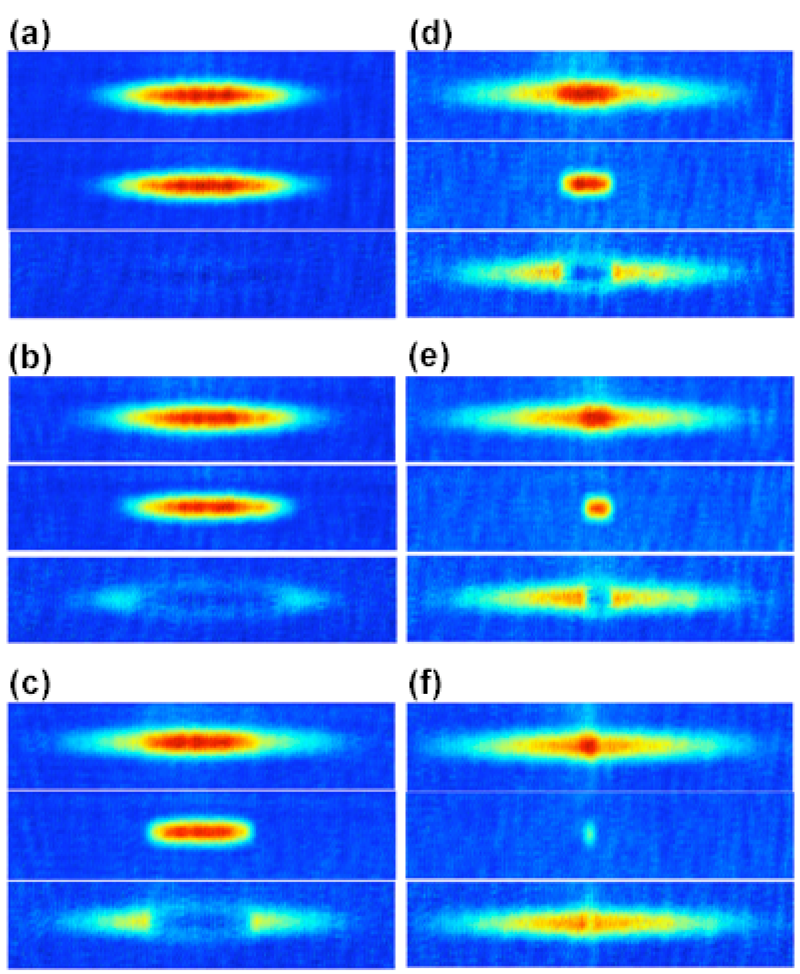
Exact result for a uniform system

Correlators are challenging.

Framework for Analysis



Nontrivial Breakdown of LDA
in highly elongated traps -
Partridge *et al.*, Science, 2006; PRL, 2006;



1D system in a trap

Bethe-Ansatz Solution *locally*

Trap effects via Local Density Approximation

Dimensional Crossover - 3D to 1D can be accessed by Bogoliubov-De Gennes approach.

For a 1D tube, QMC (Casula, Ceperley, Mueller, PRA, 2008) calculations show that deviations from LDA are small for trap profiles.

Bethe-Ansatz Method

H. Bethe, 1931

Yang-Baxter Equation:

$$S_{ik}S_{jk}S_{ji} = S_{ji}S_{jk}S_{ik} \qquad\qquad S_{ij} = \frac{(\alpha(k_i)-\alpha(k_j))I_{ij} + icP_{ij}}{\alpha(k_i)-\alpha(k_j)+ic}$$

Wavefunction:

$$\boxed{\psi_{a_1,\ldots,a_N}(x_1,\ldots,x_N) = \mathcal{A}e^{i\sum_j k_j x_j}\sum_Q S(Q)A_{a_1,\ldots,a_N}\theta(x_Q)}$$

Energy:

$$E = \sum_{j=1}^{N}\epsilon(k_j)$$

Gaudin-Yang Model

$$\epsilon(k) \to k^2$$

$$\alpha(k) \to k$$

Bethe-Ansatz Equations:

$$e^{ik_jL} = \prod_{m=1}^{M}\frac{\alpha(k_j)-\Lambda_m+ic/2}{\alpha(k_j)-\Lambda_m-ic/2},$$

$$-\prod_{m=1}^{M}\frac{\Lambda_n-\Lambda_m-ic}{\Lambda_n-\Lambda_m+ic} = \prod_{j=1}^{N}\frac{\Lambda_n-\alpha(k_j)-ic/2}{\Lambda_n-\alpha(k_j)+ic/2}$$

Bethe-Ansatz Solution

$$H = \boxed{-\sum \frac{\partial^2}{\partial z_i^2} - 4\Delta \sum \delta(z_i - z_j)} + \sum z_i^2$$

$$E = \sum_{j=1}^{N} k_j^2 \equiv eL$$

$$\Delta = -\frac{g_{1\mathrm{D}}}{2\hbar\omega_z}\sqrt{\frac{m\omega_z}{\hbar}} > 0$$

Bethe-Ansatz Equations

$$e^{ik_jL} = \prod_{m=1}^{M} \frac{k_j - \Lambda_m - i\Delta}{k_j - \Lambda_m + i\Delta},$$

Eigenvalue Equation

$$-\prod_{m=1}^{M} \frac{\Lambda_n - \Lambda_m - 2i\Delta}{\Lambda_n - \Lambda_m + 2i\Delta} = \prod_{j=1}^{N} \frac{\Lambda_n - k_j - i\Delta}{\Lambda_n - k_j + i\Delta}$$

Auxiliary Equation

$j = 1, ..., N \qquad n = 1, ..., M \qquad M \leq N/2$

Classes of Solutions $L \to \infty, N \to \infty, M \to \infty$

k_j	Unpaired particles
$k_j^{\pm} = \Lambda_j \pm i\Delta, \Lambda_j$	Bound states of two particles
$\Lambda^{(r)j} = \Lambda^{(r)} + i\Delta(r + 1 - 2j)$ $j = 1, ..., r$	Bound states of r spinons

Bethe-Ansatz Solution: Thermodynamic Limit

$$L \to \infty, N \to \infty, M \to \infty \quad N/L = \text{const} \quad M/L = \text{const}$$

$$\rho_u^{r(h)}(k,z) \qquad \rho_b^{r(h)}(k,z) \qquad \rho_{sn}^{r(h)}(\Lambda,z)$$

$$e = \int dk k^2 \rho_u^r(k) + 2\int dq (q^2 - \Delta^2)\rho_b^r(q)$$ Energy density

$$n_t = n_\uparrow + n_\downarrow = \int dk \rho_u^r(k) + 2\int dq \rho_b^r(q)$$ Particle density

$$n_s = n_\uparrow - n_\downarrow = \int dk \rho_u^r(k) - 2\sum_m m \int d\Lambda \rho_{sm}^r(\Lambda)$$ Spin density

$$s = \sum_\alpha \int dk [\rho_\alpha^r(k) f_\alpha(k) + \rho_\alpha^h(k)\bar{f}_\alpha(k)]$$ Entropy density

$$f_\alpha = \ln(1 + \rho_\alpha^h/\rho_\alpha^r) \qquad \bar{f}_\alpha = \ln(1 + \rho_\alpha^r/\rho_\alpha^h)$$

$$\mathcal{F}(z) = e(z) - Ts(z) - \mu(z)n_t(z) - hn_s(z)$$ Free energy density

Thermodynamic-Bethe-Ansatz Equations

Exact energy spectrum → Partition Function → Free Energy Functional

Minimization of the Free Energy

$$f_u - \bar{f}_u = G * f_b - G * f_{s1}$$ Unpaired particles

$$f_b - \bar{f}_b = 2[k^2 - \Delta^2 - \mu(z)]/T + K_1 * \bar{f}_u + K_2 * \bar{f}_b$$ Paired particles

$$f_{sn} - \bar{f}_{sn} = \delta_{n,1} G * \bar{f}_u + G * (f_{sn+1} + \hat{\delta}_{n,1} f_{sn-1})$$ Spin waves

$$\lim_{n\to\infty} (K_{n+1} * f_{sn} - K_n * f_{sn+1}) = -2h/T$$ Asymptotic condition

$$\mathcal{F}(z) = -\frac{T}{2\pi}\int dk \bar{f}_u(k,z) - \frac{T}{\pi}\int dk \bar{f}_b(k,z)$$ Free energy density

$$G(k) = \frac{1}{4\Delta\cosh\frac{\pi k}{2\Delta}} \qquad K_n(k) = \frac{1}{\pi}\frac{n\Delta}{k^2 + (n\Delta)^2}$$

