DISSIPATIVE DYNAMICS OF SOLITONS IN FERROMAGNETIC METALS

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OUTLINE

Landau-Lifshitz-Gilbert phenomenology of magnetization dynamics in inhomogeneous metallic ferromagnets

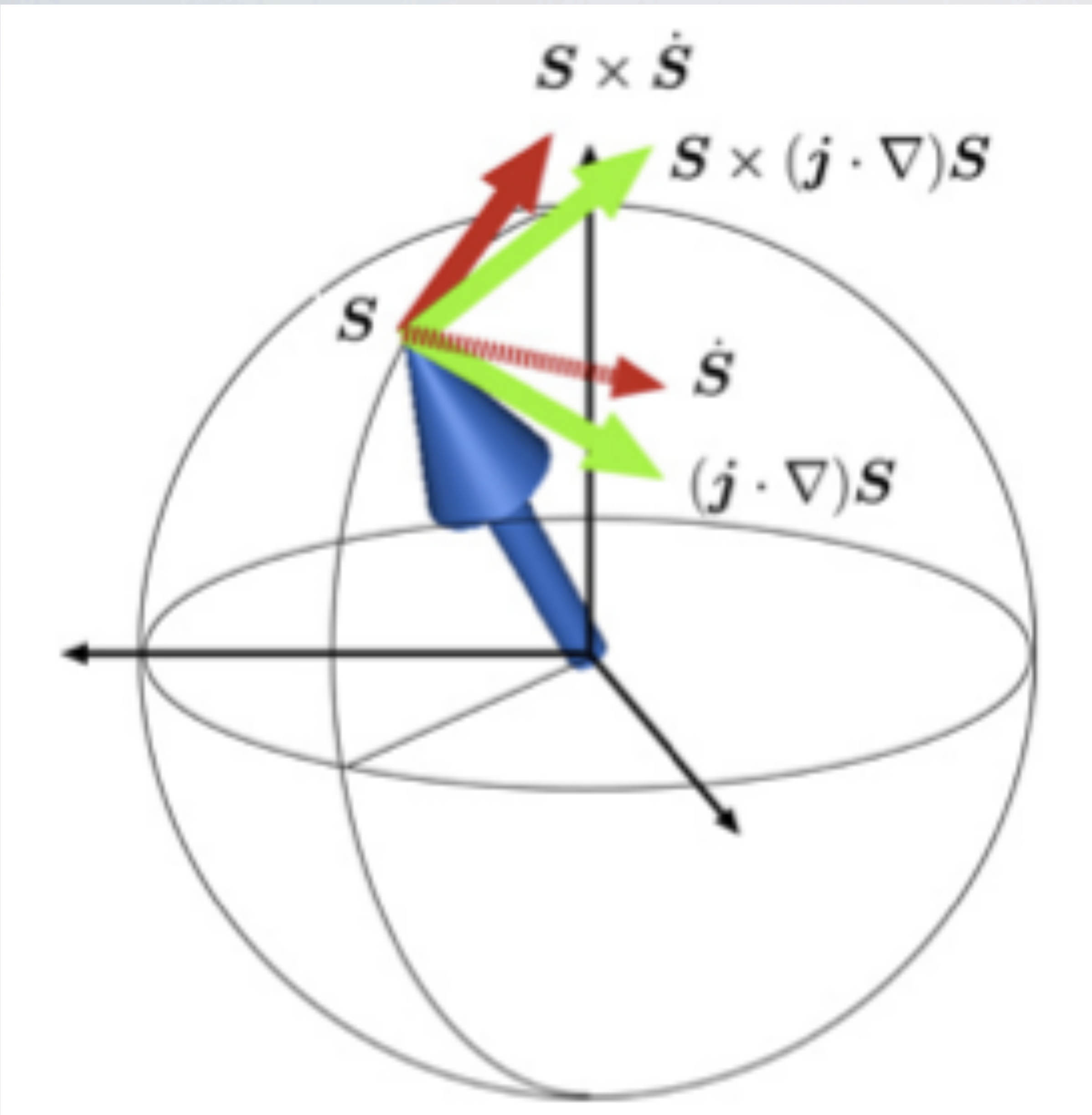
Spin-charge electronic hydrodynamics

- fictitious electrodynamics
- Topological Hall Effect
- Voltages induced by geometric motive forces

Magnetic soliton dynamics

- domain-wall motion
- Vortex gyration in spin valve

CURRENT-MAGNETIZATION DYNAMICS IN METALLIC FERROMAGNETS



H. Kohno et al., *JPSJ*, 75 (2006) 113706.

- Inhomogeneous magnetization

$$\mathbf{M} = \gamma S \mathbf{m}(\mathbf{r}, t)$$

$\gamma < 0$ effective gyromagnetic ratio

$S = \frac{\hbar}{2}(n_\uparrow - n_\downarrow)$ constant spin density, below Curie temperature

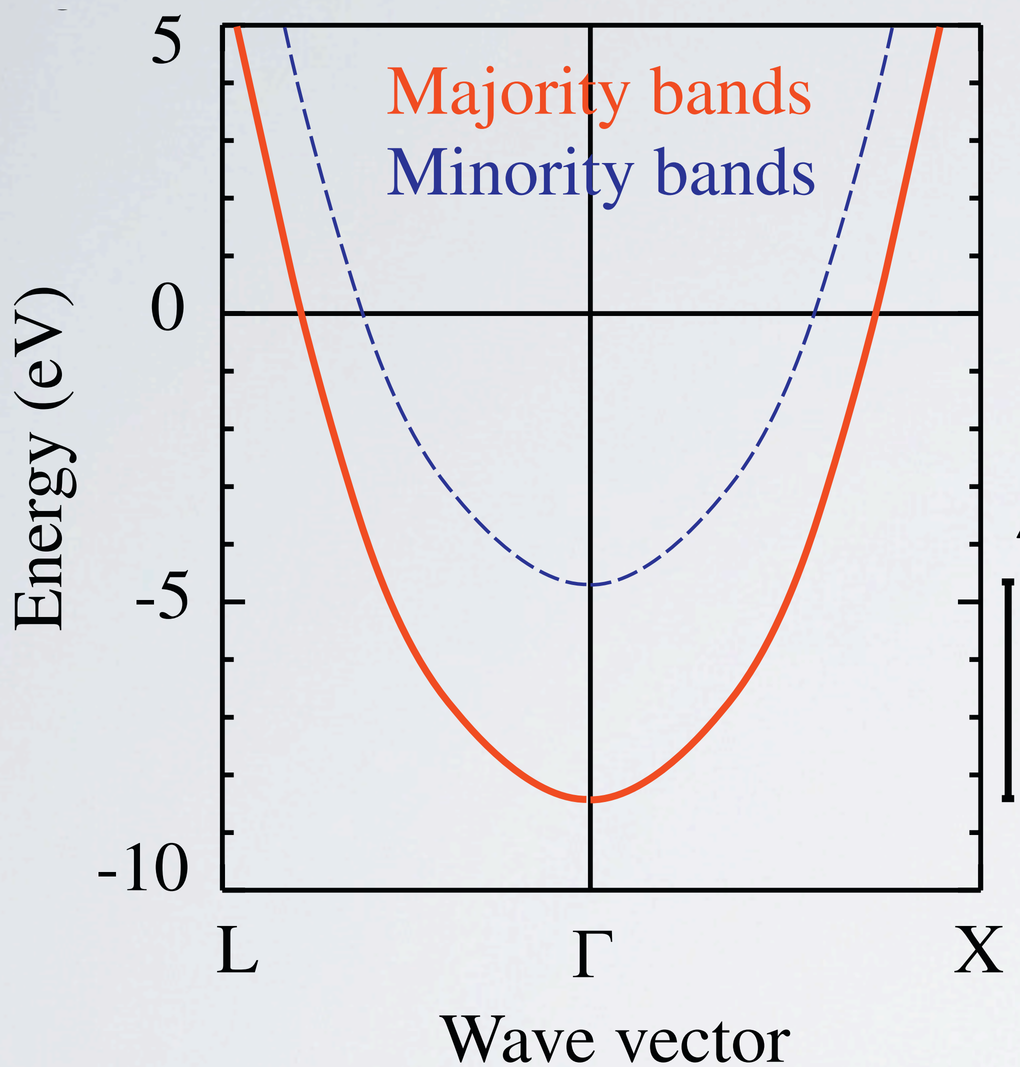
$n_{\uparrow/\downarrow}$ Majority/minority electron density.

$\mathbf{m}$ local, dynamical orientation, goldstone modes

$\dot{\mathbf{m}}, \mathbf{j}$ non-equilibrium magnetization and current

Figure of possible vector products in equations of motion of an adiabatic, near equilibrium theory

Dynamical Stoner Model



D. Ralph and M. Stiles. *JMMM*, **320** (2008)1190 .

$\Delta_{xc} \sim \mathrm{eV}$

In the semiclassical regime

$$\xi \gg \lambda_F, \hbar\omega \ll \epsilon_F$$

λ_F, ϵ_F fermi wavelength /energy

ξ, ω length scale, frequency of magnetic texture

$\omega \sim \mathrm{microwave}$

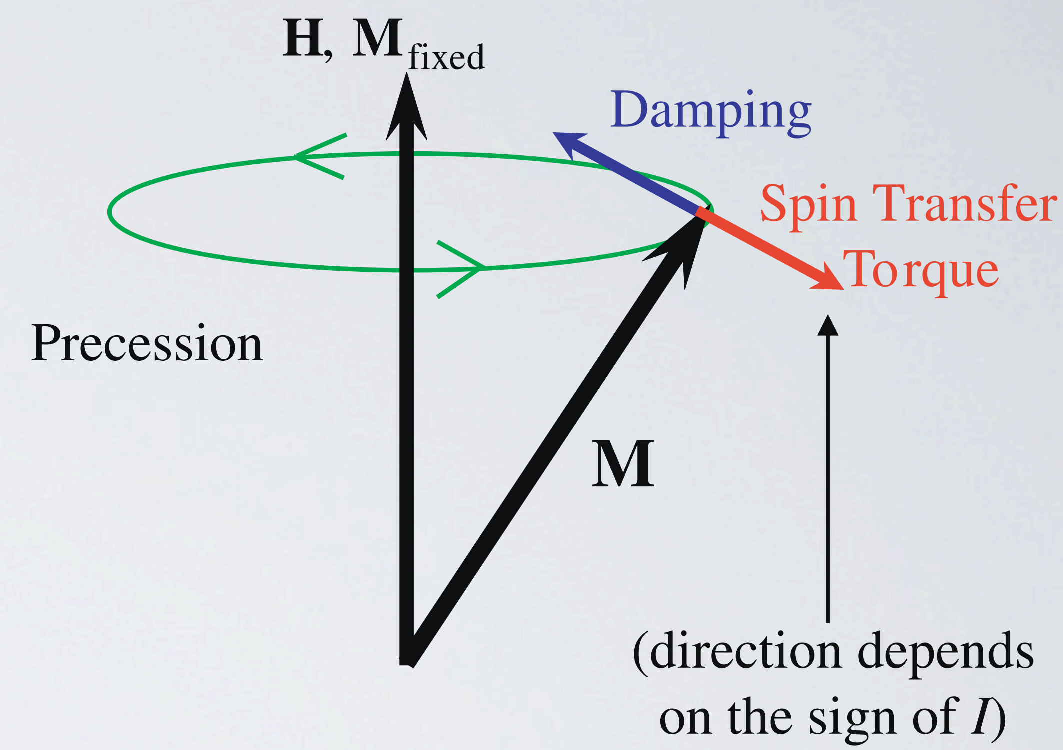
local, instantaneous band energies

$\mathbf{m}$ additional dynamical parameter specifying spin orientation fermi surfaces

ferromagnetic coherence length $l_c \equiv \dfrac{\hbar v_F}{\Delta} \approx \text{Å}$

LANDAU-LIFSHITZ-GILBERT EQUATION

$$\boxed{S(\dot{\mathbf{m}} + \alpha \mathbf{m} \times \dot{\mathbf{m}}) + \mathbf{m} \times \mathbf{H} = 0}$$



D. Ralph and M. Stiles. *JMMM*, **320** (2008)1190 .

- Precession in Effective field $\mathbf{H} \equiv \dfrac{\delta \mathcal{V}}{\delta \mathbf{m}}$
- $\mathbf{m}$ dependent free energy

$$\mathcal{V}[\mathbf{m}] = \int d^3x \, \underbrace{\frac{A}{2}(\partial_i \mathbf{m})^2}_{\text{exchange energy}} + \underbrace{U_{\text{ani}}(\mathbf{m})}_{\text{anisotropic potential}} - M \mathbf{h}_{\text{ext}} \cdot \mathbf{m}$$

external magnetic field in the usual magnetostatics

- Thermal fluctuations may be included in a stochastic (langevin) field.

