Aalto University
School of Science
and Technology

Imbalanced Fermi gases: the FFLO state, polarons, and the Josephson effect

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€: EURYI award ESF/Eurohorcs

Academy of Finland

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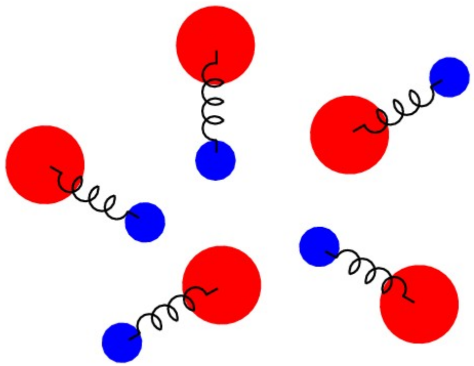
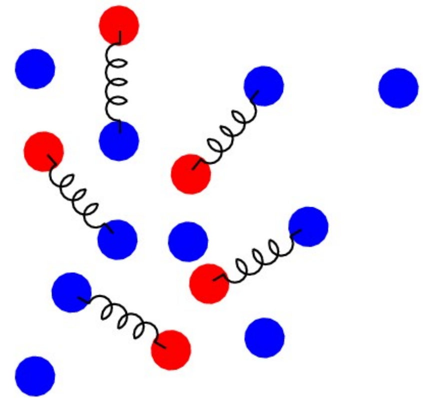
- Fermi-polaron-like effects in a 1D lattice (Leskinen, Nummi, Massel, PT, NJP 2010)
- Spin-asymmetric Josephson effect (Heikkinen, Massel, Kajala, Leskinen, Paraoanu, PT, arXiv 2009)
- Other recent work
  - Hopping modulation in 1 D lattices (Massel, Leskinen, PT, PRL 2009)
  - Gorkov correction in optical lattices and 3-component systems (Kim, PT, Martikainen, PRL 2009, Martikainen, Kinnunen, PT, Pethick, PRL 2009)
  - Fermi gases as sensors (Koponen, Pasanen, PT, PRL 2009)

Probing the FFLO state by double occupancy modulation spectroscopy

Imbalanced/Polarized Fermi gases





Polarization

$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

Pairing between particles with unequal mass or unequal total number

Related to, e.g., high energy physics (colour superconductivity of quarks)

Experiments:

M.W.Zwierlein, A.Schirotzek, C.H.Schunck, W.Ketterle, Science 2006

G.B.Partridge, W.Li,R.I.Kamar, Y.Liao, R.G.Hulet, Science 2006

G.B.Partridge, W.Li,Y.Liao, R.G.Hulet, M.Haque, H.Stoof, PRL 2006

M.W.Zwierlein, C.H.Schunck, A.Schirotzek, W.Ketterle, Nature 2006

C.H.Schunck, Y.Shin, A.Schirotzek, M.W. Zwierlein, W.Ketterle, Science 2007

Y.Shin, C.H.Schunck, A.Schirotzek, W.Ketterle, Nature 2008

A. Schirotzek, C-H. Wu, A. Sommer, M.W. Zwierlein, PRL 2009

S. Nascimbene, N. Navon, K.J. Jiang, L. Taruell, M. Teichmann, J. McKeever, F.Chevy, C. Salomon, PRL 2009

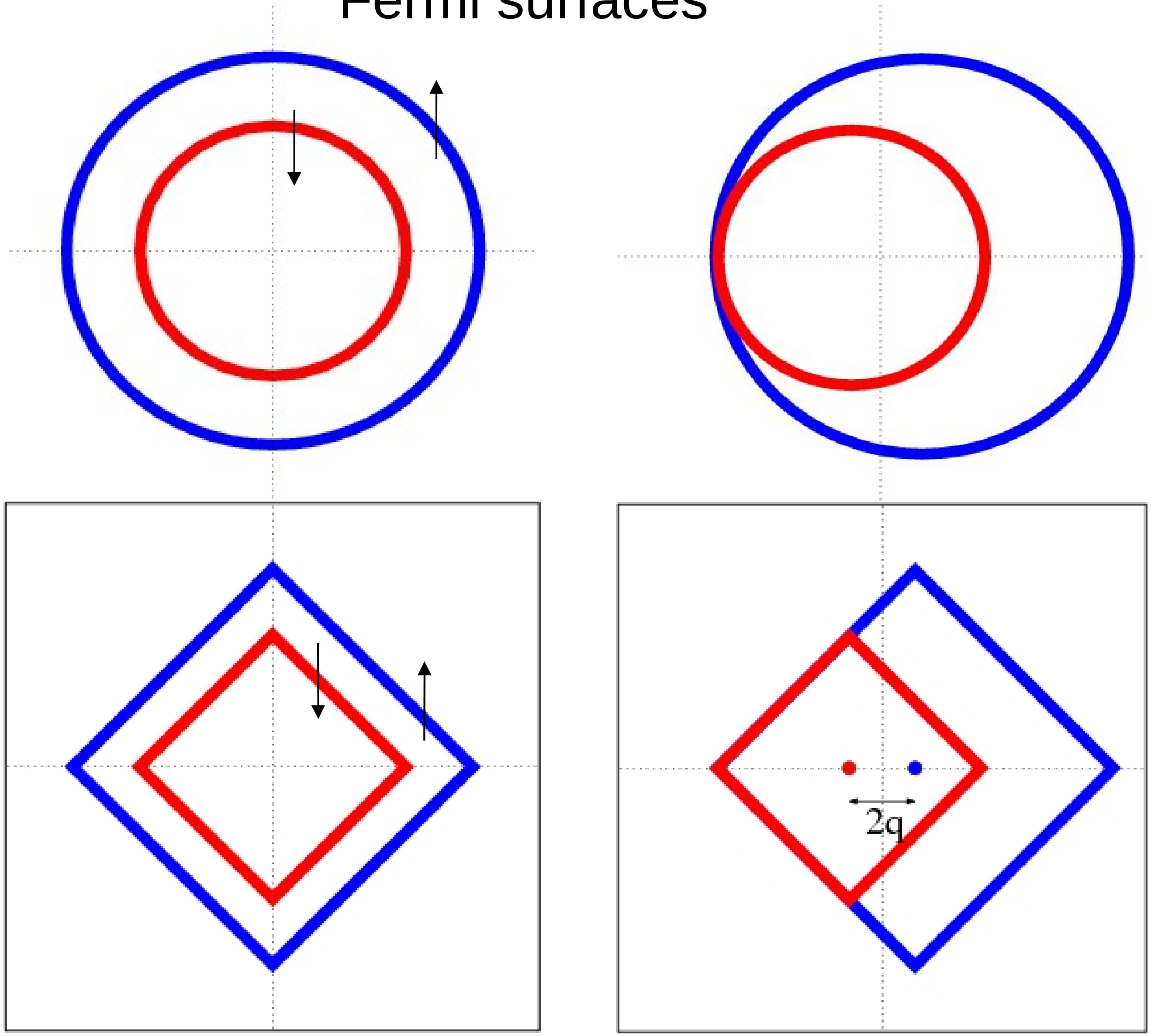
COULD ONE OBSERVE THE FFLO STATE IN ULTRACOLD GASES?

- FFLO (Fulde, Ferrel, Larkin, Ovchinnikov) state
  - Finite polazation P and superfluidity simultaneously (also at T=0)
  - Non-uniform order parameter $\Delta e^{i2\mathbf{q}\cdot\mathbf{r}}$ $\Delta \sin(2\,q\,r)$
- Observations under debate
  - H.A. Radovan, N.A. Fortune, T.P. Murphy, S.T. Hannahs, E.C. Palm, S.W. Tozer, D. Hall, Nature 2003
  - A. Bianchi, R. Movshovich, C. Capan, P.G. Pagliuso, J.L. Sarrao, PRL 2003
  - K. Kakuyanagi, M. Saitoh, K. Kumagai, S. Takashima, M. Nohara, H. Takagi, Y. Matsuda, PRL 2005
  - V.F. Correa, T.P. Murphy, C. Martin, K.M. Purcell, E.C. Palm, G.M. Schmiedeshoff, J.C. Cooley, S.W. Tozer, PRL 2007
- The parameter window for existence of this phase is exceedingly small for particles in free space, in 3D
  - Exceedingly small for particles in free space, in 3D. See e.g. D.L. Sheehy, L. Radzihovsky, PRL 2006
  - Enhanced in optical lattices, T.K. Koponen, T. Paananen, J.-P. Martikainen, P. Törmä, PRL 2007
  - … and expecially in 1D