Total particle density

$$n_t(z) = -\frac{\partial \mathcal{F}(z)}{\partial \mu(z)}$$

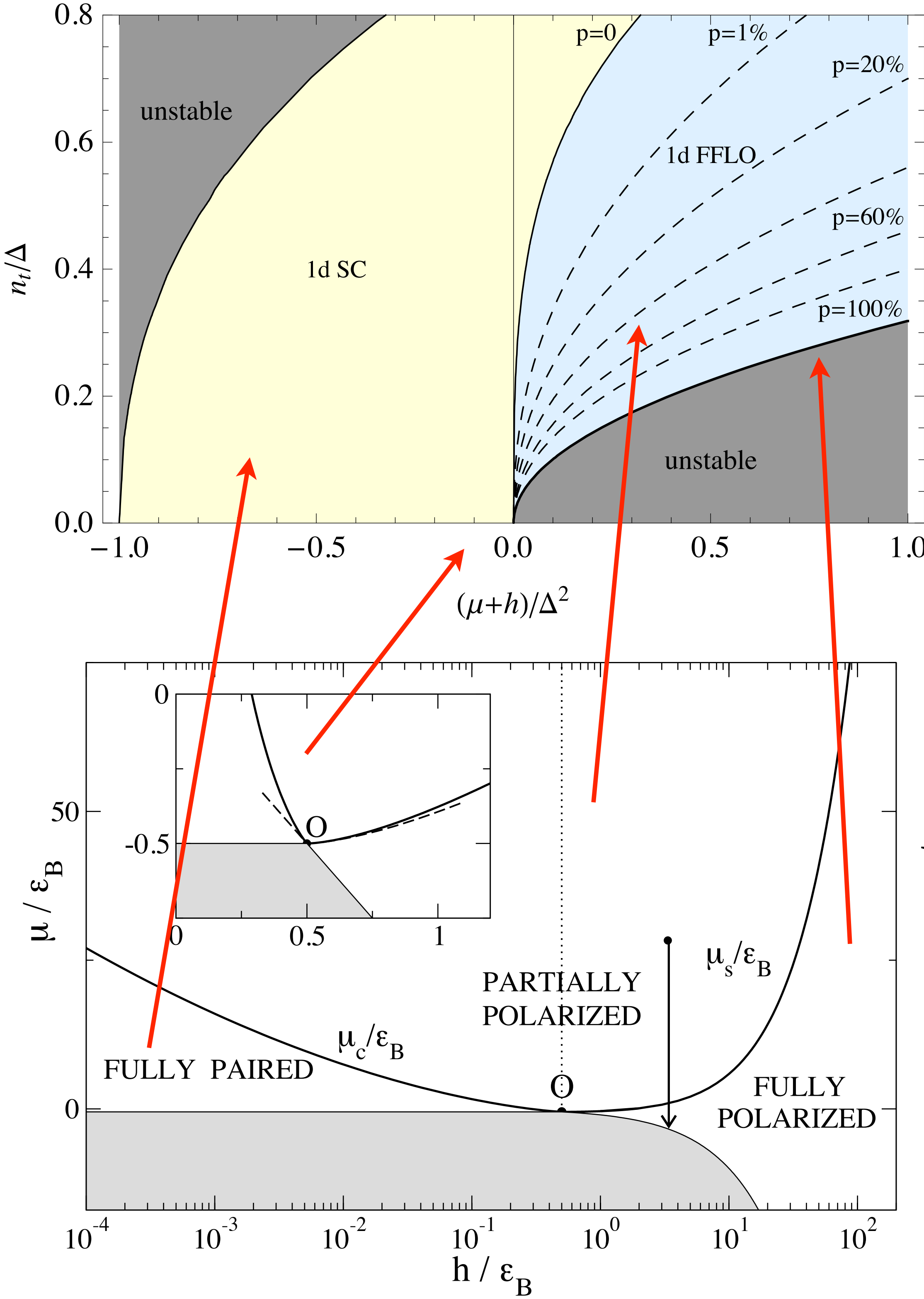
Spin density

$$n_s(z) = -\frac{\partial \mathcal{F}(z)}{\partial h}$$

Entropy density

$$s(z) = -\frac{\partial \mathcal{F}(z)}{\partial T}$$

“Phase Diagram” of a Single Tube



Is it possible to determine the “phase diagram” of a uniform system from an experiment in a trap?

We propose a scheme that requires measurements of density and polarization profiles.

Experimental Parameters and Measurements

Particle number per tube $N \sim 100$ Interaction strength $g_{1\text{D}} = 40 - 300$ Temperature $T/\mu_{\uparrow F} \sim 0.1$

