Correlations and superfluidity of a one-dimensional Bose gas in a quasiperiodic potential

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Phys. Rev. A **81**, 063635 (2010)

Introduction

- Anderson Localization
- Role of interaction
- Bogoliubov approach
- 1D Bose glass

Anderson localization

Anderson (1958)

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey

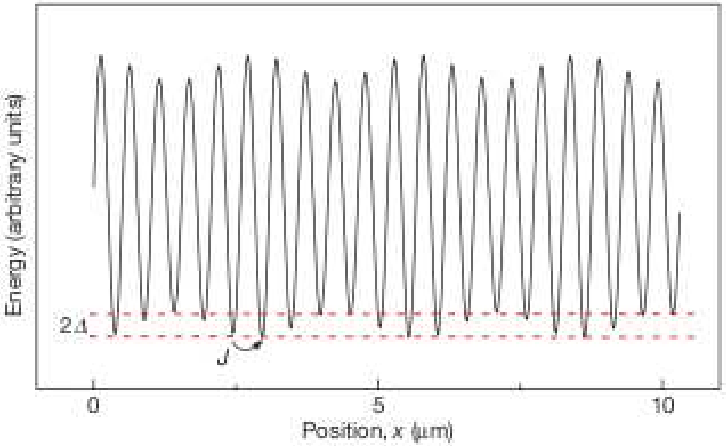
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Disordered lattice can lead to exponential localization of the wavefunction

External potential

- Random potential → groundsate is *always* localized
- $V(z) = V_1 \cos(\frac{2\pi}{\lambda_1} z) + V_2 \cos(\frac{2\pi}{\lambda_2} z)$



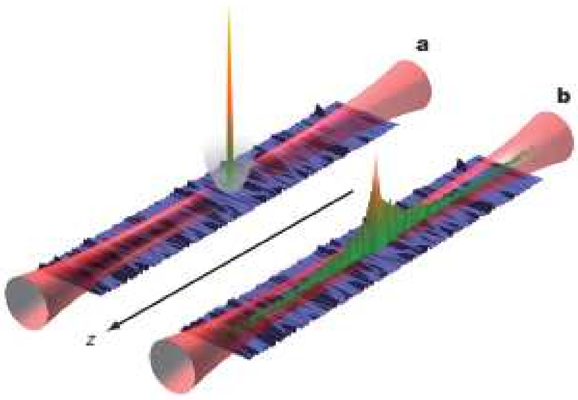
Roati *et al.*, Nature, **453**, 12 (2008)

- $\frac{\lambda_1}{\lambda_2}$ is incommensurate
- Critical strength of the quasiperiodic potential:

L. H. Eliasson, *Comm. in Math. Phys.* **146**, 447 (1992)

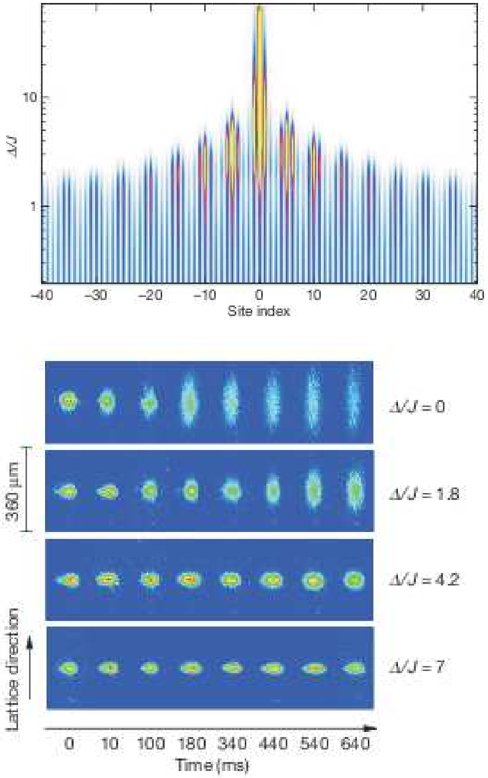
Localization in cold atoms

Speckle potential



Billy *et al.*, Nature, **453**, 12 (2008)