$$\mathbf{H} \rightarrow \mathbf{H} + \mathbf{h}$$

DYNAMICS OF ITINERANT ELECTRONS

- Mean field action $S\mathbf{m}(\mathbf{r},t) = \frac{\hbar}{2}\langle \hat{\psi}^\dagger \hat{\boldsymbol{\sigma}} \hat{\psi}\rangle$

$$I[\mathbf{m}] = \int dt d^3 r \hat{\psi}^\dagger \left[i\hbar\partial_t + \frac{\hbar^2}{2m_e}\nabla^2 + \frac{\Delta_{xc}}{2}\mathbf{m}\cdot\hat{\boldsymbol{\sigma}}\right]\hat{\psi}$$

$$\mathbf{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

Go to local spin frame

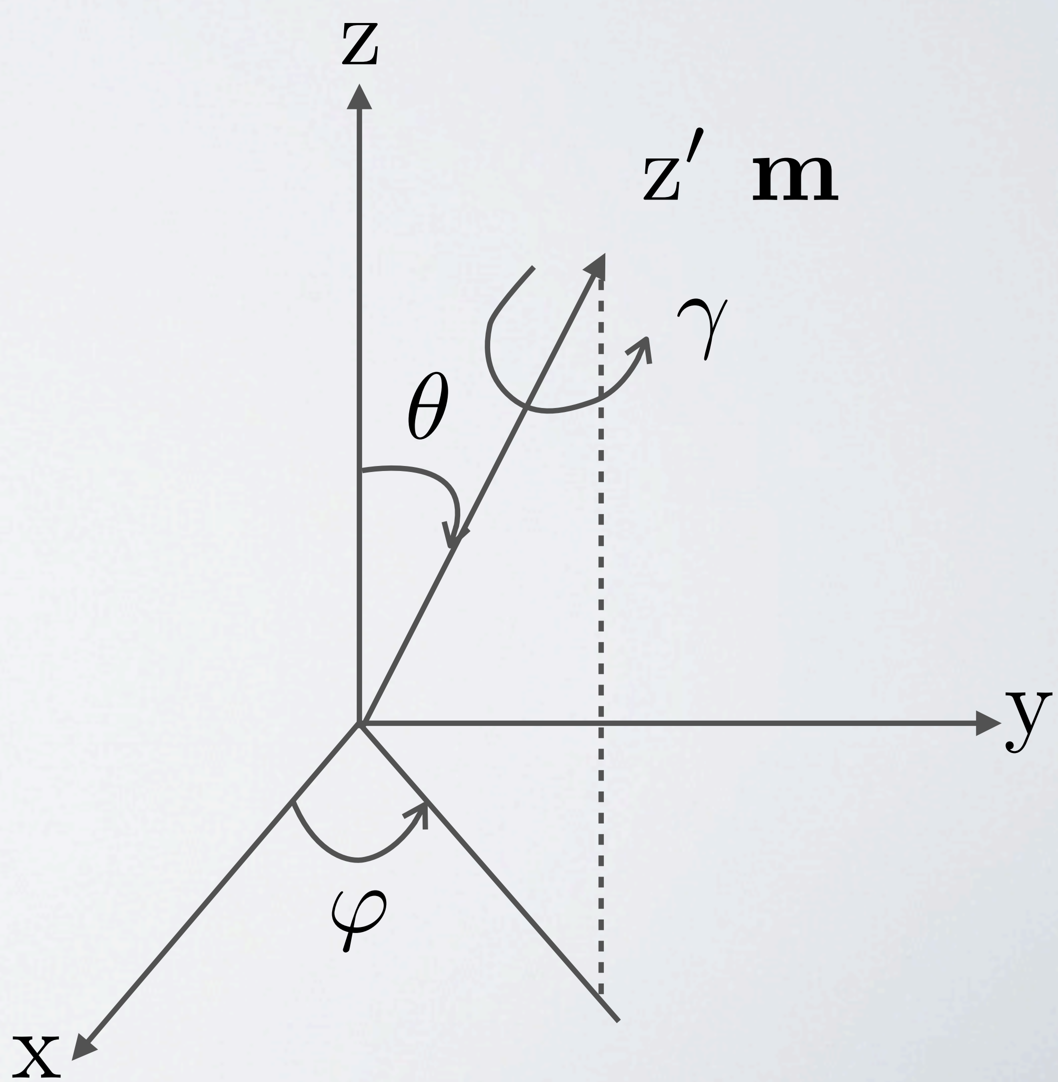
$$\hat{\mathcal{U}}^\dagger(\hat{\boldsymbol{\sigma}}\cdot\mathbf{m})\hat{\mathcal{U}} = \hat{\sigma}_z \qquad \hat{\psi} = \hat{\mathcal{U}}\hat{\psi}'$$

Parametrized by Euler angles

$$\mathcal{U} = e^{-i\varphi\sigma_z/2} e^{-i\theta\sigma_y/2} e^{-i\gamma\sigma_z/2}$$

Matrix gauge fields appear in covariant derivative

$$A^a_\mu(\mathbf{r},t)\hat{\sigma}^a \equiv i\hbar\hat{\mathcal{U}}^\dagger\partial_\mu\hat{\mathcal{U}} \quad \partial_\mu = (\partial_t, \boldsymbol{\nabla})$$



ADIABATIC LIMIT

$$\omega \ll \Delta ; \xi \gg \frac{\hbar v_F}{\Delta}$$

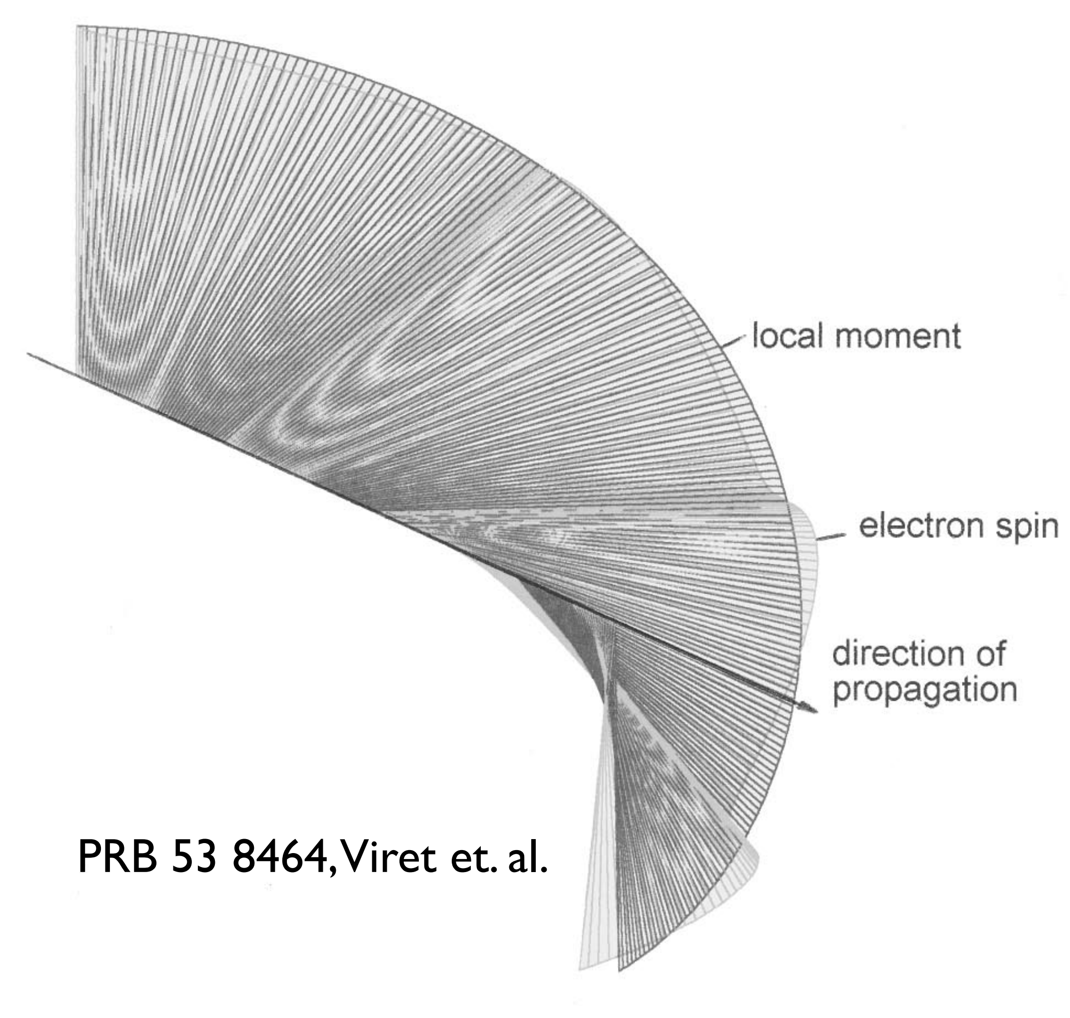
ξ, ω length scale, frequency of magnetic texture

Retain longitudinal gauge fields $\hat{a}_\mu(\mathbf{r}, t) = \hat{\sigma}_z A^z_\mu$

$$a_{\sigma\mu} = (a_\sigma, \mathbf{a}_\sigma) = q_\sigma(\cos\theta \mp 1)\partial_\mu \varphi$$

$$q_\sigma = \sigma\hbar/2, \ \sigma = \pm$$

a_σ is the Wess-Zumino term for itinerant electrons



G. E. Volovik, *JPC:*, **20**(1987)L83.

Projected action for majority/minority electrons is formally identical to electrodynamics of two species with opposite charge

$$I[\mathbf{m}] = \int dt \int d^3 r \hat{\psi}'^\dagger \left[(i\hbar\partial_t + \hat{a}) - \frac{(-i\hbar\boldsymbol{\nabla} - \hat{\mathbf{a}})^2}{2m_e} + \frac{\Delta}{2}\hat{\sigma}_z \right] \hat{\psi}'$$

$$- \int dt F[\mathbf{m}]$$

$$F[\mathbf{m}] = \frac{\hbar^2}{4m_e}\int d^3 r \hat{\rho}(\partial_i \mathbf{m})^2 \qquad \hat{\rho} = \hat{\psi}^\dagger \hat{\psi}$$

FICTITIOUS ELECTRODYNAMICS

Semiclassical Lagrangian of 2-component fluid

$$L(\mathbf{r}_p, \mathbf{v}_p; \mathbf{m}, \partial_\mu \mathbf{m}) = \sum_p \left(\frac{m_e \mathbf{v}_p^2}{2} + v_p^\mu a_{\sigma\mu} \right) - \frac{A}{2} \int d^3r (\partial_i \mathbf{m})^2$$

single particle equations

$$m_e \dot{\mathbf{v}}_p = q_p(\mathbf{e} + \mathbf{v}_p \times \mathbf{b}) \qquad q_p \equiv \sigma_p \hbar/2$$

Fictitious electromagnetic fields

$$e_i = \mathbf{m} \cdot (\partial_t \mathbf{m} \times \partial_i \mathbf{m}), \quad b_i = \frac{\epsilon^{ijk}}{2} \mathbf{m} \cdot (\partial_k \mathbf{m} \times \partial_j \mathbf{m})$$

Quantized monopole flux through a sphere (skyrmion number)

$$N = \frac{1}{4\pi} \oint_{S^2} \mathbf{b} \cdot d\mathbf{S} = \frac{1}{8\pi} \oint d^2 S_i \, \epsilon^{ijk} \, \mathbf{m} \cdot \partial_j \mathbf{m} \times \partial_k \mathbf{m} \qquad N \in \mathbb{Z} = \pi_2(S^2)$$

Faraday's law $\quad \partial_t \mathbf{b} + \mathbf{\nabla} \times \mathbf{e} = 0$

SYMMETRIES AND CONSERVATION LAWS

particle density :
(gauge invariance)

$$\boxed{\dot{n} + \boldsymbol{\nabla} \cdot \mathbf{j} = 0\,, \quad \dot{n}_s + \boldsymbol{\nabla} \cdot \mathbf{j}_s = 0}$$

will include spin relaxation which in practice fixes spin density

Angular momentum:
(rotational invariance)

$$\boxed{\partial_t (S m_i) + \partial_j \Pi_{ij} = 0}$$

nondissipative spin torque

spin-current tensor $\Pi_{ij} = S v_{sj} m_i - A(\mathbf{m} \times \partial_j \mathbf{m})_i$

itinerant collective background

linear momentum
translational invariance)

$$\boxed{m_e \sum_\sigma \rho_\sigma (\partial_t + \mathbf{v}_\sigma \cdot \boldsymbol{\nabla}) v_{\sigma i} + \partial_j T_{ij} = 0}$$

Euler equation for course grained velocities

stress tensor $T_{ij} = A \left[(\partial_i \mathbf{m}) \cdot (\partial_j \mathbf{m}) - \frac{\delta_{ij}}{2} (\partial_k \mathbf{m})^2 \right] + \frac{\delta_{ij}}{2} \rho_\sigma K^{-1}_{\sigma\sigma'} \rho_{\sigma'}$

magnetization

electronic compressibility

ENERGY AND DYNAMICS

Magnetohydrodynamic Hamiltonian:

$$H[\rho_\sigma, \mathbf{p}_\sigma; \mathbf{m}] = \int d^3r \frac{\rho_\sigma}{2m_e}(\mathbf{p}_\sigma - \mathbf{a}_\sigma)^2 + \frac{1}{2}\int d^3r \rho_\sigma \hat{K}^{-1}_{\sigma\sigma'}\rho_{\sigma'} + F[\mathbf{m}]$$

kinetic elastic exchange

The hydrodynamic Landau-Lifshitz equation:

$$S\dot{\mathbf{m}} = -\mathbf{m} \times \delta_{\mathbf{m}} F - (\mathbf{j}_s \cdot \boldsymbol{\nabla})\mathbf{m}$$

spin-transfer torque

The Euler equation of motion in the presence of fictitious fields:

$$m_e(\partial_t + \mathbf{v}_\sigma \cdot \boldsymbol{\nabla})v_{\sigma i} = -\boldsymbol{\nabla}\mu_\sigma + \frac{\sigma\hbar}{2}[\mathbf{m}\cdot(\partial_t \mathbf{m} \times \partial_i \mathbf{m}) + v_{\sigma j}\mathbf{m}\cdot(\partial_j \mathbf{m} \times \partial_i \mathbf{m})]$$

fictitious EMF topological Hall effect

There is no dissipation: the total coarse-grained energy is conserved

CONSERVATION OF ENERGY

continuity equation for energy

$$H = \int d^3r\, \mathcal{H} \qquad \partial_t \mathcal{H} = -\boldsymbol{\nabla} \cdot (\mathbf{J}_e + \mathbf{J}_m)$$

energy flux of electrons magnetic exchange energy

$$\mathbf{J}_e = \sum_\sigma \rho_\sigma \left(\frac{m_e v_\sigma^2}{2} + \mu_\sigma \right) \mathbf{v}_\sigma \qquad \mathbf{J}_m = -A \partial_t m_i \cdot \boldsymbol{\nabla} m_i$$