Experiment - R. Hulet's Group

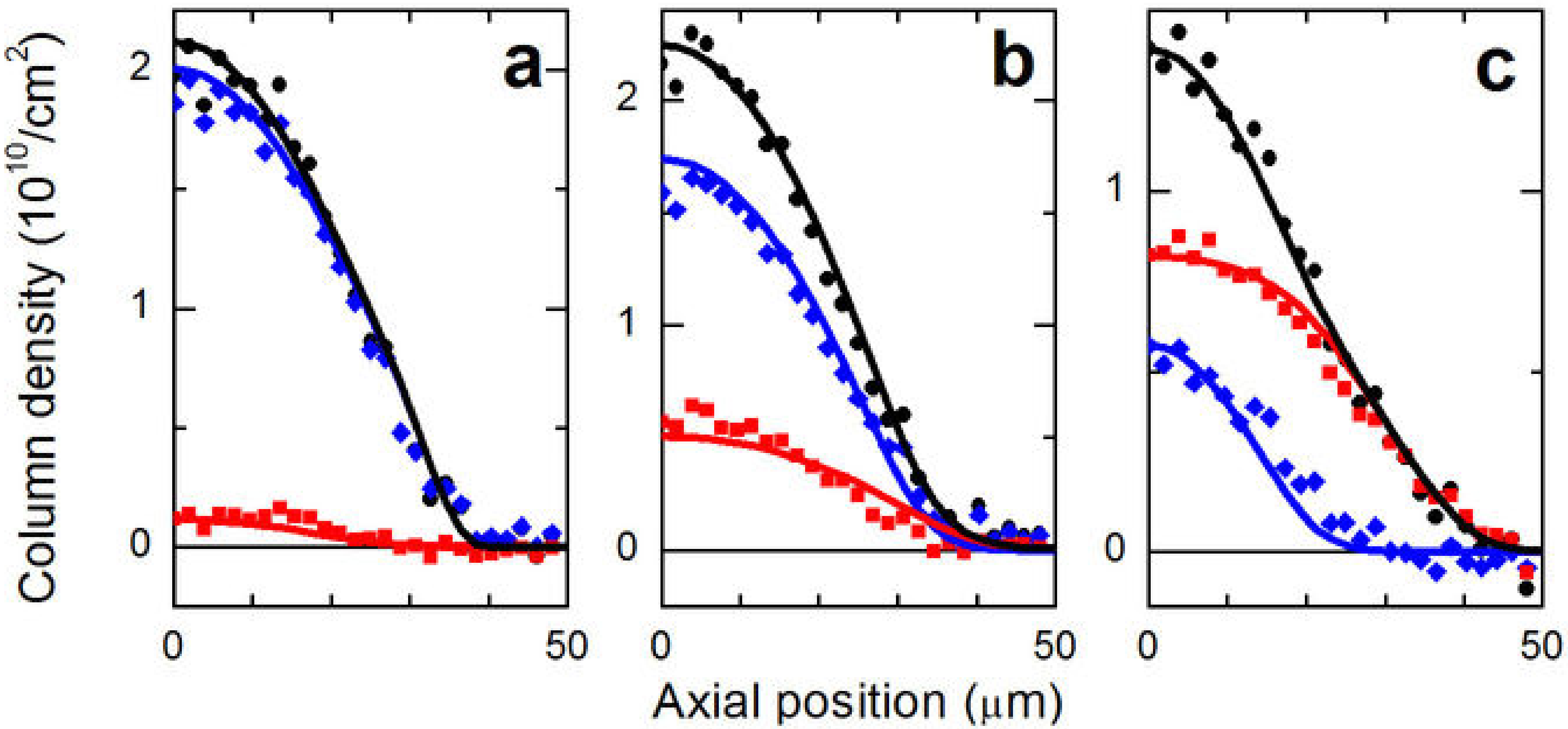
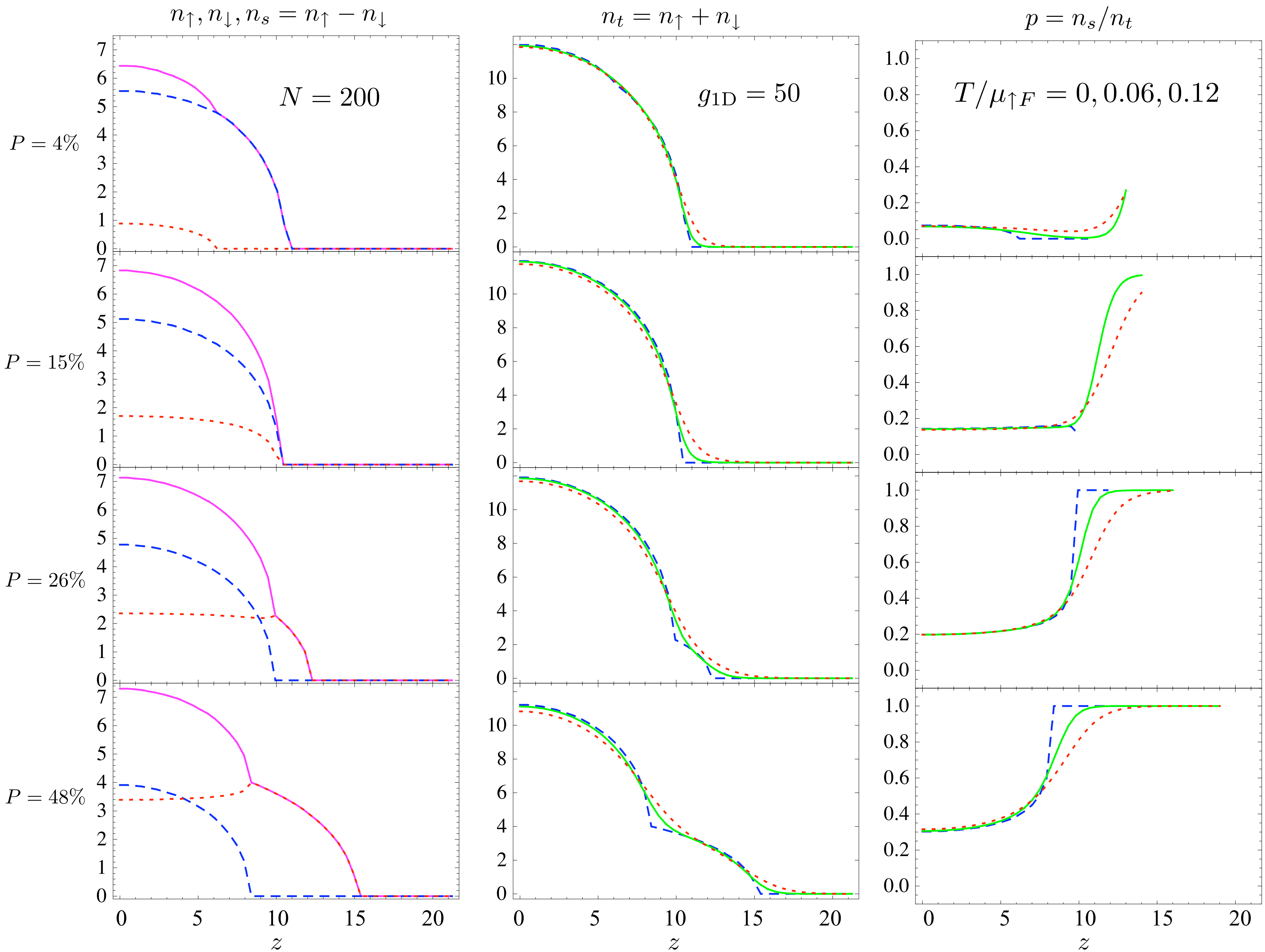
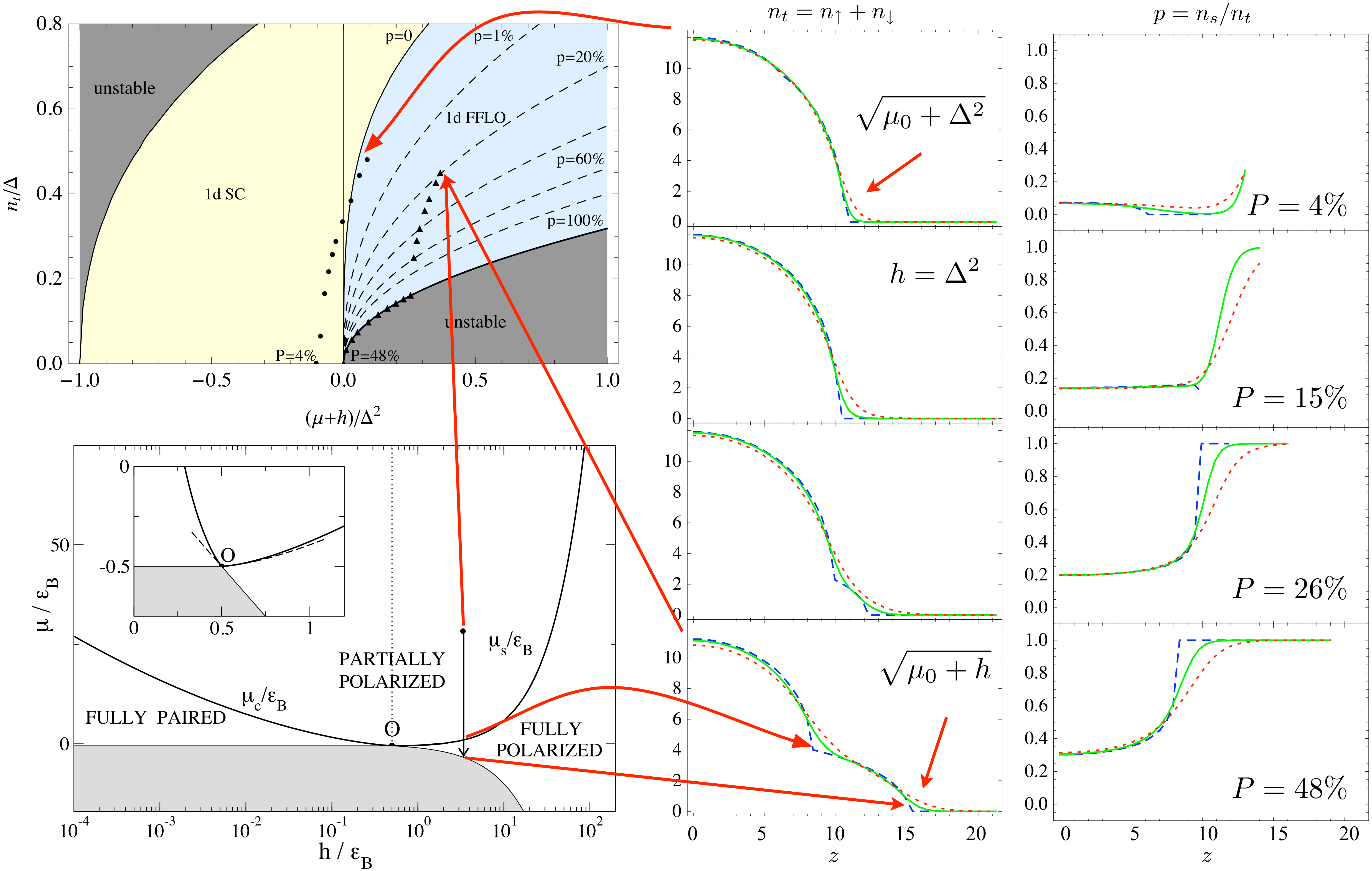