Fermi surfaces

Free space

Lattice



$$q = k_{F\uparrow} - k_{F\downarrow}$$

The FFLO state in a 1D lattice

$$H = -J\sum_{i,\sigma=\uparrow,\downarrow}^{L}\left(c_{i,\sigma}^{+}c_{i+1,\sigma} + c_{i,\sigma}^{+}c_{i-1,\sigma}\right) + \sum_{i}^{L}\left(-Un_{i\uparrow}n_{i\downarrow} + V\left(i-\frac{L}{2}\right)^{2}\left(n_{i\uparrow}+n_{i\downarrow}\right)\right)$$

$$P = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}}$$

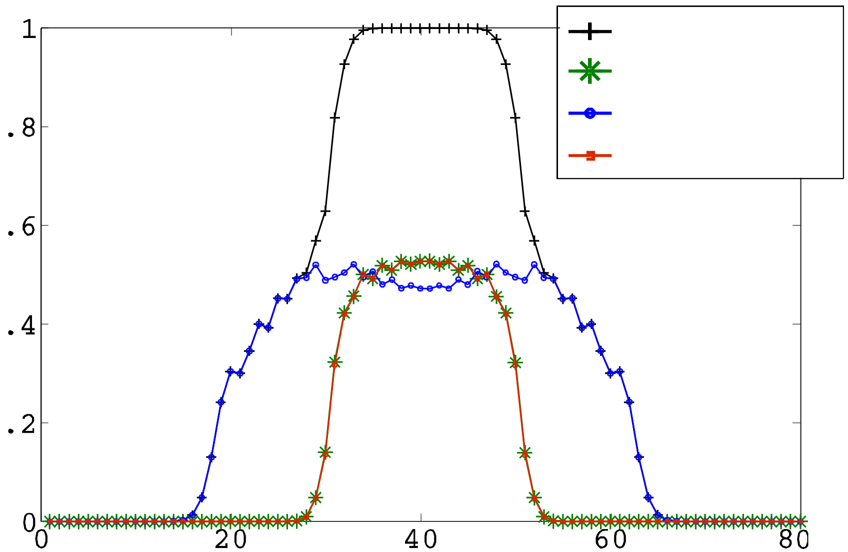
$$n_{pair}(k) \propto F\left\langle c_{j,\uparrow}^{+}c_{j,\downarrow}^{+}c_{i,\uparrow}c_{i,\downarrow}\right\rangle$$

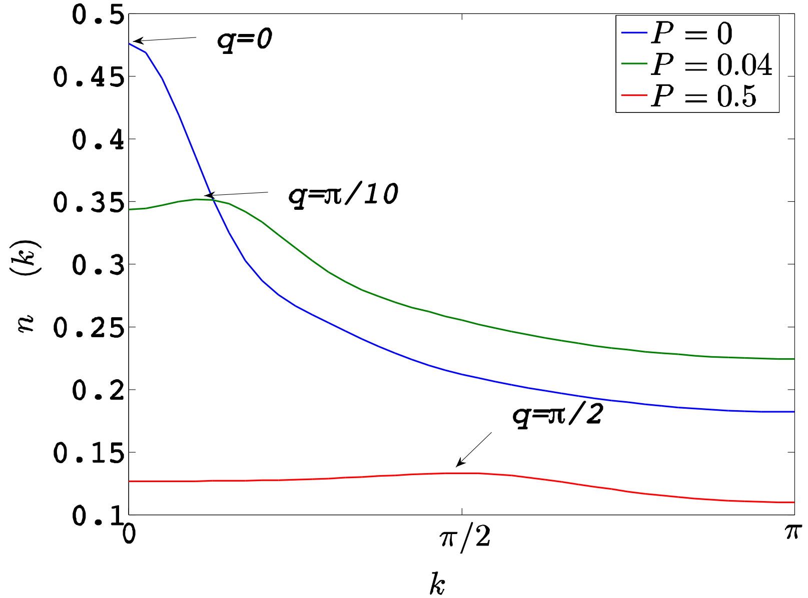
$$q = k_{F\uparrow} - k_{F\downarrow}$$???

The FFLO state in a 1D lattice

TEBD calculations:

$$n_{pair}(k) \propto \left\langle c^{+}_{j,\uparrow} c^{+}_{j,\downarrow} c_{i,\uparrow} c_{i,\downarrow} \right\rangle$$





$$U = -10, J = 1, L = 80, N_{\downarrow} + N_{\uparrow} = 40, P \geq 0.04, V = 0.005$$

$$q \cong \frac{\pi}{L(N_{\uparrow} - N_{\downarrow})} = \pi \rho P$$

A. Korolyuk, F. Massel, PT, PRL 2010

+ MANY OTHERS (using TEBD, DMRG, Bethe ansatz)

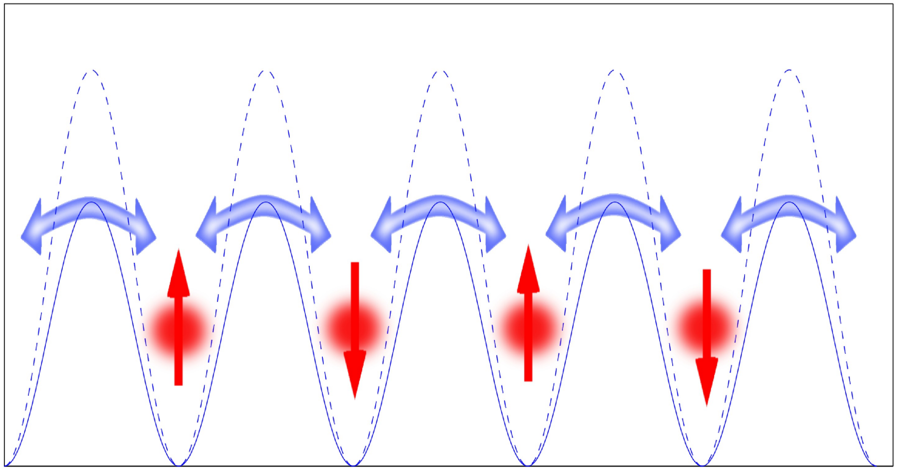
$$\rho = \frac{N_{\uparrow} + N_{\downarrow}}{L}$$

HOW TO OBSERVE THIS SUBTLE FEATURE?

A. Korolyuk, F. Massel, PT, PRL 2010

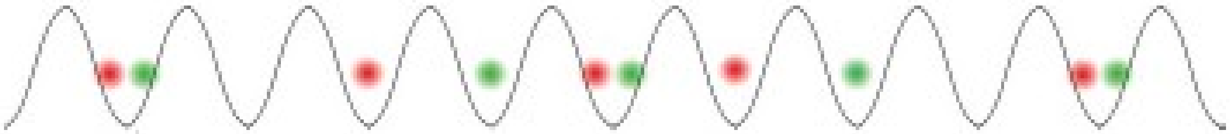
The FFLO state in 1D is directly visible as narrowing of the hopping modulation spectrum!

Hopping modulation (double occupancy) spectroscopy

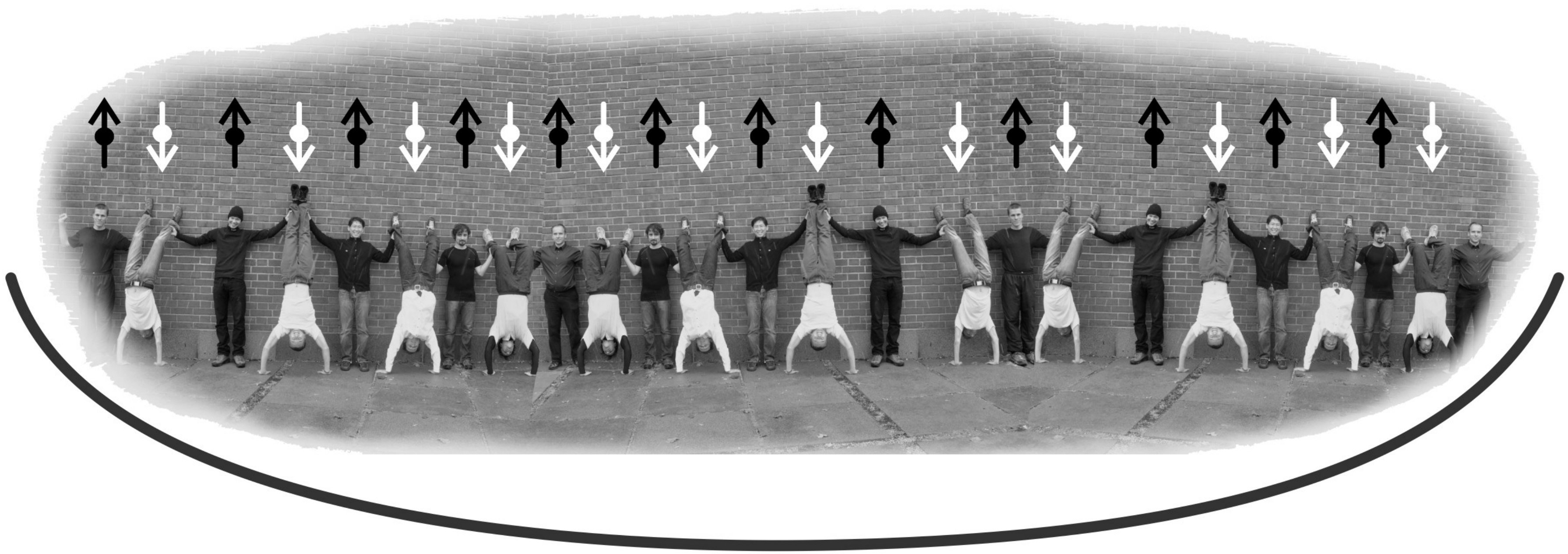


$$J(t) = J + \delta J \cos \omega t$$

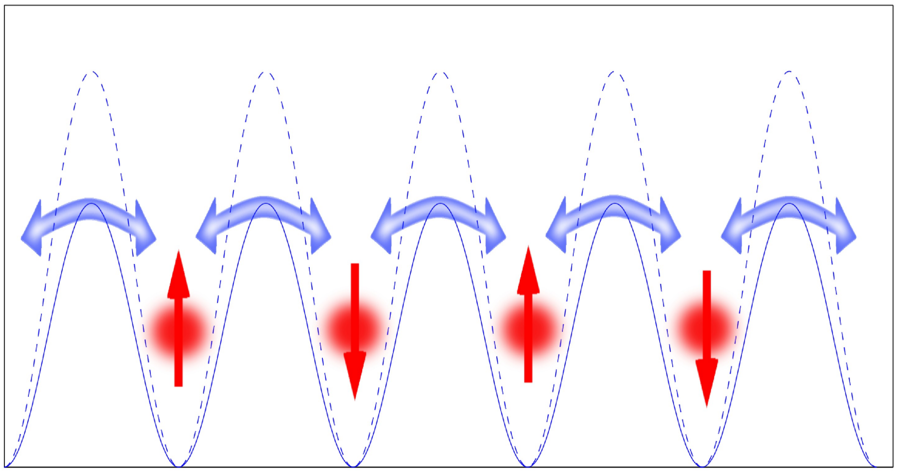
$$D_\omega(t) = \sum_{i=1}^{L} \langle n_{i\uparrow} n_{i\downarrow} \rangle$$



