RF spectroscopy, polarons, and dipolar interactions in cold gases

Georg M. Bruun
Department of Physics and Astronomy
University of Aarhus
Denmark

Outline

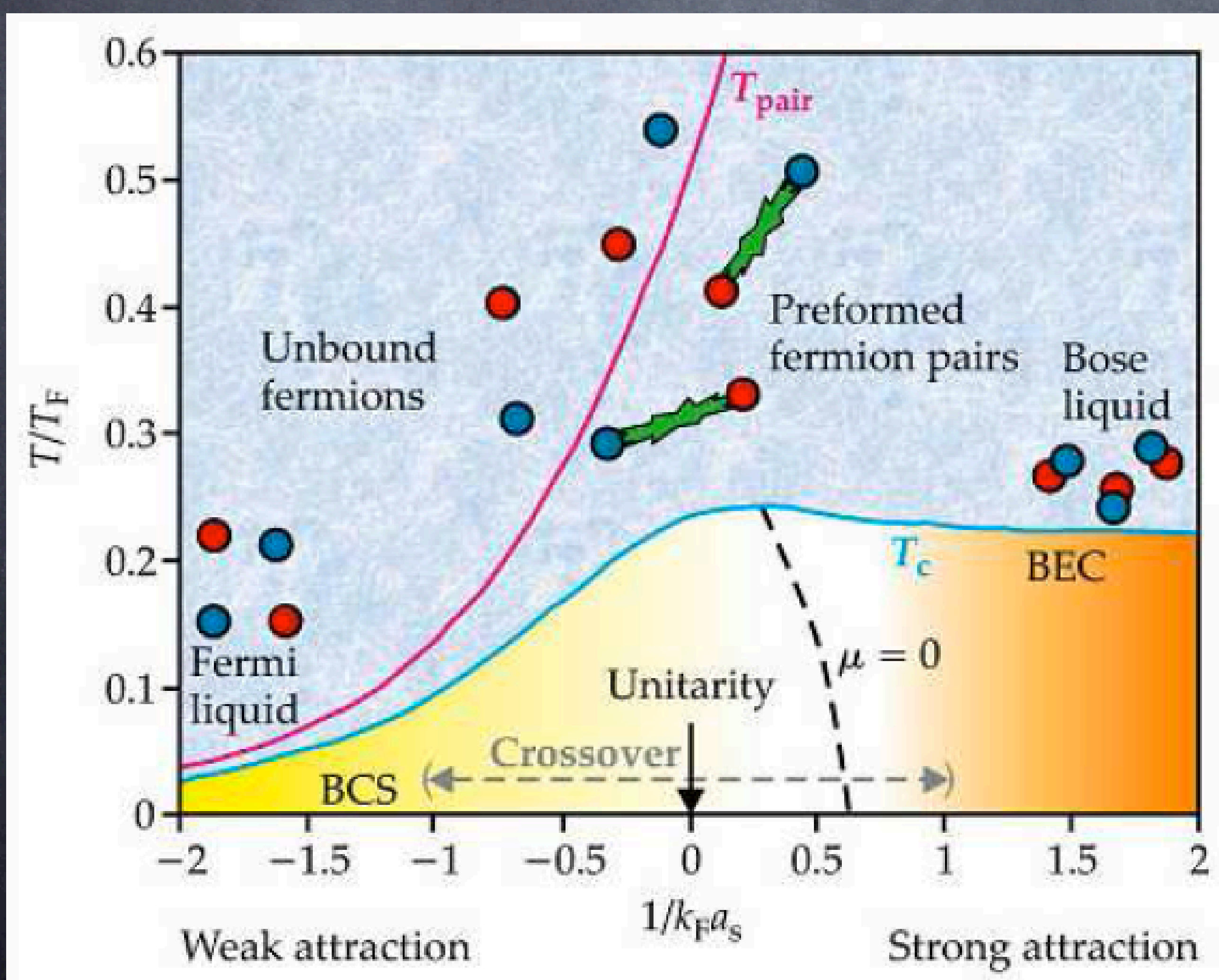
- RF spectroscopy
- Polaron and molecule coupling and decay
- Dipolar gases and p-wave pairing

Strongly Interacting Gases

Two-body collisions:

Interaction strength: a/d

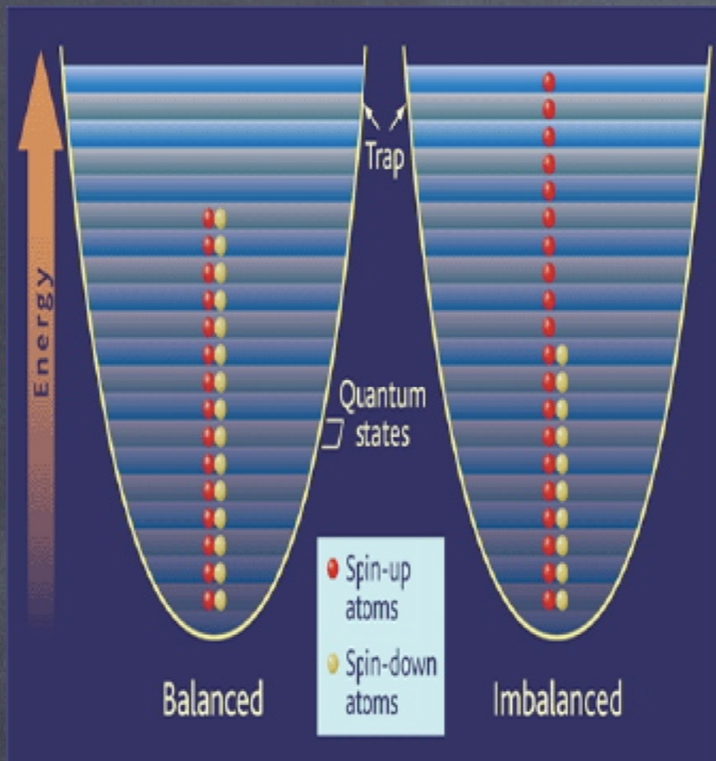
Phase Diagram

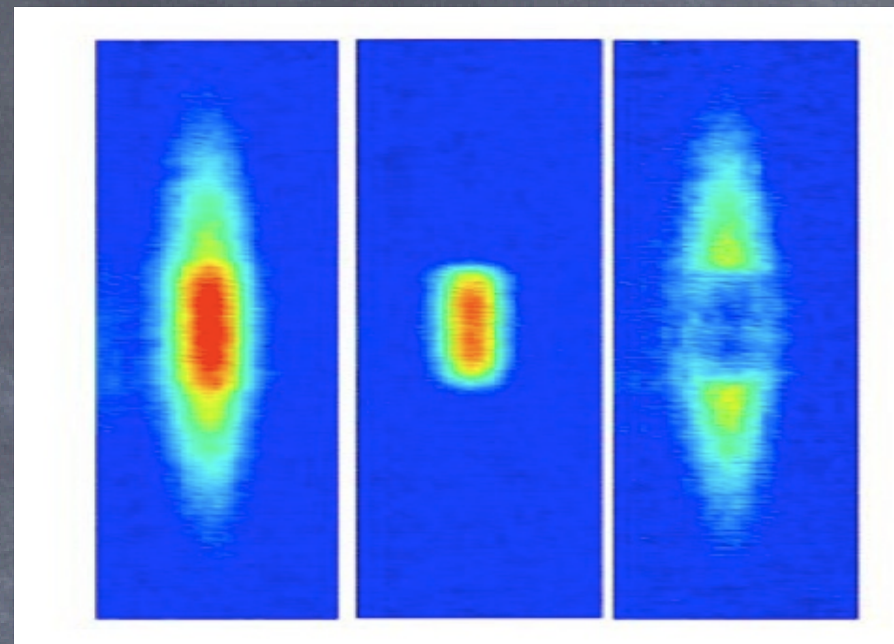


|a|»d: As strong coupling as QM allows

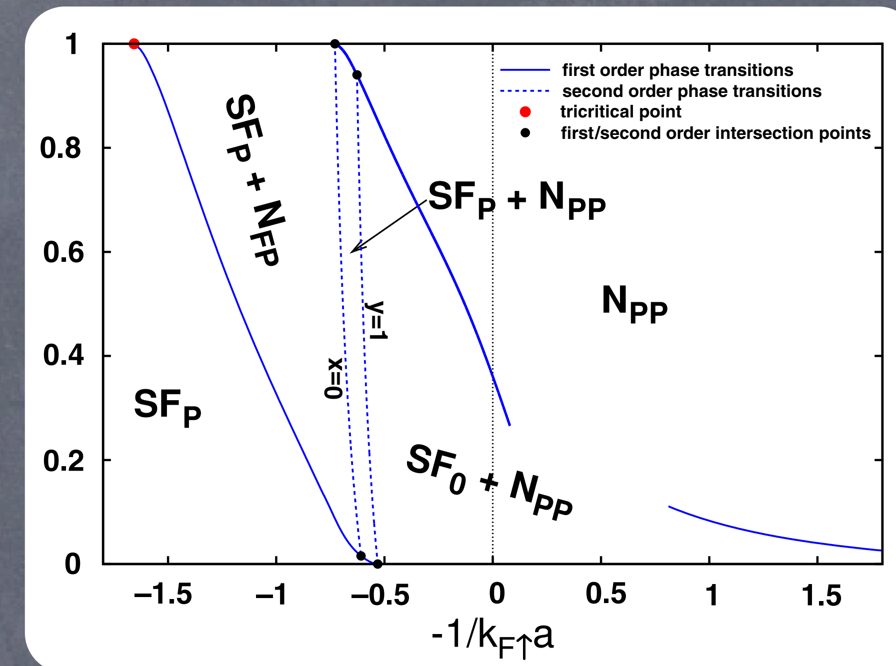
Scattering matrix $\mathcal{T} \sim \frac{a}{1 + iqa} \to \frac{1}{iq}$

Polarized Systems: $P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$

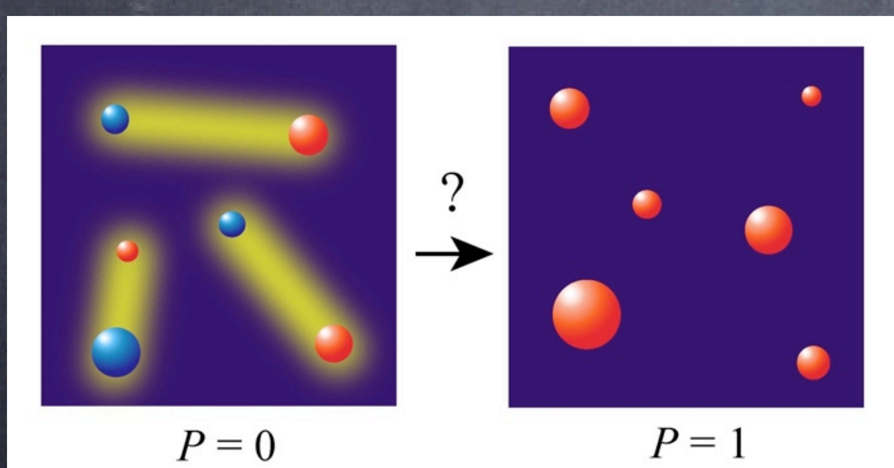


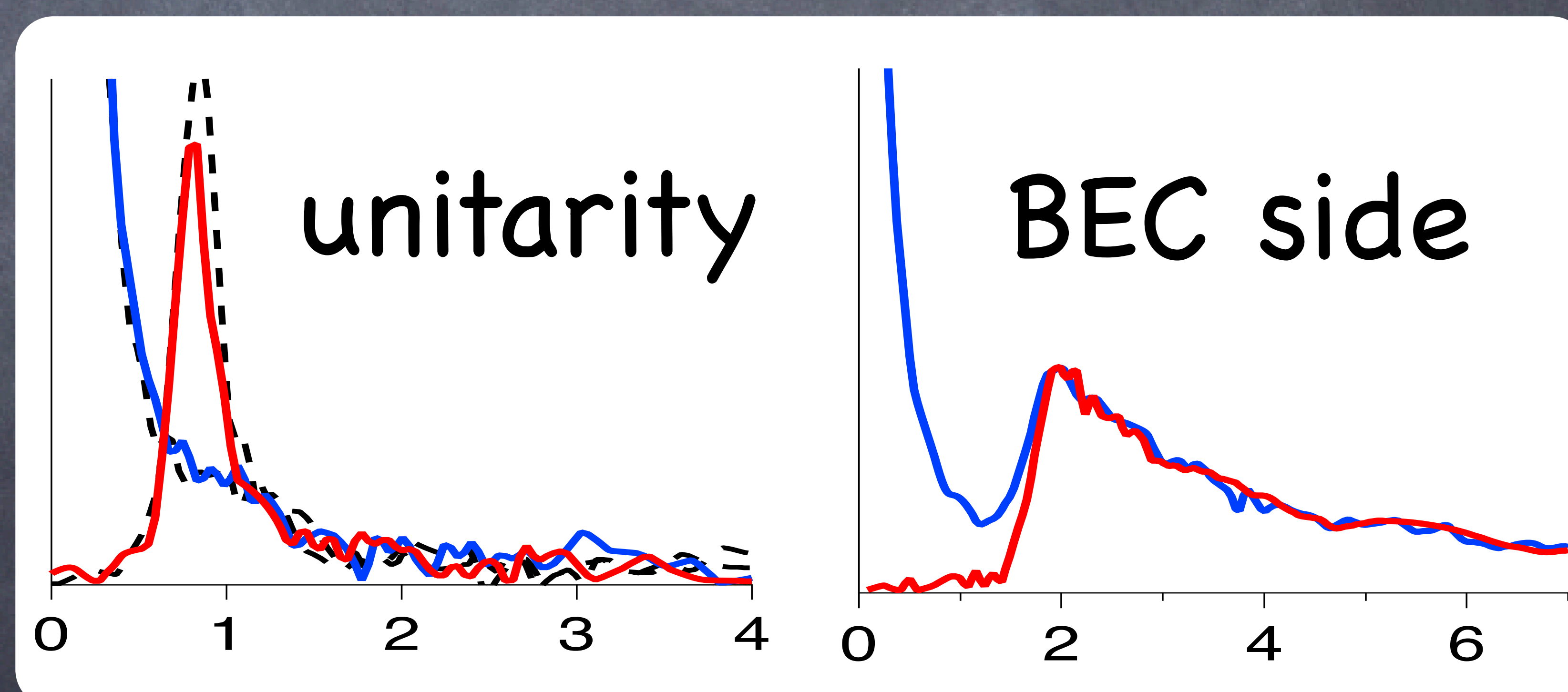


Hulet Group



Pilati & Georgini (2008)
Yip, Radzihovsky, ...

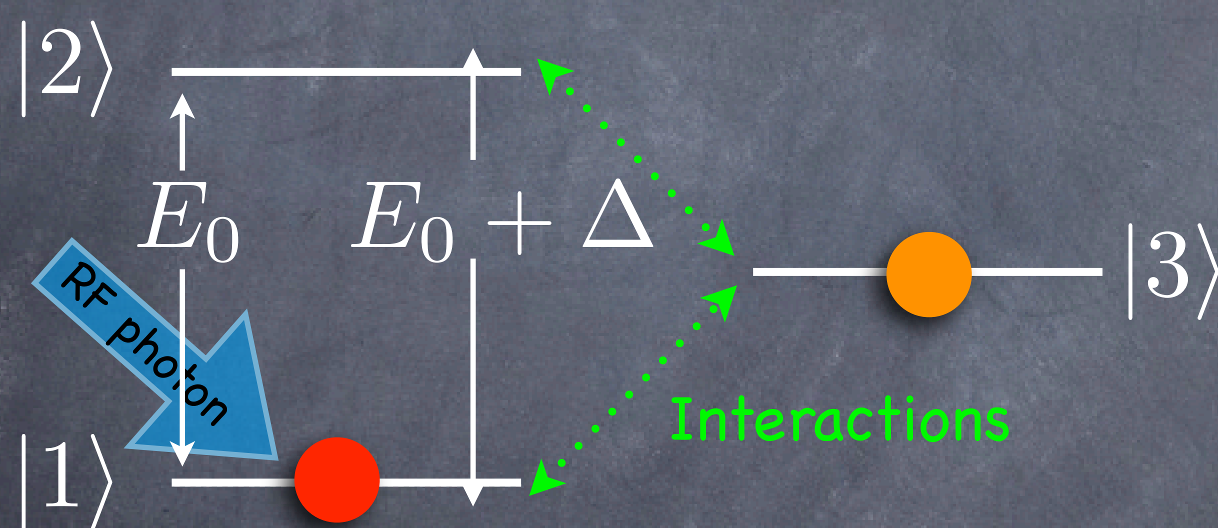


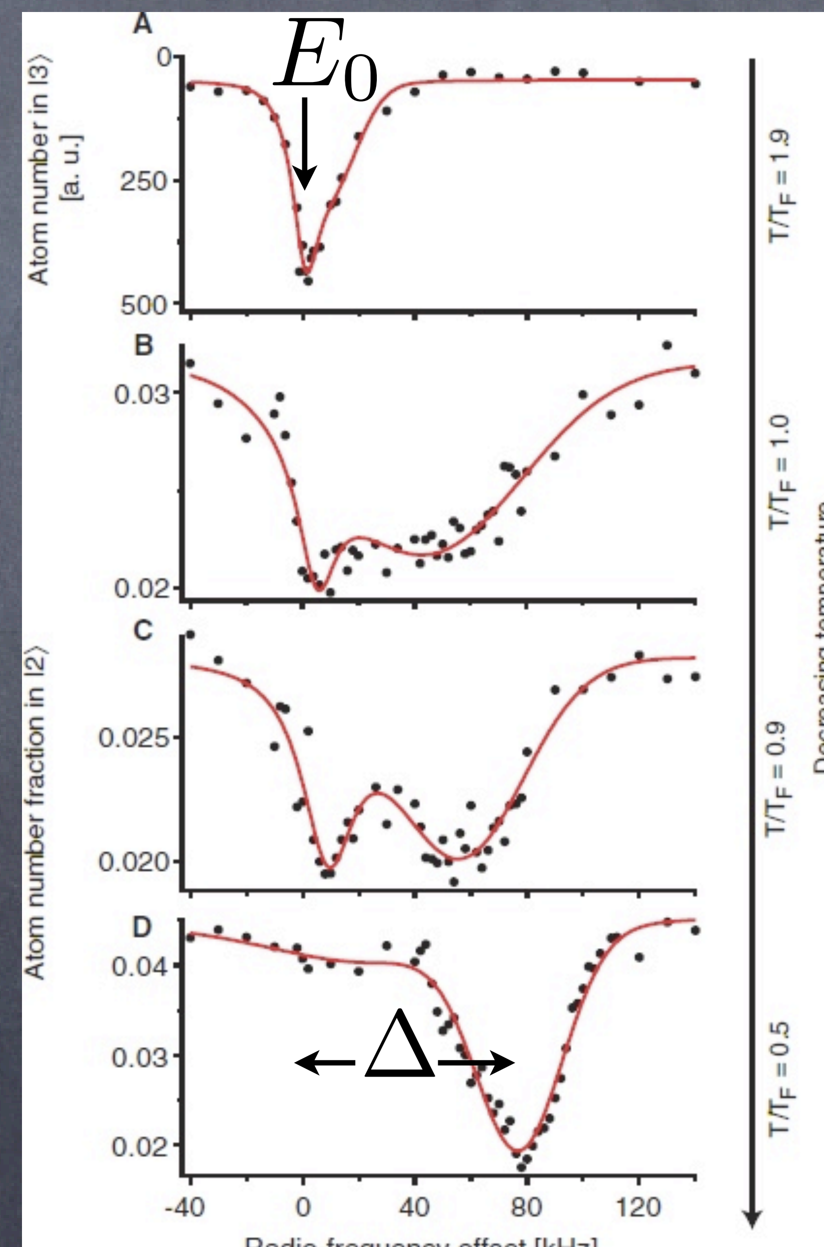


Zwierlein Group

Radio frequency spectroscopy

Atomic levels



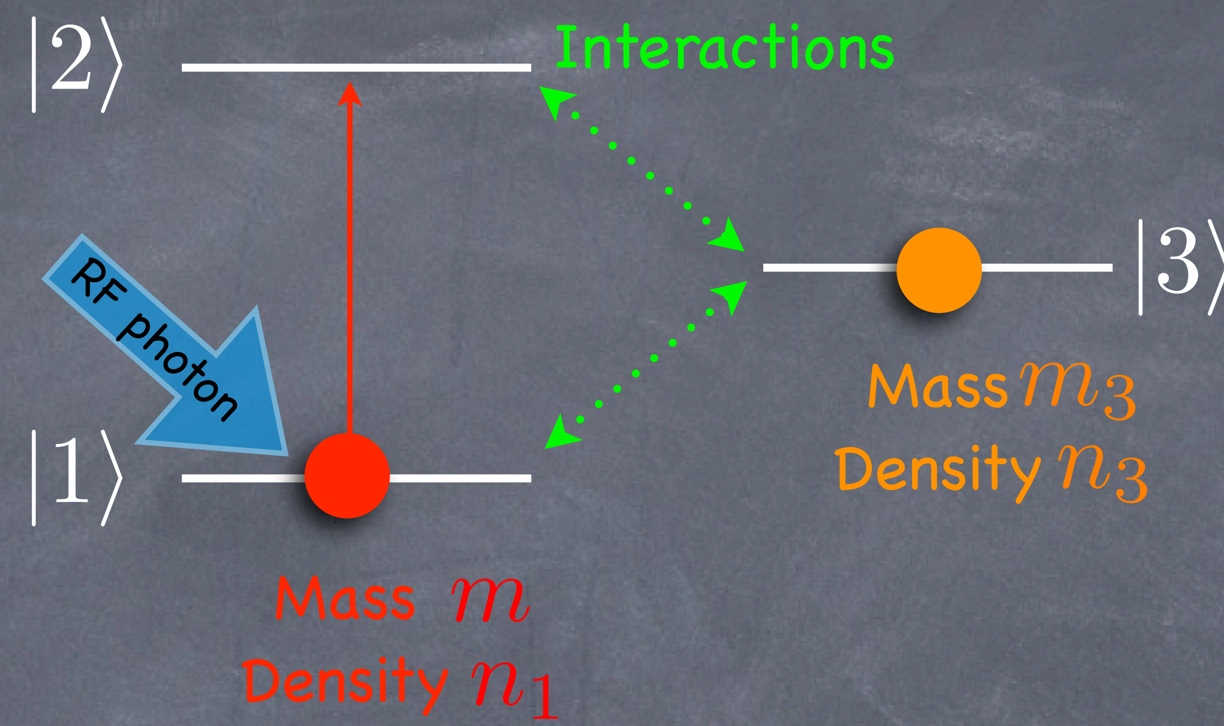


Decreasing T

Experiments:
Jin Group
Grimm Group
Ketterle group

MIT Group:
Science 316
867 (2007)