Figure 2: Column density profiles of a spin-imbalanced 1D ensemble of tubes. Column density profiles (black: majority; blue: minority; red: difference) are shown as function of *P*. Lines are theoretical predictions from the finite temperature thermodynamic Bethe ansatz (130 nK) within the local density approximation, using the measured radial 3D density distribution as input. (**a-c**) *P* corresponds to 0.05, 0.15, and 0.59, respectively. **a,** At low *P*, the edge of the cloud is fully paired and the density difference is zero. **b,** Near *P*c, nearly the entire cloud is partially polarized. **c,** Well above *P*c, the edge of the cloud becomes fully polarized and the minority density is zero. These column density profiles are averaged over a radial distance of 9.4 μm.

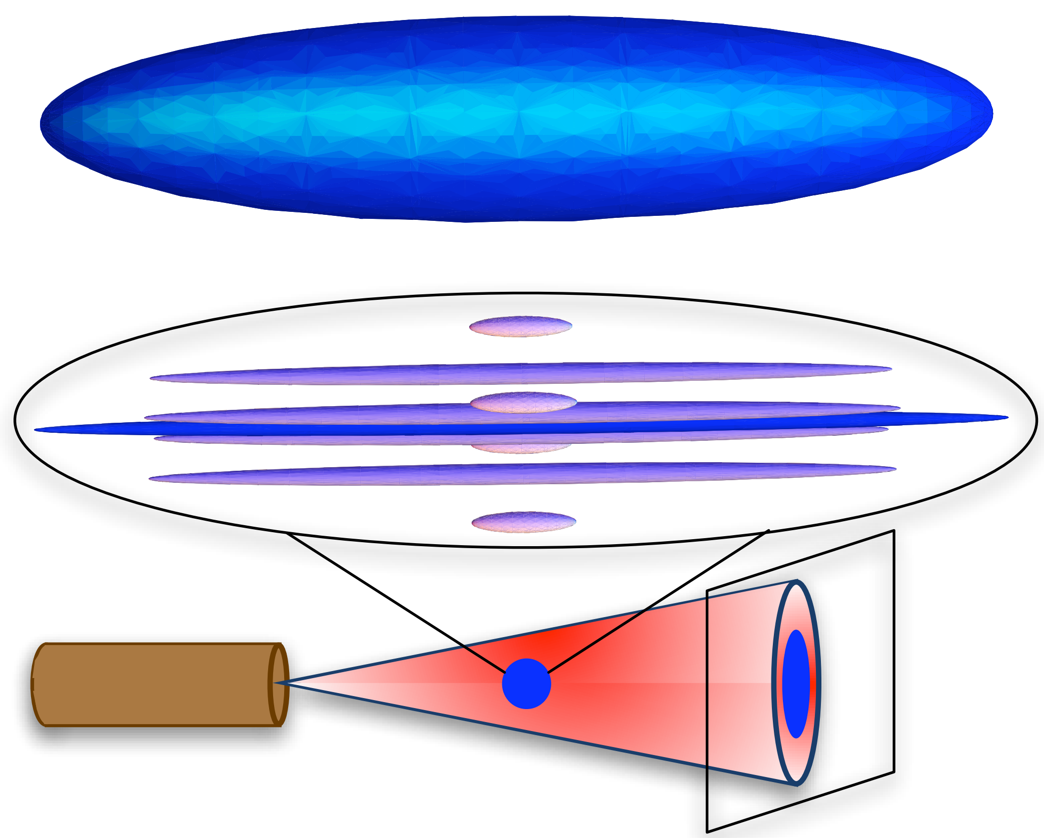
Density Profiles



"Phase Diagram" of a Single Tube



What About Many Tubes?



How to get information about individual tubes from column or axial densities?

Column Density + Cylindrical Symmetry + Inverse Abel Transform = Reconstruction of a 3D cloud.

Axial Density + Harmonic trap + LDA +Derivative =
Density profile of the central tube.

Mueller, private communication.

What About Pairing?

Single 1D tube:
No long-ranger order;
Power-law decay of the order parameter;

Many 1D tubes:
Tunneling of pairs (Josephson tunneling) between tubes;
True long-range order;

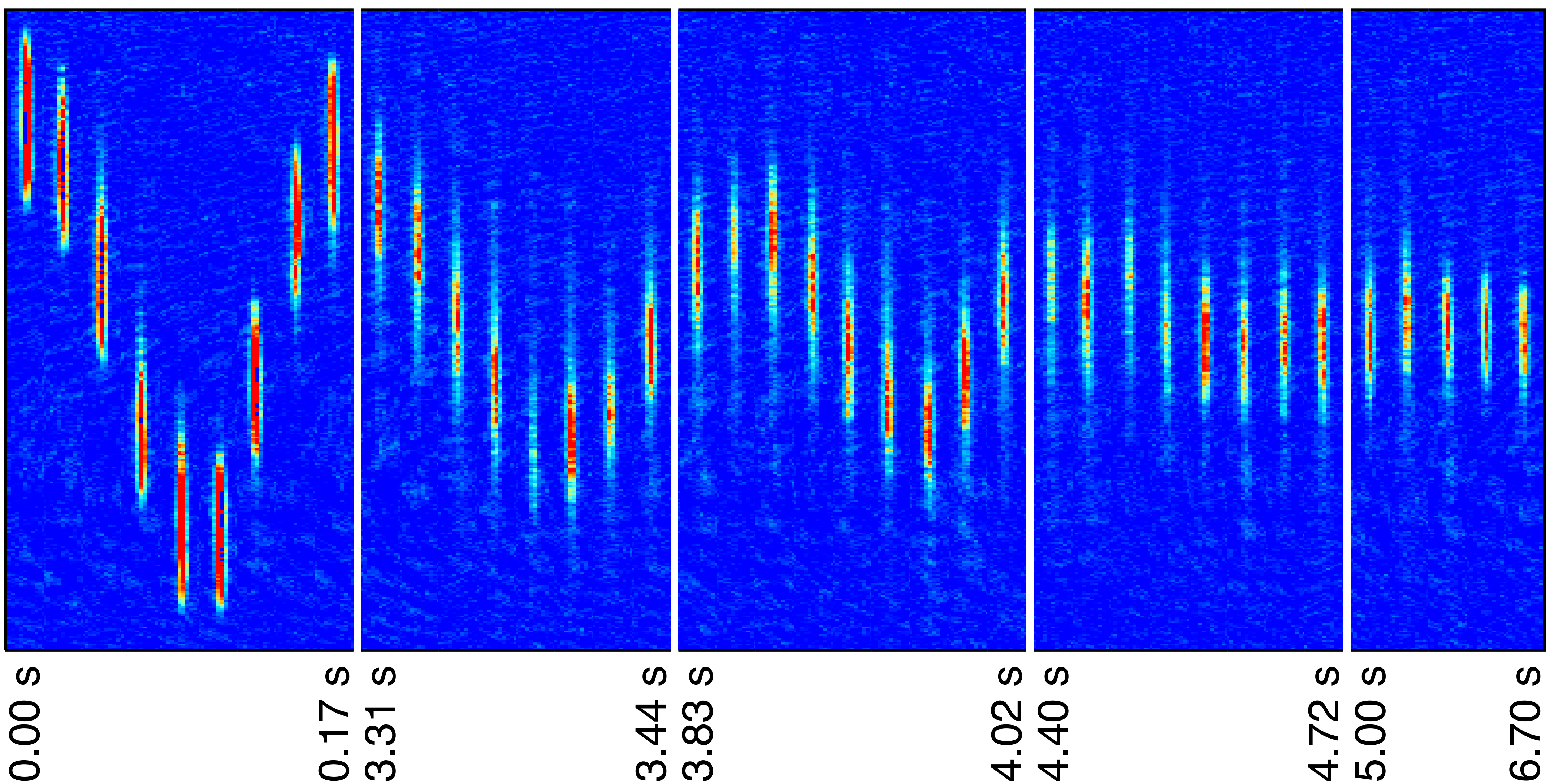
E. Zhao, W. V. Liu, PRA, 2008.