Include in free energy $\qquad \mathcal{H} \to \mathcal{F}$

ONSAGER RECIPROCITY

Casting the coarse-grained equations in the standard form of the quasistationary relaxation towards a thermodynamic equilibrium,

$$\partial_t \begin{pmatrix} n(\mathbf{r}) \\ \mathbf{p}(\mathbf{r}) \\ \mathbf{m}(\mathbf{r}) \end{pmatrix} = \int d\mathbf{r}' \, \widehat{\Gamma}(\mathbf{m}; \mathbf{r}, \mathbf{r}') \begin{pmatrix} \mu(\mathbf{r}') \\ \mathbf{j}(\mathbf{r}') \\ \mathbf{h}(\mathbf{r}') \end{pmatrix}$$

$$(\mu, \mathbf{j}, \mathbf{h}) = (\delta_n \mathcal{F}, \delta_{\mathbf{p}} \mathcal{F}, \delta_{\mathbf{m}} \mathcal{F})$$

we can employ Onsager reciprocity theorem, which constrains a phenomenological introduction of dissipation, by relating the cross terms:

$$\Gamma_{ij}[\mathbf{m}] = s_i s_j \Gamma_{ji}[-\mathbf{m}], \quad s_i = \pm$$ If the ith variable is even/odd under time reversal

SPIN HYDRODYNAMICS

Expanding to linear order in non-equilibrium quantities, quadratic order in spin texture gradients, assuming spin rotational and inversion symmetry

for $\xi > \lambda_{\mathrm{sf}}$

spin flip length

$$\dot{n} = -\boldsymbol{\nabla} \cdot \mathbf{j}$$

$$\frac{m_e}{n}\dot{\mathbf{j}} + \hat{\gamma}[\mathbf{m}]\mathbf{j} = -\boldsymbol{\nabla}\mu + q(\mathbf{m} \times \dot{\mathbf{m}} - \beta\dot{\mathbf{m}})_i \boldsymbol{\nabla} m_i$$

$$S(1 + \alpha\mathbf{m}\times)\dot{\mathbf{m}} = -\mathbf{m} \times \mathbf{h} - q(1 + \beta\mathbf{m}\times)(\mathbf{j} \cdot \boldsymbol{\nabla})\mathbf{m}$$

Wong and YT, *PRB* **80** (2009) 184411

The strength of the magnetohydrodynamic coupling is given by

$$q = p\frac{\hbar}{2}$$

in terms of the ferromagnetic conductance polarization $p \in [0, 1]$

SPIN TEXTURE CORRECTIONS TO MAGNETIC NOISE

Including Langevin sources in thermodynamic forces to describe thermal fluctuations

$$(\mu, \mathbf{j}, \mathbf{h}) \rightarrow (\mu + \delta\mu, \mathbf{j} + \delta\mathbf{j}, \mathbf{h} + \delta\mathbf{h})$$

By the fluctuation-dissipation theorem, we find nonlocal spin texture corrections to magnetic noise, to leading order in gradients

$$\begin{aligned}\langle \delta h_i(\mathbf{r}, t) \delta h_{i'}(\mathbf{r}', t') \rangle =& 2k_B T \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \\ & (\alpha S \delta_{ii'} - \frac{(q\beta)^2}{\gamma} \partial_k m_i \partial_k m_{i'})\end{aligned}$$

This implies corresponding corrections to the gilbert damping tensor.

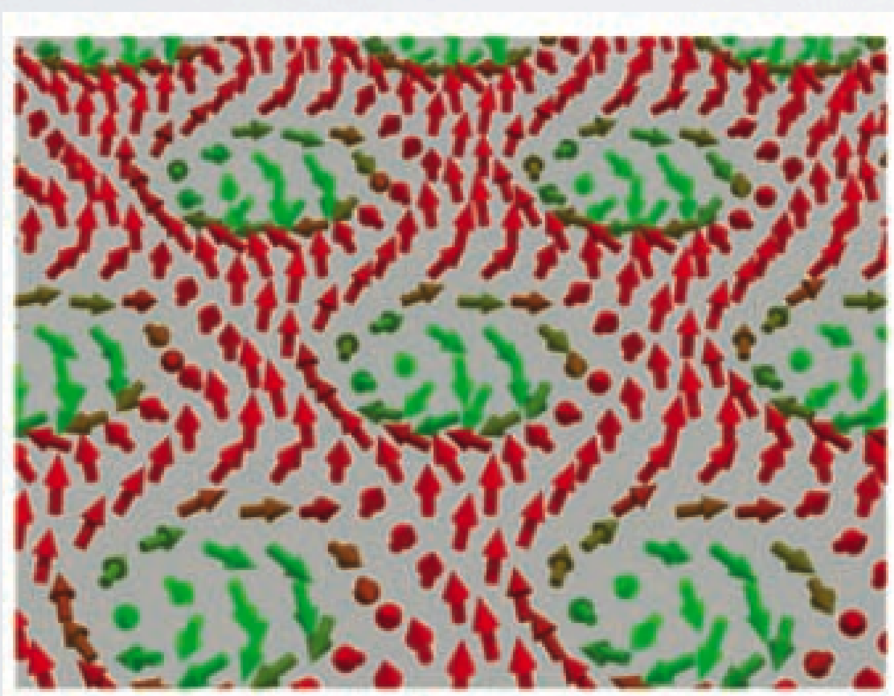
SPIN-TEXTURE RESISTIVITY

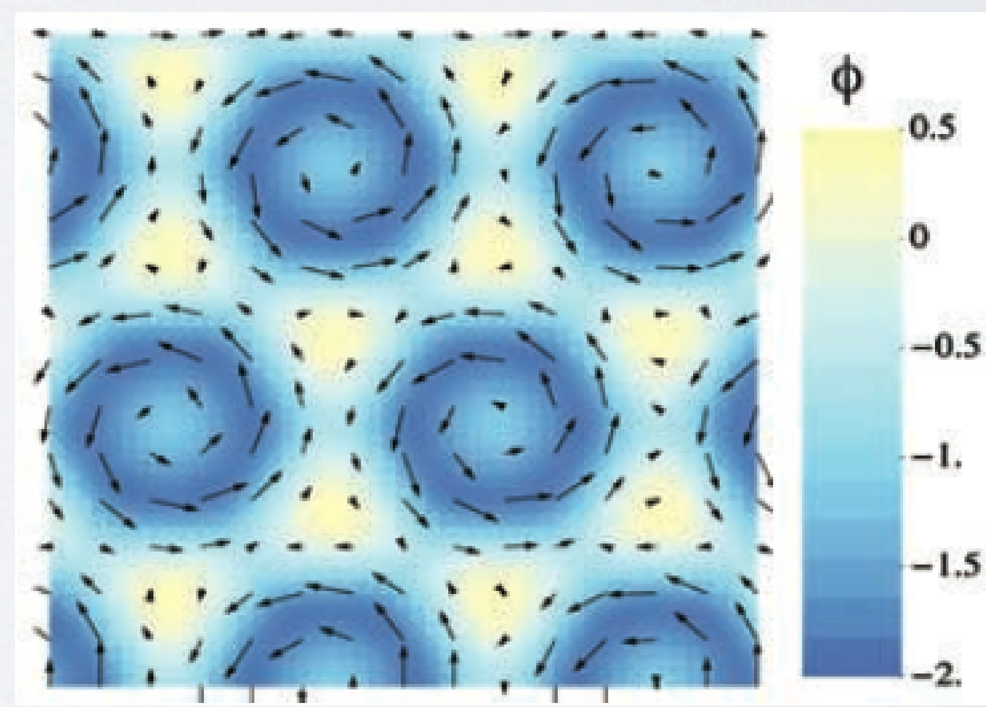
Spin texture introduces anisotropy and Hall effect into the resistivity tensor:

$$\gamma_{ij}[\mathbf{m}] = \underbrace{\delta_{ij}\left[\gamma + \eta(\partial_k \mathbf{m})^2\right]}_{\text{Drude} \quad \text{isotropic}} + \underbrace{\eta' \partial_i \mathbf{m} \cdot \partial_j \mathbf{m}}_{\text{anisotropic}} + \underbrace{\frac{q}{n} \mathbf{m} \cdot (\partial_i \mathbf{m} \times \partial_j \mathbf{m})}_{\text{topological Hall}}$$

Example: Neutron scattering shows skyrmion-lattice spin texture in MnSi:

$$\phi = \frac{1}{4\pi} \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$$





Mühlbauer *et al.*, *Science* **323** (2009) 915

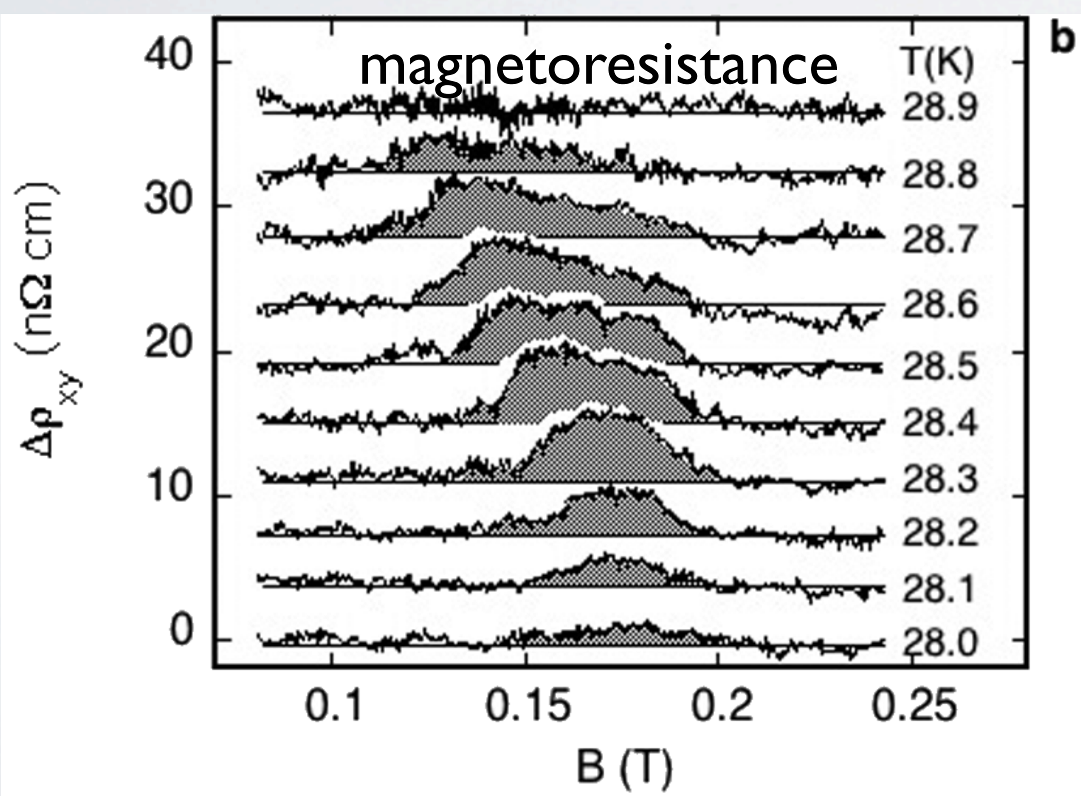
TOPOLOGICAL HALL EFFECT

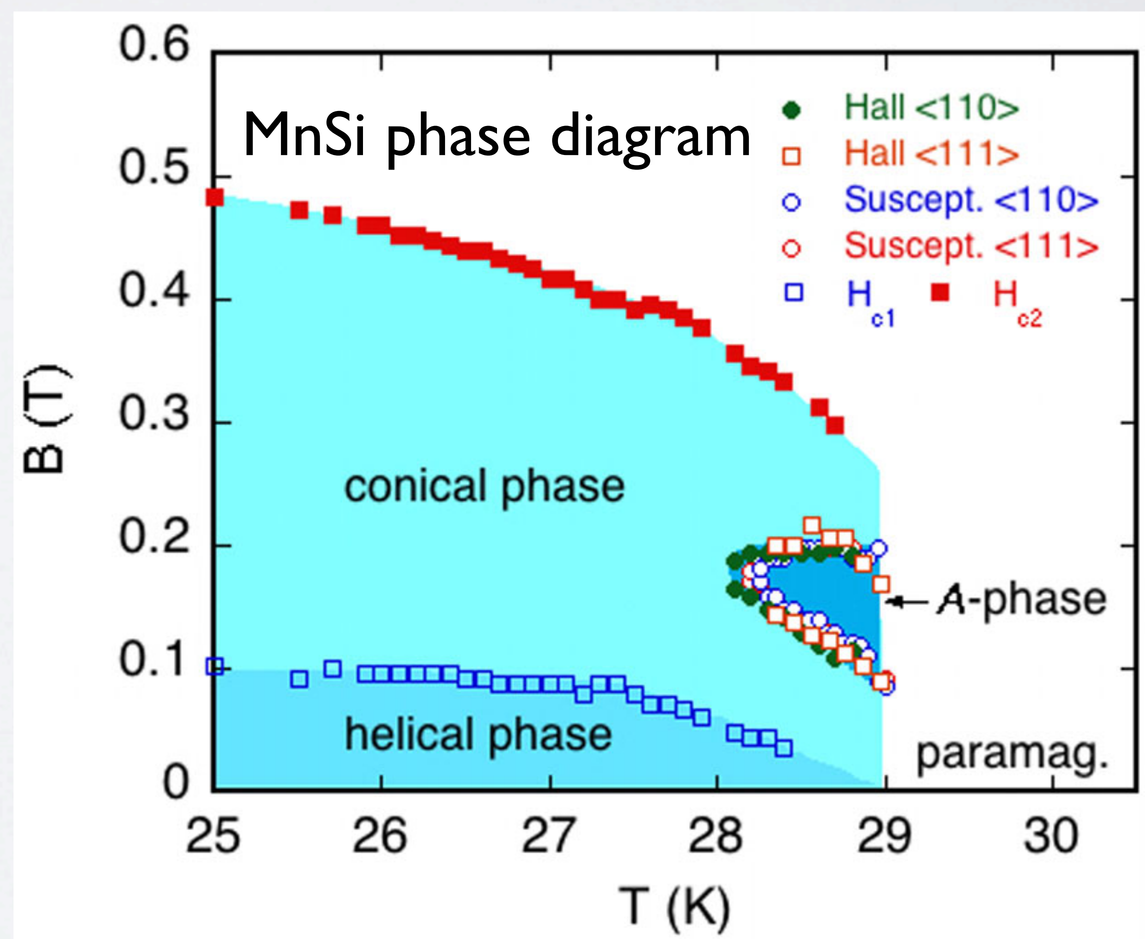
DM term arises due to broken inversion symmetry.

