NORDITA Quantum Solids, Liquids, Gases --- Stockholm, Sweden --- 22 July 2010

p-Orbital ultracold particles and Bose-Einstein crystal

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[View of Pittsburgh---source: PittsburghSkyline.com]

Acknowledgement

Our Work:

- PRA 2006
- PRL 2006
- PRL 2008a
- PRL 2008b
- PRL 2010
- rXiv:0910.2431
- Xiv:1005.4027

Done by group members:

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in collaboration with

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Joel Moore *(Berkeley)*,
Kai Sun *(Maryland)*, Congjun Wu *(UCSD)*

Thanks for Support by
U.S. ARO W911NF-07-1-0293 (individual) and
DARPA OLE W911NF-07-1-0464 (Hulet/Rice team)

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Cold Atoms/Optical Lattices Program, Santa Barbara 2010

Kavli Institute for Theoretical Physics (KITP), Santa Barbara

Program: "Beyond Standard Optical Lattices," **Sep 13—Dec 10, 2010** (3 months)

Coordinators:

Ehud Altman
Maciej Lewenstein
W. Vincent Liu*

Scientific advisors:

Immanuel Bloch
Sankar Das Sarma
Misha Lukin
William Phillips

More details are announced at the KITP webpage: http://www.kitp.ucsb.edu/

Conference: "Frontiers of ultracold atoms and molecules," **Oct 11-15, 2010**

Coordinators: Ofir Alon, Immanuel Bloch, W. Vincent Liu*, William Phillips

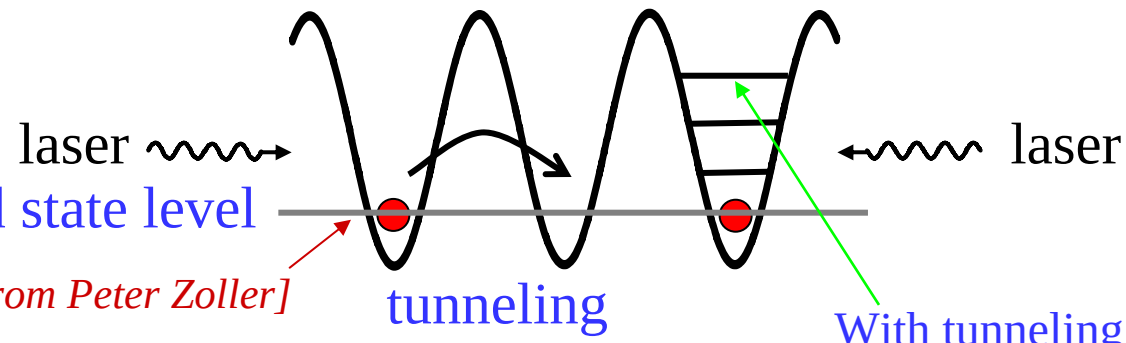
*=lead coordinator

Part 1. Introduction:
What is the p-band, Why?

p-band of an optical lattice (1D illustration)

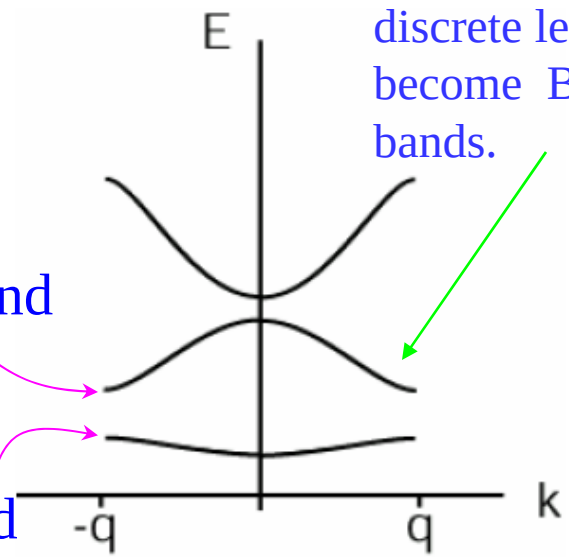
- optical lattice created by laser standing waves
- Based on AC Stark shifts

lattice spacing $\frac{\lambda_L}{2} \sim 400\text{nm}$
Level spacing $\sim 1\text{MHz} \sim 50\mu\text{K}$.



[Picture: gift from Peter Zoller]

With tunneling, discrete levels become Bloch bands.



- **Bose-Hubbard model**

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{1}{2} U \sum_i b_i^\dagger b_i^\dagger b_i b_i$$

theory: Jaksch et al, PRL 1998
exp.: Bloch et al, Nature 2002

- **This talk**

A new direction: p-orbital physics in optical lattices

- **Orbital degeneracy** (*px, py, pz orbitals*) is considerably less understood in comparison. Implies emergent symmetry.
- **Similar to spin physics** but is different fundamentally.
- **Strong anisotropy**: Anisotropy is an interesting new feature, not a problem!
- ***p*-orbitals are different than *d*-orbital in solids:** Parity ODD. New possibility---*p*-orbital bosons as opposed to *d*-electrons (fermions) in solids
- unique to cold atom systems, not previously available in conventional condensed matter systems.

→ **new quantum phases (a main motivation of our study).**

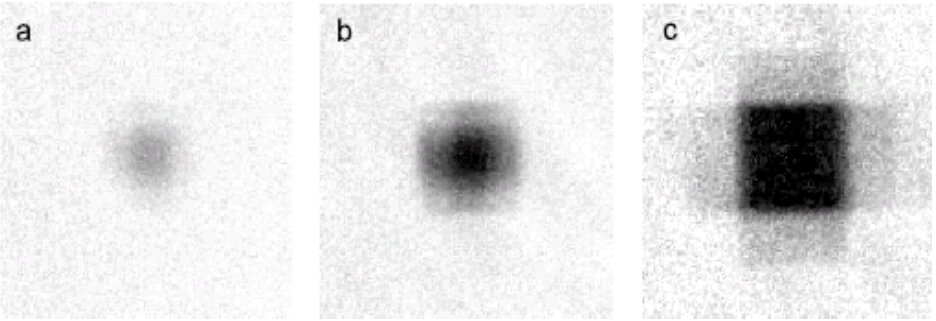
Theoretical studies on the first excited (*p-orbital*) bands

An incomplete list!! ***[red=WVL involved]***

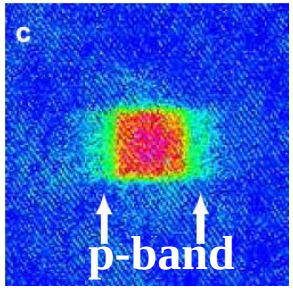
- V. Scarola, S. Das Sarma, *Phys. Rev. Lett.* (2005)
- A. Isacsson and S. Girvin, *Phys. Rev. A* 72, 053604 (2005).
- A. B. Kuklov, *Phys. Rev. Lett.* (2006)
- WVL and C. Wu, *Phys. Rev. A* (2006)
- C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006).
- A. F. Ho, arXiv:cond-mat/0603299
- C. Xu and M. P. A. Fisher, *Phys. Rev. B* 75, 104428 (2007)
- C. Wu, D. Bergman, L. Balents, and S. Das Sarma, *Phys. Rev. Lett.* (2007)
- A. Kantian, A. J. Daley, P. Törmä and P. Zoller, *New J. Phys.* (2007)
- L. Guo, Y. Zhang, and S. Chen, *Phys. Rev. A (2007)*
- E. Zhao and WVL, *Phys. Rev. Lett.* (2008);
- R. O. Umucallar and M. Ö. Oktel, *Phys. Rev. A* (2008)
- K. Wu and H. Zhai, *Phys. Rev. B* (2008)
- L. Wang, X. Dai, S. Chen, X. C. Xie, *arXiv:0805.2719* (2008)
- R. M. Lutchyn, S. Tewari, S. Das Sarma, *arXiv:0812.0815 (2008)*
- V. Stojanovic, C. Wu, WVL and S. Das Sarma, *Phys. Rev. Lett.* (2008)
- …
- K. Sun. E. Zhao, WVL, *Phys. Rev. Lett.* (2010)
- Z. Zhang, H. H. Hung, C.M. Ho, E. Zhao, WVL, *arXiv:0910.2431*
- ...

Orbital physics of interacting cold atoms

- **<u>Fermions in p-band</u>** M. Köhl et al, PRL **94**, 080403 (2005)

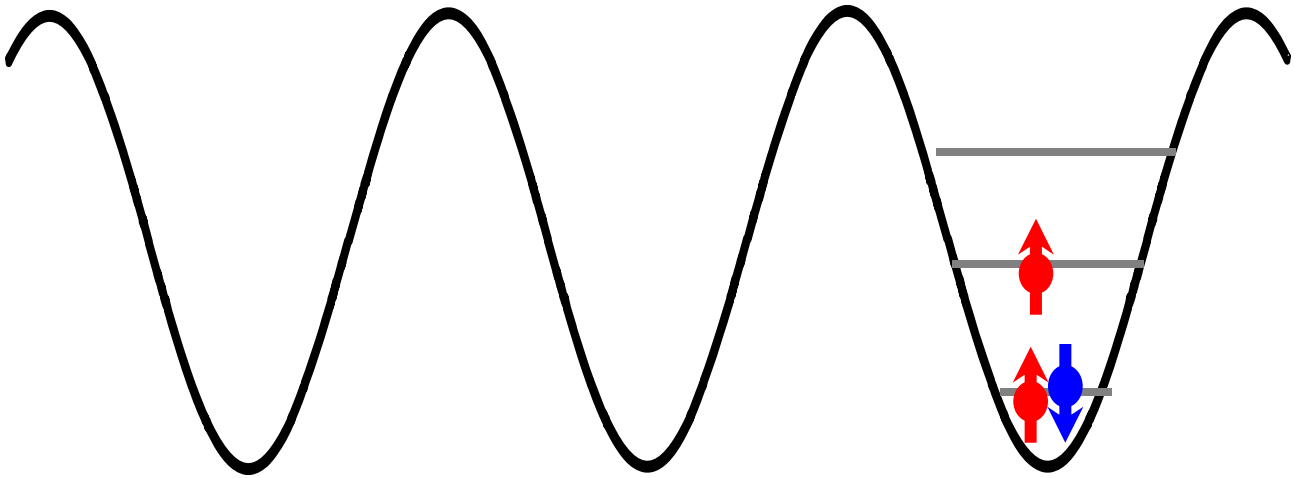


Fermi Surface vs band filling



Fermions are transferred into the p-band using a sweep across the Feshbach resonance, i.e., by strong interaction.

