Low temperature properties of solid ^{4}He: Supersolidity or quantum "metallurgy"?

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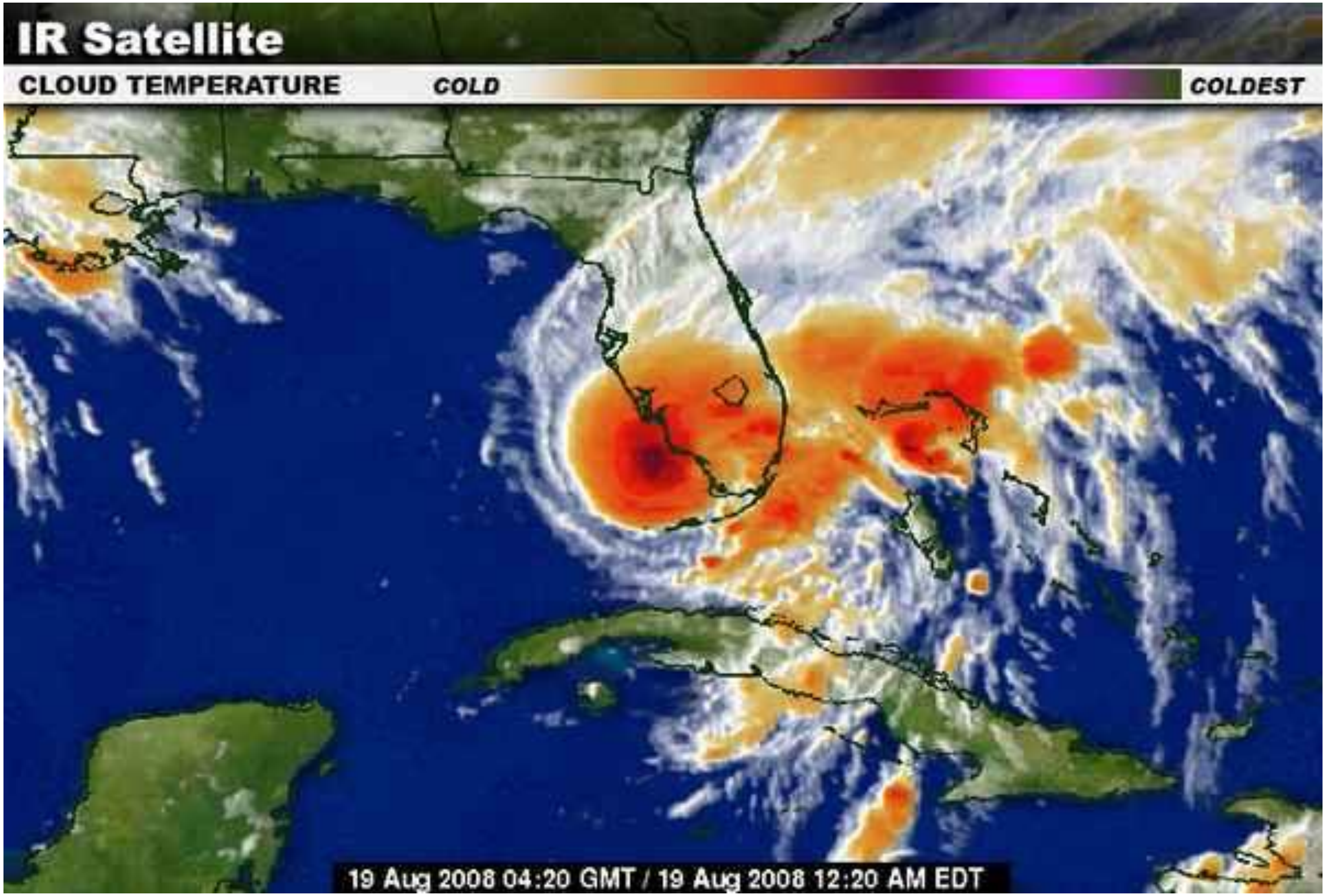
And thanks to the NSF!

Where's Gainesville?

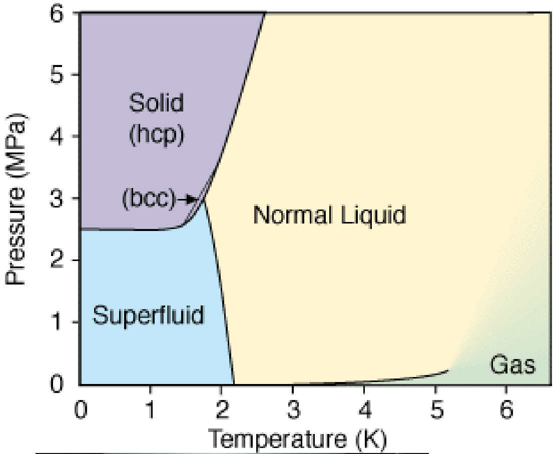


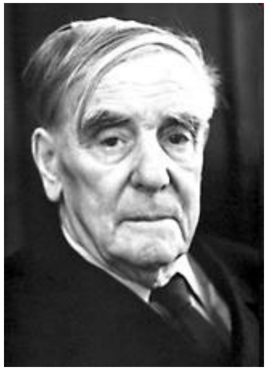
Outline

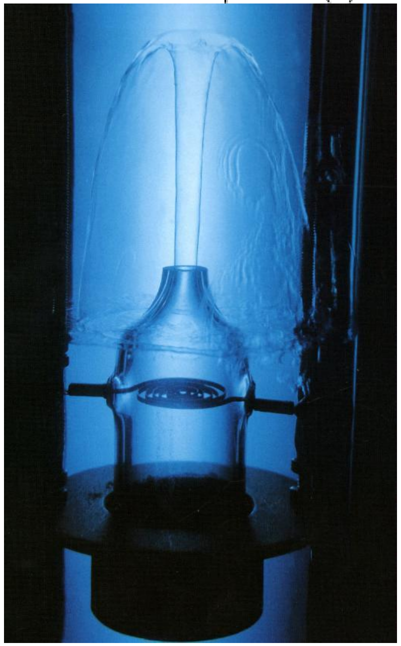
- Can BEC occur in a solid?
  - A brief history of “supersolids”
  - How would we know?
- Some curious experimental results
  - Torsional oscillator
  - Shear modulus anomalies
  - Specific heat
- Supersolidity – or quantum metallurgy? The role of dislocations and other defects.

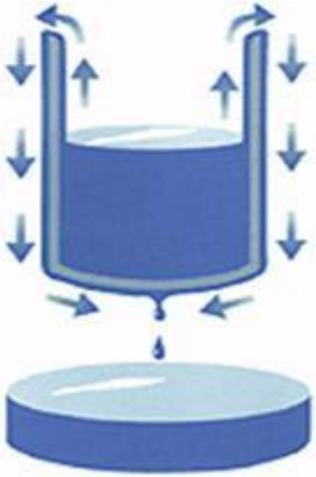
Superfluidity in ^{4}He

- Discovered in 1937 by Kapitsa, and by Allen & Misener.
- Properties:
  - Zero viscosity-flow through narrow pores
  - Fountain effect
  - Creep along (up) surfaces
  - Second sound ("thermal waves")
- Related to BEC (but strong interactions!)



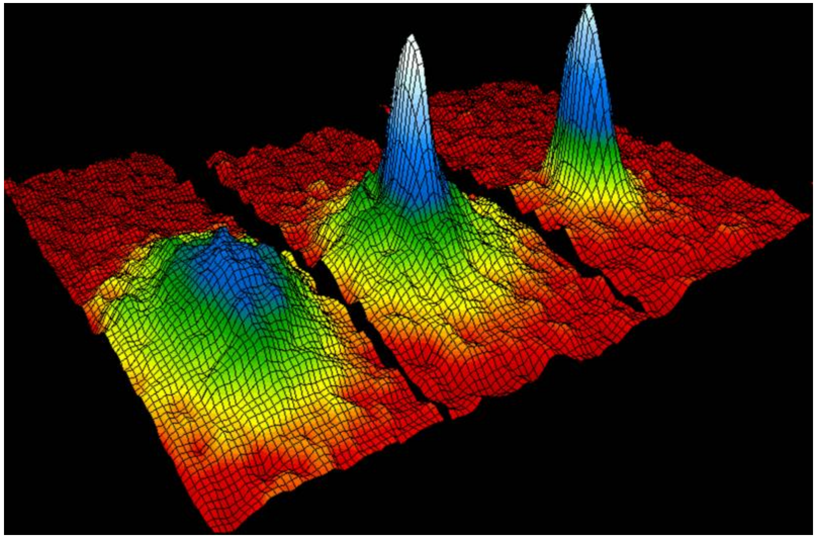


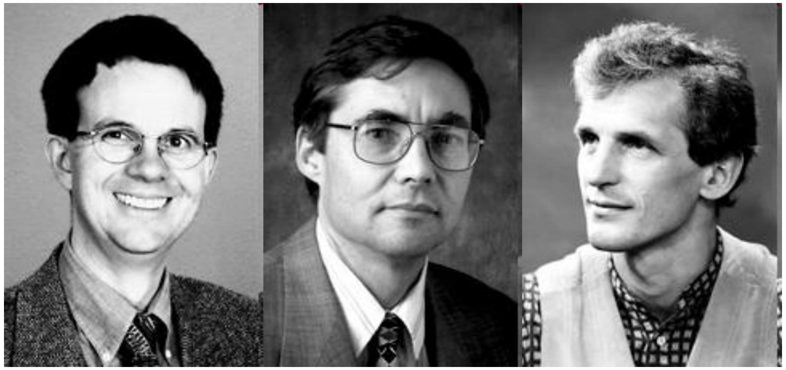




BEC in dilute atomic gases

- Discovered in 1995 by Wieman & Cornell (^{87}Rb) and Ketterle (^{23}Na).
- Temperature of 170 nK!
- Weak interactions (unlike ^{4}He).
- Fermionic condensates also possible—^{40}K in a magnetic field (D. Jin, 2003).





Can BEC occur in a solid?

- BEC occurs in the gas and liquid phases; can it also occur in a solid?
- Can off-diagonal long range order (Bose condensation) coexist simultaneously with shear rigidity (a solid)?
- No: BEC requires extended wavefunctions, solids have localized wavefunctions.
- Yes: why not?
- What do the theorists say? Are "supersolids" possible?

Penrose & Onsager (1956): no

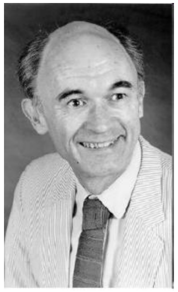
PHYSICAL REVIEW VOLUME 104, NUMBER 3 NOVEMBER 1, 1956

Bose-Einstein Condensation and Liquid Helium

Oliver Penrose* and Lars Onsager
Sterling Chemistry Laboratory, Yale University, New Haven, Connecticut
(Received July 30, 1956)

The mathematical description of B.E. (Bose-Einstein) condensation is generalized so as to be applicable to a system of interacting particles. B.E. condensation is said to be present whenever the largest eigenvalue of the one-particle reduced density matrix is an extensive rather than an intensive quantity. Some transformations facilitating the practical use of this definition are given.

An argument based on first principles is given, indicating that liquid helium II in equilibrium shows B.E. condensation. For absolute zero, the argument is based on properties of the ground-state wave function derived from the assumption that there is no "long-range configurational order." A crude estimate indicates that roughly 8% of the atoms are "condensed" (note that the fraction of condensed particles need not be identified with ρ_s/ρ). Conversely, it is shown why one would not expect B.E. condensation in a solid. For finite temperatures Feynman's theory of the lambda-transition is applied: Feynman's approximations are shown to imply that our criterion of B.E. condensation is satisfied below the lambda-transition but not above it.



Chester (1970): well, maybe

PHYSICAL REVIEW A VOLUME 2, NUMBER 1 JULY 1970

Speculations on Bose-Einstein Condensation and Quantum Crystals*

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(Received 13 May 1969)

It is shown, by almost rigorous arguments, that there exist many-body states of a system of interacting bosons which exhibit both crystalline order and Bose-Einstein condensation into the zero-momentum eigenstate of the single-particle density matrix. The implications of this result are discussed in relation to theories of superfluidity and the nature of quantum crystals.

type will give an accurate description of the exact states of solid and liquid helium four. We make this statement because the arguments presented above allow us to include in the wave function as complicated explicit correlations as we like. It is of course true that there are other states one can envisage which do not satisfy our condition, for example, a linear combination of Jastrow states. Indeed, if no state in this wide class of model wave functions adequately describes the spatial correlations of a real-quantum crystal, then these correlations must be fundamentally different from those which occur in a classical crystal. For these reasons, we believe that our speculation about real physical systems are on much firmer grounds than would appear at first sight.

Finally, we comment on the proof by Onsager and Penrose[2] that a state with crystalline order cannot have a Bose-Einstein condensate in the zero-momentum state. This proof is based on a particular class of model states in which each particle is localized on a lattice site, each site is occupied by a particle and symmetry is ignored. If either of the latter restrictions is removed, then the original proof fails.[6] In particular, if there are vacancies in the model state and symmetry is ignored, then a condensate exists and this condensation would presumably persist if symmetry were taken into account. It is now interesting to note that we expect all the model states we have discussed to lead to crystalline order with vacancies present. This is because the equivalent classical system would be expected, on physical grounds, to lead to crystallization with a finite fraction of vacancies. We may therefore add one final speculation, namely, that a quantum crystal can only have a Bose-Einstein condensate if it has a finite fraction of vacancies. We see no reason, whatsoever, to suppose that a quantum crystal cannot have a finite fraction of vacancies at absolute zero. Liquid helium exists at absolute zero and this suggests that a crystal with a finite amount of spatial disorder could exist at absolute zero. If, on the other hand, a finite fraction of vacancies can only exist at elevated temperatures, then it might be impossible to have a Bose-Einstein condensate because of the high temperature. We pointed out in Sec. IV that it is almost impossible to predict the temperature at which such a condensation might occur.