Quasiperidic potential



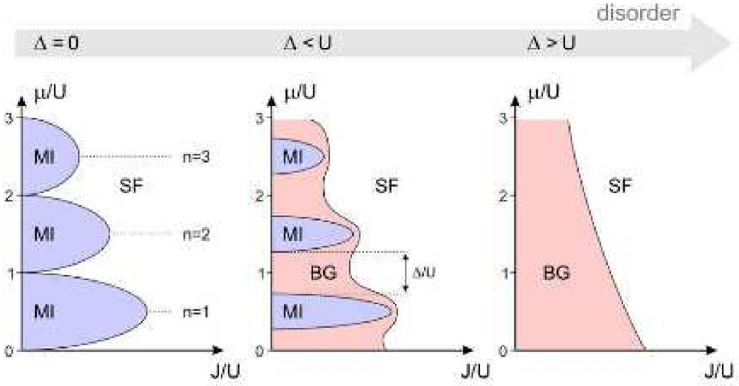
Roati *et al.*, Nature, **453**, 12 (2008)

Bose glass

- Bose-Hubbard with disorder

  $\hat{H} = -J \sum_{\langle i\, j \rangle} \hat{b}_i^\dagger \, \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i \, (\hat{n}_i - 1) + \sum_i \epsilon_i \, \hat{n}_i \qquad \epsilon_i \in [-\Delta/2, \Delta/2]$

- Interaction $\rightarrow$ New phase: Bose glass!



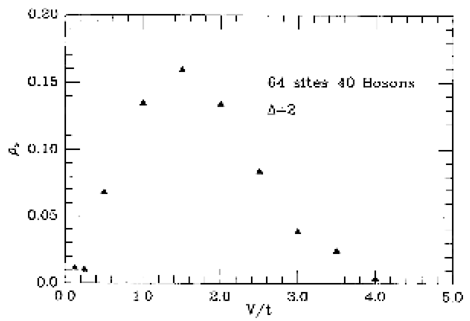
Fisher *et al.*, *Phys. Rev. B*, **40**, 546 (1989)

Fallani *et al.*, *Adv. Atomic, Mol., and Opt. Phys.*, **56**, 119 (2008)

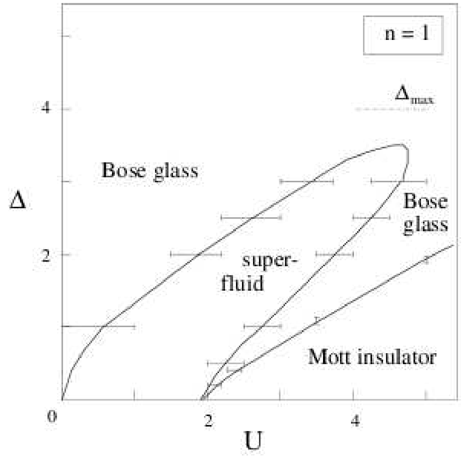
- Insulating phase
  - Exponentially decaying correlation function
  - No superfluid part
- Gapless $\rightarrow$ Compressible!

Phase diagram

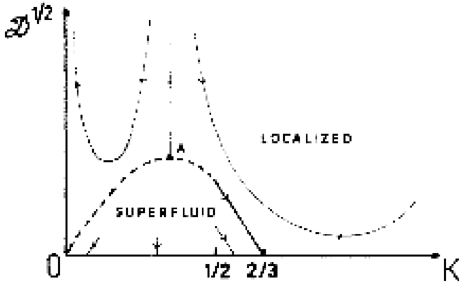
Scalettar *et al.*, *Phys. Rev. Lett.*, **66**, 3144 (1991)



Rapsch *et al.*, *Eur. Phys Lett.*, **46**, 559 (1999)



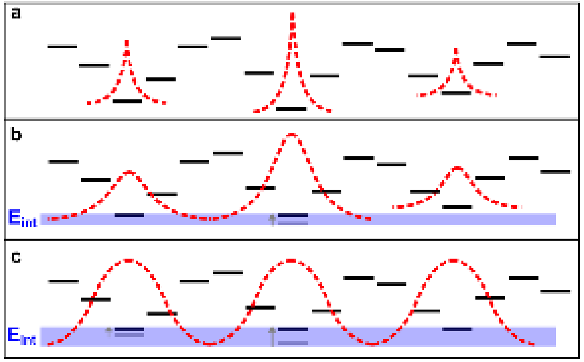
Giamarchi and Schulz, *Phys. Rev. B*, **37**, 325 (1988)

