Quantum Dynamics in Ferromagnetic and Antiferromagnetic Condensates

Nordita

Program on Quantum Solids, Liquids, and Gases

August 9, 2010

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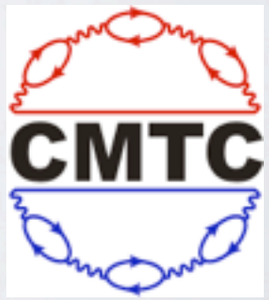

Outline for talk

1. Introduction & brief overview of spinor condensates
2. Antiferromagnetic spinor condensate dynamics (^{23}Na experiments at NIST).
3. Ferromagnetic spinor condensate dynamics (^{87}Rb quench experiments at Berkeley)

Introduction & Overview

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- However, in optical trap spin degrees of freedom are "unleashed".

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- When in magnetic trap spin degrees of freedom are frozen out.
- However, in optical trap spin degrees of freedom are "unleashed".
- BEC spin structure is determined by two body interaction.
- Leads to MF ground states having nontrivial symmetry groups.

Spinor Condensates:

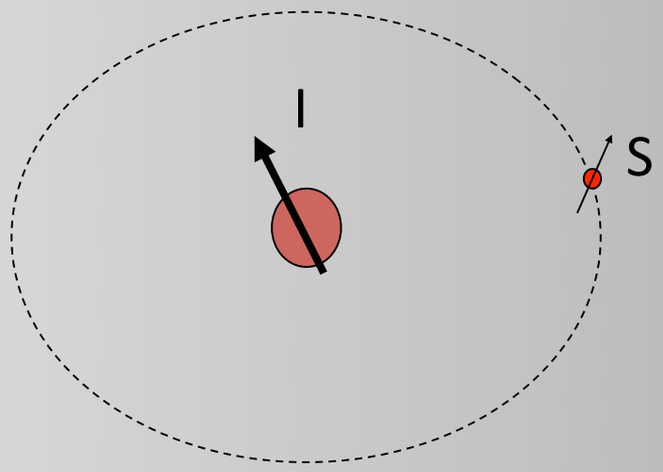
Spin structure from hyperfine interaction

I = nuclear spin = 3/2

S = electron spin = 1/2

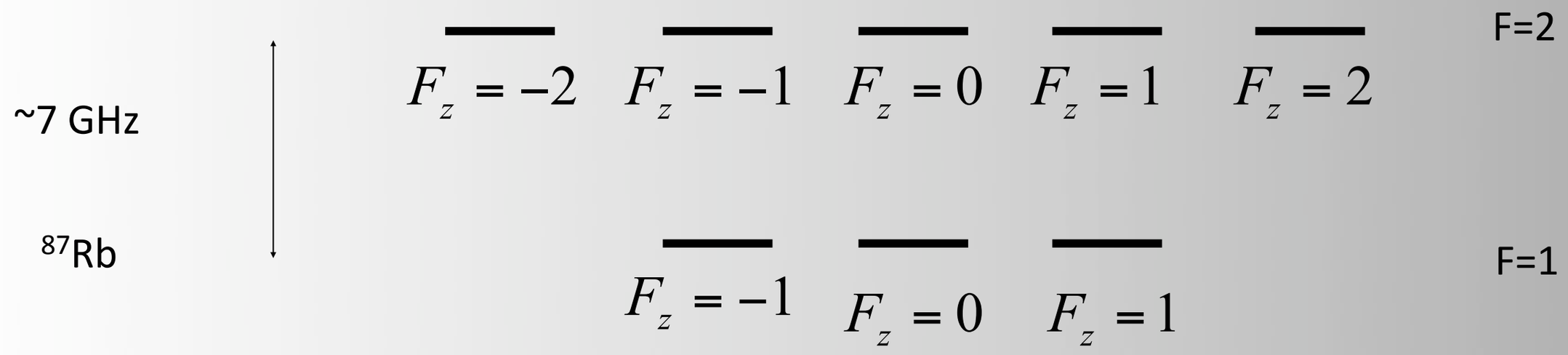
F = total spin = 1 or 2

$$\mathbf{F} = \mathbf{I} + \mathbf{S}$$



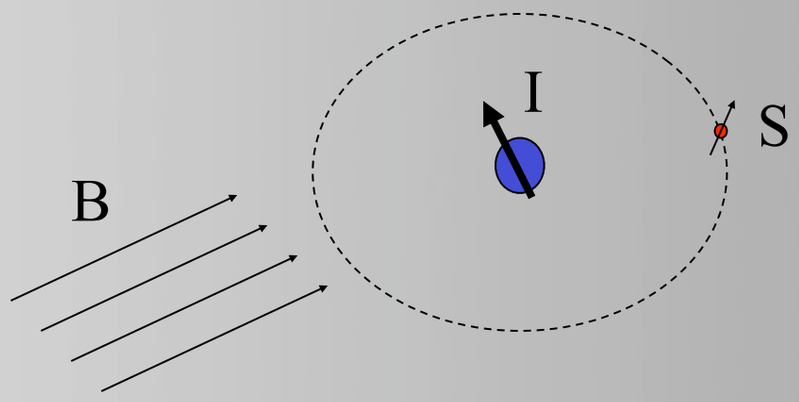
Alkalis

$$\mathcal{H}_{hf} = \gamma \mathbf{I} \cdot \mathbf{S} = \frac{\gamma}{2}(F^2 - I^2 - S^2)$$



How do external magnetic fields couple?

External B fields



$$\mathcal{H}_{hf} \propto \mathbf{I}\cdot\mathbf{S} = \frac{1}{2}(F^2 - \text{const})$$

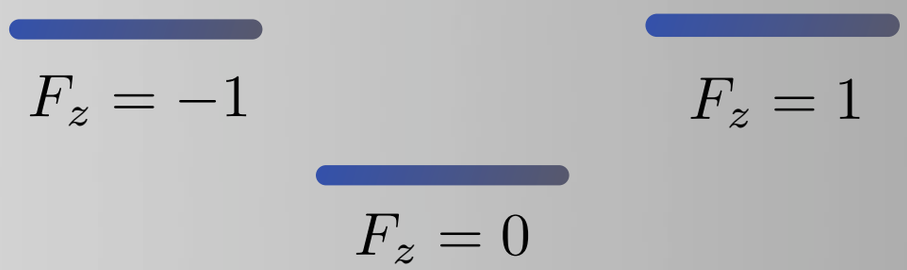
F good quantum #

$$\mathcal{H}_{\text{zem}} = -B\mu S_z$$

S good quantum #

$$\mathcal{H}_{\text{eff}} = -pF_z + qF_z^2$$

Conserved



Spinor Condensates: Order Parameter

Two-Body Interaction: $V(\mathbf{r}_1 - \mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2)(g_0\mathcal{P}_0 + g_2\mathcal{P}_2 + \ldots)$

Many body interaction: $\mathcal{H}_{\rm int} = \sum g_F \psi^\dagger_\alpha \psi_\beta < \alpha\beta|Fm >< Fm|\alpha'\beta' > \psi_{\alpha'}\psi_{\beta'}$

Order Parameter: $\psi \rightarrow \psi_\alpha$ for $\alpha = -F \ldots F$

Example: Spin one

$\mathcal{H}_{\rm int} \propto (g_2 - g_0)F^2$ where $\mathbf{F} = \psi^\dagger_\alpha \mathbf{F}_{\alpha\beta}\psi_\beta$

Ferromagnetic

$g_0 > g_2$

$$\psi \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Mean Field

Polar

$g_0 < g_2$

$$\psi \sim \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Spin Representations

The symmetry of the spin state is made most apparent by the Schwinger boson representation:

$$|\psi>=\prod_{i=1}^{2F}\left(\cos\left(\frac{\theta_i}{2}\right)a^\dagger+\sin\left(\frac{\theta_i}{2}\right)e^{i\phi_i}b^\dagger\right)|0>$$

Namely, a collection of 2F points on the unit sphere.

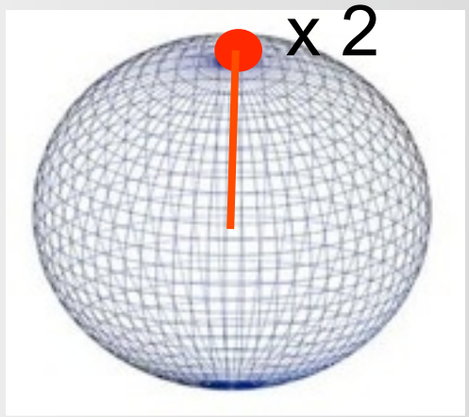
AKA “Majorana”, “Spin Node”, and “Stellar” representation

Also see work by Makela et al PRL (2007), Turner et al PRB (2009) and Lamacraft PRB (2010)

Example: Spin one -- two spin

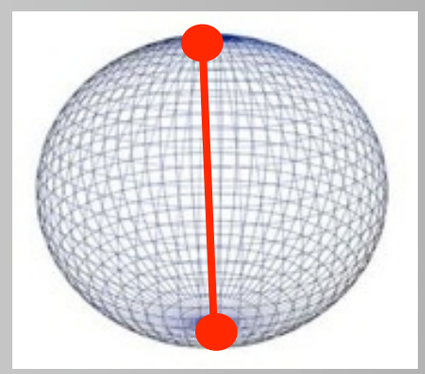
Ferromagneti $g_0 > g_2$

$$\psi\sim\begin{pmatrix}1\\0\\0\end{pmatrix}$$



Polar $g_0 < g_2$

$$\psi\sim\begin{pmatrix}0\\1\\0\end{pmatrix}$$



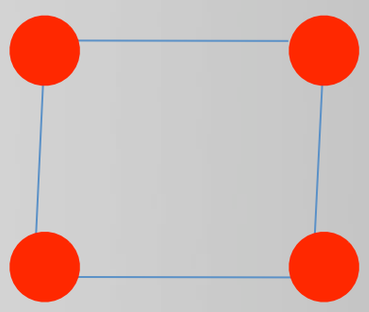
Symmetry of F=2 condensates

$$\mathcal{H}_{\text{int}} = \alpha F^2 + \beta \left(\Psi_t^\dagger \cdot \Psi \right)^2$$

$$\psi \sim (1,0,0,0,0)^T$$

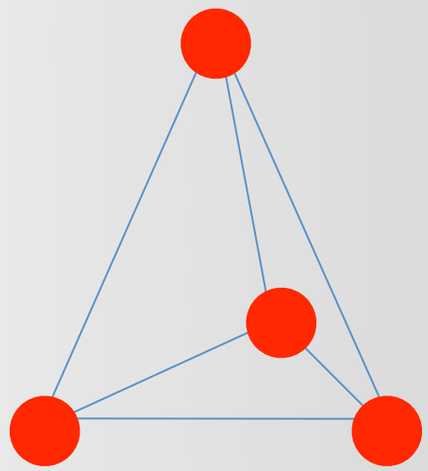
Ferromagnetic

$$\psi \sim (\sin(\eta)/\sqrt{2},0,\cos(\eta),0,\sin(\eta)/\sqrt{2})^T$$



Nematic

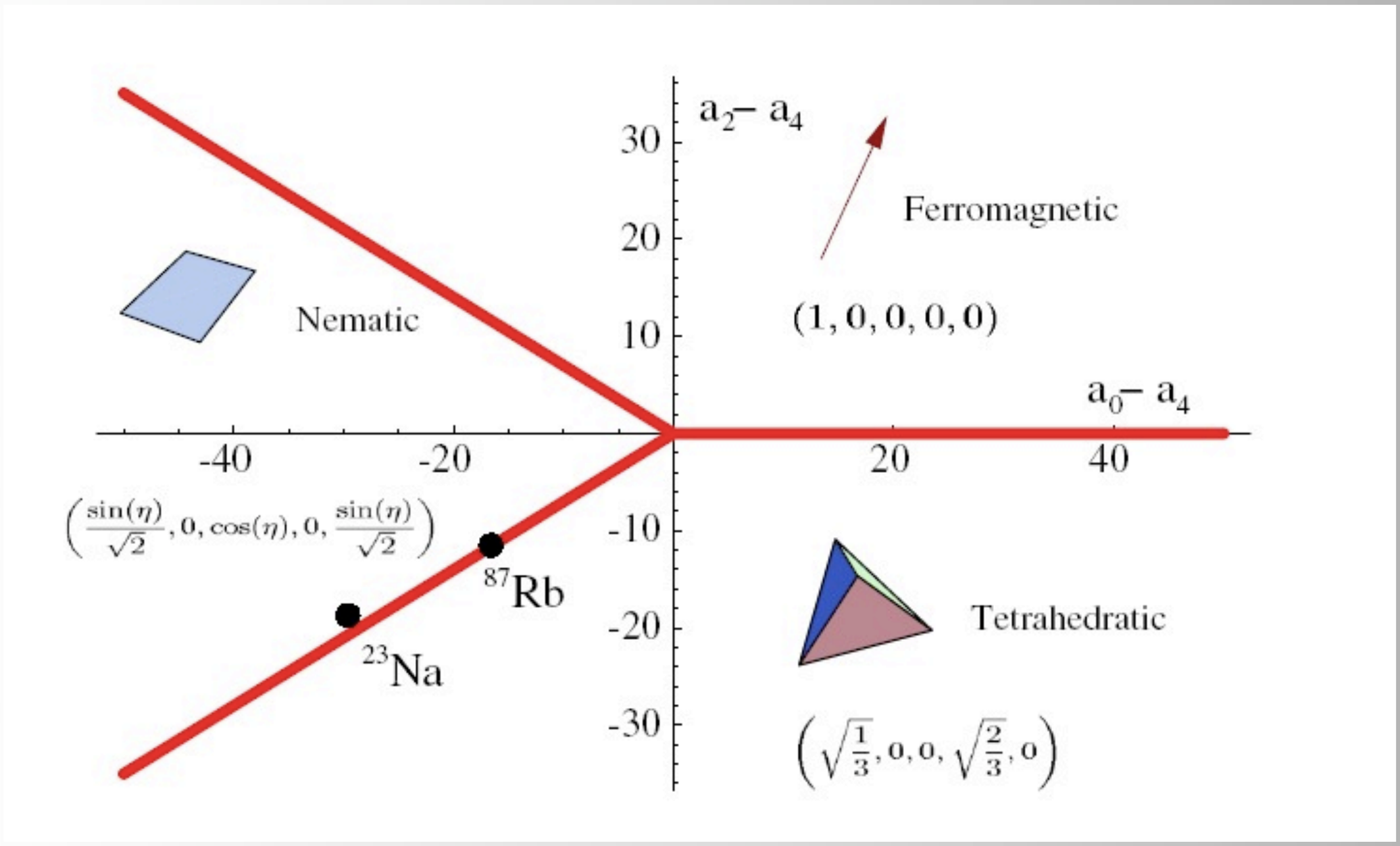
$$\psi \sim (\sqrt{1/3},0,0,\sqrt{2/3},0)^T$$