Vacancies and interstitials

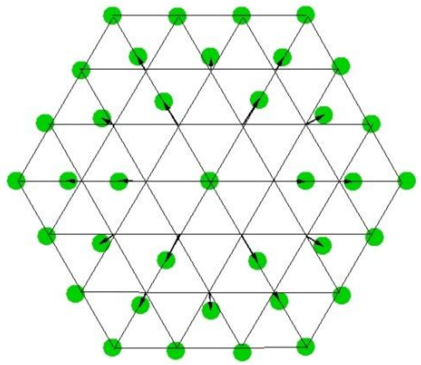
- Local density changes arise from either lattice fluctuations (with a displacement field **u**) or vacancies and interstitials.

$$\delta\rho = \delta\rho_\Delta - \rho_0 \nabla \cdot \mathbf{u}$$

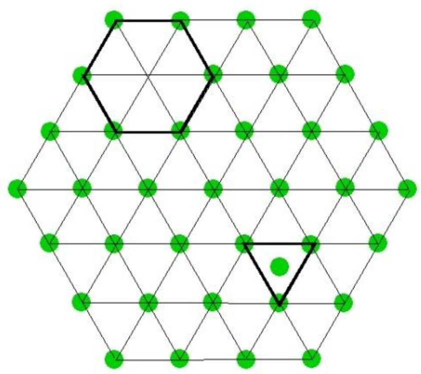
- In *classical* solids the density of vacancies is small at low temperatures.

$$n_{\text{vacancies}} = a^{-3} e^{-E_V/T}$$

- Does ^{4}He have zero point vacancies?



$$\delta\rho = -\rho_0 \nabla \cdot \mathbf{u}$$



$$\delta\rho_\Delta = (\delta\rho_{\text{interstitial}} - \delta\rho_{\text{vacancy}})$$

Supersolidity in Jastrownium™

- Jastrow variational wavefunction for N bosons:

$$\Psi_N(\mathbf{r}_1,\ldots,\mathbf{r}_N) = \prod_{i\neq j} f(r_{ij}) = \exp\left[-\frac{1}{2}\sum_{i\neq j} u(r_{ij})\right]$$

- Probability distribution is identical to the classical Gibbs distribution for N particles interacting through a potential $V(r_{ij})=T_{eff}\, u(r_{ij})$

$$P_N = \Psi_N^2 = \exp\left[-\sum_{i\neq j} V(r_{ij})/T_{\rm eff}\right]$$

- Expect a solid phase for some sufficiently low T_{eff}, with a defect density of $n_{\rm vacancies} = a^{-3}e^{-E_V/T_{\rm eff}}$
- Proof of principle: Jastrownium can have both ODLRO and crystalline order. What about ^{4}He?
- Andreev and Lifshitz (1969): defects Bose condense, producing a condensate that lives within the solid phase.

Andreev & Lifshitz: here's one way

SOVIET PHYSICS JETP VOLUME 29, NUMBER 6 DECEMBER 1969

QUANTUM THEORY OF DEFECTS IN CRYSTALS

A. F. ANDREEV and I. M. LIFSHITZ

Institute of Physical Problems, U.S.S.R. Academy of Sciences

Submitted January 15, 1969

Zh. Eksp. Teor. Fiz. 56, 2057–2068 (June, 1969)

At sufficiently low temperatures localized defects or impurities change into excitations that move practically freely through a crystal. As a result instead of the ordinary diffusion of defects, there arises a flow of a liquid consisting of "defectons" and "impuritons." It is shown that at absolute zero in crystals with a large amplitude of the zero-point oscillations (for example, in crystals of the solid helium type) zero-point defectons may exist, as a result of which the number of sites of an ideal crystal lattice may not coincide with the number of atoms. The thermodynamic and acoustic properties of crystals containing zero-point defectons are discussed. Such a crystal is neither a solid nor a liquid. Two kinds of motion are possible in it; one possesses the properties of motion in an elastic solid, the second possesses the properties of motion in a liquid. Under certain conditions the "liquid" type of crystal motion possesses the property of superfluidity. Similar effects should also be observed in quasiequilibrium states containing a given number of defectons.

Leggett (1970): look for NCRI!

Can a Solid Be "Superfluid"?

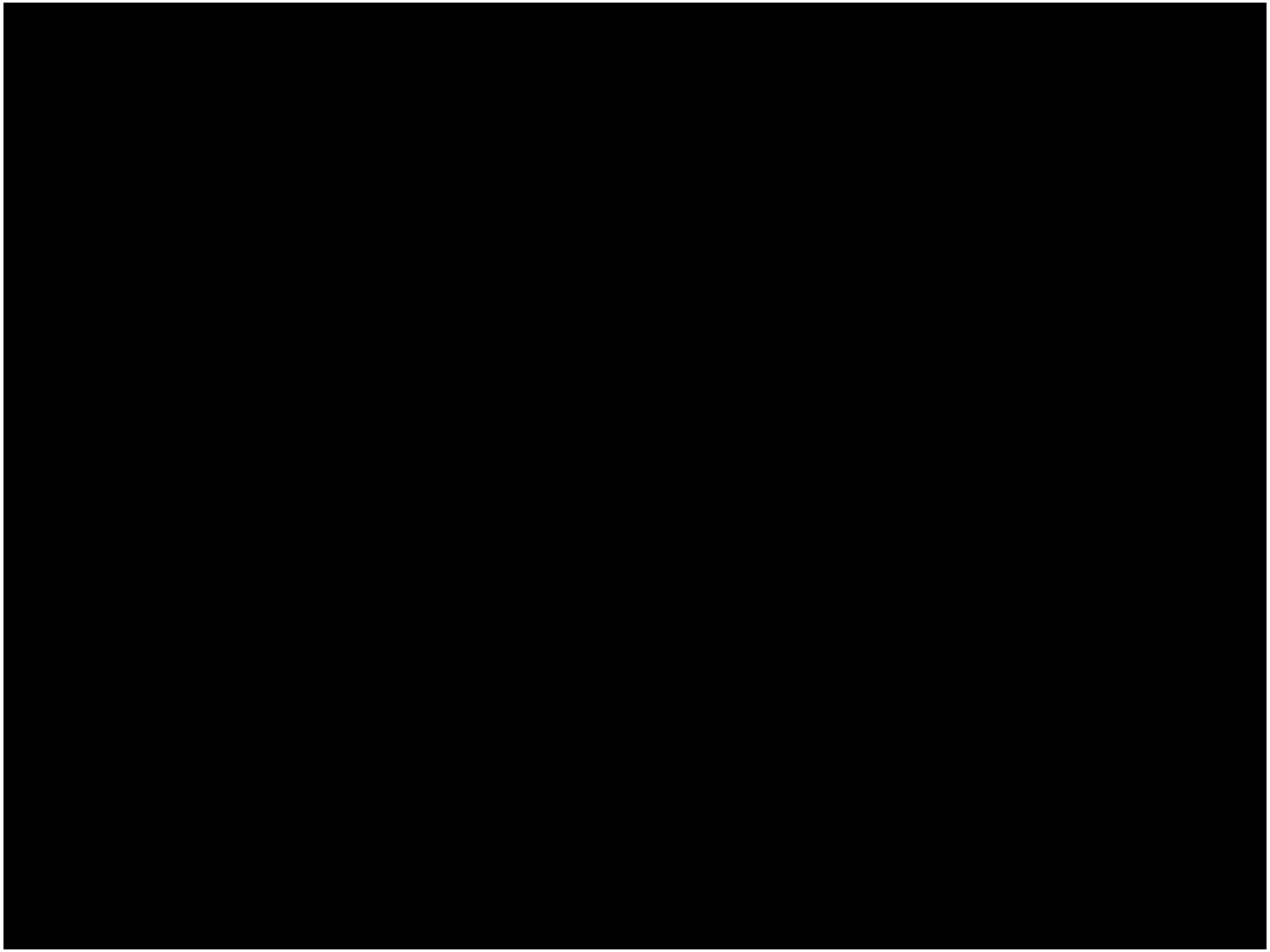
A. J. Leggett
School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton, Sussex, England
(Received 15 September 1970)

It is suggested that the property of nonclassical rotational inertia possessed by superfluid liquid helium may be shared by some solids. In particular, nonclassical rotational inertia very probably occurs if the solid is Bose-condensed as recently proposed by Chester. Anomalous macroscopic effects are then predicted. However, the associated superfluid fraction is shown to be very small (probably $\lesssim 10^{-4}$) even at $T=0$, so that these effects could well have been missed. Direct tests are proposed.

In this Letter we shall suggest that, on the contrary, it is impossible to exclude the occurence of NCRI also in insulating[1] solids (where "solid" is defined phenomenologically—see below), and that if it does occur it should produce a number of interesting phenomena analogous to those of superfluidity. However, we shall show that the associated "superfluid fraction" must be very small even at $T=0$ (probably always $\lesssim 10^{-4}$). As a result, these phenomena could well have escaped notice even if "superfluid solids" do exist at temperatures already reached, since they have not (to the best of the author's knowledge) been specifically looked for. While the ideas discussed here are somewhat speculative, an experiment to test them should be relatively simple and seems well worthwhile.

If NCRI should indeed occur in a solid, how would it manifest itself? The most direct experiment to look for it would be to rotate the solid in the form of an annulus[12] below its transition temperature; then the apparent moment of inertia should be slightly less than the classical value I_0 (and, more relevantly, presumably temperature dependent). A second test would be to rotate the solid above its presumed critical angular velocity ω_c and then bring the container to rest; if we assume that NCRI is associated with the metastability of flow states as in other superfluid systems, we should expect a persistent residual angular momentum $(\rho_s/\rho)I_0\omega_c$. In view of the small value of ρ_s/ρ, it seems highly unlikely that these effects would have been discovered by accident even if "superfluid solids" do exist at attained temperatures.

That's entertainment!

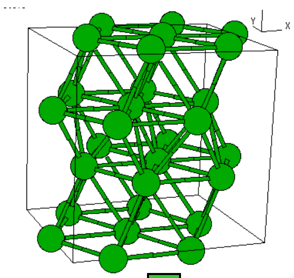


Phase diagram of ^{4}He

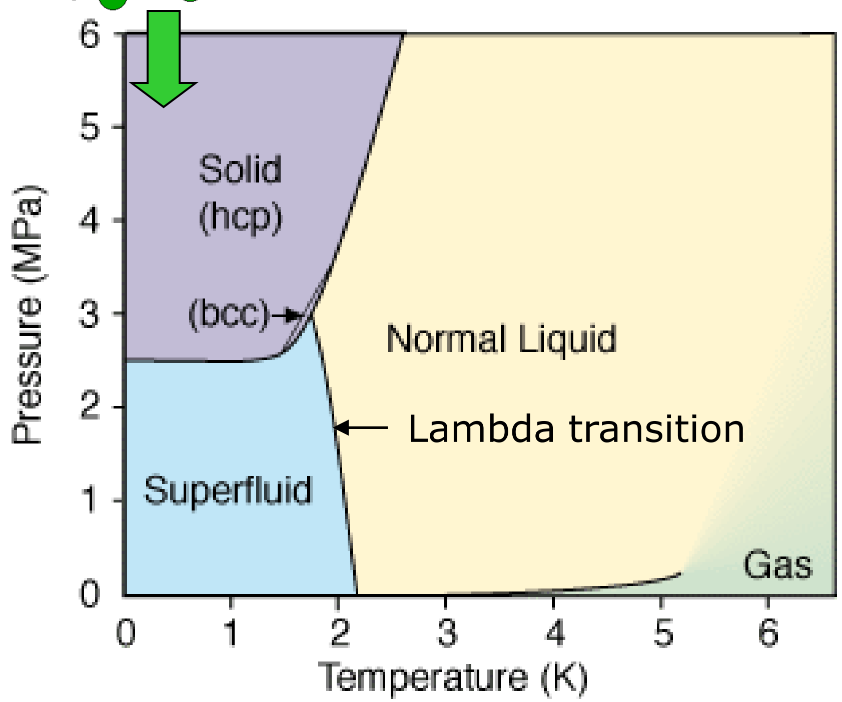
- Solid ^{4}He is *soft*: shear modulus of 20 MPa (Al is 26 GPa, butter is 5 MPa).
- Solid ^{4}He is *light*:density of 0.2 g/cm^3 (like cork or balsa wood).
- θ_{Debye} =25 K.
- de Boer parameter

$$\Lambda \equiv \lambda/(2\pi\sigma) = 0.282$$

Solid Ar has Λ=0.019. Quantum effects are important.