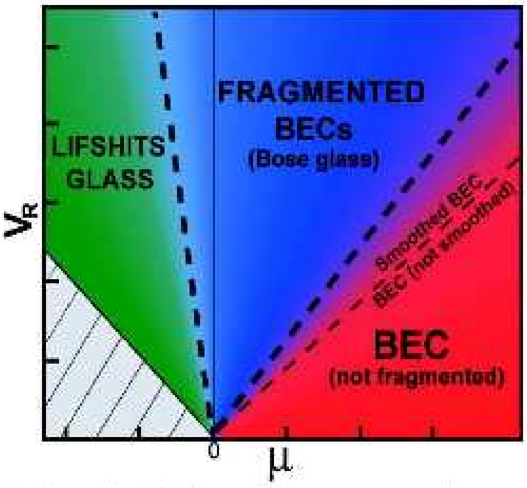


Anderson regime

Deissler *et al.*, *Nature Physics* **6**, 354 (2010)

Lugan *et al.*, *Phys. Rev. Lett.* **99**, 180402 (2007)





The Bogoliubov approach

- $g(r,r') = \langle \psi^\dagger(r)\,\psi(r')\rangle$ = ?
- Path integral approach

$$\mathcal{Z}[\psi^*,\psi] = \int \mathcal{D}[\psi^*,\psi]\, e^{-\mathcal{S}[\psi^*,\psi]}$$

$$\mathcal{S}[\psi^*,\psi] = \int d\tau\, d\mathbf{r}\, \psi^*(\mathbf{r},\tau)\left(-\hbar\,\partial_\tau + \frac{\hbar^2}{2\,m}\nabla^2 - U(\mathbf{r}) + \mu - \frac{g}{2}\,|\psi(\mathbf{r},\tau)|^2\right)\psi(\mathbf{r},\tau)$$

- Ansatz for the field

$$\begin{aligned}\psi(\mathbf{r},\tau) &= e^{i\,\theta(\mathbf{r},\tau)}\sqrt{n_0(\mathbf{r}) + \delta n(\mathbf{r},\tau)} \\ &\approx e^{i\,\theta(\mathbf{r},\tau)}\sqrt{n_0(\mathbf{r})}\left(1 + \frac{1}{2}\frac{\delta n(\mathbf{r},\tau)}{n_0(\mathbf{r})} - \frac{1}{8}\frac{\delta n^2(\mathbf{r},\tau)}{n_0(\mathbf{r})^2}\right)\end{aligned}$$

The Bogoliubov approach

- Keeping quadratic terms

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 + \mathcal{S}_2$$

- $\mathcal{S}_0$ brings the GP equation

$$\left[-\frac{1}{2}\frac{d^2}{dx^2} + g\,N\,|\Phi(x)|^2 + U(x)\right]\,\Phi(x) = \mu\,\Phi(x)$$

- Green function for the excitations is found by inverting $\mathcal{S}_2$
- $u_j, v_j = \frac{\delta n_j}{2\sqrt{n_0}} \pm i\,\sqrt{n_0}\,\theta_j$

The Bogoliubov approach

The knowledge of $\mathcal{S}$ allow us to compute

$$\begin{aligned}
\ln\, g(\mathbf{r},\mathbf{r}') &= \ln\langle\psi^*(\mathbf{r})\,\psi(\mathbf{r}')\rangle - \ln\sqrt{n\,n'} \\
&= -\frac{1}{2}\sum_{j\neq 0}\left\{\left|\frac{v_j}{\sqrt{n}} - \frac{v'_j}{\sqrt{n'}}\right|^2 + N_j\left[\left|\frac{u_j}{\sqrt{n}} - \frac{u'_j}{\sqrt{n'}}\right|^2 + \left|\frac{v_j}{\sqrt{n}} - \frac{v'_j}{\sqrt{n'}}\right|^2\right]\right\} \\
&+ i\sum_{j\neq 0}\left[\frac{1}{2}I(\mathbf{r},\mathbf{r}') + N_j I(\mathbf{r},\mathbf{r}')\right]
\end{aligned}$$

[Mora and Castin, *Phys. Rev. A* **67**, 053615 (2003)]