$$\mathcal{F} = A(\partial_i \mathbf{m})^2/2 + B\mathbf{m}\cdot(\boldsymbol{\nabla}\times\mathbf{m}) + U_{int}(\mathbf{m})$$

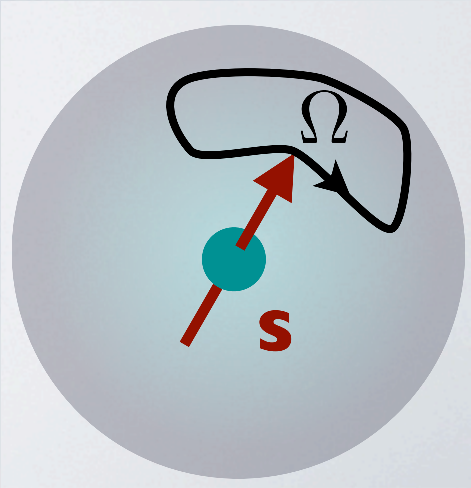
Magnetic helices form in the ground state, skyrmion lattice stabilized by quartic interaction potential

Lee *et al.*, *PRL* **102** (2009) 186601
Neubauer *et al.*, *PRL* **102** (2009) 186602

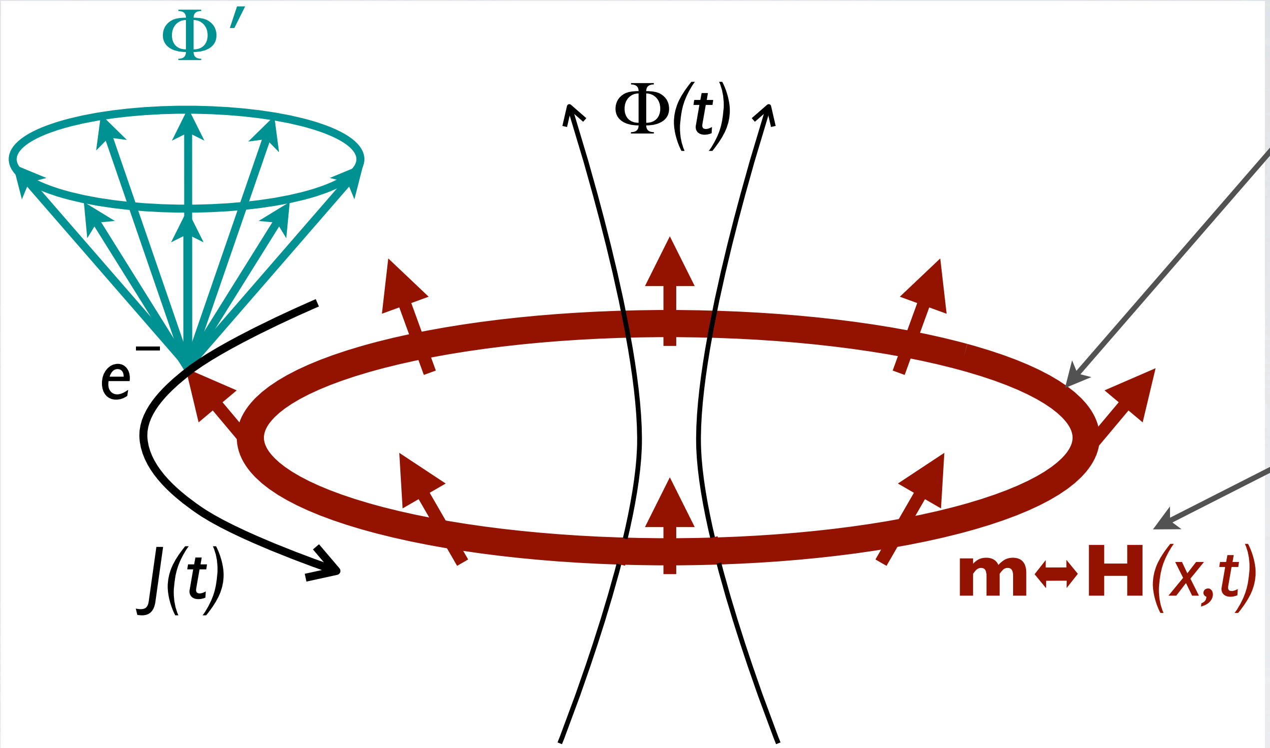




FICTITIOUS ELECTROMOTIVE FORCE



electrons pick up Berry phase Φ' proportional to solid angle Ω subtended by spin texture



spin texture with nontrivial topology

magnetization driven by applied magnetic field

Y. Tserkovnyak and C. H. Wong, *PRB* **79**(2009) 014402.

Current/magnetization coupled by fictitious (Berry) flux

S. E. Barnes and S. Maekawa. *PRL* **98**(2007)246601.

$$\Phi' = \frac{\hbar c}{2e} \oint dx (1 - \cos\theta) \partial_x \phi$$

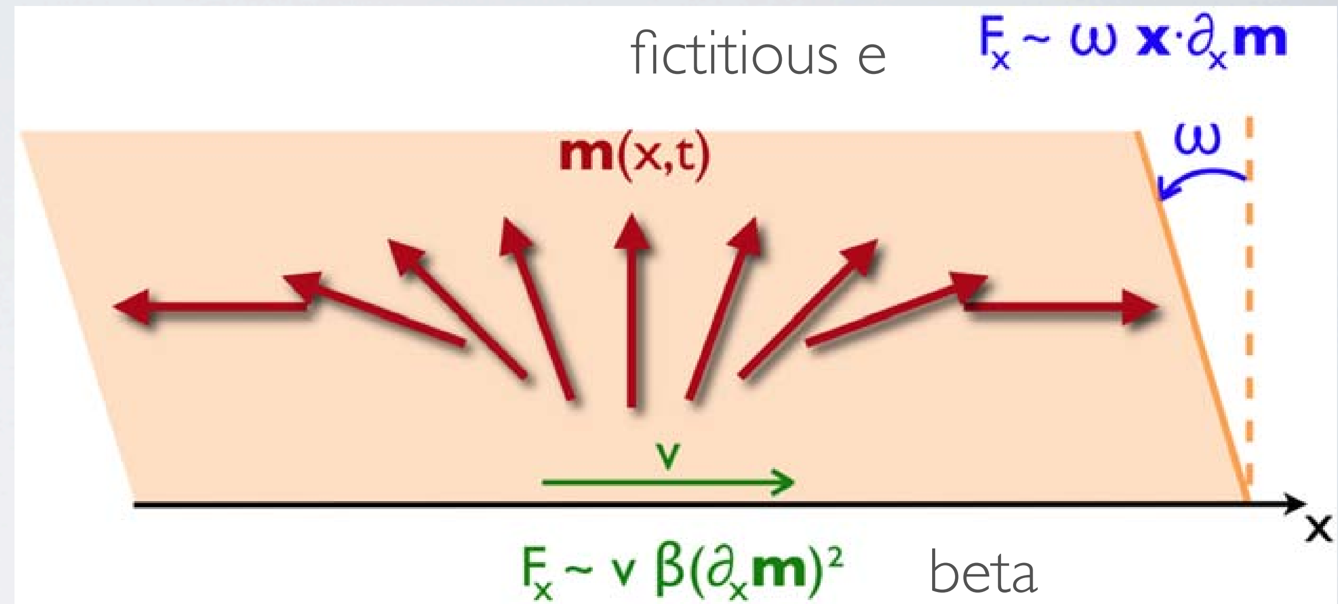
fictitious magnetic flux

$$\mathcal{E}' \equiv -p \partial_t \Phi' / c$$

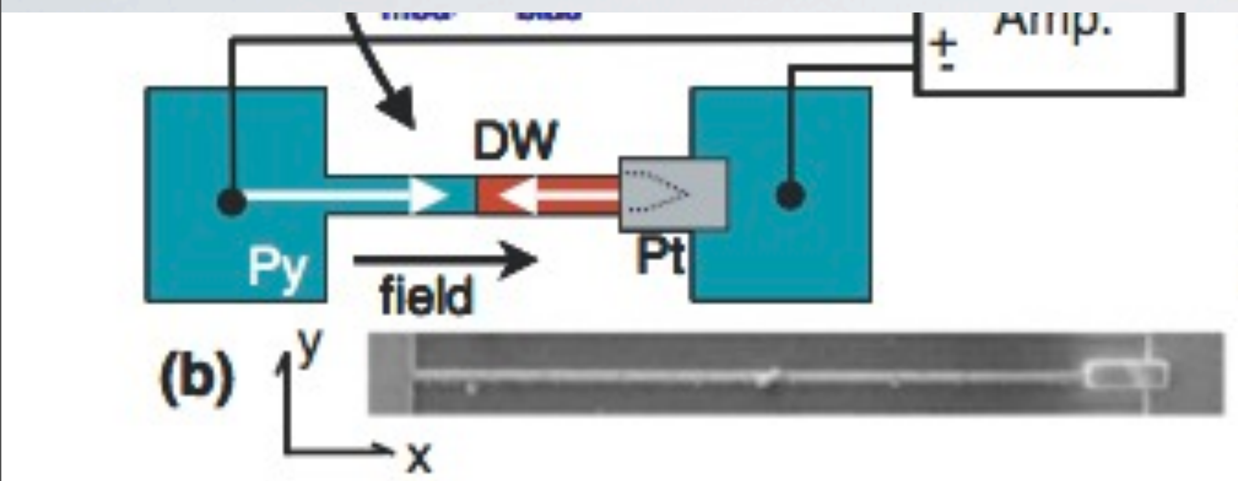
electromotive force

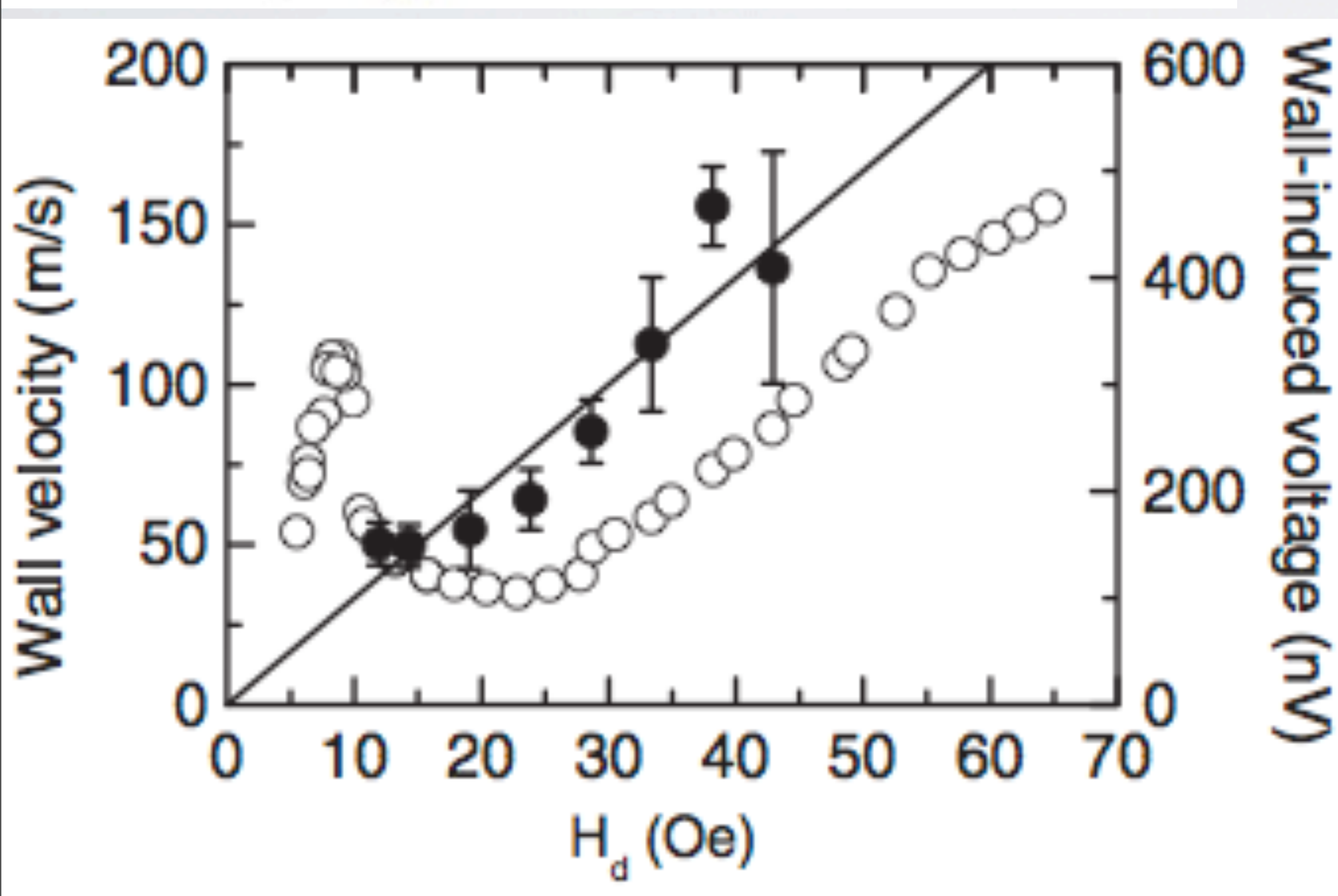
OBSERVATION OF THE F-EMF

Yang et al. ,PRL **102**, 067201(2009)