Theory



Transition rate:

$$\sum_{i,f}(P_i - P_f)\left|\int d^3r\langle f|\psi_2^\dagger(\mathbf{r})\psi_1(\mathbf{r})|i\rangle\right|^2 \delta(\omega - E_f + E_i)$$

$$\propto \mathrm{Im}\mathcal{D}(\omega) = \int d\mathbf{r}d\mathbf{r}'\mathrm{Im}\mathcal{D}(\mathbf{r},\mathbf{r}',\omega)$$

$$\mathcal{D}(\mathbf{r},\mathbf{r}',t-t') = -i\theta(t-t')\langle[\psi_2^\dagger(\mathbf{r},t)\psi_1(\mathbf{r},t),\psi_1^\dagger(\mathbf{r}',t')\psi_2(\mathbf{r}',t')]\rangle$$

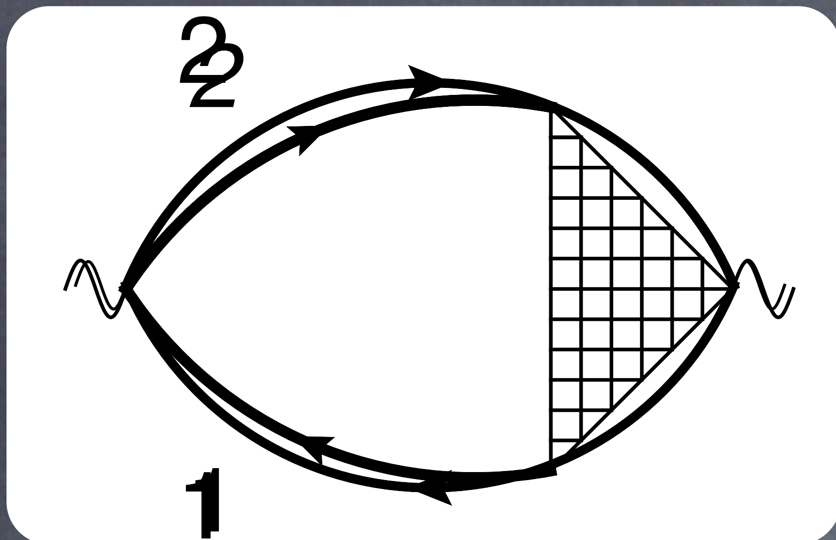
Difficult problem: Self-energies, trap, vertex corrections, pairing, ...

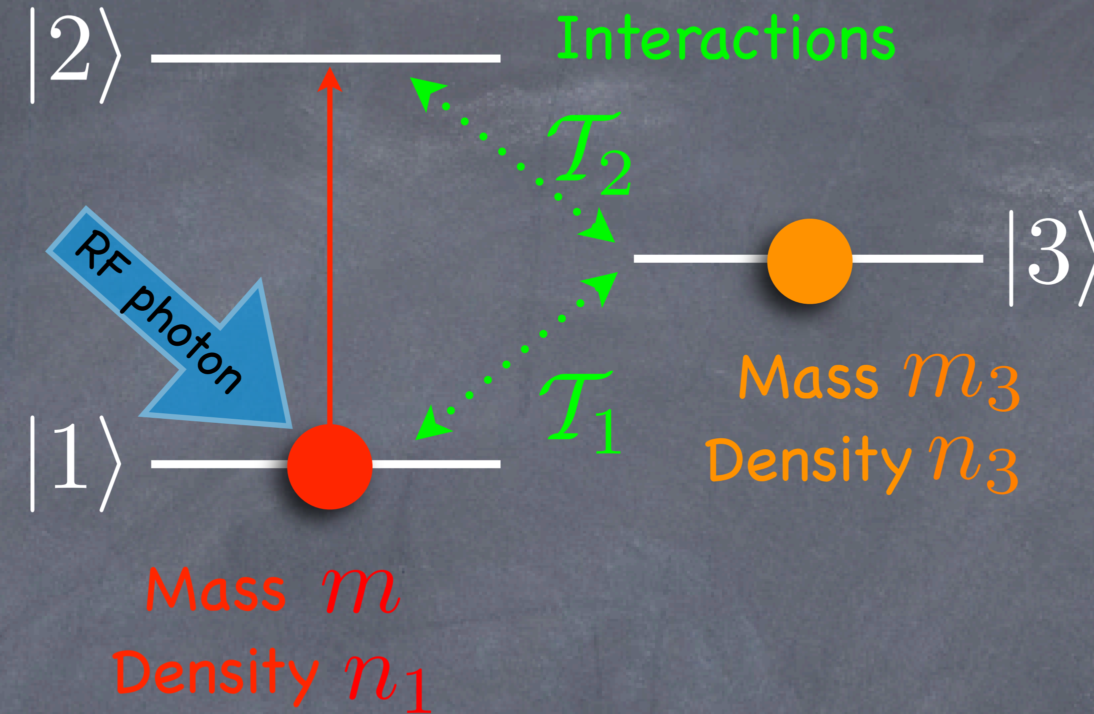
Strinati, Pieri, Perali, Levin, Sheehy, Sachdev, GMB, Stoof, Massignan, Baym, ...

Bottom lines

- $m_3 >> m_1$: Controlled conserving calculation
- Exact analytical results for $n_3 << n_1$
- Vertex corrections qualitatively change spectrum
- Resonance ≠ large line shifts
- Ladder approx. inadequate for $m_3 >> m_1$

1-loop approx.





$$\mathcal{D}(\mathbf{r},\mathbf{r}',t-t') = -i\theta(t-t')\langle[\psi_2^\dagger(\mathbf{r},t)\psi_1(\mathbf{r},t),\psi_1^\dagger(\mathbf{r}',t')\psi_2(\mathbf{r}',t')]\rangle$$

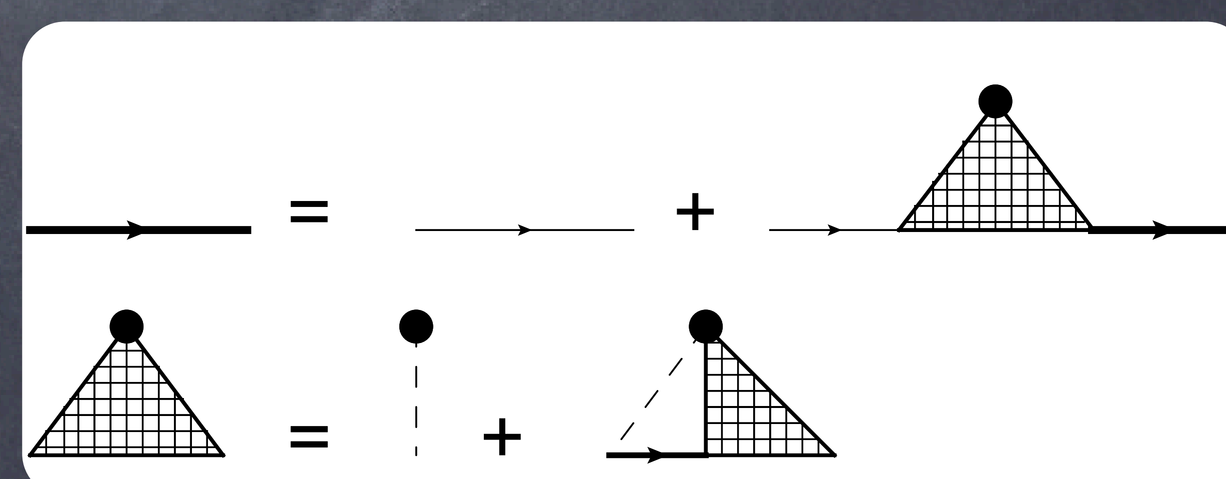
Limit $m_3 >> m$ ⇒ Static impurities

Self-energy effects for $|1\rangle$ and $|2\rangle$:

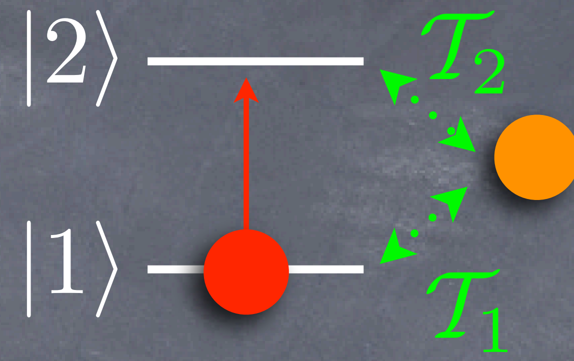
$$G_\sigma(p,z)^{-1} = G_\sigma^0(p,z)^{-1} - \Sigma_\sigma(z)$$

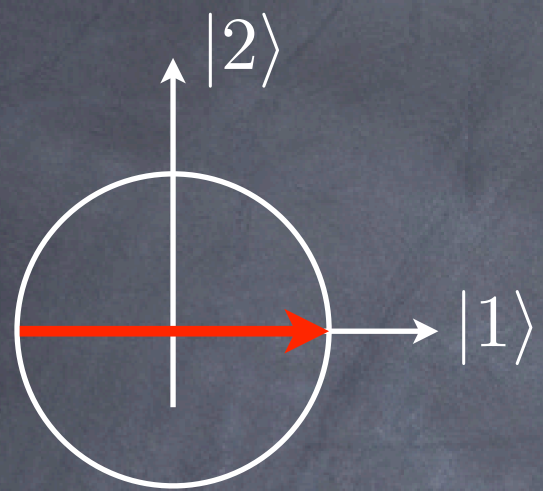
$$\Sigma_\sigma(z) = n_3\mathcal{T}_\sigma(z)$$

$$\mathcal{T}_\sigma(z) = V_\sigma + V_\sigma G_\sigma(z)\mathcal{T}_\sigma(z)$$



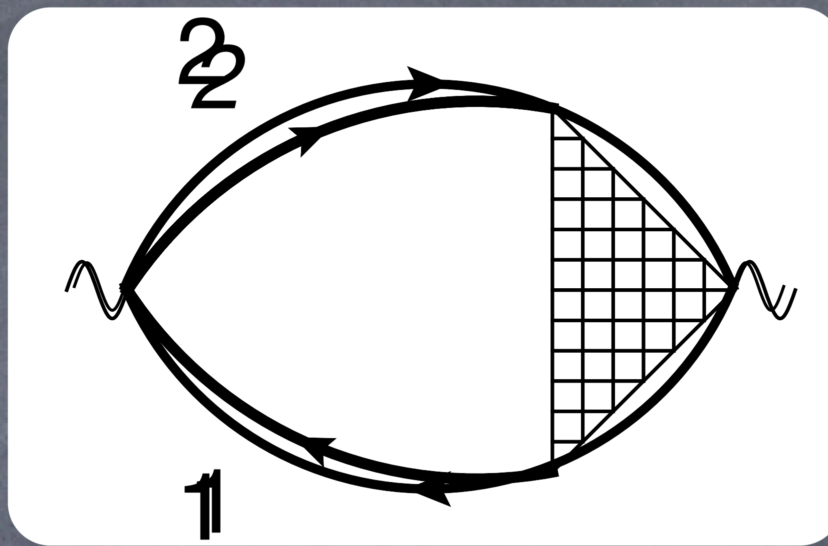
SU(2) symmetry: $\mathcal{T}_1 = \mathcal{T}_2$





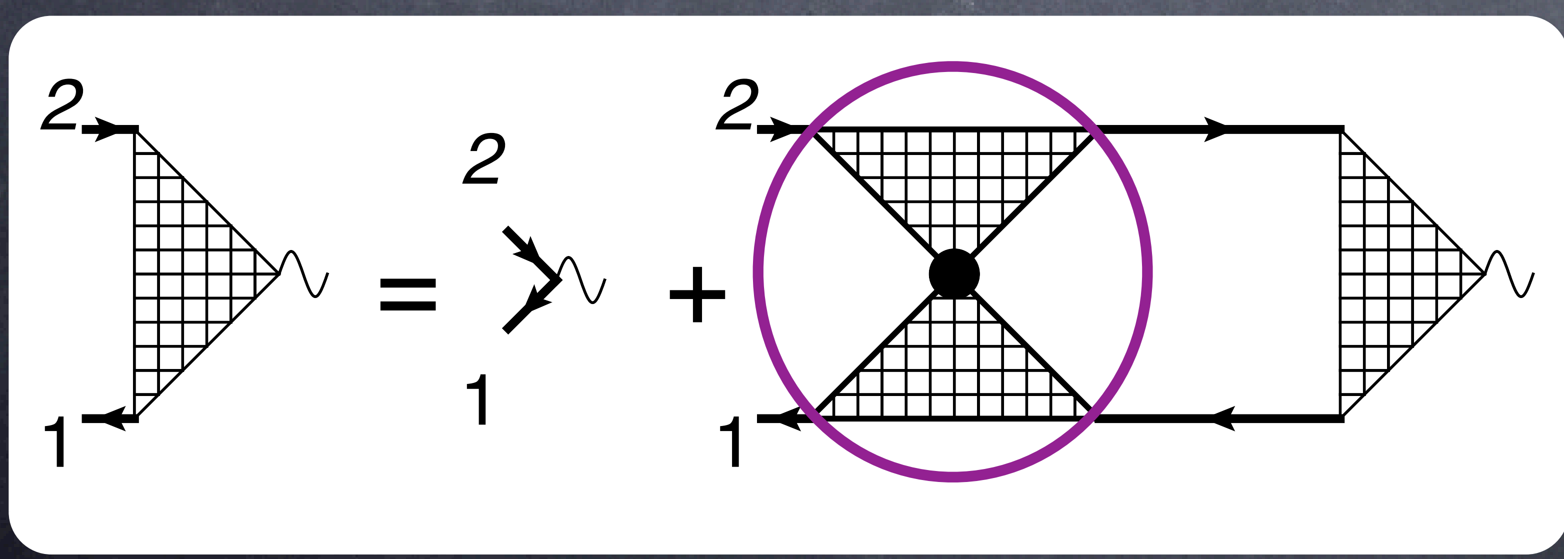
No line shift due to interactions Yu & Baym 2006

1-loop approx.



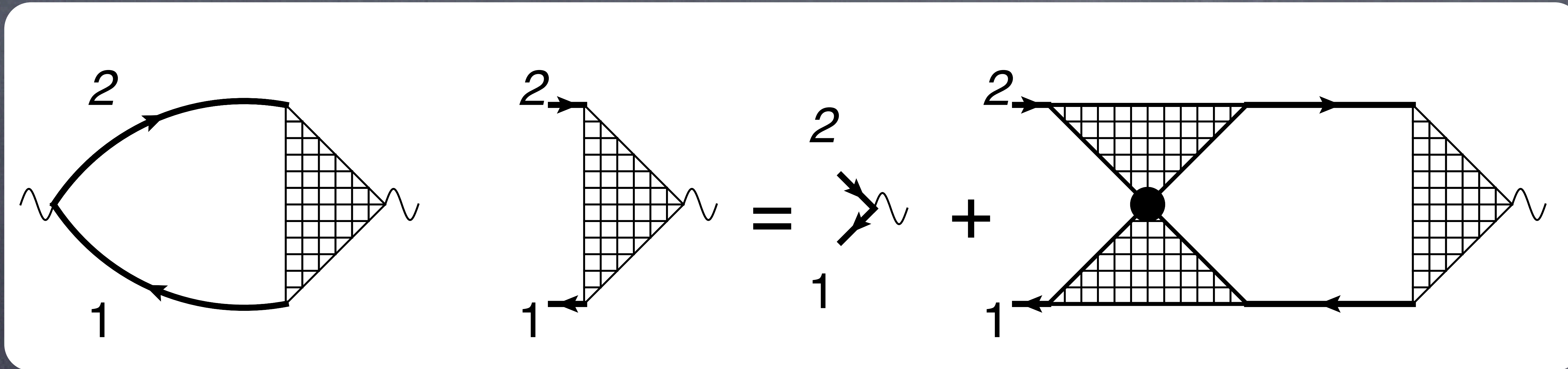
Conserving approximation: vertex corrections

Strinati group, Nat. Phys. 2009; Strinati group + Jin group, 2010



Particle-hole scattering on same impurity

$$V_{\text{eff}} = n_3 \mathcal{T}_1(i\omega_\nu)\mathcal{T}_2(i\omega_\nu + i\omega_\gamma)$$

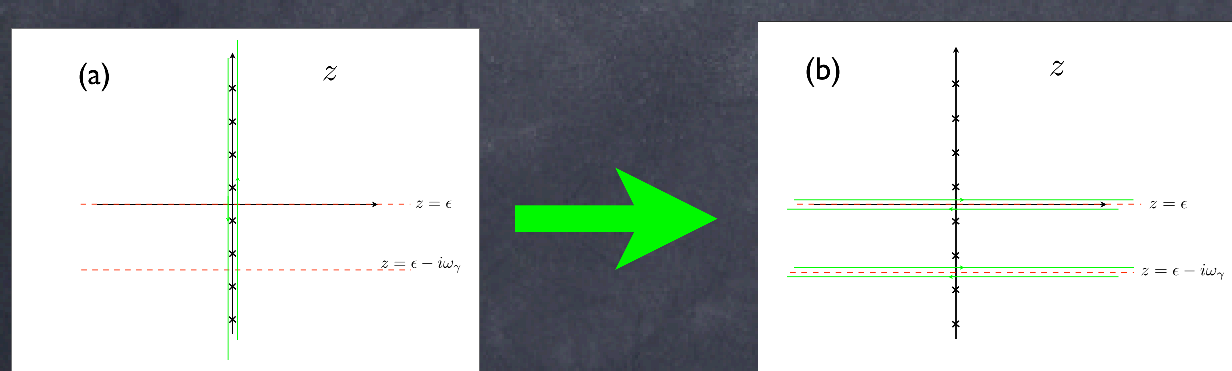


Momentum independent. Series can be summed

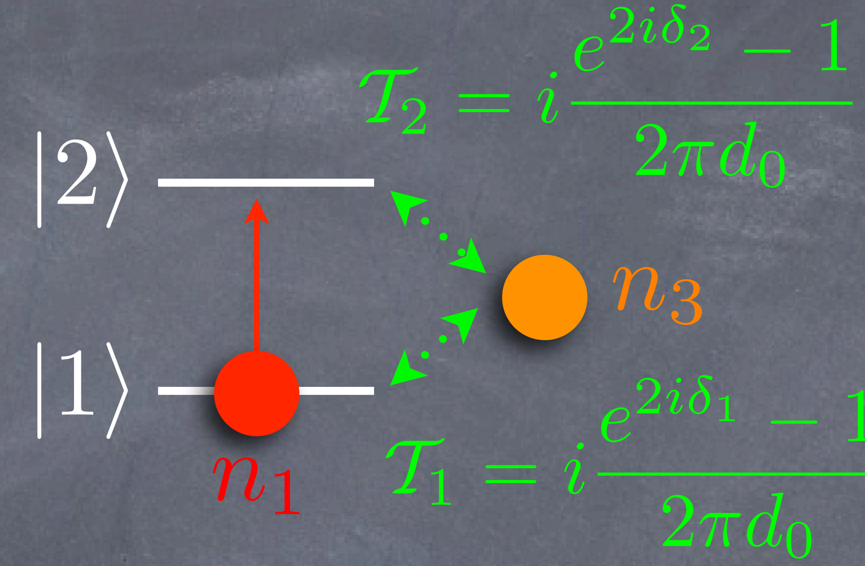
$$\mathcal{D}(i\omega_\gamma) = T\sum_{\omega_\nu} \frac{(2\pi)^{-3}\int d^3p\, G_1(p,i\omega_\nu)G_2(p,i\omega_\nu+i\omega_\gamma)}{1-n_3\mathcal{T}_1(i\omega_\nu)\mathcal{T}_2(i\omega_\nu+i\omega_\gamma)(2\pi)^{-3}\int d^3p\, G_1(p,i\omega_\nu)G_2(p,i\omega_\nu+i\omega_\gamma)}$$

$$M(z_1,z_2) = \int \frac{d^3p}{(2\pi)^3} G_1(p,z_1)G_2(p,z_2) = i\pi \frac{d_2(z_2)\mathrm{sgn}(\mathrm{Im}z_2) - d_1(z_1)\mathrm{sgn}(\mathrm{Im}z_1)}{z_2 - z_1 + \mu_2 - \mu_1 - \Delta + \Sigma_1(z_1) - \Sigma_2(z_2)}$$

