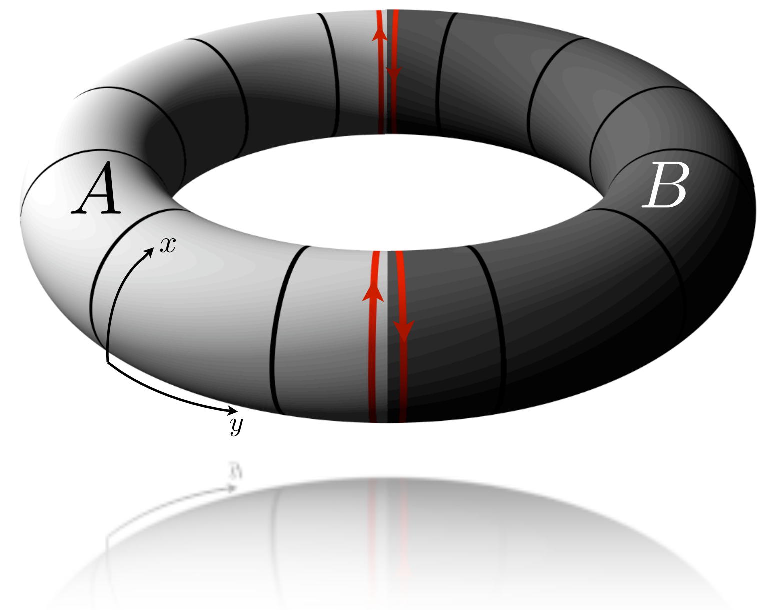


Entanglement in Fractional Quantum Hall States (on the torus)

Emil J. Bergholtz

MPI für Physik komplexer Systeme - Dresden

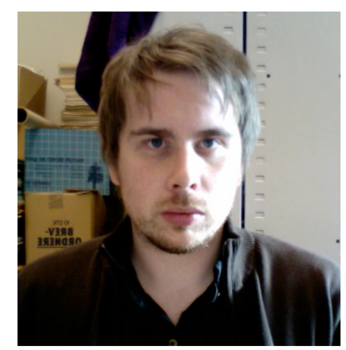
Phys. Rev. Lett. **104** 156404 (2010)
New J. Phys. **12** 075004 (2010)

Nordita, Stockholm, 14 September 2010

Collaborators

- Andreas Läuchli, Dresden

- Juha Suorsa, Oslo



- Masudul Haque, Dresden

Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

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- Introduction: Fractional Quantum Effect
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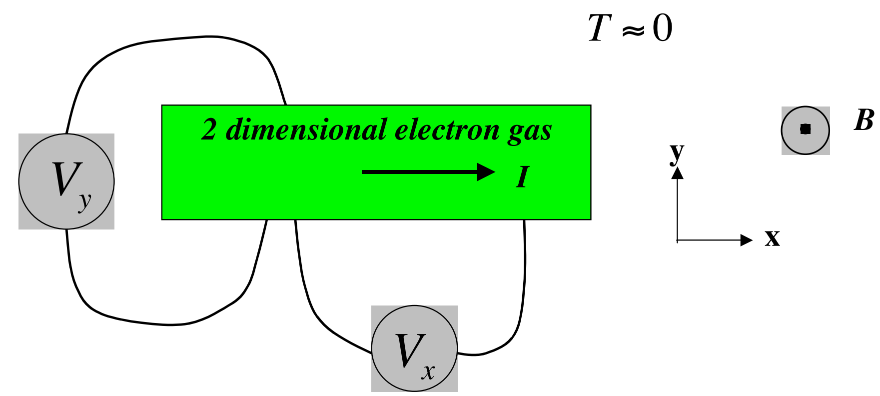
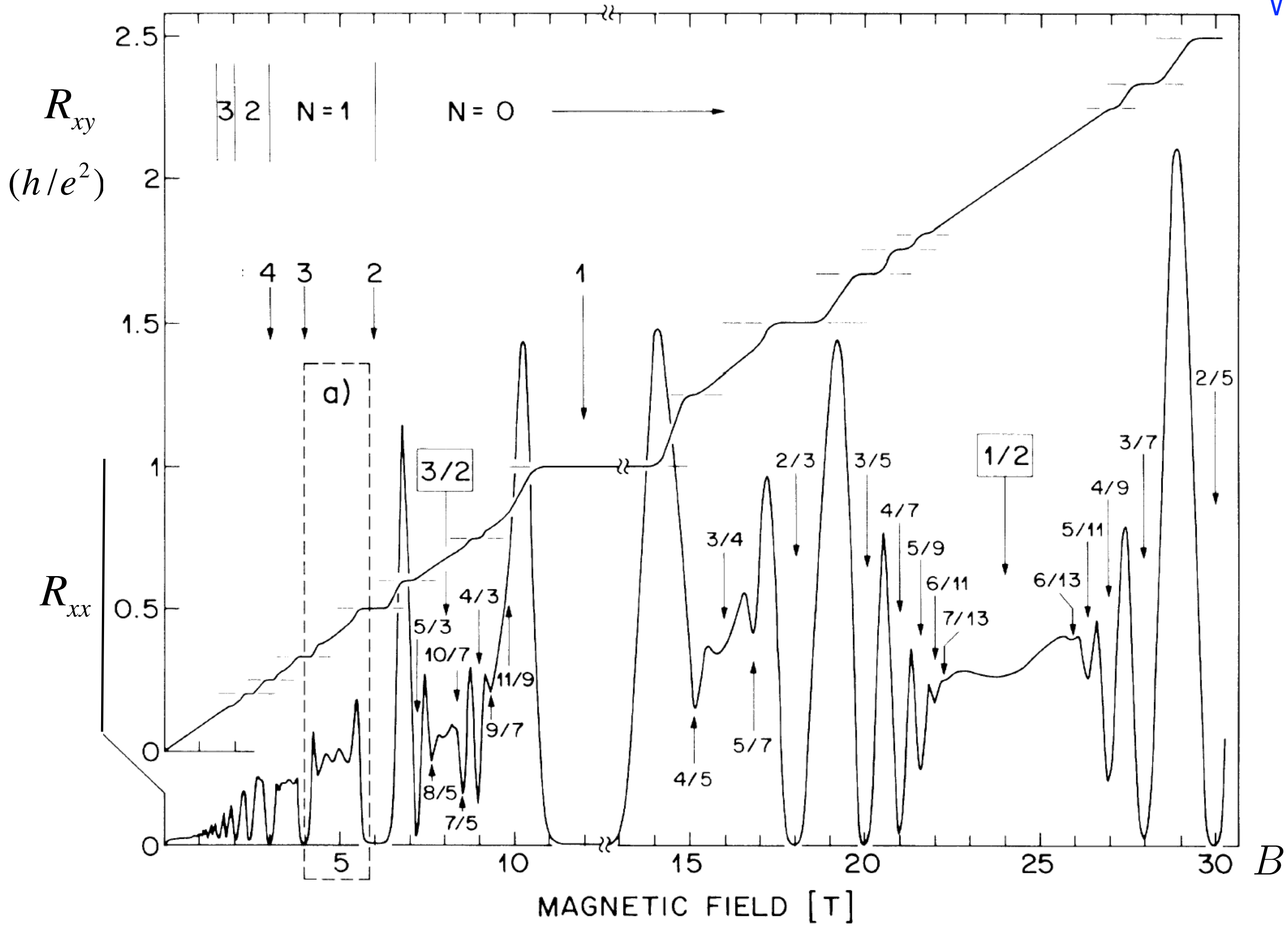
Quantum Hall Effect





Willet et al.,1987

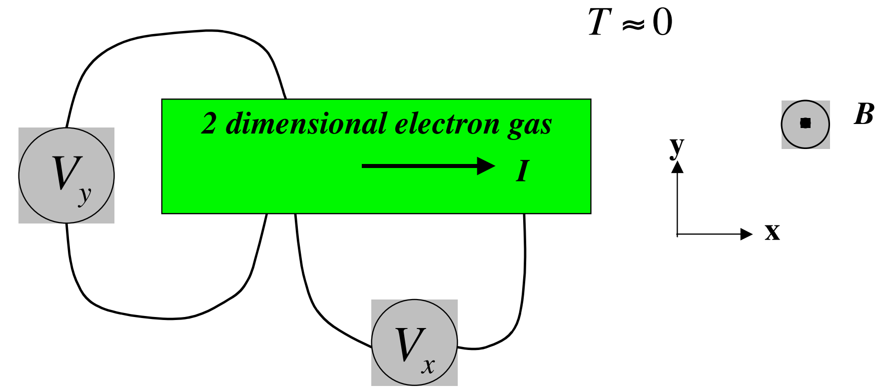
Quantum Hall states

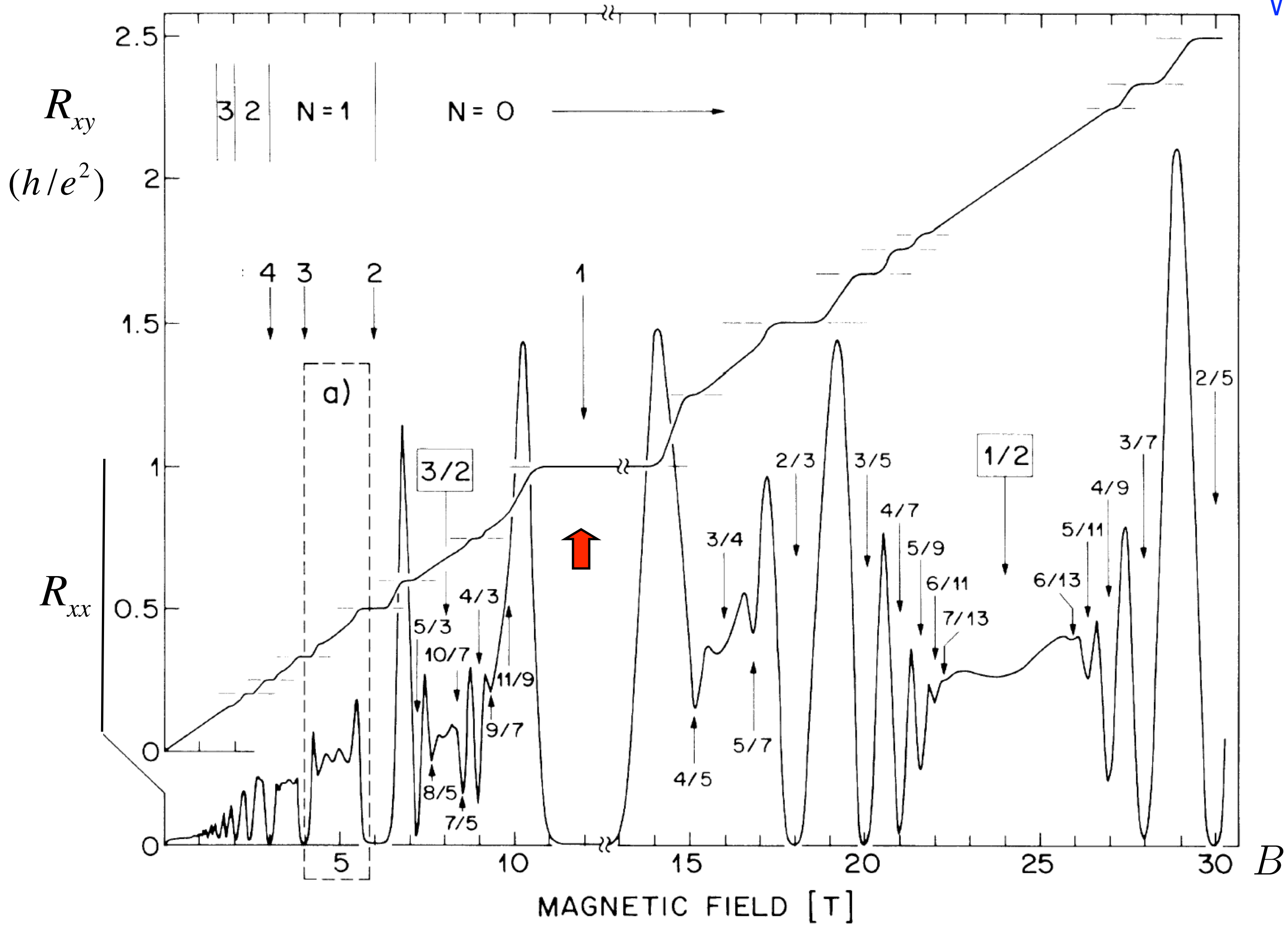
$$R_{xy} = \frac{1}{s}\frac{h}{e^2} \qquad R_{xx} = 0$$

h Planck's constant
e electron charge

mpipks

Quantum Hall Effect





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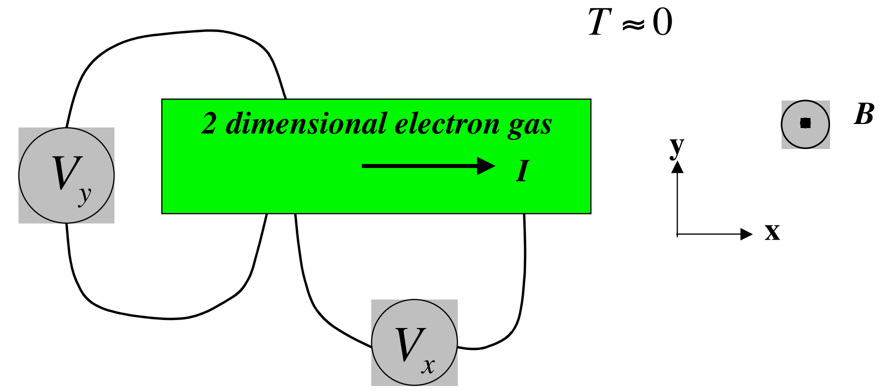

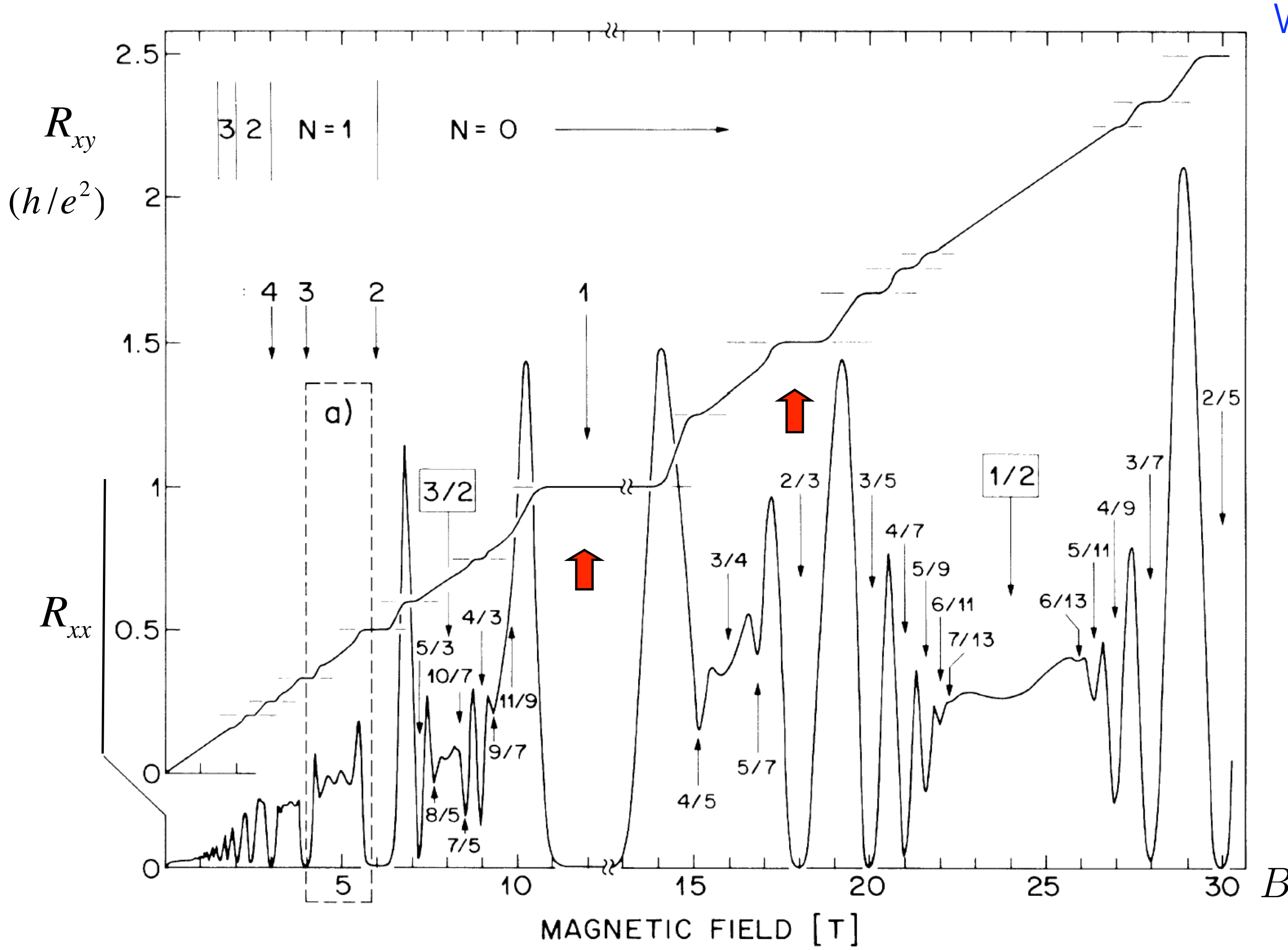

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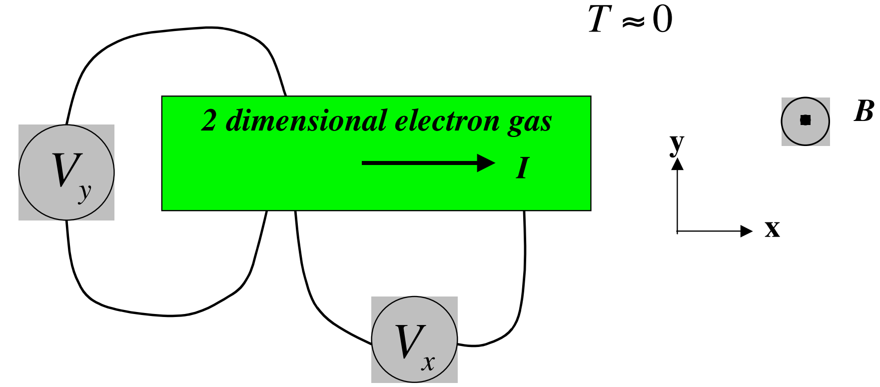

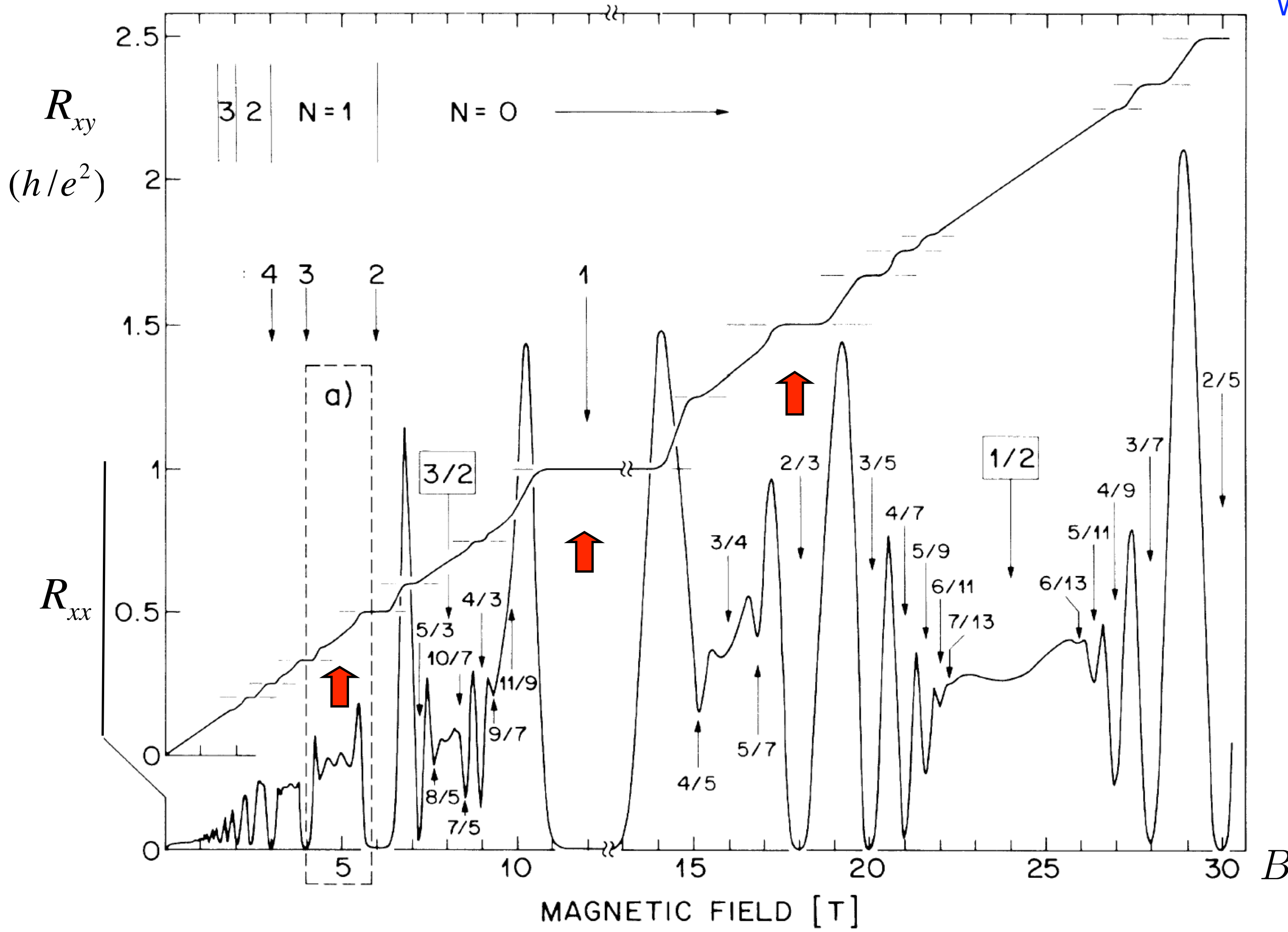

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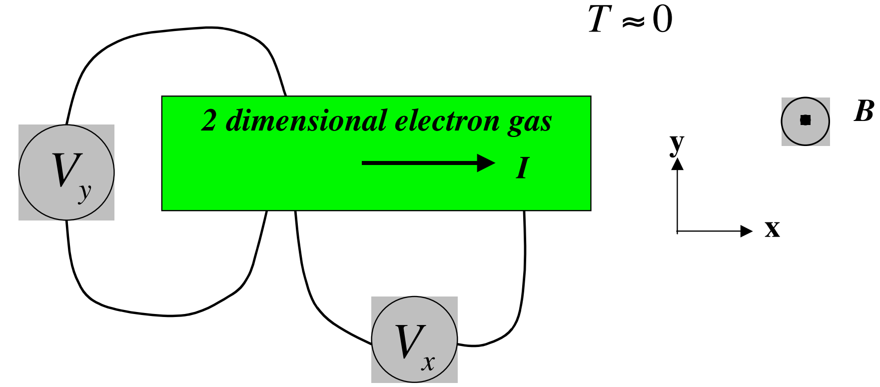