Macklenburg and Tserkovnyak ,PRB **77**





Possible existence of a Josephson effect in ferromagnets

L. Berger

Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

(Received 11 July 1985)

$$V/H = (\hbar/e)\gamma \approx 11.6\mathrm{nV/Oe}$$

DISSIPATION POWER

Phenomenological coefficients constrained by second law of thermodynamics: positive dissipation

$$P = -\int d^3 r \partial_t \mathcal{F} \geq 0$$

for high heat conductance, uniform temperature

Lower bound on texture resistivity is consistent with microscopic calculations

C. Wickles and W. Belzig, *PRB*, **80**(2009) 104435.

$$\eta + \eta' \geq \frac{(q\beta)^2}{\alpha S}$$

DYNAMICAL CORRECTIONS TO LLG

Wong, C.H. and Tserkovnyak, Y. , PRBr 81, 060303 (2010)

Gradient corrections to LLG with from current dynamics (diffusive limit)

$$\left[(1+\mathbf{m}\times\hat{K})+\mathbf{m}\times(\alpha+\hat{\Gamma})\right]\dot{\mathbf{m}}+\mathbf{m}\times\mathbf{H}/S=\boldsymbol{\tau}_D$$

Gilbert damping tensor $\mathcal{P}\equiv p\hbar/2e$ ρ drude resistivity

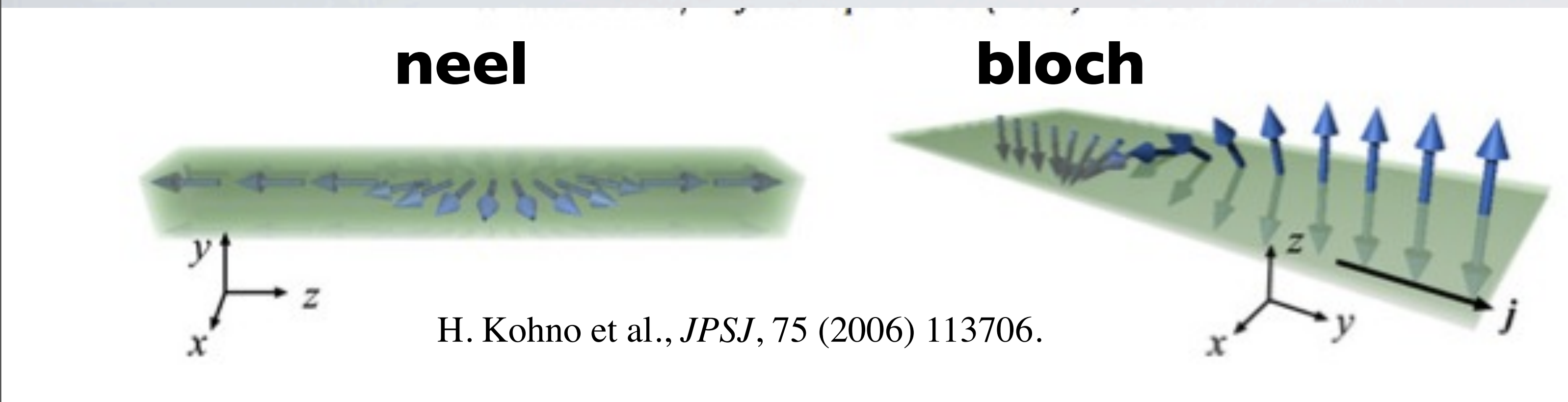
$$\hat{\Gamma}=\frac{\mathcal{P}^2}{\rho S}\left[(\mathbf{m}\times\nabla_i\mathbf{m})\otimes(\mathbf{m}\times\nabla_i\mathbf{m})-\beta^2\nabla_i\mathbf{m}\otimes\nabla_i\mathbf{m}\right]$$

Gyromagnetic tensor

$$\hat{K}=\frac{\beta\mathcal{P}^2}{\rho S}\left[(\mathbf{m}\times\nabla_i\mathbf{m})\otimes\nabla_i\mathbf{m}-\nabla_i\mathbf{m}\otimes(\mathbf{m}\times\nabla_i\mathbf{m})\right]$$

Total dissipation power $$P=\int d^3r\,\dot{\mathbf{m}}\cdot(\alpha+\hat{\Gamma})\cdot\dot{\mathbf{m}}$$

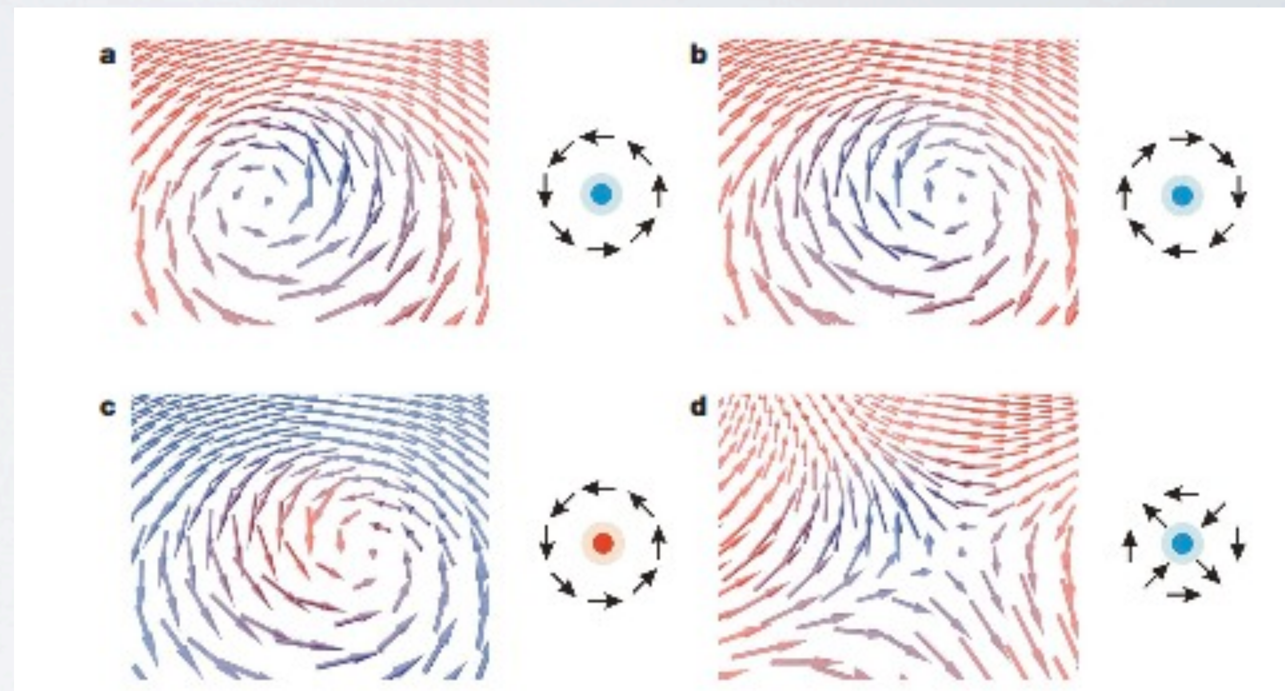
SPIN TEXTURES: TOPOLOGICAL SOLITONS



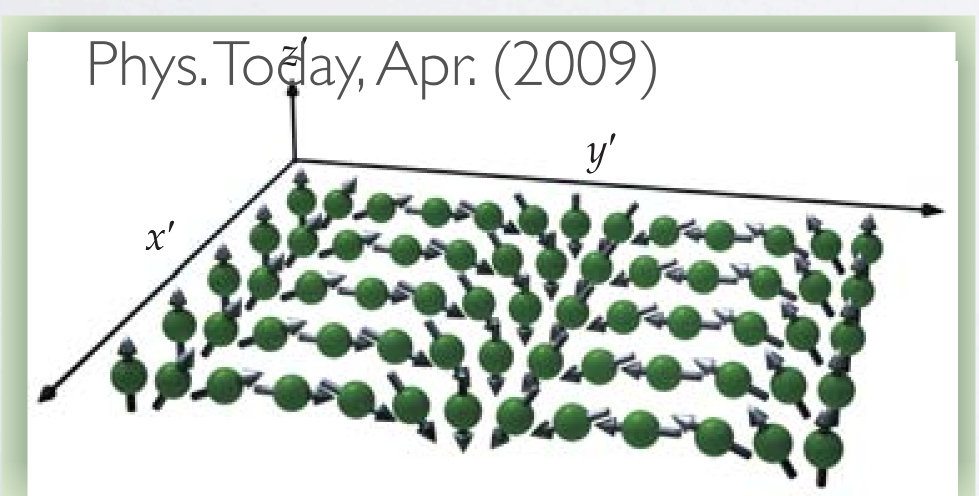
H. Kohno et al., *JPSJ*, 75 (2006) 113706.

DOMAIN WALLS

VORTEX/ANTI-VORTEX



B. Van Waeyenberge et. al., *Nature*, **444**(2006)461.



Phys. Today, Apr. (2009)

SPIN HELIX

2D MAGNETIC SOLITONS

- Magnetic solitons are particle-like objects characterized by topological charge.

$$q = \int dxdy \, \frac{1}{4\pi} \mathbf{m} \cdot (\partial_x \mathbf{m} \times \partial_y \mathbf{m})$$

- They minimize exchange energy within each topological sector.

$$\frac{U_{xc}}{A} = \frac{1}{2} \int dxdy \, (\partial_i \mathbf{m})^2 \geq 4\pi q$$

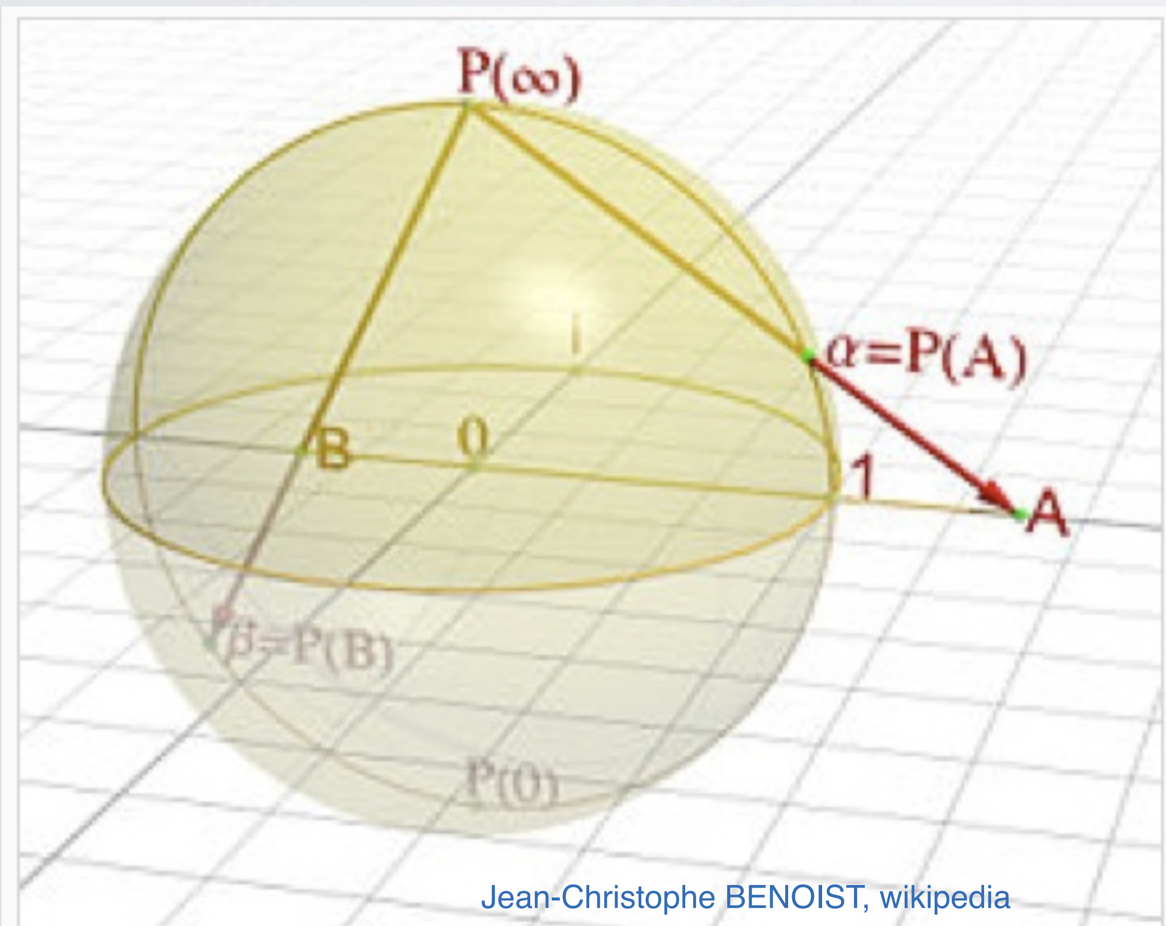
- In stereographic coordinates: $w = \tan\frac{\theta}{2} e^{i\varphi}, \; z = x + iy$