$I(\mathbf{r},\mathbf{r}')$ is completely imaginary

$$\begin{aligned}
i\,I(\mathbf{r},\mathbf{r}') &= \left(\frac{v_j^*}{\sqrt{n}}\frac{v'_j}{\sqrt{n'}} - \frac{v_j'^*}{\sqrt{n'}}\frac{v_j}{\sqrt{n}}\right) + \left(\frac{u_j'^*}{\sqrt{n'}}\frac{u_j}{\sqrt{n}} - \frac{u_j^*}{\sqrt{n}}\frac{u'_j}{\sqrt{n}}\right) \\
&+ \left(\frac{u_j'^*}{\sqrt{n'}}\frac{v'_j}{\sqrt{n'}} - \frac{v_j'^*}{\sqrt{n'}}\frac{u'_j}{\sqrt{n'}}\right) + \left(\frac{v_j^*}{\sqrt{n}}\frac{u_j}{\sqrt{n}} - \frac{u_j^*}{\sqrt{n}}\frac{v_j}{\sqrt{n}}\right)
\end{aligned}$$

The Bogoliubov approach

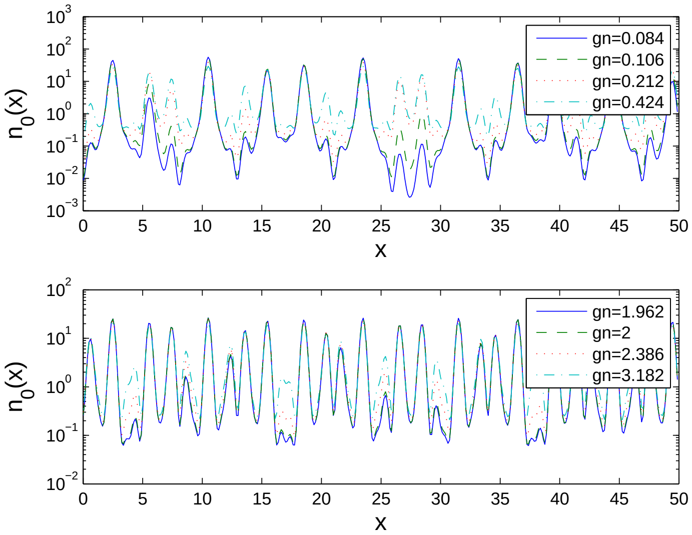
- The Bogoliubov approach allows us to study the *continuous* case
- $T = 0 \to g(\mathbf{r}, \mathbf{r}') \sim e^{-\frac{1}{2}\sum_{j\neq 0}\left\{|\frac{v_j}{\sqrt{n}} - \frac{v'_j}{\sqrt{n'}}|^2\right\}}$
- $V(z) = V\,\cos(\frac{2\,\pi}{d}\,z) + V\,\cos(\frac{2\,\pi}{\lambda}\,z)$
- λ/d is the golden ratio
- Units in $E_R = \hbar^2/m\,d^2$
- Fontanesi, Wouters, Savona
  - *Phys. Rev. Lett.* **103**, 030403 (2009)
  - *Phys. Rev. A* **81**, 053603 (2010)

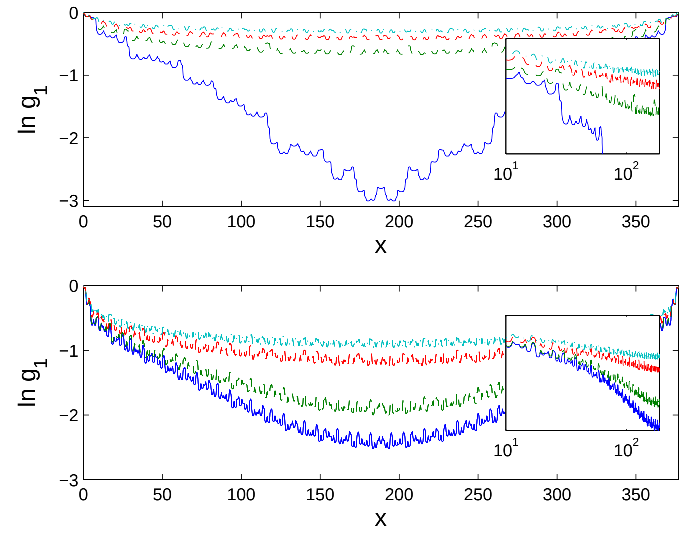
Our results

Cetoli and Lundh, *Phys. Rev. A* **81**, 063635 (2010)