Cyclic

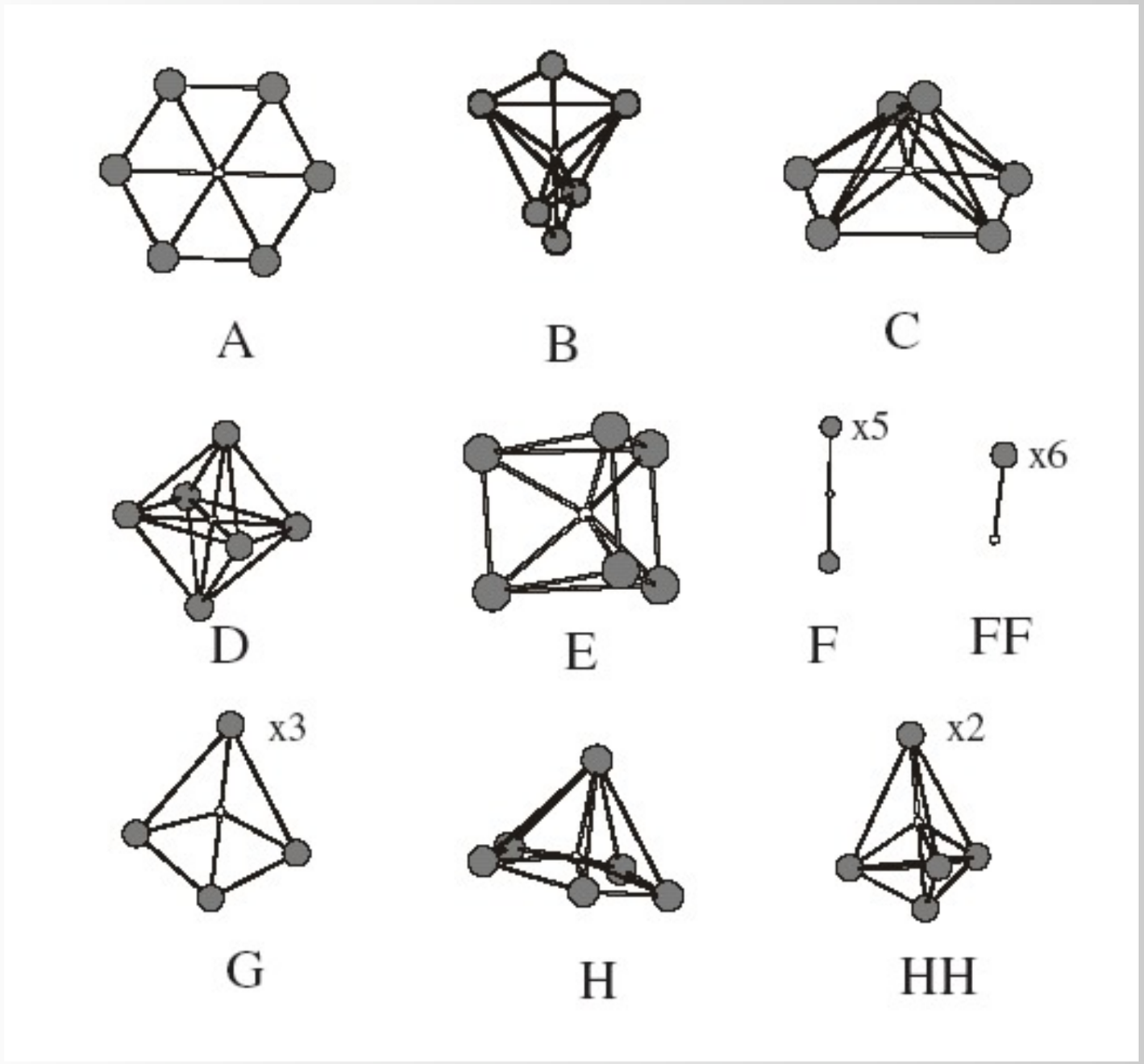
F=2 spinor condensates

Parameters (a_0, a_2, a_4)



F=3 spinor condensates

Parameters (a_0, a_2, a_4, a_6)



See also
Santos, Pfau,
PRL 96:190404 (2006);
Diener, Ho,
PRL 96:190405 (2006)

Barnett, Turner, Demler (PRA) 2007

Interesting aspects of spinor condensates not in scalar condensates

- Different types of elementary excitations (phonons)
- Exotic “fractional” topological defects (see Ari’s talk)
- Quantum effects can be important (spin two polar phase and small particle number)
- Coherent spin dynamics

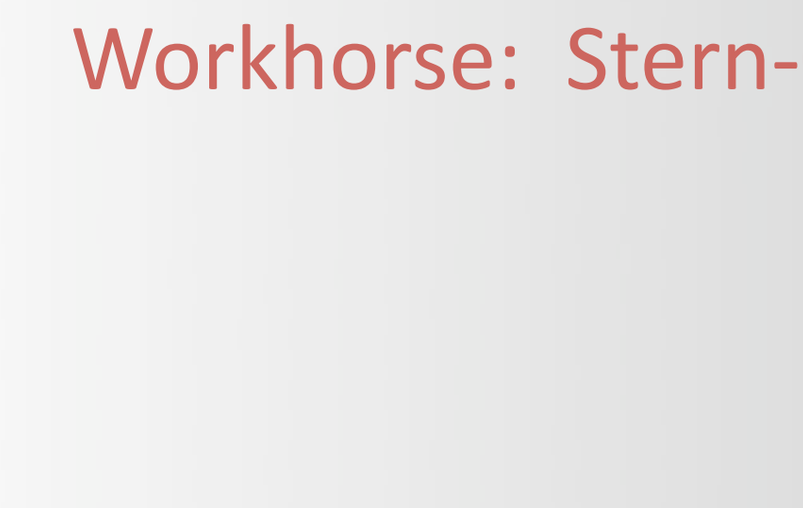
Experimental Probes

Workhorse: Stern-Gerlach Experiments

B

X X X
X X X
X X X
X X X
X X X

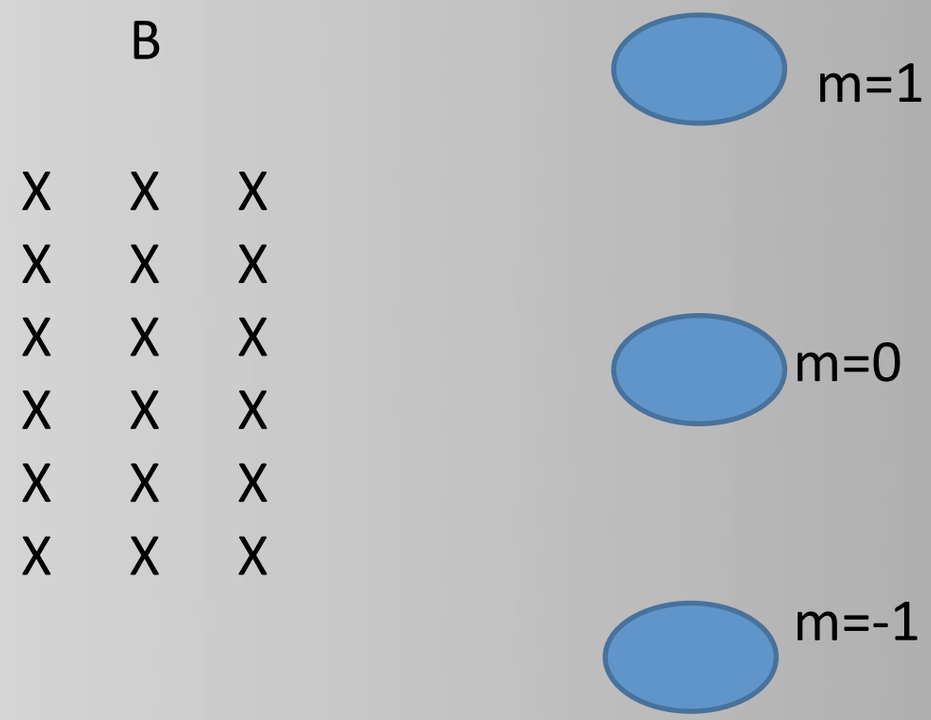
^{87}Rb

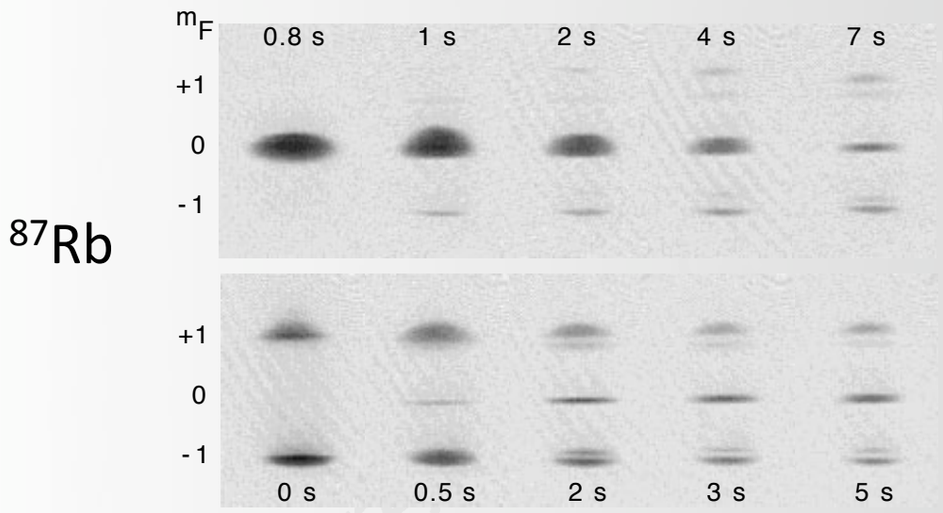


Stenger et al, Nature, 396 345 (1998)
(Ketterle Group)

Experimental Probes

Workhorse: Stern-Gerlach Experiments





Stenger et al, Nature, 396 345 (1998)
(Ketterle Group)

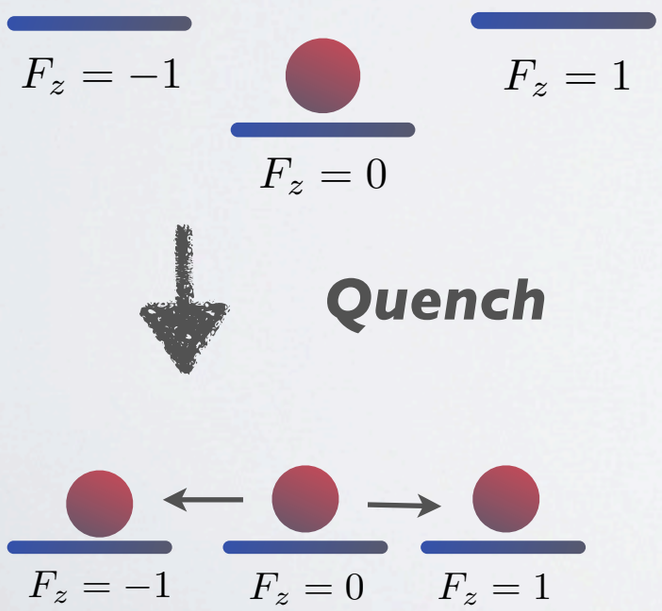
Experimental Probes

In situ imaging of spin domains

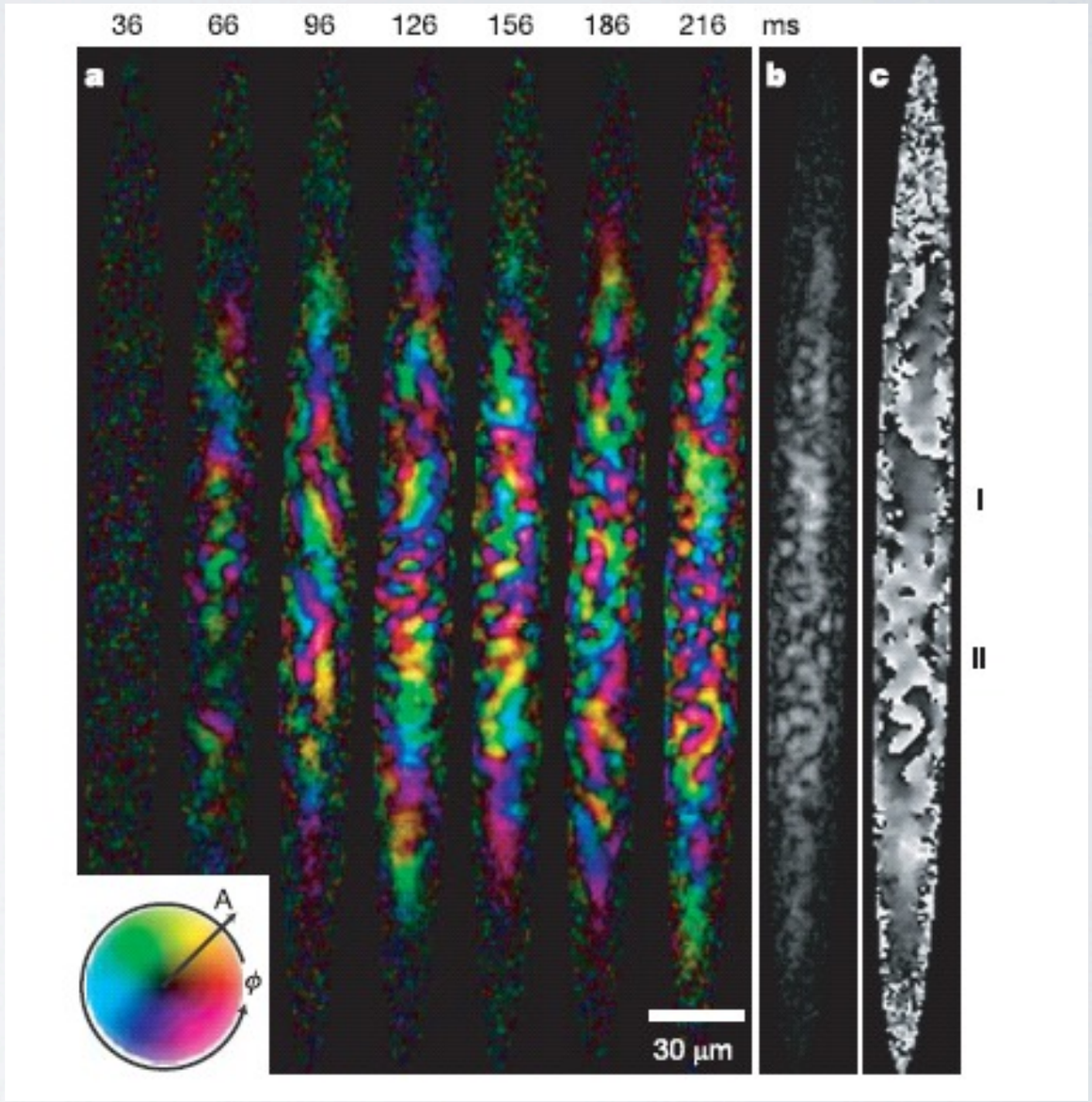
Quantum quenching experiments in spin-one ^{87}Rb

Stamper-Kurn group, UC Berkeley

Sau et al, PRA 80, 023522 (2009)
Leslie et al, PRA 79, 04363 (2009)
Cherng et al, PRL 100, 180404 (2008)
Vengalattore et al PRL, 100, 170403 (2008)
Vengalattore et al PRL, 97, 200801 (2007)
Sadler et al, Nature 443, 312 (2006)



^{87}Rb: Ferromagnetic interactions



Antiferromagnetic spinor condensate dynamics (^{23}Na experiments at NIST)

Ryan Barnett, Jay D. Sau, and S. Das Sarma, arxiv:1003.2634.