  Solitons are analytic functions: $\dfrac{\partial w}{\partial \bar{z}} = 0, \quad \dfrac{\partial}{\partial \bar{z}} \equiv \partial_x + i\partial_y$

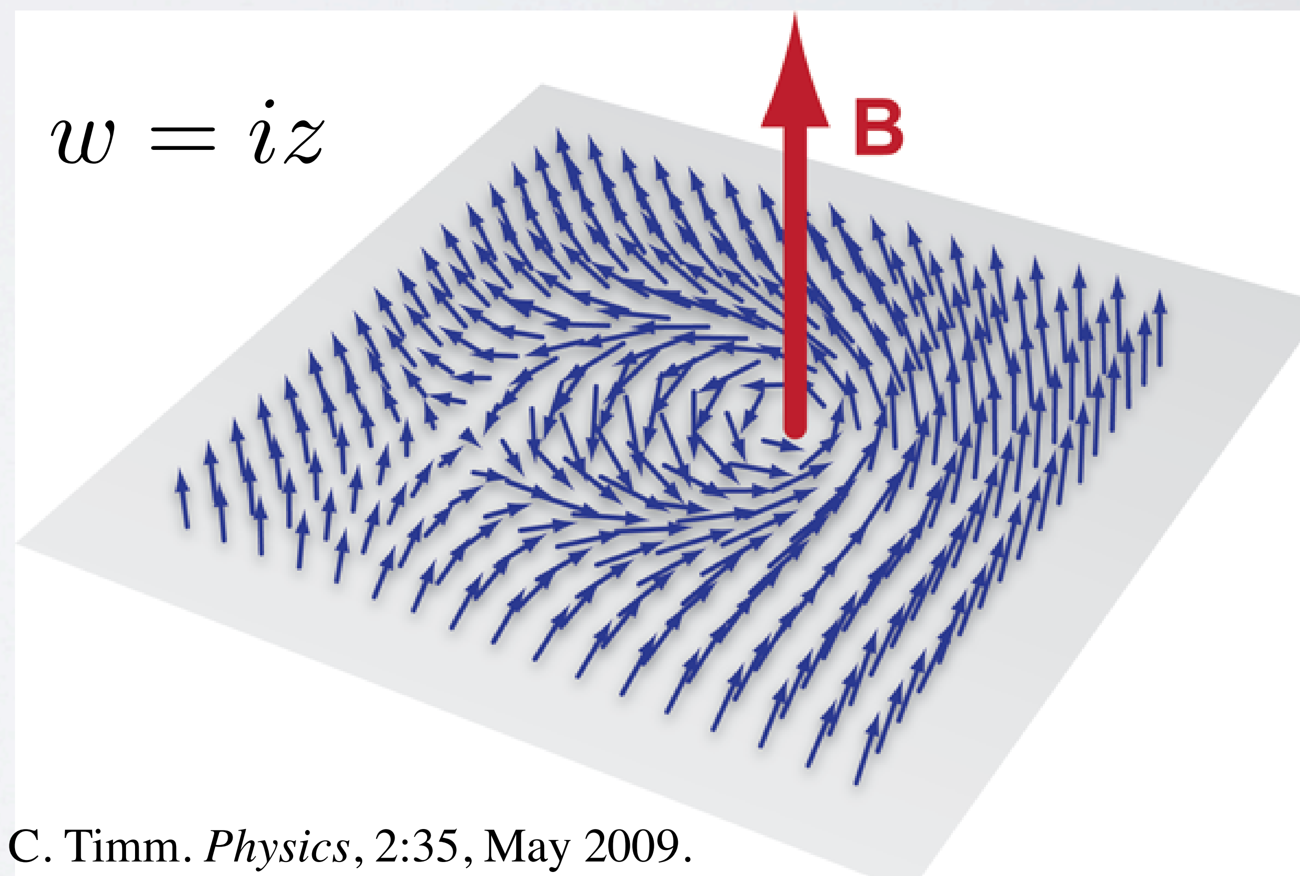
SKYRMION TEXTURES

- The stereographic coordinates projects the sphere onto the complex plane

- The simplest finite energy solution on the infinite plane is a skyrmion



Jean-Christophe BENOIST, wikipedia



C. Timm. *Physics*, 2:35, May 2009.

COLLECTIVE COORDINATES

- Parametrize magnetic texture $\mathbf{m}(\mathbf{q}(t), \mathbf{r})$ by collective coordinates, $\mathbf{q}(t)$, in configuration space.

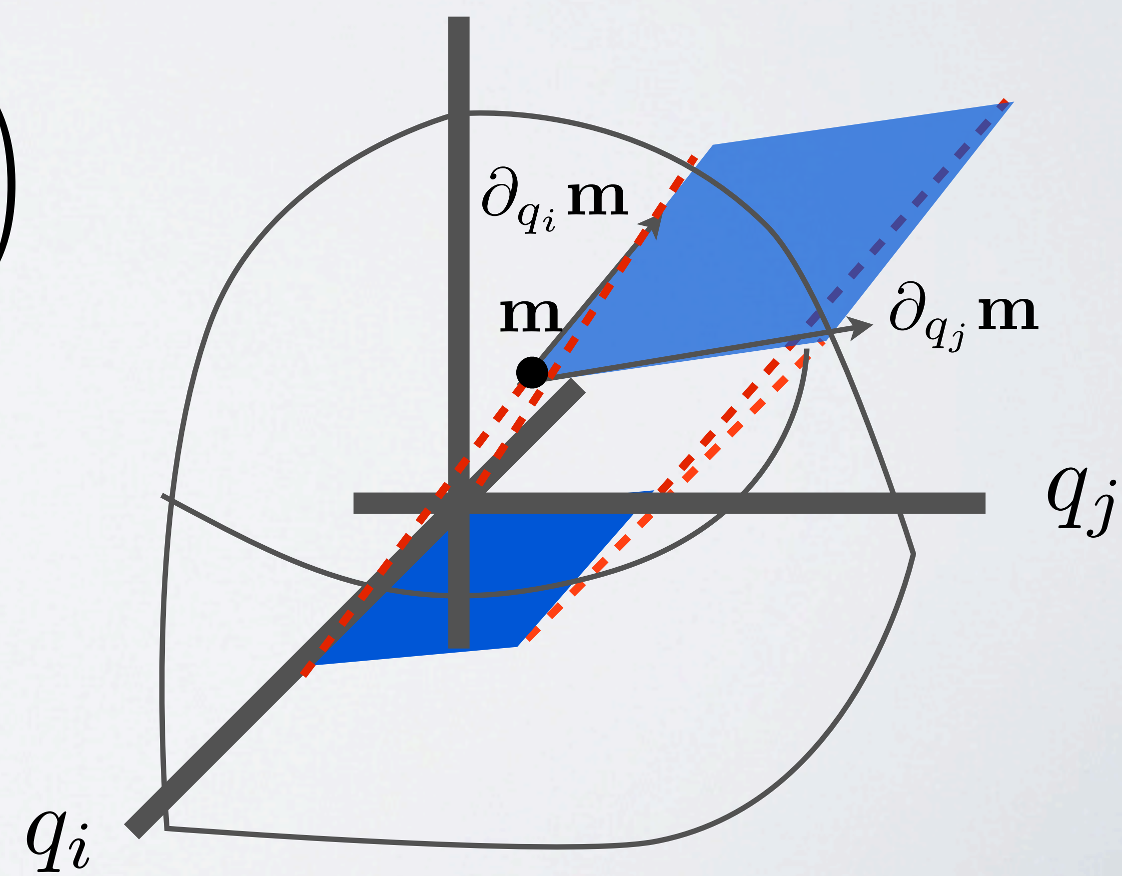
- Soliton texture geometry described by two basic tensors.

$$b_{ij} \equiv \mathbf{m} \cdot \frac{\partial \mathbf{m}}{\partial q_i} \times \frac{\partial \mathbf{m}}{\partial q_j}$$

$$= \sin\theta \left(\frac{\partial \theta}{\partial q_i} \frac{\partial \varphi}{\partial q_j} - \frac{\partial \theta}{\partial q_j} \frac{\partial \varphi}{\partial q_i} \right)$$

$$d_{ij} \equiv \frac{\partial \mathbf{m}}{\partial q_i} \cdot \frac{\partial \mathbf{m}}{\partial q_j}$$

$$= \frac{\partial \theta}{\partial q_i} \frac{\partial \theta}{\partial q_j} + \sin^2\theta \, \frac{\partial \varphi}{\partial q_i} \frac{\partial \varphi}{\partial q_j}$$



SOLITON EQUATION OF MOTION

- LLG may be approximated by an equation of motion for collective coordinates.

"massless" particle

$$\boxed{\hat{G}(\mathbf{q})\dot{\mathbf{q}} + \hat{D}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{F}(\mathbf{q})}$$

A. A. Thiele, *PRL*., 30 (1973) 230.

effective forces

$$F_i \equiv -\frac{1}{S}\int d^3r\,\frac{\partial \mathbf{m}}{\partial q_i}\cdot \mathbf{H}_{\rm st} - \frac{1}{S}\frac{\partial}{\partial q_i}\int d^3r\,\mathcal{V}\,,$$

gyrotropic and damping tensors

$$\hat{G} \equiv \int d^3r\,\hat{b}\,,\;\; \hat{D} \equiv \alpha\int d^3r\,\hat{d}$$

Dissipation power $\quad P = D_{ij}\dot{q}_i\dot{q}_j$

hierachy of hard/soft modes $\quad \tau_{q_N} < \ldots < \tau_{q_i} < \ldots \tau_{q_1}$

zero mode

- inertial terms may be generated by spin wave excitations

K. Y. Guslienko et. al., *PRB*, **81** (2010) 014414.

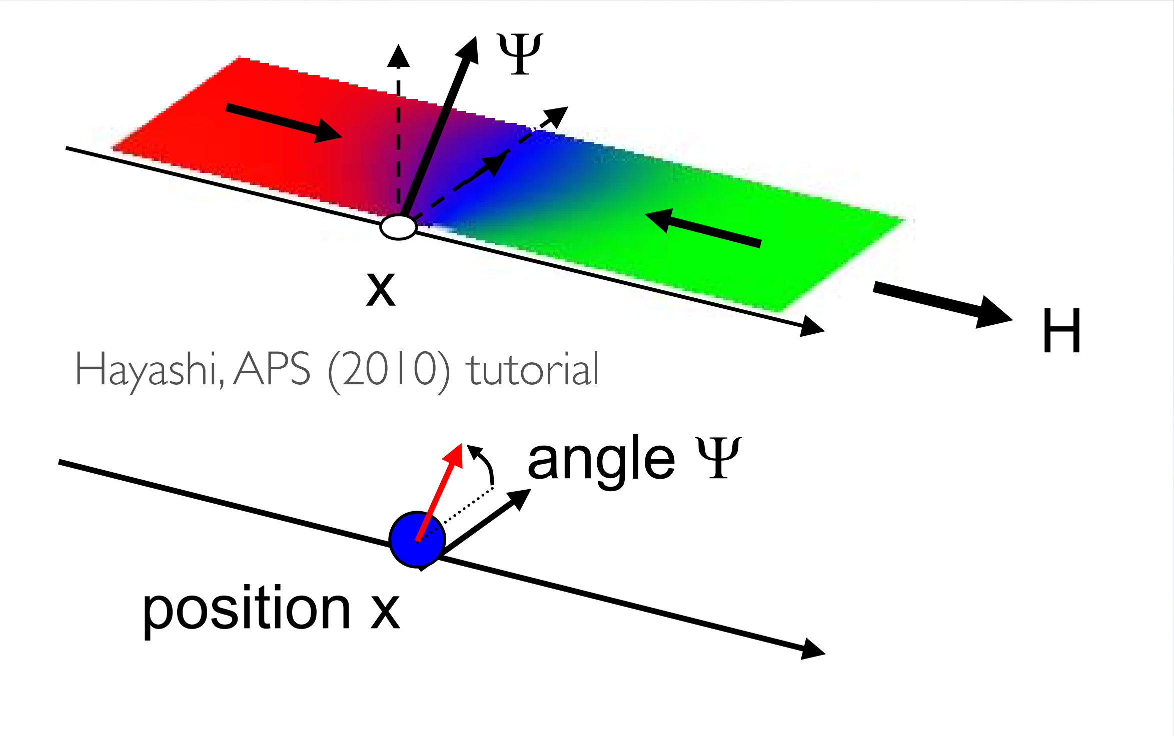
Example: Domain wall dynamics

soft modes $\mathbf{q} = (\Psi, X)$

hard mode-wall width W

$$\alpha \frac{\dot{X}}{W} - \dot{\Psi} = -\gamma H + \frac{\beta \mathcal{P} j}{SW}$$