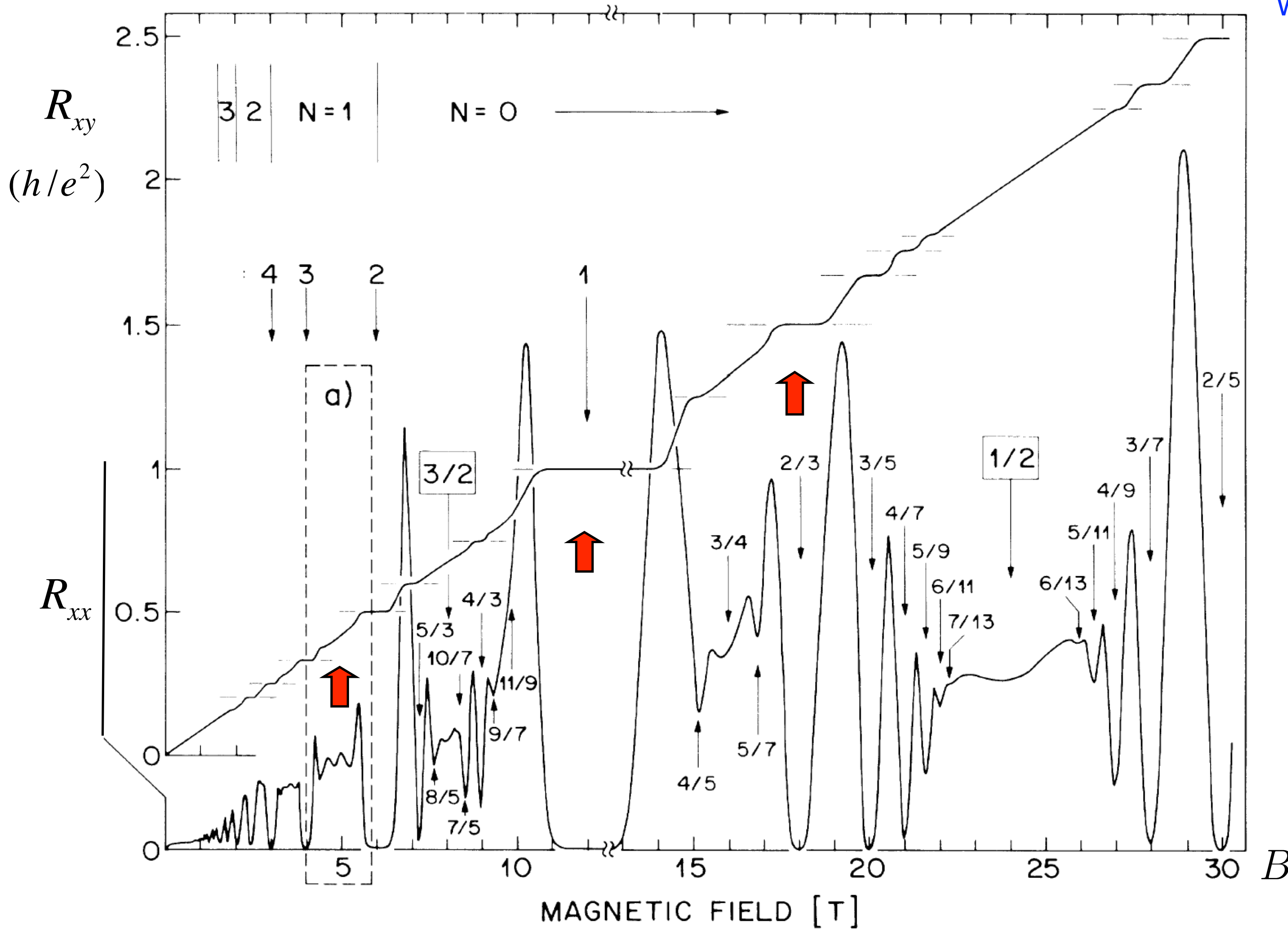

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(incompressible quantum liquids)

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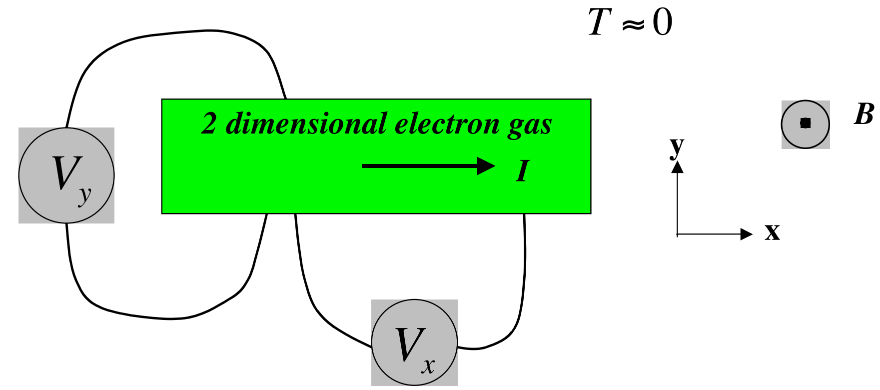

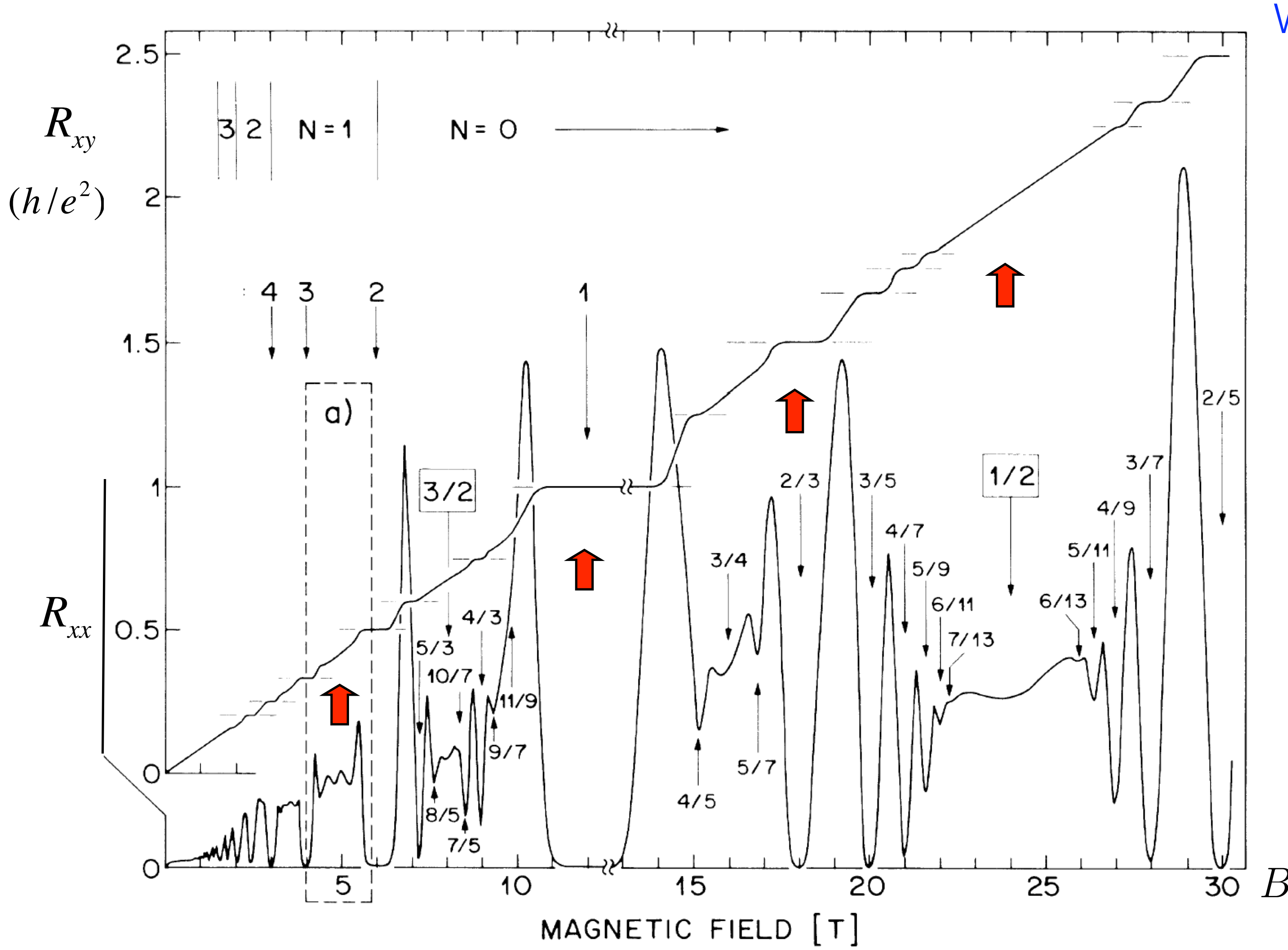

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Metallic states

$$s = \frac{1}{2}, \frac{1}{4}, \ldots$$

(compressible quantum liquids)

mpipks

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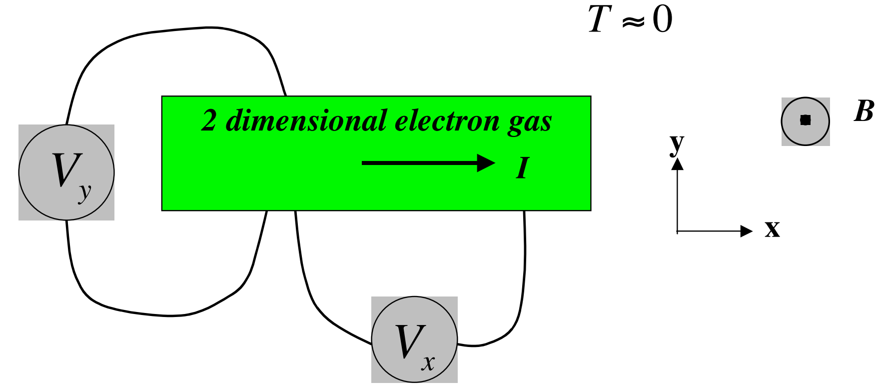

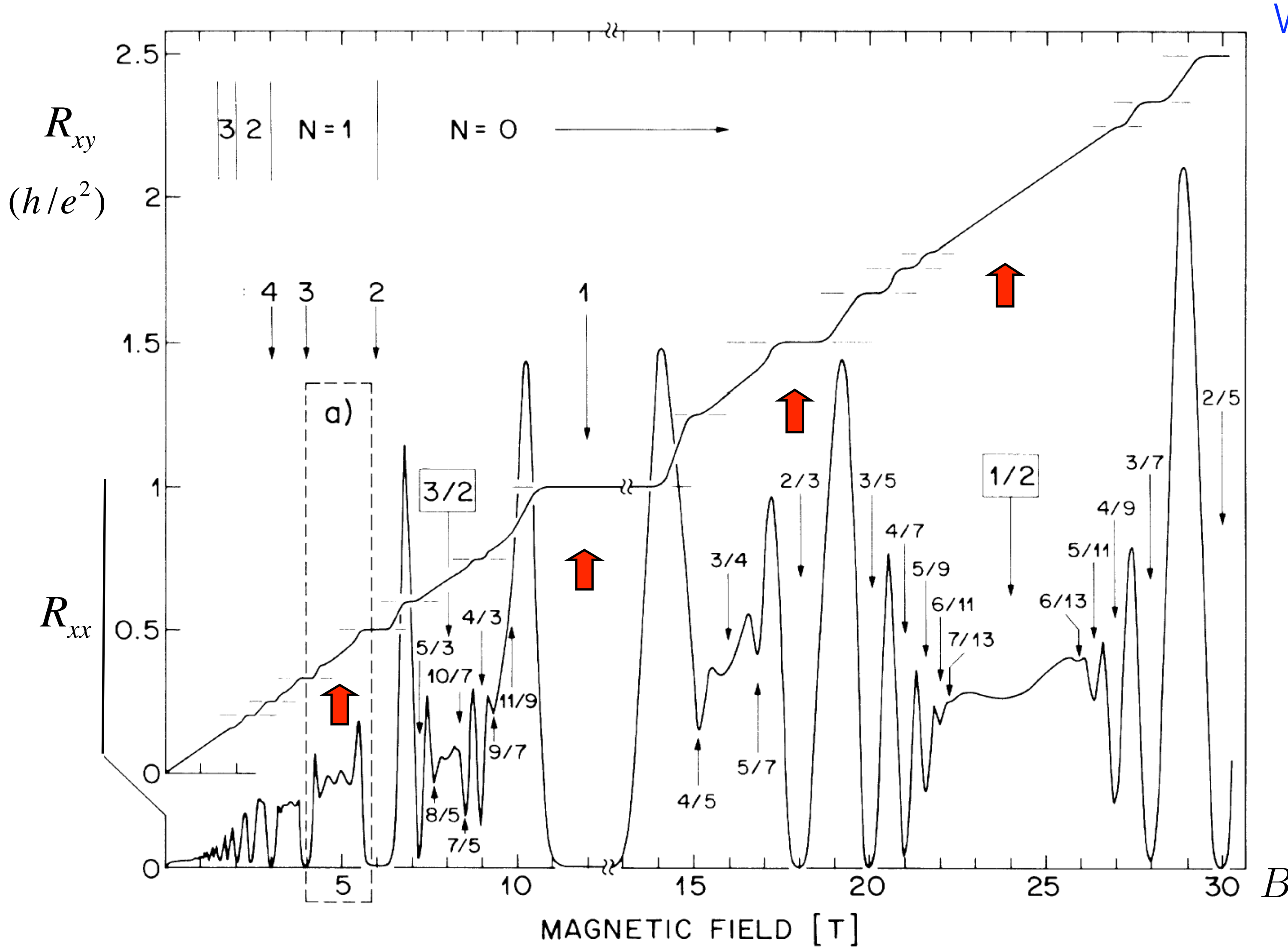

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(incompressible quantum liquids)

Metallic states

$$s = \frac{1}{2}, \frac{1}{4}, \ldots$$

(compressible quantum liquids)

....and much more

Stripes (high LL's)
Wigner Crystals (low filling)
.....

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One electron in a magnetic field (2D)

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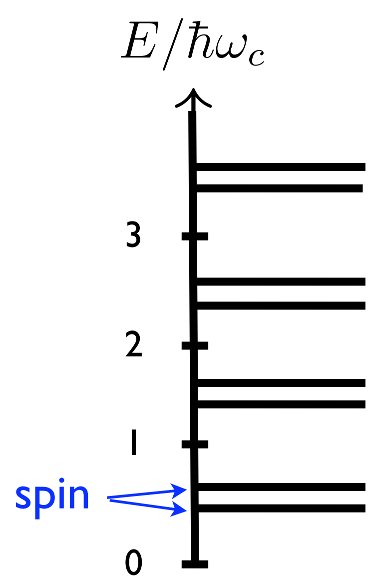

- Kinetic energy quantized $E_n = (n + \frac{1}{2})\hbar\omega_c$ $n = 0, 1, 2, 3...$

Landau levels $\omega_c = \frac{eB}{mc}$

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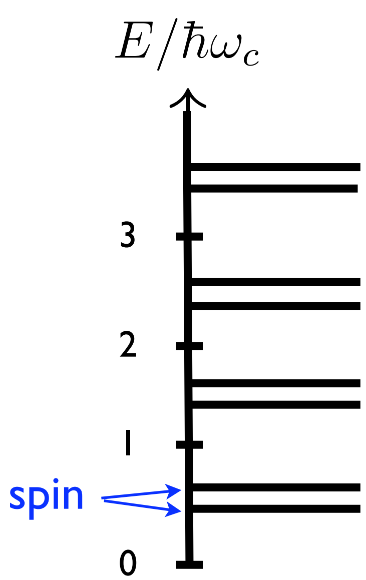

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  # of states $\propto B$

  One state per flux quantum $\varphi_0 = hc/e$

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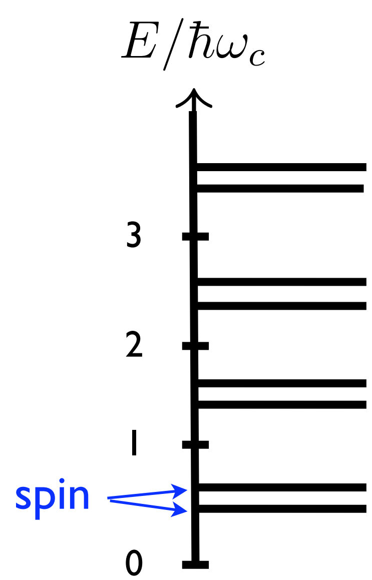

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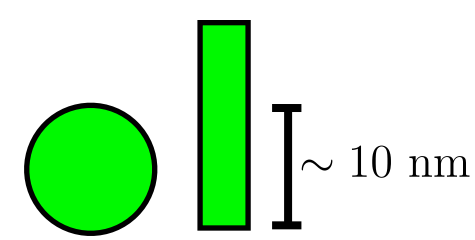
- Quantized area of electron state

$$A = 2\pi\ell^2 \;;\quad \ell \equiv \sqrt{\frac{\hbar c}{eB}} = \frac{257\text{Å}}{\sqrt{\frac{B}{1\ \text{tesla}}}}$$



(within a Landau level)

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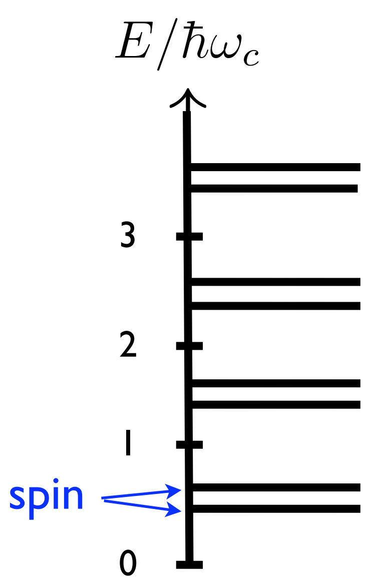

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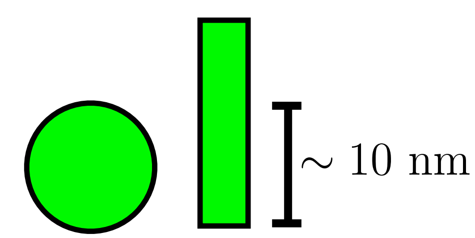

(within a Landau level)

Define ν = number of filled LL *(filling factor)*

The crucial parameter!

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Integer versus fractional filling

Fixed area per state *(one state per flux quantum* $\varphi_0 = hc/e$*)*

□ empty one electron state

■ filled one electron state

mpipks

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Fixed area per state *(one state per flux quantum* $\varphi_0 = hc/e$*)*

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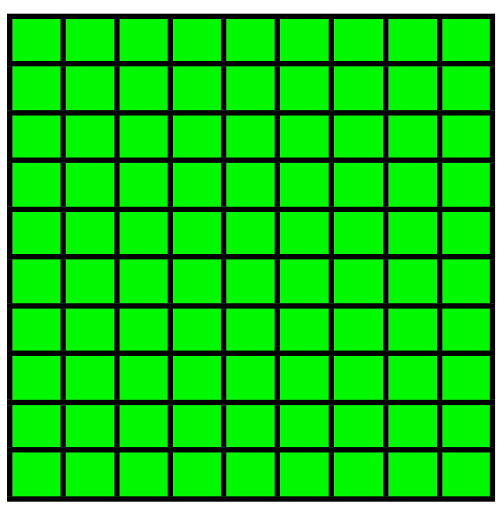
IQHE $\nu = 1$



unique state

Integer versus fractional filling

Fixed area per state (*one state per flux quantum* $\varphi_0 = hc/e$)

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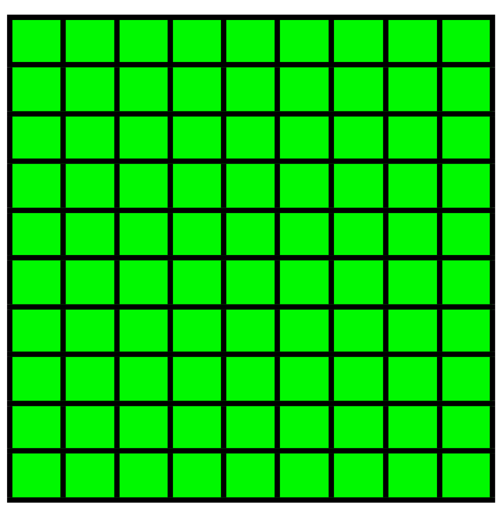

unique state

!

incompressible liquid

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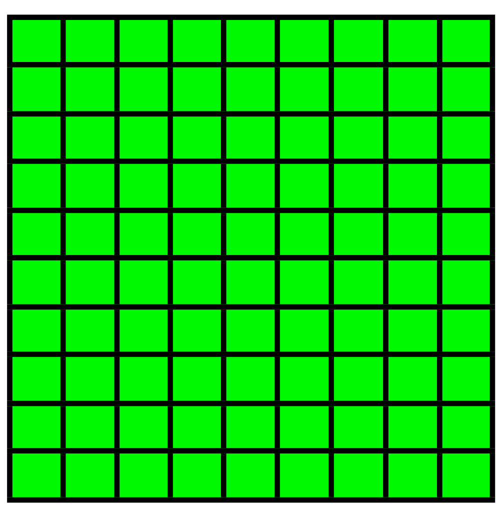
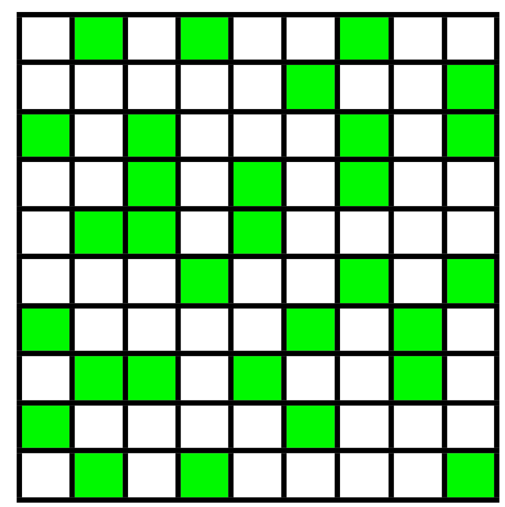


unique state

FQHE $\nu = 1/3$



~ 10 nm

(Real samples much bigger)

many, many degenerate states

!

incompressible liquid

Integer versus fractional filling

Fixed area per state *(one state per flux quantum* $\varphi_0 = hc/e$*)*

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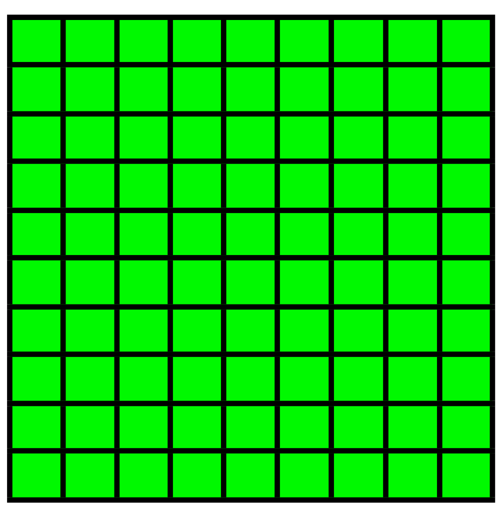

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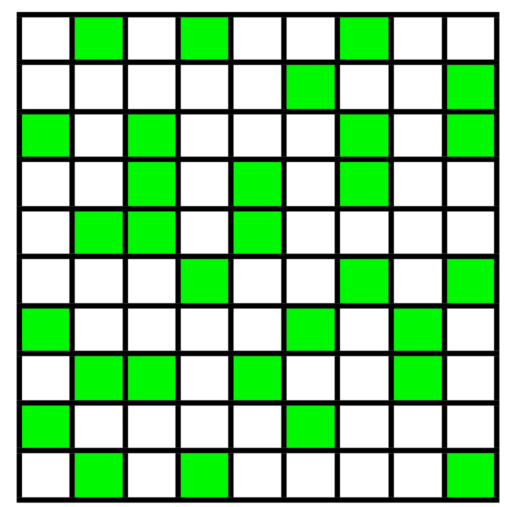

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(Real samples much bigger)

unique state

many, many
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incompressible liquid