Conserving (All propagators dressed)



The case $n_3 << n_1$: Analytical results



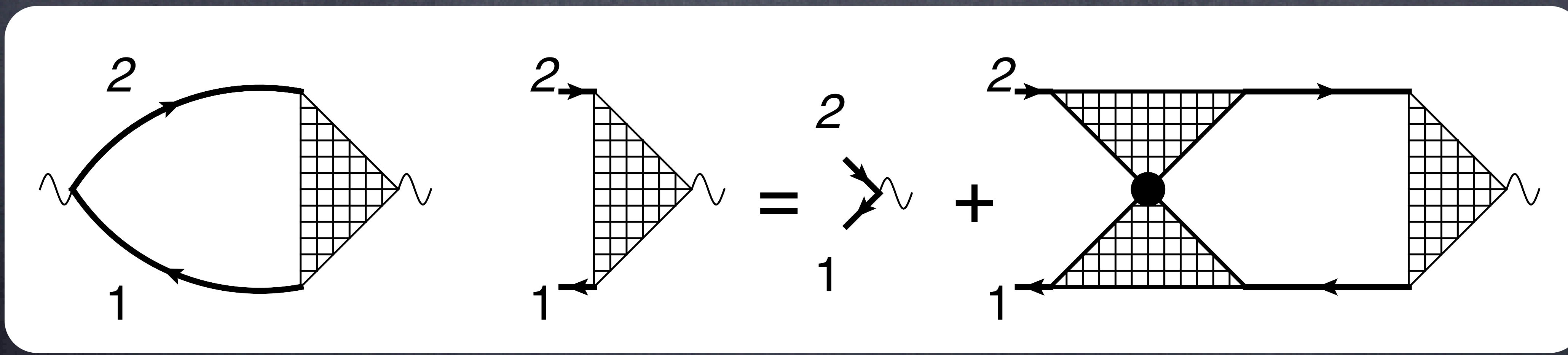
$$\mathrm{Im}\mathcal{D}(\omega) = -\mathrm{Im}\int_0^\infty d\epsilon \frac{d_0\, f(\epsilon+\epsilon_1-\mu_1)}{\omega - E_0 - in_3\left[e^{2i(\delta_1-\delta_2)}-1\right]/2\pi d_0}$$

SU(2) invariant

Ideal Gas: $\mathcal{D}(\omega) = -\frac{n_1}{\omega - E_0 + i0_+}$

2-particle and 1-hole scattering on same impurity:

$$\mathcal{T} \propto e^{2i(\delta_2-\delta_1)}-1$$



$$\mathrm{Im}\mathcal{D}(\omega) = -\mathrm{Im}\int_0^\infty d\epsilon \frac{d_0\, f(\epsilon+\epsilon_1-\mu_1)}{\omega - \Delta\omega(\epsilon) + i\Gamma(\epsilon)}$$

Shift: $\Delta\omega(\epsilon) = n_3 \frac{\pi}{m} \frac{\sin(2\delta_1 - 2\delta_2)}{k}$

Width: $\Gamma(\epsilon) = n_3 \frac{4\pi}{m} \frac{\sin^2(\delta_1 - \delta_2)}{k}$

$\tan\delta = -ka$

No vertex:

~~$n_3 \frac{\pi}{m} \frac{\sin 2\delta_1 - \sin 2\delta_2}{k}$~~

~~$n_3 \frac{4\pi}{m} \frac{\sin^2 \delta_1 + \sin^2 \delta_2}{k}$~~

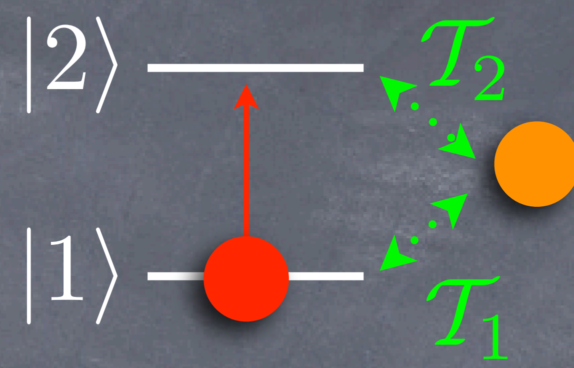
Pipkin 1964, Koelman 1988

- Weak coupling $\delta_\sigma \simeq ka_\sigma \quad \Delta\omega \simeq \frac{n_3 2\pi(a_2 - a_1)}{m}$ Mean field
- Resonant coupling $\delta_2 = \pi/2 \quad \Delta\omega = -n_3 \frac{\pi}{m} \sin(2\delta_1)$

Sign change

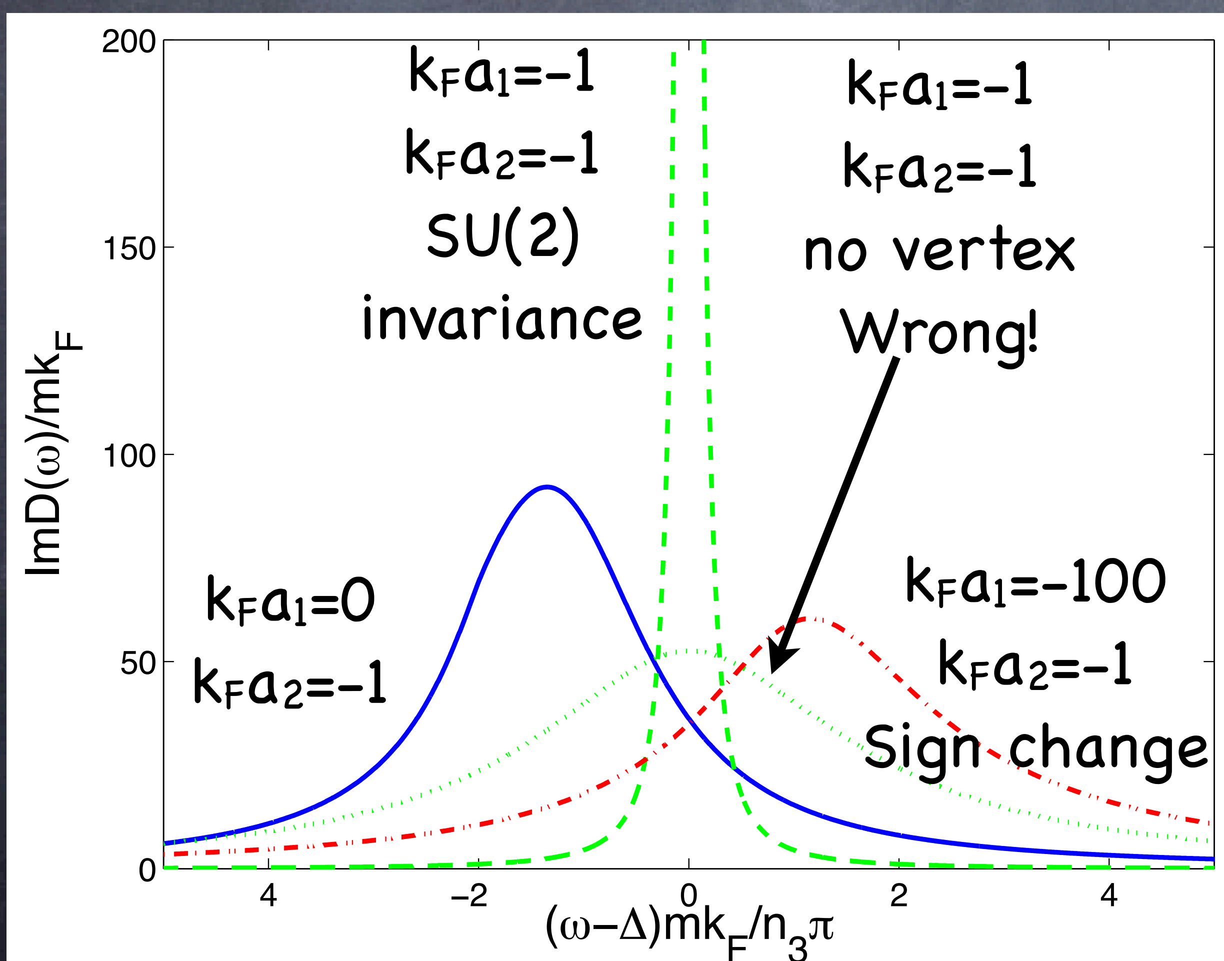
- Large shift (δ=π/4) ≠ large width (δ=π/2)

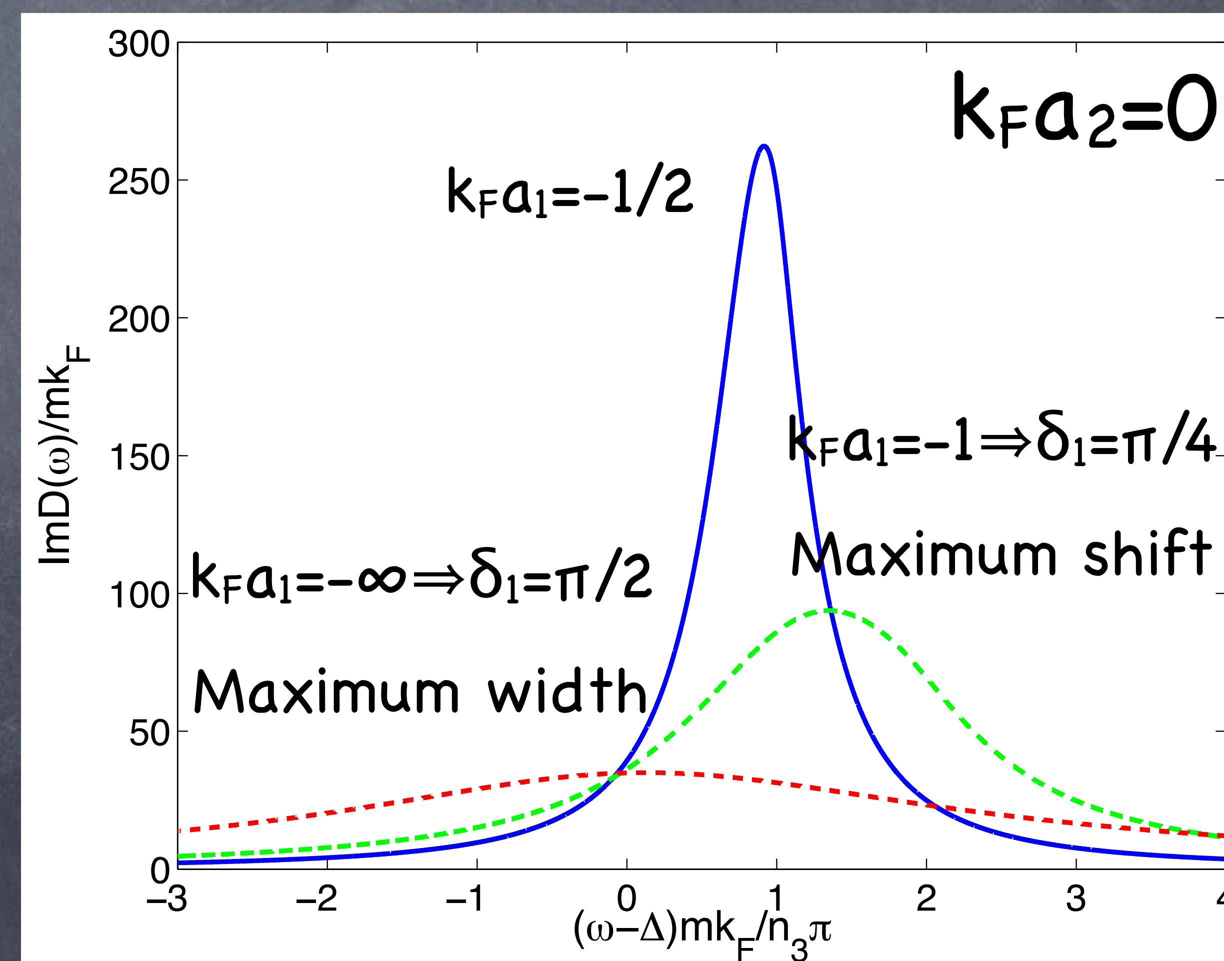
Numerics



$$\mathrm{Im}\mathcal{D}(\omega) = \int \frac{d\epsilon}{2}(f_2 - f_1)\mathrm{Im}\left[\frac{d_2 - d_1}{\omega - \Delta + n_3[\mathcal{T}_1 - \mathcal{T}_2 - i\pi\mathcal{T}_1\mathcal{T}_2(d_2 - d_1)]} - \frac{d_2 + d_1^*}{\omega - \Delta + n_3[\mathcal{T}_1^* - \mathcal{T}_2 - i\pi\mathcal{T}_1^*\mathcal{T}_2(d_2 + d_1^*}\right.$$

$n_3 << n_1$: $\Delta\omega(\epsilon) = n_3 \frac{\pi}{m} \frac{\sin(2\delta_1 - 2\delta_2)}{k}$ $\quad \Gamma(\epsilon) = n_3 \frac{4\pi}{m} \frac{\sin^2(\delta_1 - \delta_2)}{k}$



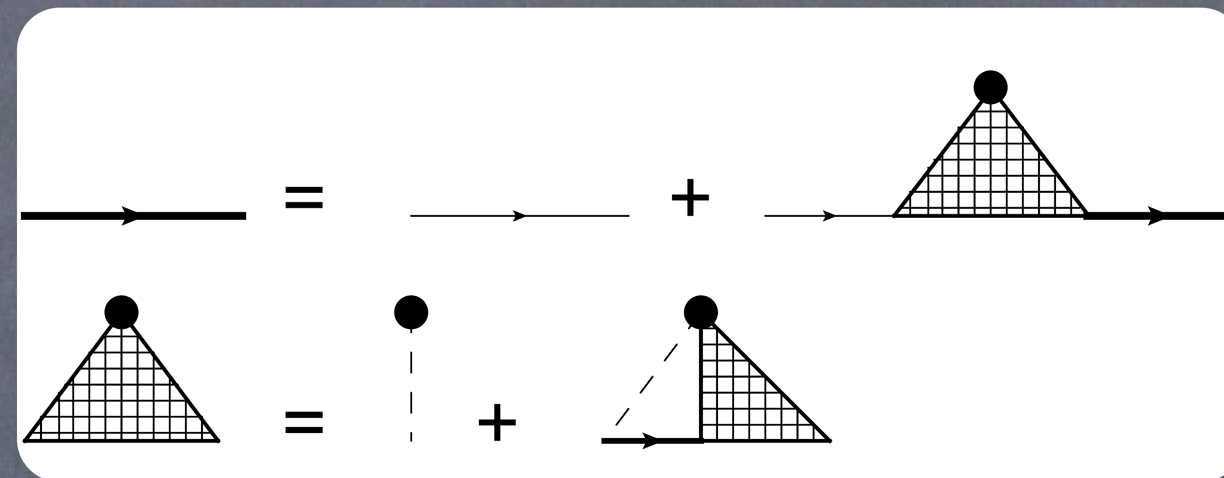


Recovery of m/m$_3$→0 limit

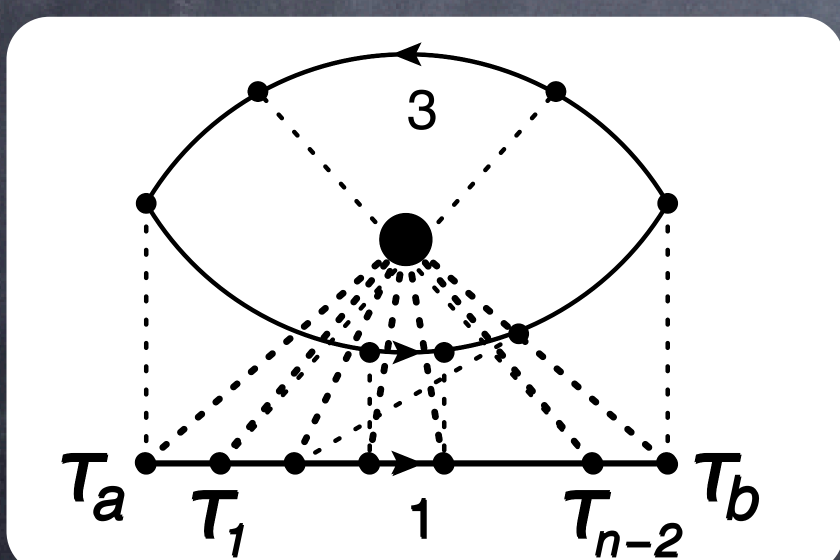
Impurity scattering

$$\Sigma_\sigma(z) = n_3 \mathcal{T}_\sigma(z)$$

$$\mathcal{T}_\sigma(z) = V_\sigma + V_\sigma G_\sigma(z)\mathcal{T}_\sigma(z)$$



Finite m$_3$: n'th order diagram



$$\Sigma_1^{(n)}(\tau_b - \tau_a) = -(-V_1)^n \int_0^\beta d\tau_1 \ldots d\tau_{n-2}$$
$$G_1(\tau_b - \tau_{n-2})G_1(\tau_{n-2} - \tau_{n-3}) \ldots G_1(\tau_1 - \tau_a)$$
$$\times[\quad G_3(\tau_b - \tau_{n-2})G_3(\tau_{n-2} - \tau_{n-3}) \ldots G_3(\tau_1 - \tau_a)G_3(\tau_a - \tau_b)$$
$$+ \text{ all } \tau \text{ permutations}]$$

$$G_3(p,\tau) = \begin{cases} f_p^3 e^{\mu_3 \tau} & \text{for } \tau < 0 \\ -e^{\mu_3 \tau} & \text{for } \tau > 0 \end{cases}$$

$$\rightarrow \Sigma_1^{(n)}(\omega) = n_3 V_1 \left[V_1 G_1(\omega)\right]^{n-1}$$

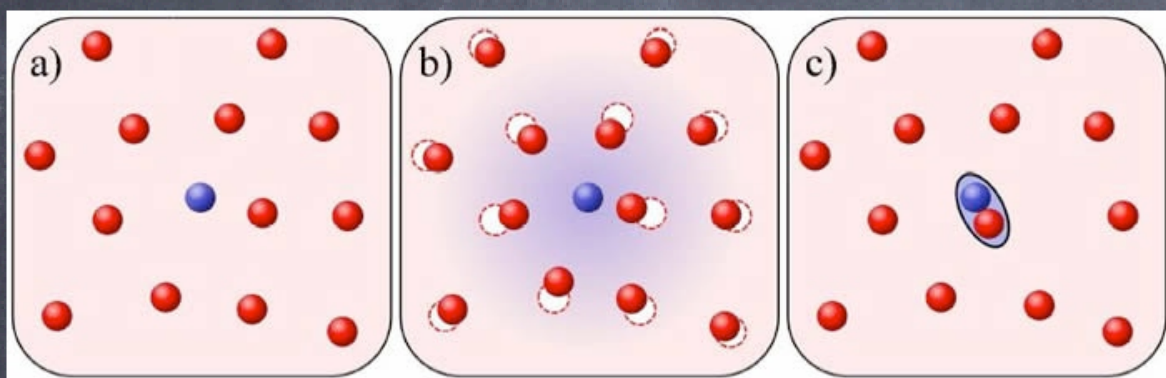
Conclusions

- Conserving calculation
- Analytical results for RF signal $m \ll m_3$
- SU(2) symmetry ⇒ Vertex corrections
- Vertex corrections change result qualitatively
- Resonant interaction ≠ large shifts

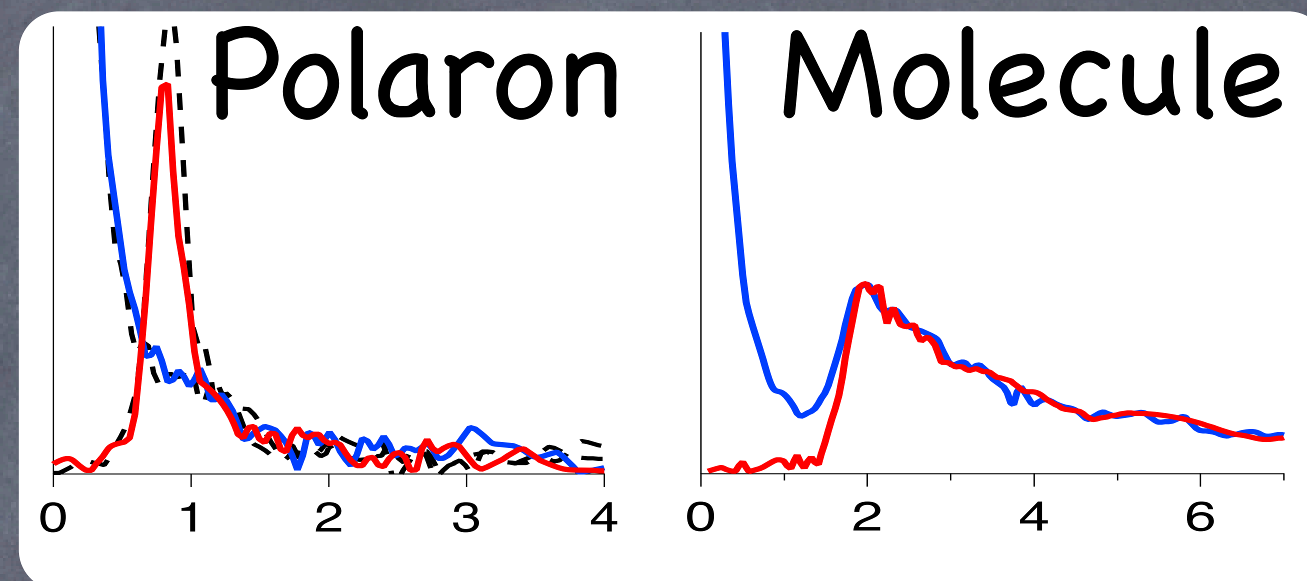
GMB, C. J. Pethick, Z. Yu, PRA 81, 033621 (2010)