Preparation of p-band fermions

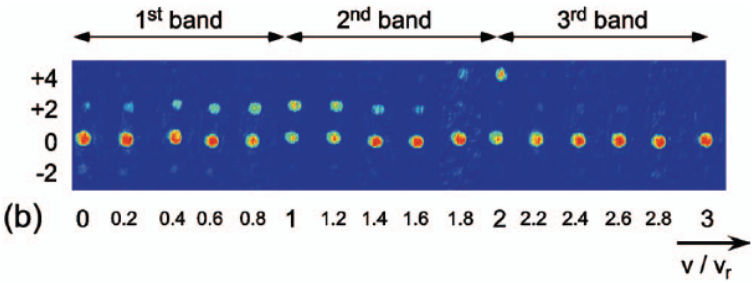


- ***Even simpler:*** Fill the lowest s-band with the specific species (say spin up). Just by having more than 2 particles/site for two-components
- p-band fermions should have no problem in lifetime (Pauli exclusion principle)

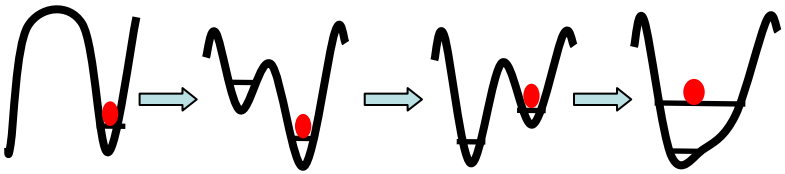
Bosons in the optical lattice p-band

Experiments

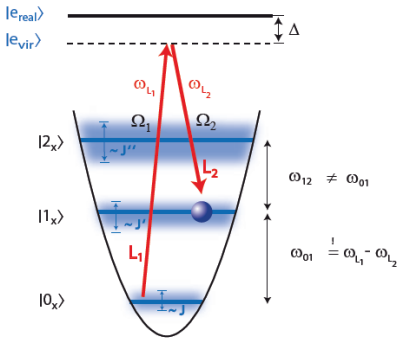
- By moving lattices [A. Browaeys, W. D. Phillips, et al, PRA **72**, 053605 (2005)]



- Dynamically deforming the double-well lattice [NIST Porto/Phillips groups: PRA (2006); J. Phys. B (2006); PRL 2007; Nature 2007; …]



- Pumping bosons by Raman transition [T. Mueller, I. Bloch et al., PRL 2007]



Theories:

- Isacsson & Girvin, PRA 05
- Kuklov, PRL 06
- WVL & Wu, PRA 06

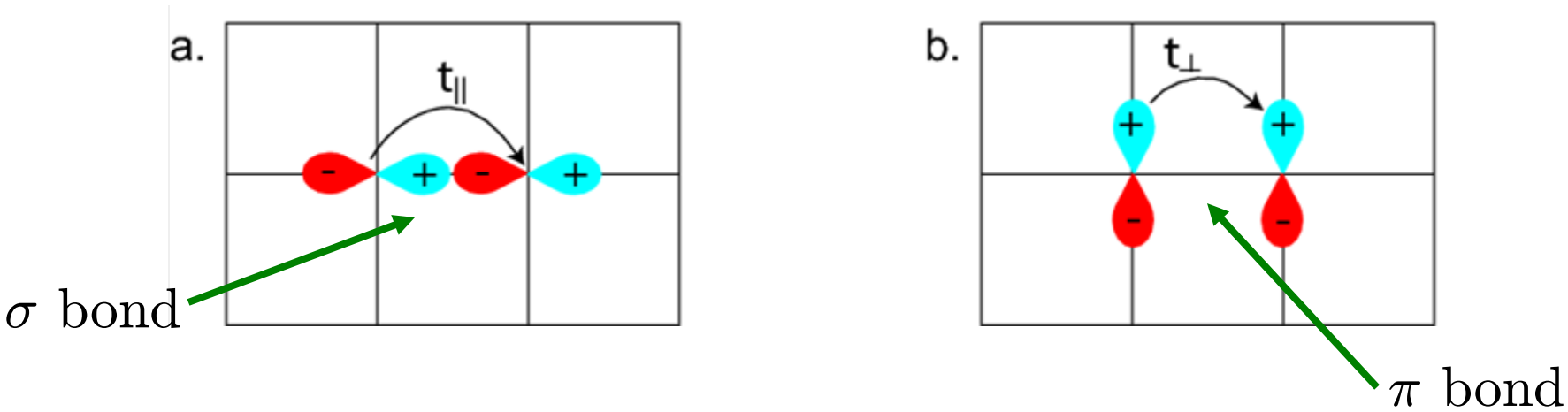
Lifetime issue: to be discussed.

Part 2.
Novel orbital BEC or topological phases

p-orbital Bose-Hubbard model: 3D cubic lattice

[derived in WVL and C. Wu, PRA (2006)]

$$H = \sum_{\mathbf{r}\mu} [t_{\parallel}\delta_{\mu\nu} - t_{\perp}(1-\delta_{\mu\nu})]\left(b^{\dagger}_{\mu,\mathbf{r}+a\mathbf{e}_{\nu}} b_{\mu\mathbf{r}} + h.c.\right) + \tfrac{1}{2}U\sum_{\mathbf{r}}\left[n^2_{\mathbf{r}} - \tfrac{1}{3}\mathbf{L}^2_{\mathbf{r}}\right]$$



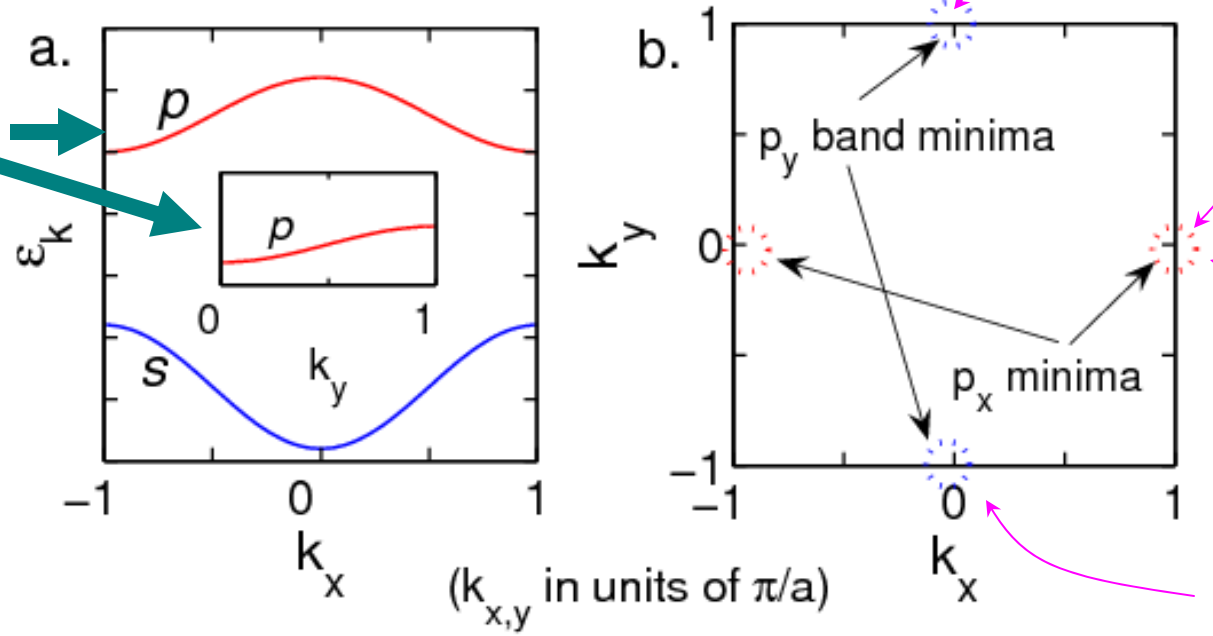
$\mu, \nu = x, y, z$ or p_x, p_y, p_z

Density field operator $n_{\mathbf{r}} = \sum_{\mu} b^{\dagger}_{\mu\mathbf{r}} b_{\mu\mathbf{r}}$

Angular momentum operator: $L_{\mu\mathbf{r}} = -i\sum_{\nu\lambda}\epsilon_{\mu\nu\lambda} b^{\dagger}_{\nu\mathbf{r}} b_{\lambda\mathbf{r}}$

New class of Bose-Einstein condensate (BEC)

p_x band



Expect BEC to take place at those minima of finite $\vec{k}$.

$$\mathbf{Q}_{p_x} = (\pi, 0, 0)$$

$$\mathbf{Q}_{p_y} = (0, \pi, 0)$$

$$\mathbf{Q}_{p_z} = (0, 0, \pi)$$

$$\boxed{a = \text{lattice constant}}$$

The *p*-orbital BEC (*p*-OBEC)

Parameterization of Order parameter:

$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \rho e^{i\varphi - i\vec{T}\cdot\vec{\theta}} \begin{pmatrix} \cos\chi \\ i\sin\chi \\ 0 \end{pmatrix}$$

U(1) phase

SO(3) orbital

T-reversal

[$\vec{T}$'s are three 3x3 matrices]