Correlation

Density Profiles



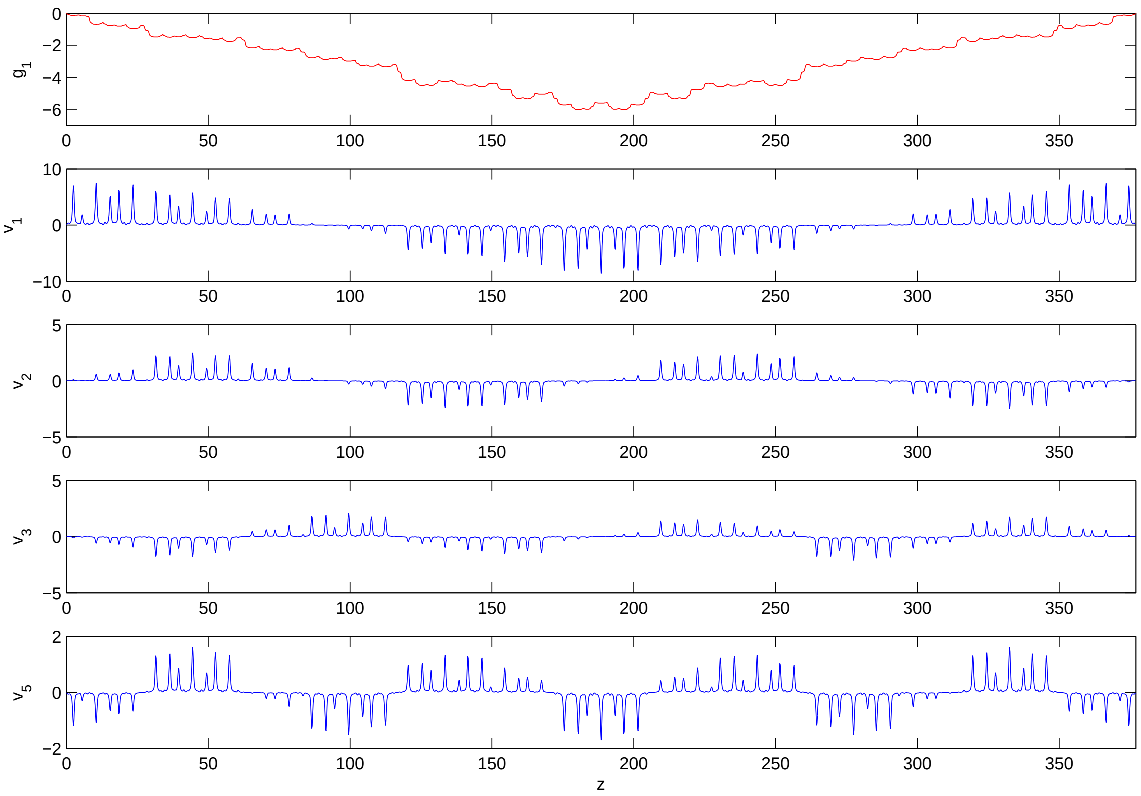


Up: $V = 5\,E_R$, down: $V = 9\,E_R$

Our results

Low Energy excitations flip the phase

$g\,n = 0.084\,E_R,\, V = 5\,E_R$



Superfluid part

- “Kicked” condensate:

$$\psi(\mathbf{r}) = e^{i\,\theta(\mathbf{r})}\,\sqrt{n_0}\left(e^{i\,\mathbf{k}_0\cdot\mathbf{r}} + \frac{1}{2}\frac{\delta n}{n_0} - \frac{1}{8}\frac{\delta n^2}{n_0^2}\right)\ ,$$