A “one-dimensional” microscopic approach

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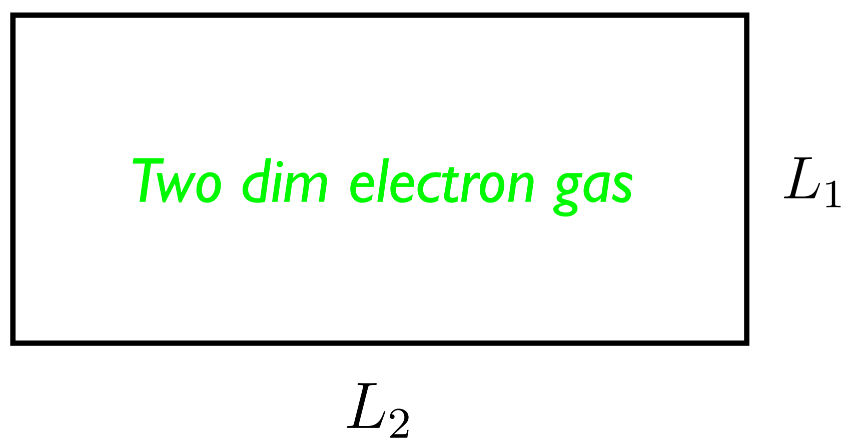
Consider sample with lengths $L_1,\ L_2$



A “one-dimensional” microscopic approach

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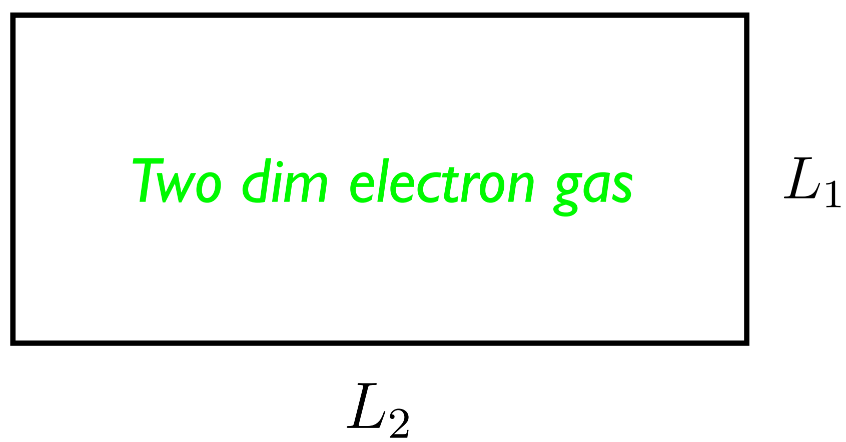
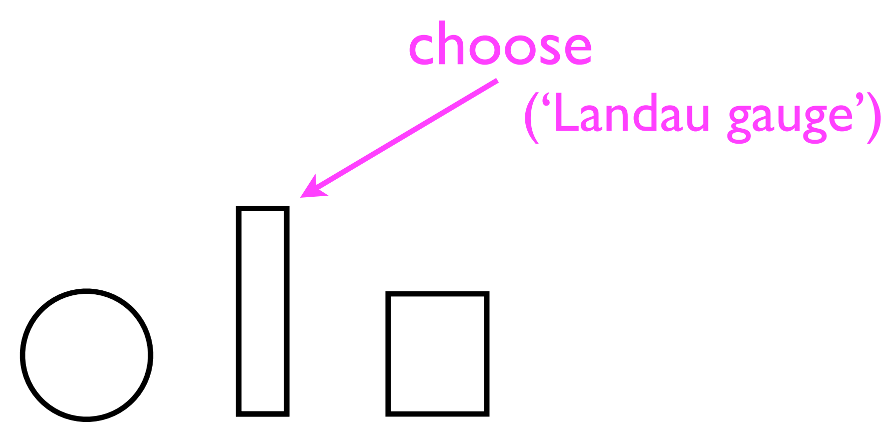


Each electron state has fixed area



(any shape is OK)

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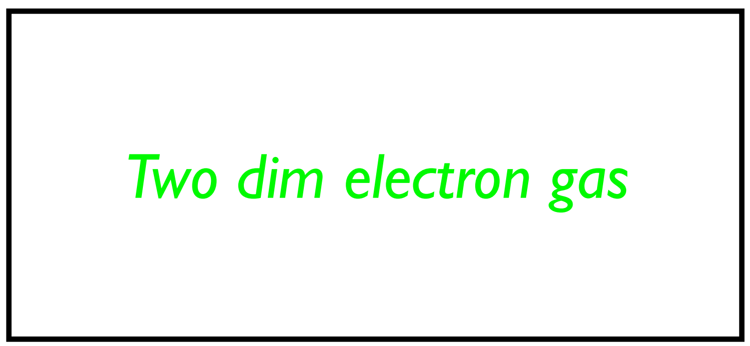

L_1

L_2

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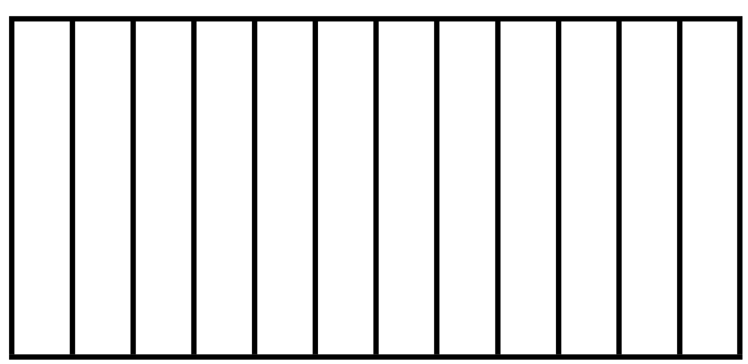
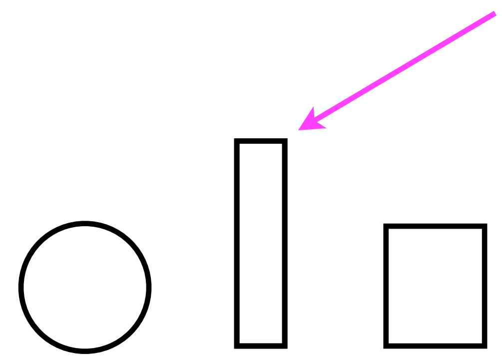


L_1

L_2

choose

(‘Landau gauge’)



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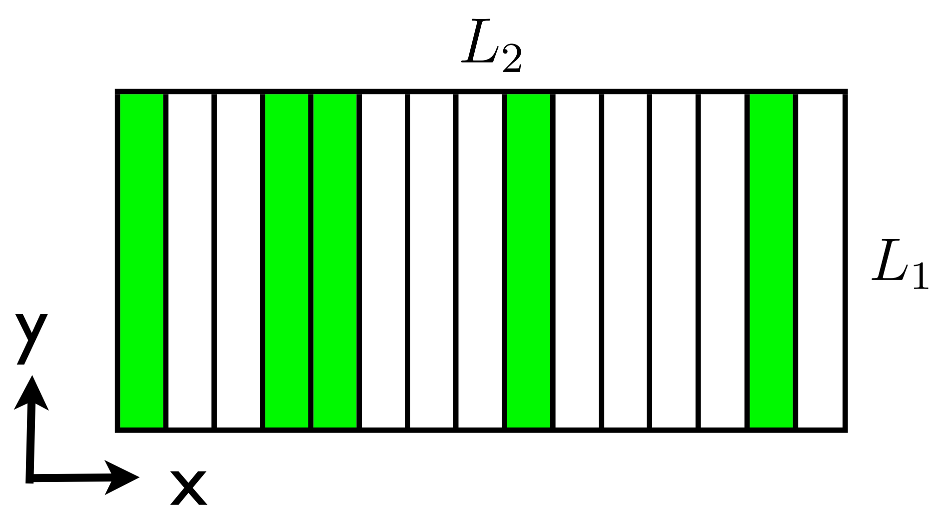

Each box is either empty 0 or filled 1

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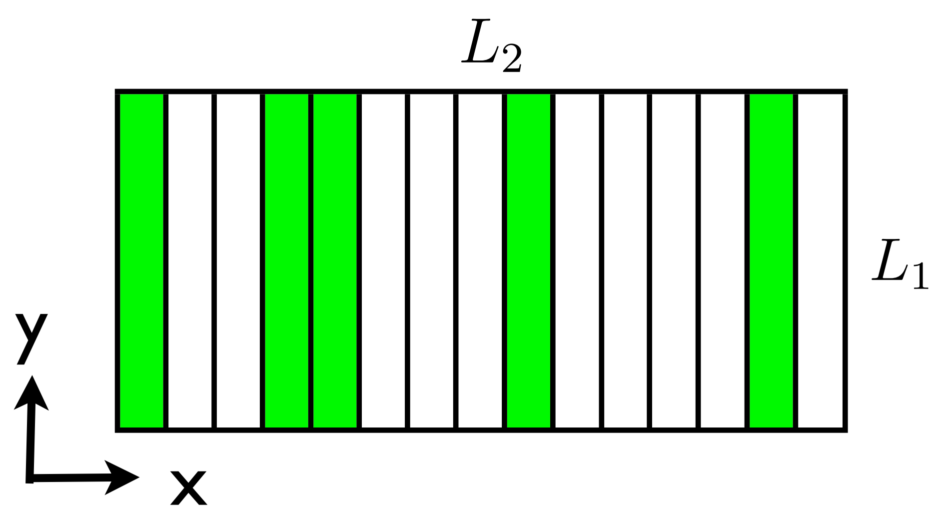

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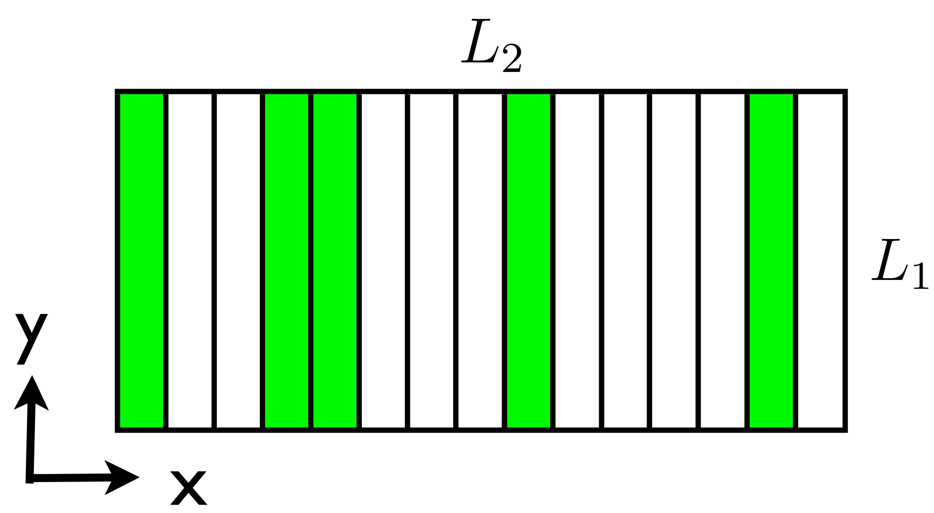
$$\psi_k \sim e^{ik\frac{2\pi}{L_1}y} e^{-(x-k\frac{2\pi}{L_1})^2/2}$$

(Landau gauge)

A "one-dimensional" microscopic approach



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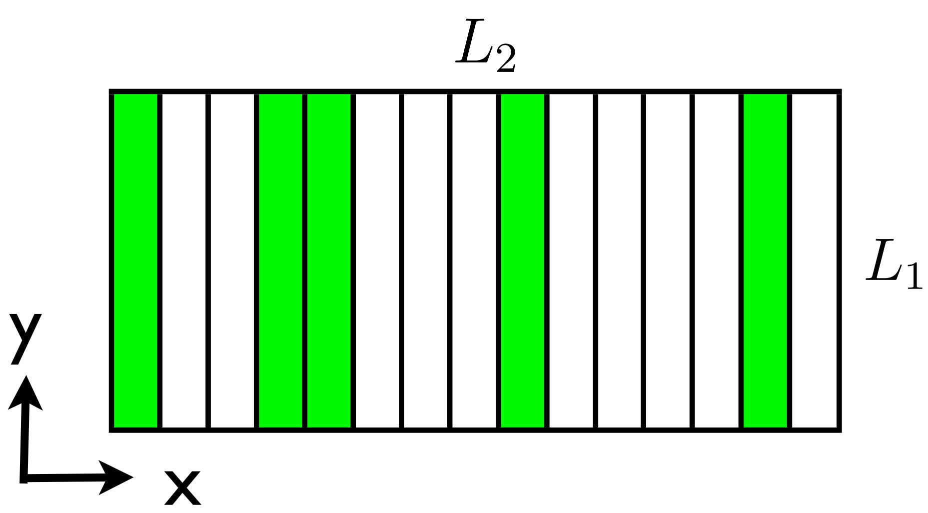
A possible state at $\nu = 1/3$

A “one-dimensional” microscopic approach



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Hamiltonian (ee-interaction)

(eg Coulomb $V(r)=e^2/r$)

No kinetic energy!

$1..0.....0..1 \leftrightarrow 0..1.....1..0$

$V_{k,m}$

k+m

k-m

(all ee-terms that preserve position of CM)

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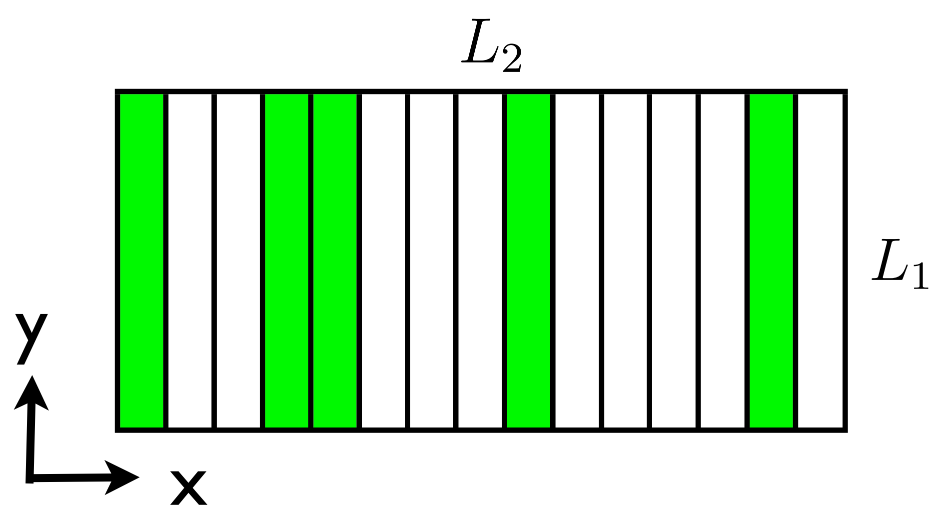

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k+m k-m

including

$1......1$ $V_{k,0}$ *(electrostatic repulsion)*

k

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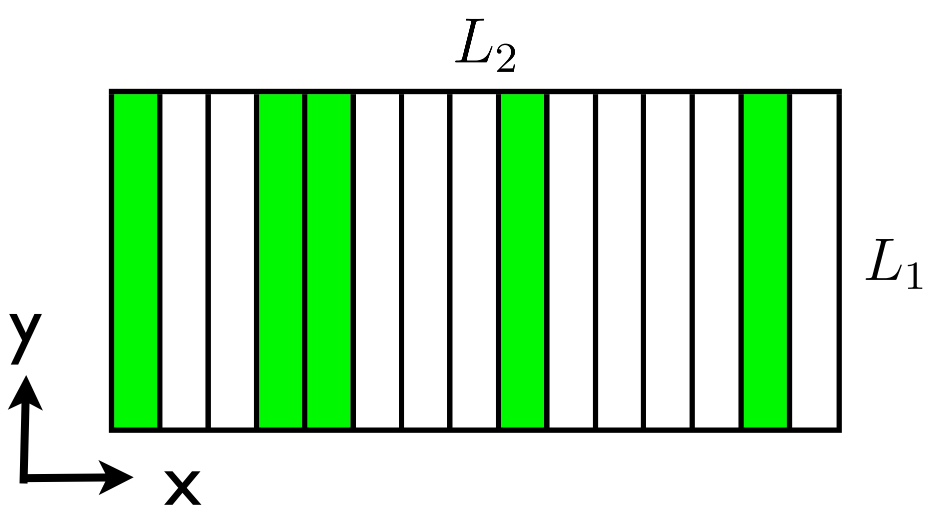

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k+m k-m

including

$1......1$ $V_{k,0}$ *(electrostatic repulsion)*

k

Exact mapping of a single Landau level!

Exact solution at $L_1 \to 0$ (Bergholtz, Karlhede et al., Seidel et al., '05-'09)

Details: E.J. Bergholtz and A. Karhede, Phys. Rev. B **77**, 155308 (2008)

Exact solution at $L_1 \rightarrow 0$ (Bergholtz, Karlhede et al., Seidel et al., '05-'09)

Hopping $1..0.....0..1 \leftrightarrow 0..1.....1..0$ makes ground state complicated.

But, when $L_1 \rightarrow 0$

hopping vanishes and only electrostatic repulsion remains: $1......1$

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States with electrons in fixed positions are the energy eigenstates - groundstate obtained by separating the electrons as much as possible:

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Unit cell

$\nu = 1/3$ 1 0 0 1 0 0 1 0 0 1 0 0 1 0 0 — 1 0 0

(Tao-Thouless (TT) states)

$\nu = 2/5$ 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 — 1 0 0 1 0

These TT states are adiabatically connected to the abelian bulk FQH states!

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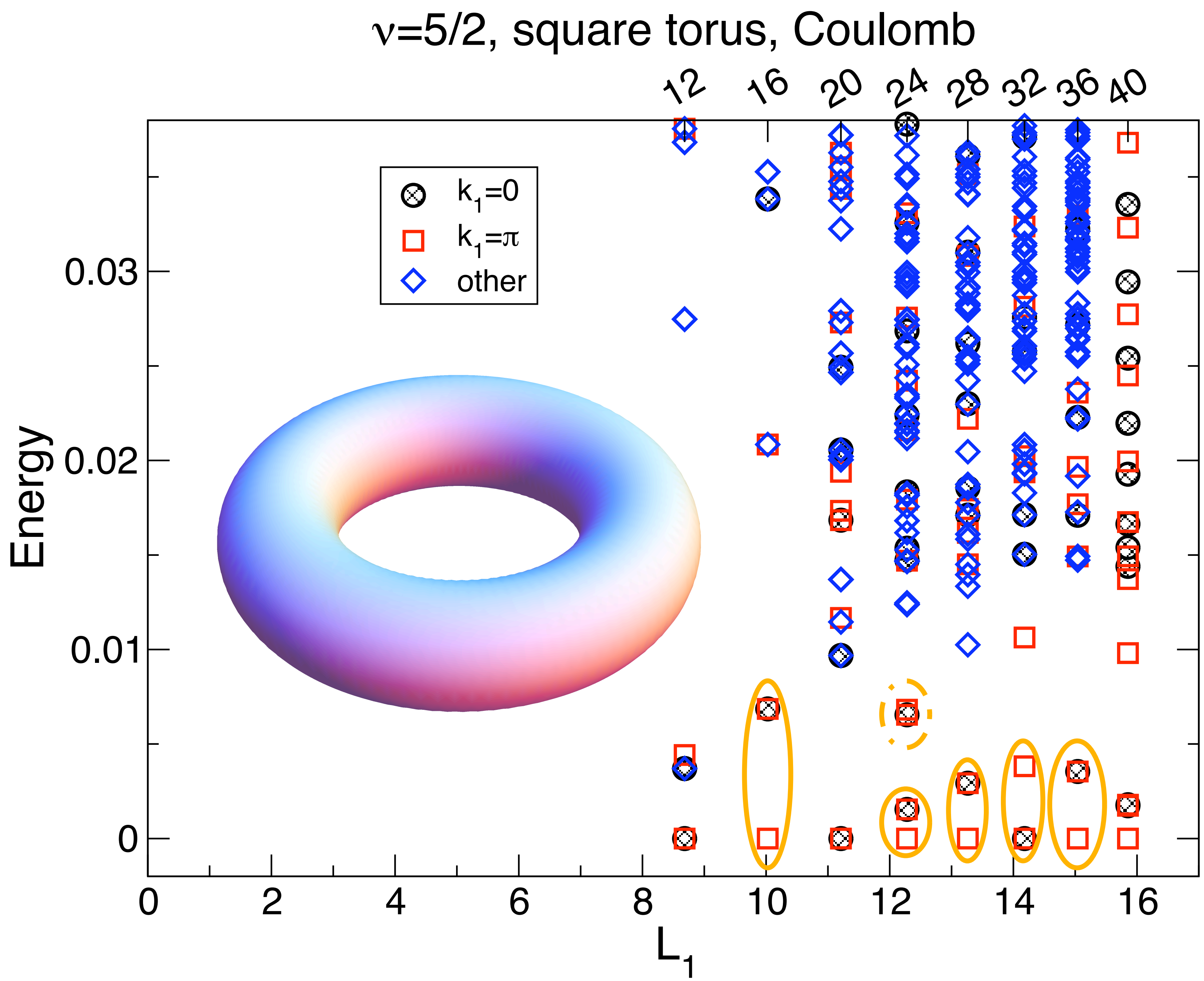
Exact Diagonalization: Main Idea

- Away from the TT limit this is a difficult problem.
- Solve the Schrödinger equation of a quantum many body system numerically

$$\mathcal{H}|\psi\rangle = E|\psi\rangle$$

- Sparse matrix, but for quantum many body systems the vector space dimension grows exponentially!

Example: ν=5/2 Fractional Hall Effect



- We solve the fully interacting problem with up to 20 particles on the torus ($L_1=L_2$)
- Matrix dimension up to 3.5 billion states (using only one quantum number)
- Topological degeneracy required for Pfaffian/ Antipfaffian ground state ?