For a dilute lattice gas of U>0, the condensate is found to be a **non-zero-momentum p_x+ip_y BEC.**

$$\begin{pmatrix} \langle b_{x\mathbf{k}=\mathbf{Q}_x} \rangle \\ \langle b_{y\mathbf{k}=\mathbf{Q}_y} \rangle \\ \langle b_{z\mathbf{k}=\mathbf{Q}_z} \rangle \end{pmatrix} = \sqrt{\frac{\mathrm{Vol.} \times n_0^b}{2}} \begin{pmatrix} 1 \\ \pm i \\ 0 \end{pmatrix}$$

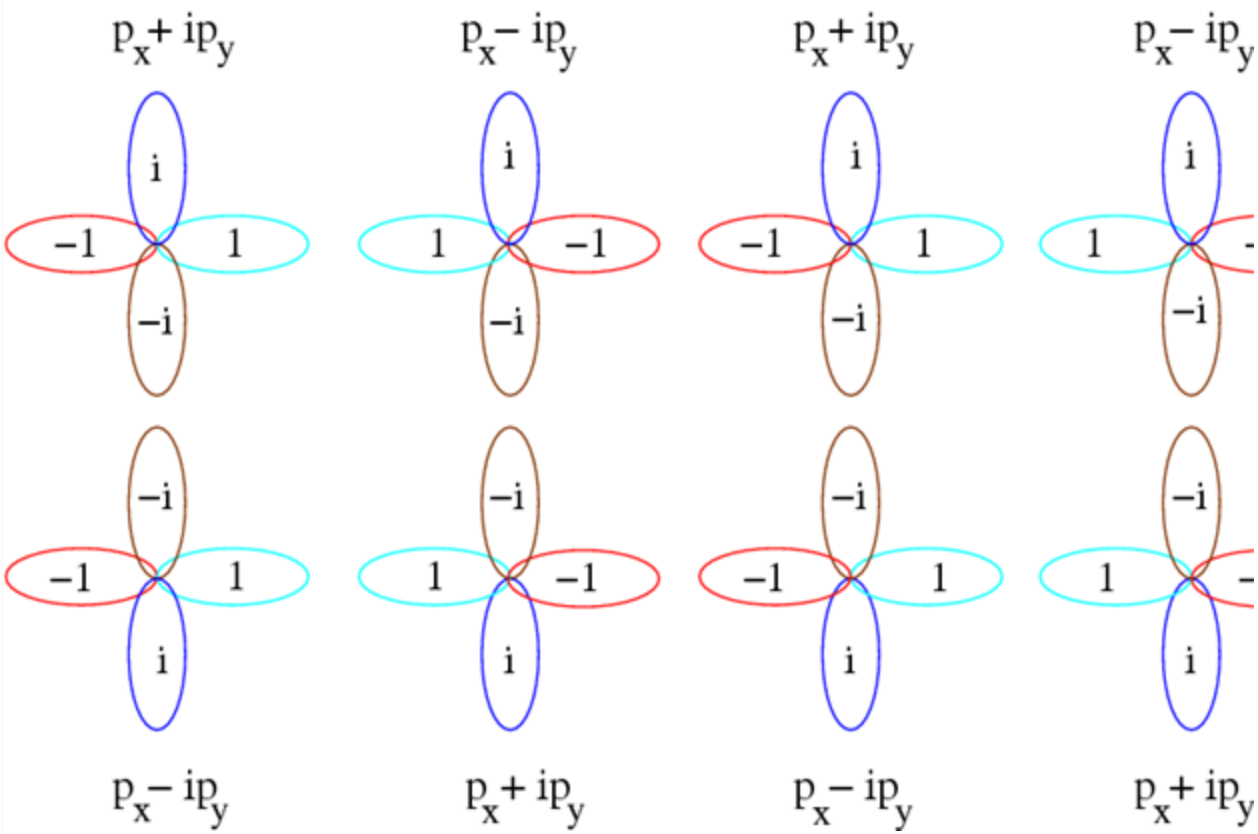
- A new concept that defies the paradigm of BEC!
- A metastable BEC wavefunction with nodes---not contradict Feynman's argument.

The state's configuration in the real space is:

Transversely Staggered Orbital Current (TSOC)

Example:

$p_x + ip_y$ state



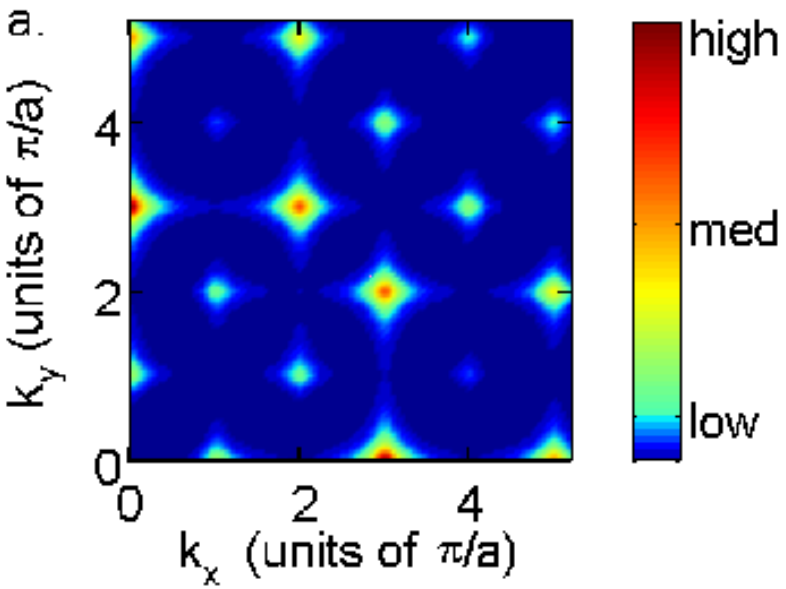
Quantitative Description of TSOC:

$$\langle L_{x\mathbf{r}} \rangle = \langle L_{y\mathbf{r}} \rangle = 0, \langle L_{z\mathbf{r}} \rangle = n_0^b (-)^{\frac{x+y}{a}} .$$

Prediction: non-zero momentum BEC of *p-orbital* atoms

Time-of-flight (TOF) experiment

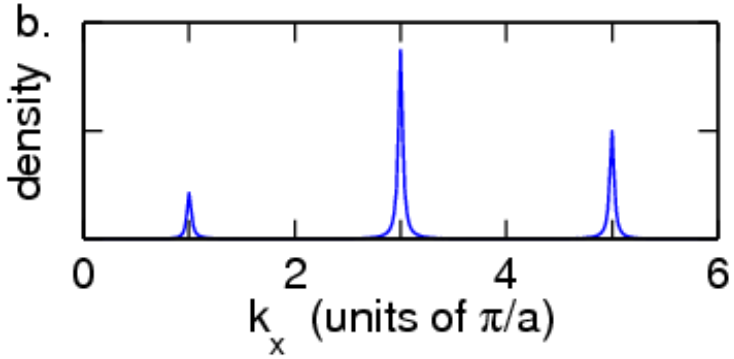
Peaks not at (0,0)!



p-orbital wavefunction imposes a *non-Gaussian* profile;

The highest moves when varying the size of the *p*-wavefunction.

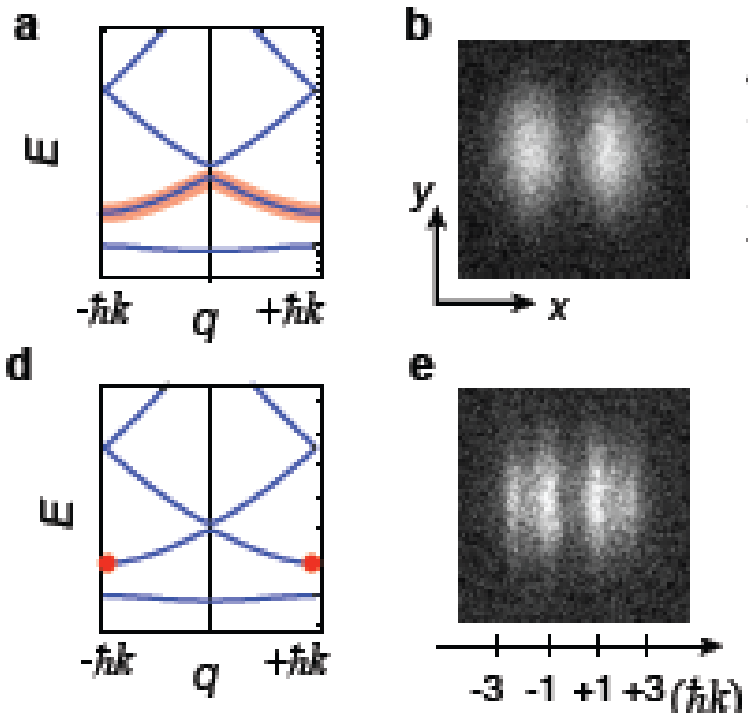
[WVL and C. Wu, PRA (2006)]



[Related results independently by: A. Isacsson, S. Girvin, PRA (2005); A. B. Kuklov, PRL 97, 110405 (2006)]

Experimental observation I: finite momentum BEC

by the Mainz/Bloch group [Mueller, Bloch, et al, PRL, 2007]



Initial time (incoherent) $20\mu s$

Later time (coherent) $1ms$

Mueller, Bloch et al [PRL 2007] confirmed the prediction of finite momentum BEC!