Polaron-molecule coupling

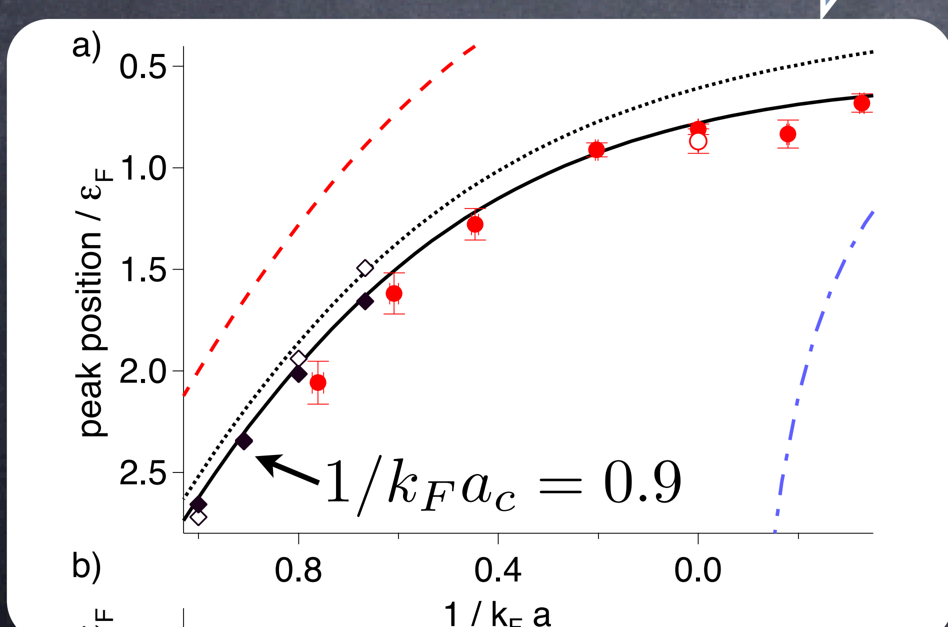
Polaron Molecule

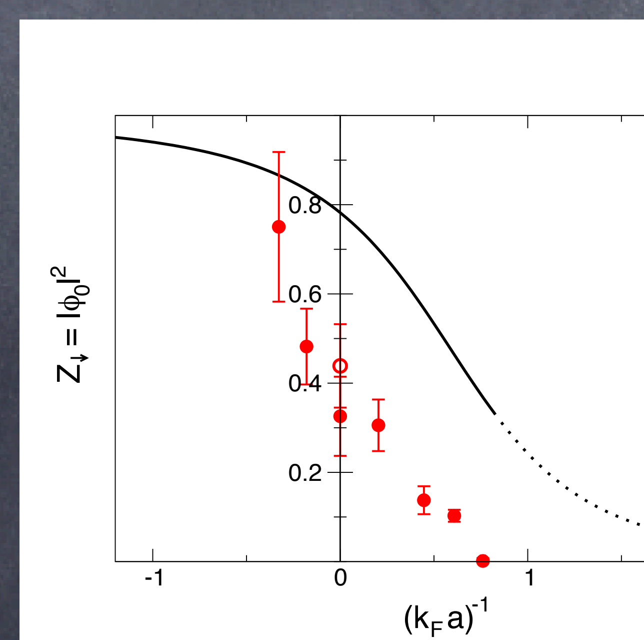


Increasing interaction



Zwierlein group, PRL 2009





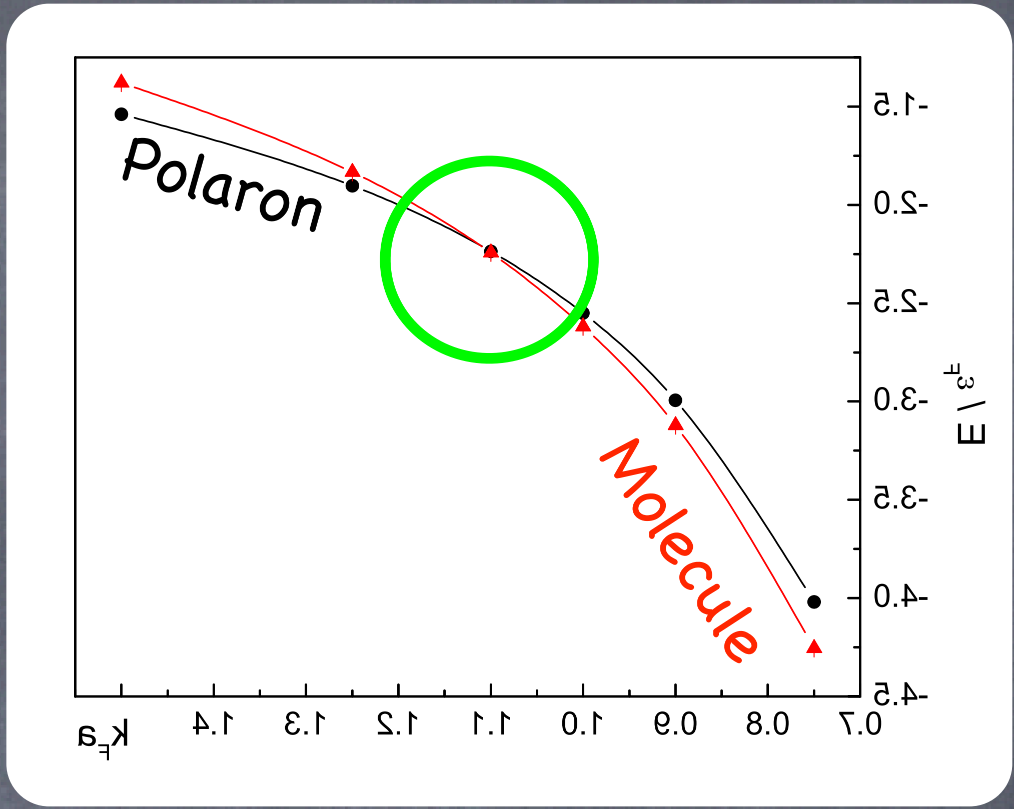
Chevy, Mora, Zwerger, Punk, Combescot, Leyronas, Recati, Lobo, Prokof'ev, Svistunov ...

Bottom Lines

- Leading process coupling molecule and polarons is 3-body
- Transition sharp
- Phase space and Fermi statistics strongly suppress decay
- Life times ~10-100ms

Cross-over region

$$E_P(p=0) - E_M(p=0) \equiv \Delta\omega \simeq 0$$



~~p=0 polaron →~~
~~p=0 molecule + p=0 hole~~

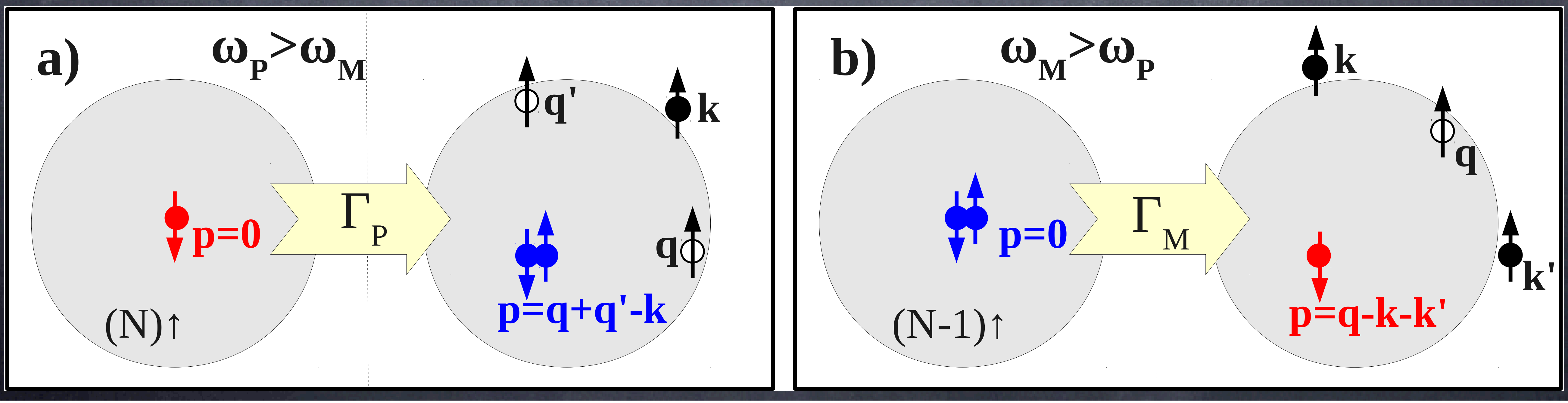
Not enough energy for $\Delta\omega \simeq < \epsilon_F/2$

Prokof'ev & Svistunov
2008

Need to look at 3-body processes

Polaron decay

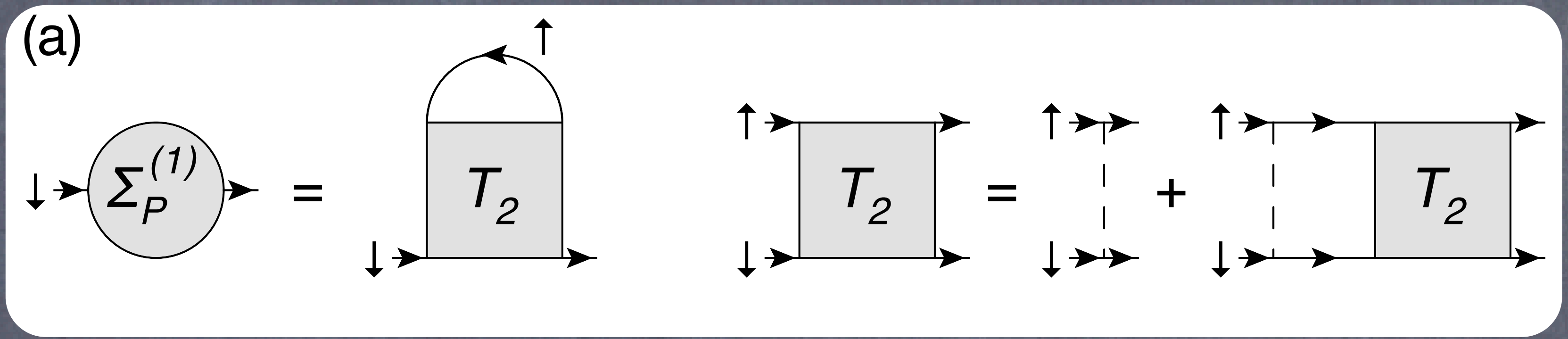
Molecule decay



Polaron: $G_\downarrow(\mathbf{p},z)^{-1} = G^0_\downarrow(\mathbf{p},z)^{-1} - \Sigma_P(\mathbf{p},z)$

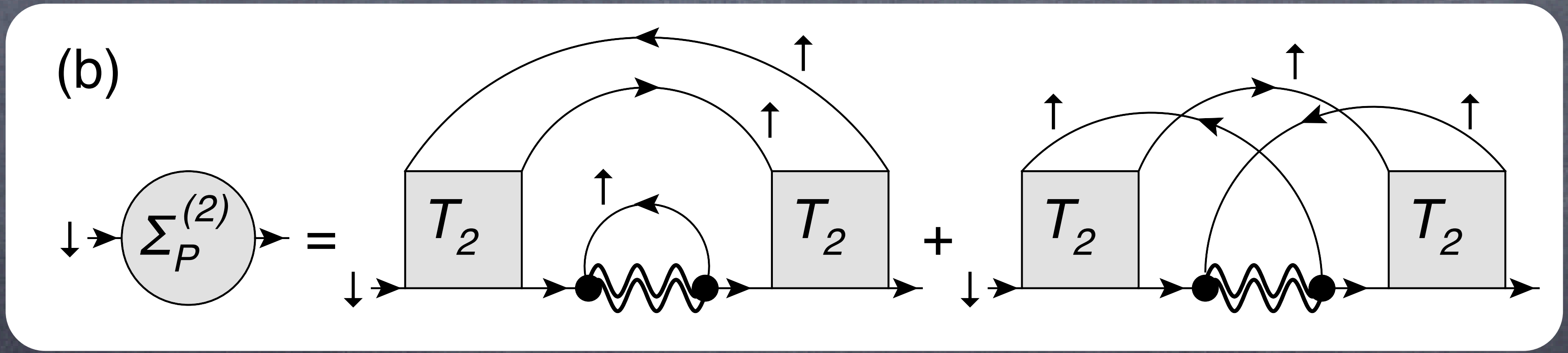
Hole expansion $\Sigma_P(\mathbf{p},z) = \Sigma^{(1)}_P(\mathbf{p},z) + \Sigma^{(2)}_P(\mathbf{p},z) + \dots .$

$\Sigma^{(1)}_P(\mathbf{p},z)$



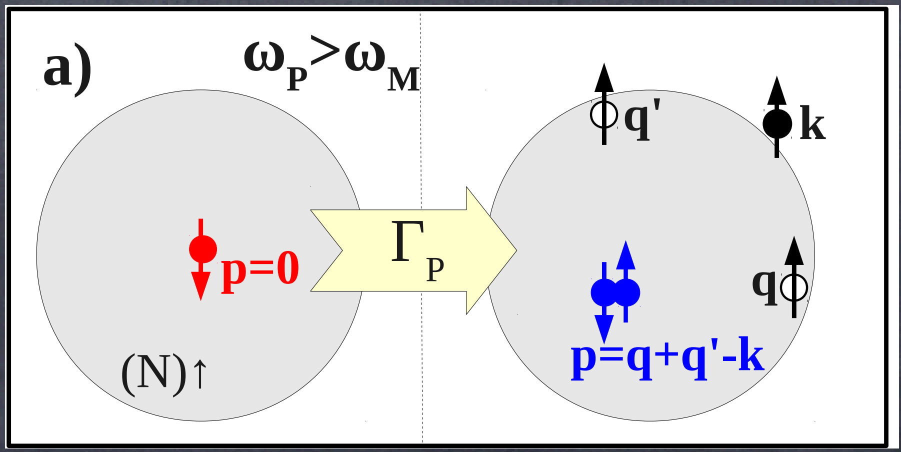
Ladder
No damping

$\Sigma^{(2)}_P(\mathbf{p},z)$



Damping rate

$\Gamma_P = -\mathrm{Im}\Sigma^{(2)}_P(0,\omega_P)$



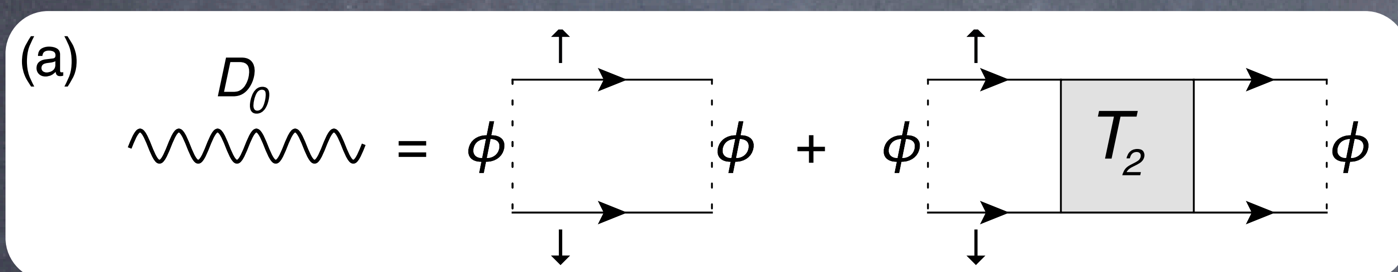
Molecule: $D(\mathbf{q},\tau) = -\langle T_\tau[\hat{b}_{\mathbf{p}}(\tau)\hat{b}^\dagger_{\mathbf{p}}(0)]\rangle$

$$\hat{b}^\dagger_{\mathbf{p}} = \int d^3\check{q}\,\phi_q \hat{a}^\dagger_{\mathbf{p}/2+\mathbf{q}\downarrow}\hat{a}^\dagger_{\mathbf{p}/2-\mathbf{q}\uparrow}$$

$$\phi_q = \frac{\sqrt{8\pi a^3}}{1+q^2a^2}$$

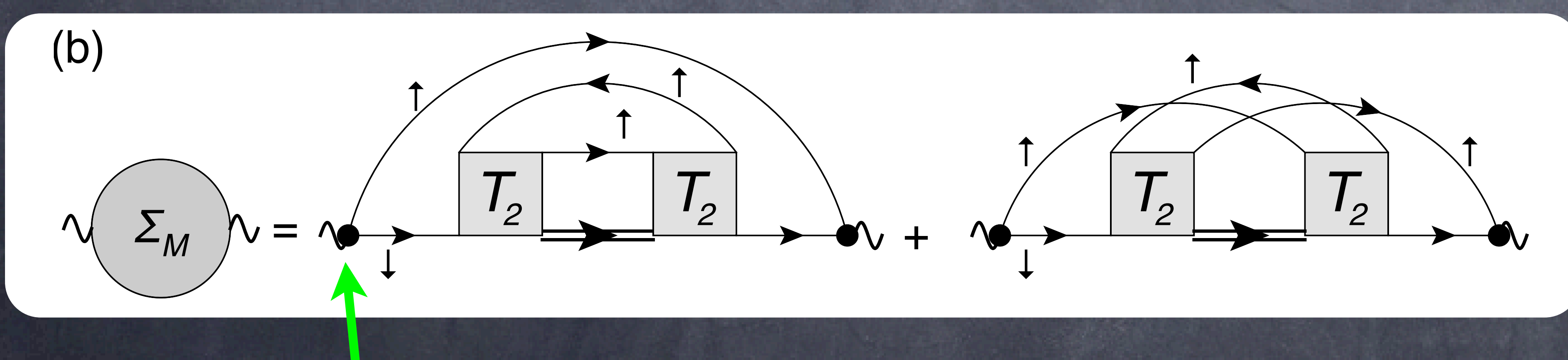
$$\phi(r) \propto \frac{e^{-r/a}}{r}$$

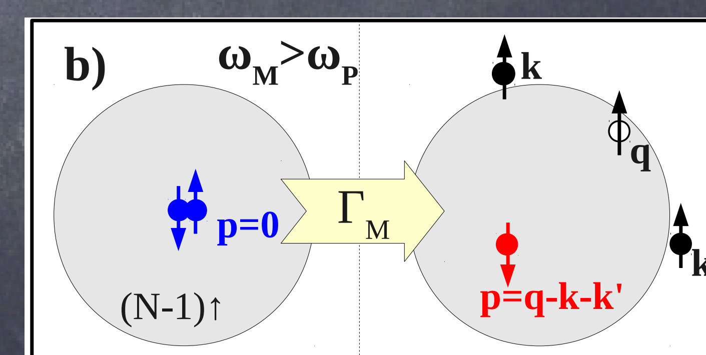
Vacuum:



$$D_0(\mathbf{p},z) = \int d^3\check{q}\,\phi_q^2 \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} + \frac{T_2(\mathbf{p},z)}{g(\mathbf{p},z)^2}$$

Medium effects:



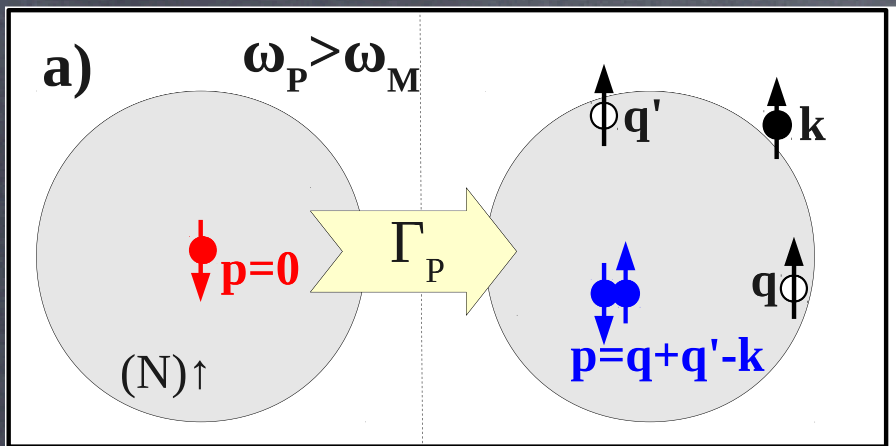


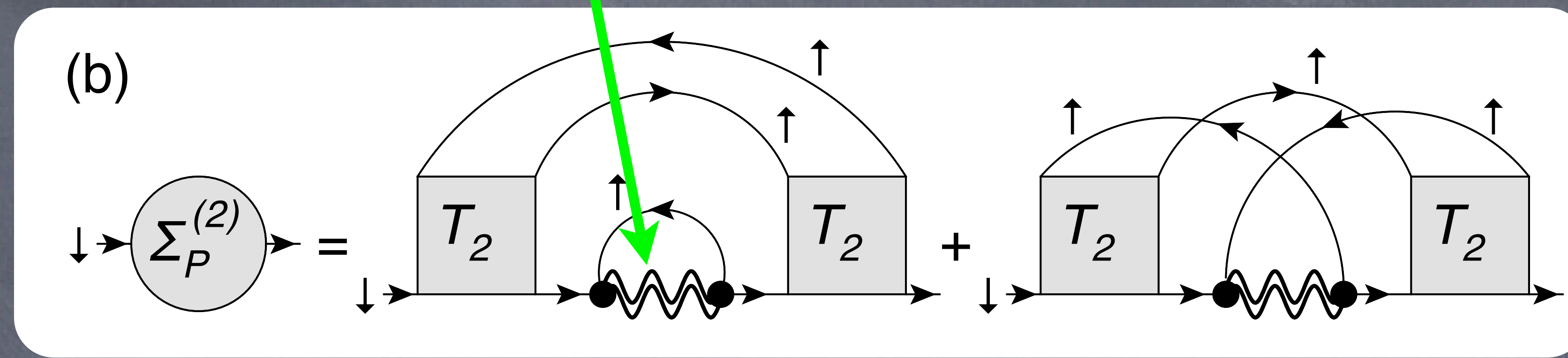
Molecule-atom coupling

$$\frac{1}{g(\mathbf{p},z)} = \int d^3\check{q}\,\phi_q \frac{1-f(\xi_{\mathbf{p}-\mathbf{q}\uparrow})}{z-\xi_{\mathbf{p}-\mathbf{q}\uparrow}-\xi_{q\downarrow}} = -\sqrt{\frac{2\pi}{m_r^2 a}}$$

Polaron Decay:

$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega - \omega_M - p^2/2m_M^*}$$





$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G^0_\downarrow(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

$$\Gamma_P = \frac{g^2 Z_M}{2}\int d^3\check{q}d^3\check{k}d^3\check{q}'\left[F(\mathbf{q},\mathbf{k},\omega_P) - F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2 \times\delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - (\mathbf{q}+\mathbf{q}'-\mathbf{k})^2/2m_M^*\right).$$

$\Delta\omega \ll \epsilon_F$:

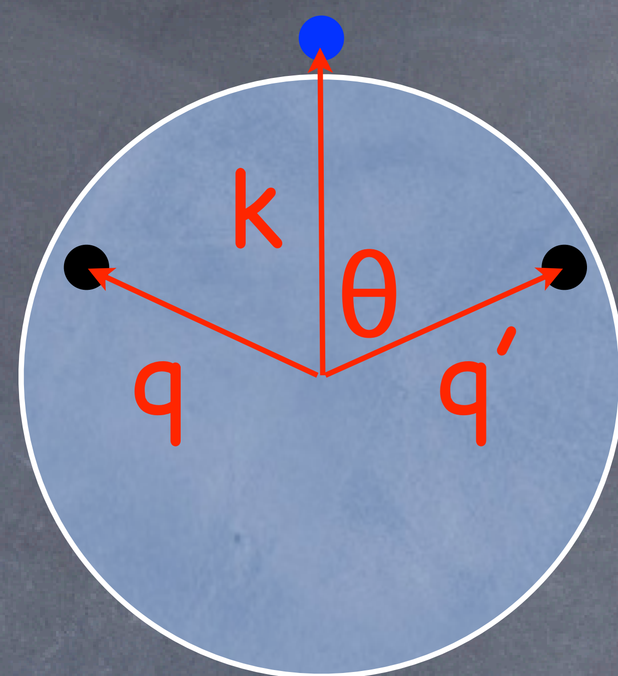
$$\int d^3\check{p}\int_0^\infty d\xi\int_{-\epsilon_F}^0 d\xi' d\xi''\delta\left(\Delta\omega + \xi' + \xi'' - \xi - \frac{p^2}{2m_M^*}\right) \propto \Delta\omega^{7/2}$$

Matrix element: $F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)$

Kinematics

$$k - q - q' \sim 0$$

Equilateral triangle



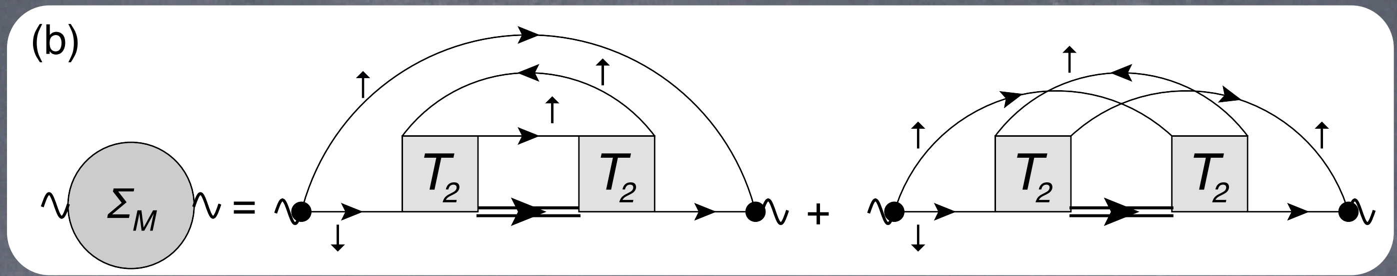
Depends only on angle θ

Expand around equilateral triangle

$$\Gamma_P \sim Z_M k_F a \left(\frac{\Delta\omega}{\epsilon_F} \right)^{9/2} \epsilon_F$$

Fermi exclusion gives extra power of Δω

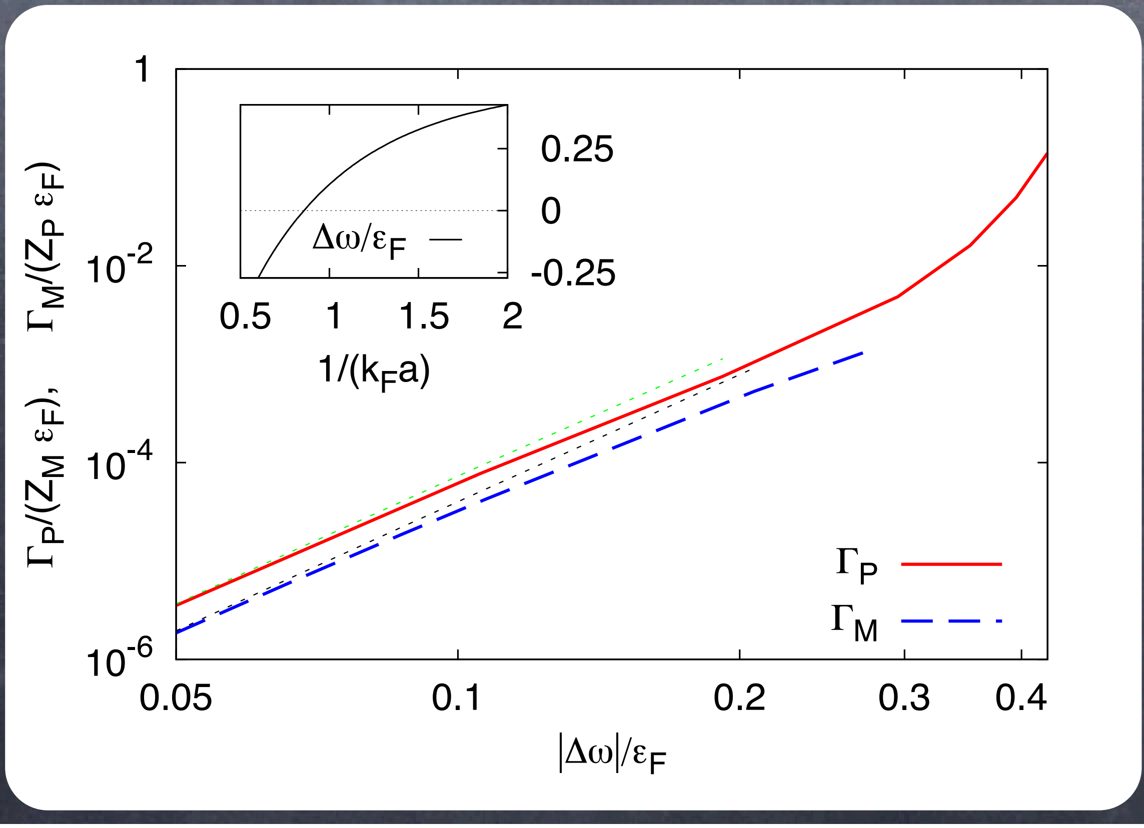
Molecule decay:



$$\Gamma_M \sim Z_P k_F a \left(\frac{|\Delta\omega|}{\epsilon_F}\right)^{9/2} \epsilon_F$$

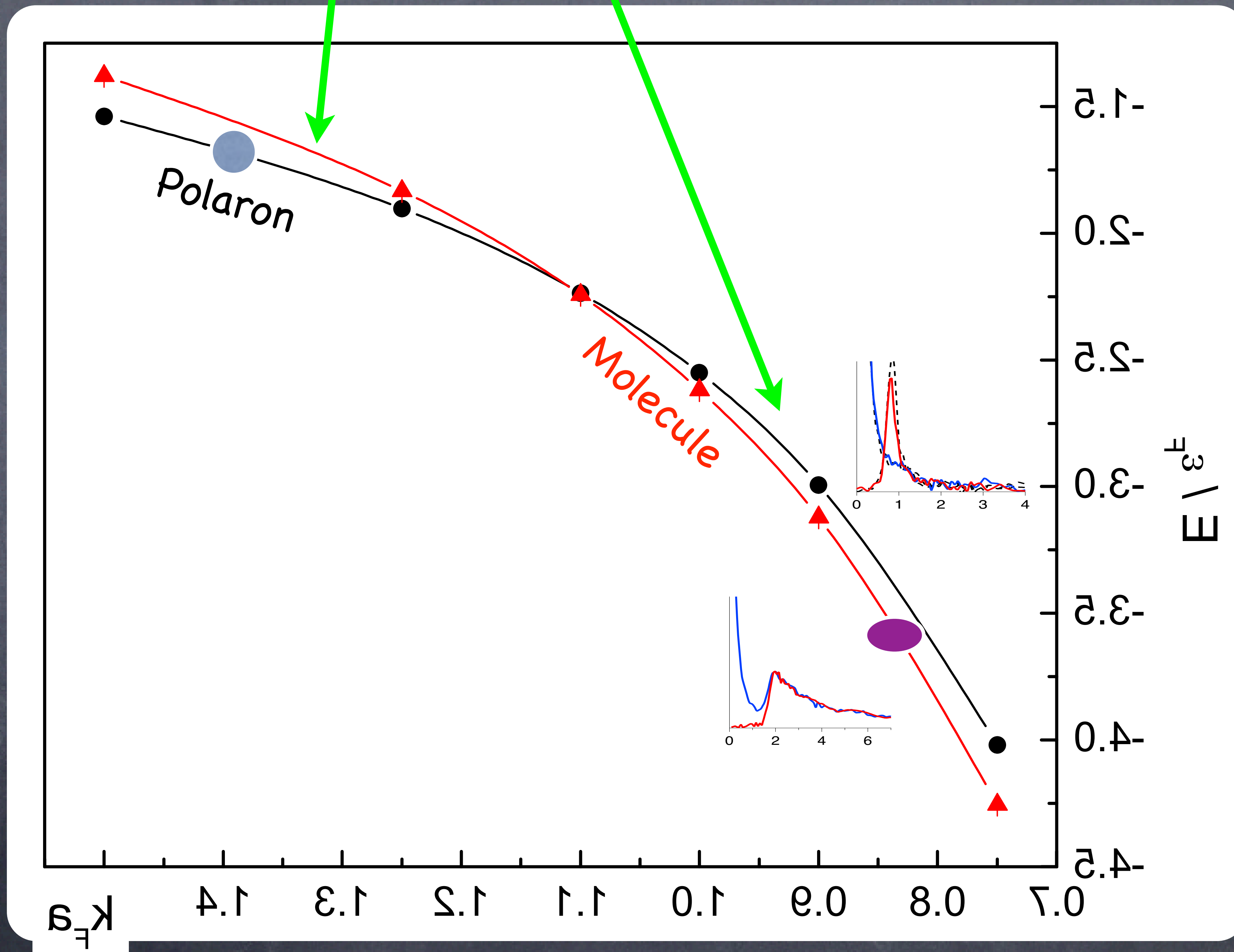
Numerics:

$$\Gamma_P = \frac{g^2 Z_M}{2} \int d^3\check{q} d^3\check{k} d^3\check{q}' \left[F(\mathbf{q}, \mathbf{k}, \omega_P) - F(\mathbf{q}', \mathbf{k}, \omega_P)\right]^2$$
$$\times \delta\left(\Delta\omega + \xi_{q\uparrow} + \xi_{q'\uparrow} - \xi_{k\uparrow} - (\mathbf{q} + \mathbf{q}' - \mathbf{k})^2/2m_M^*\right).$$



Phase space effects + Fermi statistics ⇒

Long lifetimes ~ 10-100ms



GMB, P. Massignan PRL 105, 020403 (2010)

Conclusions

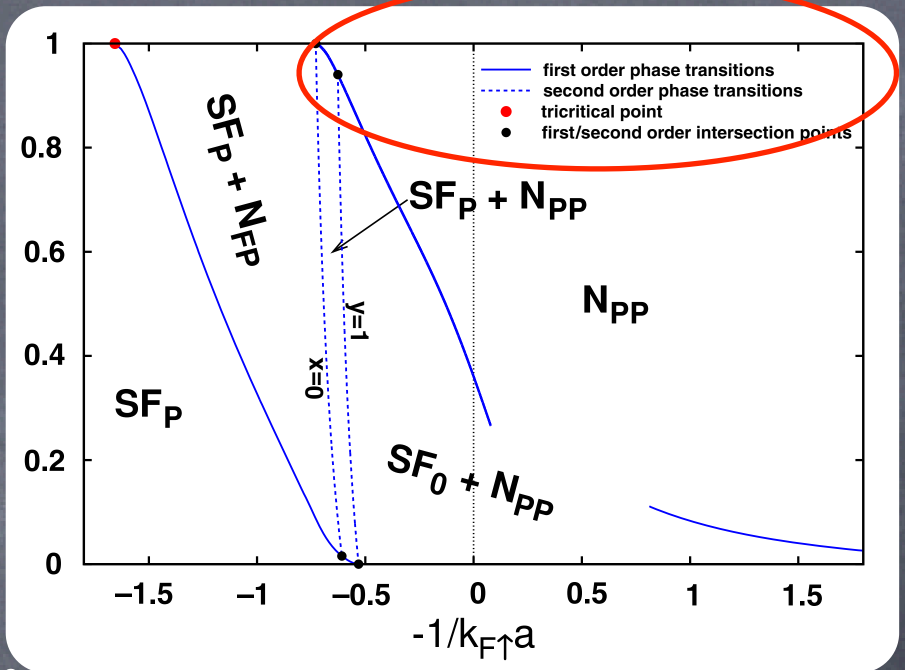
- Leading process coupling molecule and polarons is 3-body
- Transition sharp
- Phase space and Fermi statistics strongly suppress decay
- Life times ~10-100ms

Polarons at non-zero momentum

New Fermi liquid:

$|\uparrow\rangle$ ideal gas, mass $m_\uparrow$

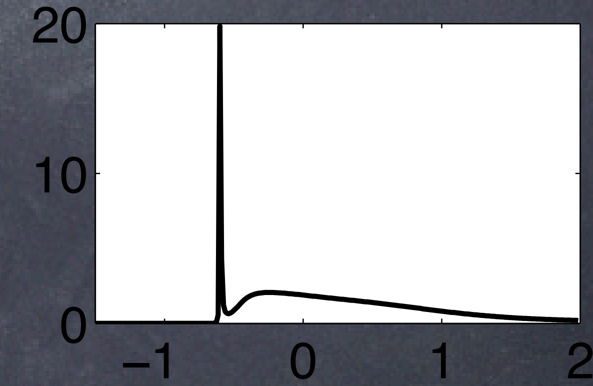
$|\downarrow\rangle$ dressed mass m^*, $Z \approx 0.7$



Rich: Can vary $m^*/m_\uparrow$, $n_\downarrow/n_\uparrow$, $k_F a$ ...

Studied experimentally by MIT group, Rice group.

New study by Paris group reveal Fermi liquid behavior Science 2010



GMB & Massignan 2008

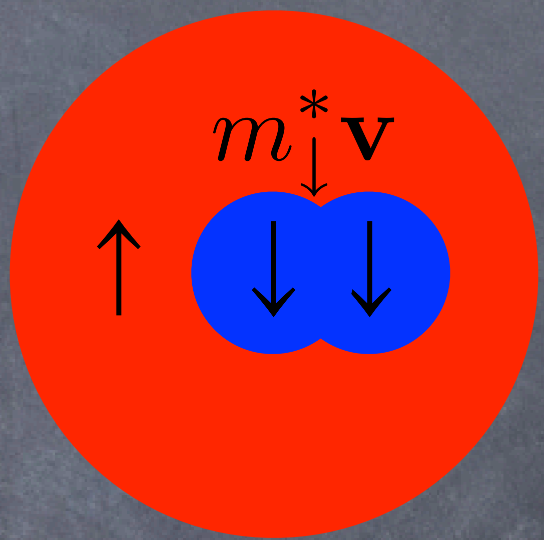
Spin dipole mode:

Damping:

$$\dot{\mathbf{P}}_{\downarrow} = -\frac{\mathbf{P}_{\downarrow}}{\tau_P}$$

Momentum relaxation time

Fermi surfaces

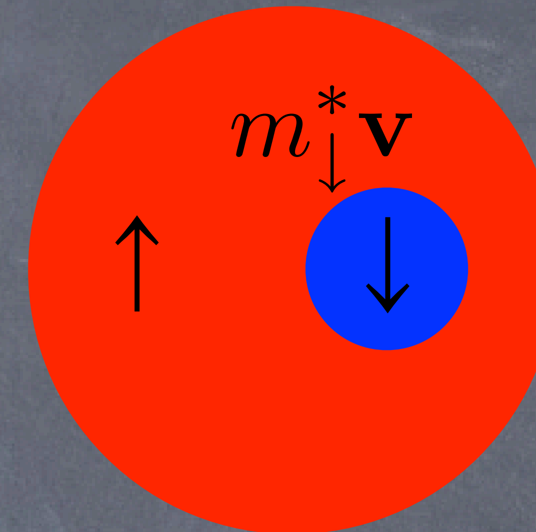


Spin modes measured for high T
by Jin group 2003

Will calculate this using thermodynamics
to find scattering rate.