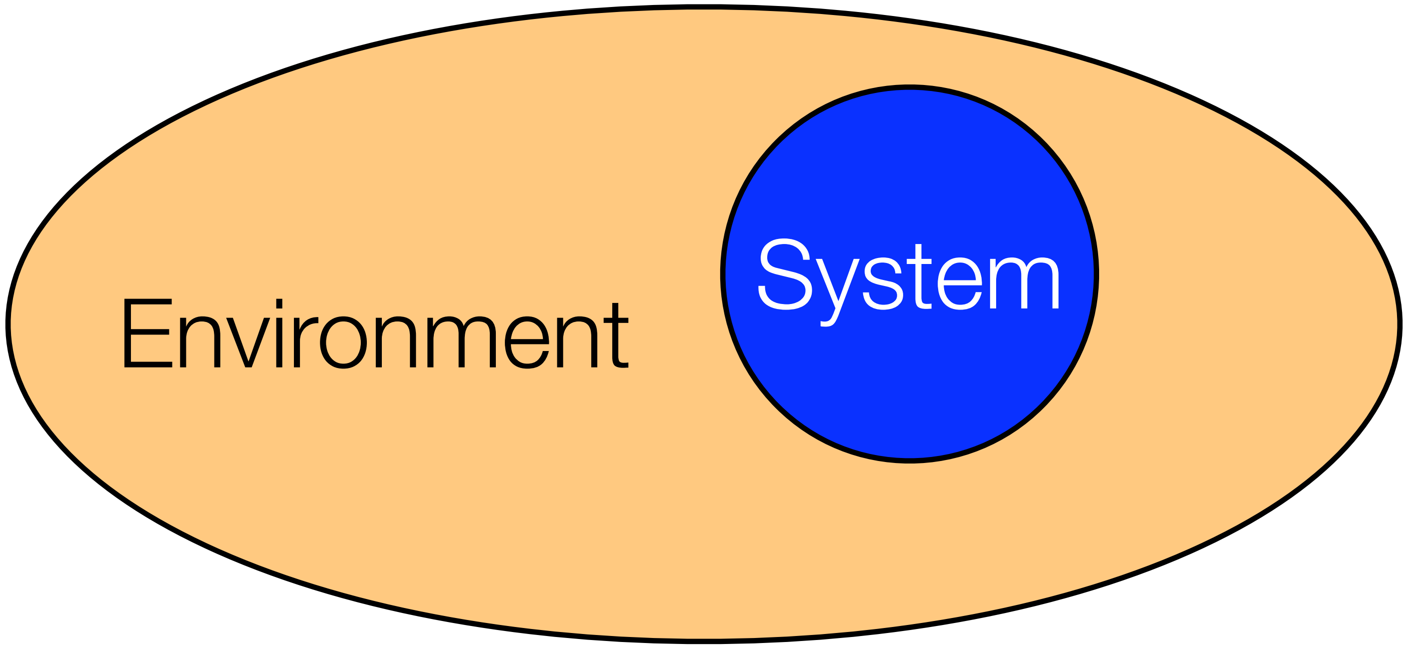
Läuchli, Bergholtz, Haque, unpublished

Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

(Topological) Entanglement Entropy

- Let us look at reduced density matrices, and their entanglement entropies



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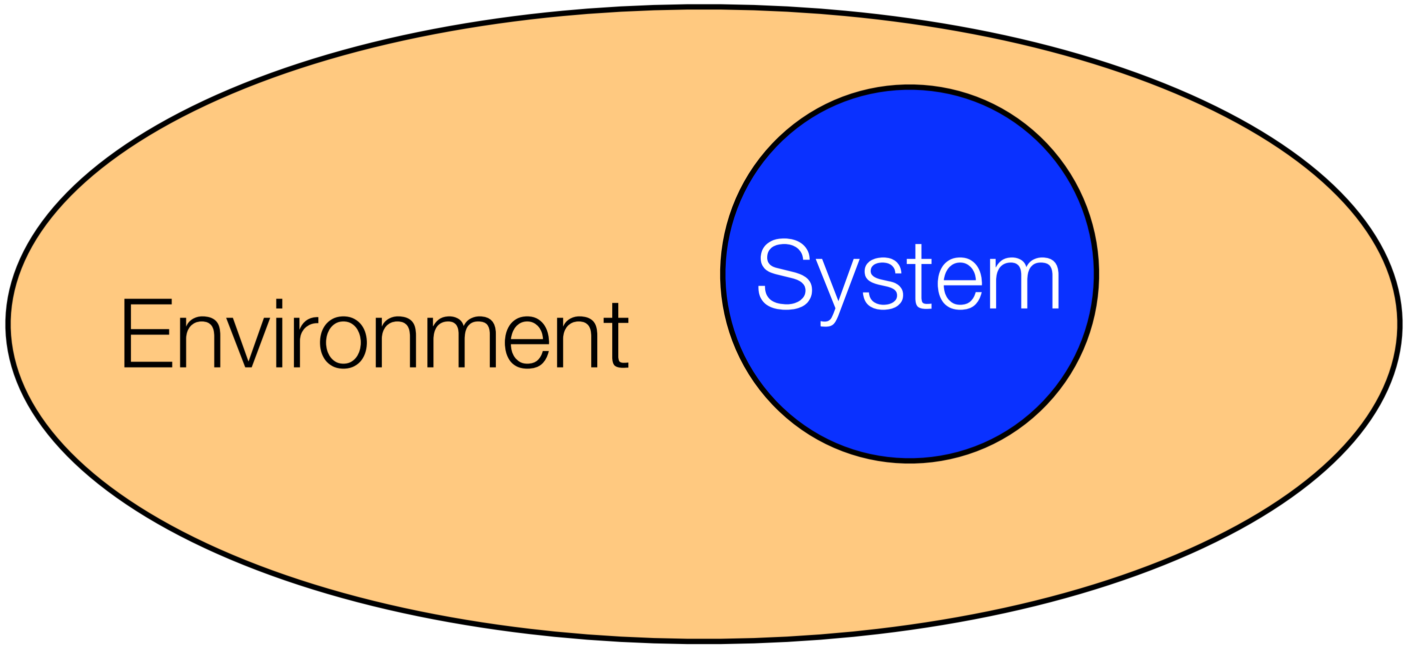

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$$S(\rho) = \alpha L - \gamma + \cdots$$

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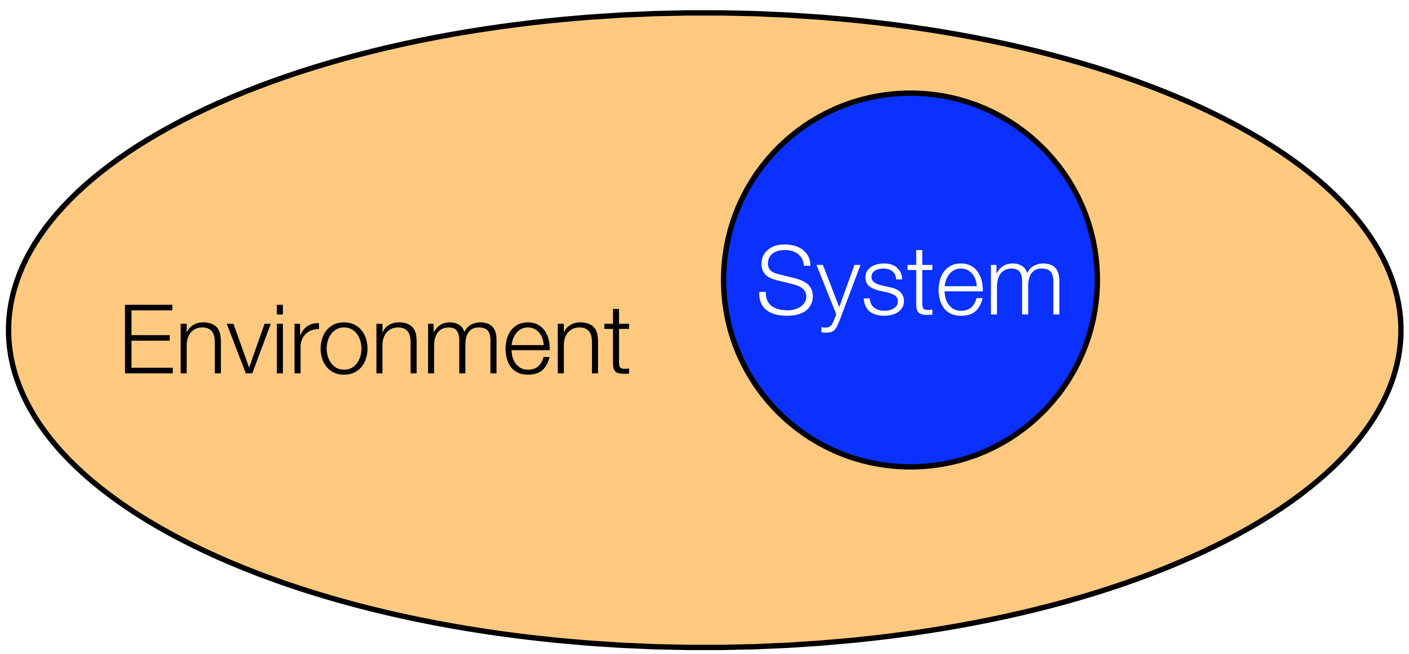

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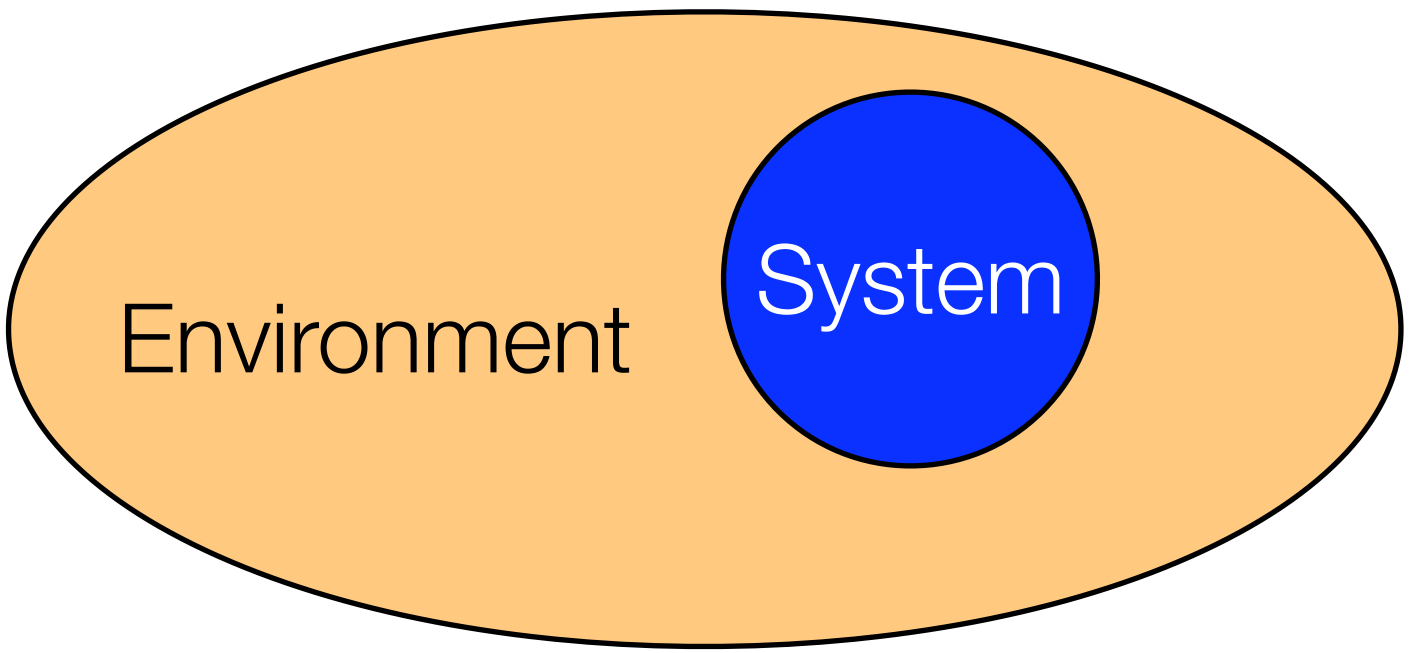

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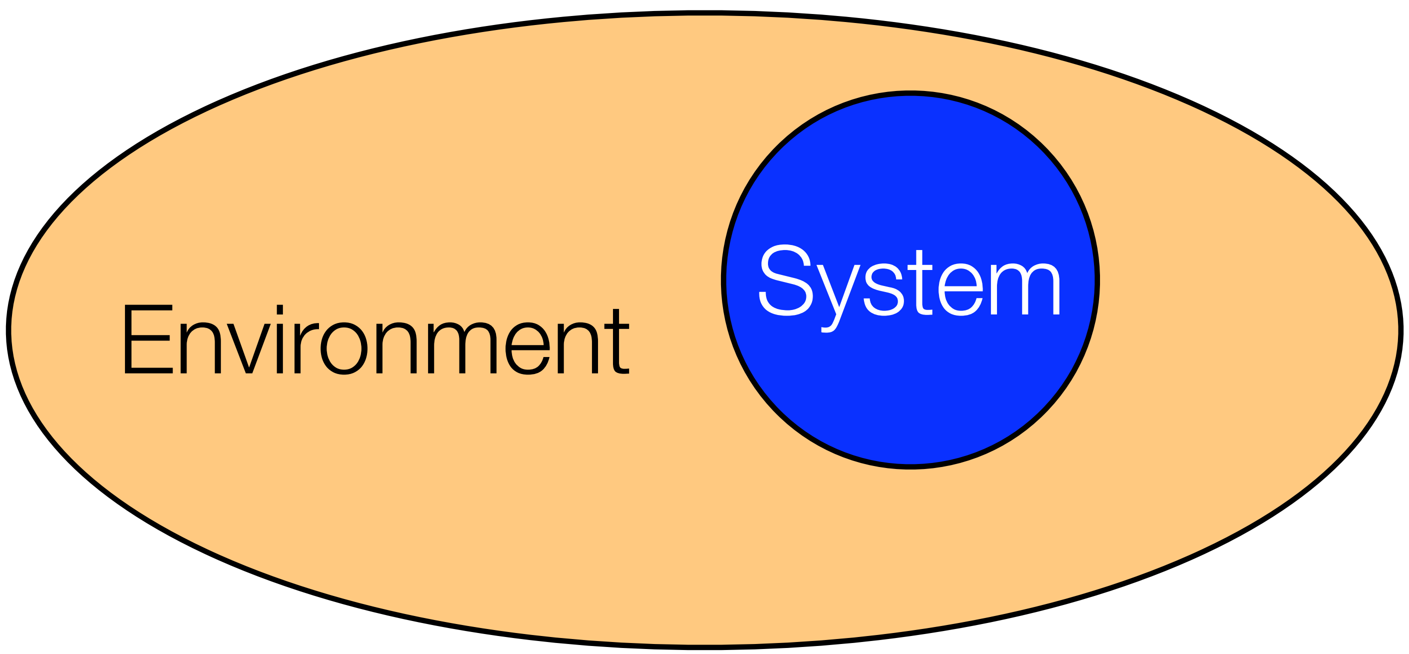

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Topological entanglement entropy

$$\gamma = \log \mathcal{D}$$

$\mathcal{D}$ Total quantum dimension

Kitaev & Preskill PRL '06
Levin & Wen PRL '06

Topological entanglement entropy for FQH states on the sphere

- Potential use:
  1) Identify topological phases.
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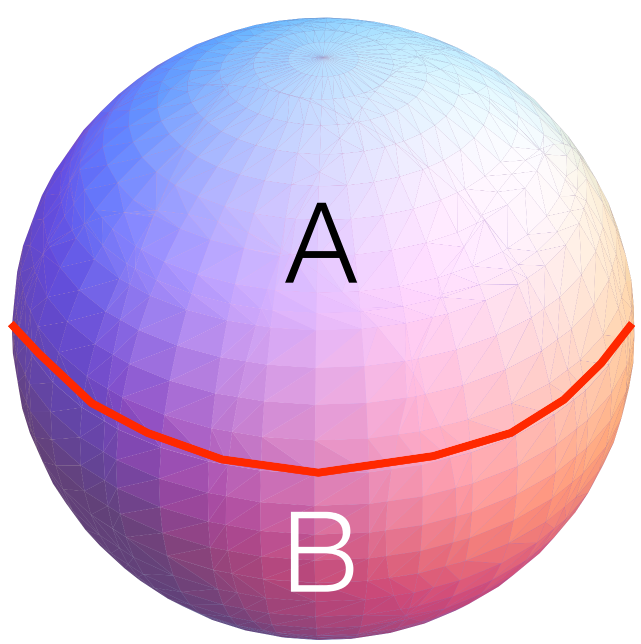
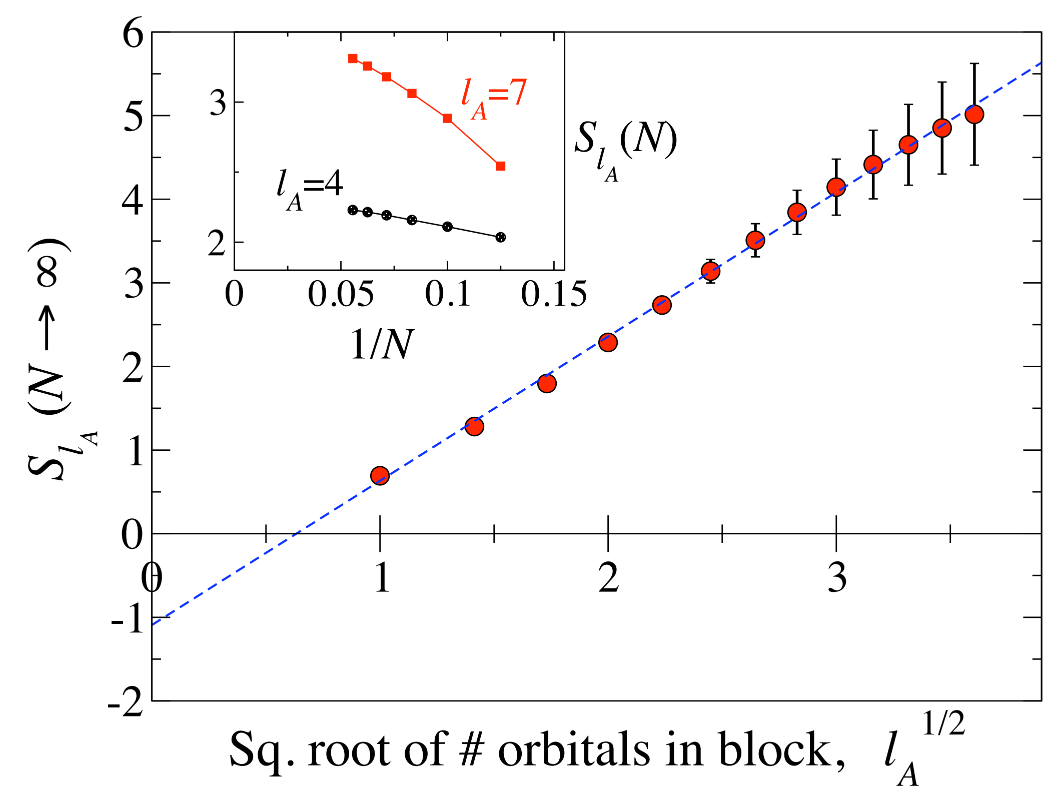
Haque, Zozulya & Schoutens, PRL '07



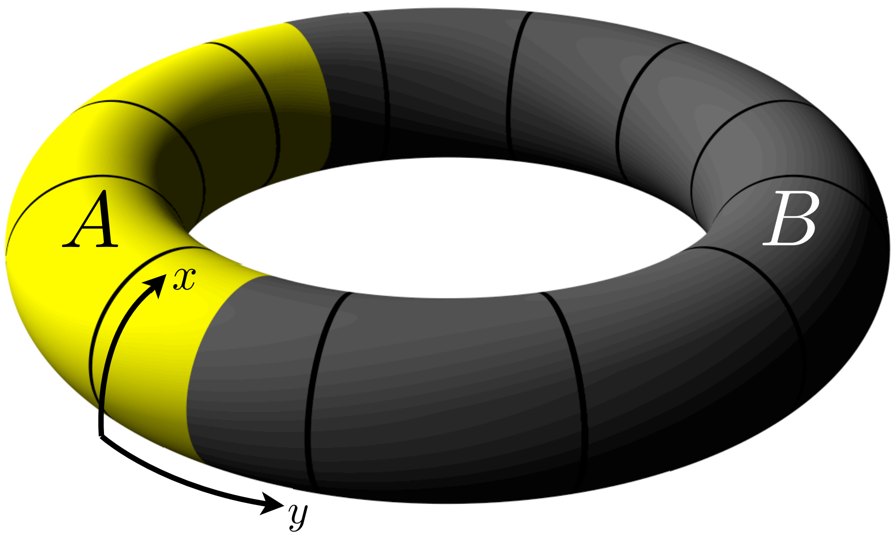


- Feasible, but tricky on the sphere.
  Complications due to varying length as a function of latitude

How to do this on the torus

Earlier (and later!!) Refs. (Friedman and Levine '08-'10) do this incorrectly

- The torus can be tuned continuously by varying L_1 and L_2 ($L_1 L_2 = 2\pi N_s$).



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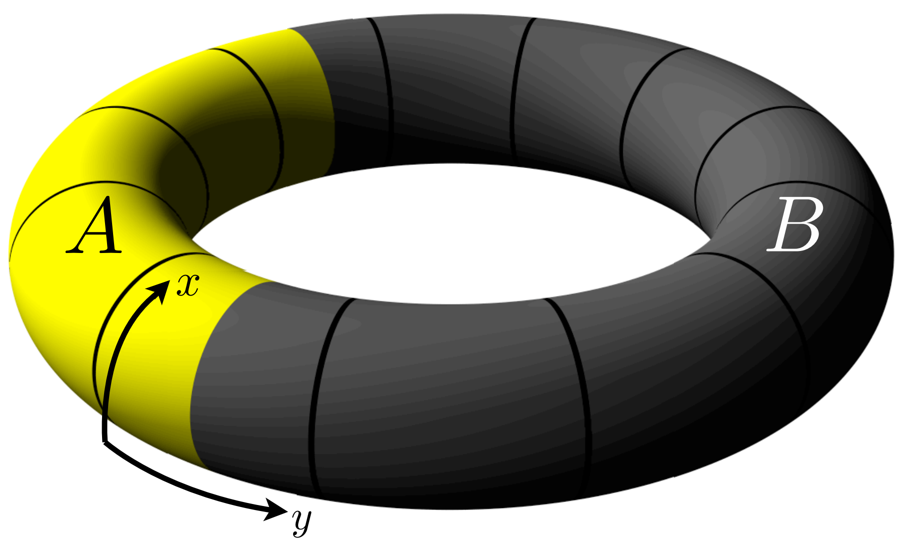
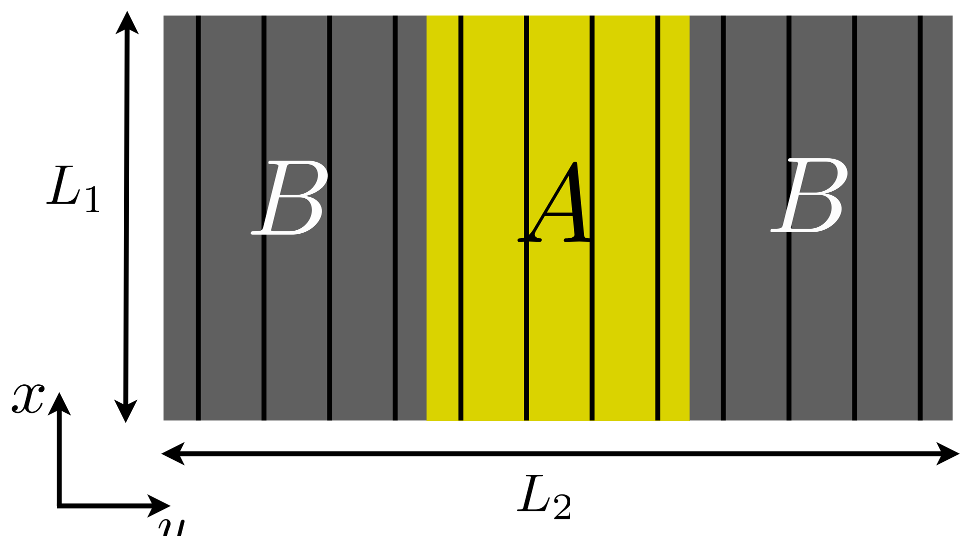


- We study orbital partitioning, which is expected to approximate to real space partitioning