Damped Dipole Oscillations

Confinement

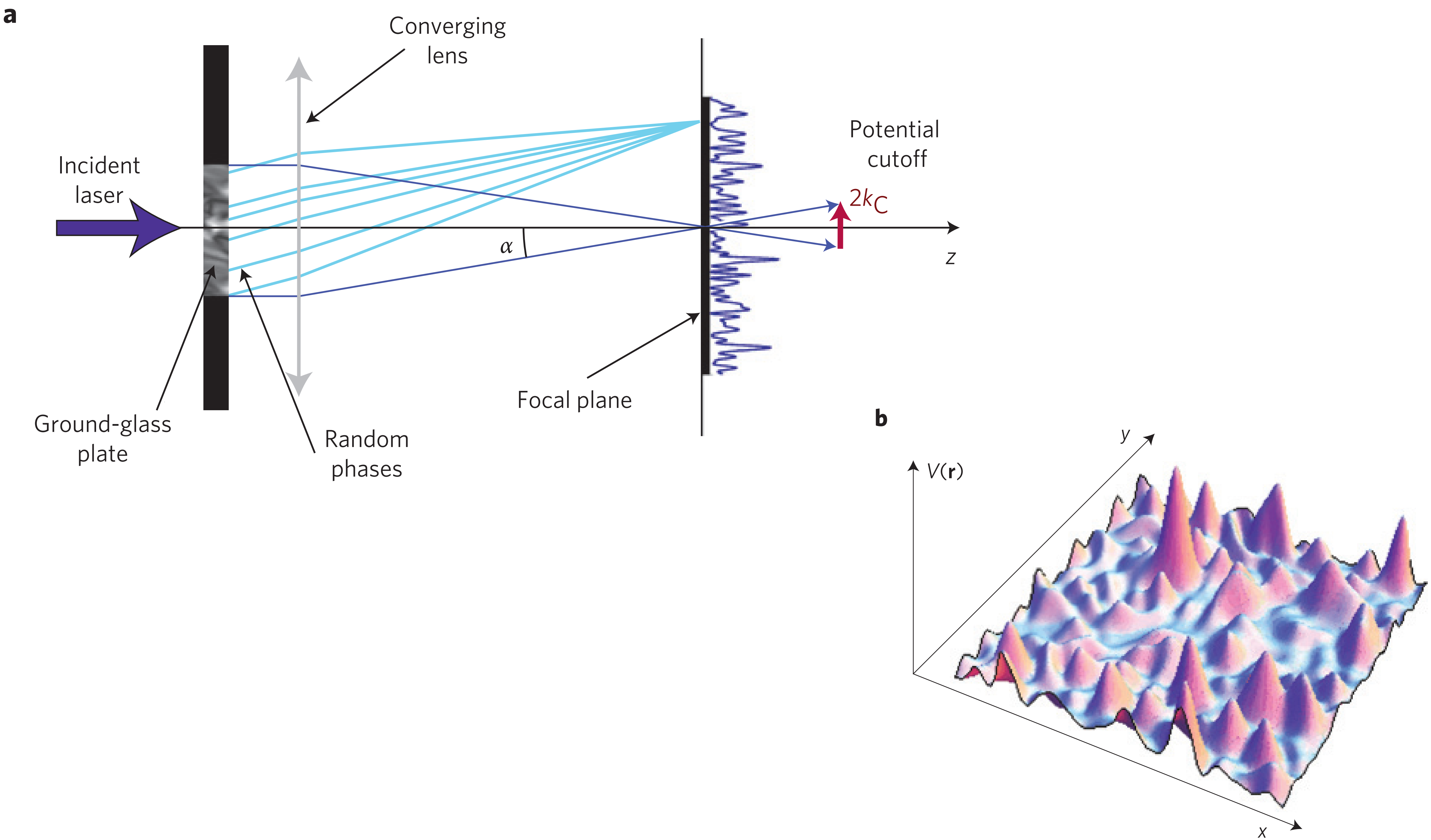
Interaction

Disorder
Correlations



Y. P. Chen, J. Hitchcock, D. Dries, M. Junker, C. Welford, and R. G. Hulet, PRA **77**, 033632 (2008);
D. Dries, S. E. Pollack, J. M. Hitchcock, and R. G. Hulet, arXiv:1004.1891;

Disorder - Optical-Speckle Potential



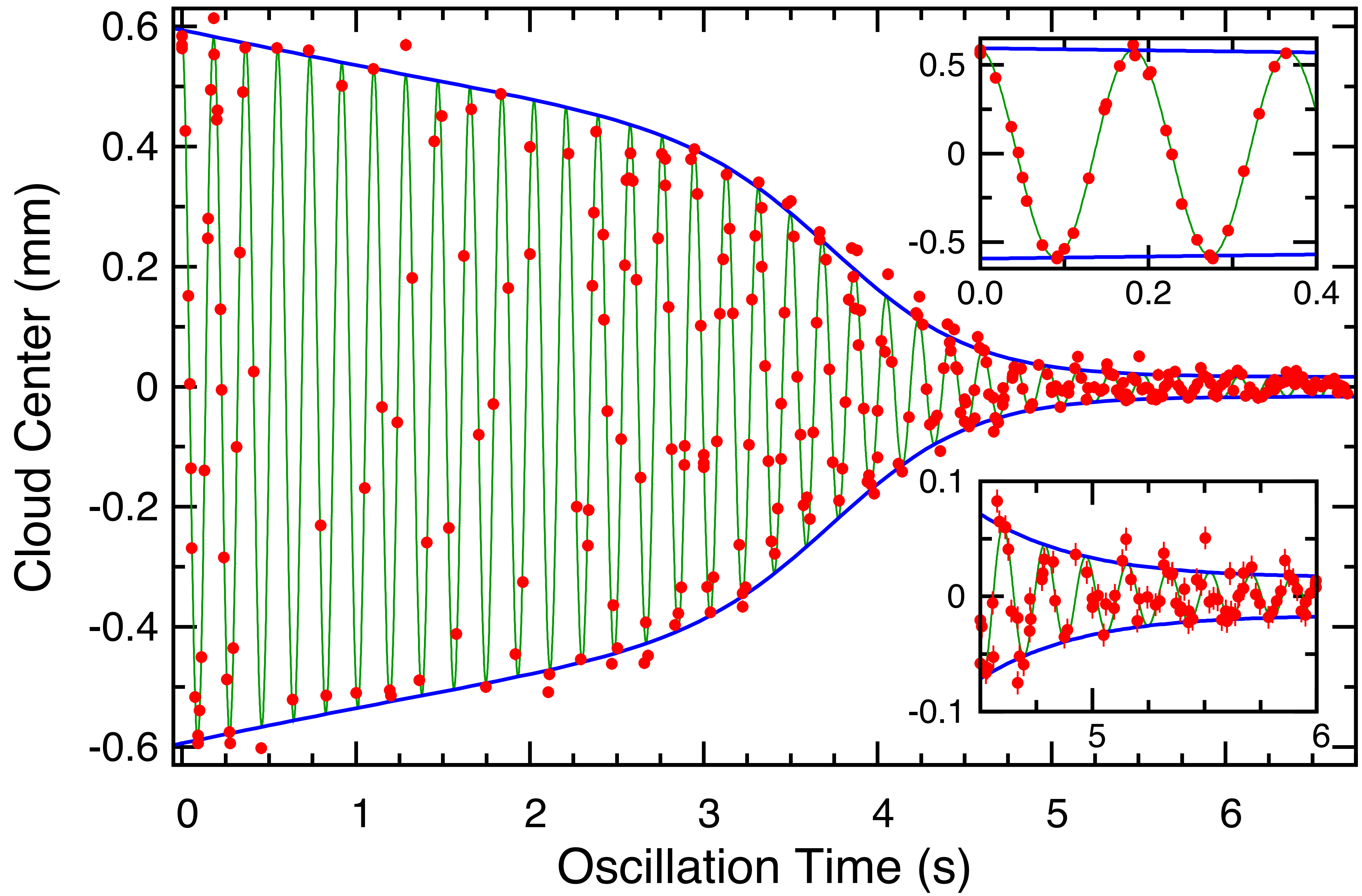
L. Sanchez-Palencia and M. Lewenstein, Nat. Phys. **6**, 87 (2010)