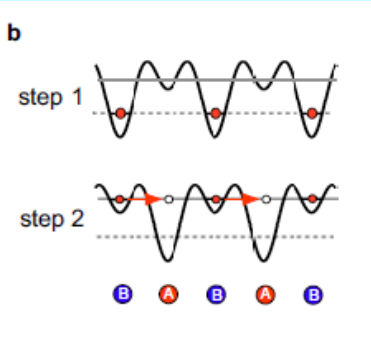
Experimental observation II: complex p-orbital superfluids

arXiv: 1006.0509

Orbital superfluidity in the P-band of an optical square lattice

Georg Wirth, Matthias Ölschläger, and Andreas Hemmerich
Institut für Laser-Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
(Dated: July 7, 2010)

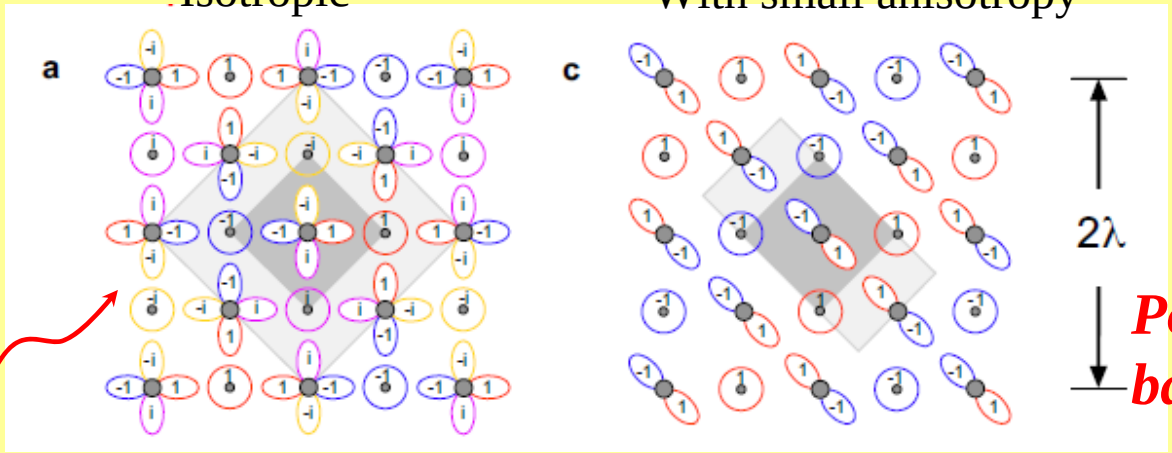
The successful emulation of the Hubbard model [1] in optical lattices [2, 3] has stimulated world wide efforts to extend their scope to also capture more complex, incompletely understood scenarios of many-body physics [4, 5]. Unfortunately, can be unexpectedly long, has further strengthened the potential significance of orbital physics in optical lattices [8, 9]. Previous experiments have demonstrated that the excitation of higher Bloch bands is in fact possible [18, 19]. In a recent experiment, investigating a homoge-



Nature of orbital order

Isotropic

With small anisotropy



complex $p_x \pm ip_y$

p_x rotated by 45°

Populate bands

Experimentally found as predicted by [WVL, C. Wu, PRA 2006]

Other lattices: finite-momentum BEC with circulating currents

- **Triangle lattices:** [C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006); more in next slide]
  - Is geometrically frustrated. Groundstate has super-lattice structure (orbital stripes). Currents circulate around neighboring triangles through nearest neighbor tunneling.
- **Double-well lattices** [V. Stojanovic, C. Wu, WVL, S. Das Sarma, *PRL 2008*]
  - *experimental system* realized in NIST (Porto group): J. Sebby-Strabley, et al, PRA (2006); M. Anderlini, et al, J. Phys. B (2006); Nature (2007); …
  - A new *supersolid-like* phase---incommensurate crystalline superfluidity with circulating super-currents. (Super-current density-wave order instead of charge).

Stripes of superfluid orbital-currents: 2D triangle lattice

Found "**quantum orbital stripes**" for all couplings [C. Wu, WVL, J. Moore, and S. Das Sarma, *Phys. Rev. Lett.* (2006).]

U(1) phase

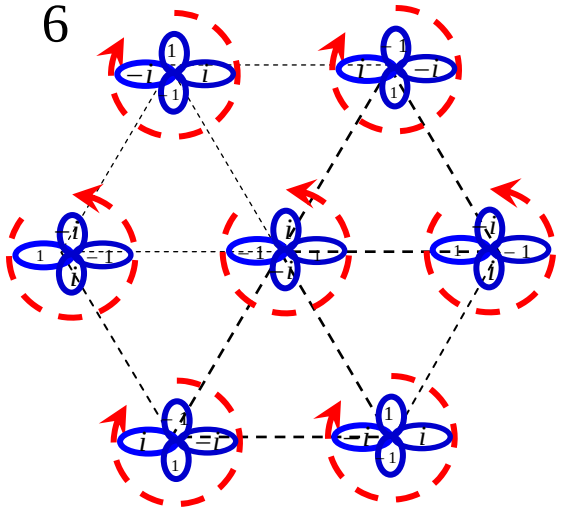
Ising variable
+1 or -1
Sign of orbital $\vec{L}$

$$e^{i\phi_r}(\cos\alpha|p_x\rangle + i\sigma_r \sin\alpha|p_y\rangle)$$

mixing angle

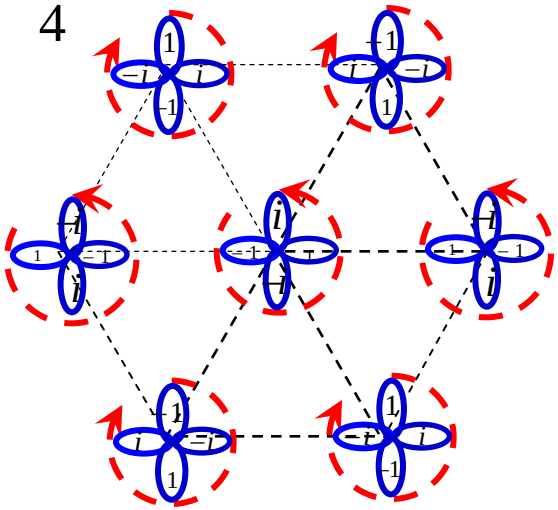
- Orbital wavefunction in lattice site ***r***:

$\alpha = \frac{\pi}{6}$



weak coupling

$\alpha = \frac{\pi}{4}$



strong coupling

[Superfluid orbital stripes are new in both AMO and condensed matter]

How do we solve the strong coupling?

$U \gg t$ i.e. Interaction >> tunneling

- Treat the lattice system as a Josephson array of $p_x + ip_y$ BECs
- Found that the effective field theory = U(1) lattice gauge theory

(not standard XY model but related!)

$$H_{\text{eff}} = -\frac{1}{2} n t_{\parallel} \sum_{\langle \vec{r}_1, \vec{r}_2 \rangle} \cos\left\{\phi_{\vec{r}_1} - \phi_{\vec{r}_2} - A_{\vec{r}_1,\vec{r}_2}(\sigma_{\vec{r}_1}, \sigma_{\vec{r}_2})\right\} + \frac{1}{3} U \sum_{\vec{r}} n_{\vec{r}}^2$$

The gauge field (as an "external flux" for ϕ)

External flux in a triangular plaquette:

$$\Phi = \frac{1}{2\pi} \sum_{\langle r, r' \rangle} A_{r,r'} = \frac{1}{6}(\sigma_{\vec{r}_1} + \sigma_{\vec{r}_2} + \sigma_{\vec{r}_3}) \bmod 1 \quad \text{must be} \quad \pm\frac{1}{6}$$

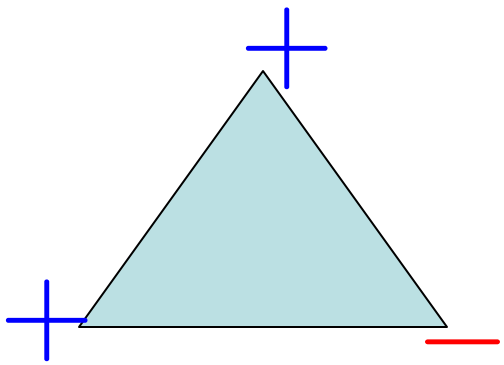
Require minimum flux in each plaquette [as shown, e.g., by Moore and Lee (2004) for a Josephson array of superconductors].

Vortex sector of U(1) gauge theory: Duality mapping to a lattice Coulomb gas

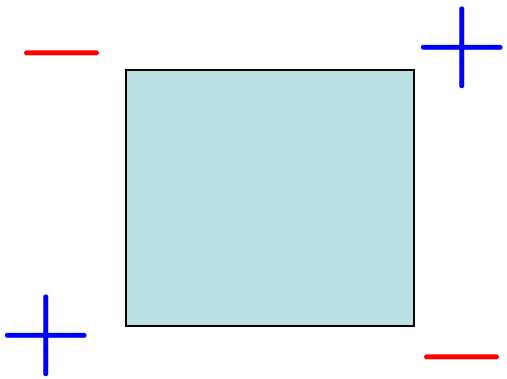
Staggered fluxes (stripe order)

Difference between square and triangular lattices

Ising variables summed around a plaquette



$$\sum_{\triangle} \sigma \neq 0$$

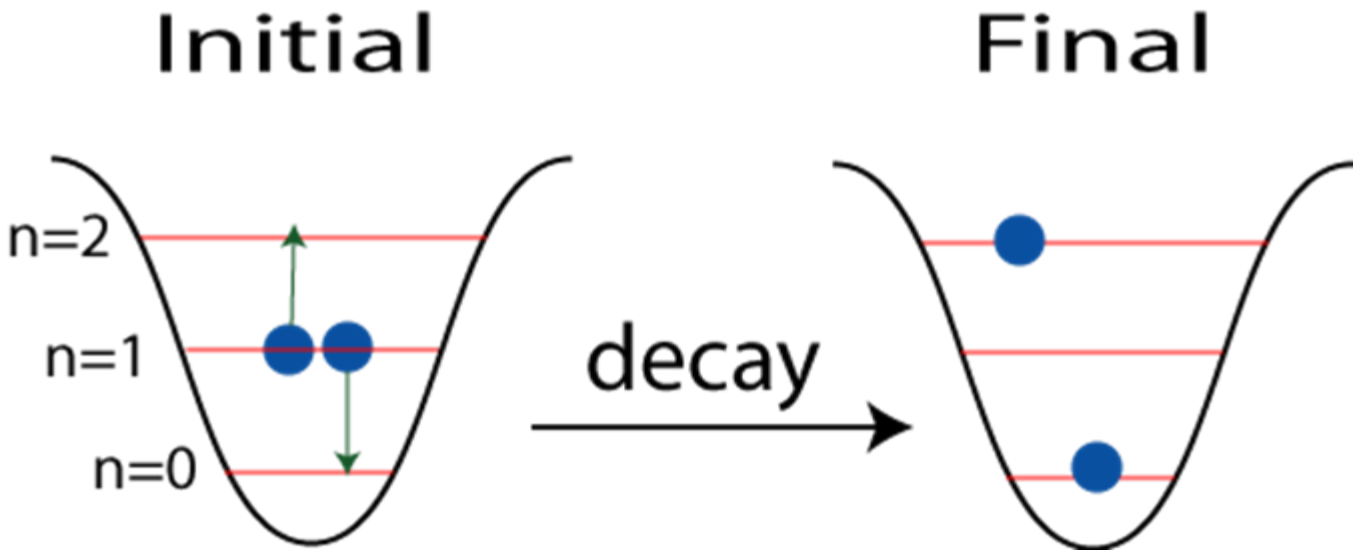


$$\sum_{\square} \sigma \quad \text{can be} \quad = 0$$

The decay problem of p-orbital bosons

The decay process where two p-bosons collide, promoting one to the 2nd excited band and bringing one down to the s-band.