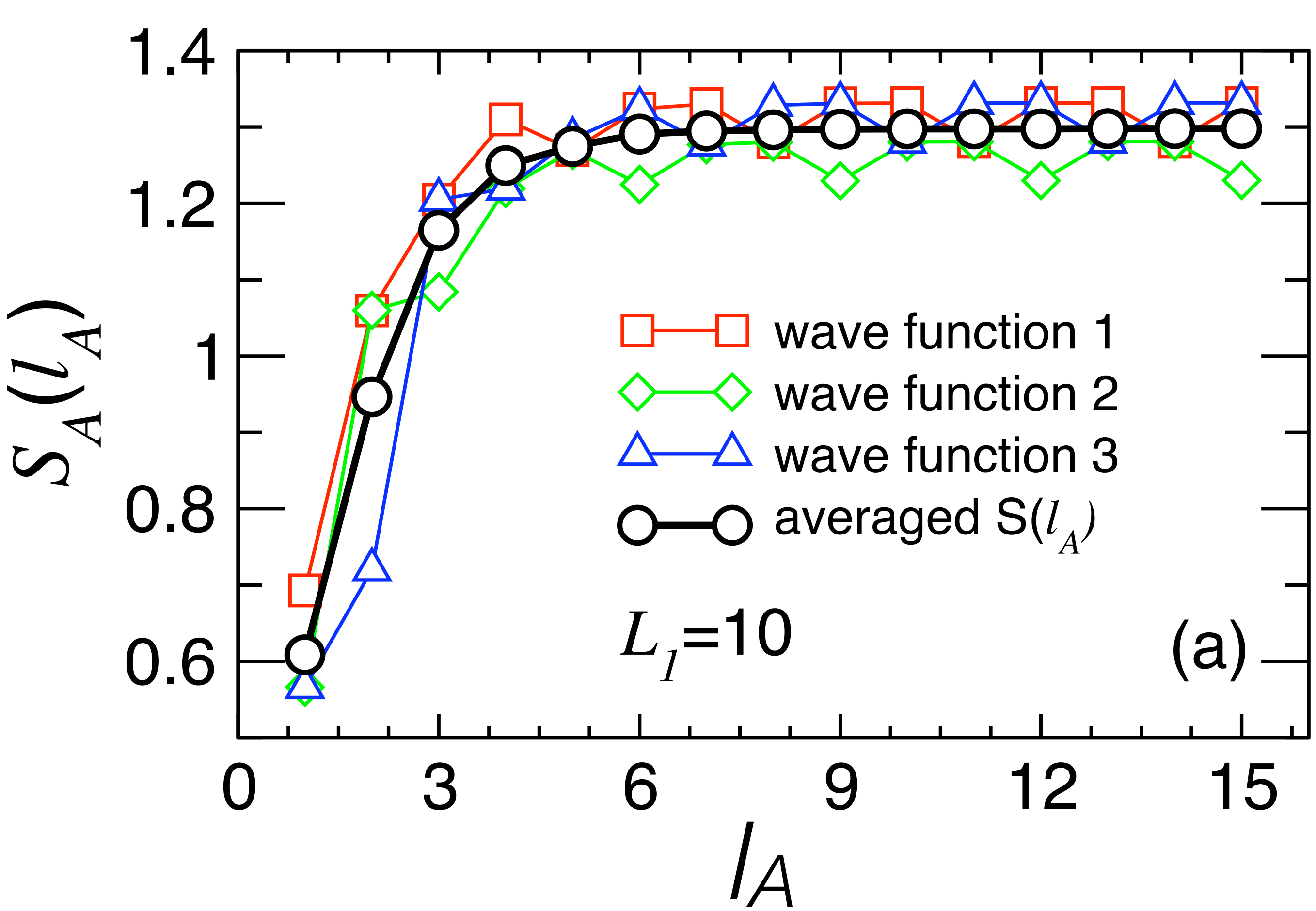


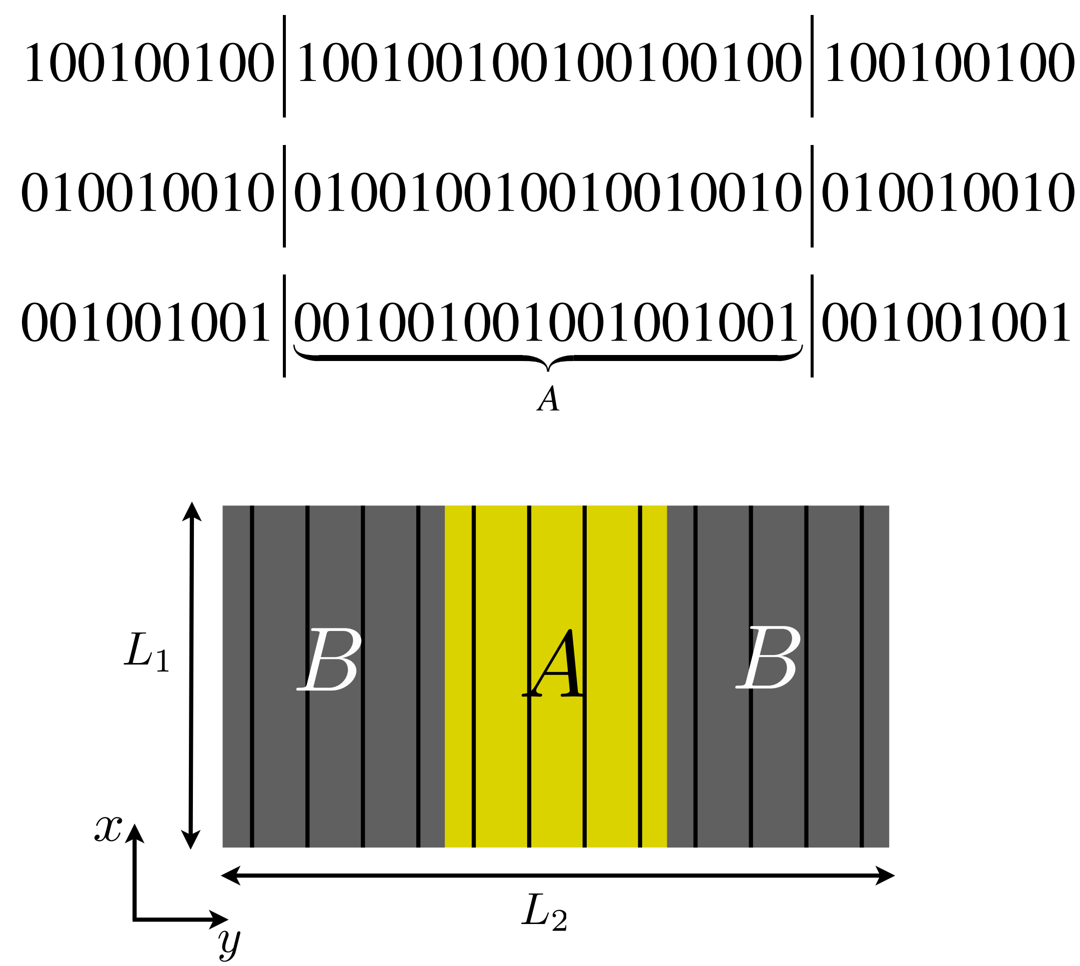
$$S(\rho) = \alpha L_1 - 2\gamma + \dots$$

A.M. Läuchli, E.J. Bergholtz & M. Haque
New J. Phys. **12** 075004 (2010)

Area law at constant L_1

- Degenerate wavefunctions ⇒ **averaging of entropies** yields smooth curve !

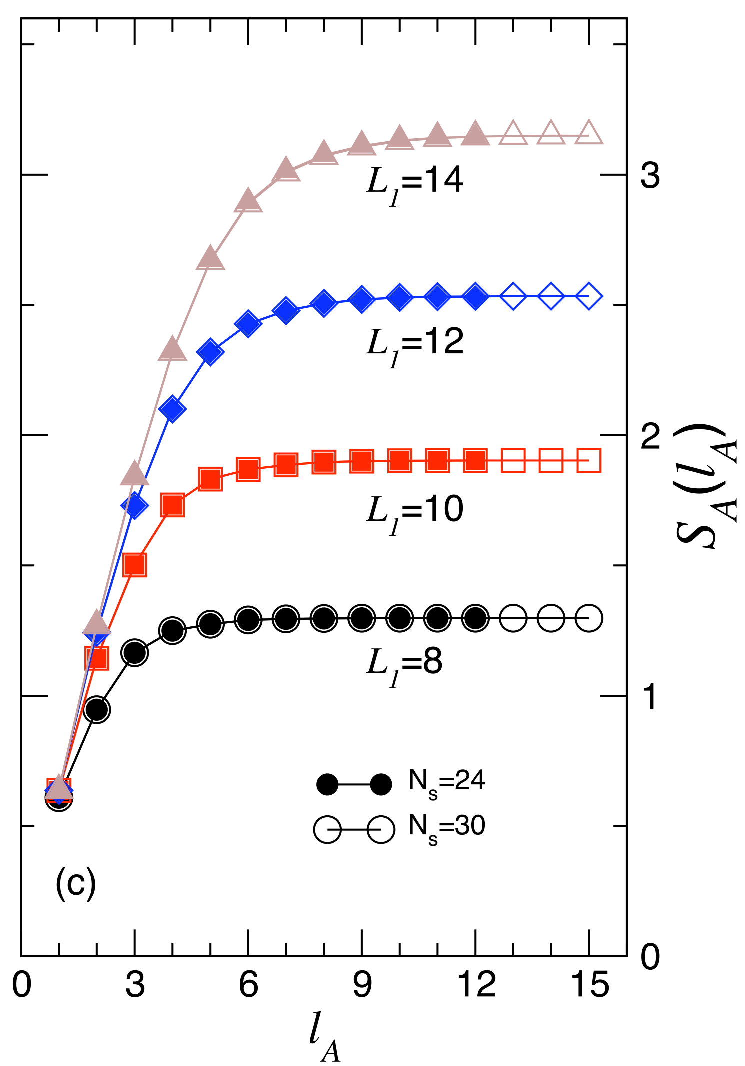


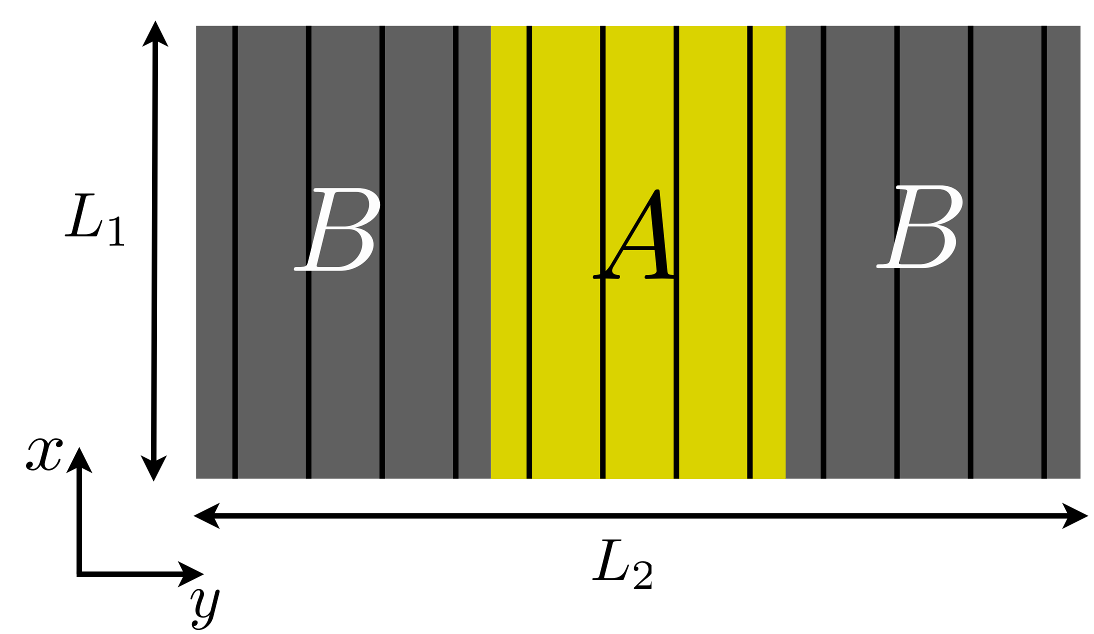


A.M. Läuchli, E.J. Bergholtz & M. Haque
New J. Phys. **12** 075004 (2010)

Area law at constant L_1

- Increasing subsystem size at constant L_1 ⇒ Saturation at large l_A



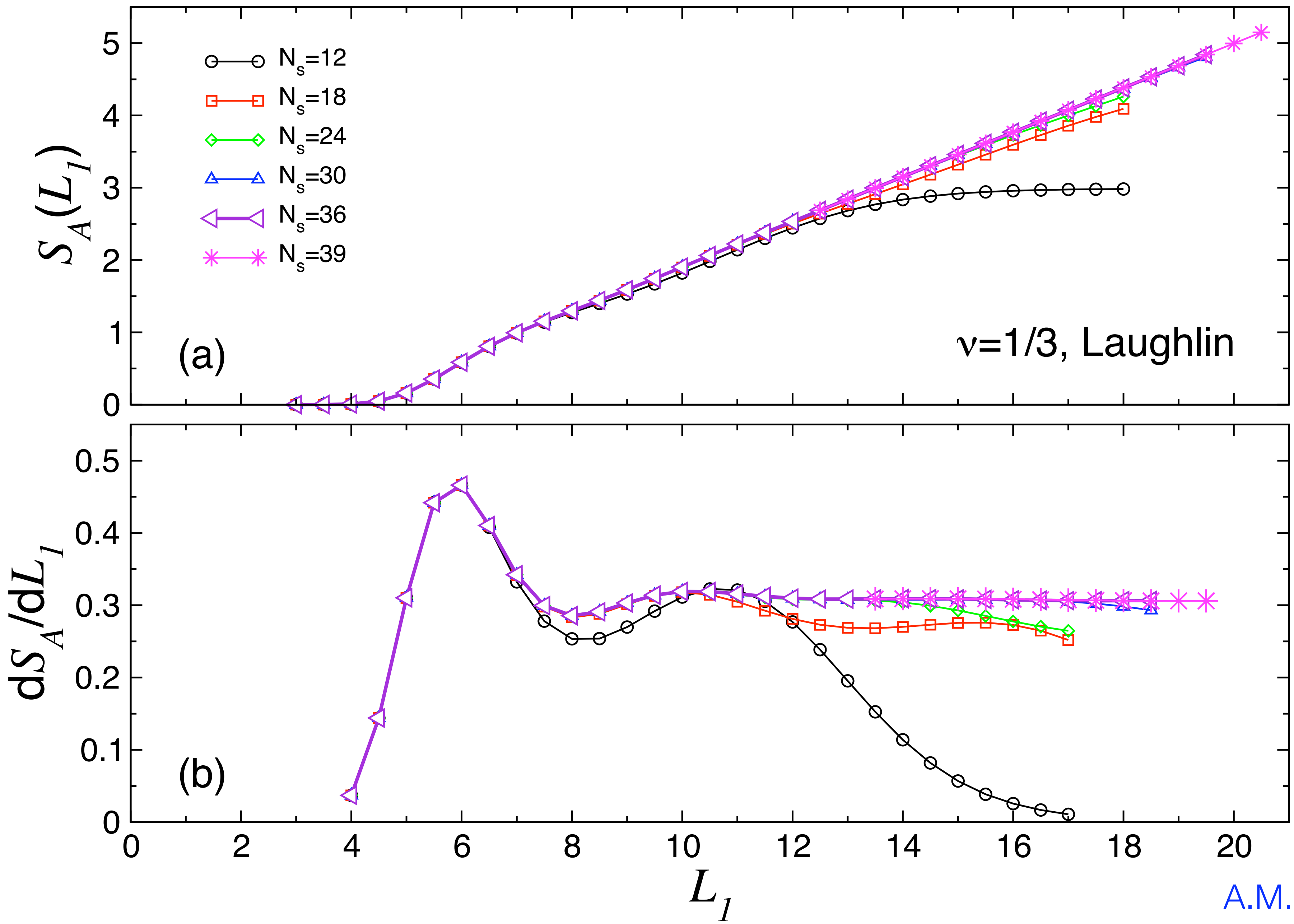


- For small enough L_1, the entropy is finite size converged for a given l_A.

A.M. Läuchli, E.J. Bergholtz & M. Haque
New J. Phys. **12** 075004 (2010)

Entanglement entropy $S(L_1)$

- For large enough N_s, $S(L_1)$ converges for each L_1



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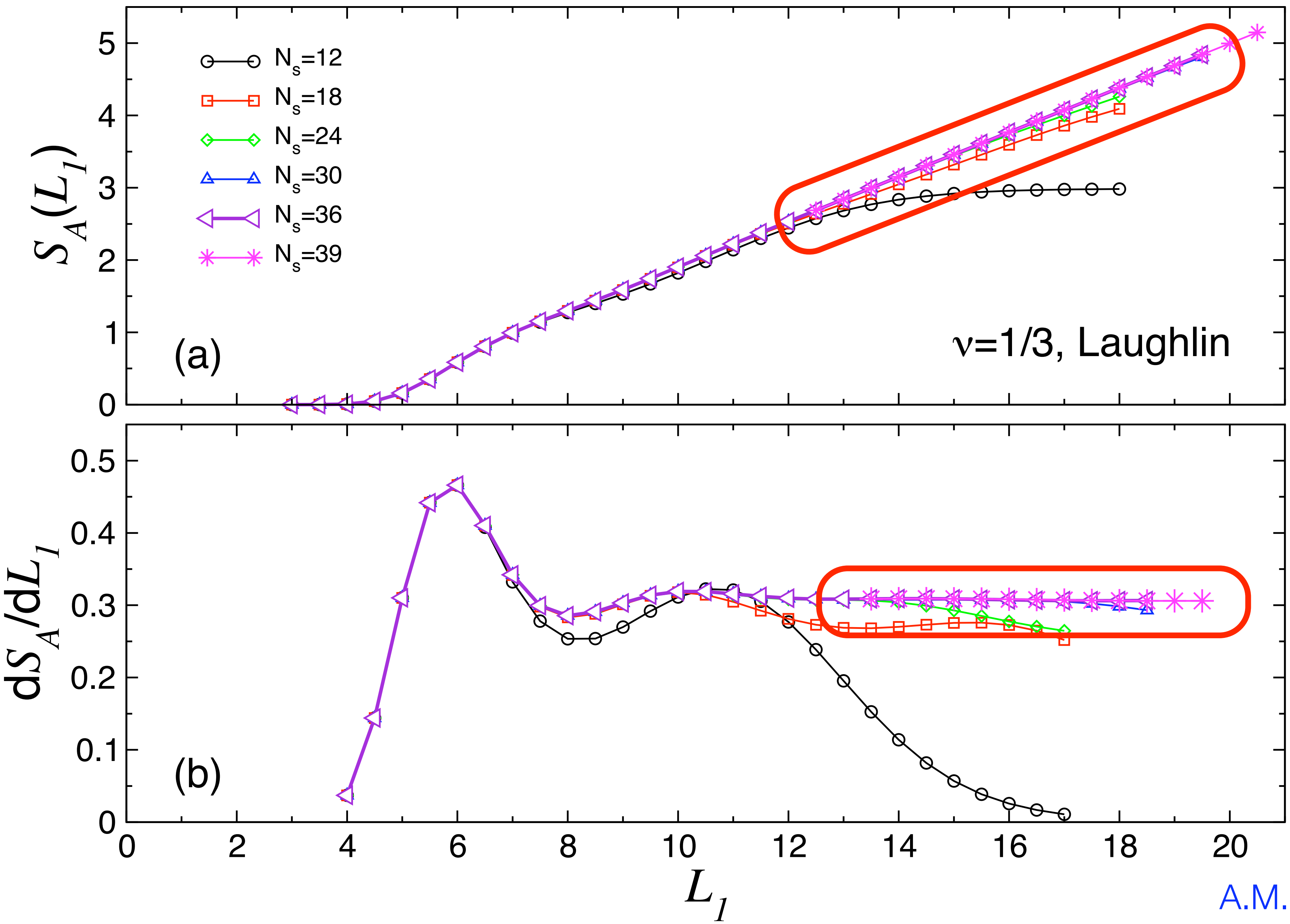


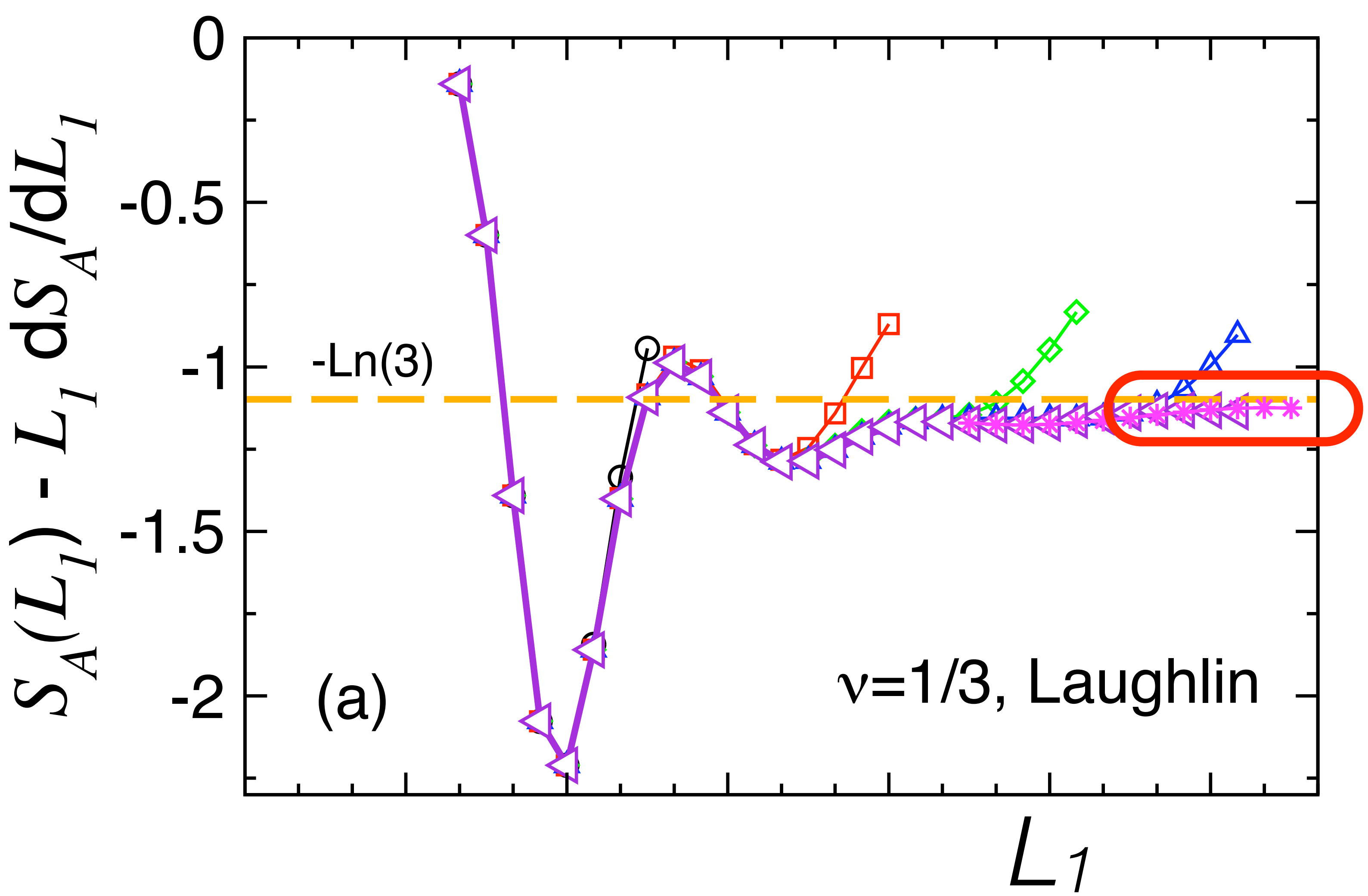
$$S(\rho) = \alpha L_1 - 2\gamma + \ldots$$

Boundary entropy density (**α**)

A.M. Läuchli, E.J. Bergholtz & M. Haque
New J. Phys. **12** 075004 (2010)

Extracting the topological entanglement entropy

- Use a running γ extraction, and monitor L_1 convergence



$\rightarrow -2\gamma$

- 2γ converges towards expected ln 3 !
  Most accurate numerical determination to date.

A.M. Läuchli, E.J. Bergholtz & M. Haque
New J. Phys. **12** 075004 (2010)

Less accurate for more complicated states

- But is still much better than earlier approaches!