Displayed Fermi surfaces:

$$n_{\mathbf{p}\uparrow} = \frac{1}{e^{\beta \xi_{\mathbf{p}\uparrow}} + 1} \qquad n_{\mathbf{p}\downarrow} = \frac{1}{e^{\beta(\xi_{\mathbf{p}\downarrow} - \mathbf{p}\cdot\mathbf{v})} + 1}$$

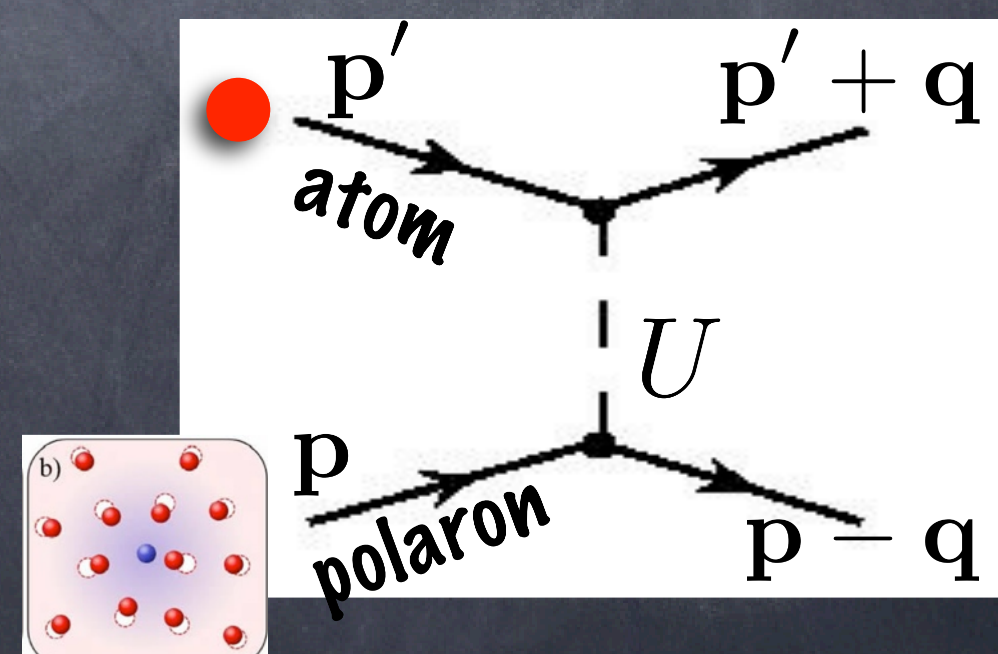


Decay rate:

$$\frac{d\mathbf{P}_\downarrow}{dt} = -2\pi \frac{|U|^2}{V^3} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q}} \mathbf{p} \Big[n_{\mathbf{p}\downarrow} n_{\mathbf{p}'\uparrow} (1 - n_{\mathbf{p}-\mathbf{q}\downarrow})(1 - n_{\mathbf{p}'+\mathbf{q}\uparrow})$$

$$- n_{\mathbf{p}-\mathbf{q}\downarrow} n_{\mathbf{p}'+\mathbf{q}\uparrow} (1 - n_{\mathbf{p}\downarrow})(1 - n_{\mathbf{p}'\uparrow}) \Big] \delta(\epsilon_{\mathbf{p}\downarrow} + \epsilon_{\mathbf{p}'\uparrow} - \epsilon_{\mathbf{p}-\mathbf{q}\downarrow} - \epsilon_{\mathbf{p}'+\mathbf{q}\uparrow})$$

Interaction between polaron and majority atoms:



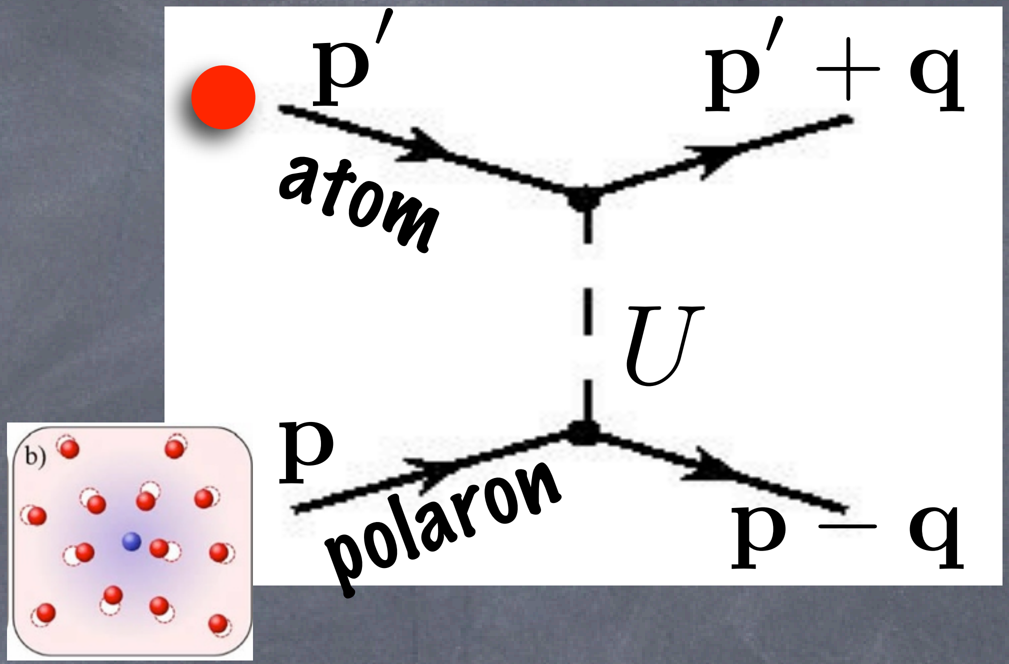
Thermodynamics:

$$U = \frac{\partial^2 E}{\partial n_\uparrow n_\downarrow} = \frac{\partial \mu_\downarrow}{\partial n_\uparrow}$$

$$= -\alpha \frac{2\epsilon_{F\uparrow}}{n_\uparrow} \propto \frac{1}{k_{F\uparrow}}$$

$$\mu_\downarrow = -\alpha \epsilon_{F\uparrow}$$

$\alpha \simeq 0.6$ Pilati & Giorgini 2006, Prokof'ev & Svistunov 2008



We get:

Lindhard function

$$\frac{d\mathbf{P}_\downarrow}{dt} = -2\pi |U|^2 \int \frac{d^3 q}{(2\pi)^3} \mathbf{q} \int_{-\infty}^{\infty} d\omega \frac{\mathrm{Im}\chi_\downarrow(q, \omega_\mathbf{q} - \omega)\mathrm{Im}\chi_\uparrow(q, \omega)}{(1 - \mathrm{e}^{\beta(\omega - \omega_\mathbf{q})})(1 - \mathrm{e}^{-\beta\omega})}$$

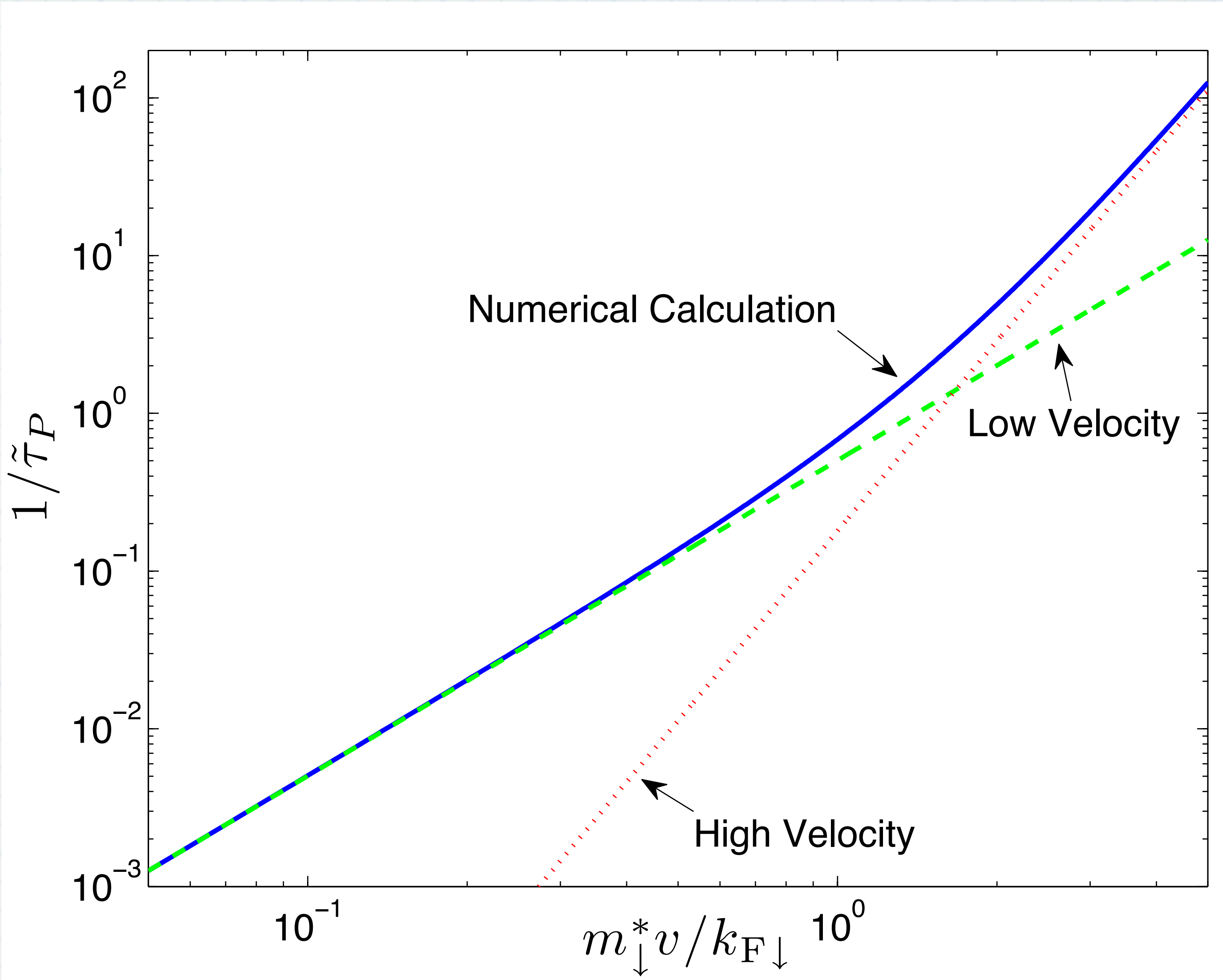
Results

T=0. Low velocity regime $m^*_\downarrow v \ll k_{F\downarrow}$

$$\frac{1}{\tau_P} = \frac{4\pi}{25}\frac{1}{\tau_0}\left(\frac{m^*_\downarrow v}{k_{F\downarrow}}\right)^2, \quad 1/\tau_0 = |\alpha|^2 k^4_{F\downarrow}/m^*_\downarrow k^2_{F\uparrow}$$

T=0. High velocity regime $k_{F\downarrow} \ll m^*_\downarrow v \ll k_{F\uparrow}$

$$\frac{1}{\tau_P} = \frac{2\pi}{35}\frac{1}{\tau_0}\left(\frac{m^*_\downarrow v}{k_{F\downarrow}}\right)^4$$



$T \ll T_{F\downarrow} \ll T_{F\uparrow}$

$$\frac{1}{\tau_P} = \frac{\pi^3}{9}\frac{1}{\tau_0}\left(\frac{T}{T_{F\downarrow}}\right)^2$$

GMB, A. Recati, C. J. Pethick, H. Smith, & S. Stringari PRL 100, 240406 (2008)

Dipole oscillation of spin polaron

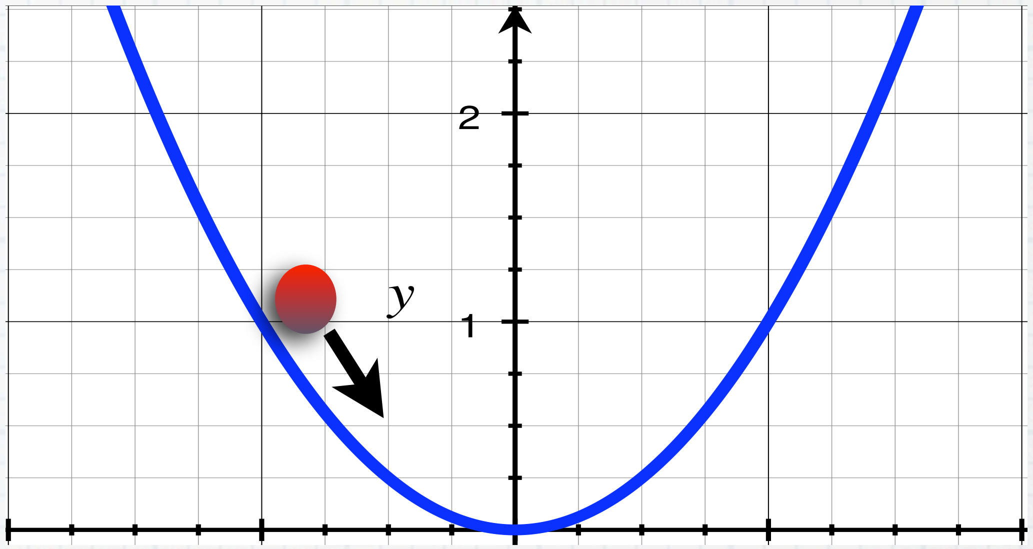
Potential seen by spin polaron

$$V_\downarrow(r) - \alpha\epsilon_{F\uparrow}(r)$$

$$V_\uparrow(r) + \epsilon_{F\uparrow}(r) = \mu_\uparrow$$

$$\Updownarrow$$

$$V_\downarrow(r) + \alpha V_\uparrow(r)$$

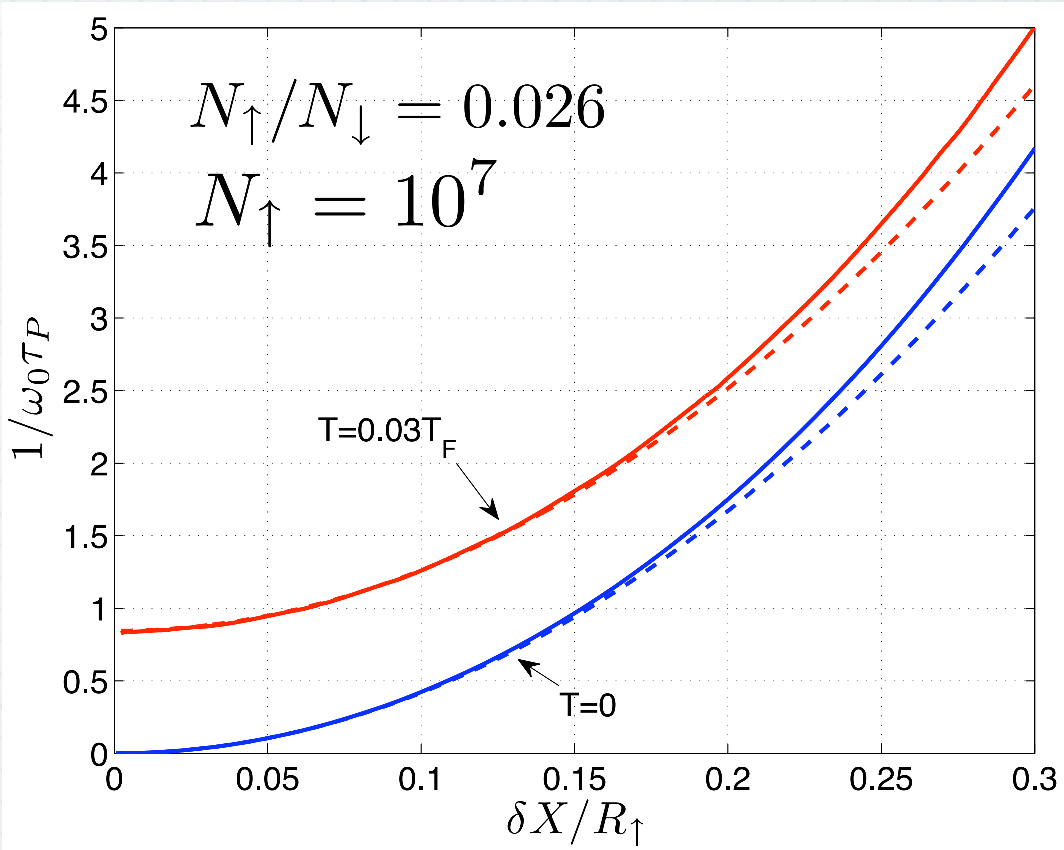


δX $\quad v = \omega_{\mathrm{D}}\delta X$

Frequency of dipole oscillation (Collisionless regime):

$$H = p^2/2m_\downarrow^* + V_\downarrow + \alpha V_\uparrow$$

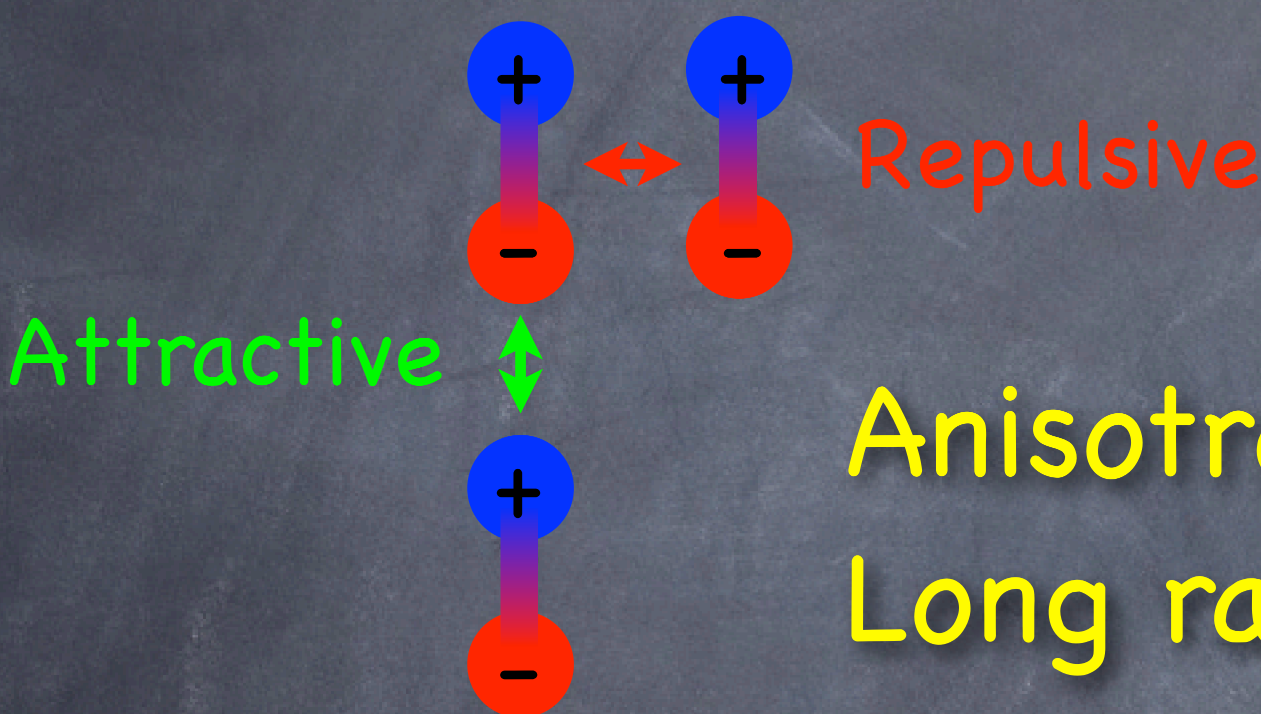
$$\omega_{\mathrm{D}} = \omega_\downarrow\sqrt{\frac{m_\downarrow}{m_\downarrow^*}\left(1 + \frac{m_\uparrow\omega_\uparrow^2}{m_\downarrow\omega_\downarrow^2}\alpha\right)}$$

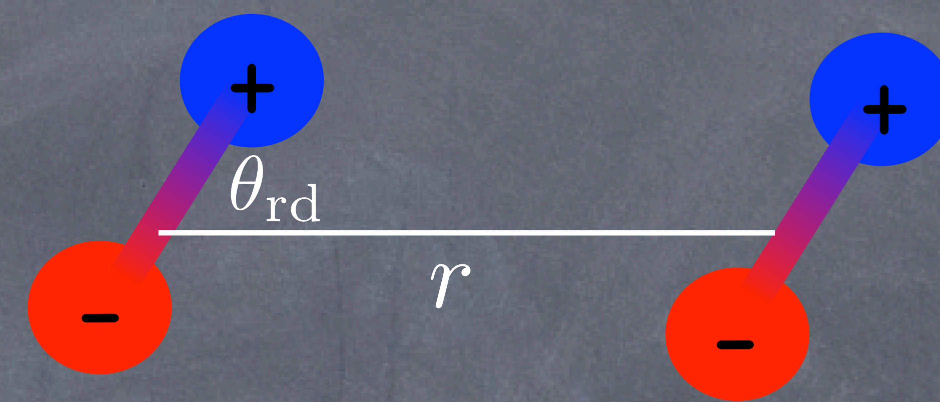


Measure interaction effects

Damping: $$\frac{1}{\omega_0\tau_P} = \frac{8\pi}{25}(6N_\uparrow)^{1/3}\alpha^2\frac{m_\downarrow^*}{m_\uparrow}\left(\frac{T_{\mathrm{F}\downarrow}}{T_{\mathrm{F}\uparrow}}\right)^2\left(\frac{\delta X}{R_\uparrow}\right)^2 + \frac{2\pi^3}{9}(6N_\uparrow)^{1/3}\alpha^2\frac{m_\downarrow^*}{m_\uparrow}\left(\frac{T}{T_{\mathrm{F}\uparrow}}\right)^2$$

Dipolar Atoms/Molecules





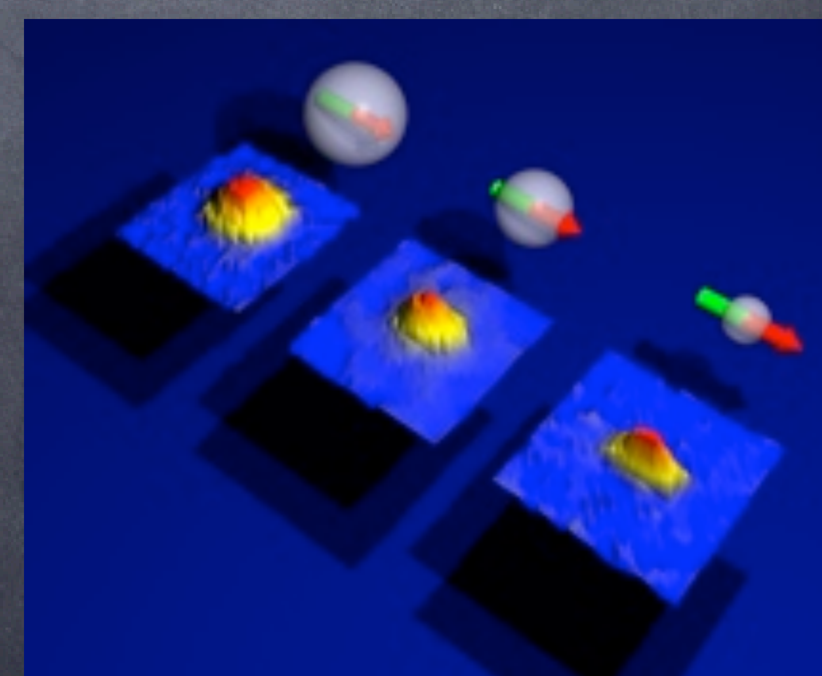
Anisotropic

Long range

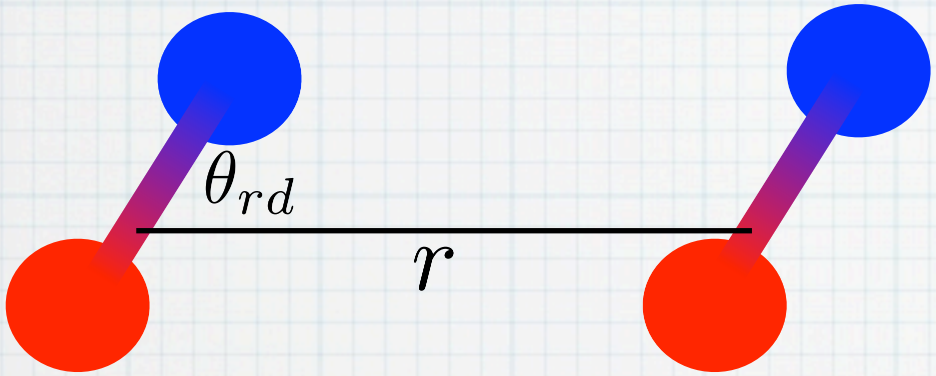
$$V(\mathbf{r}) = D^2 \frac{1 - 3\cos^2\theta_{\mathrm{rd}}}{r^3}$$