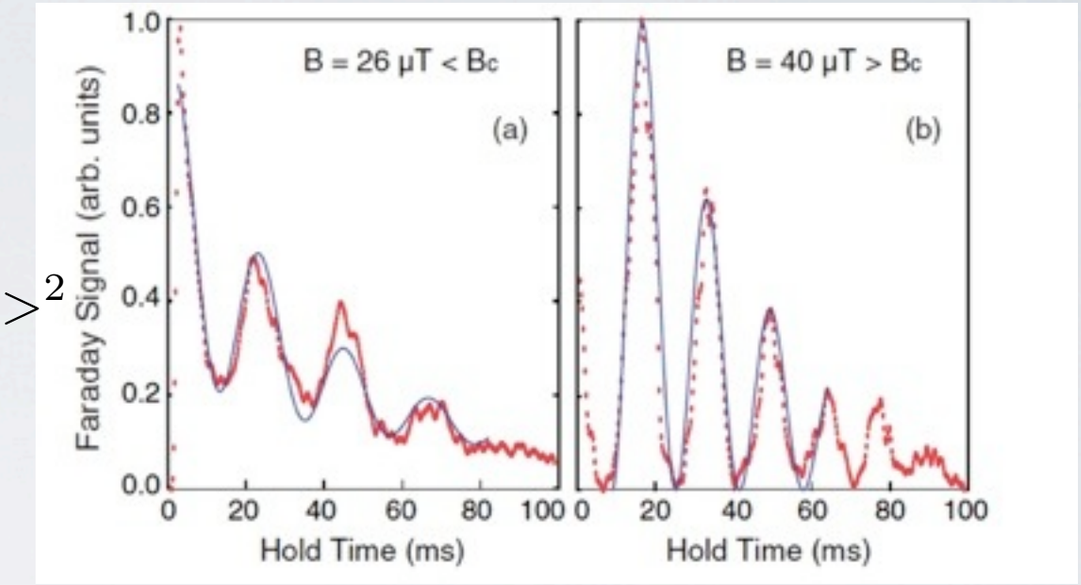
Experimental Motivation

^{23}Na: Antiferromagnetic interactions

Dynamics of spin-one ^{23}Na

Paul Lett group, NIST (Gaithersburg)

$< F_x >^2$



Liu et al, PRL 102, 225301(2009)
Liu et al, PRL 102, 125301(2009)

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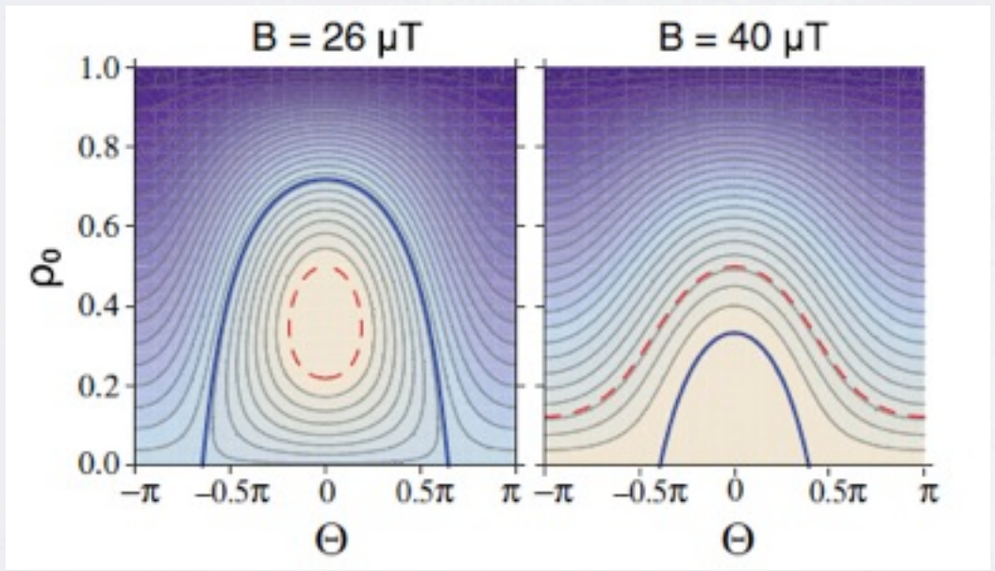
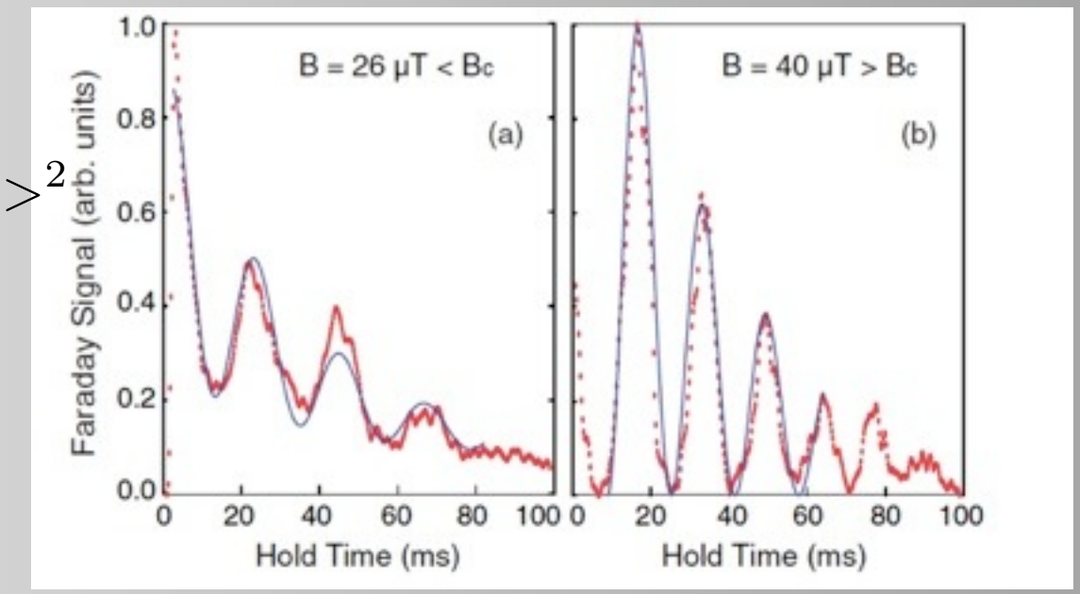


NIST Experiment

Analysis based on GPE Analysis in Single-Mode Approximation
[Pure spin dynamics]

$$
\begin{aligned}
E &= \frac{1}{2} g_s < \psi|\mathbf{F}|\psi > + q < \psi|F_z^2|\psi > \\
L &= i < \psi|\partial_t|\psi > - E \\
i\partial_t|\psi > &= g_s < \psi|\mathbf{F}|\psi > \cdot \mathbf{F}|\psi > + q F_z^2 |\psi >
\end{aligned}
$$

$< F_x >^2$



$$|\psi >= (\psi_1, \psi_0, \psi_{-1})^T$$

3-component classical spinor

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“Spin node description”

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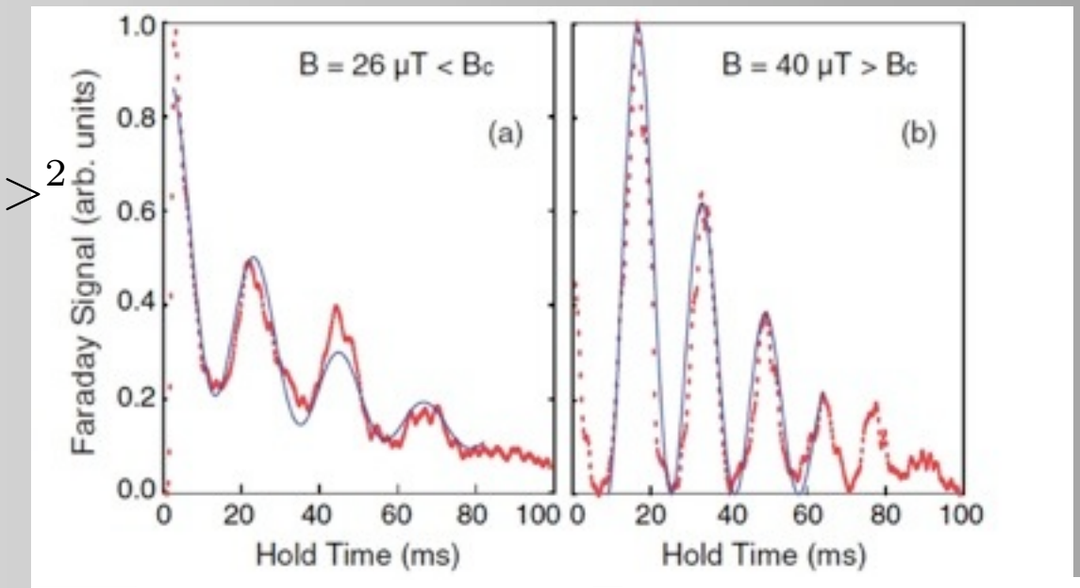

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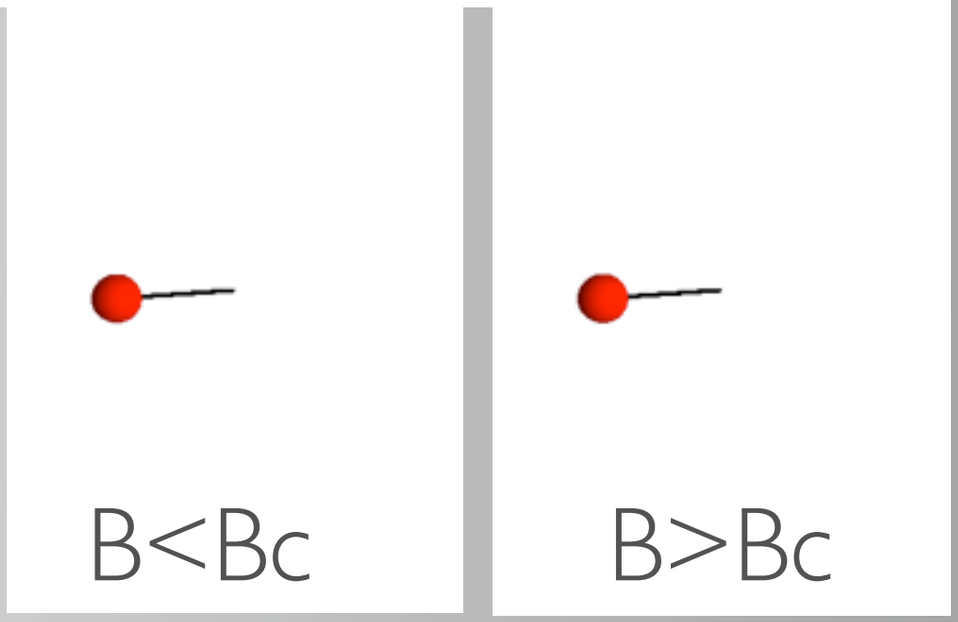
3-component classical spinor



"Spin node description"

NIST Experiment

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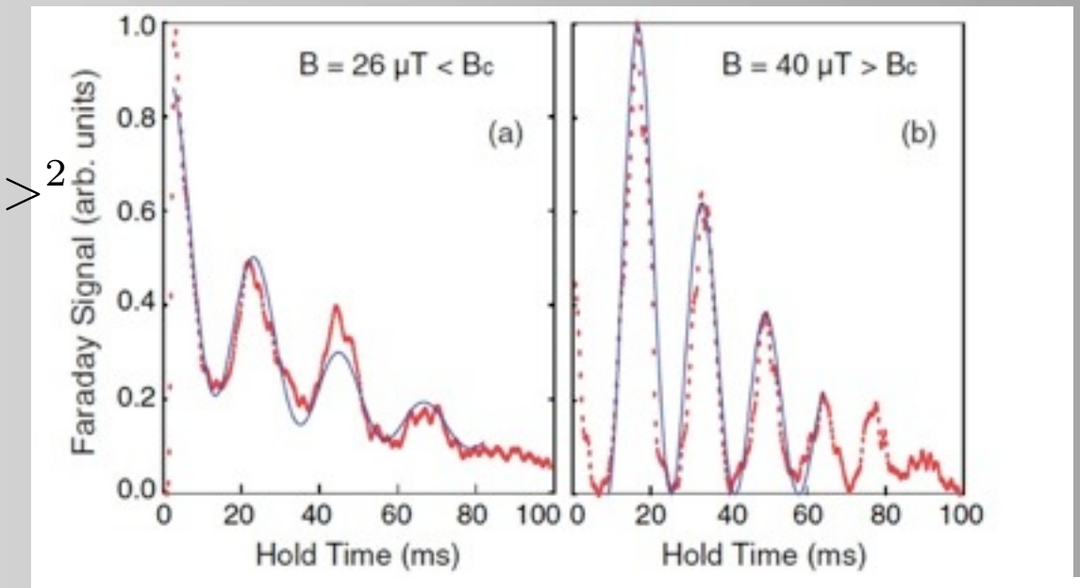
$$

$< F_x >^2$



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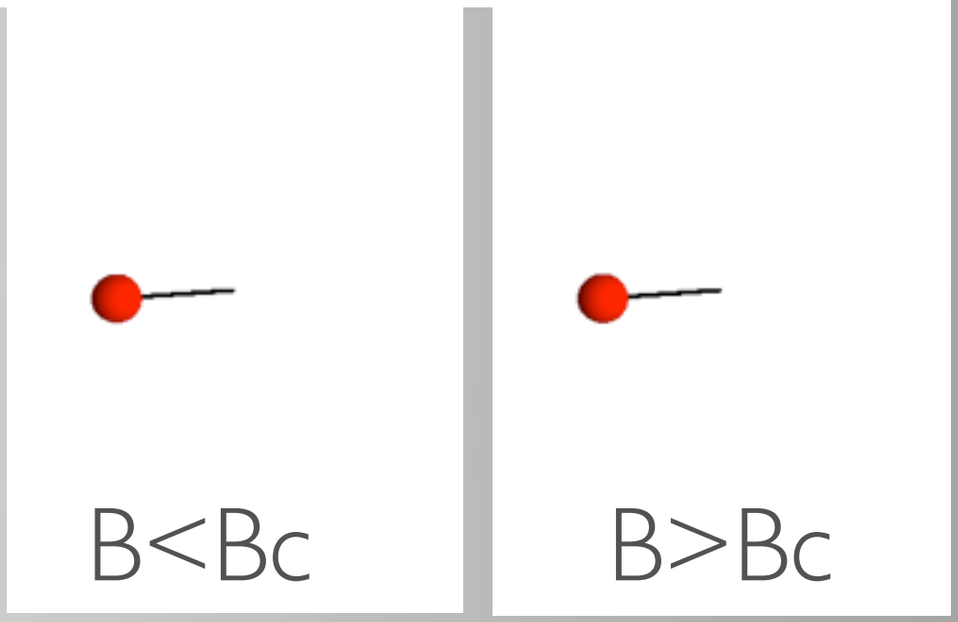


"Spin node description"

Goal: full many-body quantum treatment (within single mode). Map to Rotor model.

$$H = \frac{g}{2N}\hat{F}^2 - qa_0^\dagger a_0$$

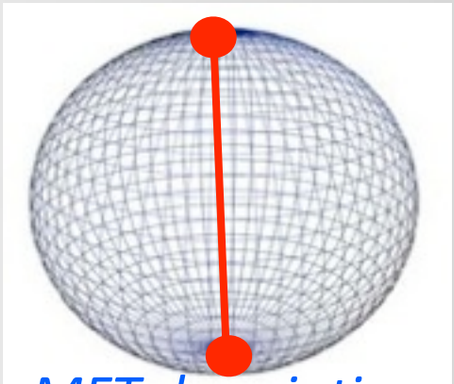
$$\hat{\mathbf{F}} = \sum a_\alpha^\dagger \mathbf{F}_{\alpha\beta} a_\beta$$

Spin-one matrices: $F_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $F_y = \frac{i}{\sqrt{2}}\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ $F_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

Ground state when g=0:

$$|\Psi> \propto (a_0^\dagger)^N |0>$$

Put all N bosons in polar state



MFT description

Ground state when q=0:

$$|\Psi> \propto \left(a_0^\dagger a_0^\dagger - 2a_1^\dagger a_{-1}^\dagger\right)^{N/2} |0>$$

Condensate of singlet pairs of bosons

"Fragmented" condensate

No mean-field theory analog

Cartesian representation of spin operators

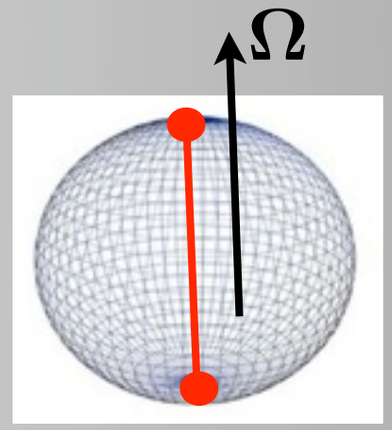
$$\begin{aligned} b_x &= -(a_1 - a_{-1})/\sqrt{2} \\ b_y &= (a_1 + a_{-1})/i\sqrt{2} \\ b_z &= a_0 \end{aligned} \qquad \hat{\mathbf{F}} = -i\mathbf{b}^\dagger \times \mathbf{b}$$

Change variables (These transform as a vector under rotations)

Define $|\mathbf{\Omega}_N >= \frac{1}{\sqrt{N!}} \left(\mathbf{\Omega} \cdot \mathbf{b}^\dagger\right)^N |0>$ $\qquad \mathbf{\Omega} \cdot \mathbf{\Omega} = 1$

Polar states pointing along unit vector $\mathbf{\Omega}$

Inner product $< \mathbf{\Omega}_N | \mathbf{\Omega}'_N >= (\mathbf{\Omega} \cdot \mathbf{\Omega}')^N$



Represent general state as superposition of polar states:

$|\Psi >= \int d\Omega |\mathbf{\Omega}_N > \psi(\mathbf{\Omega})$ ← weight

$\psi = \delta$-function $\rightarrow$ polar

$\psi =$ const $\rightarrow$ "fragmented" singlet state

Spectral Representation

General idea: find $H_{\rm rot}$ (written in terms of Ω and its derivatives) such that

$$H|\Psi>=\int d\Omega\ [H|\mathbf{\Omega}_N>]\ \psi(\mathbf{\Omega})=\int d\Omega|\mathbf{\Omega}_N>(H_{\rm rot}\psi(\mathbf{\Omega}))$$

Then $H_{\rm rot}\psi=E\psi \implies H|\psi>=E|\psi>$

$$|\Psi>=\int d\Omega|\mathbf{\Omega}_N>\psi(\mathbf{\Omega})$$

Map many particle problem onto single particle (on sphere) problem

¿ Is there a correspondence for every eigenvalue?