C. Kollath, A. Iucci, I.P. McCulloch, T. Giamarchi, PRA 2006
R. Jördens, N. Strohmaier, K. Gunter, H. Moritz, T. Esslinger, Nature 2008



Hopping modulation (double occupancy) spectroscopy



$$J(t) = J + \delta J \cos \omega t$$

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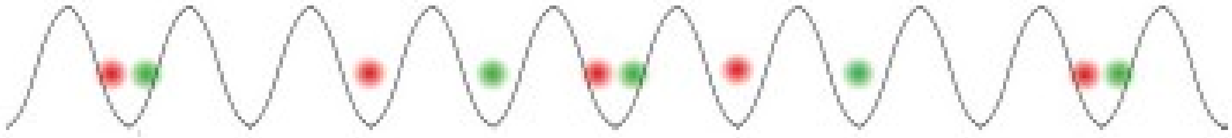
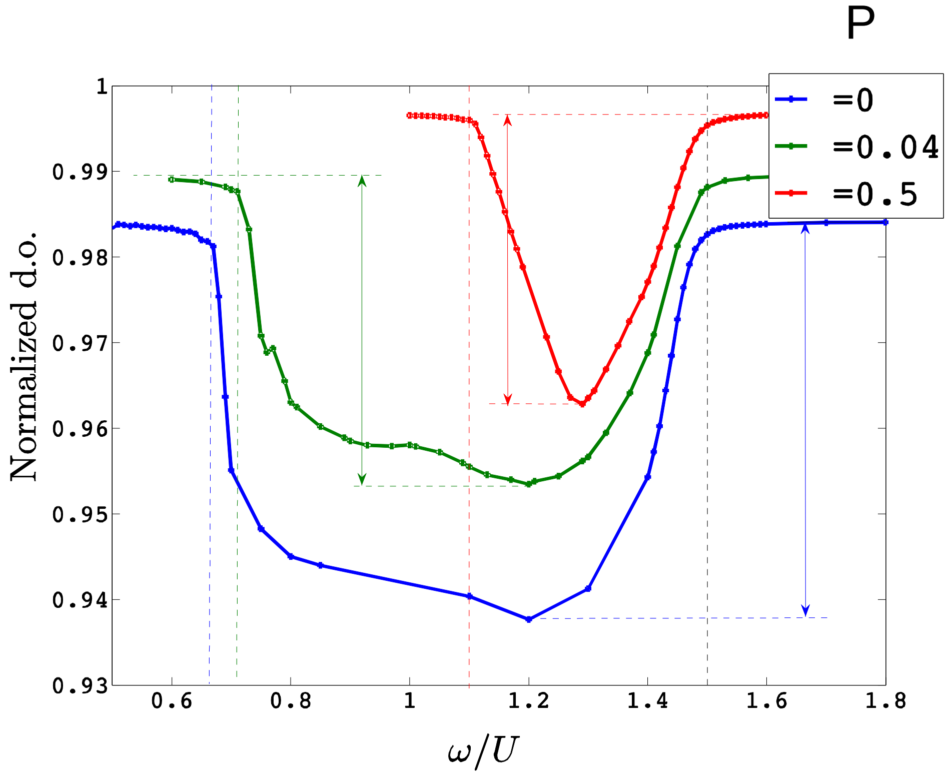


C. Kollath, A. Iucci, I.P. McCulloch, T. Giamarchi, PRA 2006
R. Jördens, N. Strohmaier, K. Gunter, H. Moritz, T. Esslinger, Nature 2008

Hopping modulation double occupancy spectra

$$D_\omega(t)\big|_{t=50}$$



$$N_\uparrow = 20$$

$$N_\downarrow = 20{,}22{,}10$$

Dramatic narrowing of the spectrum for larger imbalance!

The narrowing is directly related to the FFLO wave vector

Bethe ansatz, open boundary conditions,
the limit $\frac{U}{J} \to \infty$

[H. Asakawa, M. Suzuki, J. Phys. A 1996;
M. Ogata, H. Shiba, PRB 1990]

The system can be described by a gas of spinless Fermions,
with energy and momenta, with the U -> -U mapping

$$U \to -U \qquad \left|\uparrow\downarrow\right\rangle \leftrightarrow \left|\uparrow\right\rangle, \left|\otimes\right\rangle \leftrightarrow \left|\downarrow\right\rangle$$

$$E = -2J \sum_{j=1}^{N_\uparrow - N_\downarrow} \cos k_j \qquad k_j = \frac{\pi}{L+1} I_j$$

$$N = N_\uparrow + N_\downarrow \qquad I_j \in \mathrm{N} \qquad I_j = [1,..,L]$$

Bethe ansatz equations for open boundary conditions:

$$2Lk_j = 2\pi I_j - 2k_j - \sum_{\beta=1}^{M}\left[\Phi\left(2\frac{\sin(k_j)-\lambda_\beta}{u}\right)+\Phi\left(2\frac{\sin(k_j)+\lambda_\beta}{u}\right)\right]$$

$$\sum_{j=1}^{N}\left[\Phi\left(2\frac{\lambda_\alpha-\sin(k_j)}{u}\right)+\Phi\left(2\frac{\lambda_\alpha+\sin(k_j)}{u}\right)\right]=$$

$$2\pi J_\alpha + \sum_{\beta=1(\beta\neq\alpha)}\left[\Phi\left(\frac{\lambda_\alpha-\lambda_\beta}{u}\right)+\Phi\left(\frac{\lambda_\alpha+\lambda_\beta}{u}\right)\right]$$

where

$$j=1,\ldots,N,\ \alpha=1,\ldots,M,\ I_j\ ,\ J_\alpha\in\mathbb{N}$$

$$\Phi(x) = 2\ \tan^{-1}(2\,x)$$

The narrowing is directly related to the FFLO wave vector!

Bethe ansatz, open boundary conditions, the limit

[H. Asakawa, M. Suzuki, J. Phys. A 1996; M. Ogata, H. Shiba, PRB 1990]

The system can be described by a gas of spinless Fermions, with energy and momenta, with the U -> -U mapping

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$$N = N_\uparrow + N_\downarrow \qquad I_j \in \mathrm{N} \qquad I_j = [1,..,L]$$

Exchange of energy in the modulation spectroscopy

$$\Delta E = U - 2J\left(\cos k_1 + \cos k_2\right) \qquad -1 \le \cos k_{1,2} \le \cos q$$

$$E_{\text{low}} = U - 4J\cos q \qquad\qquad E_{\text{high}} = U + 4J$$

$$(k_1 = k_2 = q) \qquad\qquad (k_1 = k_2 = \pi)$$

The width:

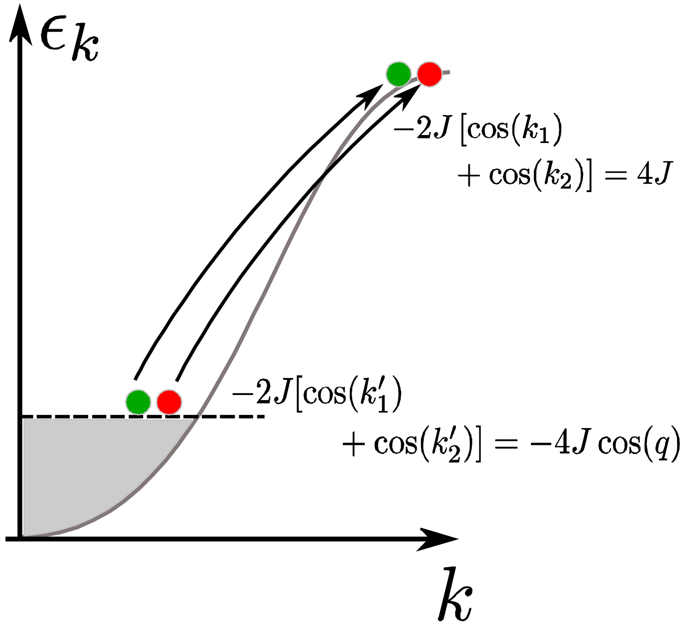
$$\boxed{\frac{\Delta\omega}{U} = \frac{4J}{U}\left(1 + \cos q\right)}$$