Experiments: ^{52}Cr $\quad$ ^{133}Cs^{85}Rb

^{133}Cs^{7}Li $\quad$ ^{40}K^{87}Rb

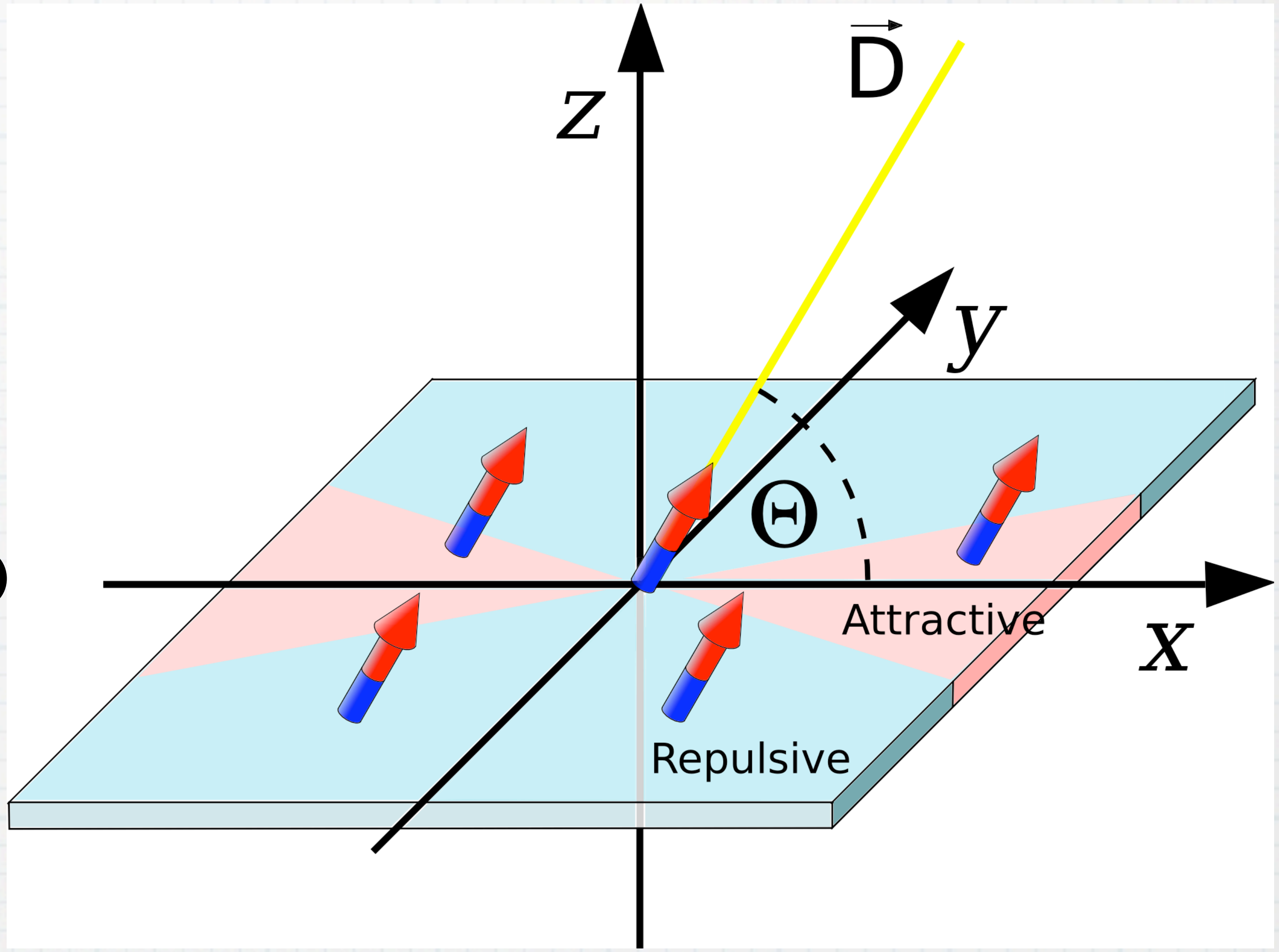


Collapse and pairing in 2D



$$V(\mathbf{r}) = D^2 \frac{1 - 3\cos^2\theta_{rd}}{r^3} \quad D^2 = \frac{d^2}{4\pi\epsilon_0}$$

State of system depends on Θ



$\Theta = \frac{\pi}{2}$ Wigner crystal

H. P. Büchler et al. PRL 98, 060404 (2007)

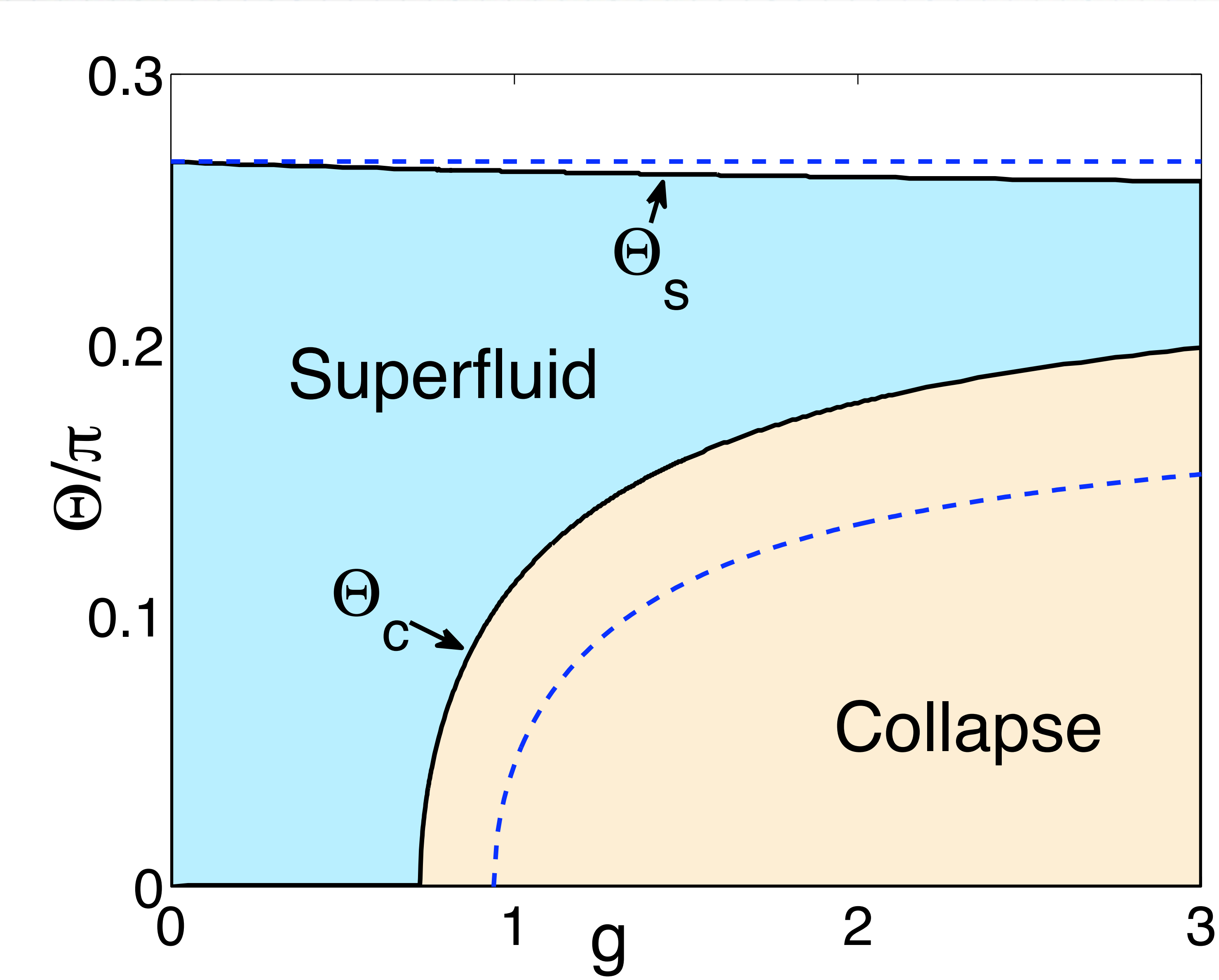
$$\Theta < \arccos(1/\sqrt{3})$$

Collapse Instability

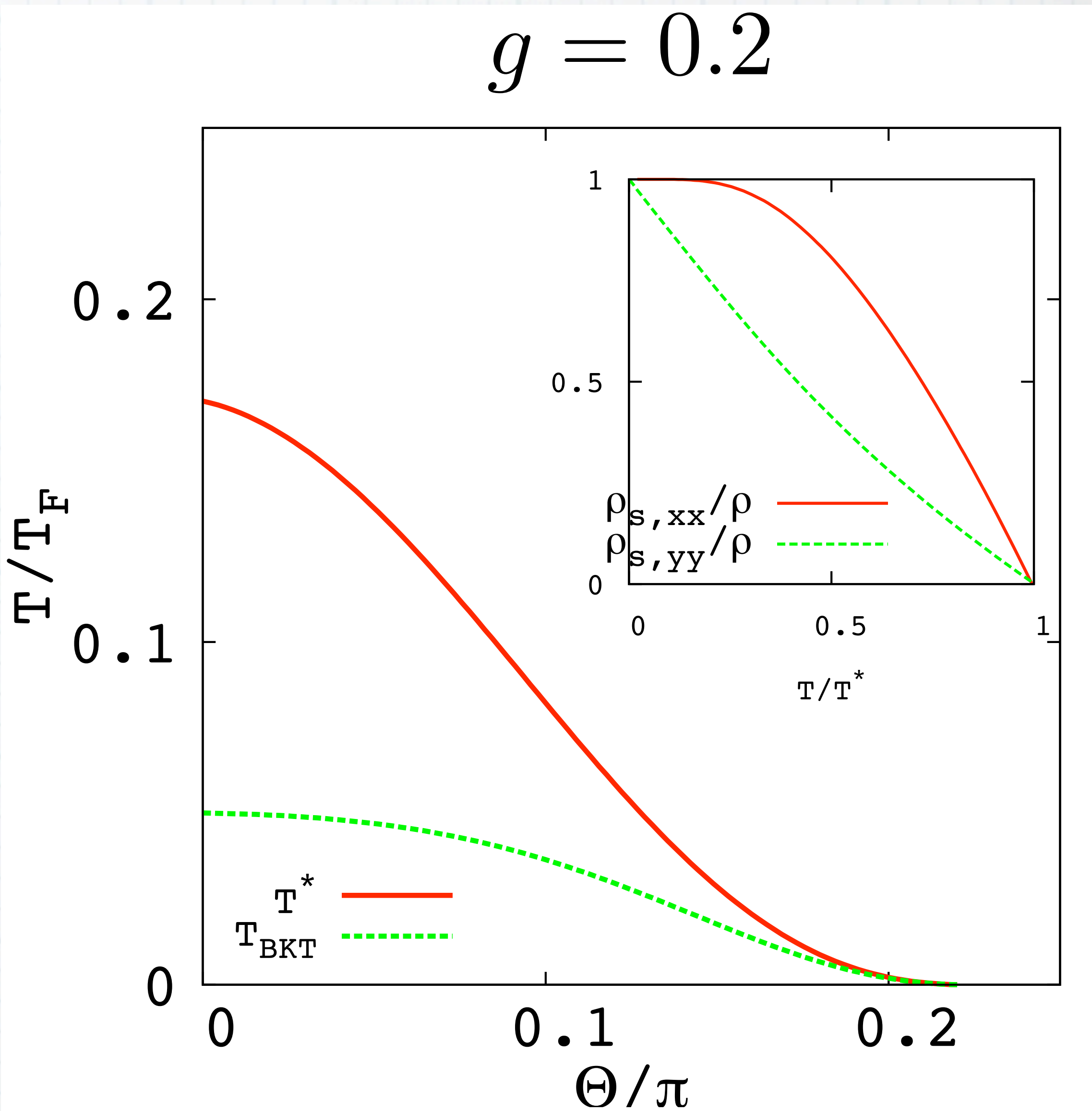
Interaction partly attractive

Superfluidity

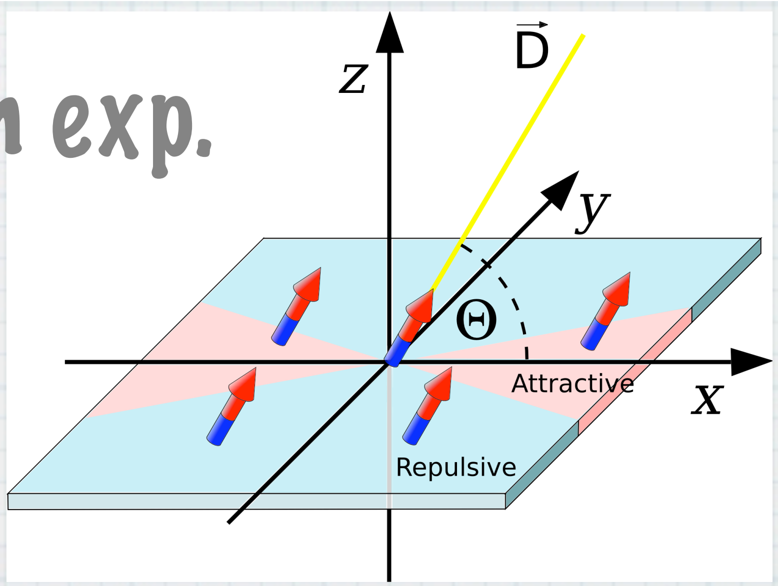
Phase Diagram



BKT transition



$$g = \frac{2D^2 k_F^3}{3\pi\epsilon_F} = 2\left(\frac{d}{1\text{Debye}}\right)^2 \frac{1\mu\text{m}}{r_0}\frac{N_A}{100} \simeq 0.8$$ Jin exp.



Significant superfluid region

Strong pairing without collapse

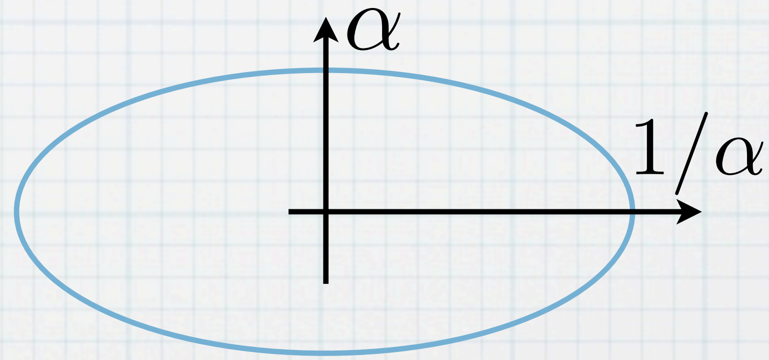
Collapse Instability

Compressibility $\kappa^{-1} = n_{2D}^2 \dfrac{\partial^2 \mathcal{E}}{\partial n_{2D}^2}$ Collapse for $\kappa < 0$

$$\mathcal{E}_{\rm n} = \mathcal{E}_{\rm kin} + \mathcal{E}_{\rm dir} + \mathcal{E}_{\rm ex}$$

Kinetic Energy $\mathcal{E}_{\rm kin} = \dfrac{1}{(2\pi)^2} \int d^2\mathbf{k} \dfrac{k^2}{2m} f_{\mathbf{k}} = \dfrac{\pi}{2} \dfrac{n_{2D}^2}{m} (\dfrac{1}{\alpha^2} + \alpha^2)$

Deformed Fermi surface



T. Miyakawa et al. PRA 77, 061603 (2008)

$$\mathcal{E}_{\rm dir} = \frac{1}{2L^2} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 n(\mathbf{r}_1, \mathbf{r}_1) V(\mathbf{r}) n(\mathbf{r}_2, \mathbf{r}_2)$$

$$n(\mathbf{r}_1, \mathbf{r}_2) \equiv \langle \hat{\psi}^\dagger(\mathbf{r}_1) \hat{\psi}(\mathbf{r}_2) \rangle$$

$$\mathcal{E}_{\rm ex} = -\frac{1}{2L^2} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 n(\mathbf{r}_1, \mathbf{r}_2) V(\mathbf{r}) n(\mathbf{r}_1, \mathbf{r}_2)$$

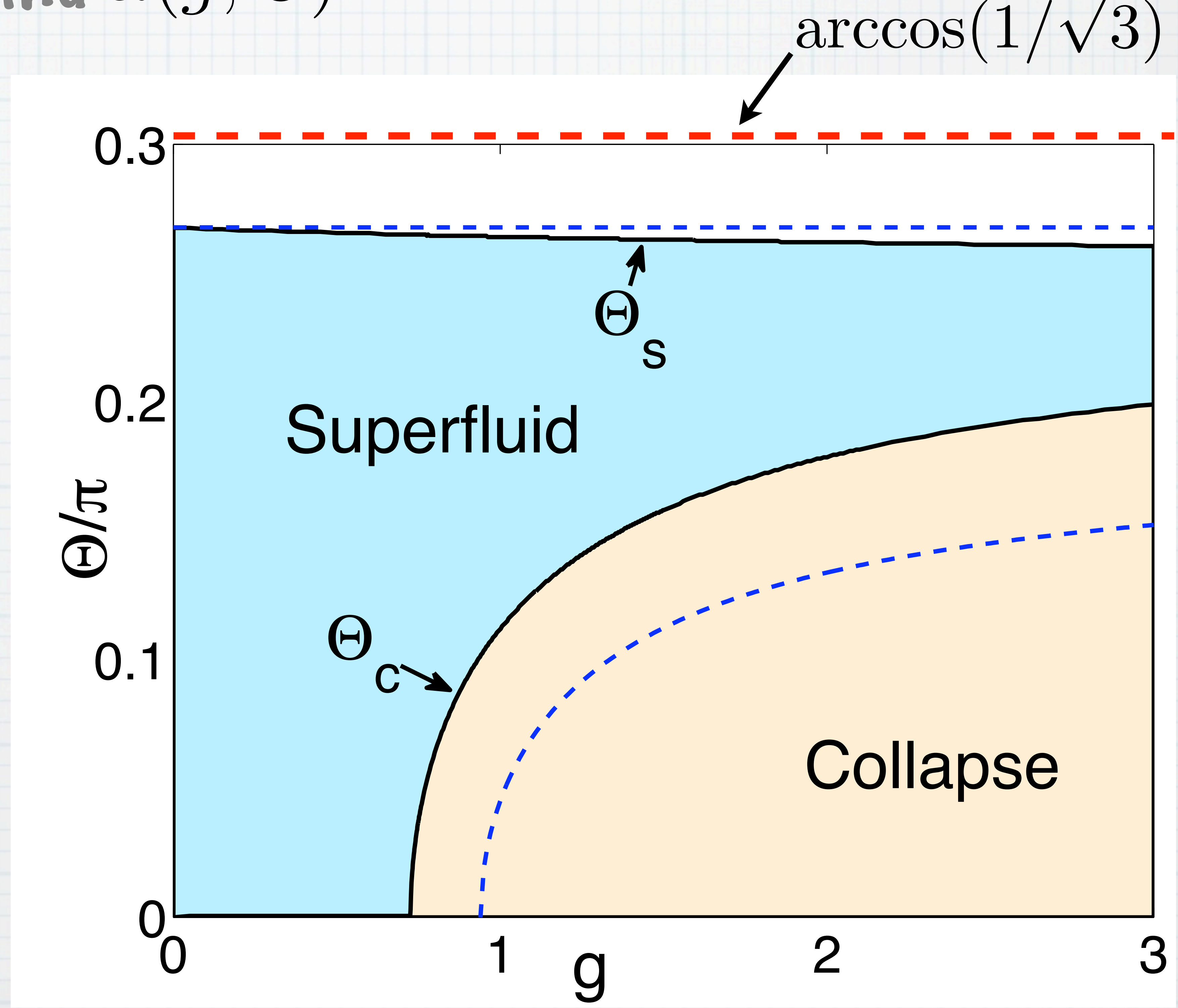
In total: $\mathcal{E} = \frac{\pi}{2}\frac{n_{2D}^2}{m}(\frac{1}{\alpha^2}+\alpha^2) - \frac{8n_{2D}^2}{3\pi m} g I(\alpha,\Theta)$