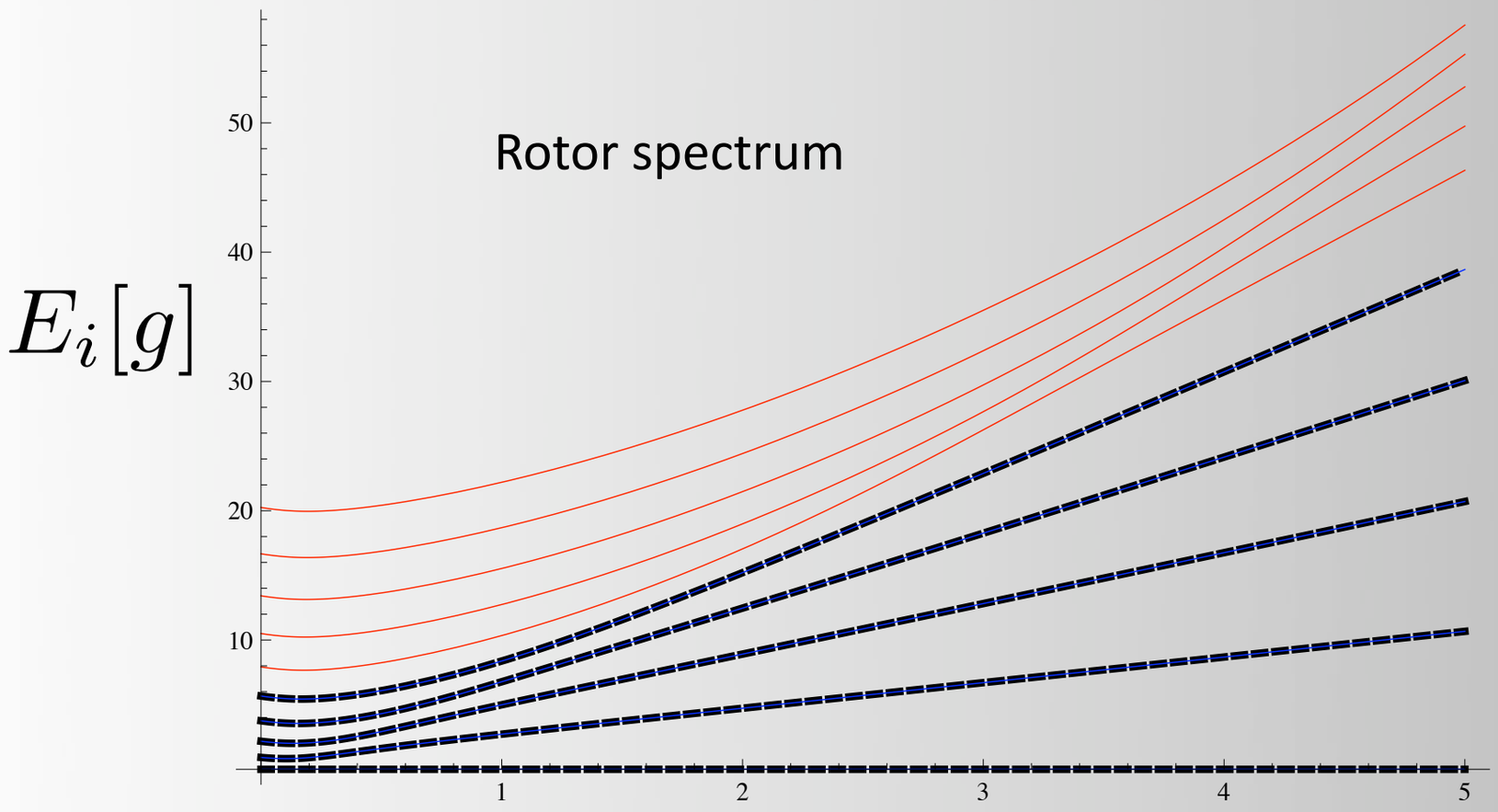
Rotor mapping: the details

- General quadratic operators can be written as $b_\alpha^\dagger b_\beta |\mathbf{\Omega}_N >= \Omega_\beta \left(\nabla_\alpha + N\Omega_\alpha\right)|\mathbf{\Omega}_N >$
- ∇_α is the gradient operator on the unit sphere: $\nabla_\alpha \Omega_\beta = \delta_{\alpha\beta} - \Omega_\alpha \Omega_\beta$
- Integration by parts can be applied to arrive at $H_{\rm rot}$
- Under this transformation, $F_\alpha \to L_\alpha = -i\varepsilon_{\alpha\beta\gamma}\Omega_\beta \nabla_\gamma$ and $b_z^\dagger b_z \to (N+3)\Omega_z^2 - \Omega_z \nabla_z$
- Thus, $H \to \tilde{H}_{\rm rot} = \frac{g}{2N\hbar^2} L^2 - q(N+3)\Omega_z^2 + q\Omega_z\nabla_z$ (not Hermitian).
- Can apply similarity transformation to render rotor hamiltonian Hermitian $H_{\rm rot} = e^F \tilde{H}_{\rm rot} e^{-F}$

The result:

$$H_{\rm rot} = \frac{g}{2N} L^2 + V(\theta)$$
$$V(\theta) = q\left(N + \frac{3}{2}\right)\sin^2(\theta) + \frac{q^2 N}{8g}\sin^2(2\theta)$$

Comparing eigenvalues of rotor and spinor Hamiltonians

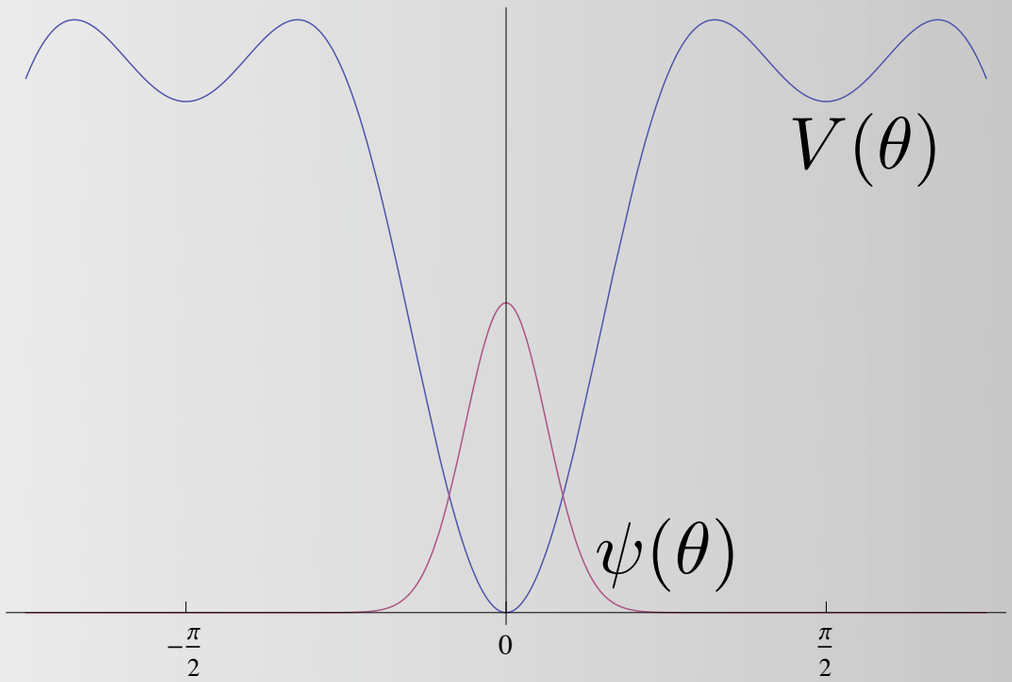


Unphysical eigenvalues

Physical eigenvalues

- The complete spectrum of the spinor problem are accounted for by rotor states.
- The rotor spectrum is unbounded from above while the spinor spectrum is not.
- Interestingly, the lowest eigenvalues of the rotor spectrum are the physical ones (see plot) which exactly agree.
- This can be understood through perturbation theory.

Harmonic Theory



- For large q, the spectrum is described by a harmonic theory about the $\theta = 0$ minimum. The ground state is the polar state.
- That is, $\psi_0(\theta) = \sqrt{\frac{1}{\pi\bar{\theta}^2}} e^{-\theta^2/2\bar{\theta}^2}$ where $\bar{\theta} = \sqrt{\frac{g}{2qN^2}}$ and $E_n = \sqrt{2gq}(n+1)$.
- When q is decreased, the state becomes delocalized: the singlet state
- However, for large particle number, the polar state wins for realistic magnetic fields.
- When $q > 2g$, a second minimum appears. Possibility of "π-states"

Semiclassical Theory Agrees with Experiment

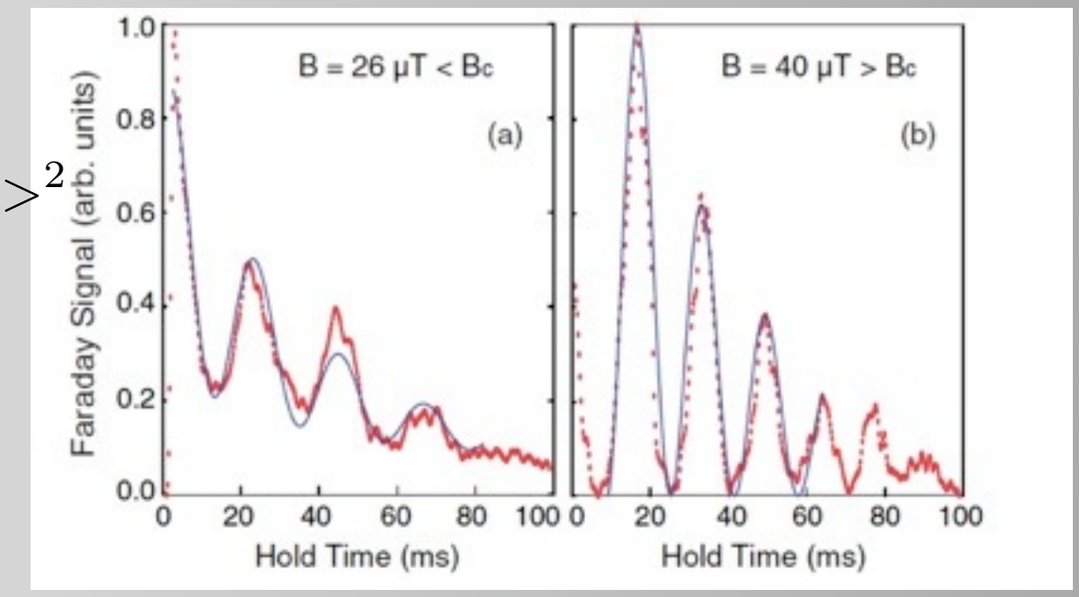
Semiclassical theory:

$$\mathcal{L} = \frac{1}{2} I \left(\dot{\theta}^2 + \sin^2(\theta) \dot{\phi}^2 \right) - V(\theta)$$

$$I\ddot{\theta} = I \frac{\cos(\theta)}{\sin^3(\theta)} p_\phi^2 - \frac{\partial V}{\partial \theta}$$

$$p_\phi = \sin^2(\theta)\dot{\phi} = \text{constant of motion}$$

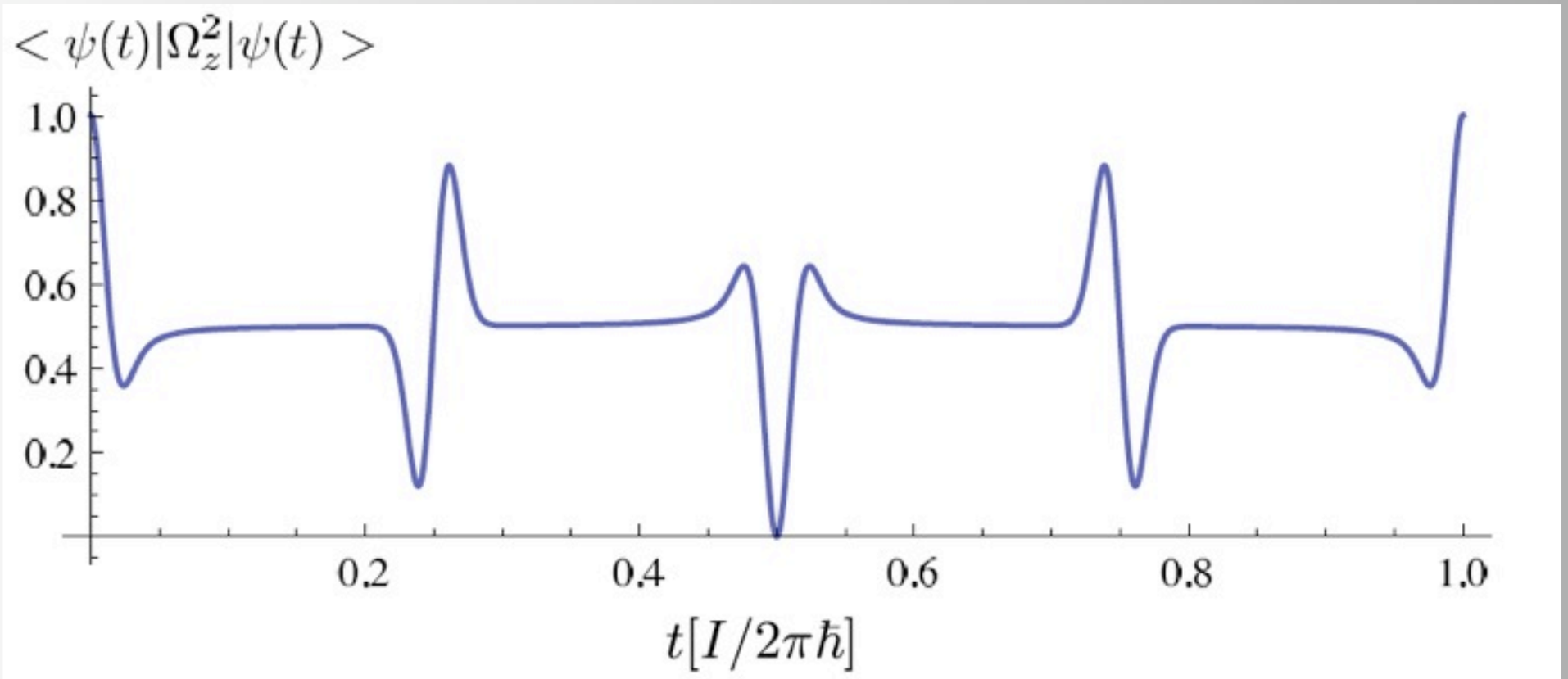
$< F_x >^2$



q<qc q>qc

- For $q > q_c$, the particle is confined to be in one hemisphere
- For $q < q_c$, the particle can explore both hemisphere
- Separatrix agrees with result from GPE energy