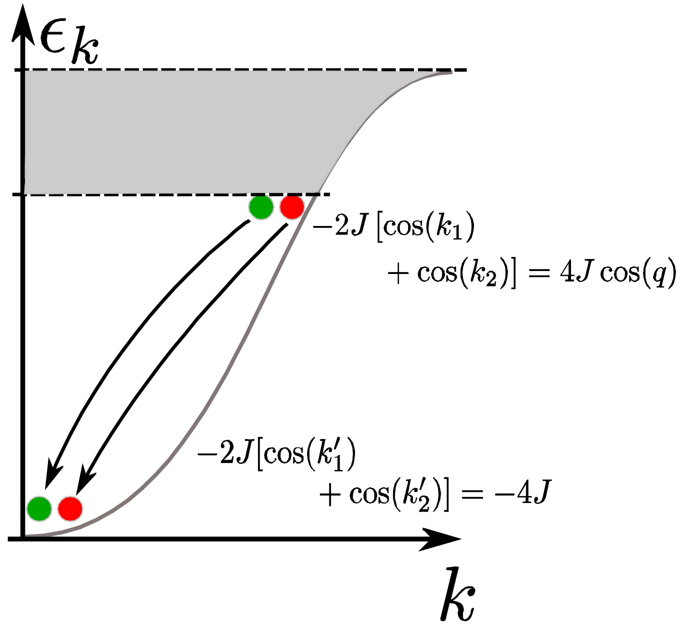
$$q \cong \frac{\pi}{L}(N_\uparrow - N_\downarrow) = \pi \rho P$$

$$\rho = \frac{N_\uparrow + N_\downarrow}{L}$$

$$E = -2J \sum_{j=1}^{N_\uparrow - N_\downarrow} \cos k_j$$

Analogy in a different system





Summary

The existence of the FFLO pair momentum restricts the available phase space in kinetic energy transfer in the pair breaking process:

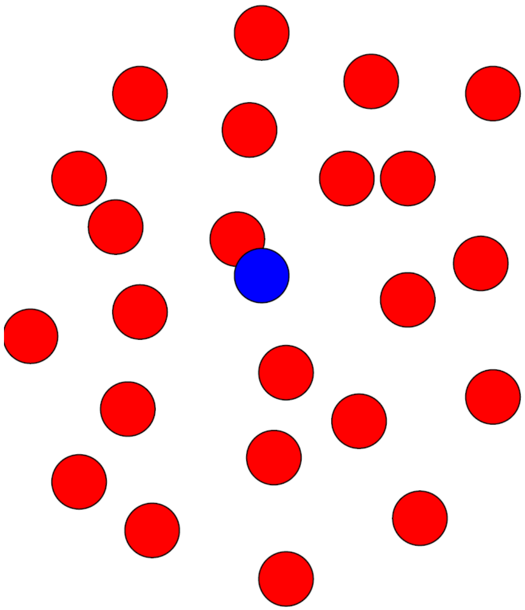
narrowing of the spectrum for larger FFLO momenta

Direct relation between q and the width:

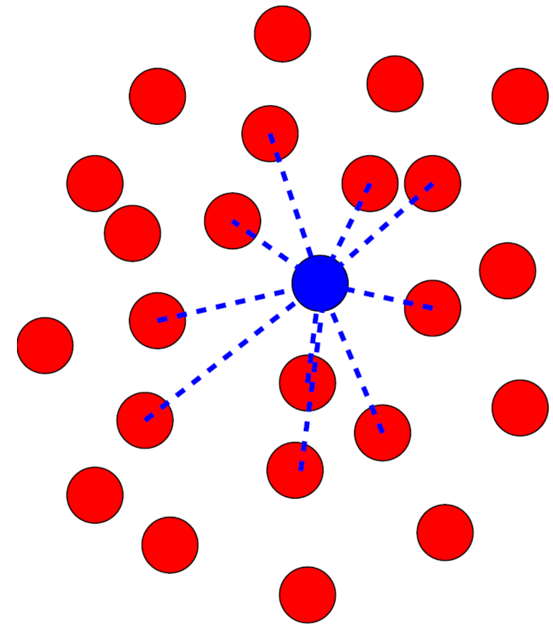
$$\frac{\Delta\omega}{U} = \frac{4J}{U}\left(1+\cos q\right)$$

Fermi-polaron-like effects in a 1D optical lattice

Fermi-polarons



$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = \frac{N_\uparrow - 1}{N_\uparrow + 1}$$



Experiments: A. Schirotzek, C-H. Wu, A. Sommer, M.W. Zwierlein, PRL 2009
S. Nascimbene, N. Navon, K.J. Jiang, L. Taruell, M. Teichmann, J. McKeever, F.Chevy, C. Salomon, PRL 2009

Experiments revealed a crossover from polaronic physics to molecular binding

Theoretical approaches in 3D

F. Chevy, PRA 2006; Polaron ansatz

$$\left|\Psi\right\rangle = \varphi_0 \hat{c}^{+}_{0\downarrow}\left|\mathrm{FS}\right\rangle_{\uparrow}\left|\otimes\right\rangle_{\downarrow} + \sum_{q<k_F^{\uparrow},k>k_F^{\uparrow}} \varphi_{kq}\hat{c}^{+}_{k\uparrow}\hat{c}_{q\uparrow}\hat{c}^{+}_{q-k\downarrow}\left|\mathrm{FS}\right\rangle_{\uparrow}\left|\otimes\right\rangle_{\downarrow}$$

M. Punk, P.T. Dumitrescu, W. Zwerger, PRA 2009;
Ansatz for the molecular limit

$$\sum_{k>k_F^{\uparrow}} \phi_k \hat{c}^{+}_{\downarrow -k}\hat{c}^{+}_{\uparrow k}\hat{c}_{\uparrow 0}\left|\mathrm{FS}\right\rangle_{\uparrow}\left|\otimes\right\rangle_{\downarrow}$$

What about 1D lattice? Does something like the polaron ansatz work at all? Moreover, what about the strongly interacting limit?

Results in 1D

M.J. Leskinen, O.H.T. Nummi, F. Massel, PT, NJP 2010

TEBD simulations
+ polaron ansatz
+ Bethe ansatz

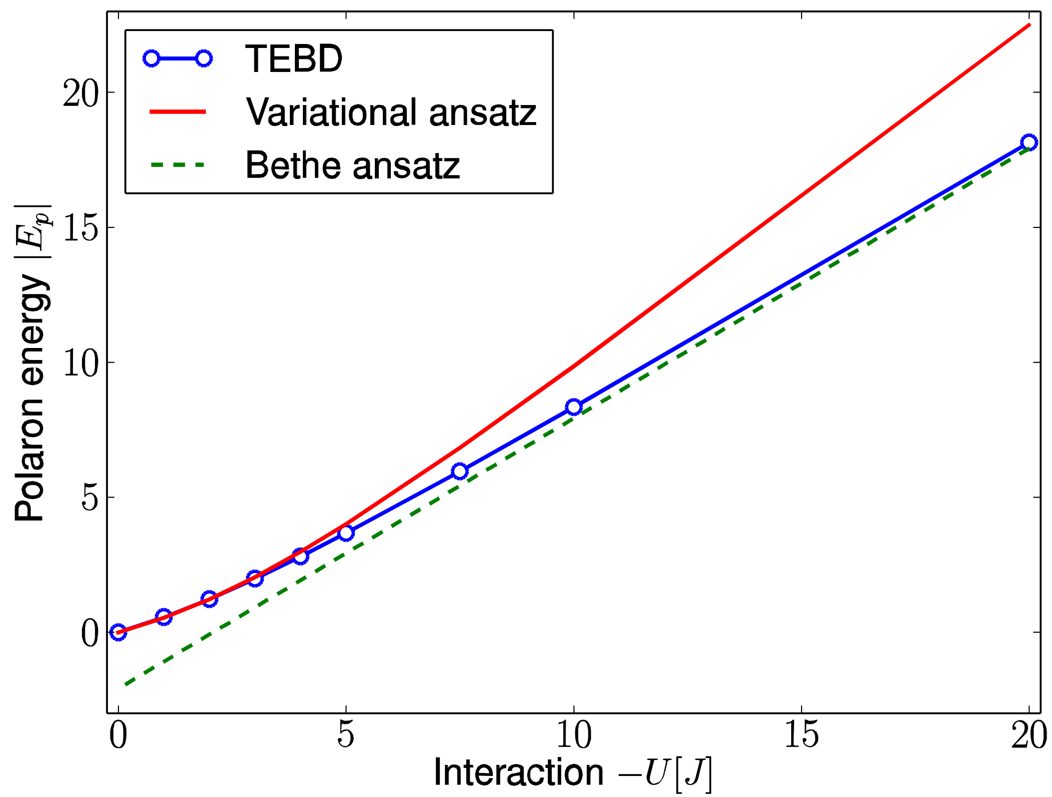
$$\hat{H}_0 = -J \sum_{\langle i,j \rangle,\sigma} \hat{c}^+_{i,\sigma} \hat{c}_{j,\sigma} + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$$