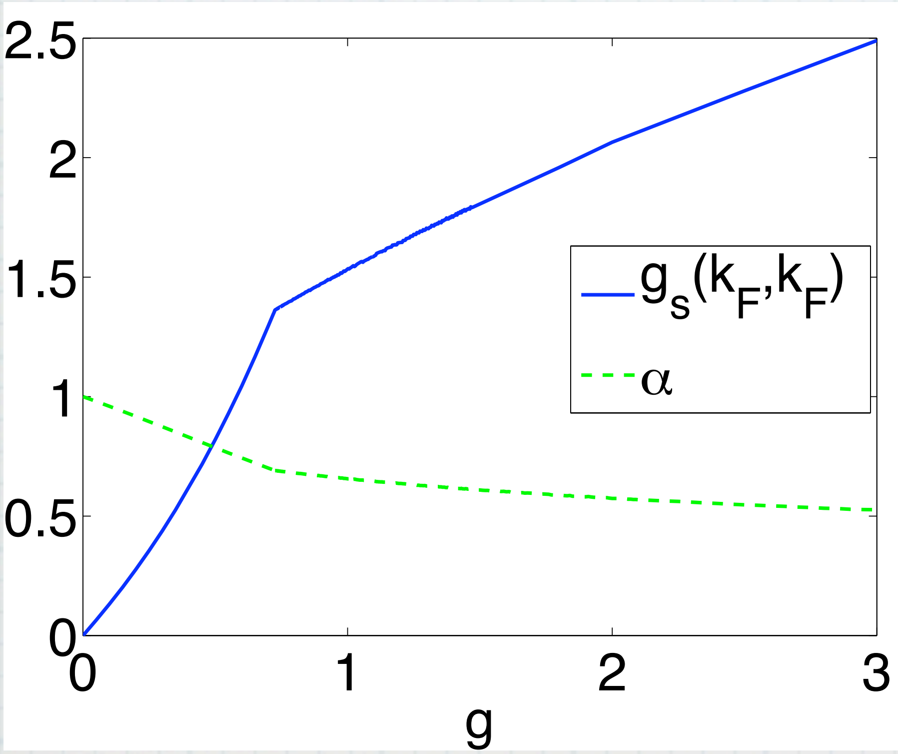
Dimensionless function

Minimize $\mathcal{E}$ to find $\alpha(g,\Theta)$

Collapse for

$$\frac{\partial^2 \mathcal{E}}{\partial n_{2D}^2} < 0$$

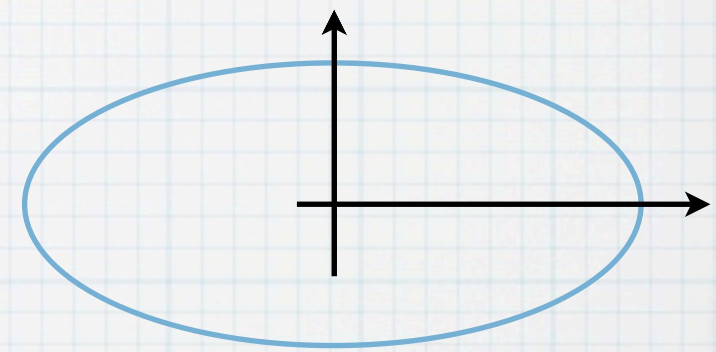




Superfluid Phase

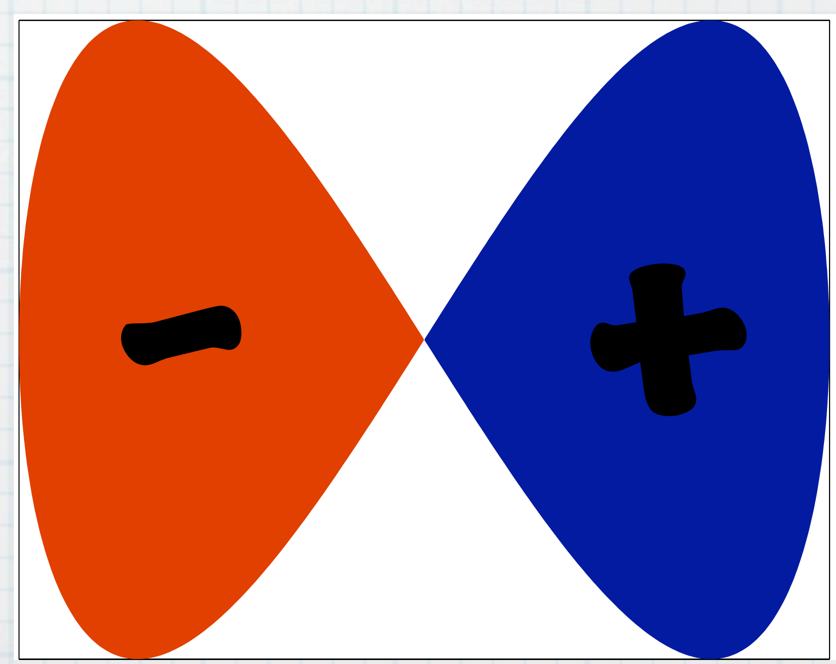
Gap equation: $\Delta_{\tilde{\mathbf{k}}} = -\int \frac{d^2\tilde{\mathbf{k}}'}{(2\pi)^2} \tilde{V}_{2D}(\tilde{\mathbf{k}}, \tilde{\mathbf{k}}') \frac{\Delta_{\tilde{\mathbf{k}}'}}{2E_{\tilde{\mathbf{k}}'}}$ $\quad \tilde{\mathbf{k}} = (\alpha k_x, k_y/\alpha)$

2D interaction $V_{2D}(\rho) = \int dz \Phi_r^2(z) V(\mathbf{r})$

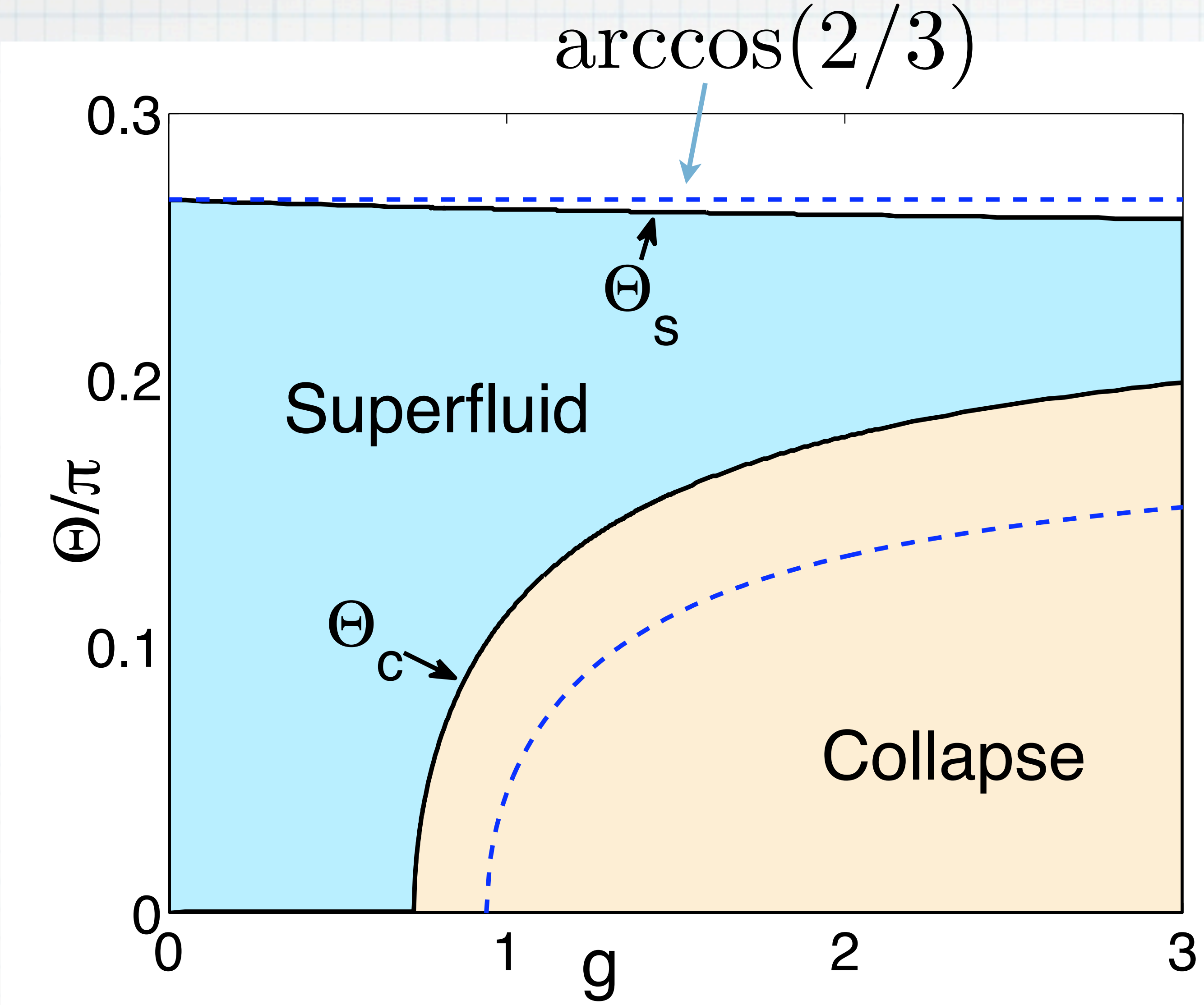


$$V_{2D}(\mathbf{k}, \mathbf{k}') = \int d^2\rho \sin(\mathbf{k}\cdot\rho) V_{2\mathrm{D}}(\rho) \sin(\mathbf{k}'\cdot\rho)$$

High energy cut-off for $k' l_z \gtrsim 1$

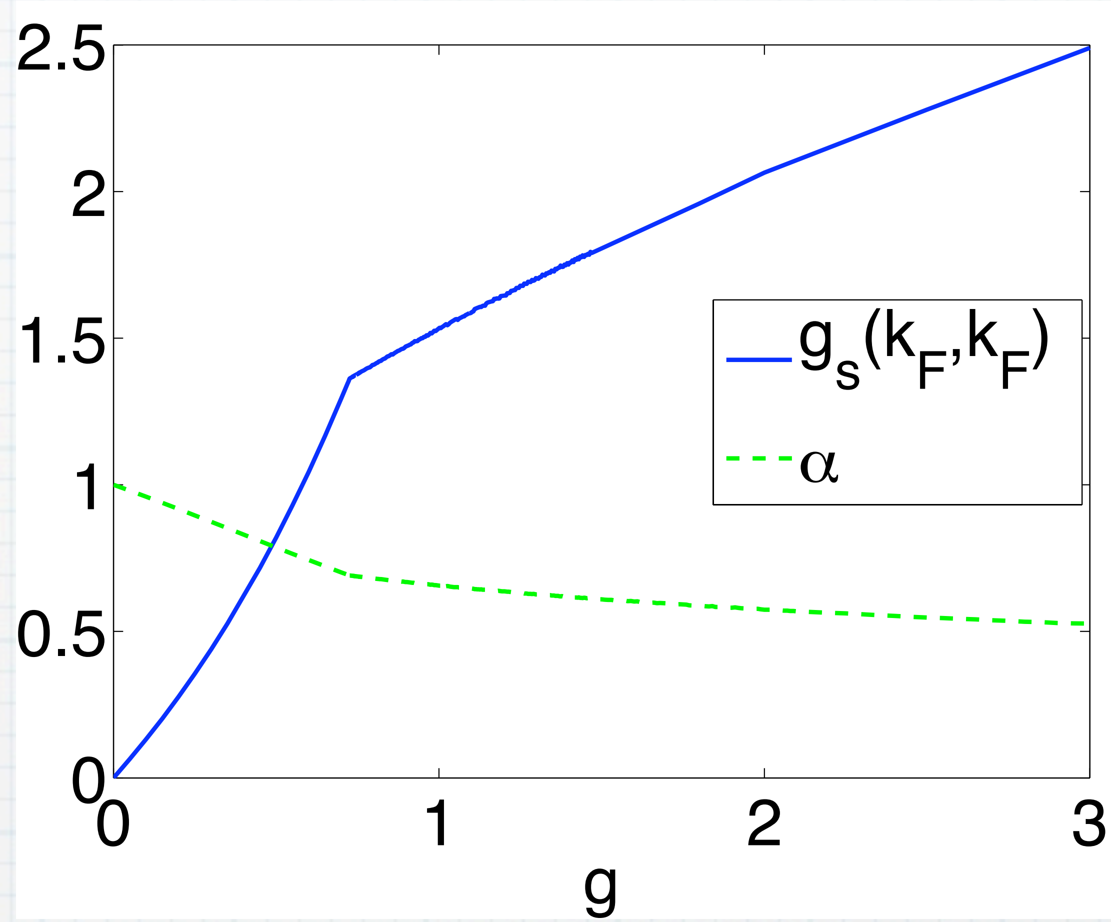


p-wave pairing $\Delta_{\tilde{\mathbf{k}}} \simeq \Delta \cos\phi$



Maximum pairing strength:

$g_s(k_F, k_F)$ along $\Theta_c(g)$



Significant superfluid region

Strong pairing without collapse

Weak coupling:

$$\Delta_0(\Theta) \propto \epsilon_F e^{1/g_s(\Theta)}$$

Infinite order QPT $$\left.\frac{\partial^n \Delta_0}{\partial \Theta^n}\right|_{\Theta_s} = 0$$

Berezinskii-Kosterlitz-Thouless transition

Phase fluctuations destroy long range order in 2D

Above T_{BKT} the vortex-antivortex pairs proliferate

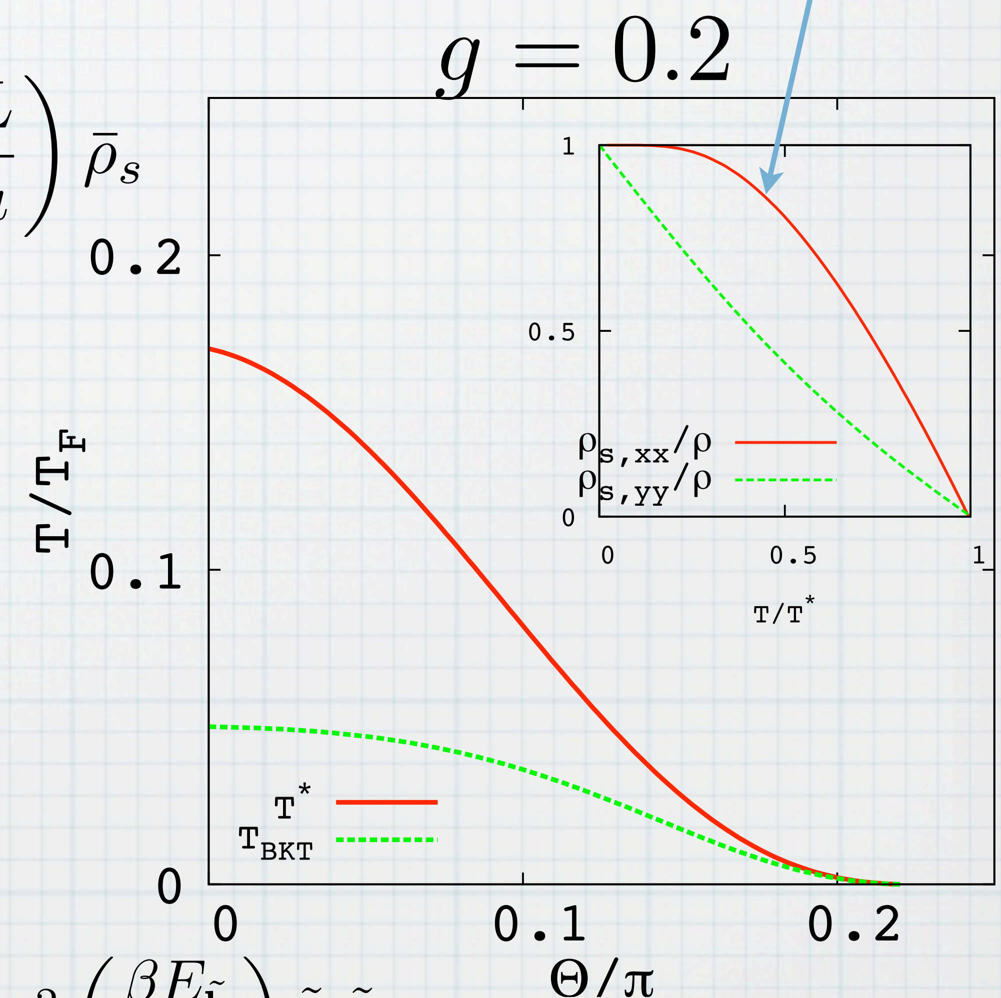
Anisotropic superfluid density

Vortex energy: $E = \frac{1}{2}\int d^2r\rho_{s,ij}v_iv_j \approx \frac{\pi\hbar^2}{4m^2}\ln\left(\frac{L}{a}\right)\bar{\rho}_s$

Entropy: $S = 2k_B\ln\left(\frac{L}{a}\right)$

Phase transition: $E - T_{\rm BKT}S = 0$

$$T_{\rm BKT} = \frac{\pi\hbar^2}{8m^2k_B}\bar{\rho}_s$$



Superfluid density: $\rho_{s,ij}(T) \;=\; mn\delta_{ij} - \frac{\beta}{4}\sum_{\tilde{\mathbf{k}}}\text{sech}^2\left(\frac{\beta E_{\tilde{\mathbf{k}}}}{2}\right)\tilde{k}_i\tilde{k}_j$

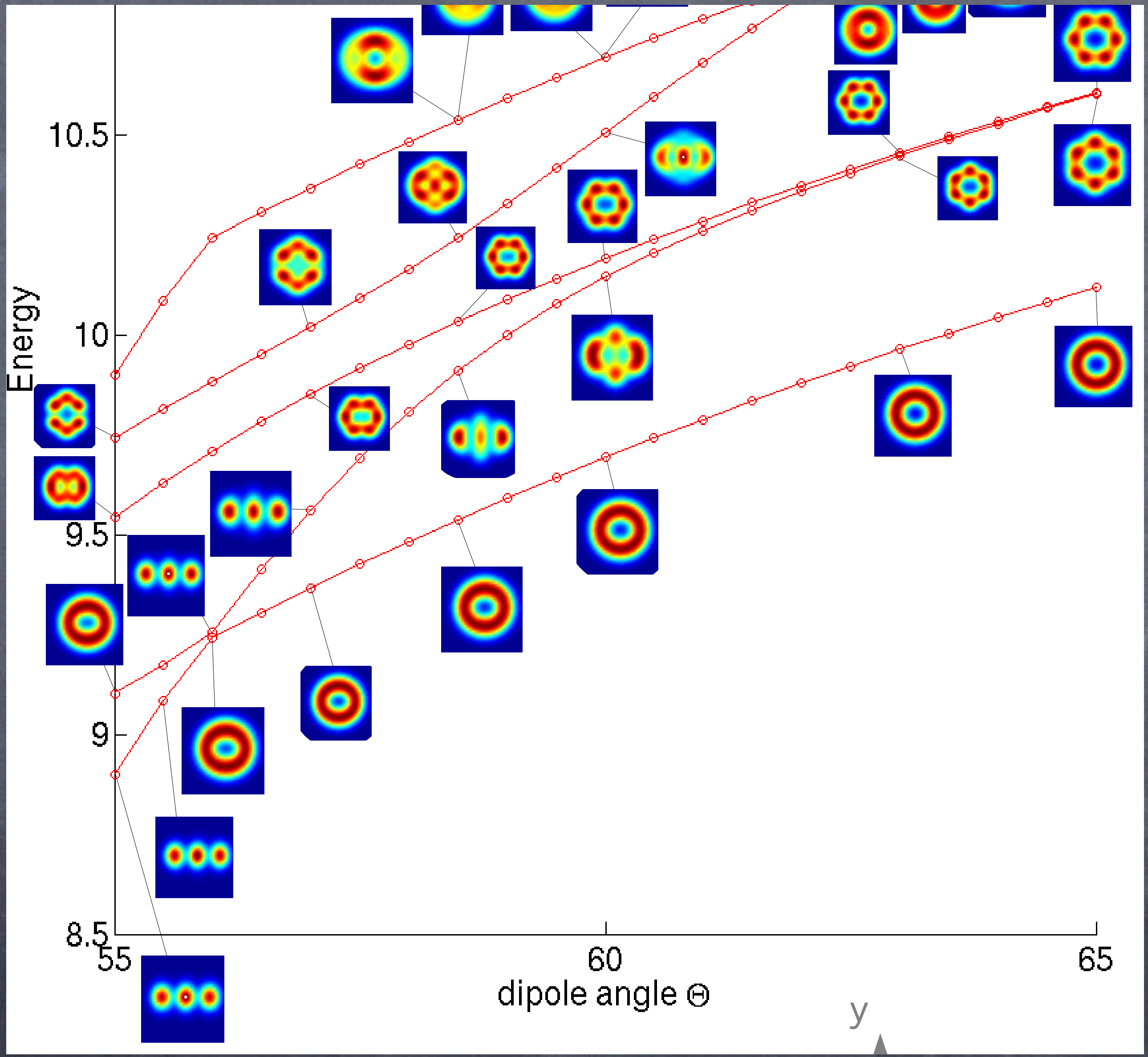
Conclusions

* 2D gas -> Angle new degree of freedom
* Pairing can be strong yet the system stable
* Berezinskii-Kosterlitz-Thouless
* Experimentally realistic with electric dipoles

GMB and E. Taylor, PRL 101, 245301 (2008)

Interesting new phase diagrams

3 dipoles in a 2D trap



Funny Wigner states