My crude cartoon picture:



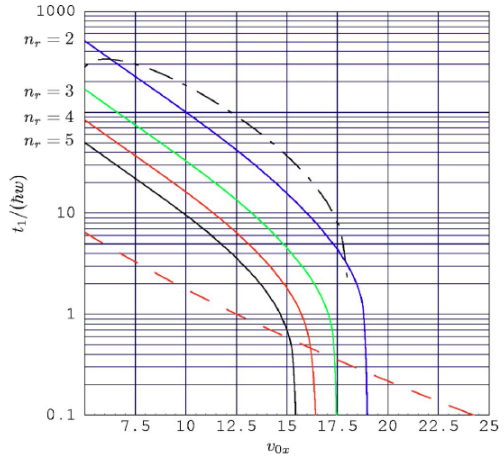


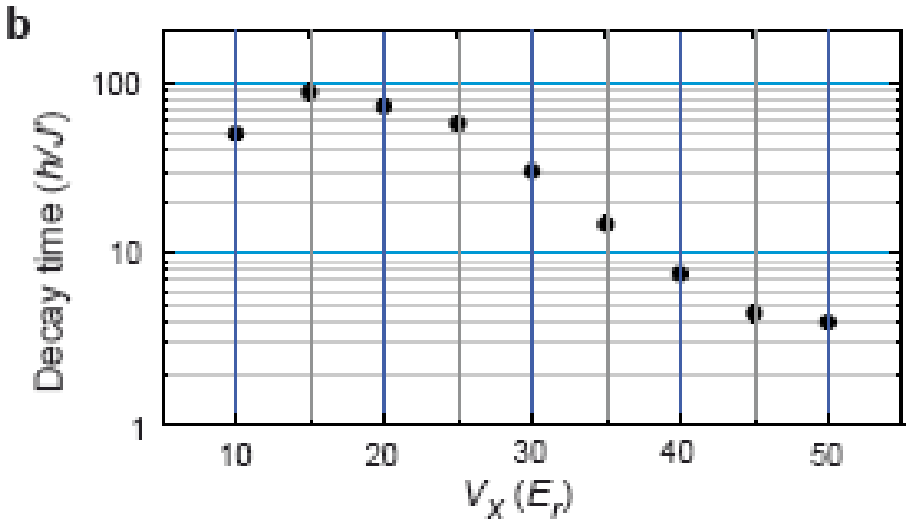
FIG. 15. (Color online) First-order lifetime w^{-1} for a 1D system with filling factor $\nu_1 = 1$ and $(a_s/a) = 1/100$ in the narrowband limit

First Studied by [Isacsson and Girvin (2005).]

p-band decay time measured by the Mainz group

[T. Mueller, I. Bloch, et al, PRL 2007]

Decay time in units of tunneling time scale
vs
lattice depth

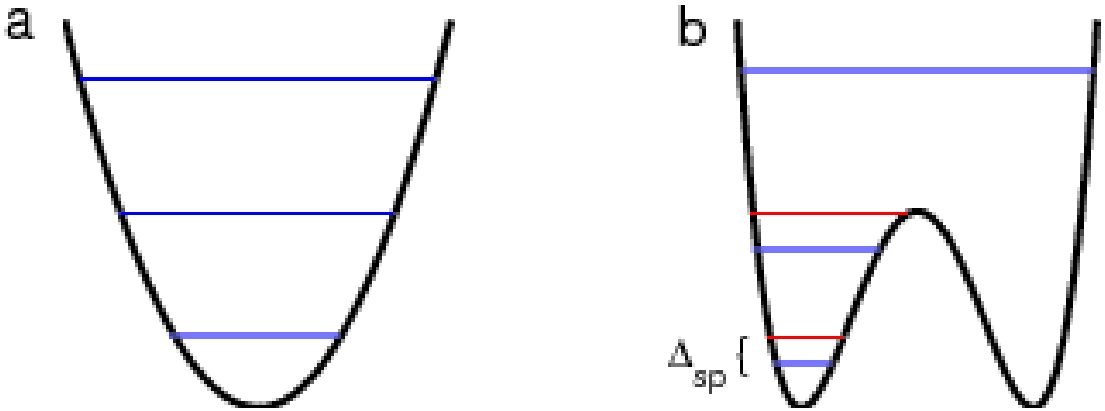


A key to slow decay as explained by Bloch et al: anharmonicity

“Energy-blocking” mechanism to suppress the decay

[WVL and C. Wu, PRA (2006)]

single well lattice ***double well lattice***



- Low energy motion of bosons is an effective two-band model;
- *p*-orbital bosons cannot decay to the “s” by energy conservation.

Another mechanism for a stable p-orbital BEC: a two-component bosonic mixture interacting with a p-wave Feshbach resonance [A. B. Kuklov, PRL 97, 110405 (2006)]

Digression to a Related Topic:

Finite-momentum BEC in the <u>groundstate.</u>

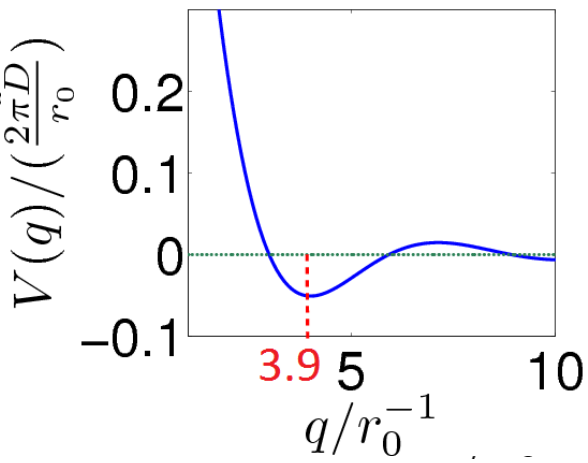
With a novel type of momentum dependent interaction

Our work:

- X. Li, WVL, C. Lin, arXiv:1005.4027

Bose-Einstein supersolid phase

Novel form of interaction



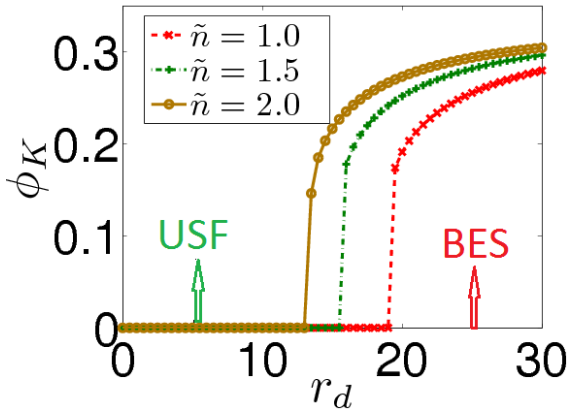
Hamiltonian

$$H = \int d^2\mathbf{x}\hat{\psi}^+(\mathbf{x})[-\frac{\hbar^2}{2m}\nabla^2 - \mu]\hat{\psi}(\mathbf{x})$$
$$+\frac{1}{2}\int d^2\mathbf{x}_1 d^2\mathbf{x}_2 \hat{\psi}^+(\mathbf{x}_1)\hat{\psi}^+(\mathbf{x}_2)V(|\mathbf{x}_1-\mathbf{x}_2|)\hat{\psi}(\mathbf{x}_2)\hat{\psi}(\mathbf{x}_1)$$

BES state

$$|BES\rangle = \exp[\sqrt{N}\sum_{\mathbf{K}}\phi_{\mathbf{K}} b_{\mathbf{K}}^{\dagger}]|\Omega\rangle$$

-phase transition



First order phase transition

-order parameter: $\phi_{\mathbf{K}} = \sqrt{\frac{1}{N}\langle b_{\mathbf{K}}^{\dagger} b_{\mathbf{K}}\rangle}$

-parameters: $r_d = \frac{2mDn^{1/2}}{h^2}, \quad \tilde{n} = n \times r_0^2$

Stability of Bose-Einstein supersolid

BES=Bose-Einstein supersolid
USF=uniform superfluid

-Bogoliubov spectrum

Fluctuations

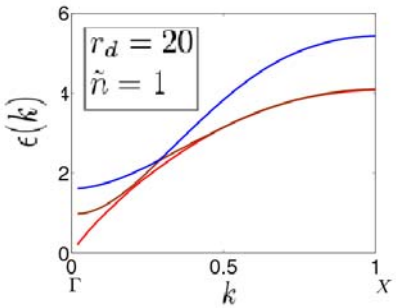
$$\psi(\mathbf{x},t) = [\rho_0(\mathbf{x}) + \delta\rho(\mathbf{x},t)]^{1/2} e^{i\varphi(\mathbf{x},t)}$$

Block diagonal effective action

$$S_{eff}[\delta\rho(\mathbf{x}),\varphi(\mathbf{x})] = \sum_{\mathbf{k}} \begin{bmatrix} \eta^\dagger(\mathbf{k}) & \varphi^\dagger(\mathbf{k}) \end{bmatrix} \mathcal{G}_k^{-1} \begin{bmatrix} \eta(\mathbf{k}) \\ \varphi(\mathbf{k}) \end{bmatrix}$$

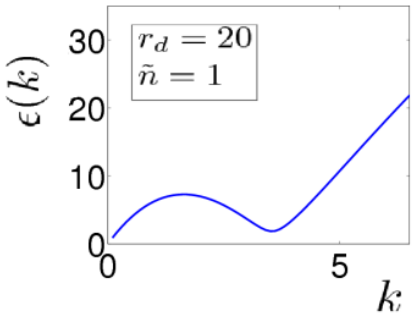
Bose-Einstein supersolid phase is stable against these fluctuations in a parameter regime to be shown in the next slide!