Transport Experiments

Confinement

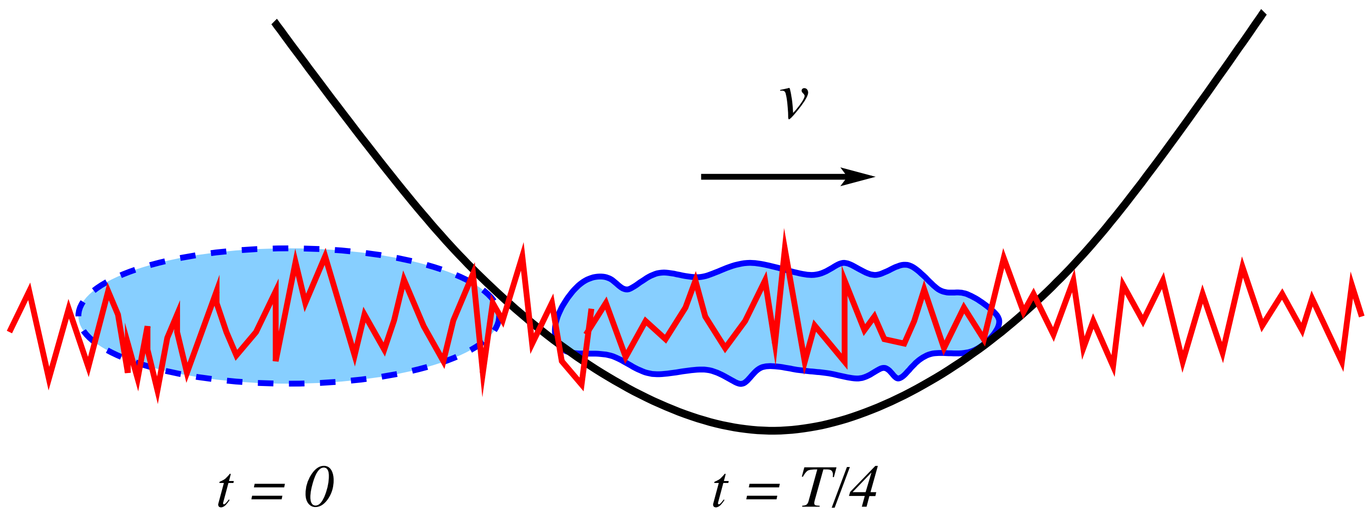
Interaction

Disorder
Correlations



Y. P. Chen, J. Hitchcock, D. Dries, M. Junker, C. Welford, and R. G. Hulet, PRA **77**, 033632 (2008);
D. Dries, S. E. Pollack, J. M. Hitchcock, and R. G. Hulet, arXiv:1004.1891;

Theoretical Approach



Interplay of Disorder, interaction and confinement

Non-linear oscillations:
- Bogoliubov modes;
- Thermal excitations;
- Solitons?

Assumptions:
- T=0;
- Non-interacting quasi-particles;
- Local density approximation;
- Delta-correlated disorder;

Trapped BEC without Disorder

Dipole Oscillation

$$H = \int \mathrm{d}\mathbf{x}\Psi^\dagger(\mathbf{x})\left[-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{x})\right]\Psi(\mathbf{x}) + \frac{\lambda}{2}\int \mathrm{d}\mathbf{x}\Psi^\dagger(\mathbf{x})\Psi^\dagger(\mathbf{x})\Psi(\mathbf{x})\Psi(\mathbf{x})$$

Hydrodynamic approach (no disorder)

$$\frac{\partial n}{\partial t} + \nabla(\mathbf{v}n) = 0$$

$$\lambda = \frac{4\pi\hbar^2 a}{m}$$

$$m\frac{\partial \mathbf{v}}{\partial t} + \nabla(\frac{m\mathbf{v}^2}{2} + V_{ext} + \lambda n) = 0$$

$$\omega = \omega_{\mathrm{ho}}\sqrt{2n_r^2 + 2n_r\ell + 3n_r + \ell}$$

Dipole Mode $\quad n_r = 0, \ell = 0, \omega = \omega_{\mathrm{ho}}$

Disorder: Replica Trick

$$< U_d(\mathbf{x})U_d(\mathbf{x}') >_{\mathrm{dis}} = \gamma^2 \delta(\mathbf{x}-\mathbf{x}')$$

$$P[U_d] = e^{-\frac{1}{2\gamma^2}\int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{x}' U_d(\mathbf{x}) K(\mathbf{x}-\mathbf{x}')^{-1} U_d(\mathbf{x}')}$$

$$K(\mathbf{x}-\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}')$$

Total average = quantum average "+" disorder average

$$\langle O \rangle_{U_d} = \frac{\int \mathcal{D}\phi O[\phi] e^{-S_{U_d}[\phi]}}{\int \mathcal{D}\phi e^{-S_{U_d}[\phi]}} \quad \Rightarrow \quad \langle O \rangle = \int \mathcal{D}U_d \, \langle O \rangle_{U_d} P[U_d]$$

$$\langle O \rangle_{U_d} = \int \mathcal{D}\phi O[\phi] e^{-S_{U_d}[\phi]} \left[\int \mathcal{D}\phi e^{-S_{U_d}[\phi]} \right]^{n-1} \qquad n \to 0$$

Trapped BEC with Disorder

$$H = \int \mathrm{d}\mathbf{x} \Psi^\dagger(\mathbf{x}) \left[-\frac{\hbar^2 \nabla^2}{2m} + V_{\mathrm{ext}}(\mathbf{x}) \right] \Psi(\mathbf{x}) + \frac{\lambda}{2} \int \mathrm{d}\mathbf{x} \Psi^\dagger(\mathbf{x}) \Psi^\dagger(\mathbf{x}) \Psi(\mathbf{x}) \Psi(\mathbf{x})$$

$$V_{\mathrm{ext}}(\mathbf{x}) = V_{\mathrm{trap}}(\mathbf{x}) + U_{\mathrm{d}}(\mathbf{x})$$

Gross-Pitaevskii Equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m} \Psi + V_{\mathrm{ext}}(\mathbf{x}) \Psi + \lambda \left| \Psi \right|^2 \Psi$$

$$\Psi = \Psi_0 + \sum_{\mathbf{k} \neq 0} e^{i\mathbf{k}\mathbf{x}} a_{\mathbf{k}}$$