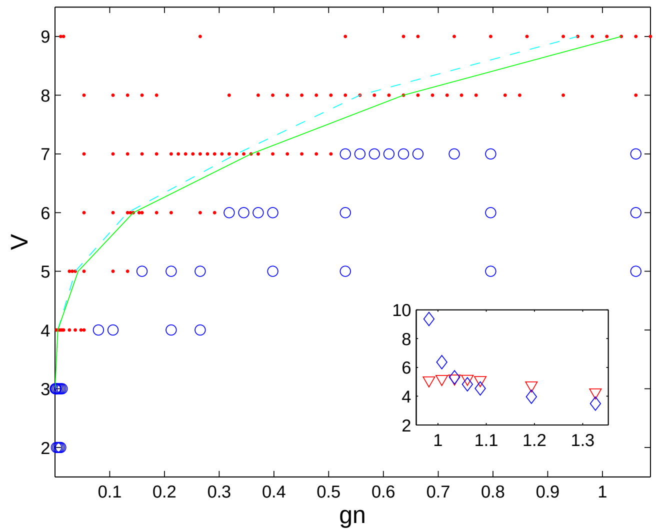
- Superfluid density ($\mathbf{k}_0 = \Theta\,\hat{e}_0/L$)

$$\rho_s \quad = \quad \frac{2\,L^2\,m^2\,N}{\hbar^2\,\Theta^2\,V}\left[F^{\Theta}(\mu,T) - F^{0}(\mu,T)\right]\ .$$

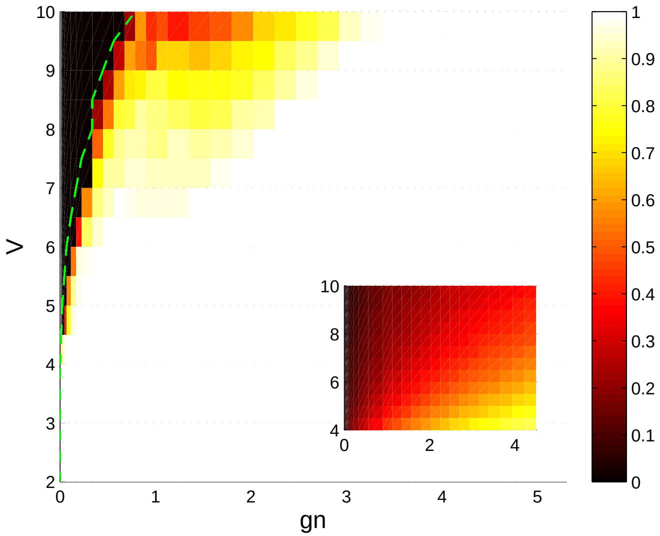
- Linked cluster theorem: $\mathcal{S}^{\Theta} \rightarrow F^{\Theta}$
- ρ_s can be computed using the Bogoliubov amplitudes!