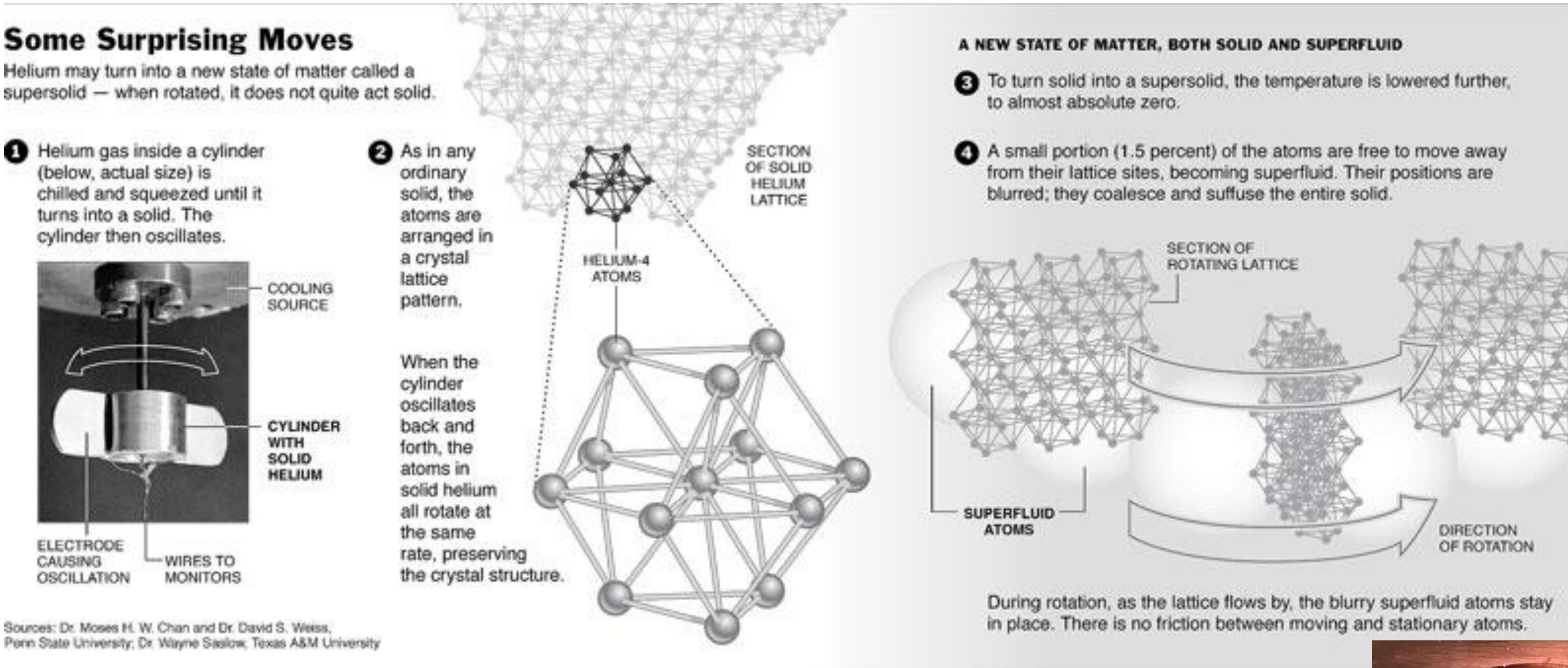
- a=3.53 A, c=5.93 A (at 2 K, 26 atm).
- c/a=1.68 [ideal HCP has c/a=$\sqrt{(8/3)}$=1.632].
- 1 atm=1.013×10^5 Pa



http://ltl.tkk.fi/research/theory/helium.html

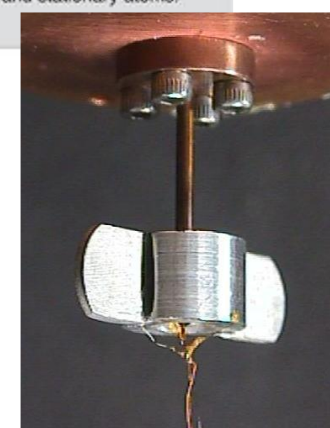
Penn State experiments

(New York Times, 21 September 2004)

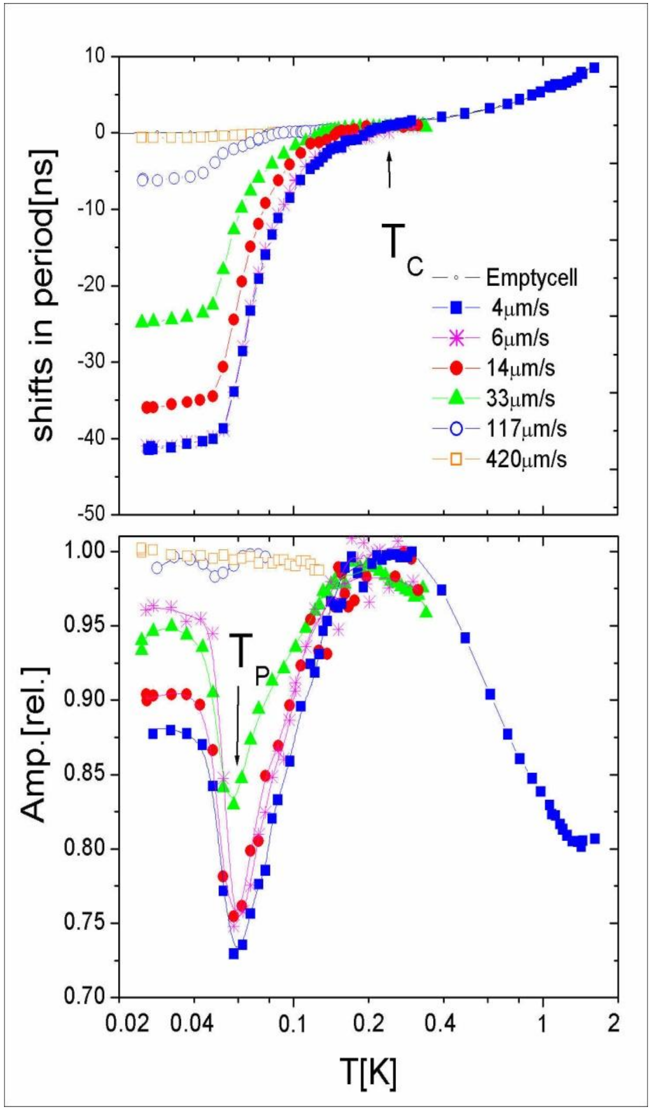


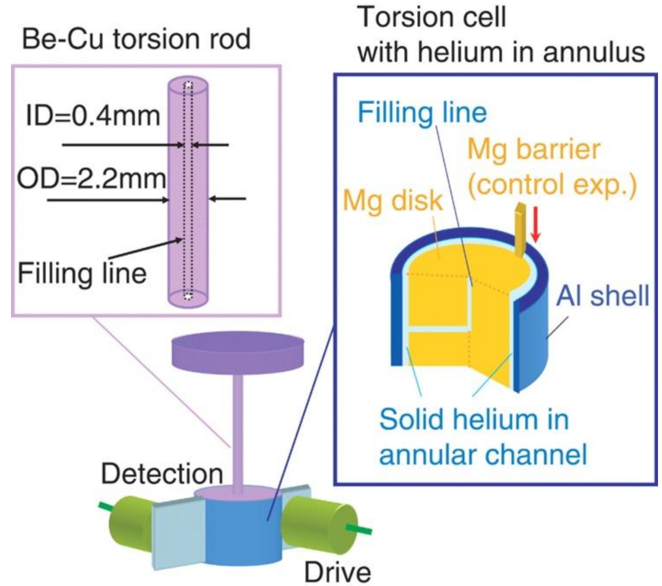
- ^{4}He in vycor glass: Nature **427**, 225 (2004).
- Bulk ^{4}He: Science **305**, 1941 (2004).
- Now reproduced by several groups: PSU, Cornell, Rutgers, Keio, Tokyo, Seoul, …

$$\tau = 2\pi\sqrt{I/\alpha}$$



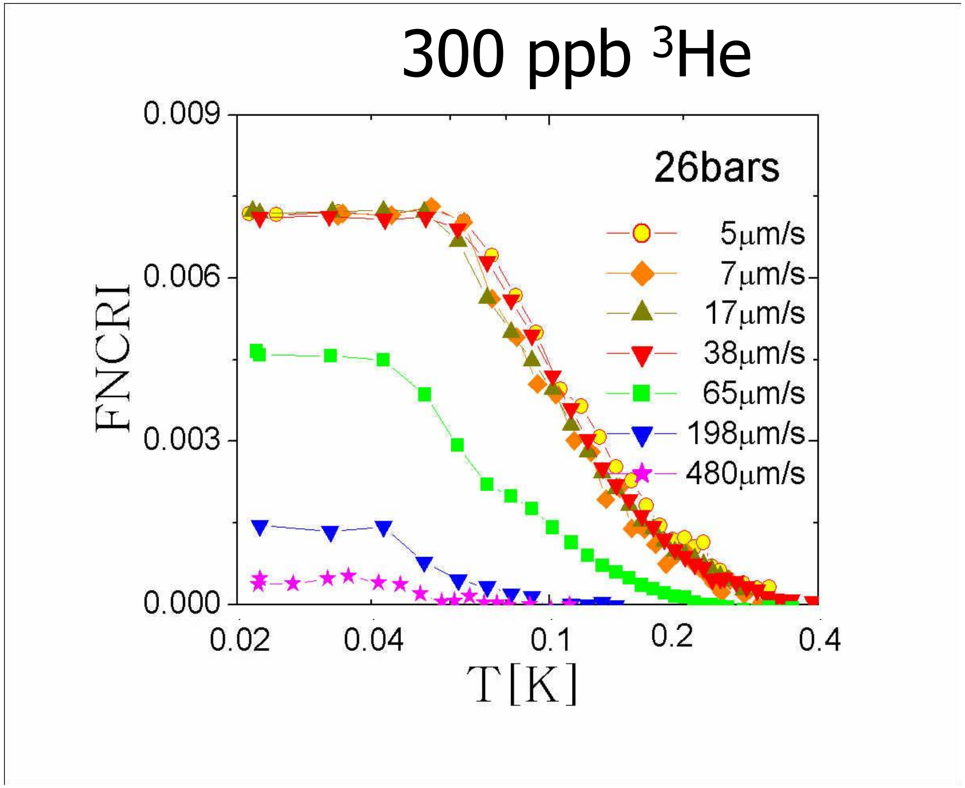
Data from PSU (Science 2004)

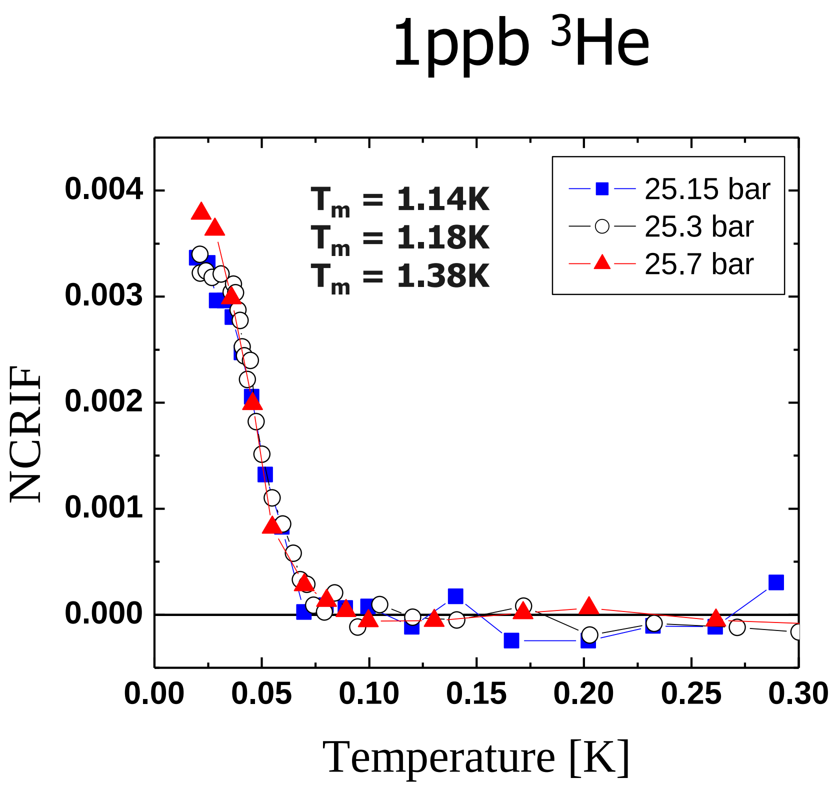




- Pressures from 26 to 66 bars.
- Amplitude of about 1 nm, maximum velocities of about 10 μ m/s.
- Frequency: 1 kHz (period of 10^6 ns). Period shifts of order 40 ns. Q of order 10^6.
- Barrier inserted: no effect.

Effects of ^{3}He impurities

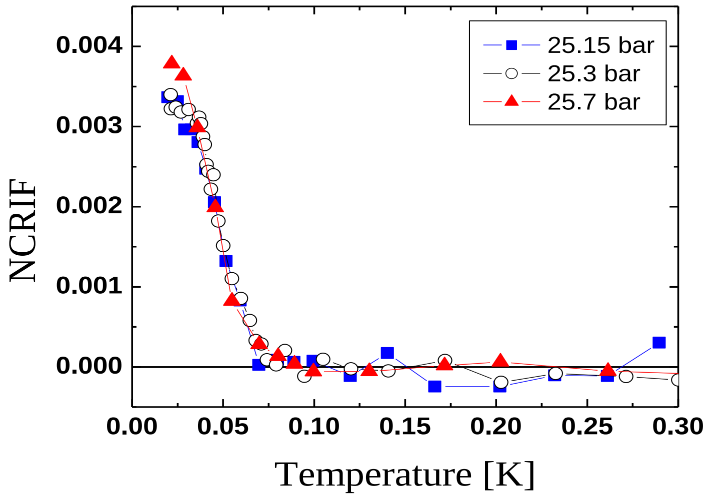


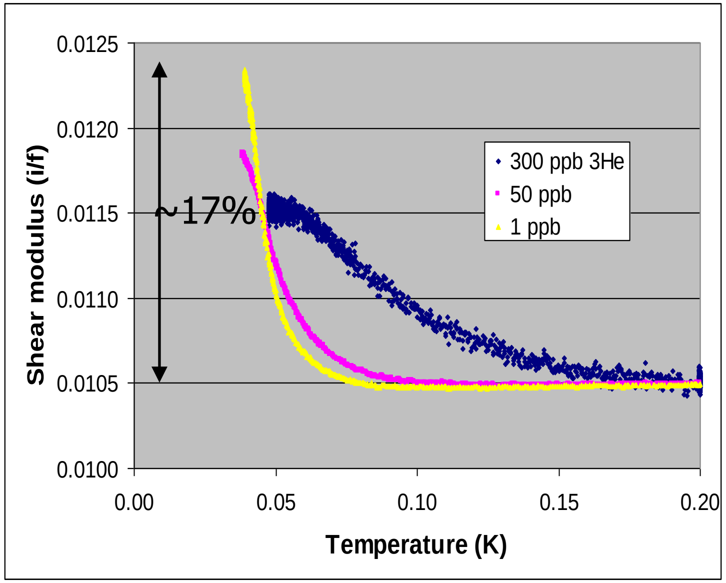


Onset T decreases 250 to 75 mK

Transition sharpens as ^{3}He is reduced

Shear stiffening





- Day and Beamish (Nature 2007): shear modulus increases by 10-20% at low temperature
- Onset temperature of stiffening for pure samples at 80 mK, similar to torsional oscillator.
- Shape of stiffening similar to period shift.
- Sensitive to ^{3}He impurities.
- See ATD and DA Huse (Nature 2007).