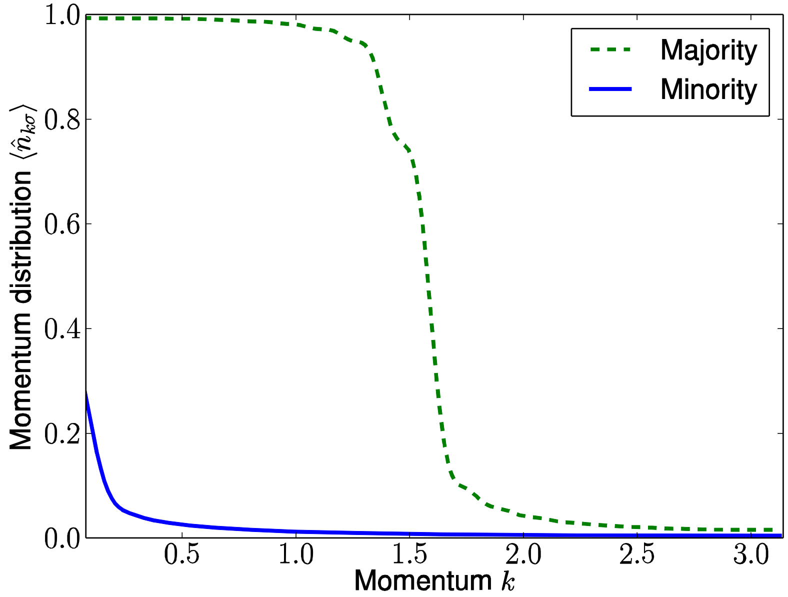
$$E_p = E_g - E_g^{\text{non-int}}$$

$$E_p = \langle g | \hat{H}_0 | g \rangle - E_g^{\text{non-int}}$$

Bethe ansatz as
in the previous topic

$$-\frac{U}{J} \to \infty \qquad E = U - 2J \sum_{j=1}^{N_\uparrow - N_\downarrow} \cos k_j$$





$$\left\langle \hat{n}_{k,\sigma} \right\rangle = \left\langle \hat{c}^{+}_{k,\sigma} \hat{c}_{k,\sigma} \right\rangle$$

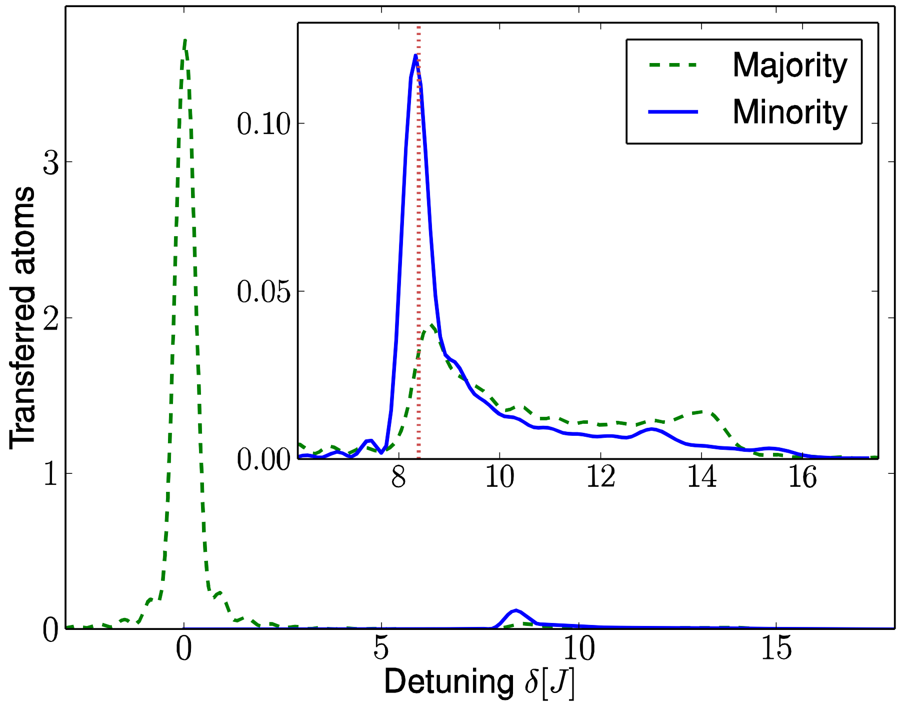
Note: the molecular limit ansatz not useful here

$$\sum_{k>k_F^{\uparrow}} \phi_k \hat{c}^{+}_{\downarrow -k} \hat{c}^{+}_{\uparrow k} \hat{c}_{\uparrow 0} \left| \mathrm{FS} \right\rangle_{\uparrow} \left| \otimes \right\rangle_{\downarrow}$$

Because it is the extreme FFLO state! Thus pairing is between k and –k+q, and for extreme polarization q = k_F

Rf photon breaking a pair

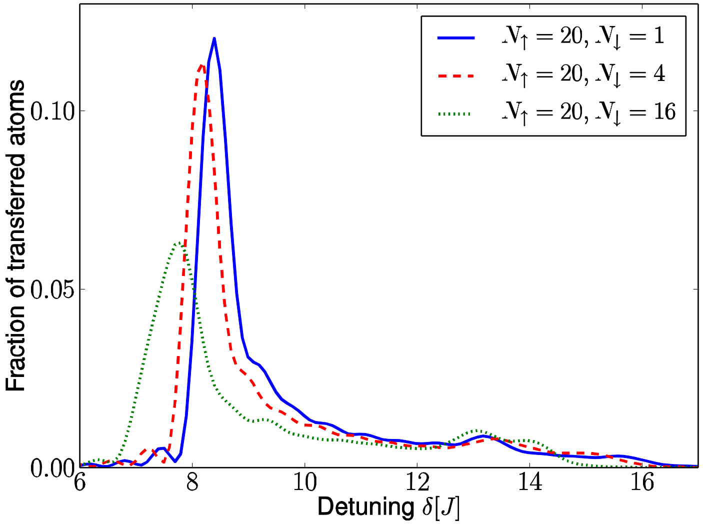
Exact RF-spectra



Ansatz: $$\mathrm{I}_{\downarrow} \propto \sum_f \left| \langle f | \hat{H}_{\mathrm{rf}}^{\downarrow} | g \rangle \right|^2 \delta^{(1d)}\left(\delta - E_f + E_g\right) =$$

$$= \left|\varphi_0\right|^2 \delta^{(1d)}\left(\delta - \left|E_p\right|\right) + \Gamma^{\mathrm{inc}}\left(\varphi_{kq}, \delta\right)$$

Also behaviour related to increasing polarization can be explained by the Bethe ansatz



$$\Delta E = E_G - E_F =$$

$$= 2U - 2J \sum_{j=1}^{18} \cos k_j -$$

$$- \left(U - 2J \sum_{j=1}^{19} \cos k_j - 2J \cos k_{\text{final}} \right) =$$

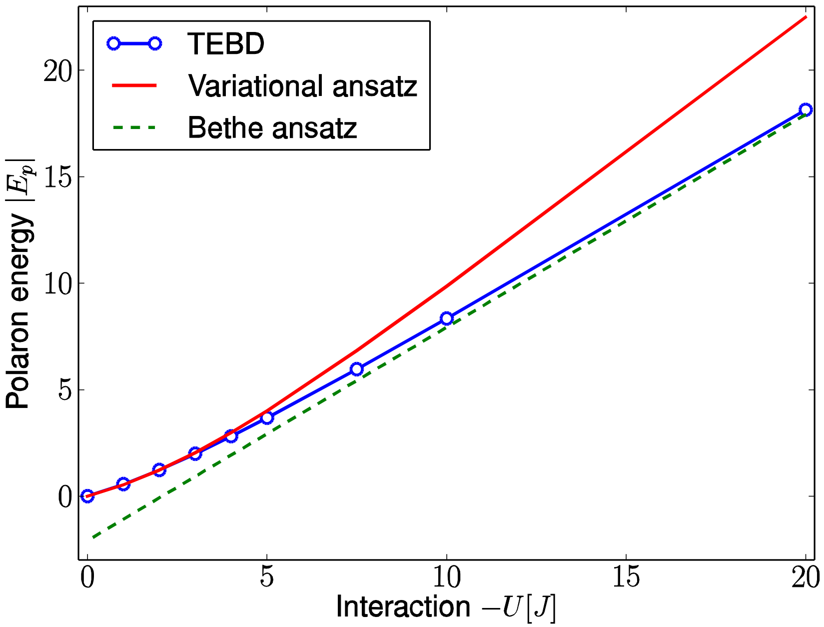
$$= U + 2J \cos k_{19} + 2J \cos k_{\text{final}}$$

Summary

Exact ground state for the Fermi polaron in 1D:
Weak interaction limit described by the variational polaron ansatz (Chevy),
the strong interaction limit by Bethe ansatz in the strong interaction limit (spinless Fermions)