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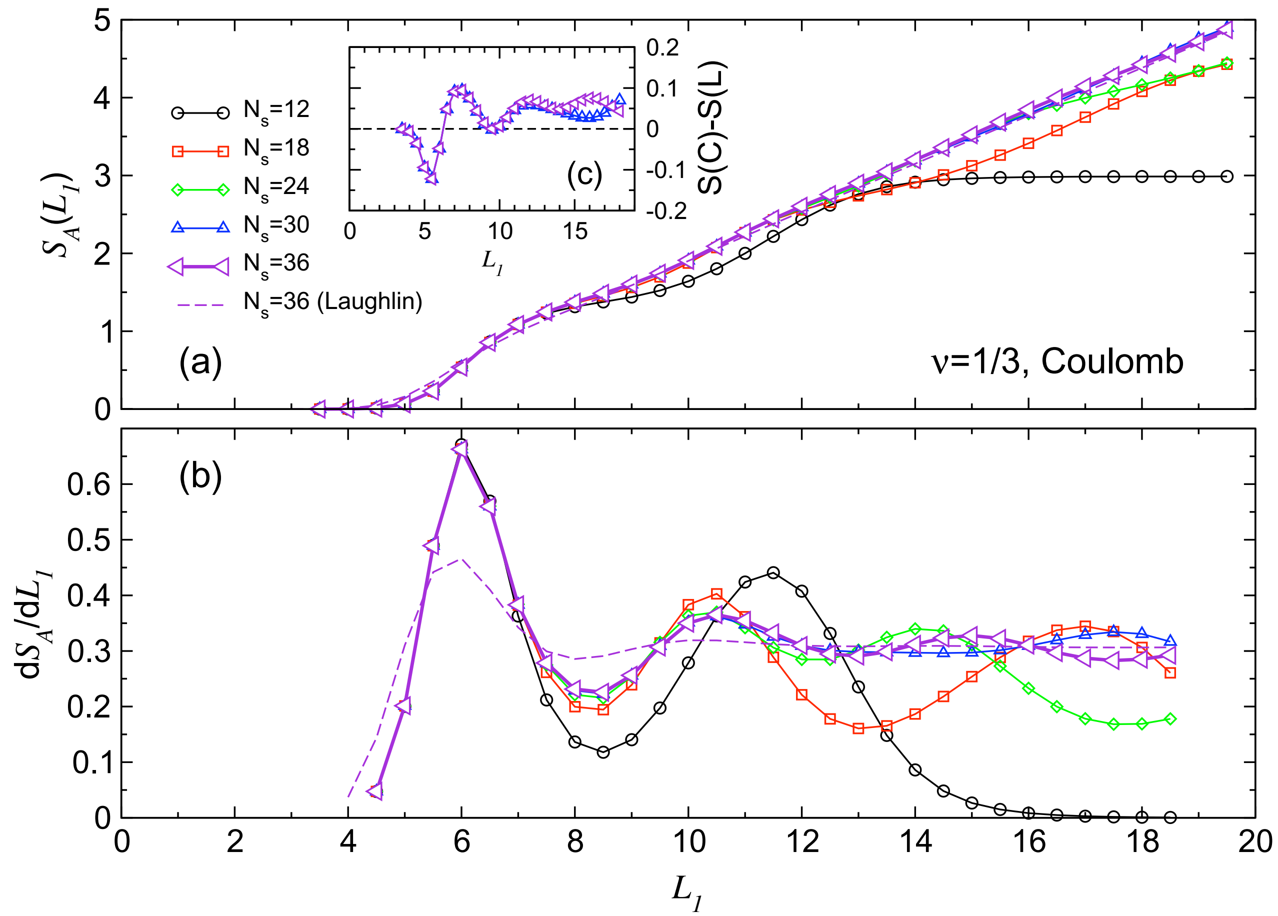

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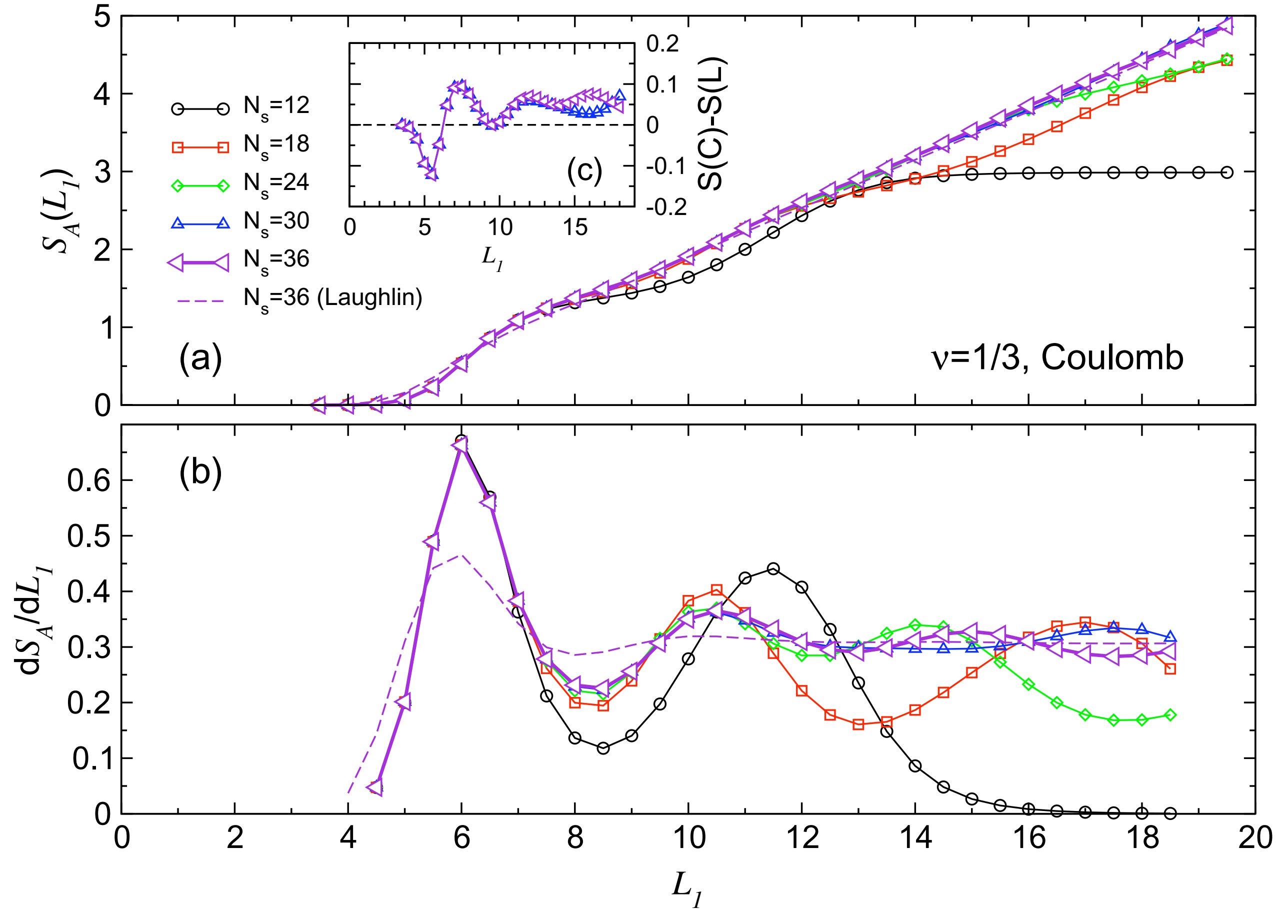
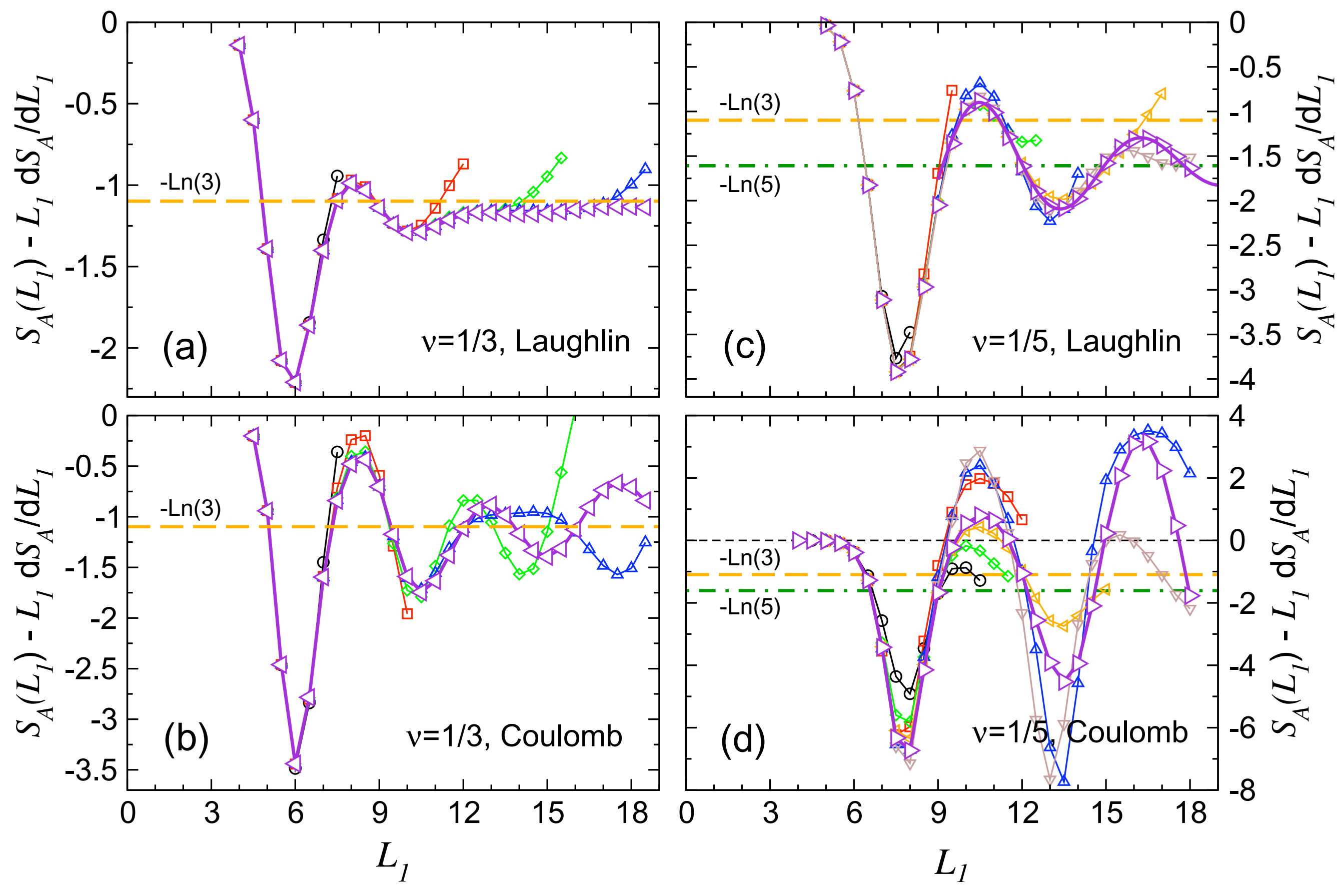
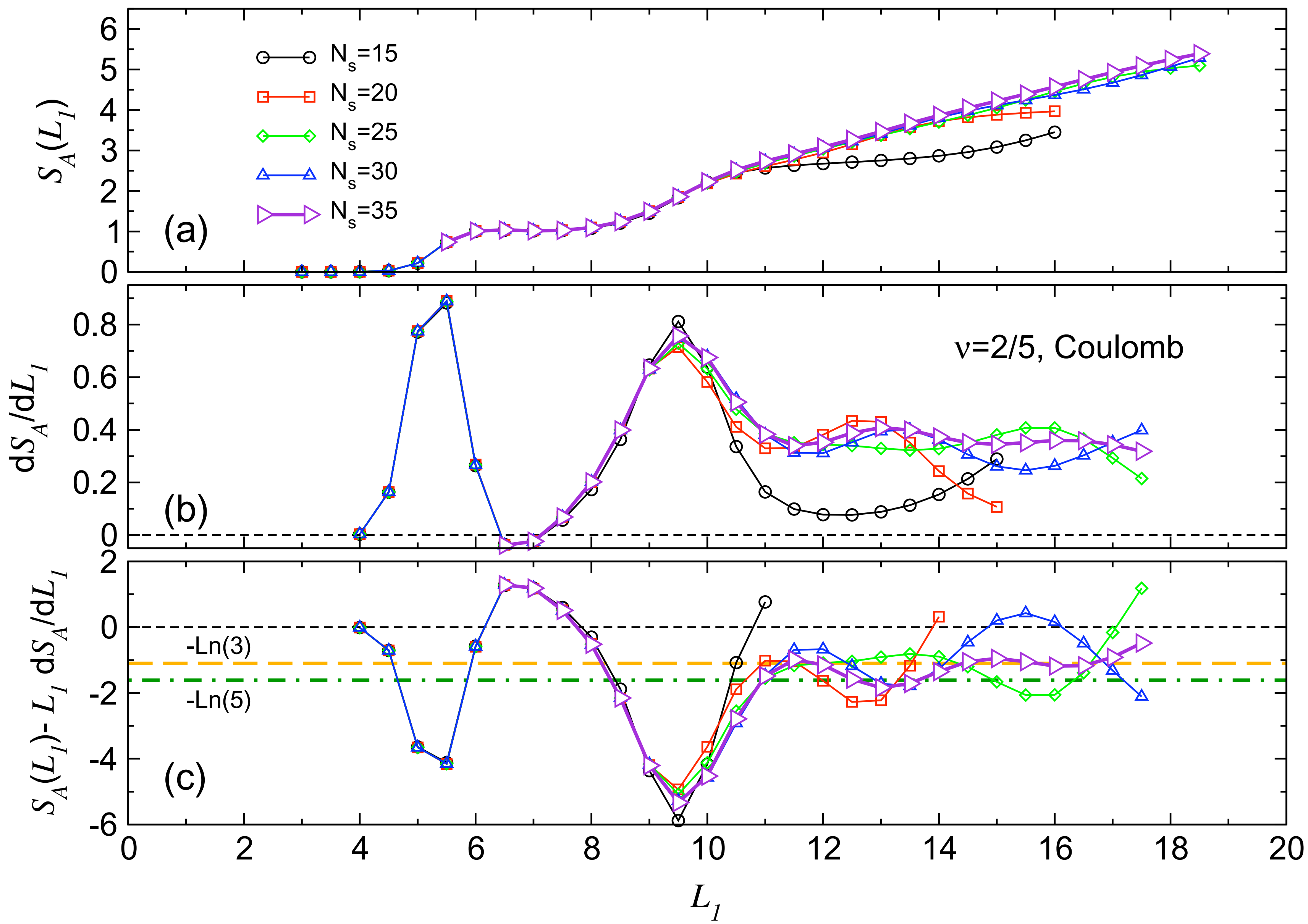




‘Complicated’ states continued...

- Hierarchy state at 2/5



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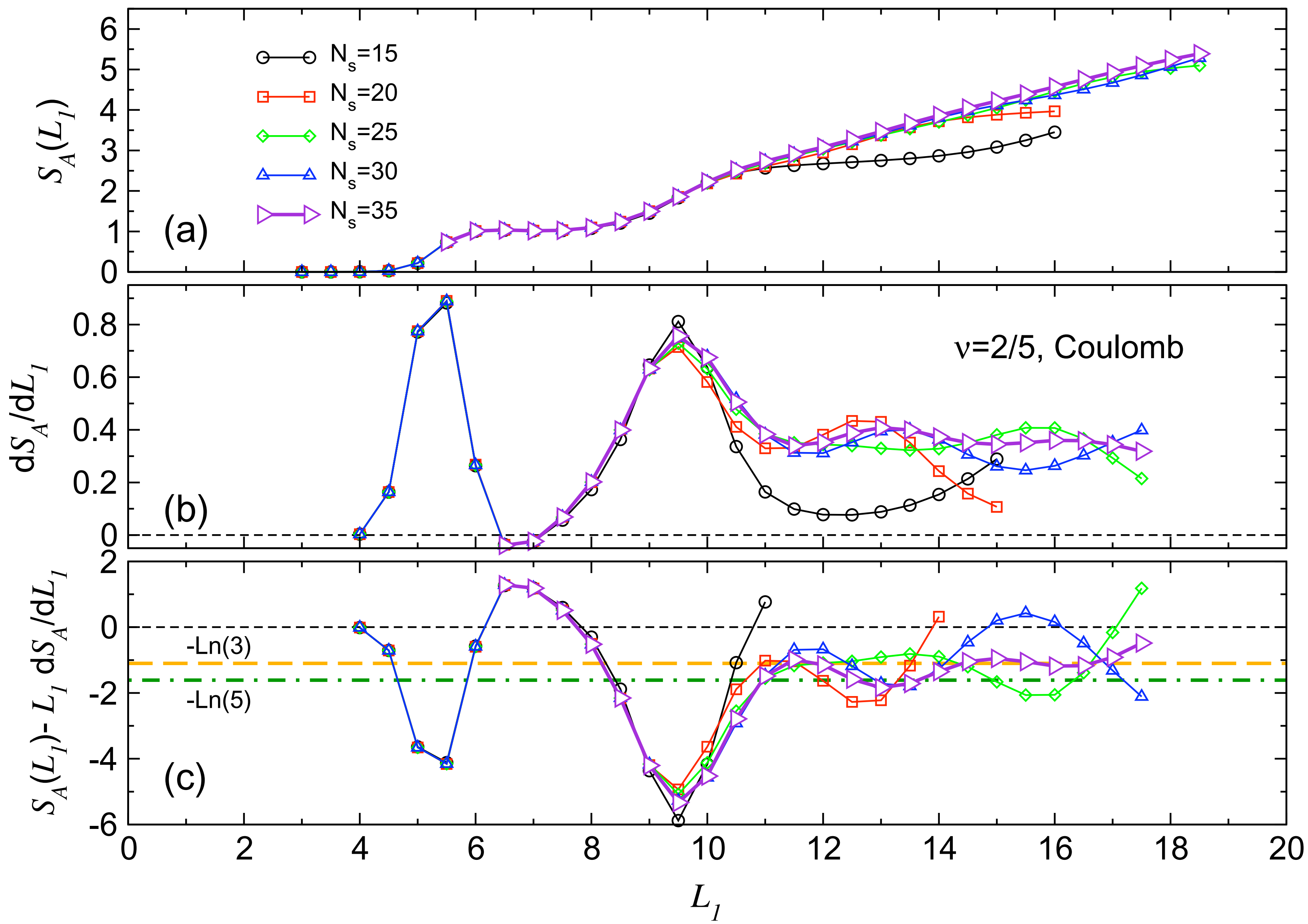
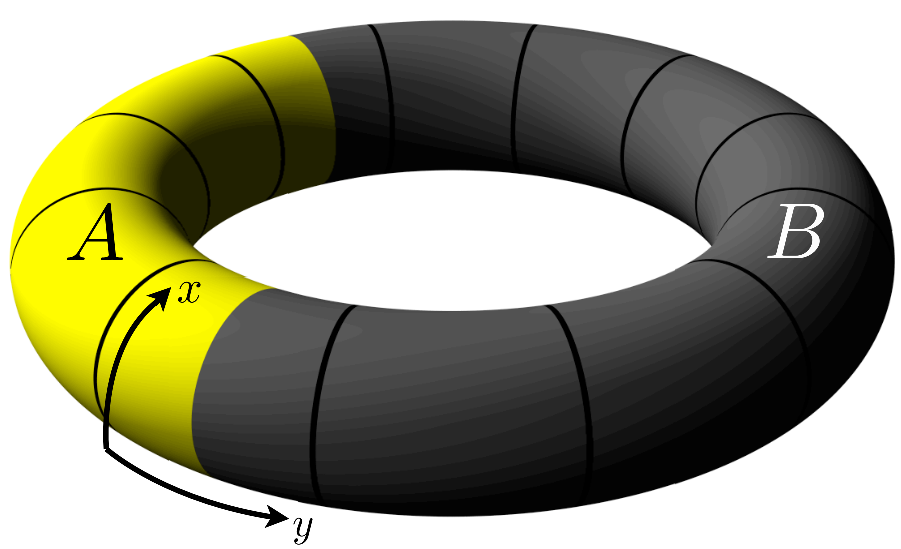
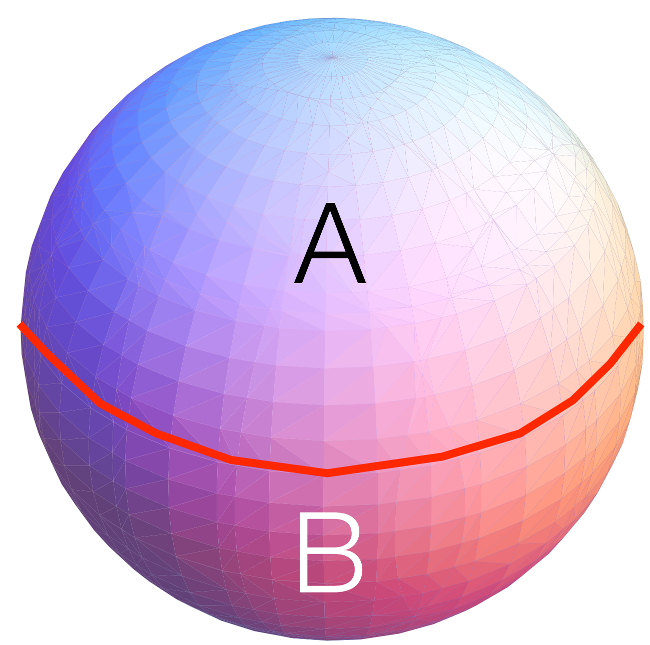


- Best fit is very close to $2\gamma = \ln 5$, but it is clearly not very convincing...

Implications: FQH using DMRG ?

- $S(L) \sim L_1$
- $m \sim \exp[S] \sim \exp[a\, L_1]$, exponential effort in the physical width of the system
- DMRG will not work easily for large 2D FQH samples (sphere approximately as hard as the torus)
- fermionic MERA/PEPS in suitable gauge?