Summary so far…

- Effect observed by several different groups.
- Sample preparation is important: annealing reduces effect (Rittner & Reppy).
- Small quantities of ^{3}He affect the magnitude of NCRI.
- No pressure driven flow (Day & Beamish), but chemical potential driven flow (Ray and Hallock)?
- Mechanical changes with reduced temperature: shear stiffening.
- Monte Carlo: perfect crystal is insulating. Vacancy creation energy about 13K.
- Are the mechanical changes and NCRI related?
- What is the role of defects—dislocations or grain boundaries?
- Why is ^{3}He important?
- What is the origin of the low critical velocity?
- For non-superfluid explanations, what about the c-shaped sample cells?

Torsional oscillator: rigid body

- Equation of motion for a rigid solid:

$$\left[(I_{\rm cell}+I_{\rm He})\frac{d^2}{dt^2}+\gamma\frac{d}{dt}+\alpha\right]\theta(t)=\tau_{\rm ext}(t)$$

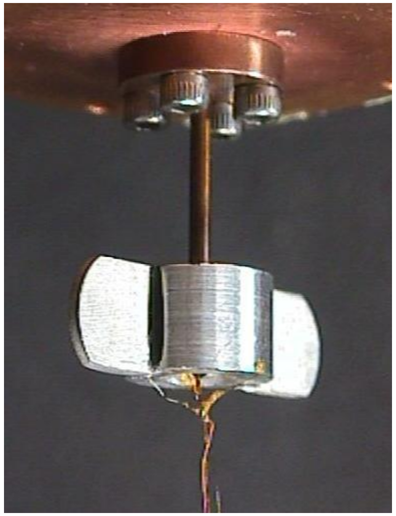
- Resonant period:

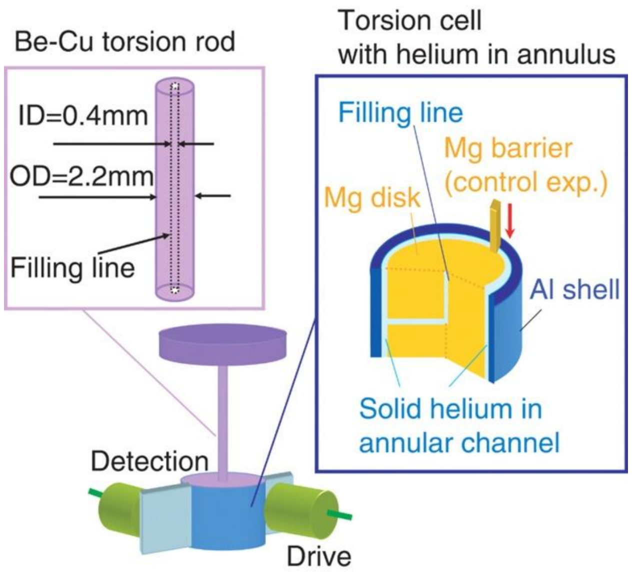
$$P=P_0\left(1+\frac{1}{8Q^2}+\ldots\right),\quad Q=\frac{\sqrt{\alpha I_{\rm total}}}{\gamma}=\mathcal{O}(10^6)$$

$$P_0=2\pi\sqrt{\frac{I_{\rm cell}+I_{\rm He}}{\alpha}}$$

$$\frac{\Delta P}{P_0}=\frac{1}{2}\frac{\Delta I_{\rm tot}}{I_{\rm tot}}=\mathcal{O}(10^{-5})$$

- What happens if the solid ^{4}He is not rigid?





Torsional oscillator: elastic solid

- Equation of motion for a TO containing an elastic solid [Nussinov et al. (2007)]:

$$\underbrace{\left(I_{\rm cell}\frac{d^2}{dt^2}+\gamma\frac{d}{dt}+\alpha\right)\theta(t)=\tau_{\rm ext}(t)}_{\text{equation of motion for unloaded cell}} \quad + \quad \underbrace{M(t)}_{\text{back reaction from elastic solid}}$$

- Back action: moment that the solid ^{4}He exerts on the walls of the cell (linear response):

$$M(t)=\int dt' g(t-t')\theta(t'), \quad M(\omega)=g(\omega)\theta(\omega)$$

- Oscillator response f $\chi(\omega)=\theta(\omega)/\tau_{\rm ext}(\omega)$

$$\chi^{-1}(\omega)=-I_{\rm cell}\omega^2-i\gamma\omega+\alpha-g(\omega)$$

- The complex poles of the response function determine the resonant frequency and dissipation of the system.

Elastic response of the solid

- *All* of the information about the solid ^{4}He is contained in $g(\omega)$. It has the following properties:
  - analytic in upper half frequency plane;
  - real and imaginary parts obey Kramers-Kronig relations;
  - low frequency behavior must be a rigid solid:

$$g(\omega) = I_{\mathrm{He}}\omega^2 + \mathcal{O}(\omega^3)$$

- To calculate $g(\omega)$ we need to solve the equation of motion for an elastic solid:

$$\rho\partial_t^2 u_i = \partial_j \sigma_{ij}, \quad u_{ik} = \left(\partial_k u_i + \partial_i u_k\right)/2$$

- Hooke's Law (nonlocal in time):

$$\sigma_{ij}(t) = \int dt'\, K_{ijkl}(t-t')u_{kl}(t'), \quad \sigma_{ij}(\omega) = K_{ijkl}(\omega)u_{kl}(\omega)$$

Viscoelasticity

- Isotropic elasticity:

$$K_{ijkl}(\omega) = \lambda(\omega)\delta_{ij}\delta_{kl} + \mu(\omega)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$$

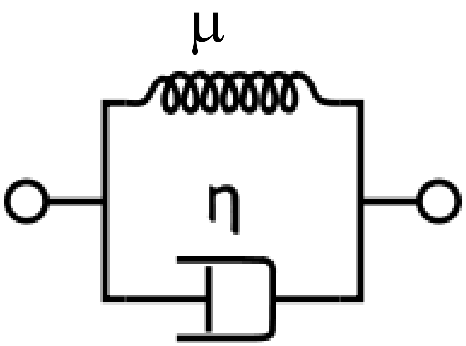
$$-\rho\omega^2\mathbf{u} = B(\omega)\nabla(\nabla\cdot\mathbf{u}) - \mu(\omega)\nabla\times\nabla\times\mathbf{u}, \qquad B = \lambda + 2\mu$$

- Shear motion of an elastic solid:

$$\rho\partial_t^2\mathbf{u} = \mu_0\nabla^2\mathbf{u}$$

- Navier-Stokes for a viscous fluid:

$$\rho\partial_t\mathbf{v} = \eta\nabla^2\mathbf{v} \quad \overset{\mathbf{v}=\partial_t\mathbf{u}}{\Longrightarrow} \quad \rho\partial_t^2\mathbf{u} = \eta\partial_t\nabla^2\mathbf{u}$$



- Combining (in "parallel"):

$$\rho\partial_t^2\mathbf{u} = (\mu_0 + \eta\partial_t)\,\nabla^2\mathbf{u}$$

- Kelvin-Voigt model (internal friction):

$$\mu(\omega) = \mu_0 + i\eta\omega = \mu_0(1 + i\omega\tau), \qquad \tau = \eta/\mu_0$$

Boundary value problem

- Cylindrical geometry, no slip boundary conditions (assume long cylinder):

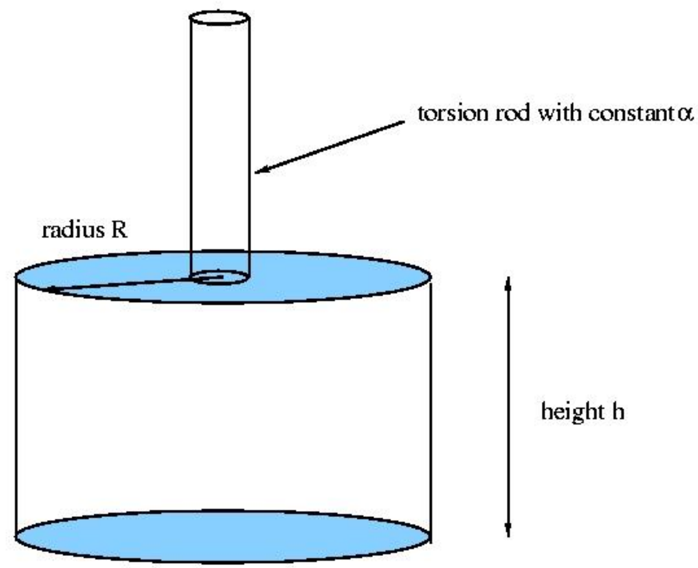
$$\mathbf{u} = u_\theta(r) e^{i\omega t}\,\hat{\theta}, \quad u_\theta(r = R) = R\theta_0$$

- Equation of mo

$$-\rho\omega^2 u_\theta = \mu(\omega)\left(\partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\right)u_\theta$$

- Solution:

$$u_\theta(r) = R\theta_0 \frac{J_1(kr)}{J_1(kR)}, \quad k^2 = \omega^2\rho/\mu(\omega)$$



- Shear stress exerted by the solid on the cell:

$$\sigma_{\theta r} = \mu(\omega)\left(\partial_r - \frac{1}{r}\right)u_\theta\Big|_{r=R} = -\theta_0 R^2 \rho\omega^2 \frac{J_2(kR)}{kRJ_1(kR)}$$

Properties of results

- TO is a probe of the shear modulus. The period shift and the dissipation are related!

$$\frac{\Delta P}{P} \propto \mathrm{Re}\left[\frac{1}{\mu(\omega)}\right], \quad \Delta Q^{-1} \propto \mathrm{Im}\left[\frac{1}{\mu(\omega)}\right]$$

  - Corrections vanish for a rigid solid.

- The peak value of Δ Q^{-1} is independent of τ :

$$\Delta Q^{-1}\big|_{\max} = \underbrace{\frac{1}{48}}_{10^{-2}} \underbrace{\frac{I_{\mathrm{He}}}{I_{\mathrm{tot}}}}_{10^{-2}-10^{-3}} \underbrace{\left(\frac{\omega_0 R}{c_T}\right)^2}_{10^{-2}} = \mathcal{O}(10^{-6} - 10^{-7})$$

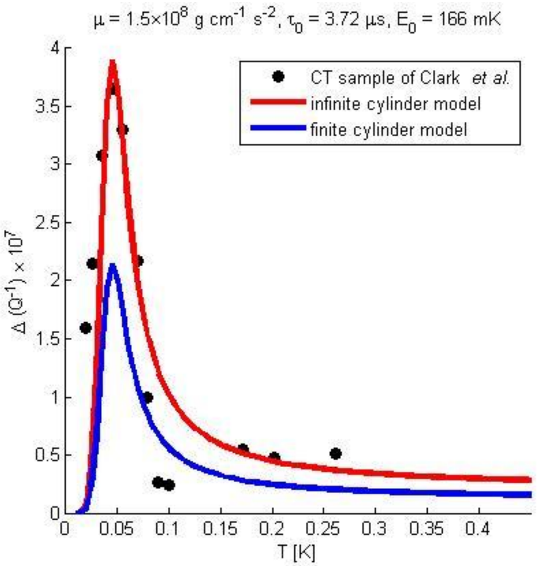
  - At the peak,

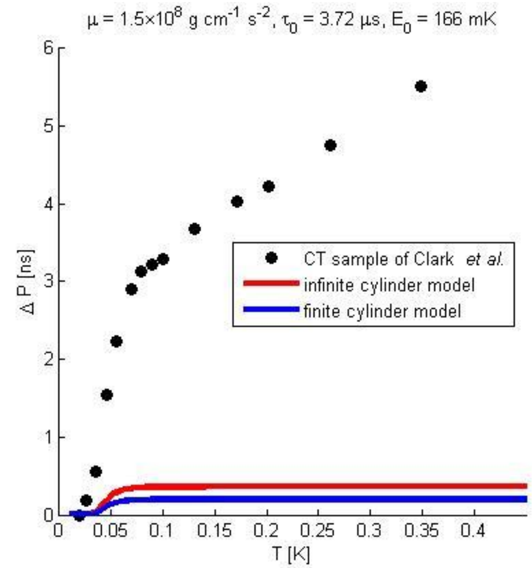
$$\Delta Q^{-1}\big|_{\max} = 2\Delta P/P\big|_{\max}$$

- For no dissipation, changing the shear modulus changes the period (inertial overshoot):

$$\frac{\Delta P}{P} = -\frac{1}{48}\left(\frac{\omega_0 R}{c_T}\right)^2 \frac{I_{\mathrm{He}}}{I_{\mathrm{tot}}} \frac{\Delta\mu}{\mu}$$

Data fits (Yoo & ATD, PRB 2009)





$$\Delta Q^{-1} \simeq \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{24 \mu I_{\rm tot}} \frac{\omega_0 \tau}{1+\tau^2\omega_0^2}$$

$$P \simeq \frac{2\pi}{\omega_0}\left[1 + \frac{\rho R^2 \omega_0^2 I_{\rm He} F(R/h)}{48 \mu I_{\rm tot}} \frac{1}{1+\tau^2\omega_0^2}\right]$$

- Dissipation peak identifies long relaxation time on the order of 1ms $\tau = \tau_0 \exp(E_0/T)$. Dislocations?
- Model seems to only account for 10% of the period shift. Suitable *distributions* of relaxation times allow a fit of both the period shift and dissipation peak (Su *et al.*, PRL 2010).
- Can dislocations be superfluid?