Experimental Proposal: Collapse and Revival



- Start with polar state (with external magnetic field)
- Quench field to zero $q = 0$.
- Most likely to be realized in optical lattice with no hopping.
- Specific parameters: $N \sim 50$, $B_{\text{final}} < 1\mu T$, revival time $\sim$ ms

Ferromagnetic spinor condensate dynamics (^{87}Rb quench experiments at Berkeley)

Ryan Barnett, Mukund Vengelattore, Anatoli Polkovnikov (to appear)

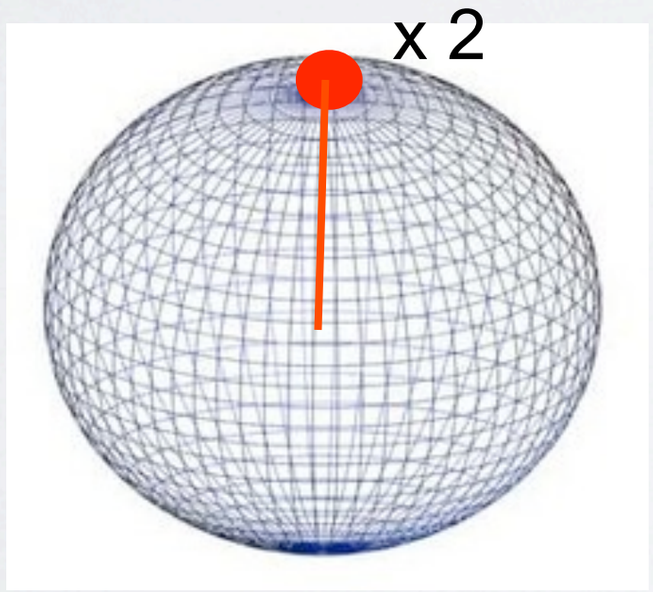
Anti vs. Ferromagnetic Condensate

$$\mathcal{E} = c_2 n_0 < \chi|\mathbf{F}|\chi >^2 + q < \chi|F_z^2|\chi >$$

$c_2 < 0$

Ferromagnetic interaction

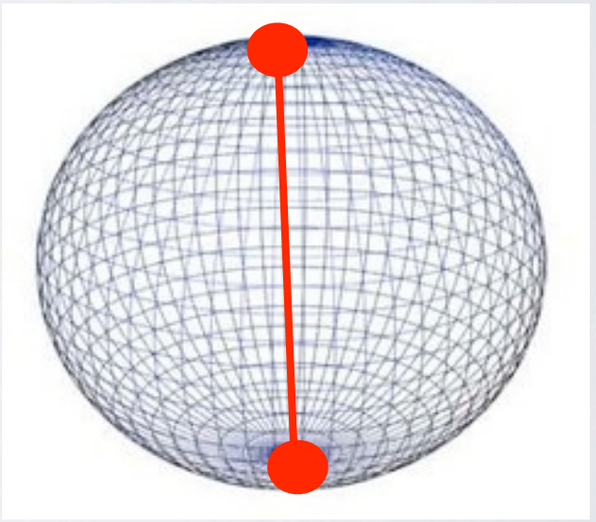
$$\chi = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$



$c_2 > 0$

Antiferromagnetic interaction

$$\chi = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

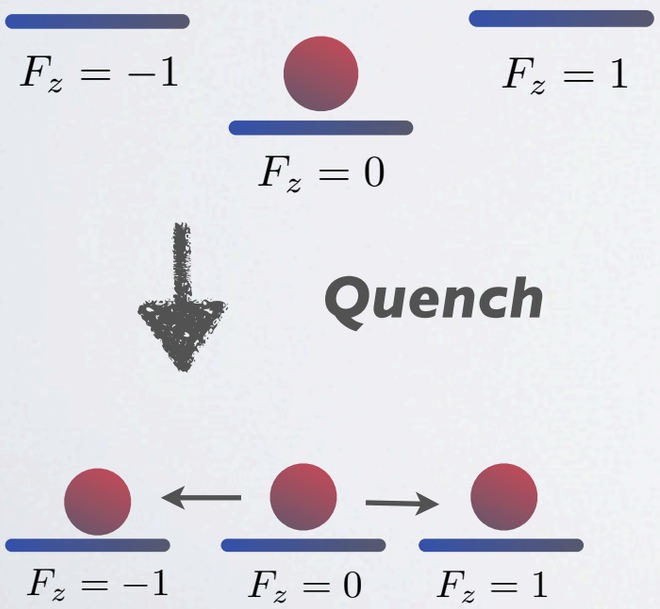


Experimental Motivation

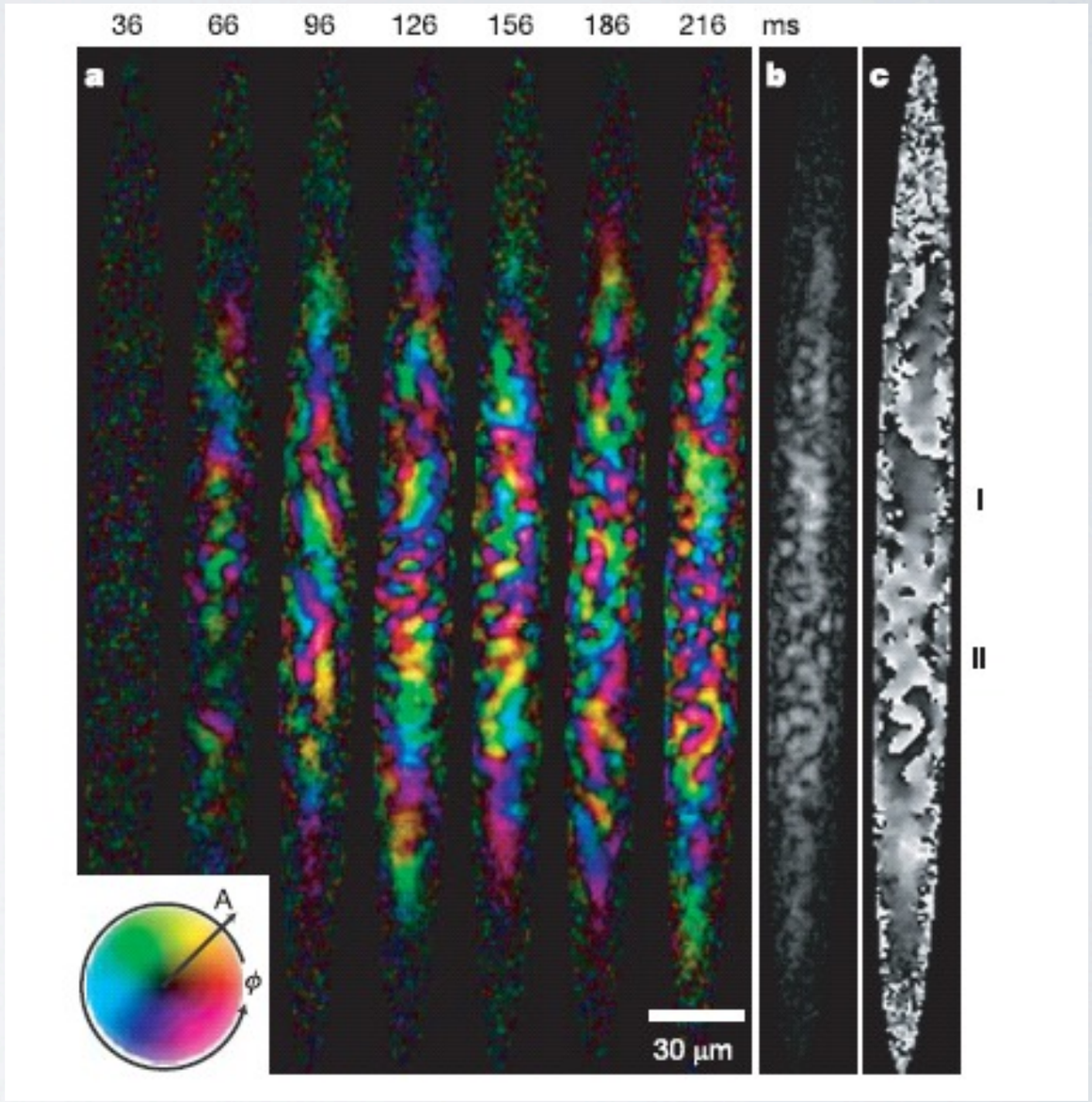
Quantum quenching experiments in spin-one ^{87}Rb

Stamper-Kurn group, UC Berkeley

Sau et al, PRA 80, 023522 (2009)
Leslie et al, PRA 79, 04363 (2009)
Cherng et al, PRL 100, 180404 (2008)
Vengalattore et al PRL, 100, 170403 (2008)
Vengalattore et al PRL, 97, 200801 (2007)
Sadler et al, Nature 443, 312 (2006)



^{87}Rb: Ferromagnetic interactions

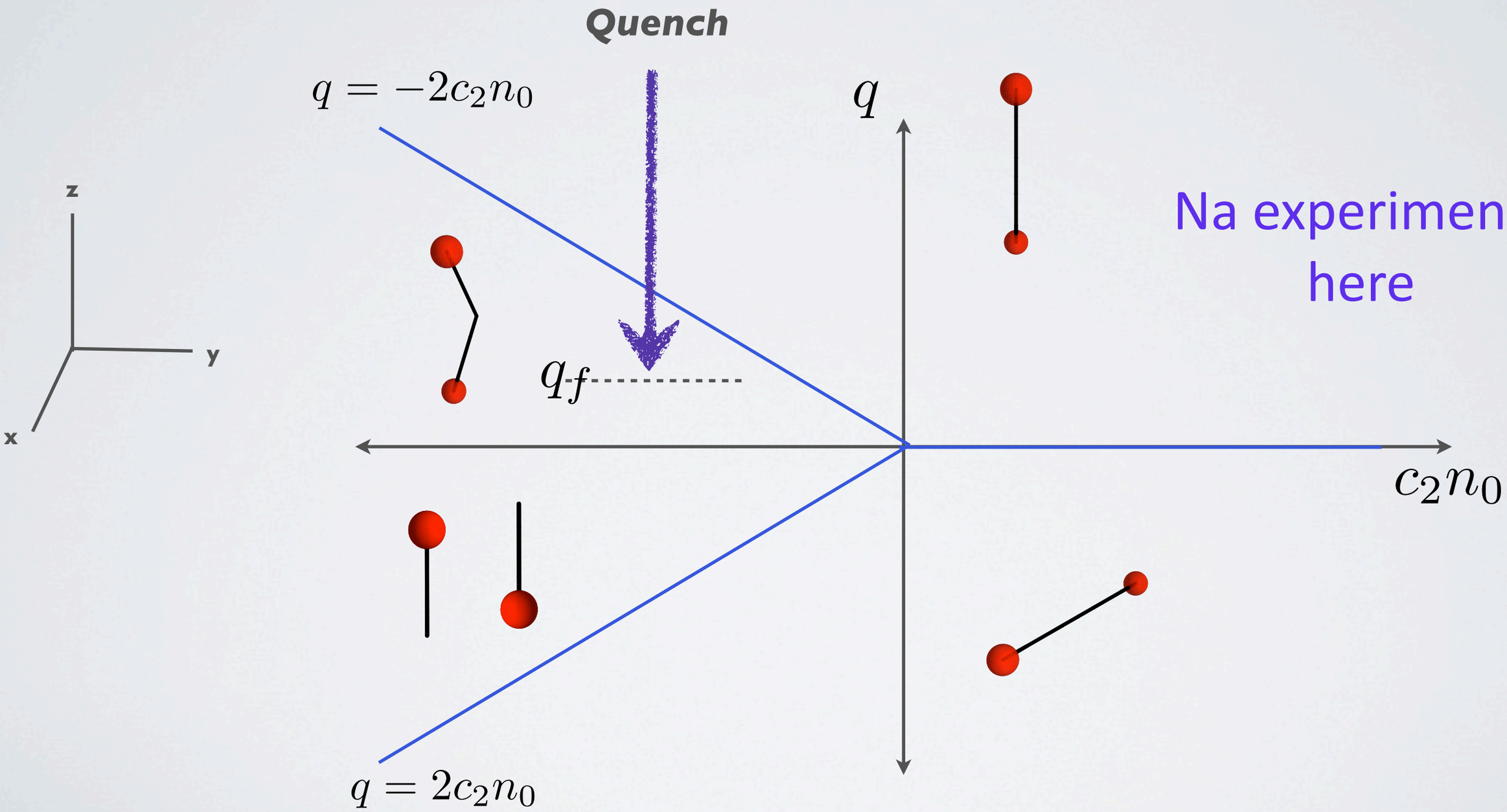


Differences with NIST experiments

- Ferromagnetic Interactions
- Much larger trap --> SMA is not realistic
- No periodic behavior
- Topological defects important
- Mean field ground state changes with q

Phase Diagram with Quadratic Zeeman

$$\mathcal{E} = c_2 n_0 < \chi|\mathbf{F}|\chi >^2 + q < \chi|F_z^2|\chi >$$

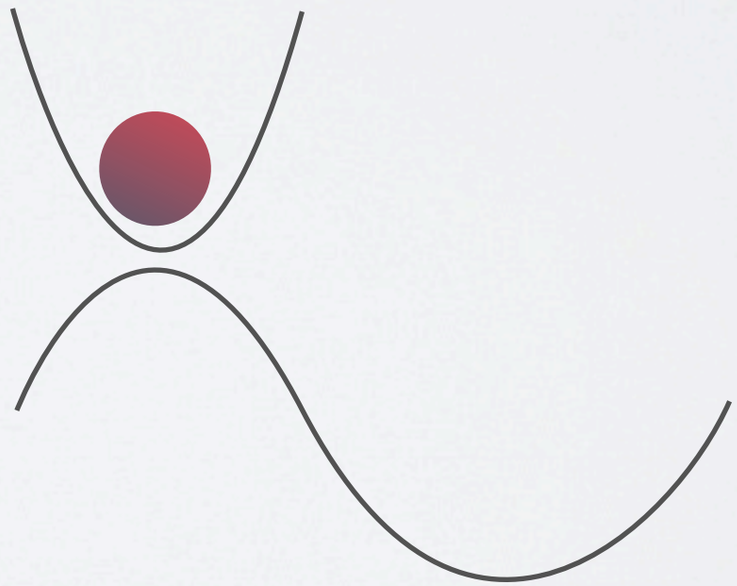


Na experiments here

Method of Simulation: Truncated Wigner Approximation

General method

Simplest example:
particle in potential



Method of Simulation: Truncated Wigner Approximation

General method

- Obtain initial wavefunction and take Wigner transform

$$W(x,p) = \int d\xi < x + \xi/2|\psi_0 >< \psi_0|x - \xi/2 > e^{-ip\xi}$$

Simplest example:
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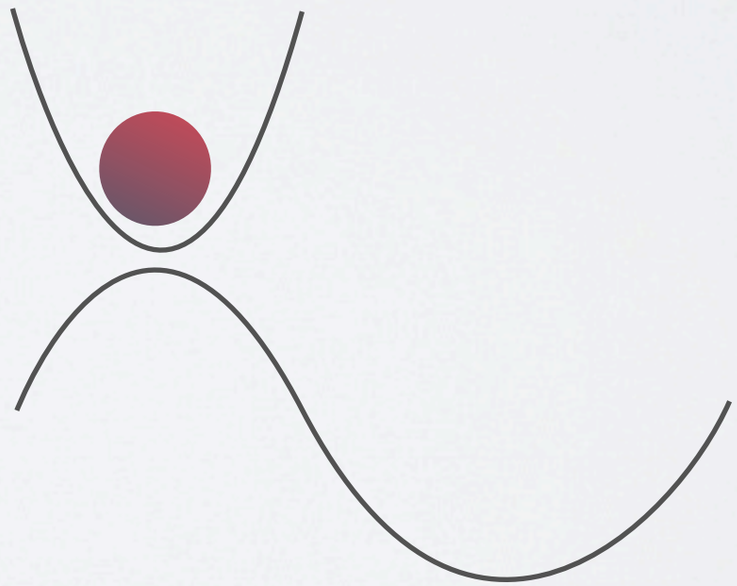