Outlook:
Dynamics! Is the polaronic quasiparticle stable e.g. when propagating through a cloud of majority particles?
Ongoing work…

$$\lim_{t\to\infty} \left| G_{\downarrow}(\mathbf{k}=0,t) \right| \neq 0$$



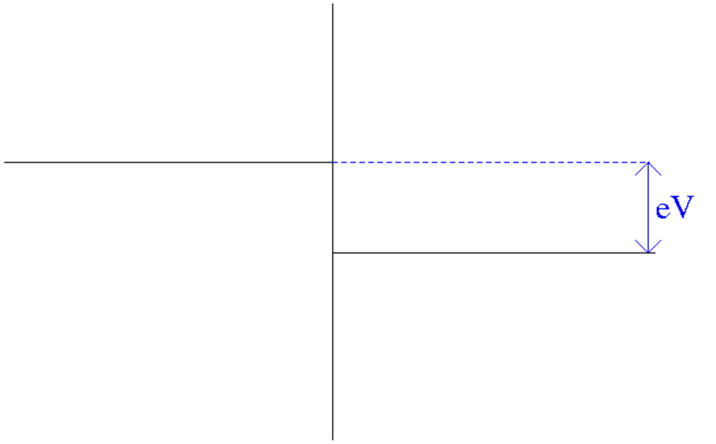

Quasiparticles in degenerate (ultracold) Fermi sea

Spin-asymmetric Josephson effect

Josephson effect

$$I = I_0(eV)\sin(2eVt)$$

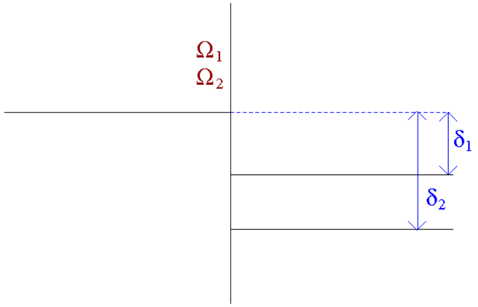
$$I_0(eV) \propto T_{tunneling}^2$$

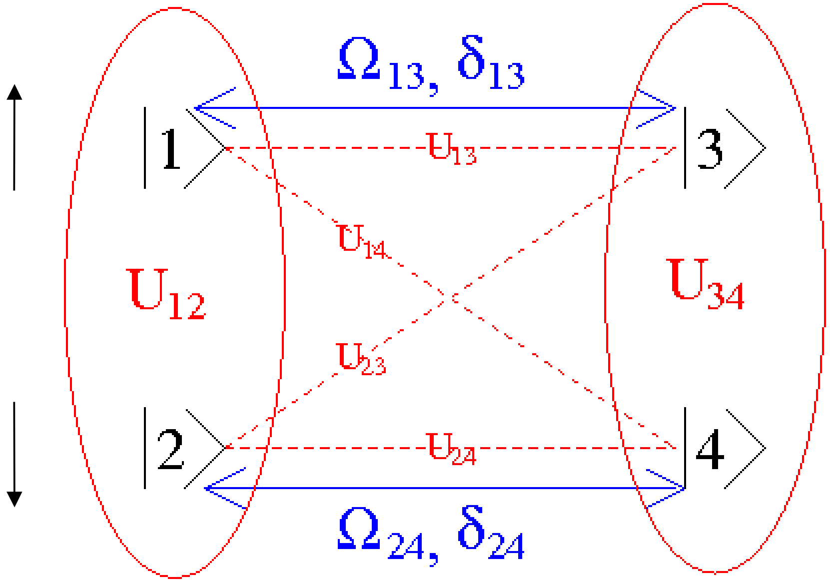


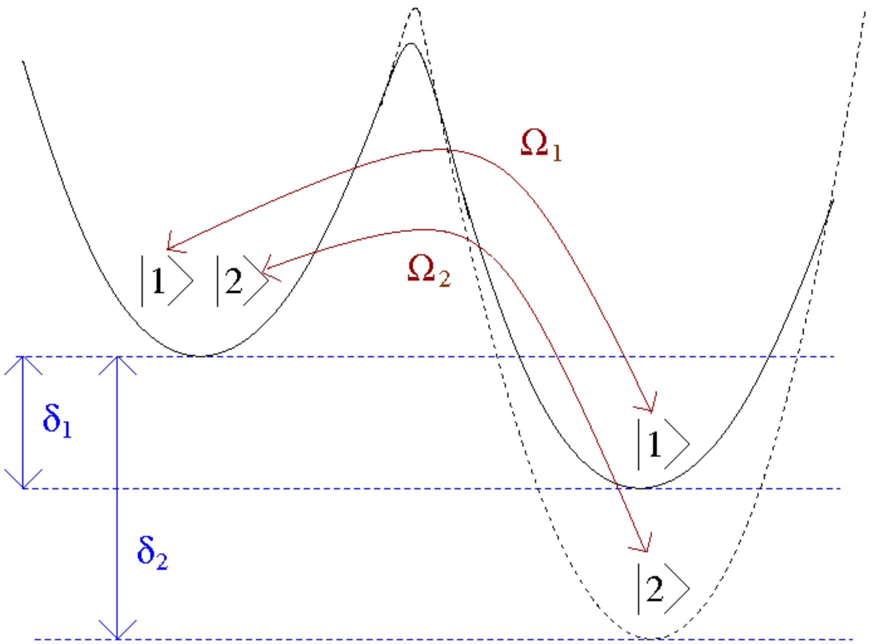
”usual” pair tunneling: $T_{tunneling}^4$

Josephson effect: ”coherent pair tunneling”: what does that mean?

Spin-asymmetric Josephson effect

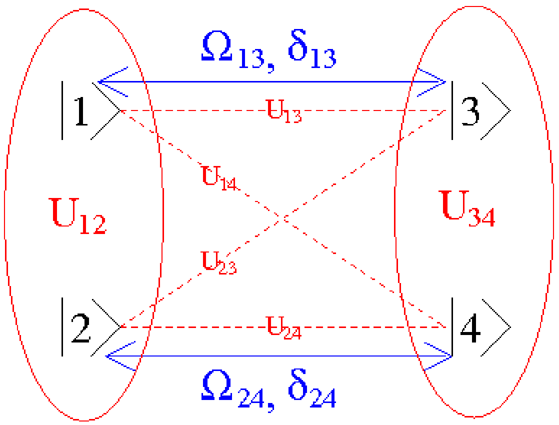






Asymmetric ”voltage” and ”tunneling”!!!

The Hamiltonian



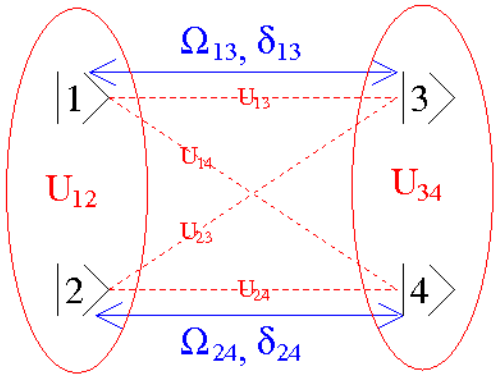
$$H = H_0 + H_t$$

$$H_0 = \int d^3 r \left(-\sum_{\sigma} \psi_{\sigma}^{\dagger}(r) \nabla^2 \psi_{\sigma}(r) \right) + \frac{1}{2} \sum_{\sigma\nu} U_{\sigma\nu} \psi_{\sigma}^{\dagger}(r) \psi_{\nu}^{\dagger}(r) \psi_{\nu}(r) \psi_{\sigma}(r)$$

$$H_t = \int d^3 r \left(\frac{\delta_{13}}{2} \left(\psi_3^{\dagger}(r)\psi_3(r) - \psi_1^{\dagger}(r)\psi_1(r) \right) + \frac{\delta_{24}}{2} \left(\psi_4^{\dagger}(r)\psi_4(r) - \psi_2^{\dagger}(r)\psi_2(r) \right) \right)$$
$$+ \int d^3 r\, \Omega_{13} \psi_1^{\dagger}(r) \psi_3(r) + h.c. + \int d^3 r\, \Omega_{24} \psi_2^{\dagger}(r) \psi_4(r) + h.c.$$

M.O.J. Heikkinen, F. Massel, J. Kajala, M.J. Leskinen, G.-S. Paraoanu, PT, arXiv:0911.4678

The results



Self-consistent linear response
(Kadanoff-Baym)

$$I_{13} = I_0(\delta_{24})\sin((\delta_{13}+\delta_{24})t)$$

$$I_{24} = I_0(\delta_{13})\sin((\delta_{13}+\delta_{24})t)$$

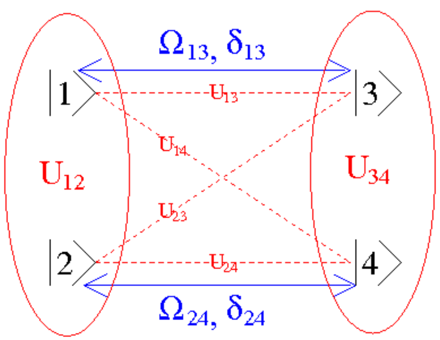
$$I_0(\delta) \propto \Omega_{13}\Omega_{24}$$