Landau theory for a super*fluid*

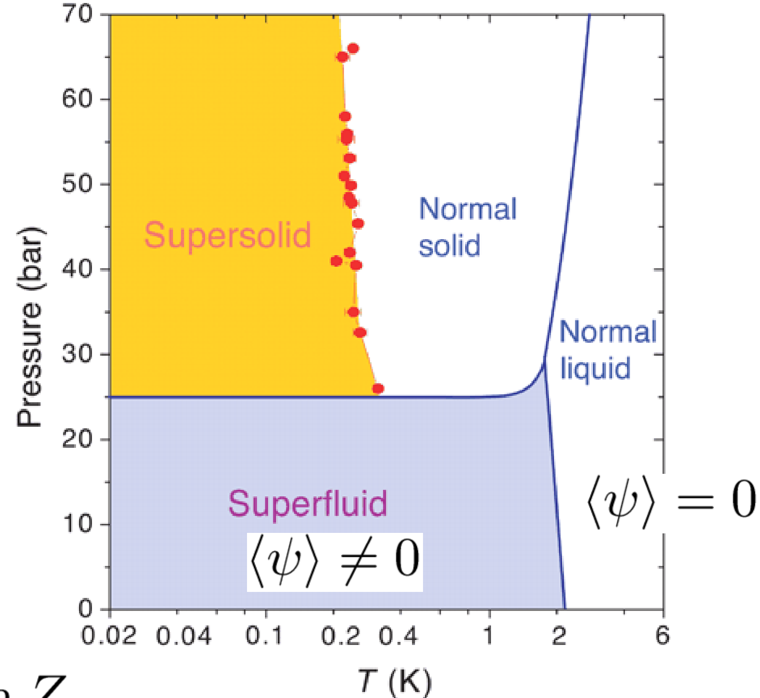
- Symmetry of order parameter

$$\psi(\mathbf{x}) \to \psi(\mathbf{x}) e^{i\phi}$$

- Broken U(1) symmetry for $T<T_c$.
- Coarse-grained free energy:

$$\mathcal{F}_{\rm sf} = \int_{\mathbf{x}} \{ \tfrac{1}{2} c |\nabla\psi|^2 + \tfrac{1}{2} a(T) |\psi|^2 + \tfrac{w}{4!} |\psi|^4 \},$$

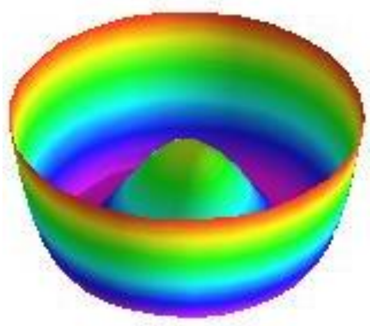
$$a(T) = a_0 (T - T_0)$$



- Average over configurations:

$$Z = \int D\psi D\psi^* e^{-\mathcal{F}_{\rm sf}/T}, \qquad F = -T \ln Z$$

- Fluctuations shift $T_0 \to T_C$, produce singularities as a function of the reduced temperature $t = |(T-T_c)/T_c|$.
- Universal exponents and amplitude ratios.



Coupling superfluidity & elasticity

- ATD, Goldbart & Toner (PRL 2006): Landau model with coupling between superfluidity and elasticity (strain dependent T_c):

$$\mathcal{F}_{\rm ss} = \int_{\mathbf{x}} \left\{ \frac{1}{2} c_{ij}\, \partial_i \psi\, \partial_j \psi^* + \frac{1}{2} a^{(0)} |\psi|^2 + \frac{w}{4!} |\psi|^4 + \frac{1}{2} K_{ijkl}\, u_{ij}\, u_{kl} + \frac{1}{2} a^{(1)}_{ij}\, u_{ij} |\psi|^2 \right\}.$$

- Predictions
  - XY anomaly in specific heat (lambda transition)
  - Anomalies in elastic constants; shows up as a dip in the sound speed at the transition:

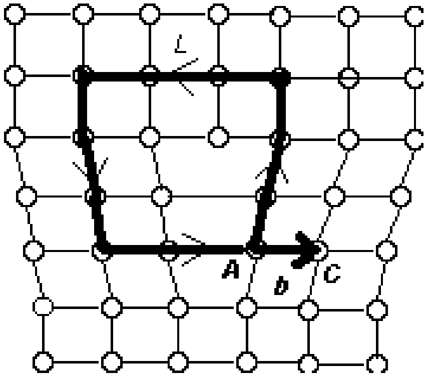
$$K_{ijkl} = -T \frac{\partial^2 F}{\partial u_{ij} \partial u_{kl}} = K^{(0)}_{ijkl} - \frac{1}{4T}\, a^{(1)}_{ij}\, a^{(1)}_{kl} \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle_0$$

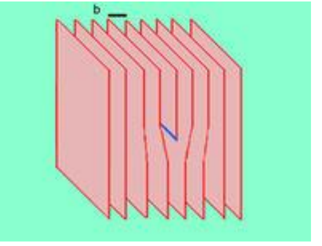
Supersolidity from dislocations?

- Dislocations: topological defects in a crystal (a string). Orowan, Polanyi and Taylor (1934) proposed that plastic deformation of solids can be described by dislocations.
- Dislocations can promote superfluidity. Recall model:
  $\mathcal{F}_{\rm ss} = \int_{\mathbf{x}} \left\{ \frac{1}{2} c \, |\nabla \psi|^2 + \frac{1}{2} t(\mathbf{r}) |\psi|^2 + \frac{w}{4} |\psi|^4 \right\},$
  $t(\mathbf{r}) = t_0 + g u_{ii}$
- Quenched dislocations produce large, long-ranged strains. For an edge dislocation (isotropic elasticity)

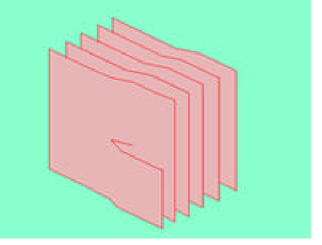
$$u_{ii} = \nabla \cdot \mathbf{u} = \frac{4\mu}{2\mu + \lambda} \frac{a \cos\theta}{r_\perp}$$

- Even if $t_0>0$ (PIMC), can have $t<0$ near the dislocation! Solve linearized Landau equation.





Edge dislocation

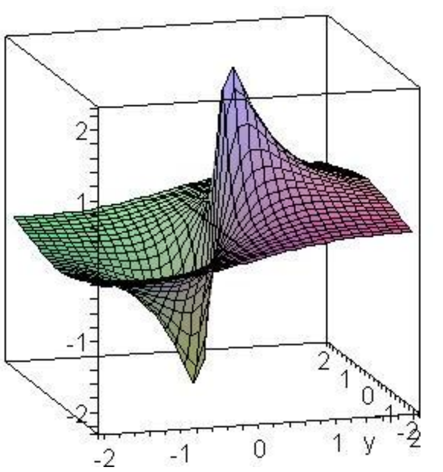


Screw dislocation

Details: 2d quantum dipole

Schrodinger equation for edge dislocation:

$$-\frac{\hbar^2}{2m_3}\nabla^2\psi + p\frac{\cos\theta}{r}\psi = E\psi$$



- Numerical methods (see K. Dasbiswas *et al*., PRB 2010)

$$E_0 = -0.28 m_3 p^2/\hbar^2 = -0.28\frac{m_3}{\hbar^2}\left(\frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\delta V\right)^2 \simeq -1\ \mathrm{K}$$

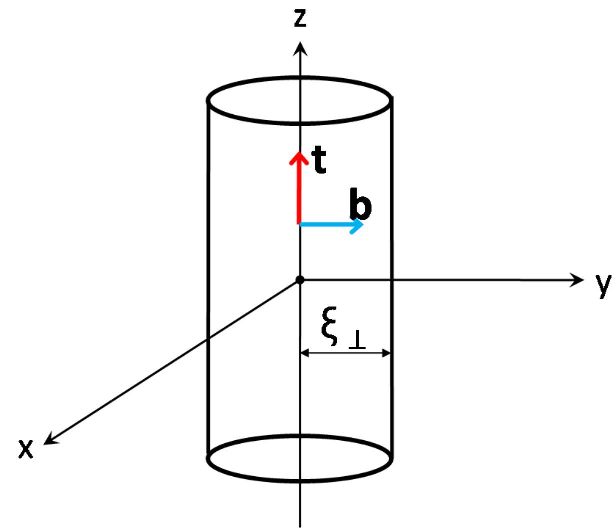
- What about screw dislocations? Need nonlinear strains, $U(r) \sim 1/r^2$. See recent Corboz et al., PRL 2008; binding energy $-E_0 = 0.8 \pm 0.1\ \mathrm{K}$.

Reduction to a 1D model

- Integrate out gapped transverse degrees of freedom in time-dependent Landau theory

$$\frac{\partial \psi}{\partial t} = \bar{\epsilon}\psi + \hat{L}\psi + \partial_z{}^2\psi - \epsilon|\psi|^2\psi + \zeta$$

$$\hat{L} \equiv \nabla_T^2 - u(x,y) + \epsilon_0$$



- Threshold *Ansatz:*

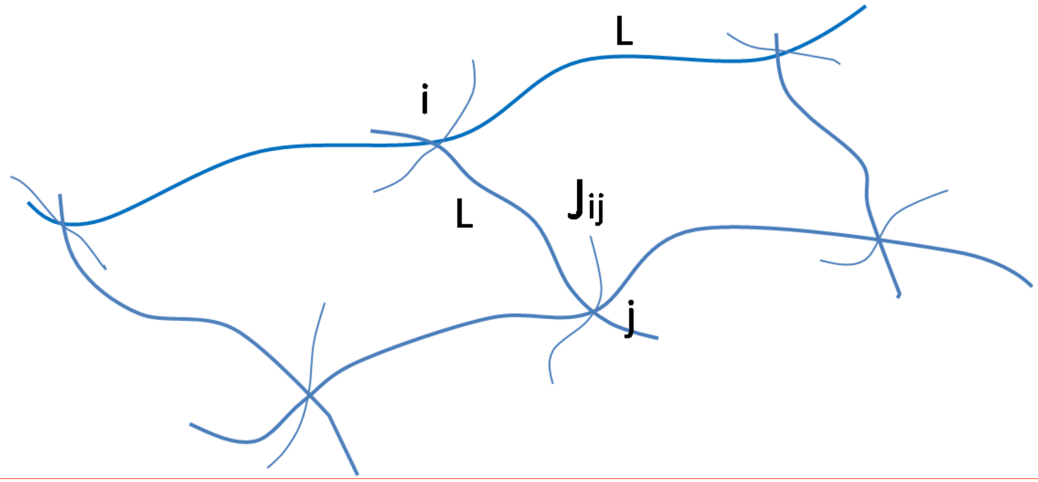
$$\psi(x,y,z,t) = A(z,t)\phi_0(x,y)$$

- Amplitude equation:

$$\frac{\partial A}{\partial t} = \bar{\epsilon}A + \partial_z^2 A - g|A|^2 A + \zeta_1 \qquad g = \epsilon \int dxdy|\phi_0|^4$$

Network model

- 1D superfluid order along a single dislocation, can overlap with neighboring dislocations
- Network of dislocations in solid ^{4}He bulk superfluid order (Shevchenko 1988, Toner 2008)



Derivation of the network model

- Superfluid coupling between sites i, j
- Integrate over fluctuations along 1D
- Effective coupling between neighboring sites obtained from a "propagator":

$$K(\psi_i, \psi_j; L) = \int D\psi D\psi^* \exp\left\{ -\frac{\beta}{2} \int_0^L dz \left[a|\psi|^2 + \frac{1}{2} b|\psi|^4 + c\left|\frac{d\psi}{dz}\right|^2 \right] \right\}$$

- Effective XY model:

$$H(\psi_i, \psi_j; L) = J_{ij}(L)\cos(\theta_i - \theta_j) \qquad J_{ij}(L) = \frac{c|\psi_i||\psi_j|}{\xi \sinh(L/\xi)}$$

- Exponential dependence of coupling on separation between nodes.

Where were we…

- A dislocation in a Bose solid may have a superfluid core (Landau theory, PIMC).
- Dislocation motion leads to shear softening, dislocation pinning to stiffening.
- Dislocations bind solute atoms (e.g., ^3He impurities)—could lead to specific heat anomalies.
- Perhaps the NCRI and shear stiffening are correlated—superfluid dislocations?

Summary

- New experiments have revealed a novel phenomena in solid ^{4}He that is likely connected to the superfluid behavior of extended defects (low dimensional superfluidity).
- This superfluidity is coupled to changes in the elastic properties, due to defect motion. Impurities can pin the defects and enhance the superfluid response.
- Dislocation motion + BEC = "quantum metallurgy"?