Phase diagram

$L = 377\,d$

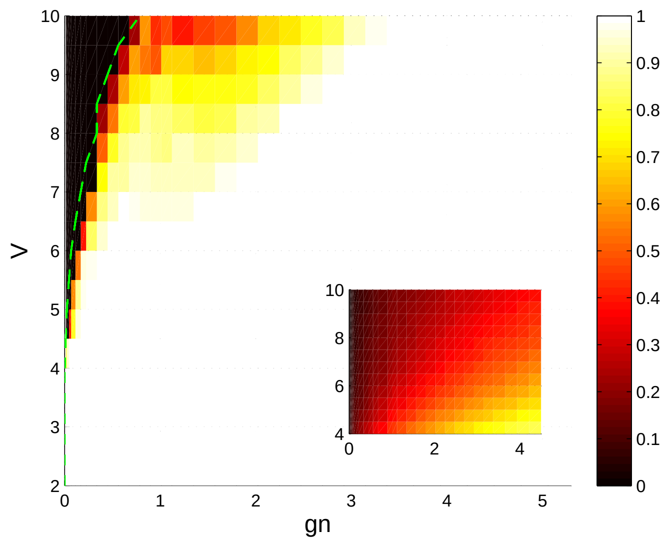


$L = 89\,d$

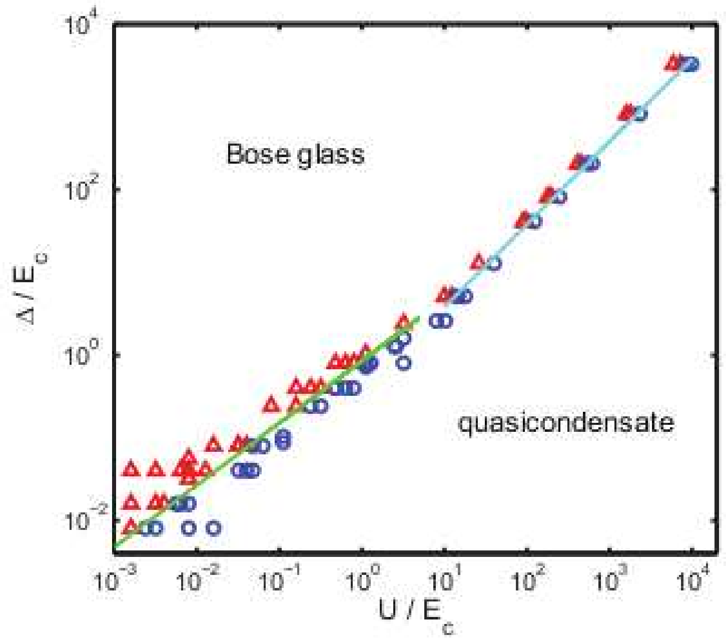


Comparison

Cetoli and Lundh



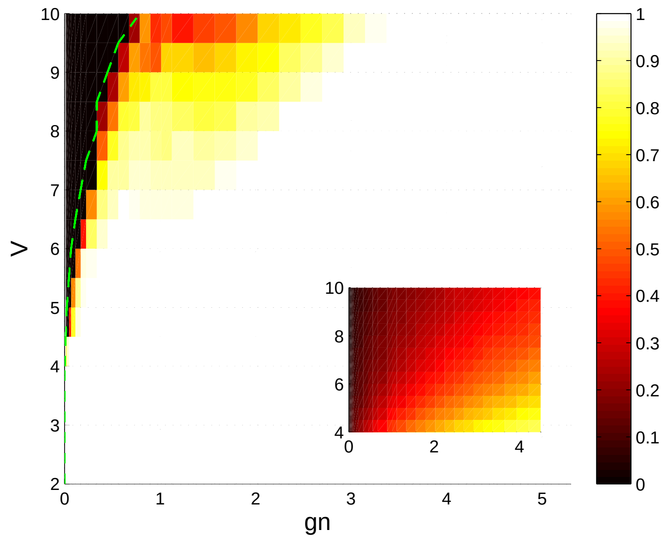
Fontanesi et al.



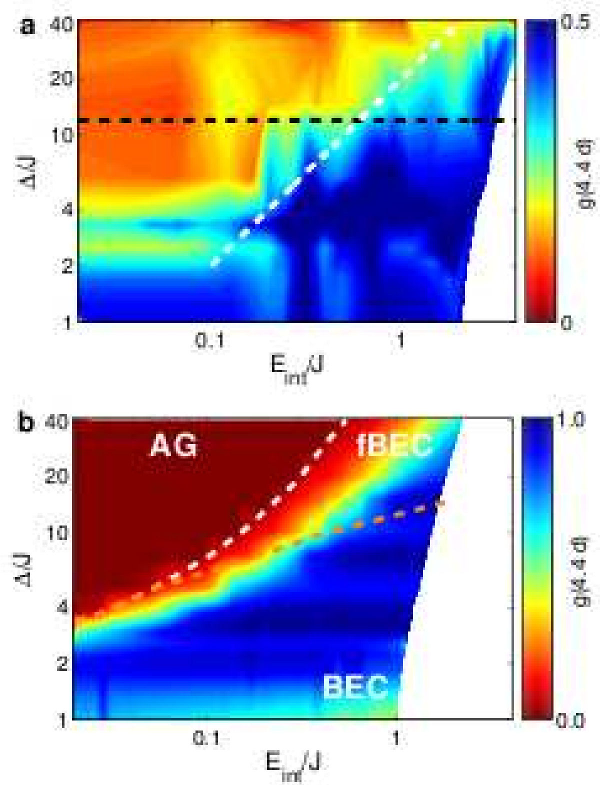
Inset: $P = \frac{1}{L} \frac{(\int dx |\Phi(x)|^2)^2}{\int dx |\Phi(x)|^4}$

Comparison

Cetoli and Lundh



Deissler *et al.*



Conclusions

- Mean field approach can give BG transition
- Threshold for the quasi periodic potential
- Bogoliubov fails to give reentrance
- Can it be applied to a 2D system?