Method of Simulation: Truncated Wigner Approximation

General method

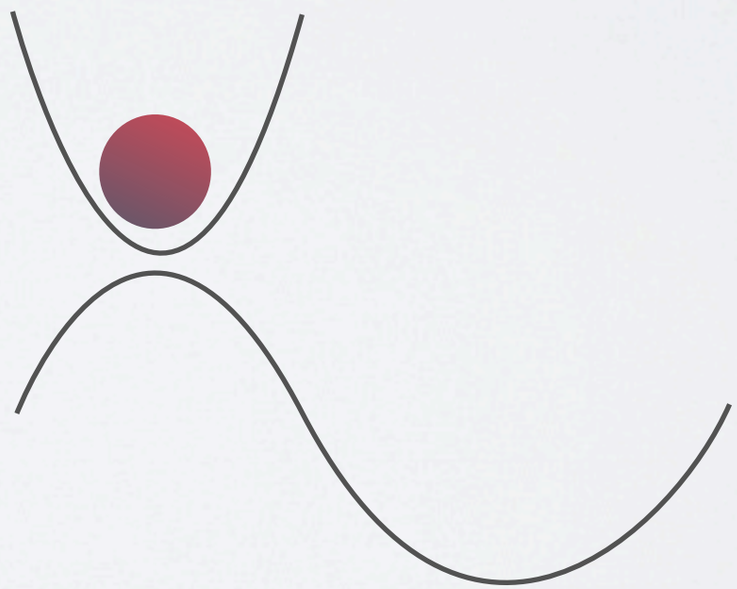
- Obtain initial wavefunction and take Wigner transform

$$W(x,p) = \int d\xi < x+\xi/2|\psi_0 >< \psi_0|x-\xi/2 > e^{-ip\xi}$$

- To evaluate expectation values of operators, average over classical trajectories with Wigner function.

$$< \Omega(t) >= \int dx_0 dp_0 W(x_0,p_0)\Omega_W(x_{\rm cl}(t),p_{\rm cl}(t))$$

Simplest example:
particle in potential



Method of Simulation: Truncated Wigner Approximation

Application to Spinor Condensate

- The ground state is the vacuum of Bogoliubov modes about the polar state

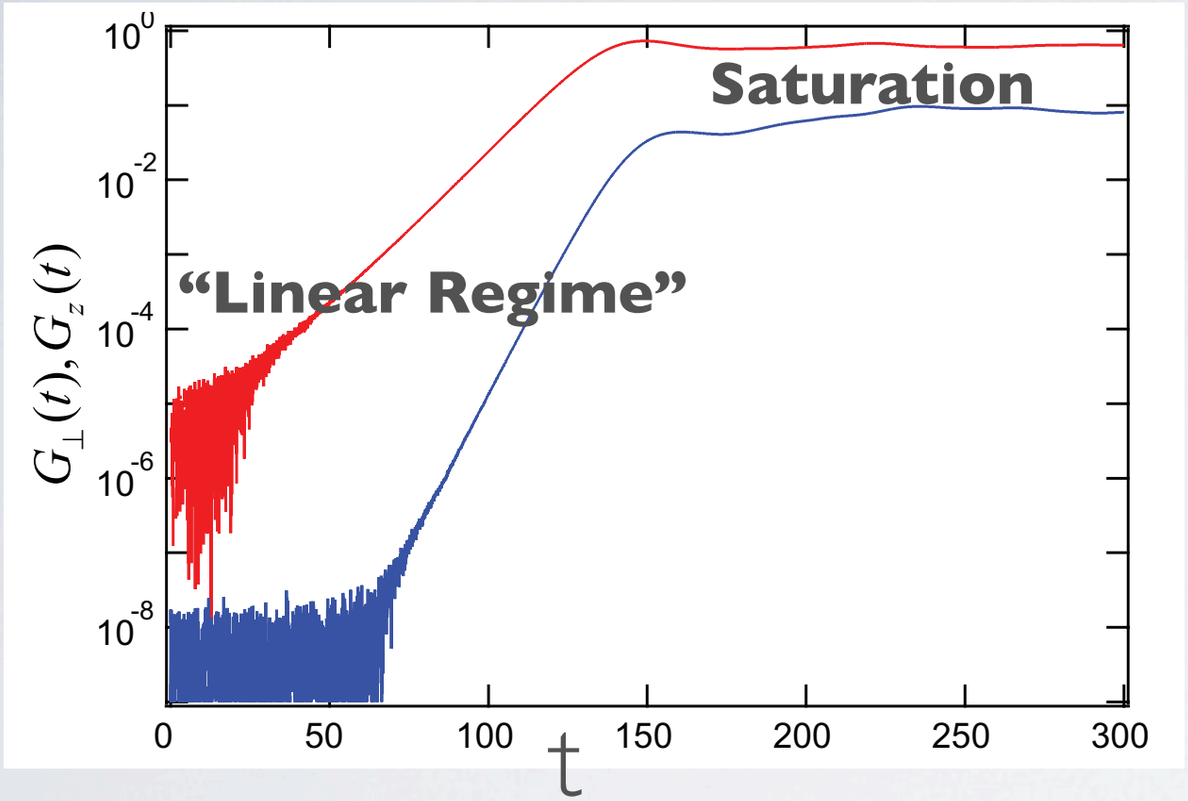
- Classical equations of motion are the Gross-Pitaevskii equation for spinor condensates. System is propagated using a second-order split operator method

$$i\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + g(\psi^\dagger\cdot\psi)\psi + c_2(\psi^\dagger\mathbf{F}\psi)\cdot\mathbf{F}\psi + qF_z^2\psi$$

$$\psi = (\psi_1, \psi_0, \psi_{-1})^T$$

$$\psi(t+\Delta t) = e^{-i\mathcal{H}\Delta t}\psi(t) \approx e^{-iV\Delta\frac{t}{2}}e^{-iT\Delta t}e^{-iV\Delta\frac{t}{2}}\psi(t) \;\; ; \;\; \mathcal{H} = T + V$$

TWA results



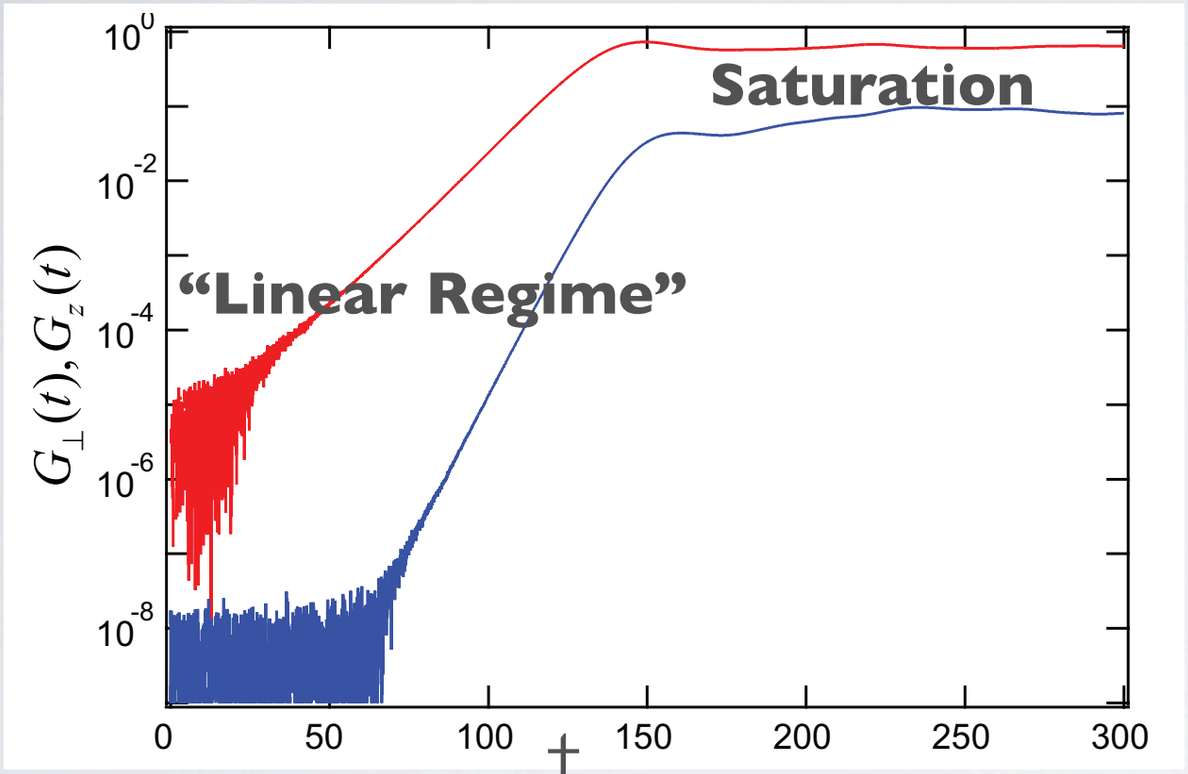
$$G_\perp(t) =< (F_x^2 + F_y^2) >$$

$$G_z(t) =< (F_z^2) >$$

See also Sau et al, PRA 80, 023522 (2009)

Color gives angle in xy plane
Brightness gives spin length

TWA results



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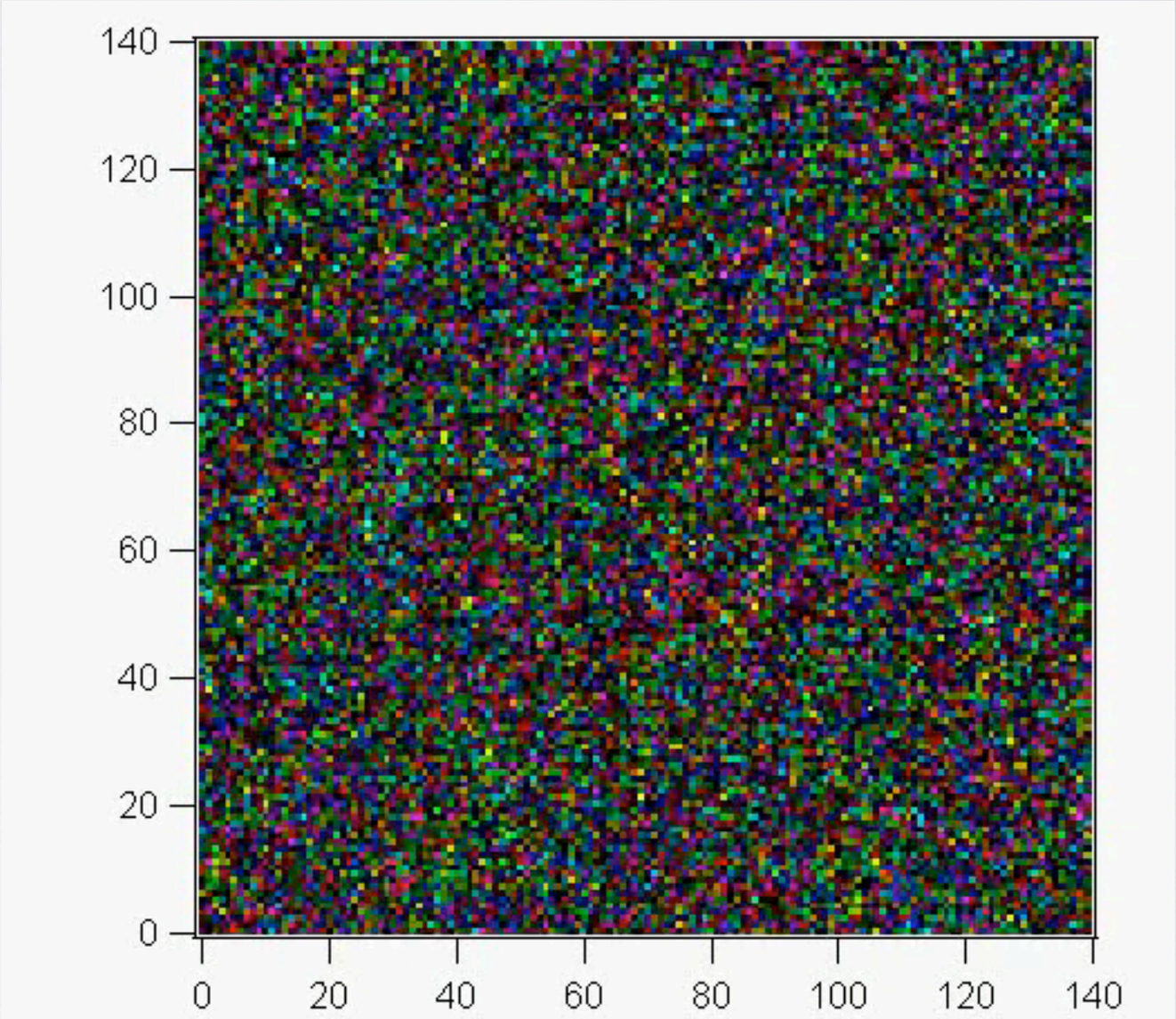