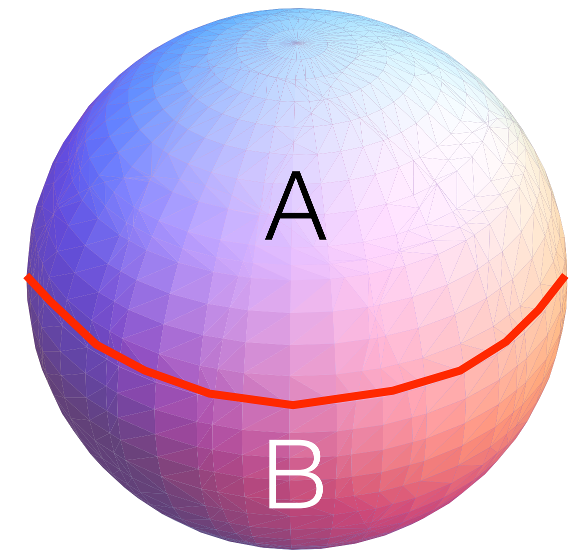


Outline

- Introduction: Fractional Quantum Effect
- Topological Entanglement Entropy
- Entanglement Spectra

Entanglement Spectra (Li & Haldane PRL '08)

- The entanglement entropy is a single number !
- Is there more one can extract from the reduced density matrix ?
- One can always write



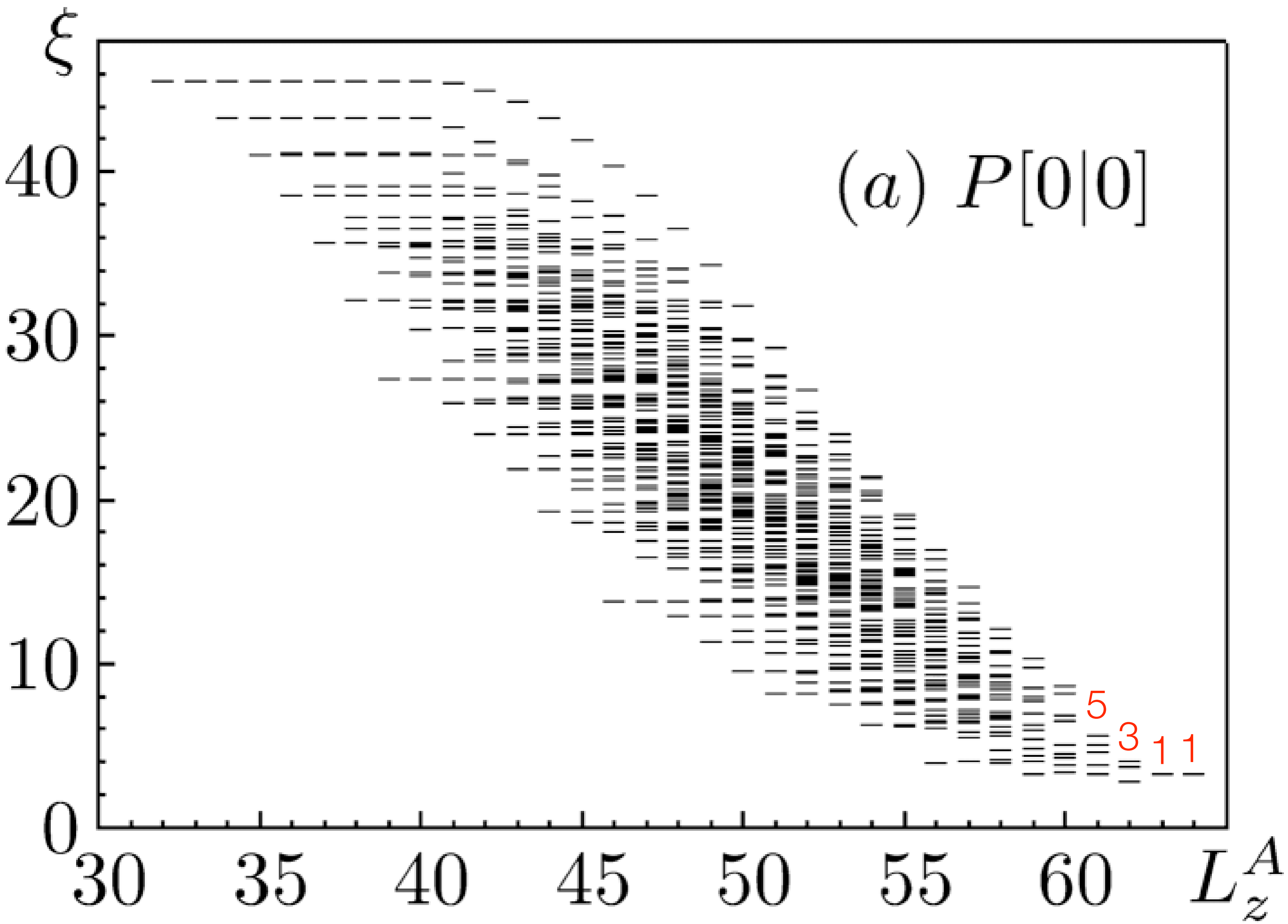
$$\rho =: \exp[-H_{\text{Entanglement}}]$$

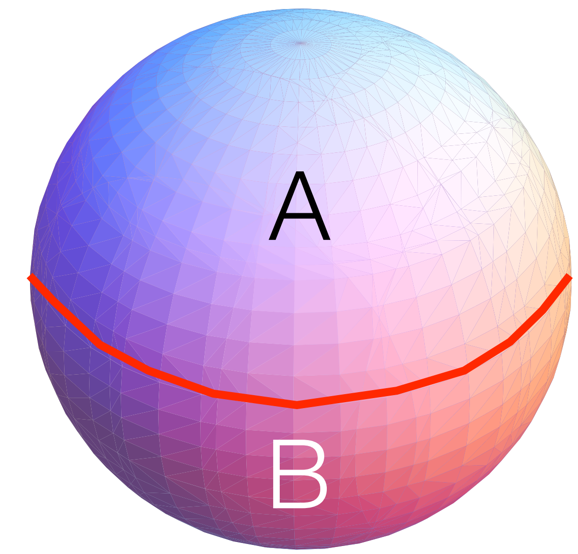
$$\mathcal{S} = \sum_i \xi_i \exp(-\xi_i),$$

- Assuming that the entanglement Hamiltonian and the physical Hamiltonian are "similar" (e.g. as in free fermionic systems), then one expects to see features related to the open boundary structure in the spectrum of the reduced density matrix
- FQH states have interesting edge physics, visible in entanglement spectrum ?

Moore-Read state on the sphere (Li & Haldane, PRL '08)

- Entanglement spectrum has dispersive structure



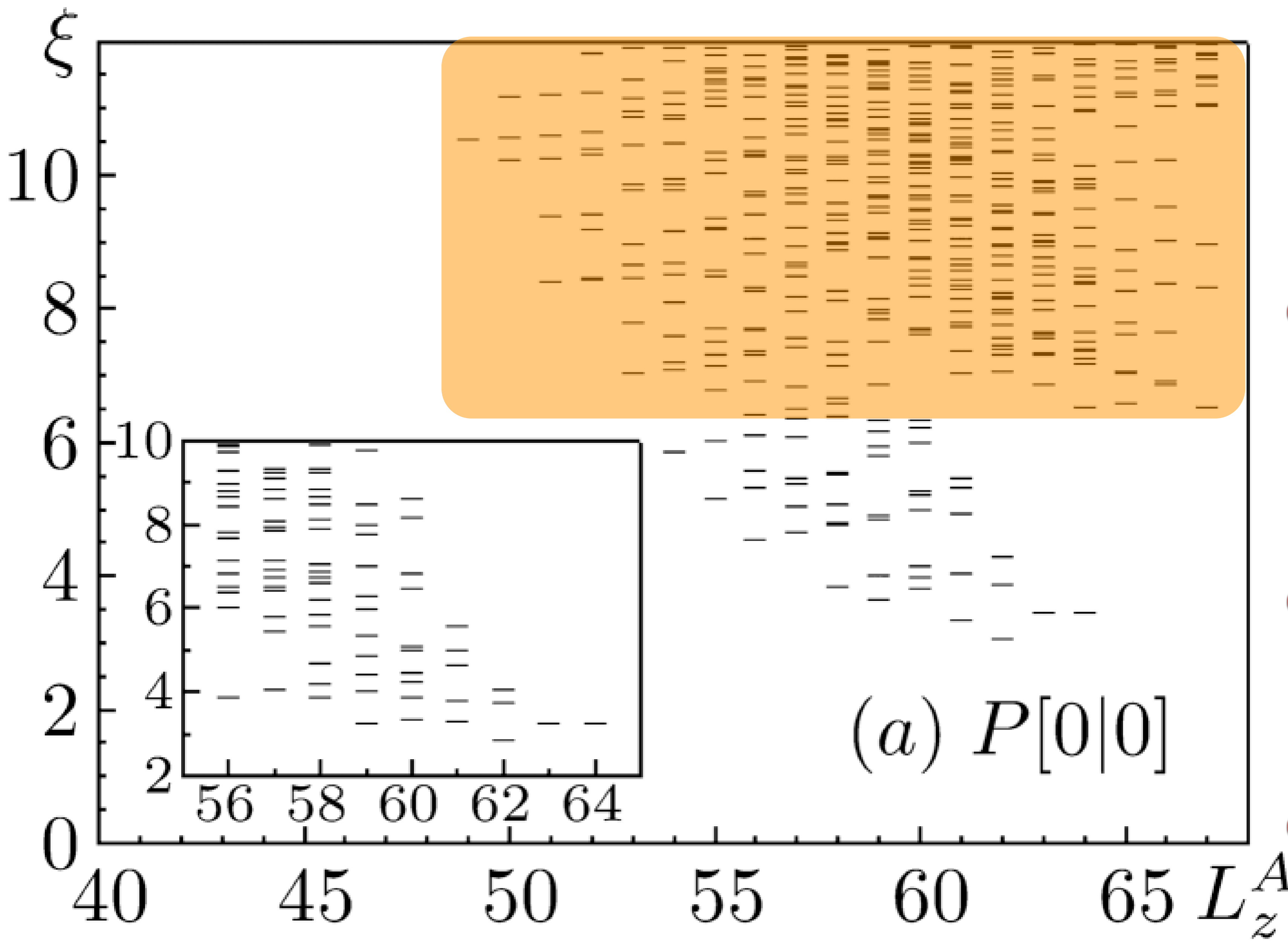


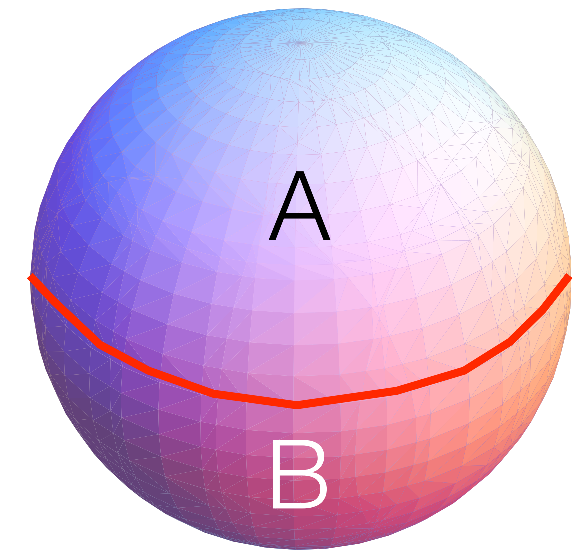
- Degeneracy at large momenta follows CFT counting rule (edge theory of the Pfaffian is U(1)+Majorana)

Wen, PRL '93

Moore-Read state on the sphere (Li & Haldane, PRL '08)

- Now for "realistic" Coulomb Hamiltonian at **ν**=5/2

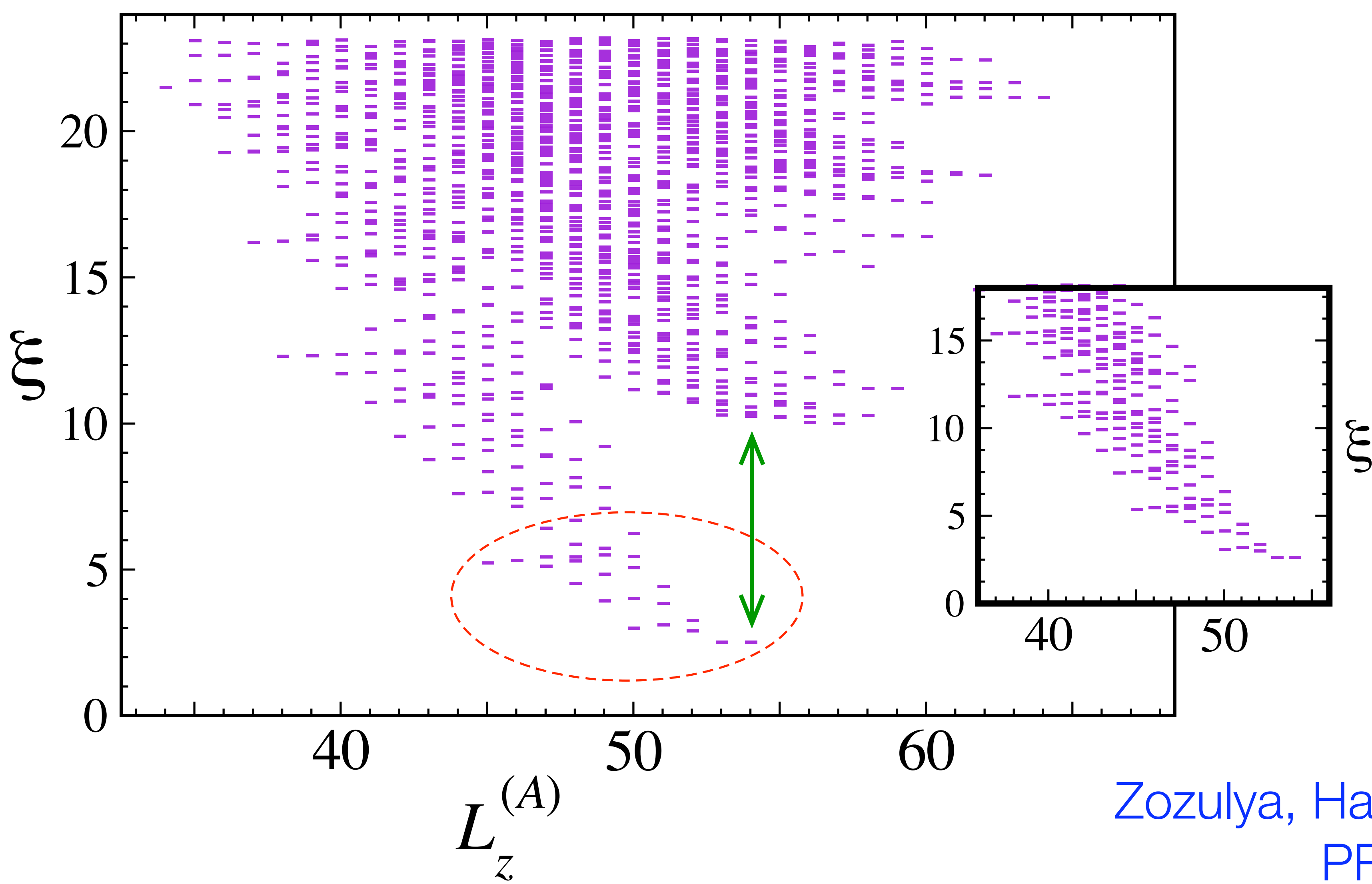




- Lower part of entanglement spectrum similar to model state
- Pollution by generic levels above "entanglement gap"
- Energetics not understood!

Entanglement Spectrum at ν=1/3 (Coulomb)

- Chiral low energy mode with an entanglement gap to generic levels
- Satisfies degeneracy count for a chiral U(1) theory (1-1-2-3-5-7-11-....)



Zozulya, Haque & Regnault
PRB '09

Entanglement Spectra studies: some examples

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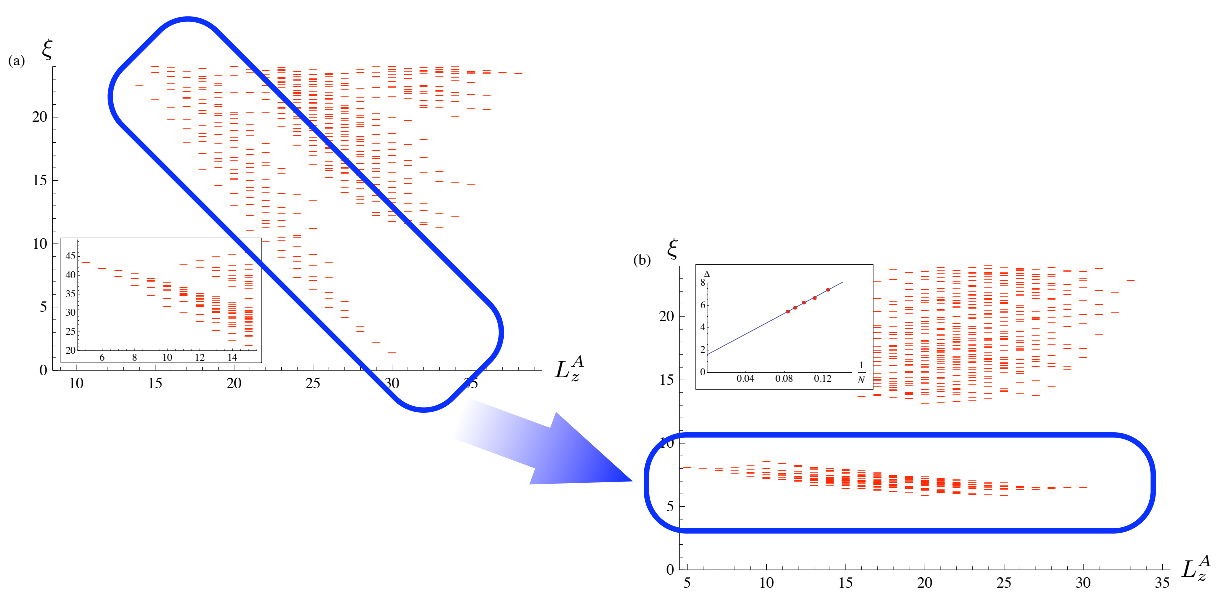

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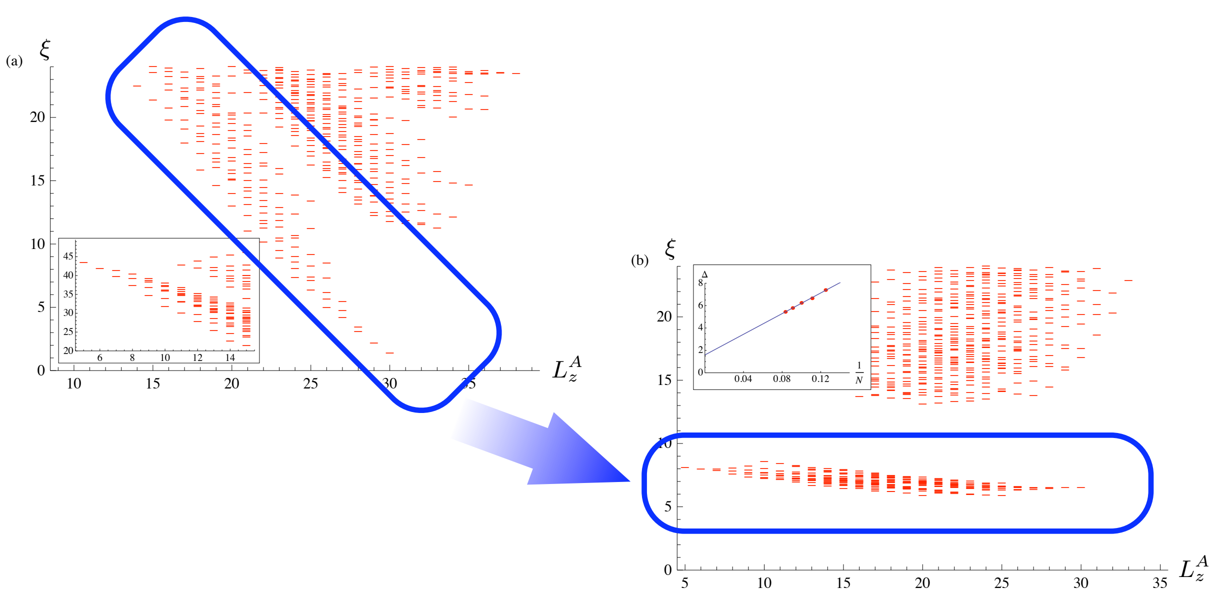

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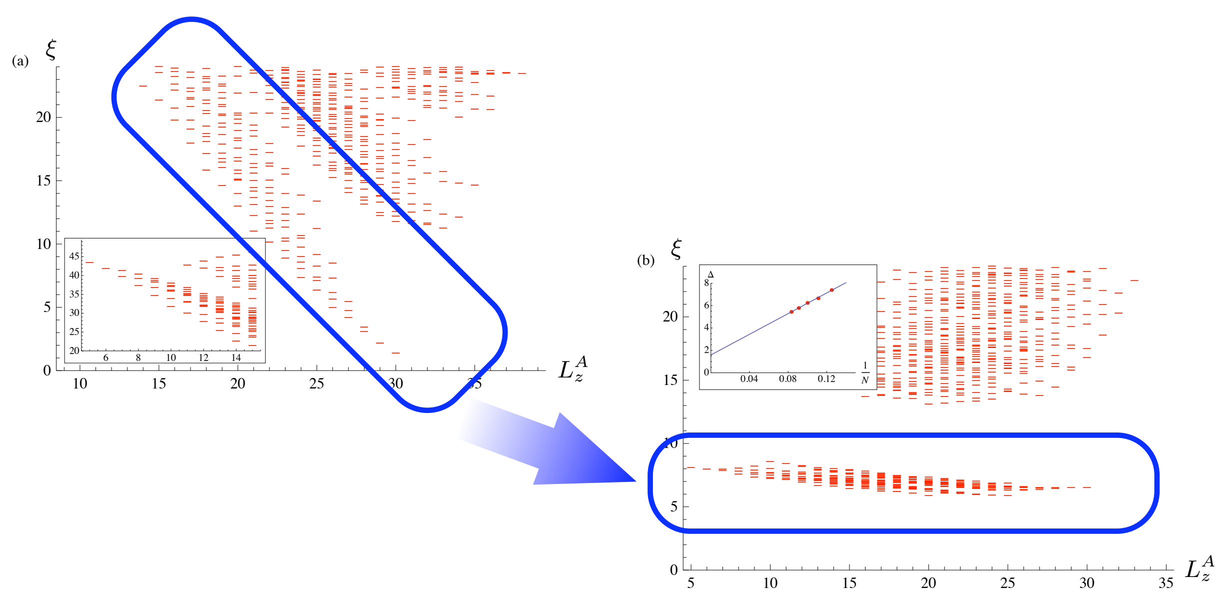

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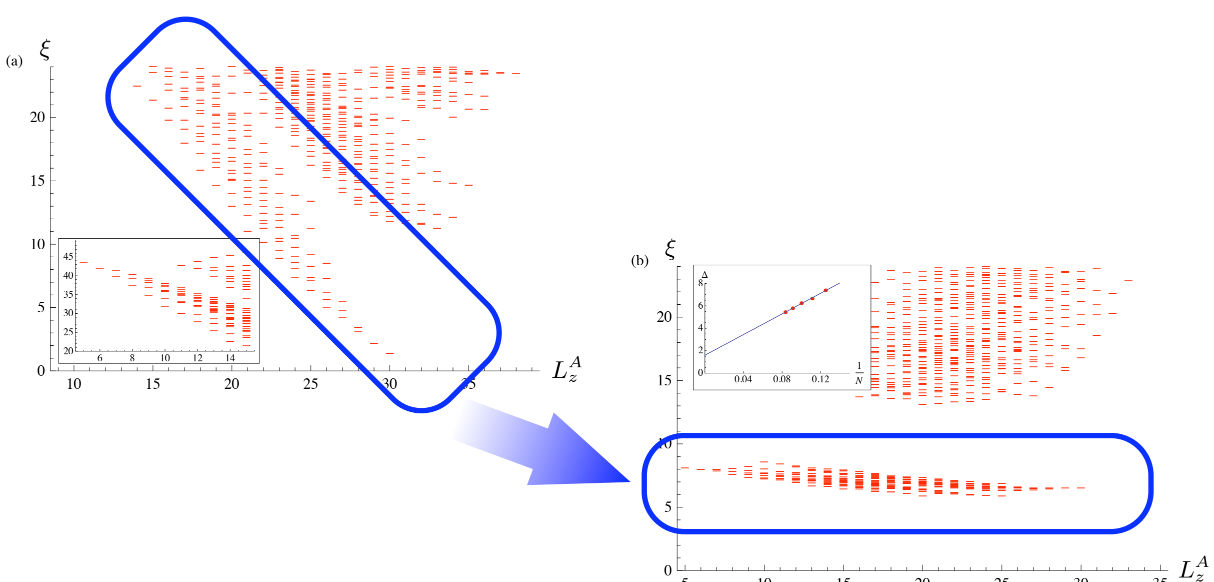

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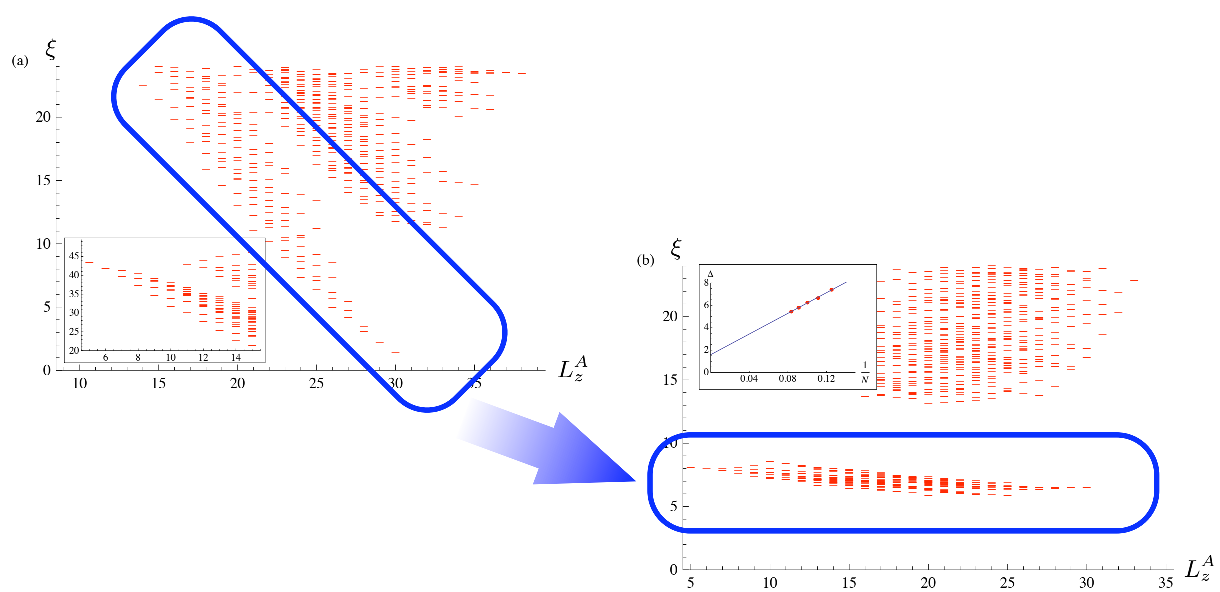