BES

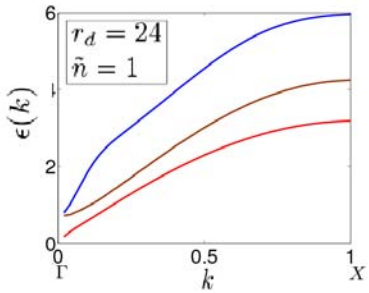


'BES' is stable

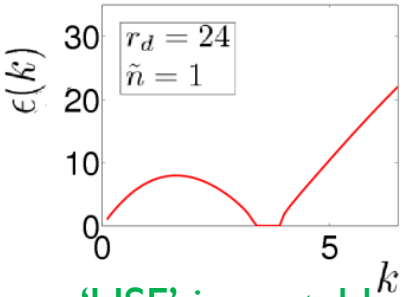
USF



'USF' stable



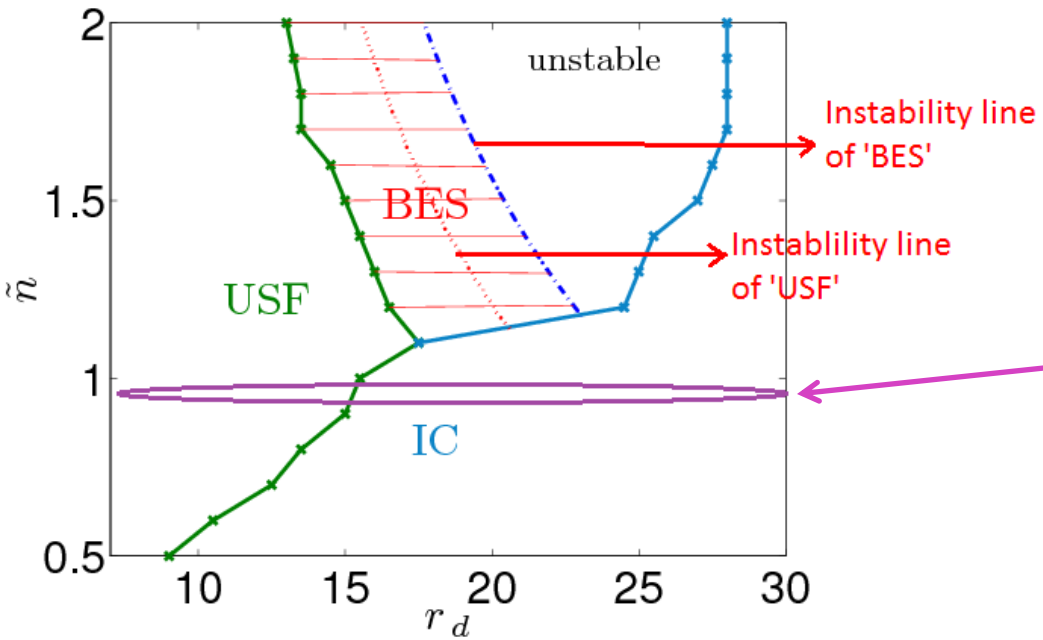
'BES' is stable



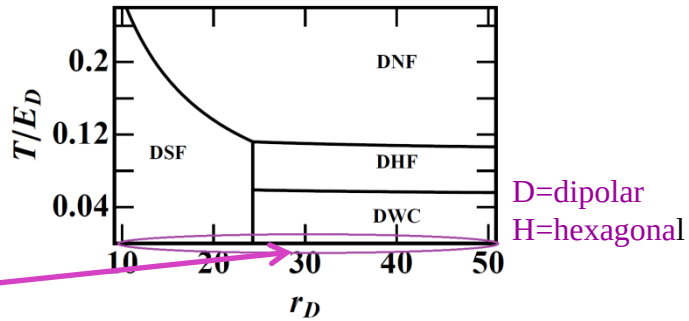
'USF' is unstable

Phase Diagram

-our result

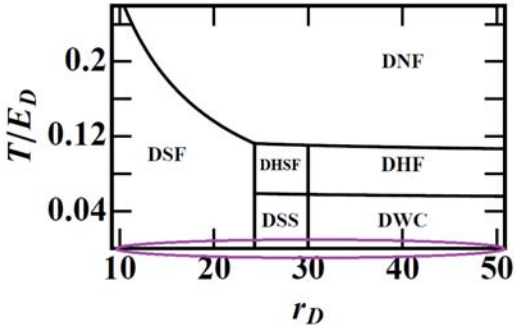


-established phase diagram



K. Mitra, et al (2009), arXiv:0903.4655

-speculated phase diagram



K. Mitra, et al (2009), arXiv:0903.4655

In the 'red shaded' regime, the BES phase is stable against fluctuations and has lower energy than crystal.

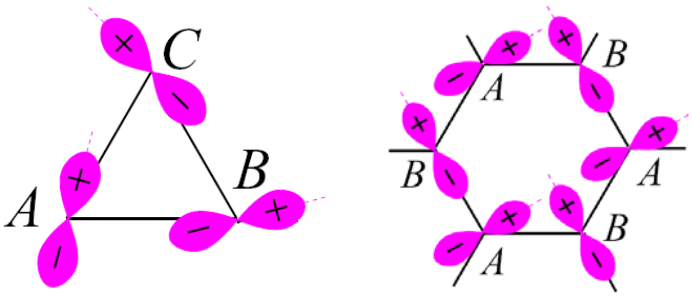
Part 3.
Study important quantum condensed matter models with lattice orbitals

Novel aspects of spinless fermions in p-bands

Our proposal to realize the ***orbital quantum 120° model***

[E. Zhao and WVL, PRL (2008); See also independent work by C. Wu, PRL 2008.]

Pseudo-spin operators on frustrated lattices (triangular, honeycomb, Kagome, …)



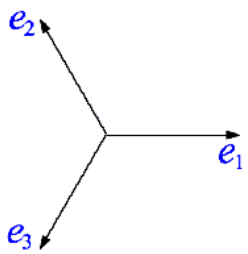
$|p_y\rangle \quad |p_x\rangle$

$$T_x = \frac{1}{2}\left(c_y^\dagger c_x + c_x^\dagger c_y\right)$$

$$T_y = \frac{i}{2}\left(c_y^\dagger c_x - c_x^\dagger c_y\right)$$

$$T_z = \frac{1}{2}\left(c_x^\dagger c_x - c_y^\dagger c_y\right)$$

The orbital exchange Hamiltonian is:



$$H_{120} = J_z \sum_{\mathbf{R},j} T_j(\mathbf{R}) T_j(\mathbf{R}+\hat{e}_j)$$

lattice sites

j =1,2,3

where $T_3 = -\frac{1}{2}T_z + \frac{\sqrt{3}}{2}T_x$

$$T_1 = T_z,\ T_2 = -\frac{1}{2}T_z - \frac{\sqrt{3}}{2}T_x$$

This quantum 120° model is closely related to the compass model and Kitaev model.

Quantum 120° model of e_g orbitals in cubic lattice: van den Brink, New J. Phys. 6, 201 (2004).

Topological phase of dipolar bosons in elongated Wannier orbitals

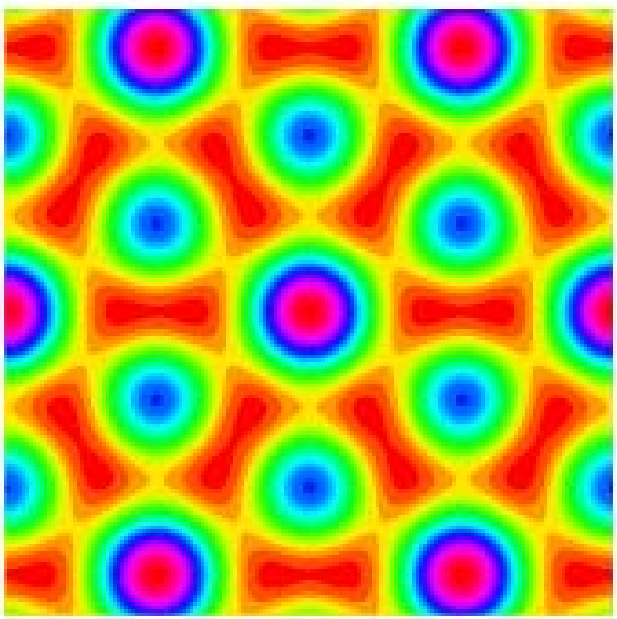
Orbital + Dipolar interaction to realize a quantum dimer model

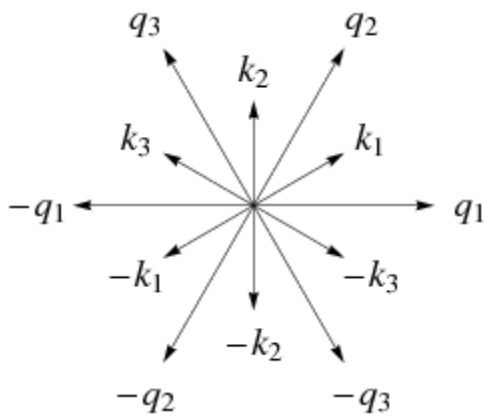
[Kai Sun, Erhai Zhao, WVL, Phys. Rev. Lett. 104, 165303 (2010)]