The three amigos

Debajit, Kinjal and Chi-Deuk

Torsional oscillator: rigid body

- Equation of motion for a rigid solid:

$$\left[(I_{\mathrm{cell}} + I_{\mathrm{He}}) \frac{d^2}{dt^2} + \gamma \frac{d}{dt} + \alpha \right] \theta(t) = \tau_{\mathrm{ext}}(t)$$

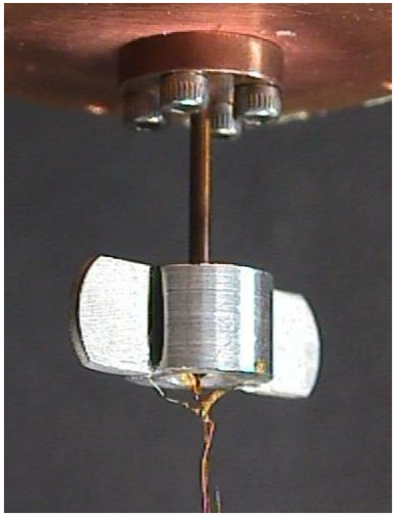
- Resonant period:

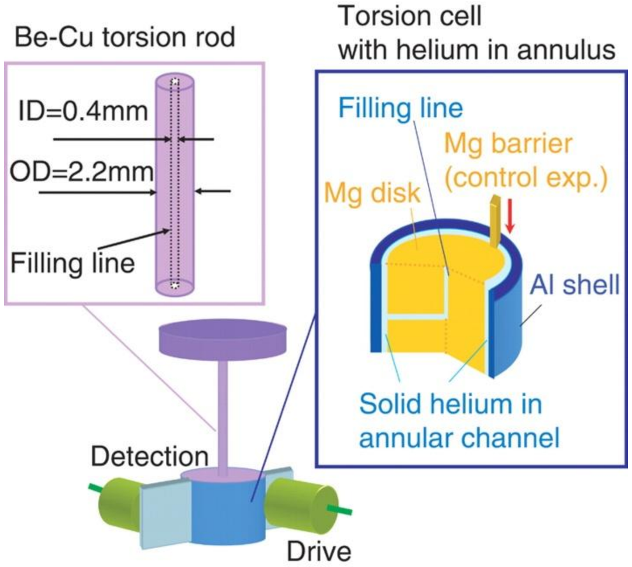
$$P = P_0 \left(1 + \frac{1}{8Q^2} + \ldots \right), \quad Q = \frac{\sqrt{\alpha I_{\mathrm{total}}}}{\gamma} = \mathcal{O}(10^6)$$

$$P_0 = 2\pi \sqrt{\frac{I_{\mathrm{cell}} + I_{\mathrm{He}}}{\alpha}}$$

$$\frac{\Delta P}{P_0} = \frac{1}{2} \frac{\Delta I_{\mathrm{tot}}}{I_{\mathrm{tot}}} = \mathcal{O}(10^{-5})$$

- What happens if the solid ^{4}He is not rigid?





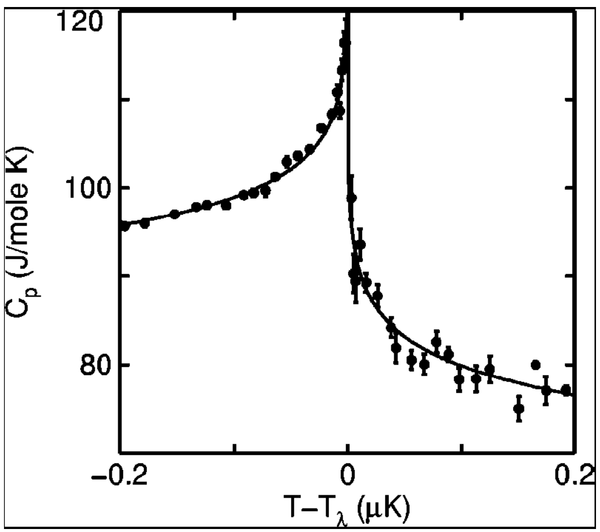
Specific heat near the λ transition

- The singular part of the specific heat is a correlation function:

$$S = -\partial F/\partial T \propto -\partial F/\partial a(T) = \int_{\mathbf{x}} \langle |\psi(\mathbf{x})|^2 \rangle$$

$$C = T(\partial S/\partial T) \propto \int_{\mathbf{x}} \int_{\mathbf{x}'} \langle |\psi(\mathbf{x})|^2 |\psi(\mathbf{x}')|^2 \rangle \sim A_{\pm} |t|^{-\alpha}$$

- For the λ transition, $\alpha = -0.0127$.



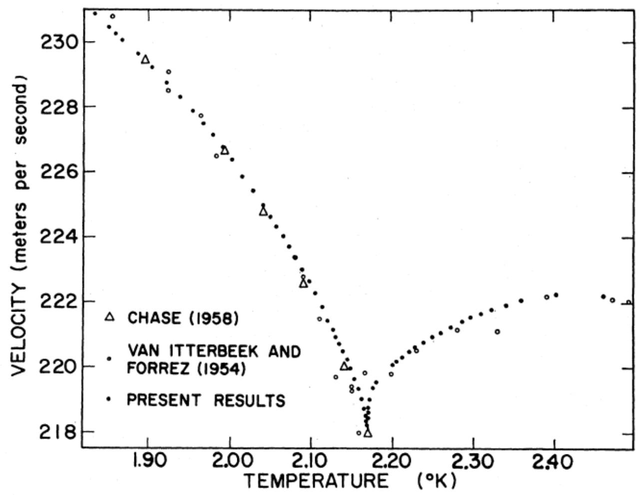


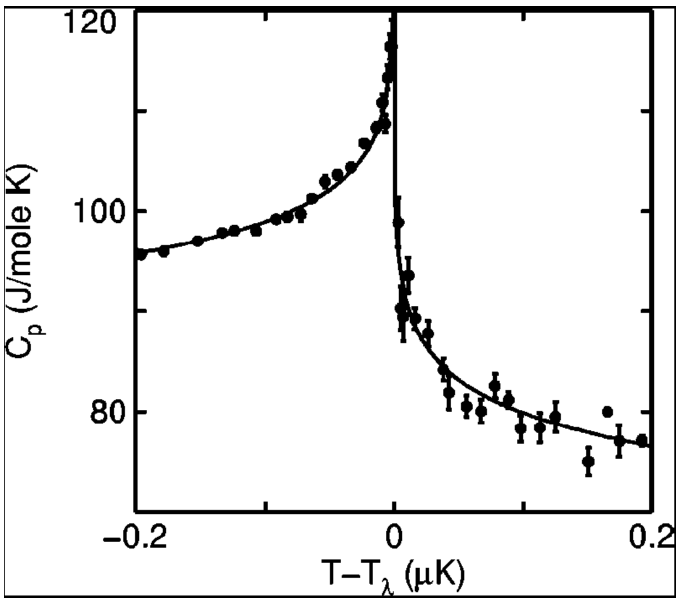
Fig. 7. Temperature dependence of the sound velocity in an extended region about the λ point.

Lipa et al., Phys. Rev. B (2003).

Barmatz & Rudnick, Phys. Rev. (1968)

Specific heat I

High resolution specific heat measurements of the lambda transition in zero gravity.



J.A. Lipa et al.,
Phys. Rev. B **68**, 174518 (2003).

Specific heat near the putative supersolid transition in solid ^{4}He.

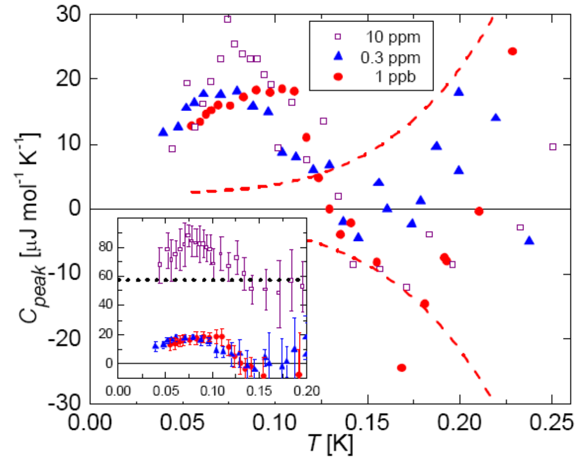
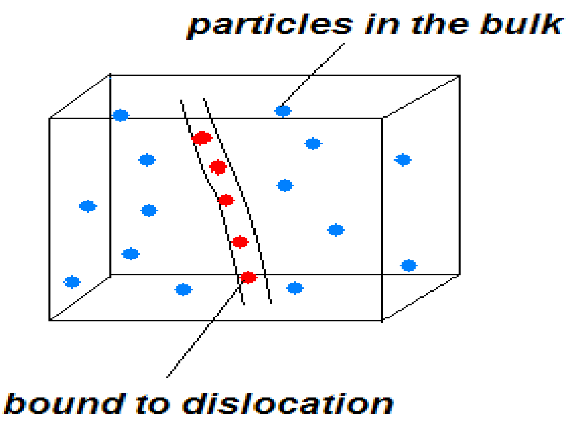


Figure 5 Specific heat peak of the 1 ppb, 0.3 ppm, and 10 ppm samples. The x_3-independent peak centred around 75 mK is revealed when the phonon contribution is subtracted. The red dashed lines indicate the uncertainty in the 1 ppb data. The uncertainty for x_3 = 0.3 ppm is comparable. For x_3 = 10 ppm, it is similar above 200 mK but decreases more gradually with decreasing temperature. For T < 100 mK the uncertainty is four times larger than that of the 1 ppb sample. The inset compares the specific heat of the three samples without the subtraction of the impuriton term of the 10 ppm sample (dotted line, 59 μJ mol^{-1} K^{-1}).

Lin, Clark, and Chan,
Nature (2007)

An alternative: lattice gas model

- Edge dislocations in ^{4}He provide an attractive potential for ^{3}He impurities.
- Bound ^{3}He impurities “evaporate” from the dislocations, increasing entropy and producing a bump in the specific heat.
- Divide the impurities into bound and free; two systems are in chemical equilibrium.
- Treat both systems classically.
- See T. N. Antsygina et al., Low Temp. Phys. 21, 453 (1995).

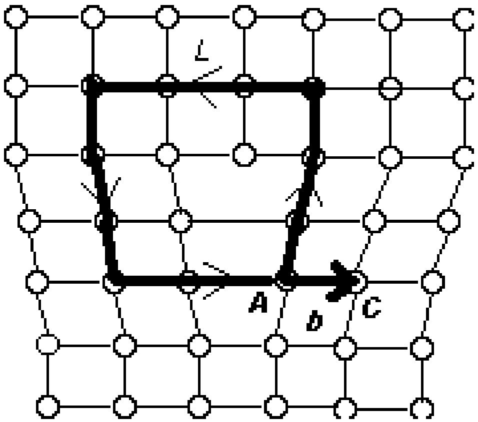


Binding of ^{3}He to dislocations

- Hydrostatic pressure due to an edge dislocation (continuum theory):
  $p = -\frac{1}{3}\sigma_{ii} = \frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\frac{\sin\theta}{r}$
- Effective potential due to a volume defect δV (Cottrell "atmosphere"):

$$U(r,\theta) = p\delta V = U_0 \frac{\sin\theta}{r}, \quad U_0 = \frac{\mu b}{3\pi}\frac{1+\nu}{1-\nu}\delta V$$

- Breaks down in the core due to diverging strains; need a cut off. The cut off will reduce the binding energy.



Specific heat II: some details

- N ^{3}He impurities, M defect sites that bind the impurities with energy ε .
- The defect sites have 0 or 1 ^{3}He impurities (two level system). Ignore correlations among sites and quantum statistics.
- Assume particles that have evaporated form a noninteracting gas.

$$N = \langle N_{\mathrm{gas}} \rangle + M \langle n_{\mathrm{site}} \rangle, \quad \langle n_{\mathrm{site}} \rangle = \left[1 + e^{-\beta(\epsilon+\mu)} \right]^{-1}$$

- Equate chemical potentials of the gas and the adsorbed particles:

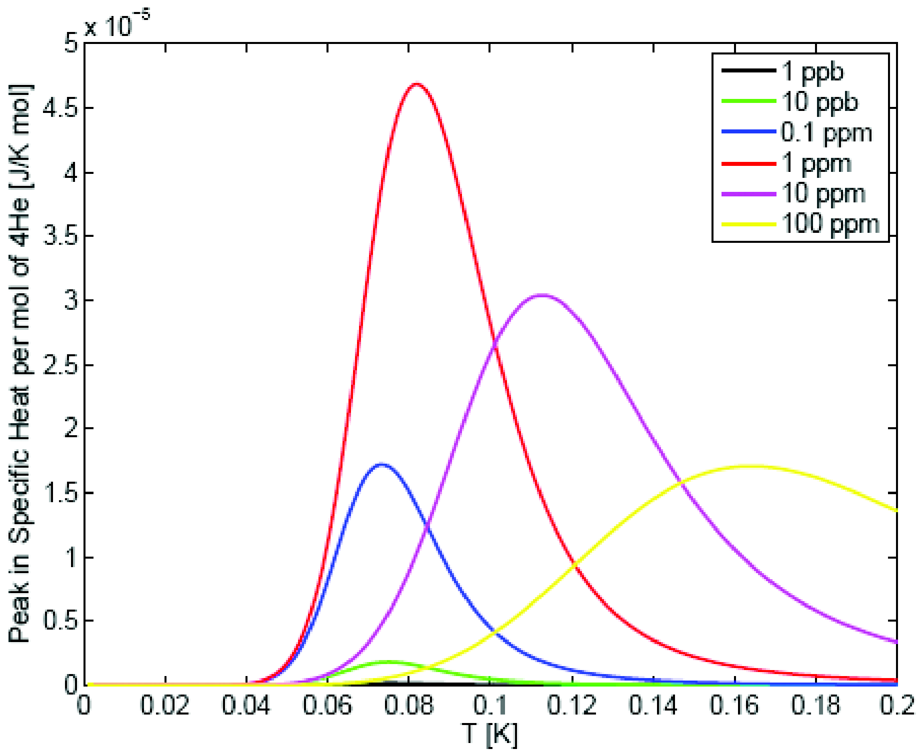
$$\langle N_{\mathrm{gas}} \rangle = \frac{1}{2} \left[N - M - p(T) + \sqrt{(N - M - p(T))^2 + 4Np(T)} \right]$$

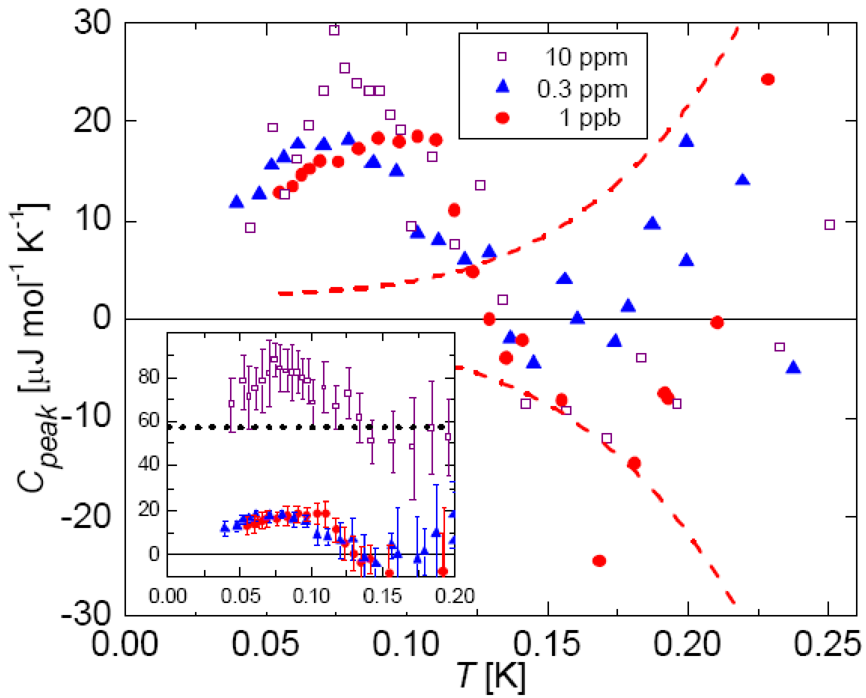
$$p(T) = V e^{-\epsilon/T} \left(\frac{2\pi m_3 T}{h^2} \right)^{3/2}$$

Properties

- Calculate the molar specific heat at constant N; complicated expression. Roughly, there is a background piece (from the gas particles) and a bump (Schottky anomaly) from the adsorbates.
- Two limits:
  - N>M ("saturated" case): size of the bump scales as M.
  - N<M ("unsaturated" case): size of bump scales with N.
- Peak appears at a temperature T^* such that $\epsilon + \mu(T^*) = T^*$. The peak position depends on the ^{3}He concentration (weakly).