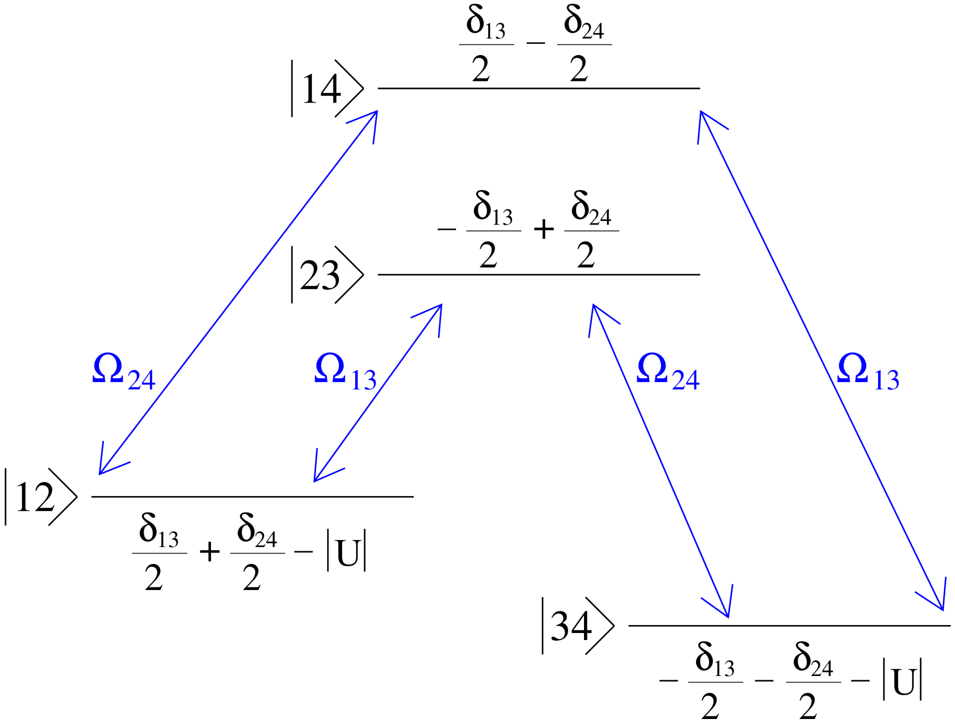
WHY ASYMMETRY IN CRITICAL CURRENTS???

THIS IS NOT CONSISTENT WITH THE TRADITIONAL "PAIRS TUNNELING TOGETHER" INTERPRETATION

Exact and perturbative studies of small systems

- 1D lattice with L = 1, 2, 3, sites (1, 2, 3 momentum states)
- Discussed here: L = 1
  - A four-component system without motional degree of freedom
  - Basically, study dynamics of one pair





Results

Initial state $\left|\phi(0)\right\rangle = \left|12\right\rangle$

No Josephson oscillations, only "usual" pair tunneling in order Ω^4

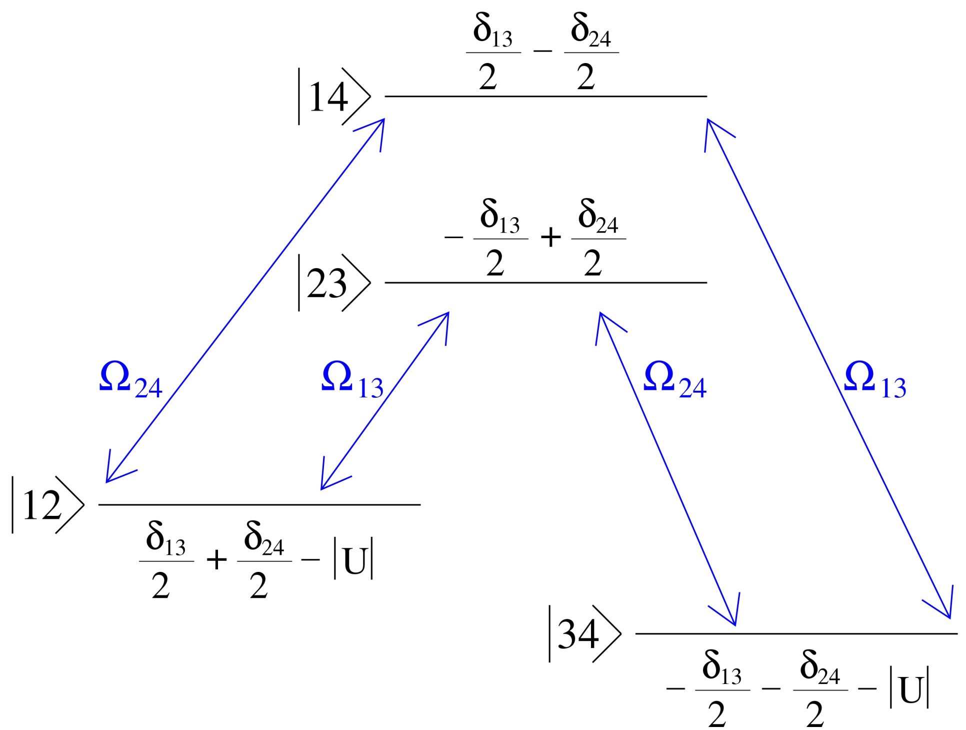
Initial state

$$\left|\phi(0)\right\rangle = \alpha_I \left|12\right\rangle + \beta_{II}\left|34\right\rangle$$

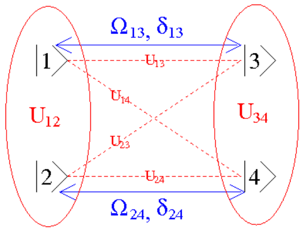
Josephson oscillations, same as linear response!

$$I_{13} = I_0(\delta_{24})\sin((\delta_{13}+\delta_{24})t)$$

$$I_0(\delta) \propto \Omega_{13}\Omega_{24}$$



Why a superposition?



- The BCS ansatz for two superfluids

$$\begin{aligned} |\Psi\rangle &= |BCS\rangle_{12} \otimes |BCS\rangle_{34} \\ &= \prod_k \left(u_k|0\rangle_k + v_k|12\rangle_k\right) \prod_k \left(u_{k'}|0\rangle_{k'} + v_{k'}|34\rangle_{k'}\right) \end{aligned}$$

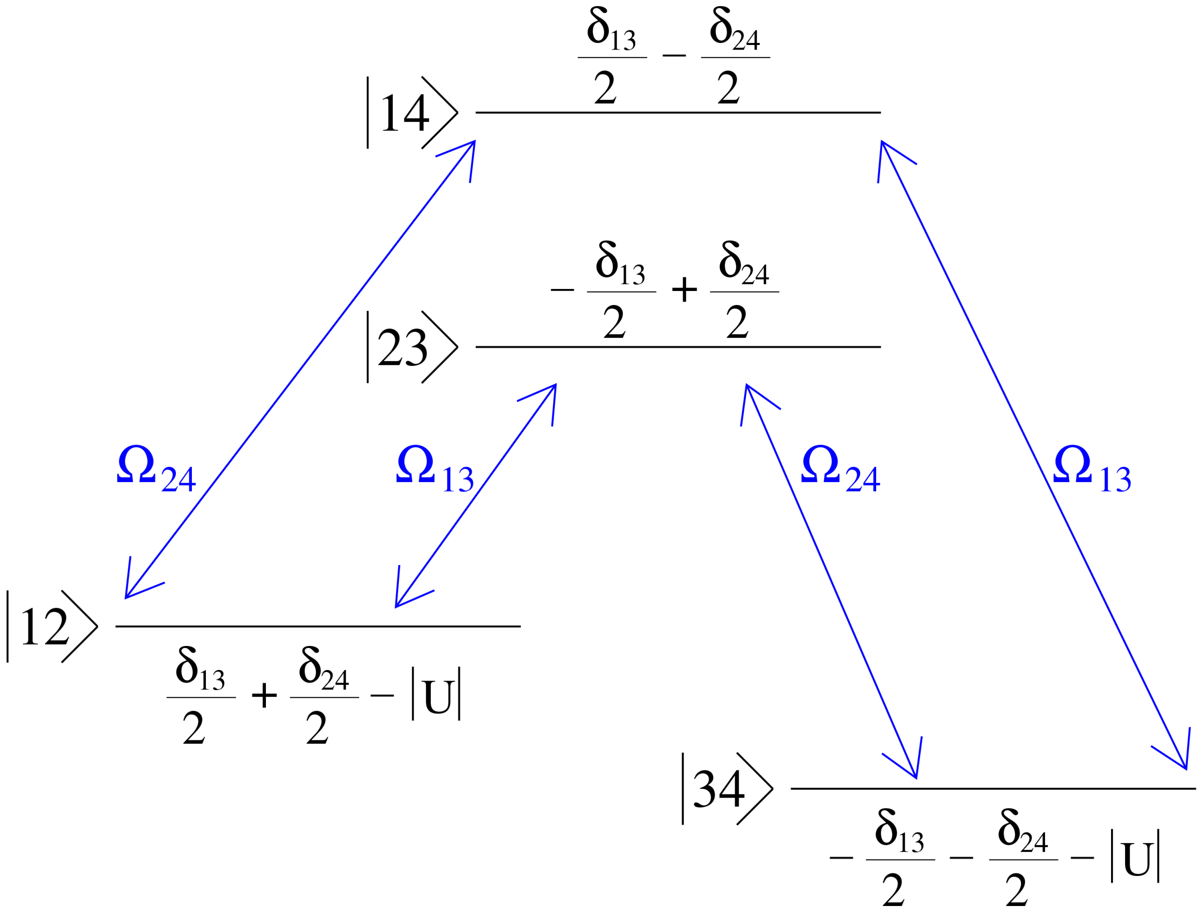
$$u^2|0\rangle + \boxed{uv(|12\rangle + |34\rangle)} + v^2|1234\rangle$$