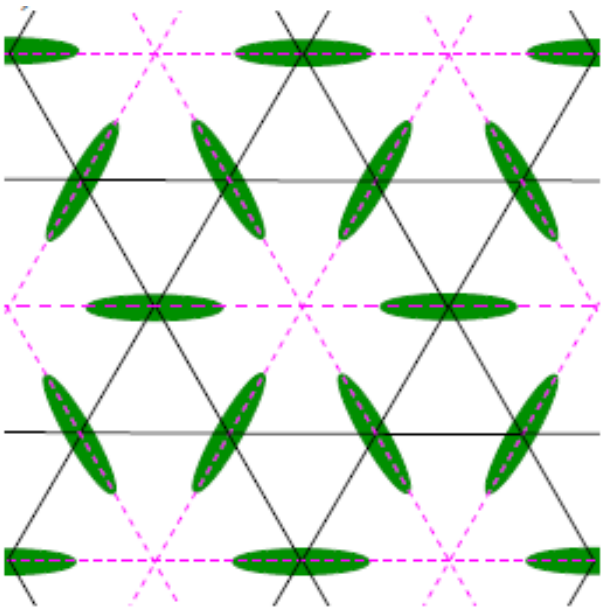
2D Optical lattice with elongated orbital

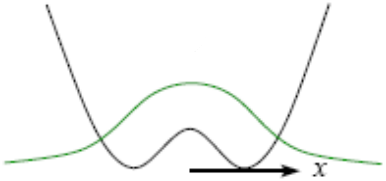
$$I = I_1 \sum_{i=1,2,3} \cos^2(\mathbf{k}_i \cdot \mathbf{r}) + I_2 \sum_{j=1,2,3} \cos^2(\mathbf{q}_j \cdot \mathbf{r})$$

$$|\mathbf{q}_j| = \sqrt{3}|\mathbf{k}_i|,\ \theta_{\mathbf{q}_j} = 2j\pi/3,\ \theta_{\mathbf{k}_i} = (2i-1)\pi/6$$









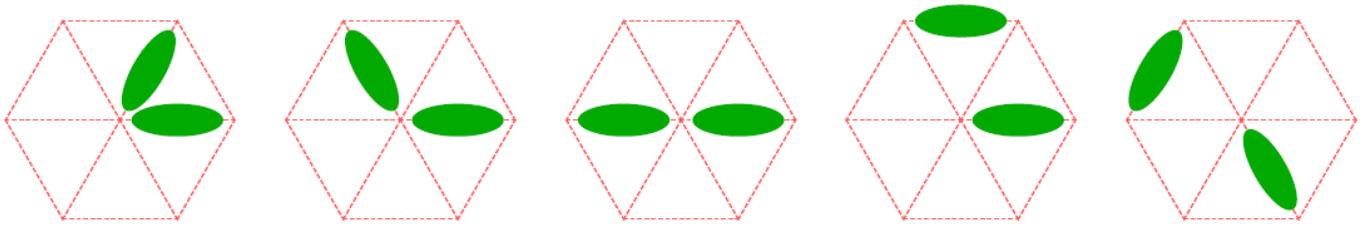
- Elongated Wannier orbital wave-function is a "dimer"
- "shape" of atom ⟺ spin-singlet of two neighboring spins (electrons)
- Site of the Kagome lattice ⟺ bond of the triangular lattices
- Hard-core constraint? use interaction.

[Kai Sun, Erhai Zhao, WVL, Phys. Rev. Lett. 104, 165303 (2010)]

Polar Molecules

to enforce 'no double occupancy' constraint

Dipoles aligned in z direction (2D $1/r^3$ repulsive interaction)

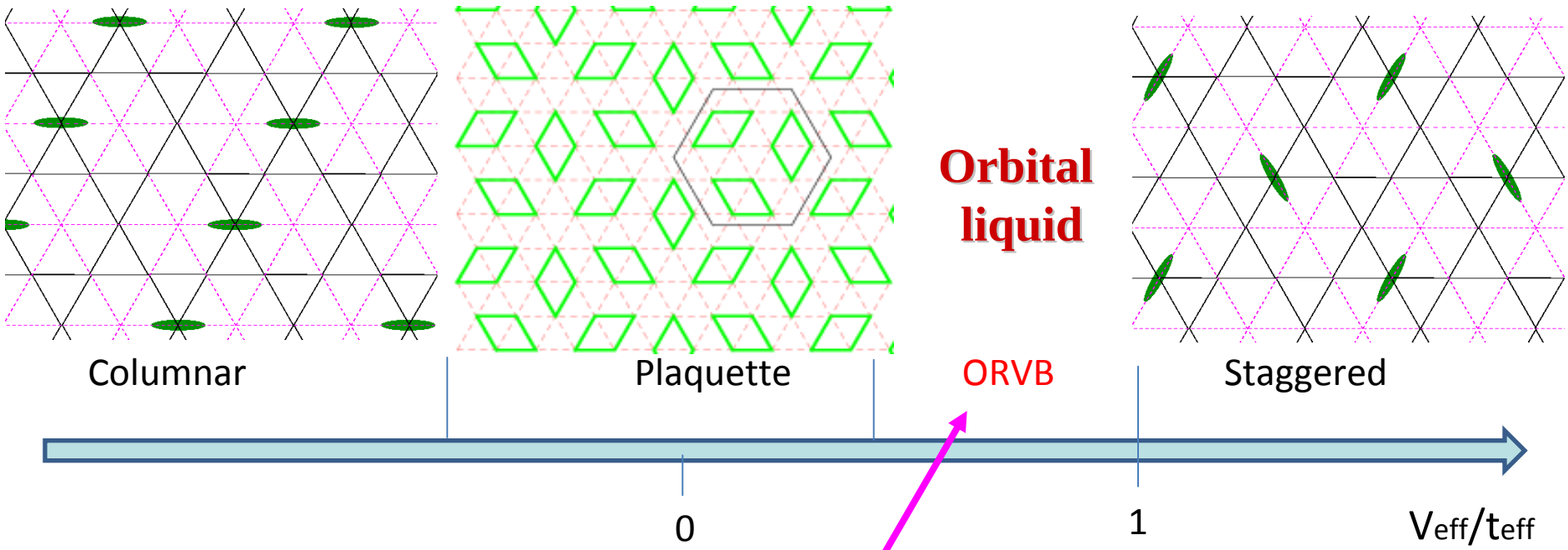


V_1 V_2 V_3 >> $V_{//}$ V_S

1/6 filling mapped to Orbital Quantum Dimer Model

- Energy scale: 1nk (by our estimate)
- Charge (no spin)
- Either Fermion or Boson? (Having fermionic dimers is new and unexplored.)

Phase Diagram for Bosons



- Breaks no symmetry (no local order parameter), outside the paradigm of Landau-Ginsburg-Wilson theory
  - No symmetry-related signature, c.f. other three phases
- Fractional excitation: holons
  - carries ½ quantum number of the particle
  - can be fermions or bosons (microscopic details)
    - Fermionic quasiparticles in a bosonic system!
- Topological degeneracy

my other current interest (I)

Mismatched Fermi surfaces / Mass and population imbalance / from 3D to 1D

- 3D: **Breached pair superfluidity** [proposed in WVL and F. Wilczek, *Phys. Rev. Lett. 2003a.* And with Wilczek, Zoller, et al. *Phys. Rev. Lett. 2003b; Phys. Rev. A 2004; Phys. Rev. Lett. 2005; Annals of Physics 2008*]

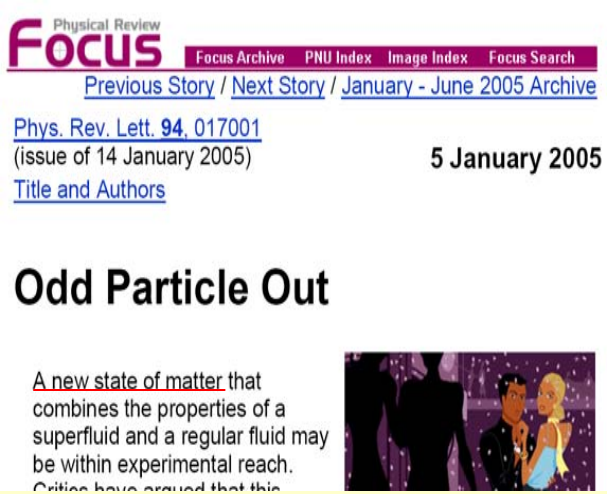
[2003a PRL citation: ***178*** as of Jan 2010]

- 1D: **A new effective field theory of the 1D FFLO phase based on Bethe ansatz and conformal invariance.** Generalized to coupled 1D tubes [Zhao, WVL, *Phys. Rev. A 2008]*

- **Thermodynamic Bethe ansatz**, simplified solution, and thermometry for 1D strongly interacting Fermi gases (the Gaudin-Yang model) [Zhao, Guan, WVL, Batchelor, Oshikawa, *Phys. Rev. Lett. 2009]*

- Related studies on resonant superfluids (topological chiral bound states, effective field theory description of strongly interaction) [WVL, *Phys. Rev. A 2005;* WVL, *Phys. Rev. Lett. 2005]*

Reported in Phys. Rev. Focus


Physical Review Focus
Focus Archive PNU Index Image Index Focus Search
Previous Story / Next Story / January - June 2005 Archive
Phys. Rev. Lett. 94, 017001
(issue of 14 January 2005)
5 January 2005
Title and Authors

Odd Particle Out

A new state of matter that combines the properties of a superfluid and a regular fluid may be within experimental reach.

my other current interest (II)