Sample comparison with data





ε=0.6K ; defect site concentration= 0.3 ppm

Lin *et al*., Nature **449**, 1025 (2007)

Hydrodynamics I: simple fluid

- Conservation laws and broken symmetries lead to long-lived "hydrodynamic" modes (lifetime diverges at long wavelengths).
- Simple fluid:
  - Conserved quantities are ρ, g_i, e.

$$\partial_t \rho + \partial_i g_i = 0 \qquad \text{(conservation of mass)},$$

$$\partial_t g_i + \partial_j \sigma_{ij} = 0 \qquad \text{(conservation of momentum)},$$

$$\partial_t e + \partial_i J_i^Q = 0 \qquad \text{(conservation of energy)}.$$

  - No broken symmetries.
  - 5 conserved densities $\Rightarrow$ 5 hydrodynamic modes.
    - 2 transverse momentum diffusion modes $\omega = -iD_t q^2$.
    - 1 longitudinal thermal diffusion mode $\omega = -iD_T q^2$
    - 2 longitudinal sound modes . $\omega = \pm c_1 q$

Light scattering from a simple fluid

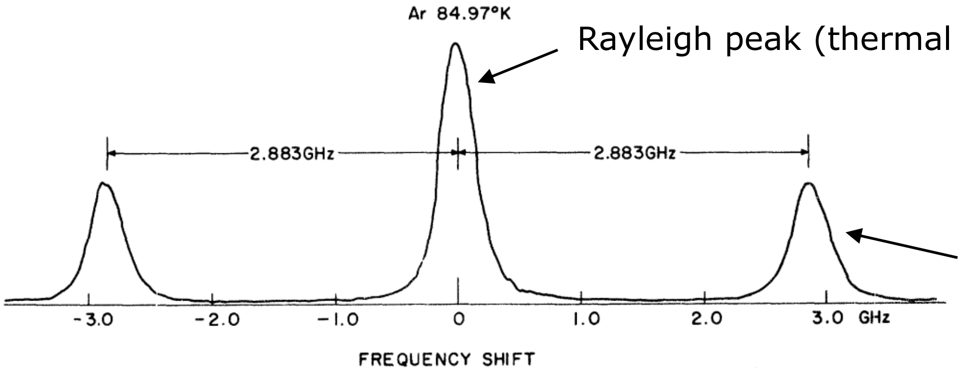


FIG. 3. Brillouin spectrum of liquid argon, $T = 84.97\,^\circ\mathrm{K}$, $\theta = 90°\ 14'$, laser wavelength is 5145 Å.

P. A. Fleury and J. P. Boon, Phys. Rev. **186**, 244 (1969)

- Intensity of scattered light:

$$I(\mathbf{q},\omega) \propto S(\mathbf{q},\omega) \qquad S(\mathbf{q},\omega) = \langle \delta\rho(\mathbf{q},\omega)\delta\rho(-\mathbf{q},-\omega)\rangle$$

- Longitudinal modes couple to density fluctuations.
  - Sound produces the Brillouin peaks.
  - Thermal diffusion produces the Rayleigh peak (coupling of thermal fluctuations to the density through thermal expansion).

Hydrodynamics II: superfluid

- Conserved densities ρ, g_i, e .
- Broken U(1) gauge symmetry

$$\psi(\mathbf{r}) = |\psi(\mathbf{r})| e^{i\theta(\mathbf{r})}, \qquad \mathbf{v}_s = \frac{\hbar}{m}\nabla\theta.$$

- Another equation of motion:

$$\partial_t \theta = \mu/\hbar \qquad \text{Josephson relation.}$$

- 6 hydrodynamic modes:
  - 2 transverse momentum diffusion modes.
  - 2 longitudinal (first) sound modes.
  - 2 longitudinal second sound modes.
- Central Rayleigh peak splits into two new Brillouin peaks.

Light scattering in a superfluid

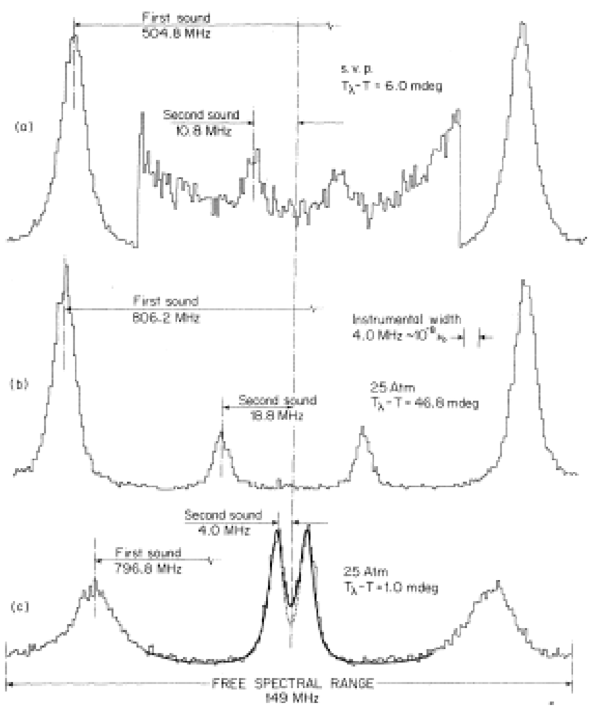


FIG. 1. Brillouin spectra showing first and second sound in superfluid He^4. In the central region of the upper trace (a) the gain is increased by a factor of 10. The counting rate in the second sound in (a) is about 3 Hz.

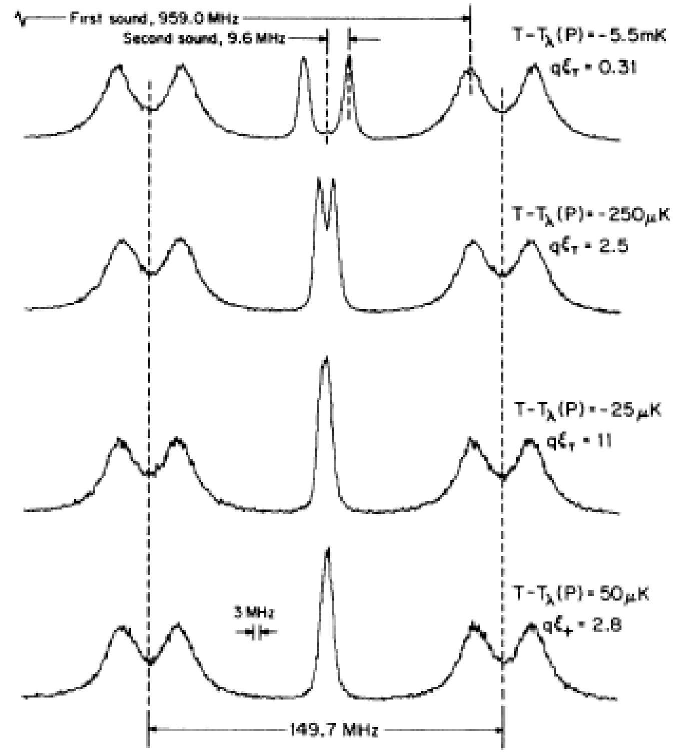


FIG. 6. Four Brillouin spectra taken at $P_\lambda = 23.1$ bars near the λ transition of liquid ^4He. In the top spectrum the frequency shifts of first and second sound are marked. For second sound, zero-frequency shift is at the center of the marked free spectral range (149.7 MHz). The instrumental linewidth (3 MHz) is also shown.

Winterling, Holmes & Greytak PRL 1973

Tarvin, Vidal & Greytak 1977

Solid "hydrodynamics"

- Conserved quantities: ρ, g_i, e .
- Broken translation symmetry: u_i, i=1,2,3
- Mode counting: 5 conserved densities and 3 broken symmetry variables$\Rightarrow$ 8 hydrodynamic modes. For an isotropic solid (two Lame constants λ and μ):
  - 2 pairs of transverse sound modes (4),
  - 1 pair of longitudinal sound modes (2),
  - 1 thermal diffusion mode (1).
- What's missing? Martin, Parodi, and Pershan (1972): diffusion of vacancies and interstitials.

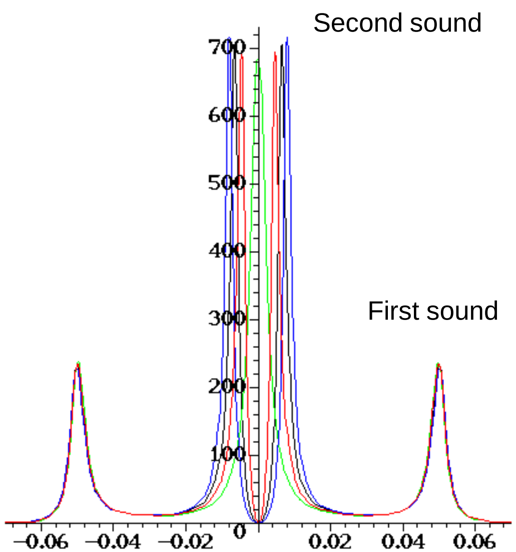
Supersolid hydrodynamics

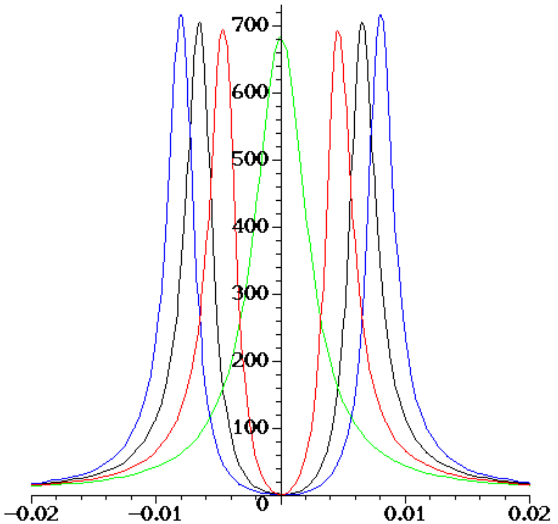
- Conserved quantities: ρ, g_i, e
- Broken symmetries: u_i, gauge symmetry.
- Mode counting: 5 conserved densities and 4 broken symmetry variables $\Rightarrow$ 9 hydrodynamic modes.
  - 2 pairs of transverse sound modes (4).
  - 1 pair of longitudinal sound modes (2).
  - 1 pair of longitudinal “fourth sound” modes (2).
  - 1 longitudinal thermal diffusion mode.
- Use Andreev & Lifshitz hydrodynamics to derive the structure function (isothermal, isotropic solid). New Brillouin peaks below T_c.

Structure function for supersolid

$$\frac{c_L^4}{D_\Delta \rho_0}\frac{\beta S(\tilde{\mathbf{q}},\tilde{\omega})}{2} \simeq \frac{\left[\delta+\Omega\left(2-\delta\right)\right]\tilde{q}^4}{\left[\tilde{\omega}^2-\left(1+\Omega\right)\tilde{q}^2\right]^2+\left[\left(\delta+2\Omega\right)\tilde{q}^2\tilde{\omega}\right]^2}+\frac{\delta^2\Omega\tilde{q}^4}{\left(\tilde{\omega}^2-\delta\Omega\tilde{q}^2\right)^2+\left[\left(1-2\Omega\right)\tilde{q}^2\tilde{\omega}\right]^2}$$
$$+\frac{\tilde{q}^2\left[\delta+2\Omega\left(2-\delta\right)\right]\left(\tilde{\omega}^2-\delta\Omega\tilde{q}^2\right)}{\left(\tilde{\omega}^2-\delta\Omega\tilde{q}^2\right)^2+\left[\left(1-2\Omega\right)\tilde{q}^2\tilde{\omega}\right]^2},$$

$$\tilde{\omega}=D_\Delta\omega/c_L^2,\ \tilde{q}=D_\Delta q/c_L,\ \Omega=\rho_{s0}/\rho_0,\ \delta=1/(\rho_0\chi c_L^2).$$





Supersolid Lagrangian

- Lagrangian
  - Reversible dynamics for the phase and lattice displacement fields
  - Lagrangian coordinates R_i, Eulerian coordinates x_i, deformation tensor $R_{ij} = \partial R_j / \partial x_i$
  - Respect symmetries (conservation laws): rotational symmetry, Galilean invariance, gauge symmetry.

$$\mathcal{L}_{SS} = -\rho \partial_t \phi - \frac{1}{2}\rho_s (\nabla\phi)^2 + \frac{1}{2}\rho_n v_n^2 - f(\rho, \rho_s, T, R_{ij}) - \rho_n \mathbf{v}_n \cdot \nabla\phi$$

$$\rho = \rho_n + \rho_s, \qquad v_{ni} = -R_{ij}^{-1} \partial_t R_j$$

- Reproduces Andreev-Lifshitz hydrodynamics. Agrees with recent work by Son (2005) [disagrees with Josserand (2007), Ye (2007)].
- Good starting point for studying vortex dynamics in supersolids (Yoo and Dorsey, unpublished). Question: do vortices in supersolids behave differently than in superfluids?