Bogoliubov Excitations

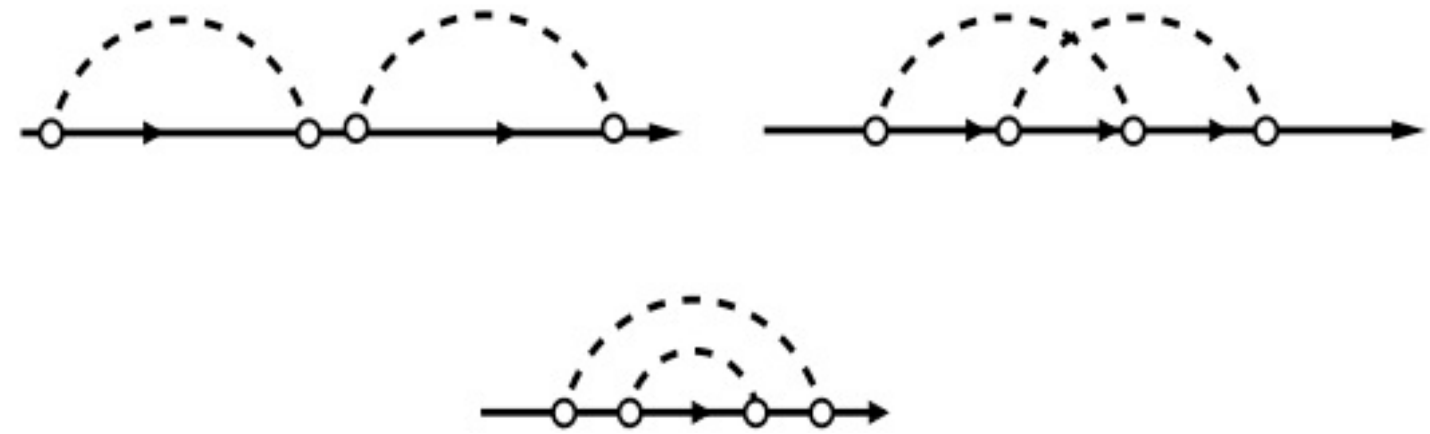
$$b_{\mathbf{k}} = u_{\mathbf{k}} a_{\mathbf{k}} + v_{\mathbf{k}} a^\dagger_{-\mathbf{k}} \qquad u_{\mathbf{k}}^2 = (e_{\mathbf{k}} / \Lambda_{\mathbf{k}} + 1)/2 \qquad v_{\mathbf{k}}^2 = (e_{\mathbf{k}} / \Lambda_{\mathbf{k}} - 1)/2$$

$$\Lambda_{\mathbf{k}} = \sqrt{e_{\mathbf{k}}^2 - (\lambda \rho_0)^2} \qquad e_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m} + \lambda \rho_0$$

Damping Rate

2nd Order

4th Order



$$\Gamma = \frac{2 \times 2\pi\gamma^2}{\hbar(2\pi)^3} \int \mathrm{d}\mathbf{k} \int \mathrm{d}\mathbf{r}_\perp \int_0^{R[r_\perp]} \mathrm{d}z\, \mu(\mathbf{r})\rho(\mathbf{r})$$
$$\times \frac{\hbar^2 k^2}{2m\Lambda_k[\mathbf{r}]} \delta(\Lambda_k[\mathbf{r}] - \hbar\mathbf{k}\cdot\mathbf{v}),$$

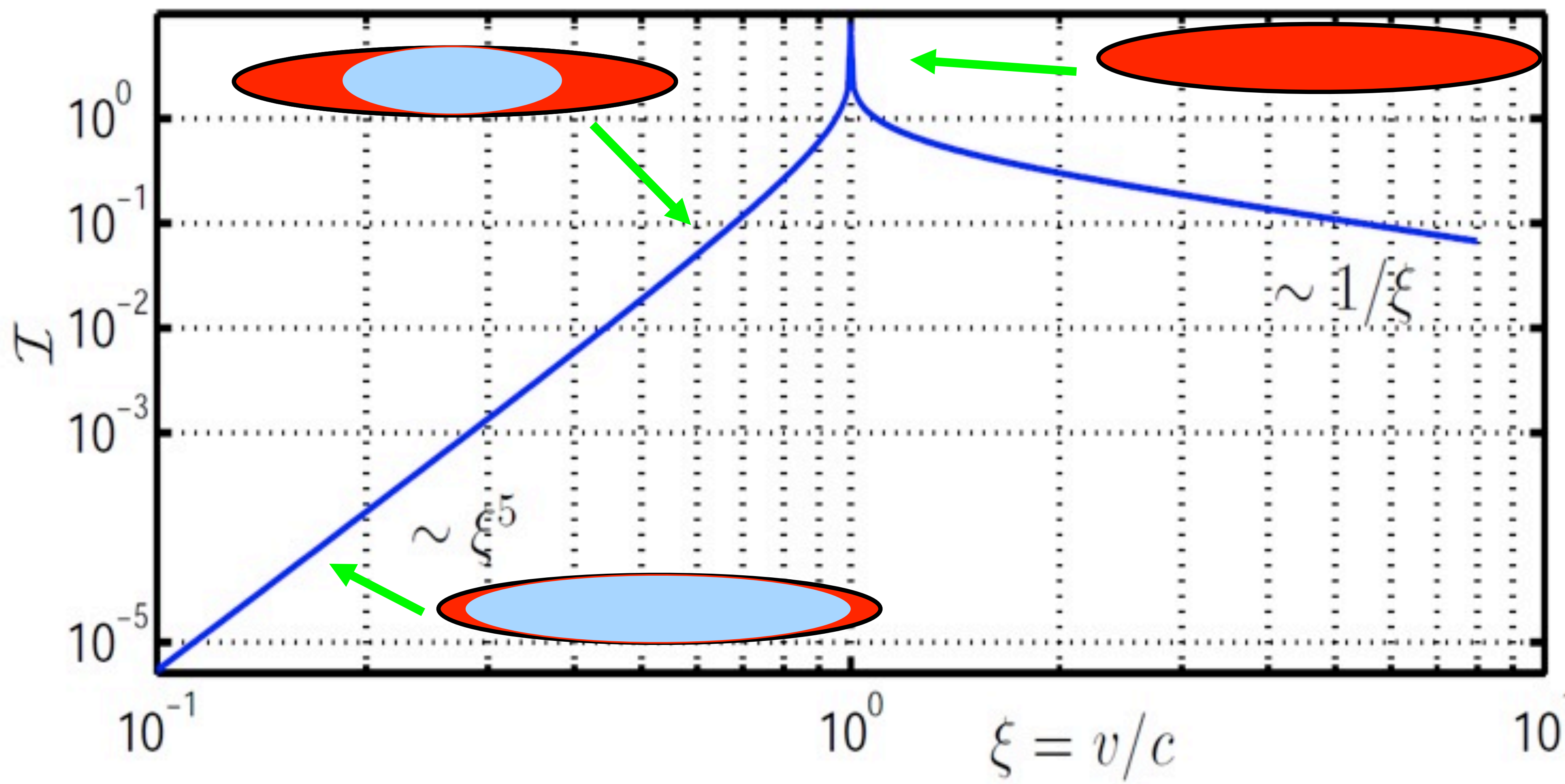
- Quasi-1D, excitations along the axes;
- Weak disorder;
- No condensate depletion;