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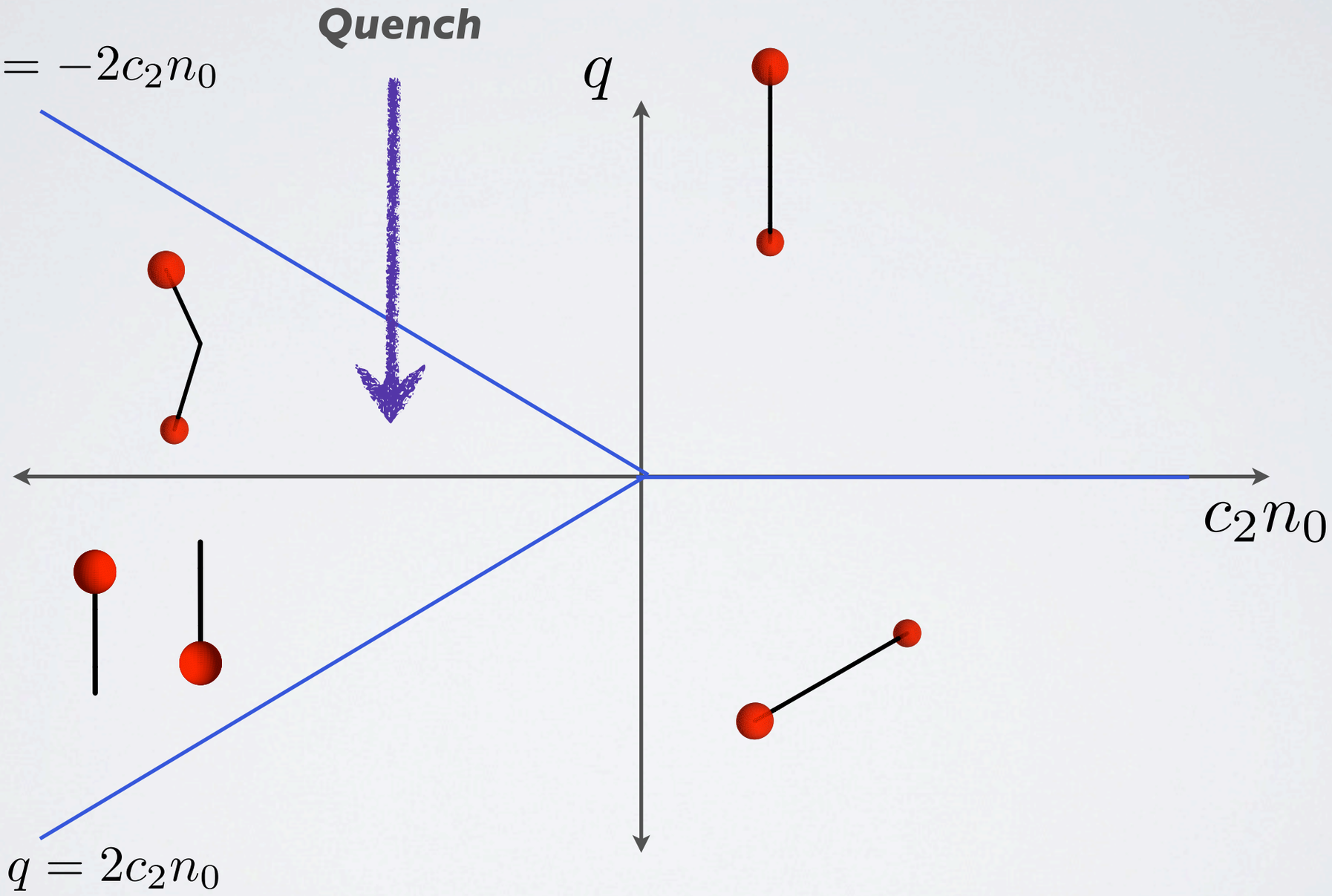
Linearized Theory

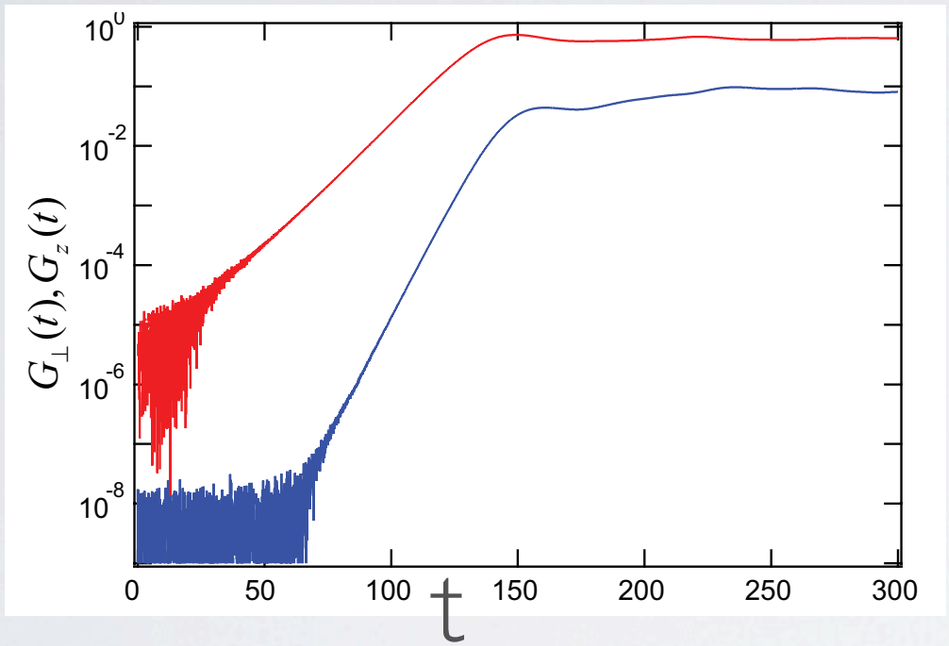
Linearized Theory

- Bogoliubov expansion about polar (unstable) polar state



Linearized Theory

- Bogoliubov expansion about polar (unstable) polar state
- Compute observables using linearized theory keeping only unstable modes

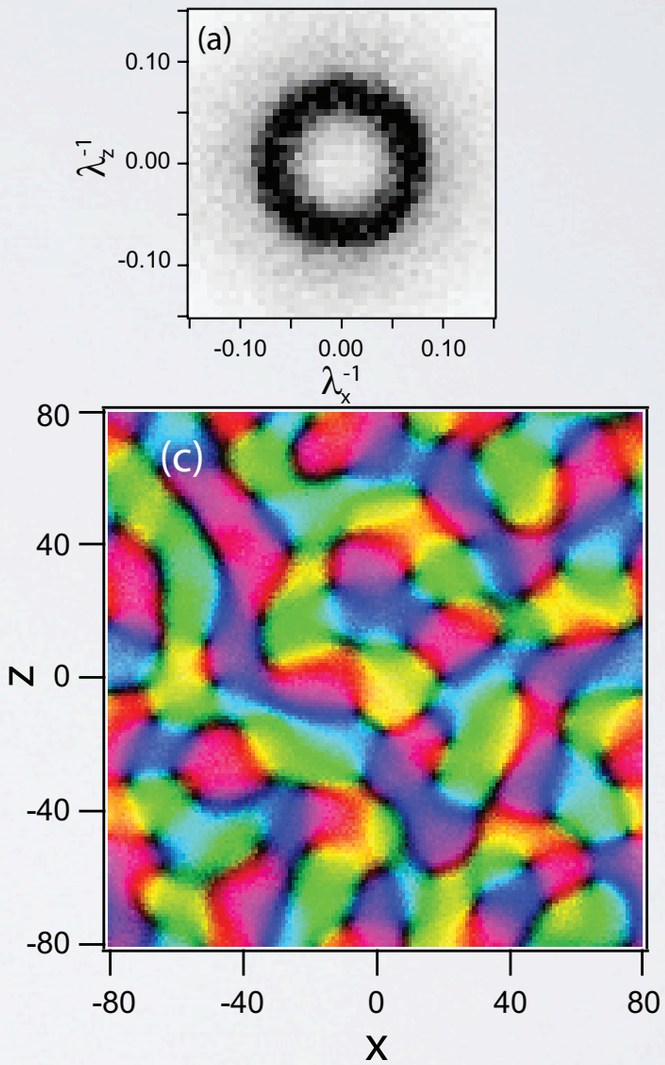


$$G_\perp(t) =< (F_x^2 + F_y^2) >= \frac{1}{\sqrt{8\pi q_0 t}} \frac{1}{n_0 \xi_s^2} e^{q_0 t}$$

$$q_0 = 2|c_2|n_0$$

Lamacraft PRL 98, 160404

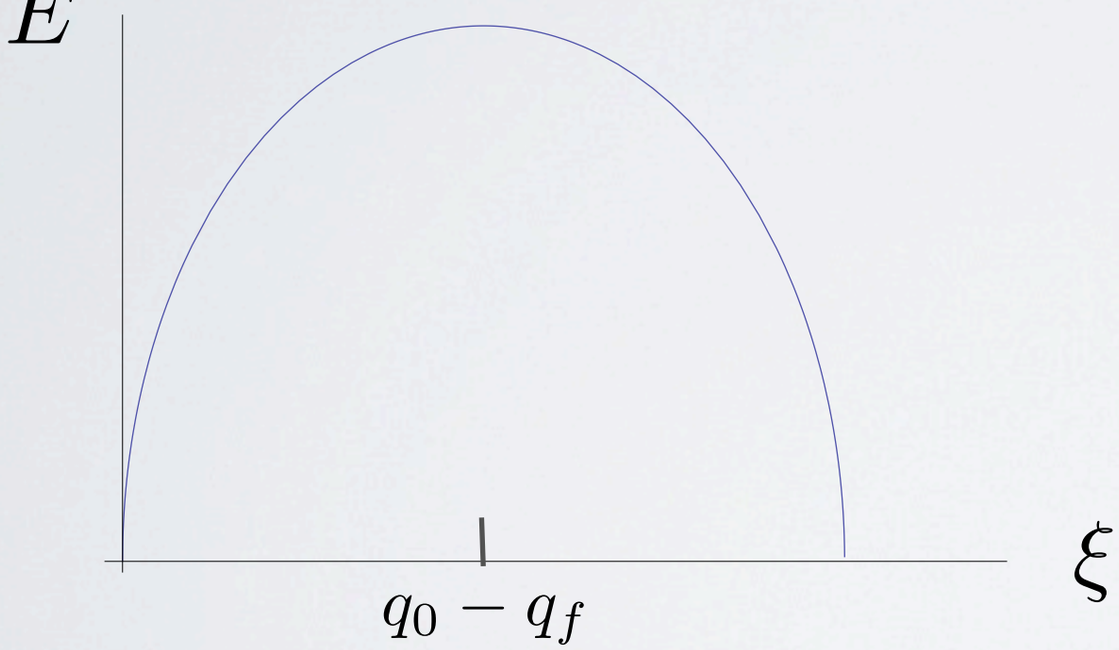
$$G_z(t) =< F_z^2 >= \frac{1}{64\pi q_0^2 t^2} \frac{1}{(n_0 \xi_s^2)^2} e^{2q_0 t}$$

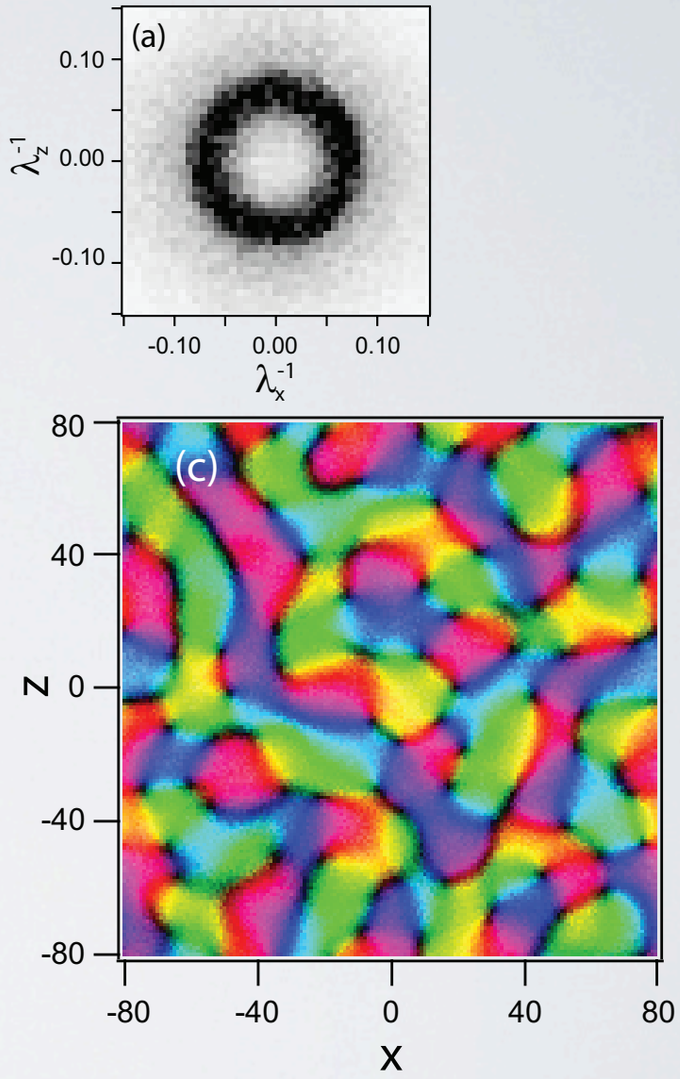


Spin domains have characteristic size given by most unstable mode

Linearized Theory: spectrum of unstable modes

$$E_k = \sqrt{(\xi_k + q_f)(q_0 - q_f - \xi_k)}$$

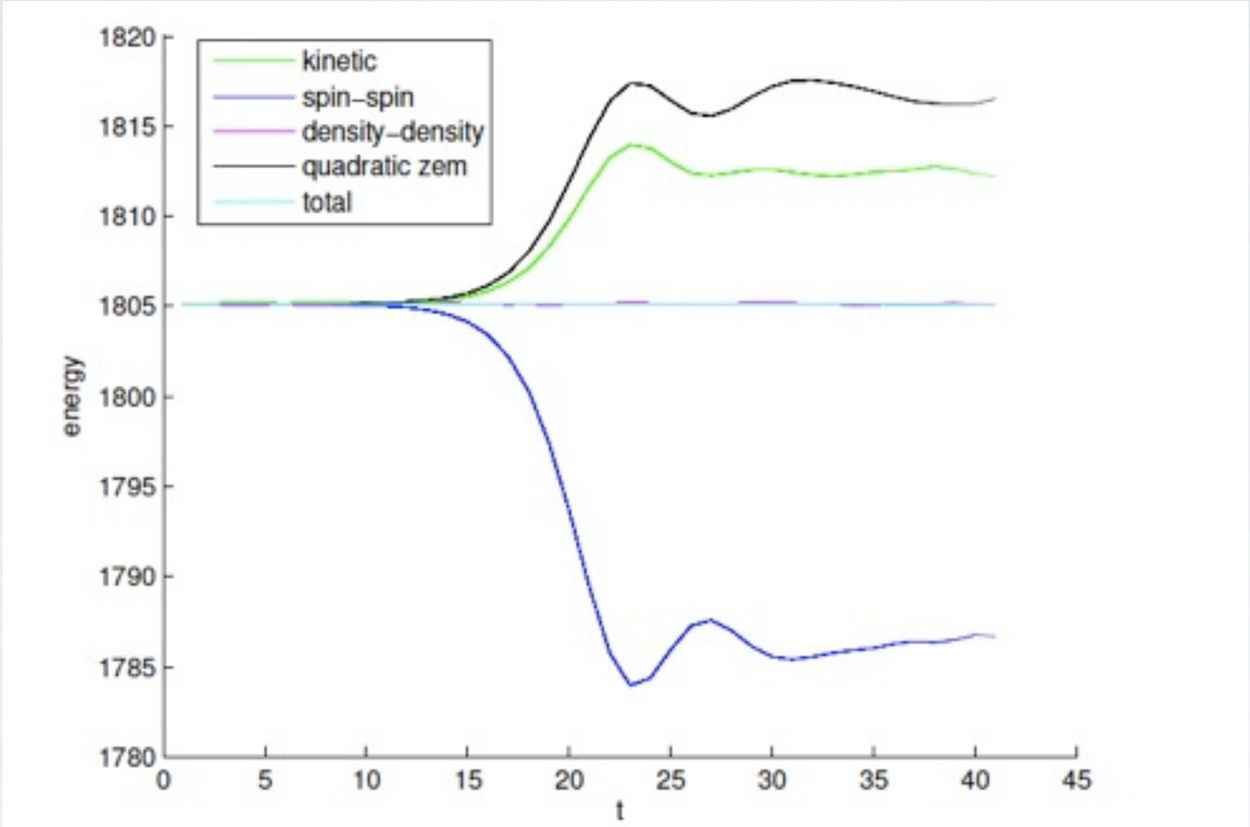




Spin domains have characteristic size given by most unstable mode

Natural Separation of Energy Scales

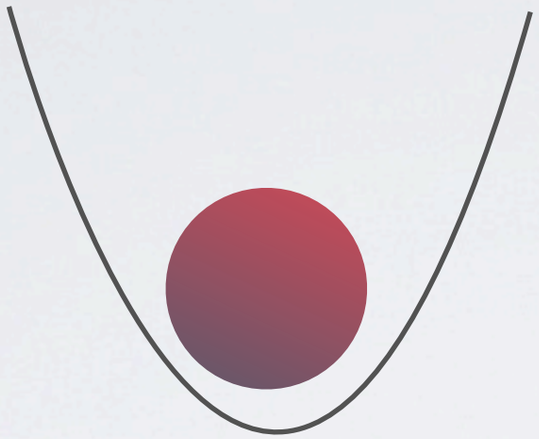
Density-density interactions are two orders of magnitude larger than spin-spin interaction