Effective Euclidean Action I

- We start from the Lagrangian density of non-dissipative Andreev and Lifshitz hydrodynamics (Yoo & ATD, PRB 2010)
  - For simplicity, consider 2-dim isotropic supersolids
  - Expand up to quadratic order and integrate out the density fluctuations

$$
\begin{aligned}
S_{\mathrm{E}}[\phi,\mathbf{u}] \;=\; \frac{1}{2}\int d\tau \int d^2x \Big[& \underbrace{2i\rho\partial_\tau\phi + 2\lambda_{ij}u_{ij}}_{\text{linear terms}} + \underbrace{\rho^2\chi(\partial_\tau\phi)^2 + \rho_{\mathrm{s}}(\partial_i\phi)^2}_{\text{superfluid}} \\
& + \underbrace{\rho_{\mathrm{n}}(\partial_\tau u_i)^2 + (\tilde{\lambda} - \rho^2\gamma^2\chi)u_{ii}^2 + 2\tilde{\mu}u_{ij}^2}_{\text{lattice}} \\
& + \underbrace{i(\rho_{\mathrm{n}} - \rho^2\gamma\chi)(\partial_\tau u_i\partial_i\phi + u_{ii}\partial_\tau\phi)}_{\text{coupling}} \Big]
\end{aligned}
$$

Effective Euclidean Action II

- Incorporate vortices and edge dislocations

$$\phi^{\mathrm{V}} = \frac{\hbar}{m} \sum_{\alpha} \mathrm{e}^{\alpha} \arctan \left[\frac{y - y_{\mathrm{V}}^{\alpha}(\tau)}{x - x_{\mathrm{V}}^{\alpha}(\tau)} \right], \qquad u_i^{\mathrm{D}} = \sum_{\bar{\alpha}} \frac{b_i^{\bar{\alpha}}}{2\pi} \arctan \left[\frac{y - y_{\mathrm{D}}^{\bar{\alpha}}(\tau)}{x - x_{\mathrm{D}}^{\bar{\alpha}}(\tau)} \right]$$

- Integrate out the degrees of freedom from the superfluid ϕ^{S} and the lattice $\mathbf{u}^{\mathrm{S}}$

$$S_{\mathrm{E}}[\mathbf{x}_{\mathrm{V}}(\tau), \mathbf{x}_{\mathrm{D}}(\tau)] = S_{\mathrm{vortex}}[\mathbf{x}_{\mathrm{V}}(\tau)] + S_{\mathrm{dislocation}}[\mathbf{x}_{\mathrm{D}}(\tau)] + S_{\mathrm{coupling}}[\mathbf{x}_{\mathrm{V}}(\tau), \mathbf{x}_{\mathrm{D}}(\tau)]$$

Vortex Dynamics I

- Action for vortices: $S_{\text{vortex}}[\mathbf{x}_{\text{V}}(\tau)] = S_{\text{Magnus}}[\mathbf{x}_{\text{V}}(\tau)] + S_{\text{V}}[\mathbf{x}_{\text{V}}(\tau)]$

$$S_{\text{Magnus}} = -\frac{2i\pi\hbar\rho}{m}\sum_{\alpha} \mathrm{e}^{\alpha}\int d\tau \frac{dx_{\text{V}}^{\alpha}(\tau)}{d\tau} y_{\text{V}}^{\alpha}(\tau)$$

$$\begin{aligned} S_{\text{V}} = & -\underbrace{\frac{\pi\hbar^2(\rho_{\text{n}}-\rho^2\chi\gamma)^2}{8m^2\rho_{\text{n}}c_T}\sum_{\alpha\beta}\mathrm{e}^{\alpha}\mathrm{e}^{\beta}\int d\tau\int d\tau' \frac{c_T^2}{\sqrt{c_T^2(\tau-\tau')^2+|\mathbf{x}_{\text{V}}^{\alpha}(\tau)-\mathbf{x}_{\text{V}}^{\beta}(\tau')|^2}}}_{\text{coupling to the transverse sound modes}} \\ & + \underbrace{\frac{\pi\hbar^2 A_L}{2m^2\rho_{\text{n}}c_L}\sum_{\alpha\beta}\mathrm{e}^{\alpha}\mathrm{e}^{\beta}\int d\tau\int d\tau' \frac{s_i^{\alpha}(\tau)s_i^{\beta}(\tau')+c_L^2}{\sqrt{c_L^2(\tau-\tau')^2+|\mathbf{x}_{\text{V}}^{\alpha}(\tau)-\mathbf{x}_{\text{V}}^{\beta}(\tau')|^2}}}_{\text{coupling to the longitudinal first sound modes}} \\ & + \underbrace{\frac{\pi\hbar^2 A_2}{2m^2\rho_{\text{n}}c_2}\sum_{\alpha\beta}\mathrm{e}^{\alpha}\mathrm{e}^{\beta}\int d\tau\int d\tau' \frac{s_i^{\alpha}(\tau)s_i^{\beta}(\tau')+c_2^2}{\sqrt{c_2^2(\tau-\tau')^2+|\mathbf{x}_{\text{V}}^{\alpha}(\tau)-\mathbf{x}_{\text{V}}^{\beta}(\tau')|^2}}}_{\text{Eckern-Schimd action (1989) for vortices in superfluid}}, \end{aligned}$$

- Similarly, action for edge dislocations

$$S_{\text{dislocation}}[\mathbf{x}_{\text{D}}(\tau)] = S_{\text{P-K}}[\mathbf{x}_{\text{D}}(\tau)] + S_{\text{D}}[\mathbf{x}_{\text{D}}(\tau)]$$

Vortex Dynamics II

- For small frequencies $c_{T,L,2}^2(\tau-\tau')^2 \gg |\mathbf{x}_{\rm V}^{\alpha}(\tau)-\mathbf{x}_{\rm V}^{\beta}(\tau')|^2$

$$S_{\rm vortex} \simeq S_{\rm static} + \int \frac{d\omega}{2\pi}\left[-2\pi\rho\omega x_{\rm V}(\omega) y_{\rm V}(-\omega) + \frac{1}{2} M_{\rm V}(\omega)\omega^2 |\mathbf{x}(\omega)|^2\right]$$

$$M_{\rm V}(\omega) = \underbrace{M_{c_T}(\omega) + M_{c_L}(\omega)}_{\text{due to coupling with elasticity}} + M_{c_2}(\omega)$$

$$M_{\rm V}(\omega) \simeq \frac{3\pi\hbar^2(\rho_{\rm n}-\rho^2\chi\gamma)^2}{4m^2\rho_{\rm n}c_T^2} - \frac{\pi\hbar^2}{m^2}\left[\rho^2\chi + \frac{1}{4}(\rho_{\rm n}-\rho^2\chi\gamma)^2\left(\frac{1}{\tilde{\mu}} + \frac{1}{\lambda-\rho^2\chi\gamma^2}\right)\right]\left[\tilde{\gamma} + \frac{3}{2} + \ln(|\omega|\epsilon)\right]$$

- Similarly, for dislocations

$$\begin{aligned} M_{ij}^{\rm D} \simeq\; & 4\pi^3 \ln(|\omega|\epsilon)\left\{\frac{2b_ib_j}{\rho_{\rm s}(\tilde{\Lambda}+2\tilde{\mu})^2}\left[\rho_{\rm s}\rho_{\rm n}(\tilde{\Lambda}+\tilde{\mu})^2 + \frac{(\rho_{\rm n}-\rho^2\chi\gamma)^2}{4}\tilde{\Lambda}^2\right]\right. \\ & \left. -\delta_{ij}b_kb_k\left[2\rho_{\rm n} + \frac{\rho_{\rm n}(\tilde{\Lambda}^2-\tilde{\mu}^2)}{(\tilde{\Lambda}+2\tilde{\mu})^2} + \frac{(\rho_{\rm n}-\rho^2\chi\gamma)^2}{2\rho_{\rm s}}\frac{\tilde{\Lambda}^2+2\tilde{\mu}^2}{(\tilde{\Lambda}+2\tilde{\mu})^2}\right]\right\} \end{aligned}$$

Summary

- Dissipation peak in the TO response is well described by a simple viscoelastic model. A long time scale is identified, probably from dislocation physics.
- The viscoelastic model only accounts for about 10% period shift. Is the rest of the “spectral weight” at zero frequency? Is there a superfluid response?
- Specific heat feature appears to have a natural interpretation as a Schottky anomaly due to evaporation of ^{3}He impurities from dislocations. Is the binding energy related to the Arrhenius behavior of the viscoelastic model?

Experiments (circa 1992)

Review: M. W. Meisel, Physica B **178**, 121 (1992).

Table 1
Summary of experimental searches for supersolid ^{4}He.

Group	Method	$^3He/^4He$ concentration	Temperature range	Pressure range	Conclusion
Andreev et al. (1969) [23]	Plastic flow	Unstated	$T_{min} = 500$ mK	25–154 bar	$\frac{\rho_s}{\rho} < 1 \times 10^{-3}$ $v_c < 2 \times 10^{-7}$ cm/s
Suzuki (1973) [24]	Plastic flow	Unstated	1.38–2.12 K	29–51 bar	$v_c < 10^{-4}$ cm/s
Tsymbalenko (1976) [25]	Plastic flow	Unstated	0.6–2.1 K	25–40 bar	$v_c < 5 \times 10^{-10}$ cm/s
Dyumin et al. (1989) [28]	Plastic flow	Unstated	1.0–1.4 K	On the melting curve	$v_c < 10^{-9}$ cm/s
Greywall (1977) [26]	ΔP across weak-link	Unstated	$T_{min} = 30$ mK	25–50 bar	$\frac{\rho_s}{\rho} v_c < 2.5 \times 10^{-9}$ cm/s
Bishop et al. (1981) [27]	Torsional oscillator	$0.3–411 \times 10^{-6}$	$T_{min} = 25$ mK	25–48 bar	$\frac{\rho_s}{\rho} < 5 \times 10^{-6}$ $v_c < 5 \times 10^{-8}$ cm/s
Bonfait et al. (1989) [29]	Cylindric "U"-tube	Standard commercial ^{4}He	$T_{min} = 4$ mK	On the melting curve	$\frac{\rho_s}{\rho} v_c < 1.8 \times 10^{-9}$ cm/s
Adams et al. (1990) [31]	$P_v(T)$	$\sim 10^{-6}$	$T_{min} = 1$ mK	26 bar	$\frac{\rho_s}{\rho} < 10^{-5}$
van de Haar et al. (1991) [32]	$P_v(T)$	$\sim 10^{-7}$	1.5–120 mK	On the melting curve and 26 bar	$\frac{\rho_s}{\rho} < 2 \times 10^{-5}$ at 26 bar; $< 6 \times 10^{-7}$ on melting curve
Lengua and Goodkind (1990) [30]	Ultrasound	1.5×10^{-9}	30 mK–1.0 K	~25 bar	$T_c = 117$ mK(?) $\frac{\rho_s}{\rho} \sim 10^{-3}$(?)

The initial attempts by Andreev et al. [23], Suzuki [24] and Tsymbalenko [25] and a recent effort by Dyumin et al. [28] to observe a supersolid ^{4}He state involved the study of plastic flow in crystals in which physical objects were moved. Later, techniques more reminiscent of superfluid helium studies were employed. Greywall [26] attempted to detect mass flow through a weak link which was subjected to a chemical potential difference between two mass reservoirs. Bishop et al. [27] used a sensitive torsional oscillator to search for a change of the moment of inertia of the system. Bonfait et al. [29] searched for mass flow in a cylindric "U"-tube experiment. Although some of the experiments [24, 25, 27, 28, 57] were able to study dislocations in the crystal and other effects, none of the investigations resulted in a positive identification of the supersolid state. The experimental conditions and results are summarized in table 1.

There are always a number of possible explanations for a null result; however, the most obvious explanation for the results of the aforementioned experiments involves the critical temperature, T_c, and the critical velocity, v_c. Either the work was not performed at temperatures below T_c, or the mass transport was being driven by a velocity field higher than v_c. Although Bonfait et al. [29] argue that their ex-

searches. As emphasized above, thermodynamic studies, which avoid problems associated with the critical velocity, are preferable to ones which are intrinsically sensitive to v_c. Finally, the experimentalists should prepare for the extension of the temperature range down to the submillikelvin region where new challenges in cooling the samples and in measuring the temperature of the system will be encountered.

Details

- Resonant period of oscillation is $\tau = 2\pi\sqrt{I/\alpha}$
- Changes in the period can be due to either I or α.
- Pressures ranged from 26 to 66 bars.
- Decoupling observed below 230 mK.
- Amplitude of about 1 nm, maximum velocities of about 10 μ m/s.
- Frequency: 1 kHz (period of 10^6 ns). Period shifts of order 40 ns. Q of order 10^6.
- Cell size: 10 mm OD, annulus 0.63 mm width, 5 mm height.
- Barrier inserted: no effect.
- 400 ppm of ^{3}He quenches effect.