- Corresponds to the superposition initial state in the small system

$$|\phi(0)\rangle = \alpha_I|12\rangle + \beta_{II}|34\rangle$$

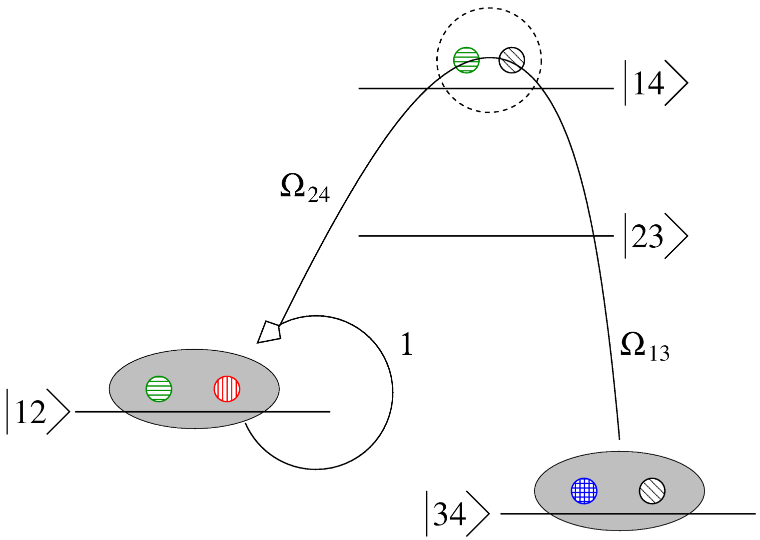
- The small system and the linear response calculation for a large system give essentially the same results
- Can the small system help us to explain the asymmetry in critical currents, and the Josephson effect in general? YES!!!
- Next: perturbation theory to second order in Ω in the small system

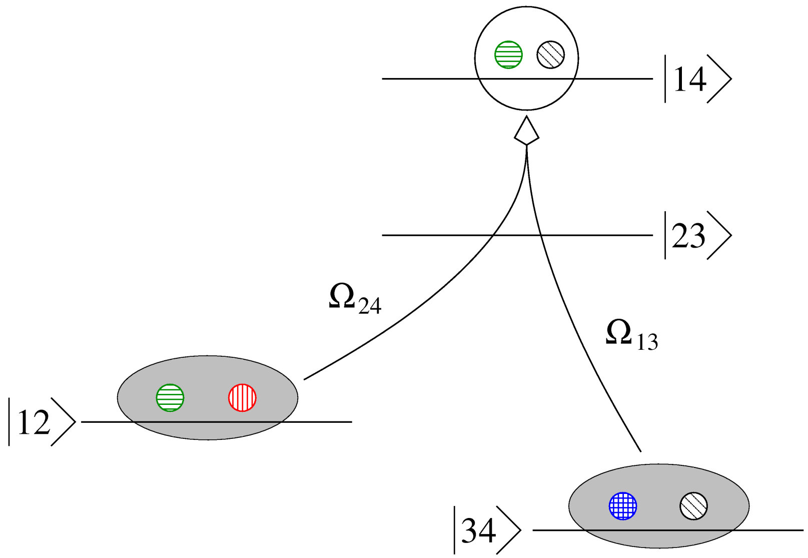
Perturbation theory to this system

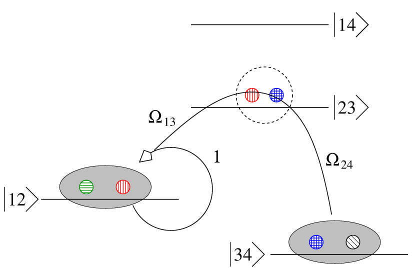


Perturbation calculation in the small system: results

The Josephson current is composed of two contributions







$$\left|\phi(0)\right\rangle = \alpha_I \left|12\right\rangle + \beta_{II} \left|34\right\rangle$$

Interference in pair tunneling

Interference in single particle tunneling (causes the asymmetry!!!)

$$\left|\phi(0)\right\rangle = \alpha_I \left|12\right\rangle + \beta_{II}\left|34\right\rangle$$

$$\left\langle\phi(t)\right|N_1\left|\phi(t)\right\rangle = \left|\alpha_I\right|^2 \left\langle 12\right|N_1\left|12\right\rangle + \left|\beta_{II}\right|^2 \left\langle 34\right|N_1\left|34\right\rangle + \alpha_I \beta_{II}^* \left\langle 34\right|N_1\left|12\right\rangle + h.c.$$

The interference terms: PAIR and SINGLE PARTICLE interference

$$\left|\phi_I(t)\right\rangle = \exp(-iHt)\left|12\right\rangle$$

$$\left|\phi_{II}(t)\right\rangle = \exp(-iHt)\left|34\right\rangle$$

$$\left|\phi_I(t)\right\rangle = \underline{\left[1 + A_I^{(2)}(t)\right]}\left|12\right\rangle + \underline{B_I^{(1)}(t)}\left|14\right\rangle + C_I^{(1)}(t)\left|23\right\rangle + D_I^{(2)}(t)\left|34\right\rangle$$

$$\left|\phi_{II}(t)\right\rangle = \left[1 + A_{II}^{(2)}(t)\right]\left|34\right\rangle + B_{II}^{(1)}(t)\left|23\right\rangle + \underline{C_{II}^{(1)}(t)}\left|14\right\rangle + \underline{D_{II}^{(2)}(t)}\left|12\right\rangle$$

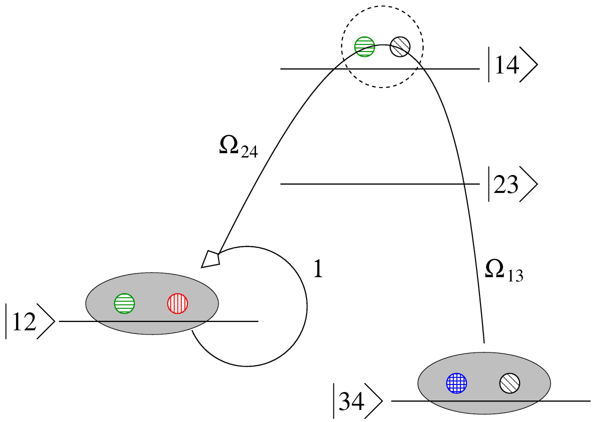
Non-interference terms: usual PAIR tunneling (4th order) and usual SINGLE PARTICLE (quasiparticle, above-gap) tunneling

$$\left|\phi_I(t)\right\rangle = \exp(-iHt)\left|12\right\rangle$$

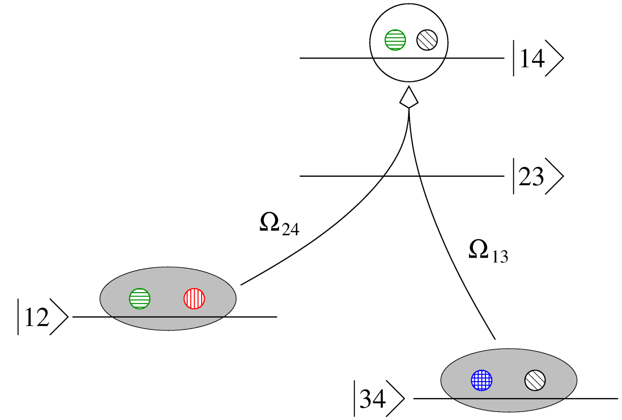
$$\left|\phi_{II}(t)\right\rangle = \exp(-iHt)\left|34\right\rangle$$

$$\left|\phi_I(t)\right\rangle = \left[1 + A_I^{(2)}(t)\right]\left|12\right\rangle + \underline{\underline{B_I^{(1)}(t)}}\left|14\right\rangle + C_I^{(1)}(t)\left|23\right\rangle + D_I^{(2)}(t)\left|34\right\rangle$$

$$\left|\phi_{II}(t)\right\rangle = \left[1 + A_{II}^{(2)}(t)\right]\left|34\right\rangle + B_{II}^{(1)}(t)\left|23\right\rangle + \underline{\underline{C_{II}^{(1)}(t)}}\left|14\right\rangle + \underline{\underline{D_{II}^{(2)}(t)}}\left|12\right\rangle$$



Conclusions



- Josephson effect comes from interferences in the pair and single particle tunneling contributions (interference terms of pair and single particle Rabi oscillations); single: asymmetry in critical currents
- Interference; lack of which-way information; particle number fluctuations; macroscopic phase
- Implication: the Josephson frequency is not δ but $\sqrt{\Omega^2+\delta^2}$!!!

Phase coherence is essential for the Josephson effect

Face Coherence

Summary

- Probing the FFLO state by modulation spectroscopy (Korolyuk, Massel, PT, PRL 2010)
  - Narrowing of the spectrum (pair momentum restricts phase space)
- Fermi-polaron-like effects in a 1D lattice (Leskinen, Nummi, Massel, PT, NJP 2010)
  - ”Polaron” and spinless Fermion regimes; crossover
- Spin-asymmetric Josephson effect (Heikkinen, Massel, Kajala, Leskinen, Paraoanu, PT, arXiv 2009)
  - Josephson effect as interference in pair and single particle tunnelings; also single particle contributions exist; asymmetry possible!

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