Damping of Collective Excitations in BCS/BEC crossover superfluids

my other current interest (II)

Damping of Collective Excitations in BCS/BEC crossover superfluids

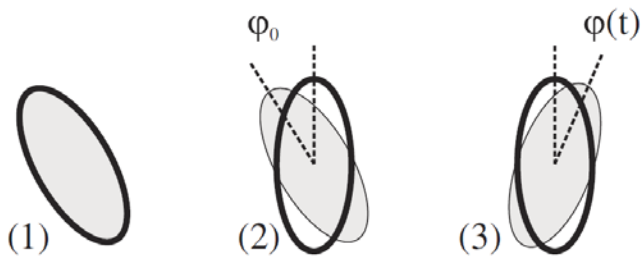
Introduction to the puzzling damping

A. Motivation: PRL 99, 150403 by M. J. Wright *et al. (2007)*

B. Unpolarized ultracoldFermi gases: BCS-BEC crossover

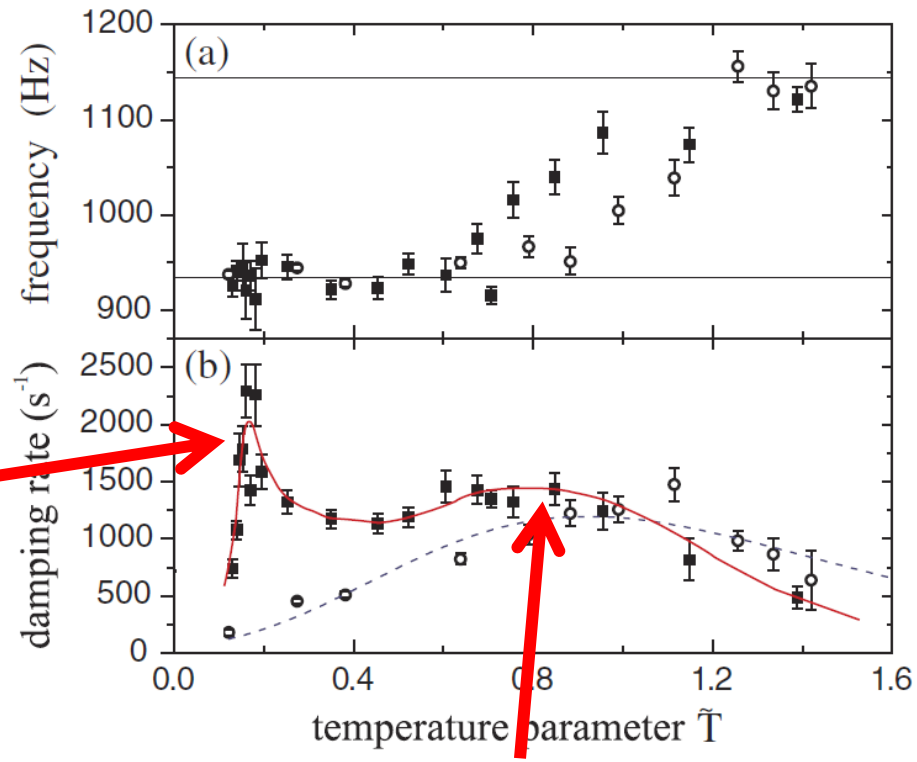
C. Oscillation frequency and damping rate measured by *varying the temperature* at (BCS side)

$$B = 895G \; (1/k_F a = -0.45)$$



Scissor Mode

A sharp peak in superfluid regime



A broad peak in Fermi liquid regime

* $\tilde{T}$ is the effective temperature parameter

Our damping mechanism and results

[Z. Zhang and WVL, arXiv:1007.3694]

A. Our model says: the mechanism is not due to fermion pair breaking, but is Landau damping at finite temperature. (Collective modes scatter with thermally excited fermionic quasi-particles.)

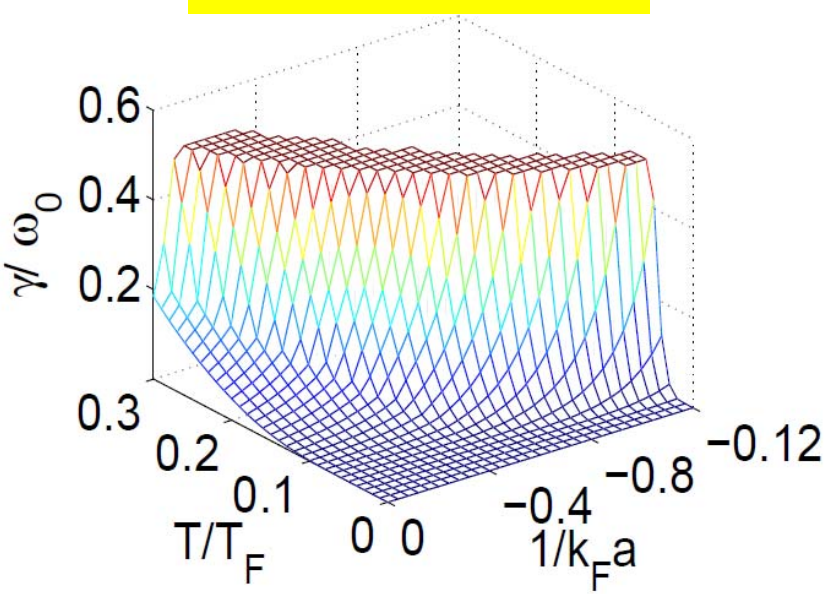
B. Our method:

1) At high T, above the phase transition, use Boltzmann equation to calculate the damping rate. [Phys. Rev. A, 76, 045602 by G. M. Bruun and H. Smith (2007)]

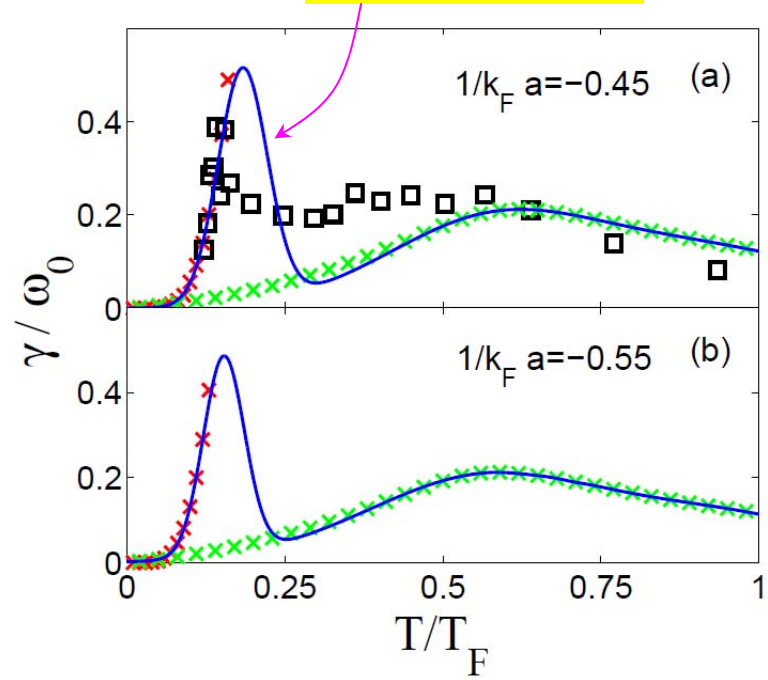
2) At low T, use our effective field theory.

3) Intermediate T: interpolate two limits

Low T EFT result

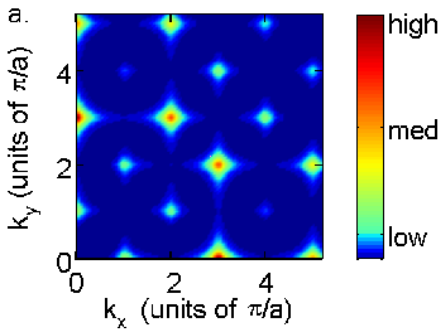


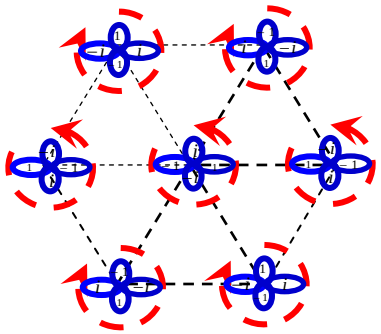
interpolation



Conclusion and summary of main results

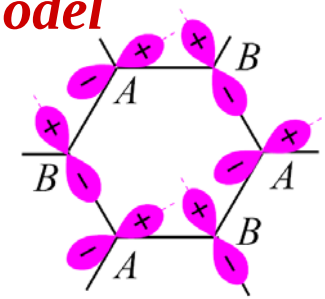
p-band bosons



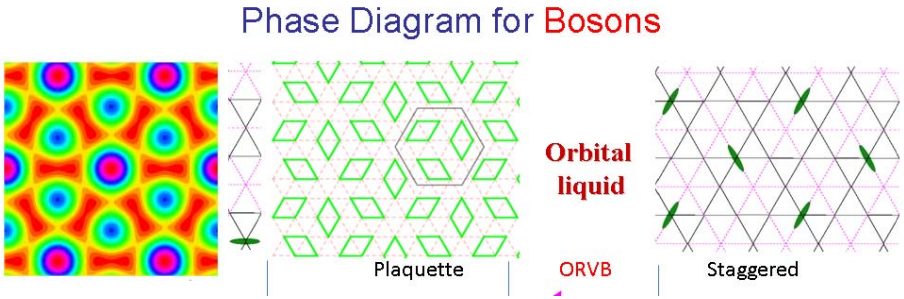


Vortex sector of U(1) gauge theory: Duality mapping to a lattice Coulomb gas

p-band fermions: 120° model



Elongated orbitals → quantum dimers



Novelty of lattice p-band Bose gases

- Non-zero momentum BEC---defying the paradigm
- A single order parameter for both superfluidity and crystalline order