Local density approximation

Galilean transformation

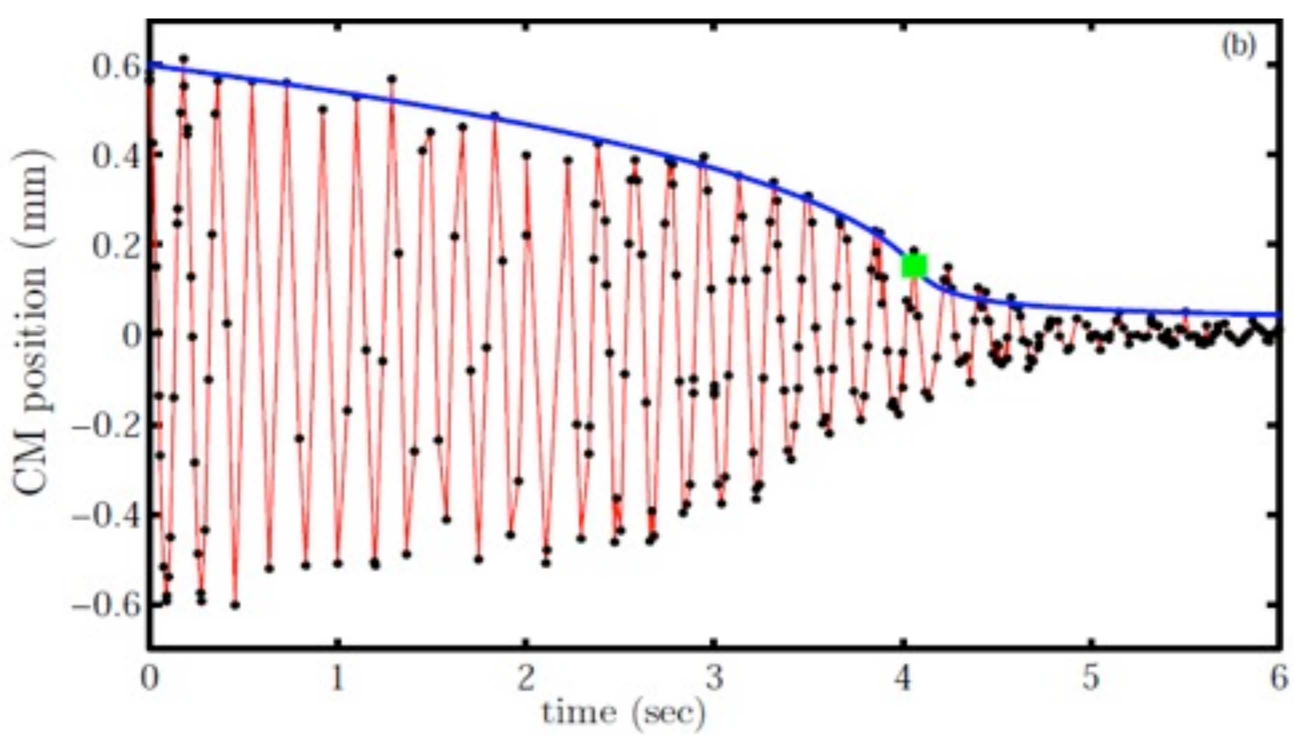
Damping Rate

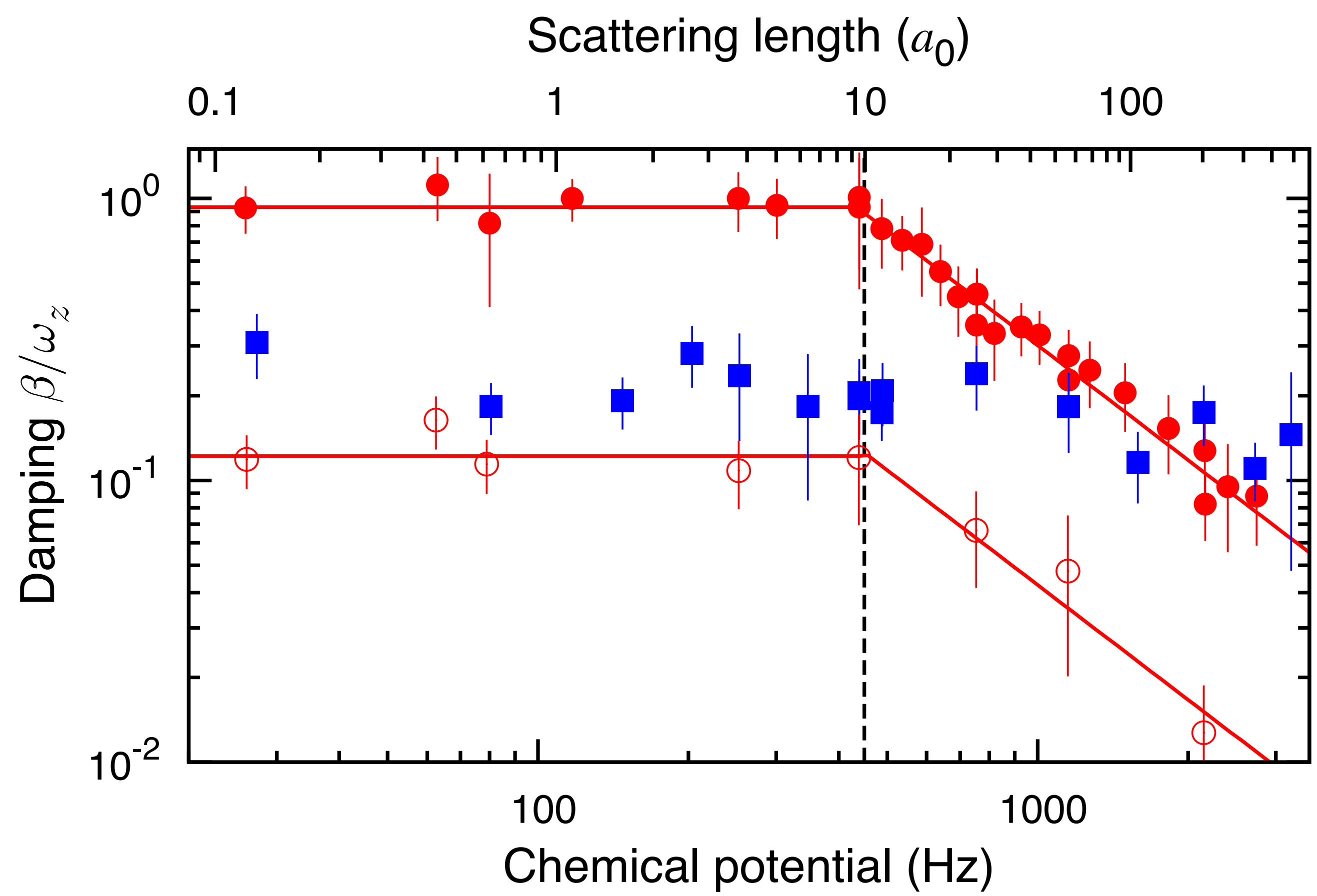
$$\bar{\Gamma}[\xi] = \frac{\bar{\gamma}^2 \bar{R}^4}{4\sqrt{2}\pi\bar{a}} \int_0^1 t^2\, \mathcal{I}[\xi/\sqrt{t}, \xi_{\min}/\sqrt{t}]\, \mathrm{d}t.$$



$$\frac{\partial}{\partial t}\xi_{\text{peak}} = -\beta\xi_{\text{peak}} \qquad \beta = -\frac{2\tilde{\Gamma}[\xi_{\text{peak}}]}{N\bar{R}^2\xi_{\text{peak}}^2} \qquad \tilde{\Gamma} \;=\; (1/T)\int_0^T \Gamma \mathrm{d}t$$

Damping Rate - Comparison with Experiments





- Single free parameter γ;
- Interactions reduce the effect of disorder - weak disorder;
- Disorder correlations - some quantitative corrections;
- Good qualitative agreement with the experiment;

$$\beta \sim a^{-\frac{6}{5}} \text{ for } \xi \ll 1$$

$$\beta \sim a^{0} \text{ for } \xi \gg 1$$

Conclusion

- Luttinger Liquid Physics;
  - ★ Distinguish between Spin-coherent and -incoherent Luttinger Liquids;
  - ★ Density noise correlations;
- Interplay of Spin Imbalance and Attractive interaction in 1D;
  - ★ Different paired phases and their stability for finite temperatures;
  - ★ Experimental scheme to determine the phase diagram of the uniform system from trap profiles.
- Dissipative Dipole Oscillations;
  - ★ Interplay of interaction, disorder and confinement;
  - ★ Remarkable agreement with experiment;

P. K., S. G. Bhongale, H. Pu and C. J. Bolech, Phys. Rev. A **78**, 041602(R) (2008);
P. K. and C. J. Bolech, Phys. Rev. A **79**, 041603(R) (2009);
S. G. Bhongale, P. K., C. J. Bolech and H. Pu, arXiv:1003.2608;

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