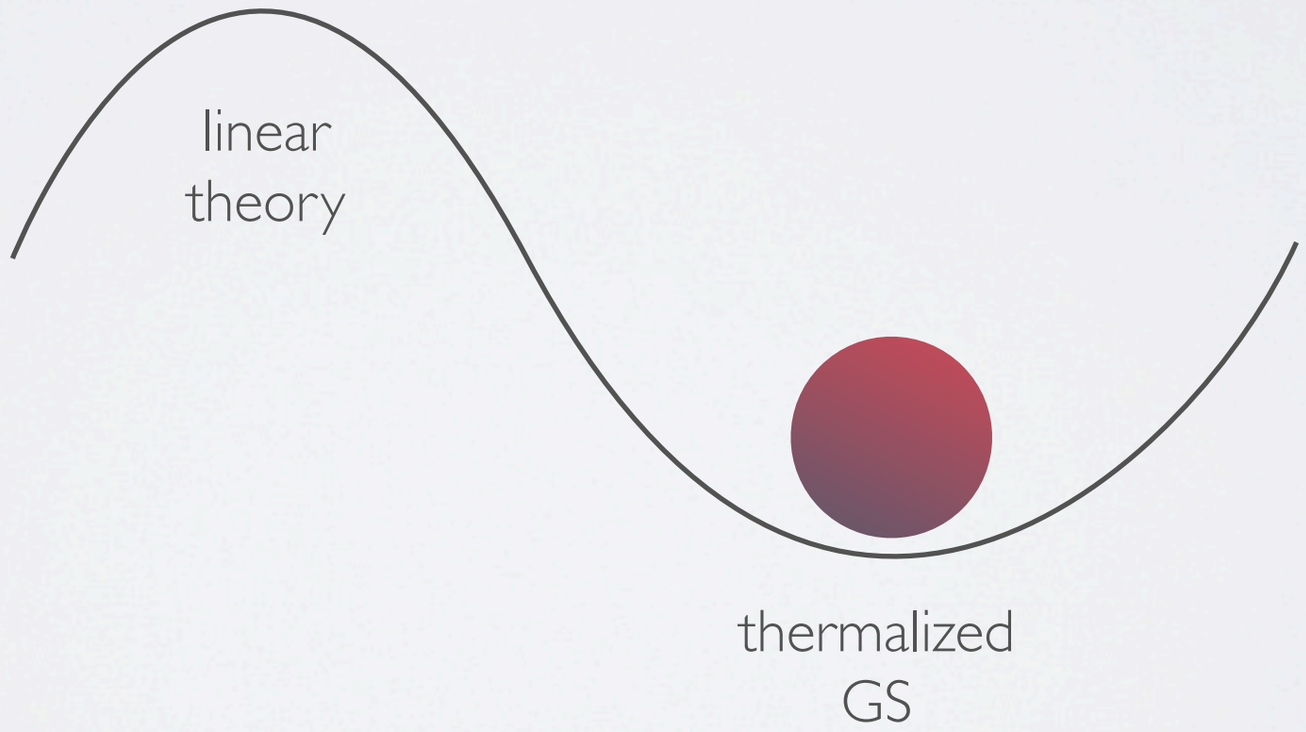
Justification for dropping density fluctuations in linearized theory

$$\mathcal{H} = \sum_{\mathbf{k}} \Big((\varepsilon_{\mathbf{k}} + c_2 n_0 + q_f)(\psi^\dagger_{1,\mathbf{k}} \psi_{1,\mathbf{k}} + \psi^\dagger_{-1,\mathbf{k}} \psi_{-1,\mathbf{k}}) + c_2 n_0 (\psi^\dagger_{1,\mathbf{k}} \psi^\dagger_{-1,-\mathbf{k}} + \psi_{-1,-\mathbf{k}} \psi_{1,\mathbf{k}}) \Big) .$$

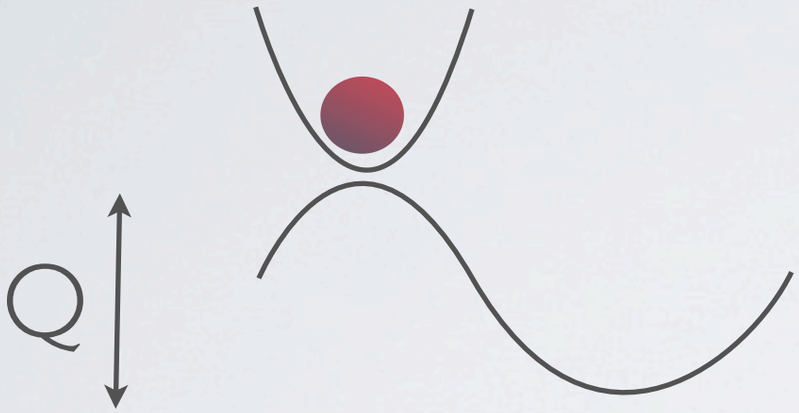
Long Times: thermalization??



Long Times: thermalization??



Assuming thermalization, develop theory to compare with numerics.



$$Q = < \mathcal{H} >_i - < \mathcal{H} >_{\rm gs}$$

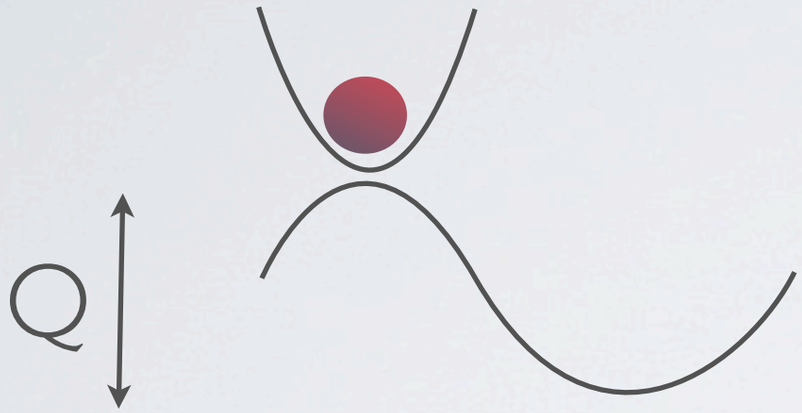
$$Q = \frac{1}{4} N q_0 \left(1 - \frac{q_f}{q_0}\right)^2$$

$$U(T) = \frac{1}{z} \mathrm{Tr}\left(e^{-\beta \mathcal{H}} \mathcal{H}\right) \qquad U(T) = Q \rightarrow T$$

- In general hard to find U. But possible when thermal depletion of final state is small (Bogoliubov theory is applicable)

Assuming thermalization, develop theory to compare with numerics.

- Heating (Q) is the difference in energies of initial and ground states.



$$Q = < \mathcal{H} >_i - < \mathcal{H} >_{\mathrm{gs}}$$

$$Q = \frac{1}{4} N q_0 \left(1 - \frac{q_f}{q_0}\right)^2$$

- Determine temperature by equating heating Q with thermal energy U

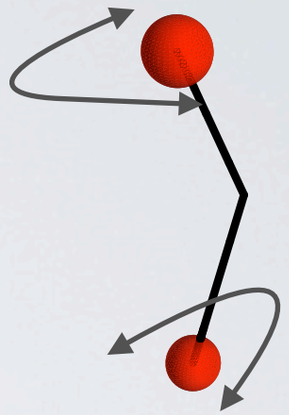
$$U(T) = \frac{1}{z} \mathrm{Tr}\left(e^{-\beta \mathcal{H}} \mathcal{H}\right) \qquad U(T) = Q \rightarrow T$$

- In general hard to find U. But possible when thermal depletion of final state is small (Bogoliubov theory is applicable)

Bogoliubov theory about symmetry broken state

- Bogoliubov theory by expanding $\Psi_\alpha = \bar{\Psi}_\alpha + \delta\psi_\alpha$ was done in Uchino, Kobayashi, and Ueda, Phys. Rev. A 81, 063632 (2010).
- We alternatively write

$$\Psi = \sqrt{n}\mathrm{e}^{i\alpha} \begin{bmatrix} \sin\eta \ \cos\phi \ \mathrm{e}^{i\xi} \\ \cos\eta \mathrm{e}^{i\gamma} \\ \sin\eta \ \sin\phi \ \mathrm{e}^{-i\xi}. \end{bmatrix}$$



- Such description does not restrict variations of massless degrees of freedom to be small. (Important for 2d).
- By expanding the Berry's phase $i\Psi^\dagger\partial_t\Psi$ about the MF ground state, conjugate variables can be read off.
- Due to "phonon-magnon" coupling, the modes are complicated.
- They aquire a simple form in the (physical) large density-density interaction limit.

The modes:

$$\xi_k^{(\perp)} = \sqrt{\varepsilon_k(\varepsilon_k + q_0)}$$

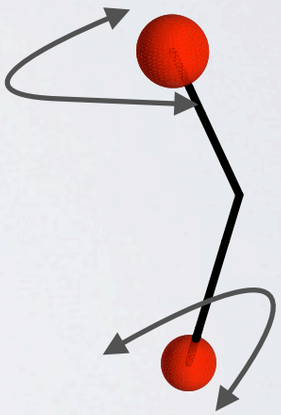
$$\xi_k^{(z)} = \sqrt{(\varepsilon_k + q_0)\left(\varepsilon_k + \frac{q_0^2 - q_f^2}{q_0}\right)}$$

Obtaining T and correlation functions

- Can use spectrum to compute $U(T) = \frac{N\pi T^2}{6(n_0\xi_s^2)q_0}$ (quadratic dispersions dominate).
- Setting $U = Q$ gives $\boxed{T = \sqrt{\frac{3}{2\pi} n_0 \xi_s^2}(q_0 - q_f).}$
- Verify that thermal depletion is small.

- Transverse magnetization correlation function:

$$G_\perp(\mathbf{r}, t) \equiv \frac{1}{n_0^2} < F_+(\mathbf{r}) F_-(0) >$$



- Within Bogoliubov theory, the dominant contribution is from the massless mode:

$$G_\perp(\mathbf{r}) = \left(1 - \left(\frac{q_f}{q_0}\right)^2\right) e^{-<\Delta\xi^2>/2}$$

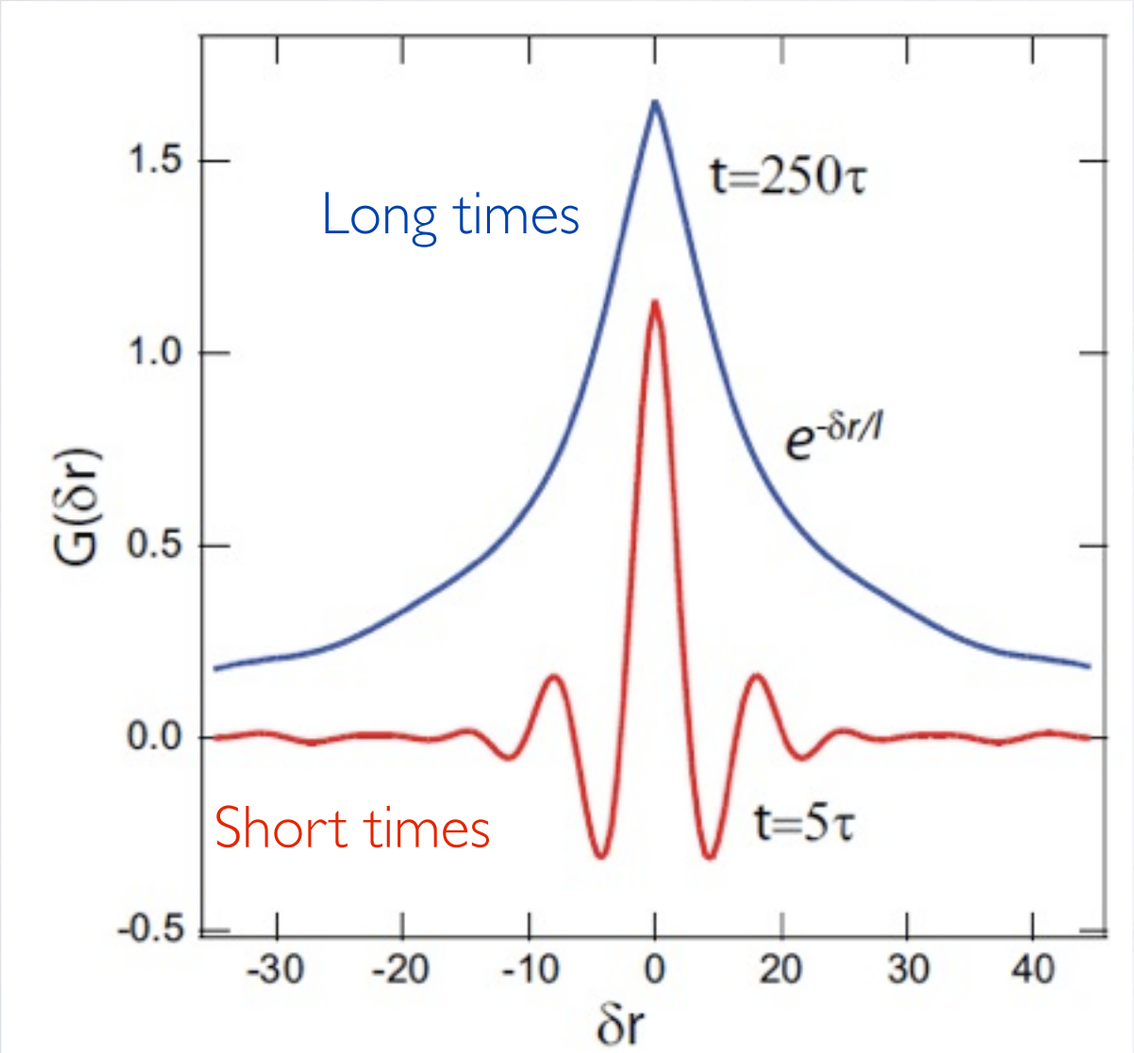
where $< \Delta\xi^2 > \equiv < (\xi(\mathbf{r}) - \xi(0))^2 >$.

- 2d (Mermin Wagner) gives power law behavior.

Comparing with TWA results

$$G_\perp(\mathbf{r}) = \left(1 - \left(\frac{q_f}{q_0}\right)^2\right) r^{-\alpha} \qquad n_0\xi_s^2 \sim 10^4 \qquad \alpha = \sqrt{6/\pi^3 n_0 \xi_s^2}$$

$n_0\xi_s^2$ large $\rightarrow$ essentially no decay of $G_\perp(\mathbf{r})$.



- For short times, the results quantitatively agree.
- For long times the results qualitatively diagree.
- No thermalization over realistic times
- From conserved topological quantities?

Conclusions / Open Issues

- Spinor condensates are interesting
- Showed mapping of spin-one system onto rotor model.
- Used to address NIST experiments and to discuss collapse and revival.
- Possible for higher spin?
- Used TWA to discuss Berkeley experiments
- Analytics agree for short times but not for long times.
- No thermalization over realistic times.
- From topological defects?

Thank You