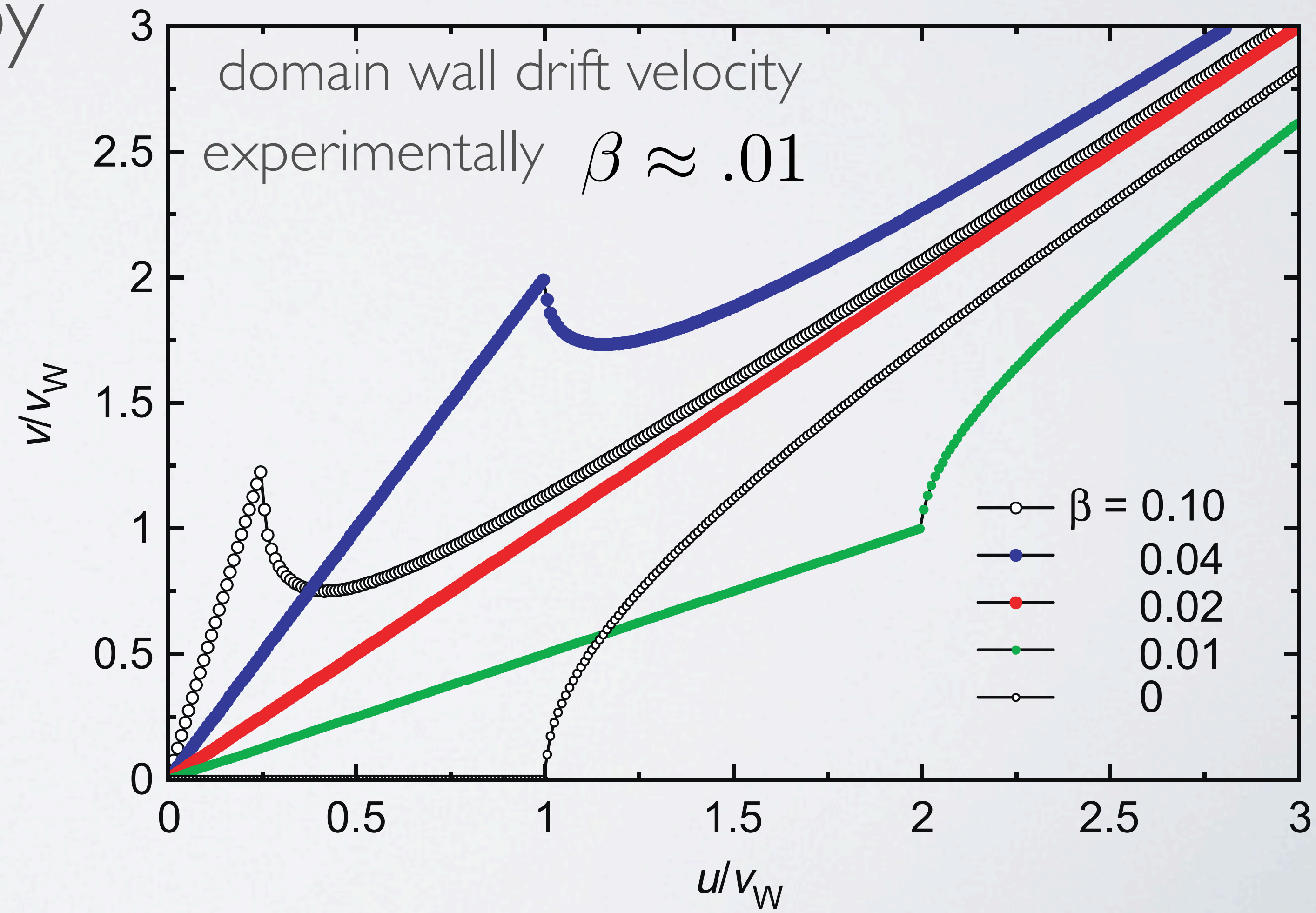
$$\frac{\dot{X}}{W} + \alpha \dot{\Psi} = \frac{K_\perp}{2S} \sin 2\Psi - \frac{\mathcal{P} j}{SW}$$



Tserkovnyak et al., JMMM 320,1282(2008)

- beta may be determined by measuring domain wall velocity.

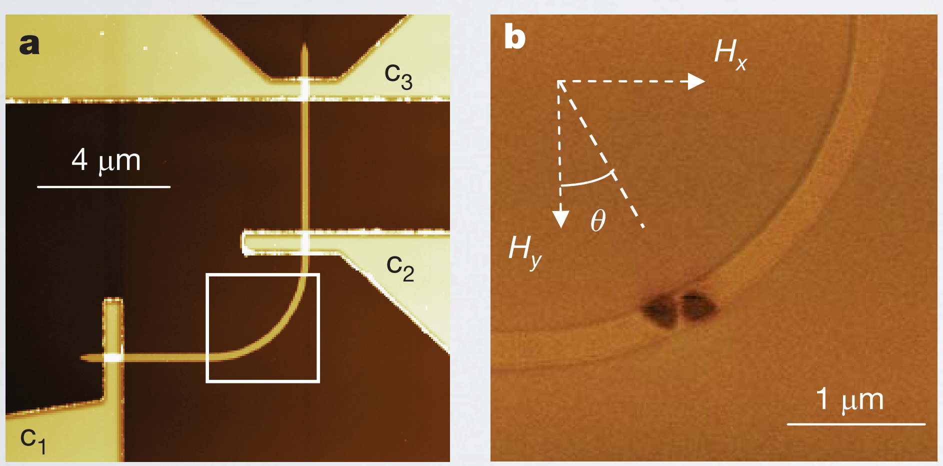
$$\dot{\Psi} \to 0, \; \dot{X} \to \frac{\mathcal{P}\beta}{\alpha} j$$

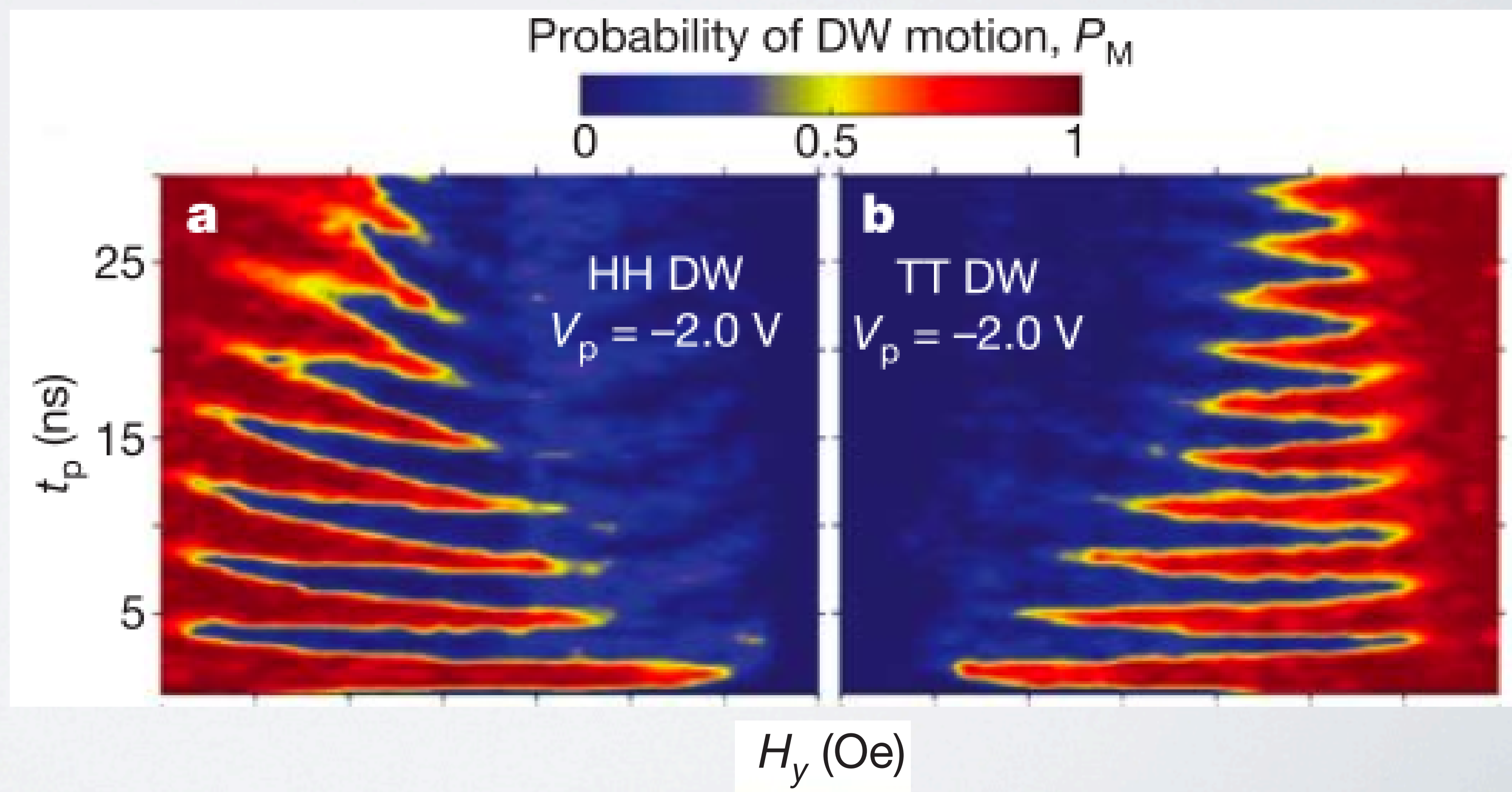


DYNAMICAL DEPINNING

Probability of escape from pinning potential has oscillatory dependence on applied current pulse length

Exact behavior depends on internal structure of domain wall





Thomas *et al.*, *Nature* **443** (2006) 197

CORRECTIONS TO SOLITON EQUATION

Wong, C.H. and Tserkovnyak, Y. , PRBr **81**, 060303 (2010)

- Consider translational mode $\mathbf{R}(t)$ of rigid soliton, $\mathbf{m}(\mathbf{r}-\mathbf{R}(t))$
- Current backaction lead to corrections to gyrotropic and damping tensor

$$\boxed{\begin{aligned}\delta\hat{G} &= -\frac{\mathcal{P}^2}{\rho S}\int d^3r\beta(\hat{b}\hat{d}+\hat{d}\hat{b}),\\ \delta\hat{D} &= -\frac{\mathcal{P}^2}{\rho S}\int d^3r(\hat{b}^2+\beta^2\hat{d}^2)\end{aligned}}$$

- Leading correction to damping is significant relative to Gilbert damping

$$\boxed{|\delta\hat{D}| \sim \frac{\mathcal{P}^2}{\rho S\lambda^2} \sim \alpha}$$

for typical values in transition metals

$\lambda \sim 3\ \mathrm{nm}$ exchange length

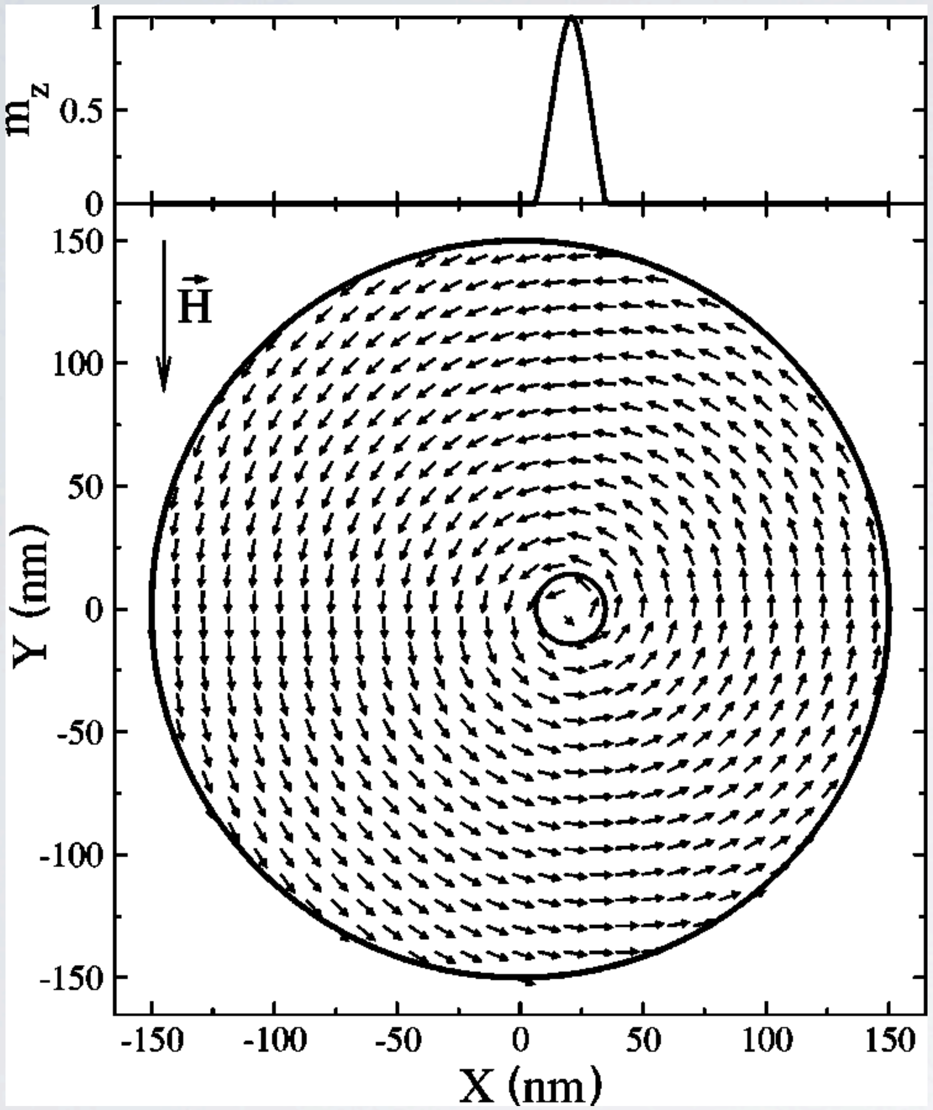
$\gamma \approx -1.8\times 10^{11}\ \mathrm{rad/s\,T}$

$M \sim 10^6\ \mathrm{A/m}$

$\rho \sim 100\ \Omega\,\mathrm{nm}, \quad p \sim 1$

$\alpha, \beta \sim 10^{-2}$

VORTEX PROFILE



- Skyrmion in the core to minimize exchange energy
- Meron outside the core to minimize magnetostatic energy by avoiding magnetic charges on surfaces

K. Y. Guslienko and K. L. Metlov, *PRB*, **63**(2001)100403.

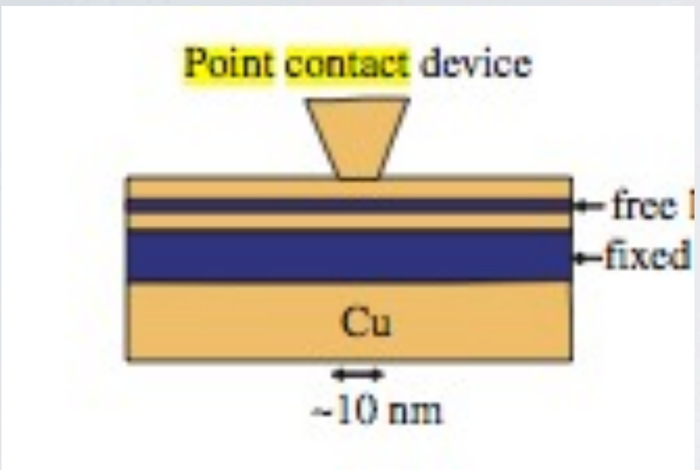
$$w = \begin{cases} iz/\lambda & |z| \leq \lambda \\ iz/|z| & |z| > \lambda \end{cases}$$

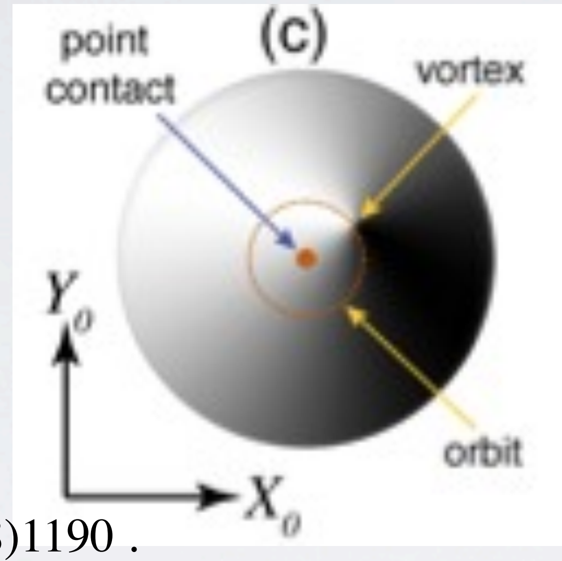
vortex core size ~ $\lambda \equiv \sqrt{A_{\rm xc}/\mu_0 M^2} \sim 3$ nm,

VORTEX GYRATION IN SPIN VALVE

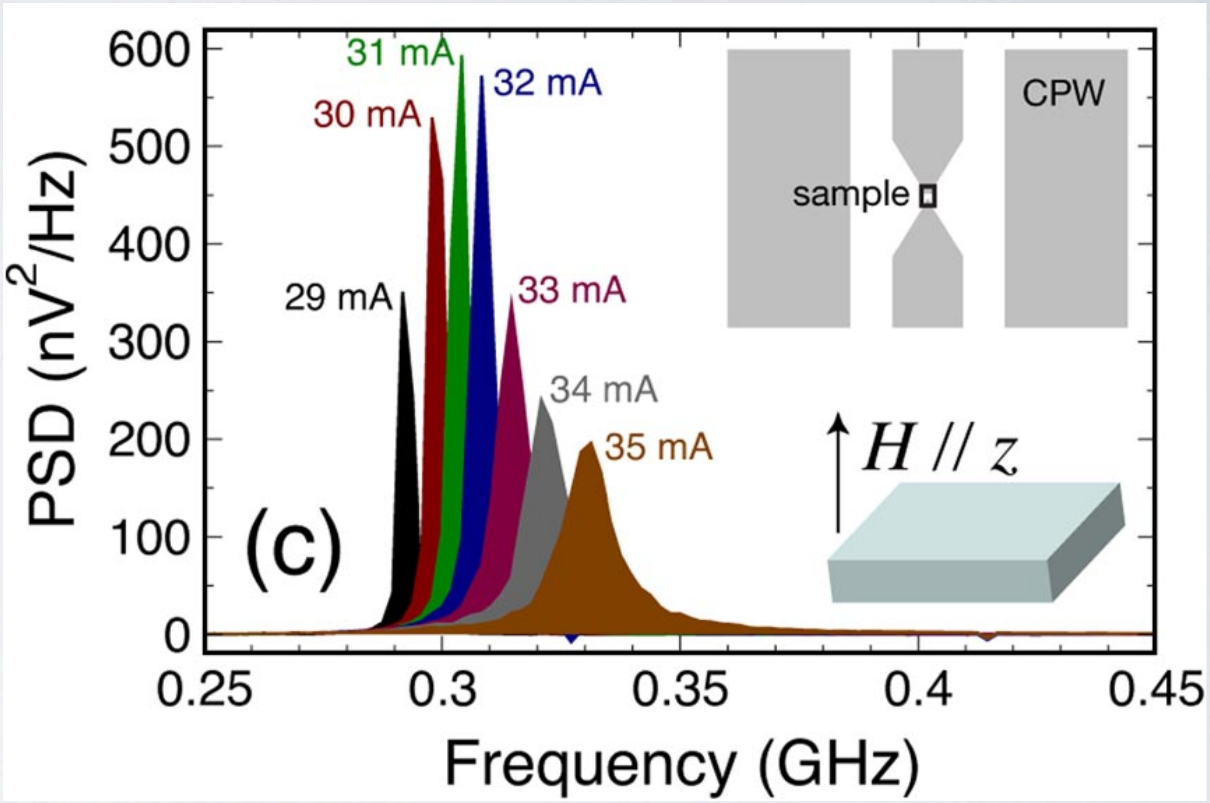
Applied perpendicular DC current drives vortex motion via Slonczewski spin torque

Q. Mistral et. al., *PRL*, **100**(2008) 257201.





D. Ralph and M. Stiles. *JMMM*, **320** (2008)1190 .



A simple model of spin transfer

$$\mathbf{H}_{\mathrm{st}} = \left(\frac{\sigma \mathcal{P} I}{\pi a^2}\right) \mathbf{p} \times \mathbf{m}$$

σ spin transfer efficiency

$\mathbf{p}$ spin polarization direction of applied current

Consider large amplitude vortex dynamics in point contact device, neglecting boundary effects

VORTEX ORBITAL MOTION

viscosity

spin transfer

$$G\hat{\mathbf{z}} \times \dot{\mathbf{R}} - D\dot{\mathbf{R}} - AI\hat{\mathbf{R}} - \frac{BI}{R}\hat{\mathbf{z}} \times \hat{\mathbf{R}} = 0$$

magnus force

confining potential

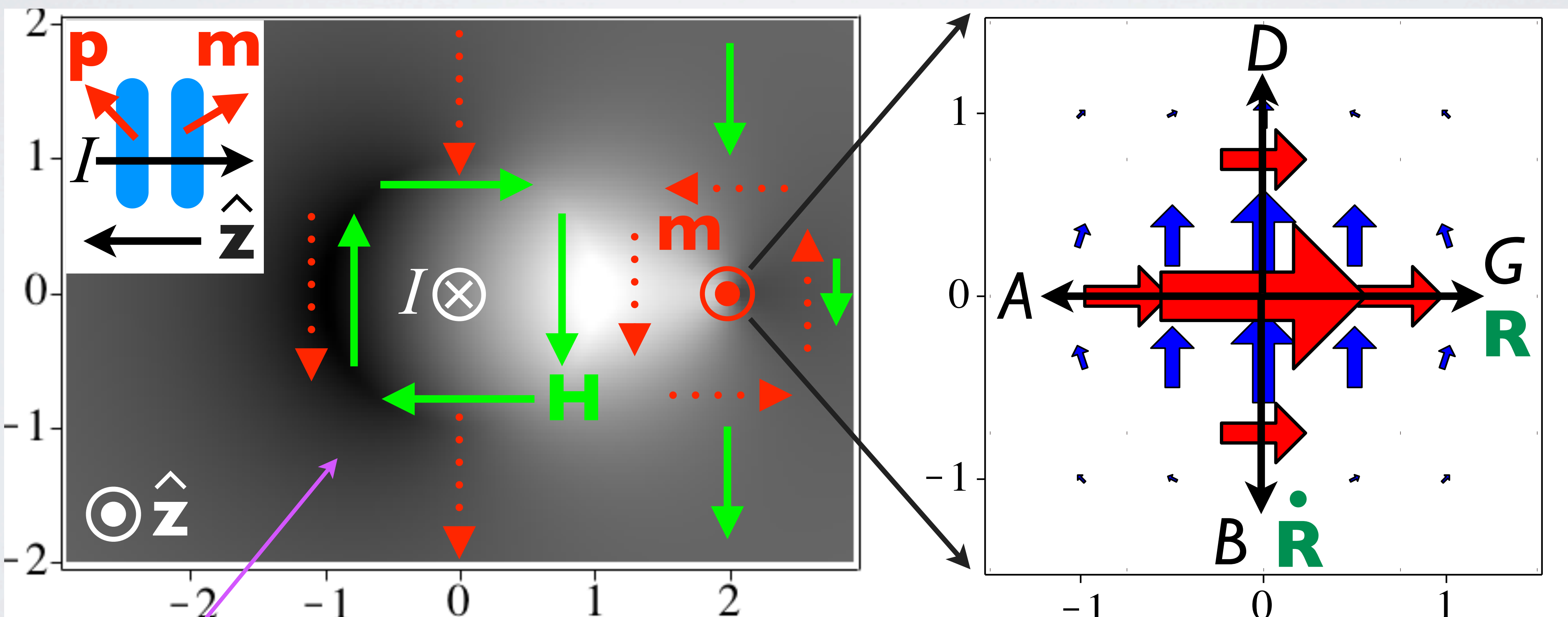
$$U(\mathbf{R}) = -\int d^2r\, \mathbf{H}_{oe} \cdot \mathbf{M}/S \approx AIR$$

spin transfer coupling

$$B = \sigma \mathcal{P} p_\perp / S$$

C. H. Wong and Y. Tserkovnyak, *PRB*, **81** (2010) 060404.

currents inside vortex core



balance of forces

vortex energy density $\mathcal{V} = -\mathbf{H}_{oe} \cdot \mathbf{M}$

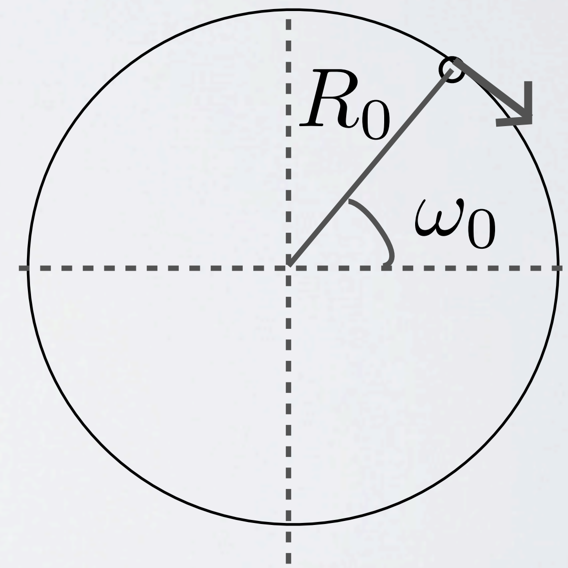
CORRECTIONS TO CIRCULAR ORBIT

Applied DC current generates Oersted field which creates linear confining potential, resulting in stable orbits, while spin torque drives precession.

$$R_0 = \frac{BG}{AD},\ \omega_0 = -\frac{AI}{GR_0} \quad P = D\left(\frac{AI}{G}\right)^2$$

In-plane "Eddy" currents circulating in vortex core causes additional damping which is measurable in orbit radius and frequency, and dissipation power

$$\frac{\delta D}{D} = \frac{\delta\omega_0}{\omega_0} = \frac{\delta P}{P} = -\frac{\delta R_0}{R_0}$$



$d\omega_0/dI \sim 10\ \mathrm{MHz/mA}$

Application as a DC tunable, high Q oscillator

Mistral et al. PRL **100** 257201,

INDUCED CHEMICAL POTENTIAL

Diffusive current exerts a counter spin-torque

$$\boldsymbol{\tau}_D = \mathcal{P}(1 + \beta \mathbf{m}\times)(\mathbf{j}_D \cdot \boldsymbol{\nabla})\mathbf{m} \qquad \mathbf{j}_D = \boldsymbol{\nabla}\mu/e\rho$$

In the incompressible limit

$$\nabla^2 \mu = \mathcal{P} e \boldsymbol{\nabla} \cdot \mathbf{F} \qquad \mathbf{F} = \mathbf{e} + \mathbf{f}$$

The spin forces are

$$\mathbf{e} = \dot{\mathbf{R}} \times \mathbf{b} \quad \mathbf{f} = \beta \hat{d} \cdot \dot{\mathbf{R}}.$$

Wong, C.H. and Tserkovnyak, Y. , PRBr **81**, 060303 (2010)

CONCLUSION

- In spin-textured metallic ferromagnets, a geometrical interaction arises between collective spin density and itinerant electrons
- The phenomenology is captured by the theory of "Spin magnetohydrodynamics"
- Hydrodynamic back action generate gradient corrections to the Landau-Lifshitz-Gilbert equation and corresponding soliton equations
- Corrections are sensitive to spin texture geometry and can be observed in soliton dynamics in spin valve structures.