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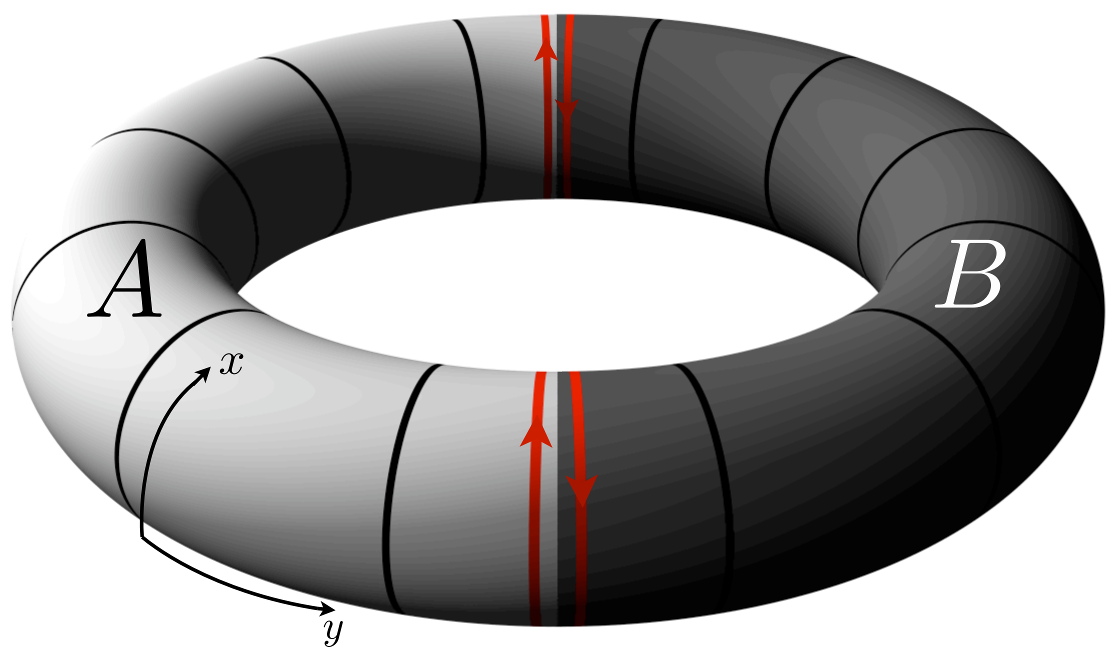
And now for something different, the torus

- The natural partition of Landau level orbitals leads to blocks having two edges

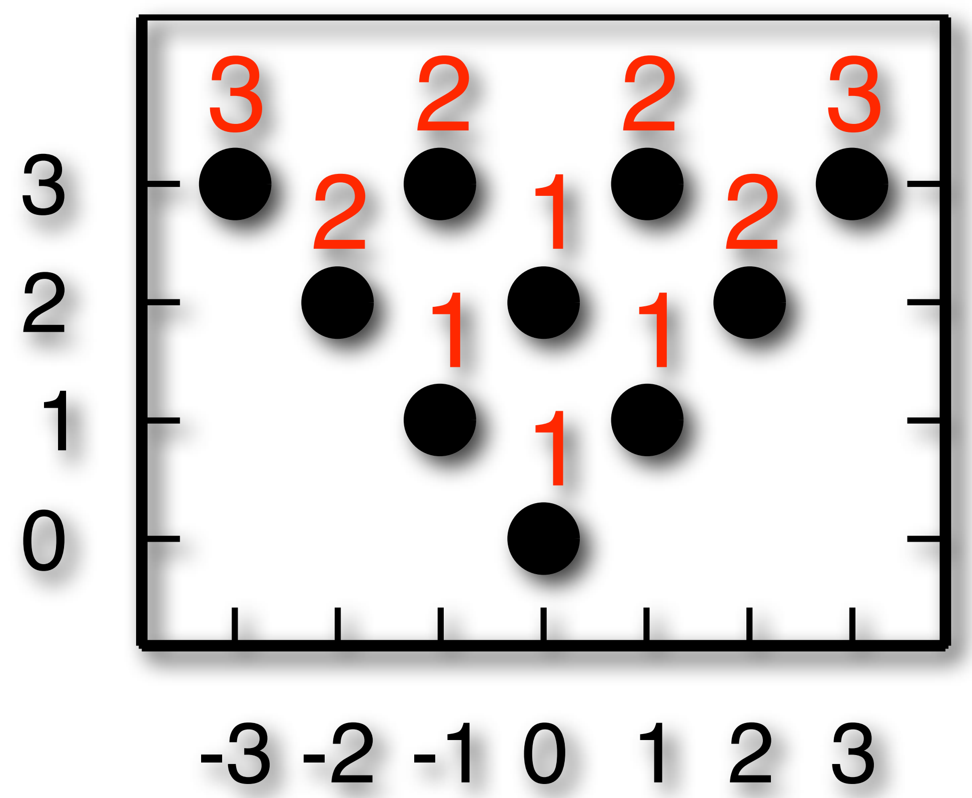


- How do the two chiral edges combine in the entanglement spectrum ?

- Can we exploit the tunability of the aspect ratio to understand the entanglement spectrum quantitatively ?

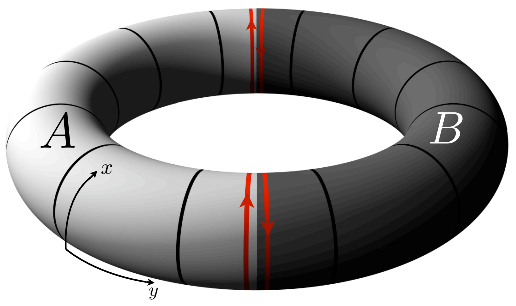
Combining two chiral U(1) edges

- What do we expect to see when there are two linearly dispersing chiral U(1) modes?

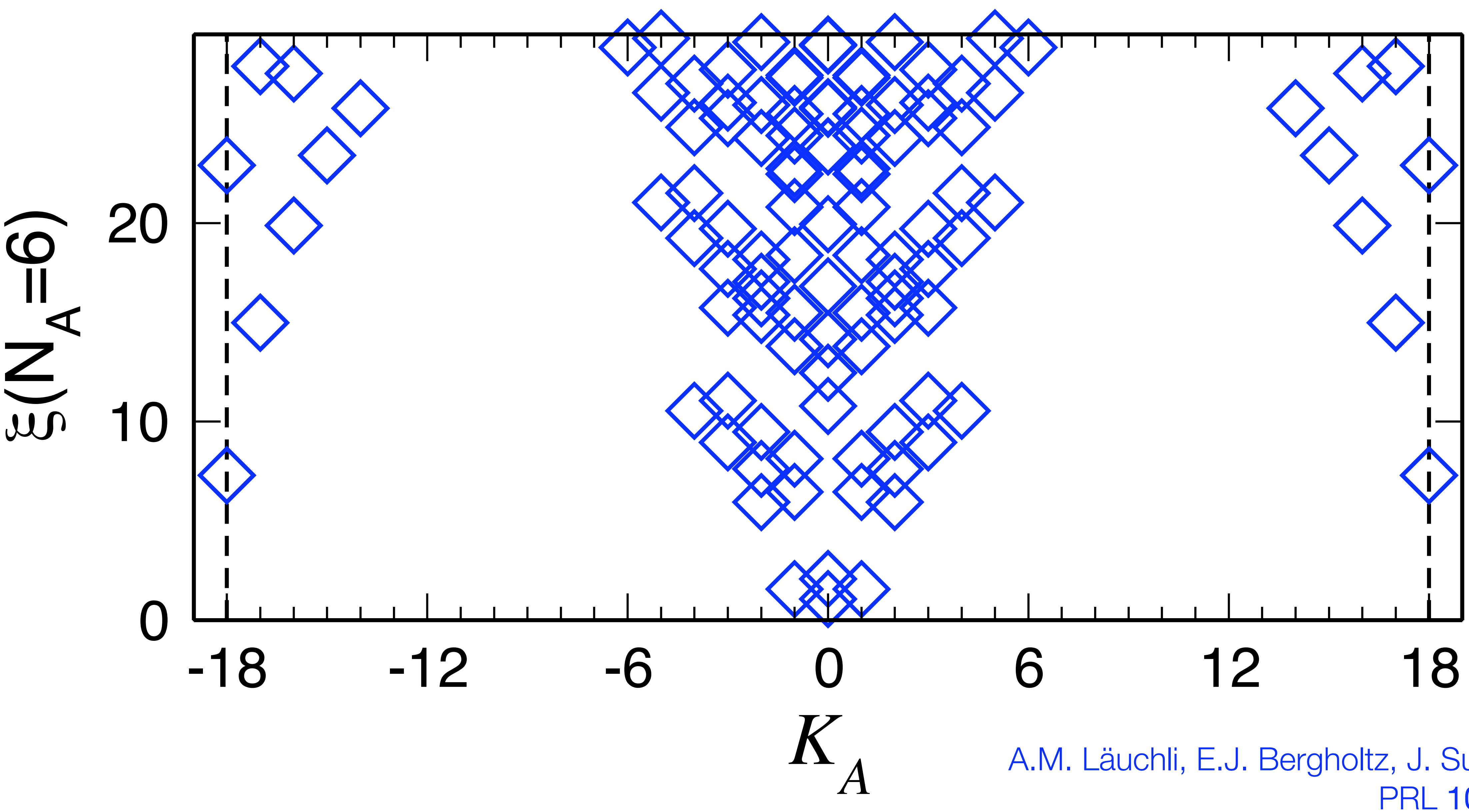


- This well known in the excitation spectrum of e.g. Luttinger liquids in spin chains

Torus entanglement spectrum

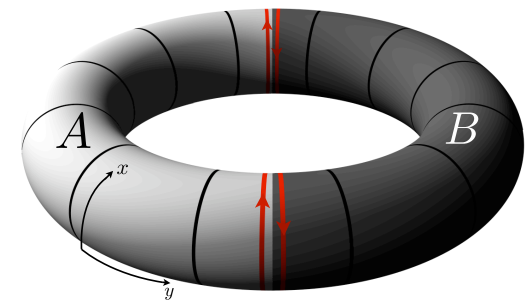


- $\nu=1/3$ Laughlin state, $N_S=36$, $L_1=10$, $L_A=N_S/2=18$, $N_A=6$

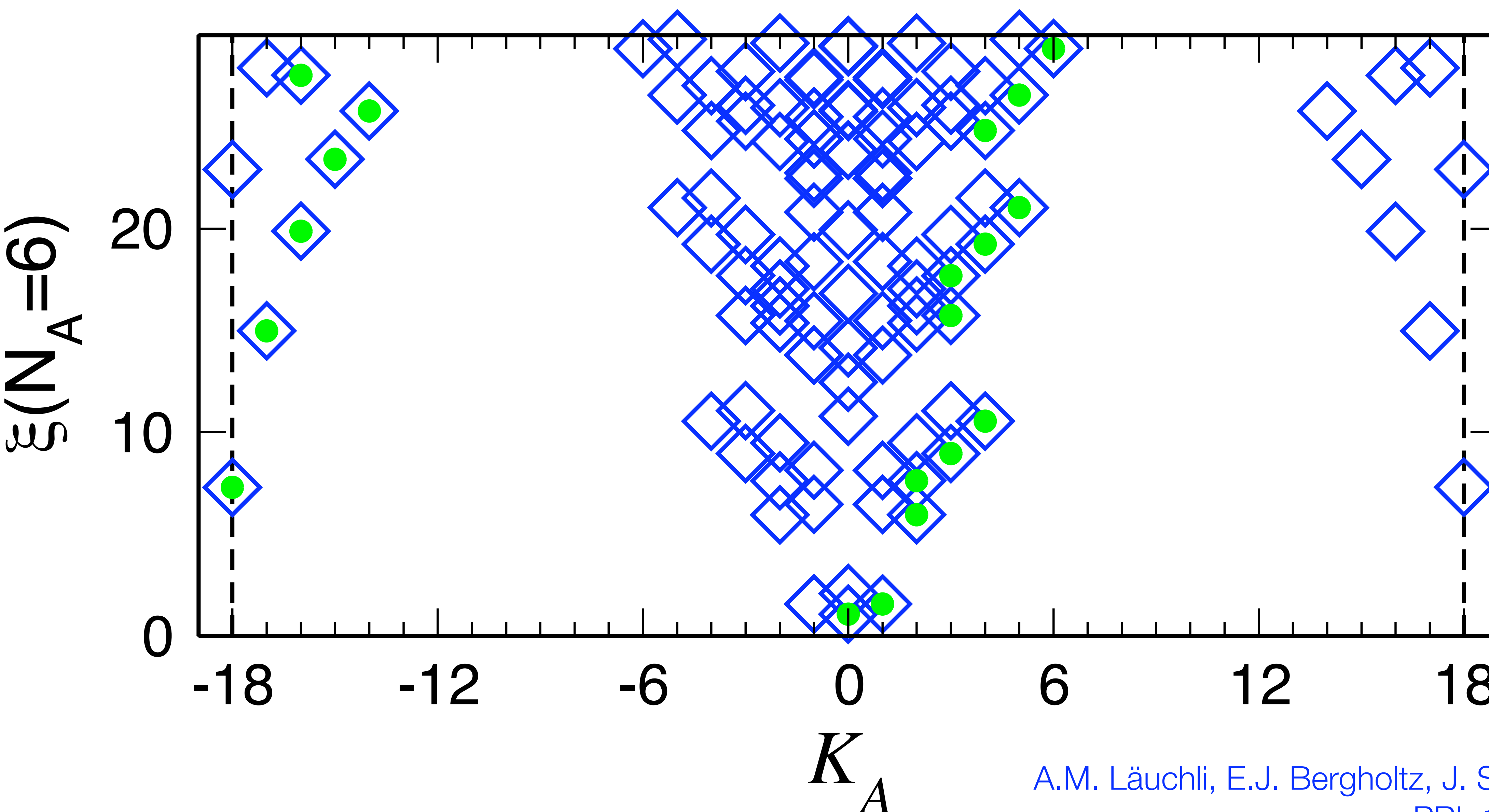


A.M. Läuchli, E.J. Bergholtz, J. Suorsa & M. Haque
PRL **104** 156404 (2010)

Assigning edge levels



- Key step: find algorithmically all edge levels by relying on a two independent edge hypothesis



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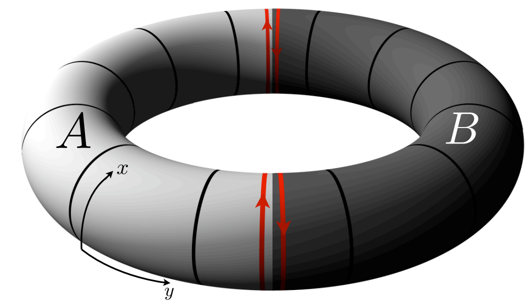
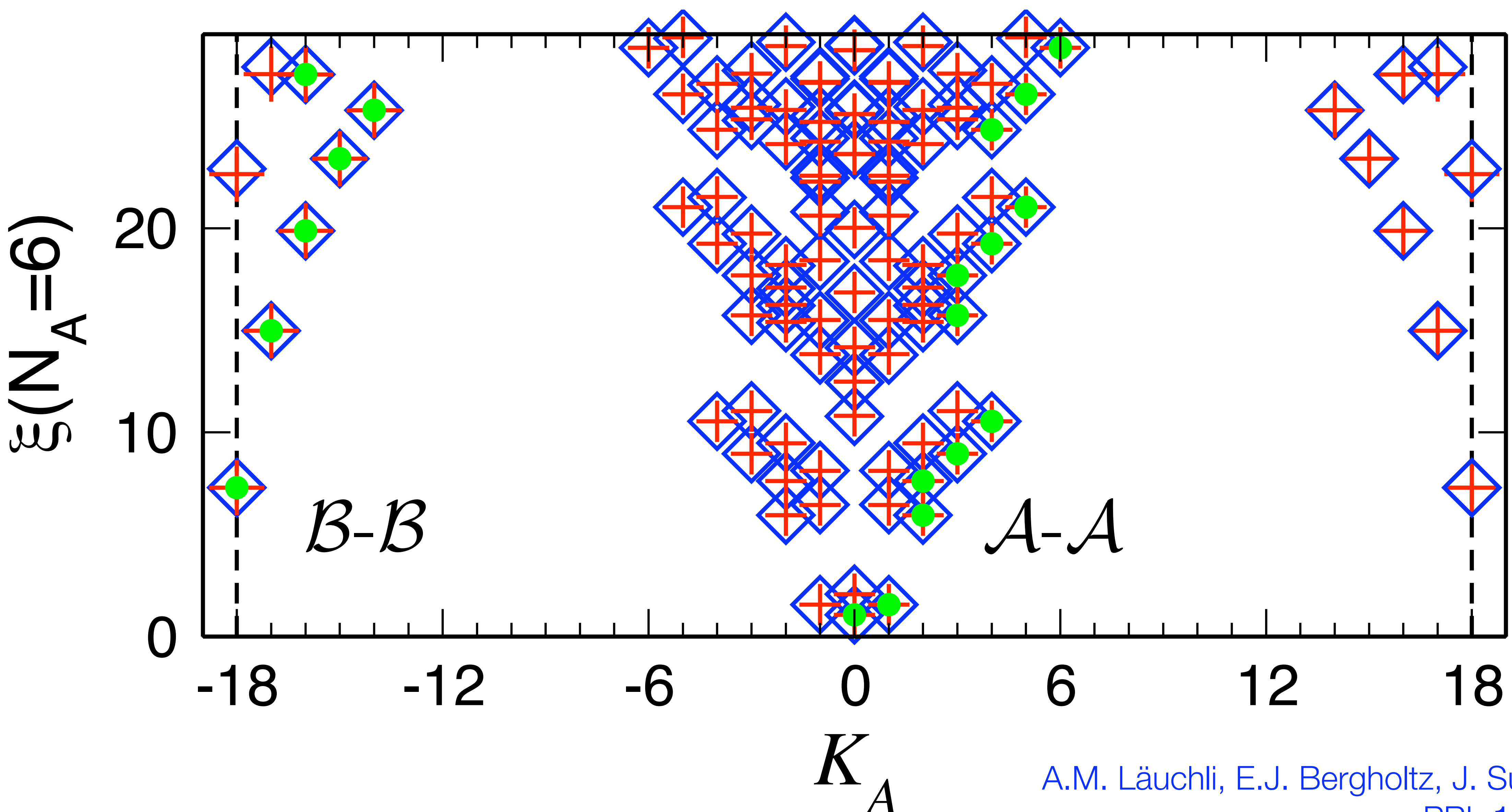
The two edge hypothesis at work



- Excellent match between the actual entanglement spectrum (tilted squares) and the two edge prediction (crosses)



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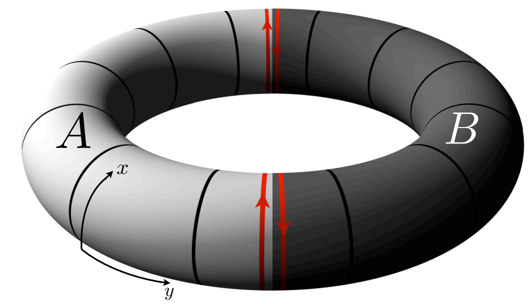
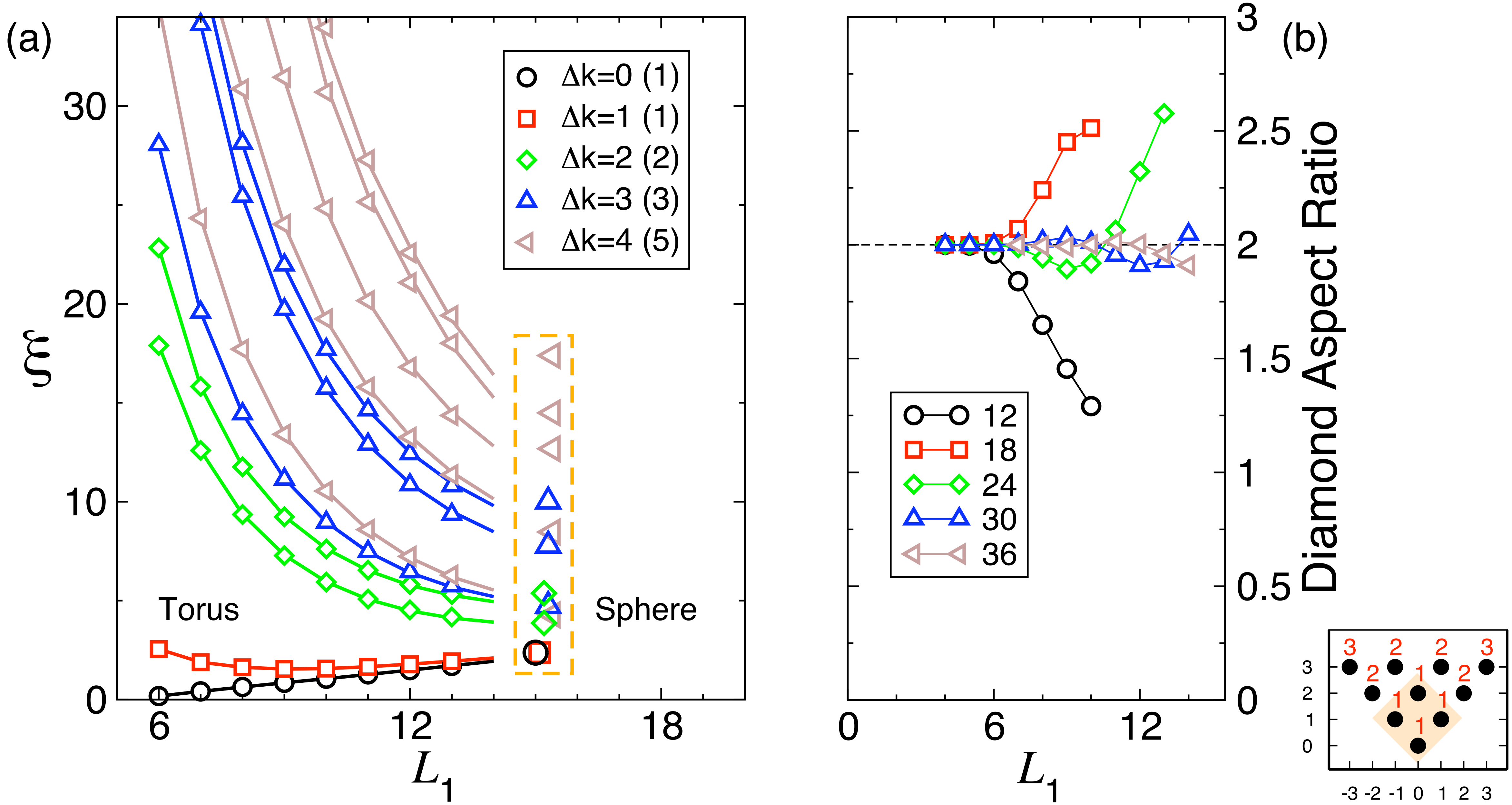
L_1 dependence of chiral edge levels



- Chiral edge theory has the correct U(1) count [1-1-2-3-5-....] (not enforced) !



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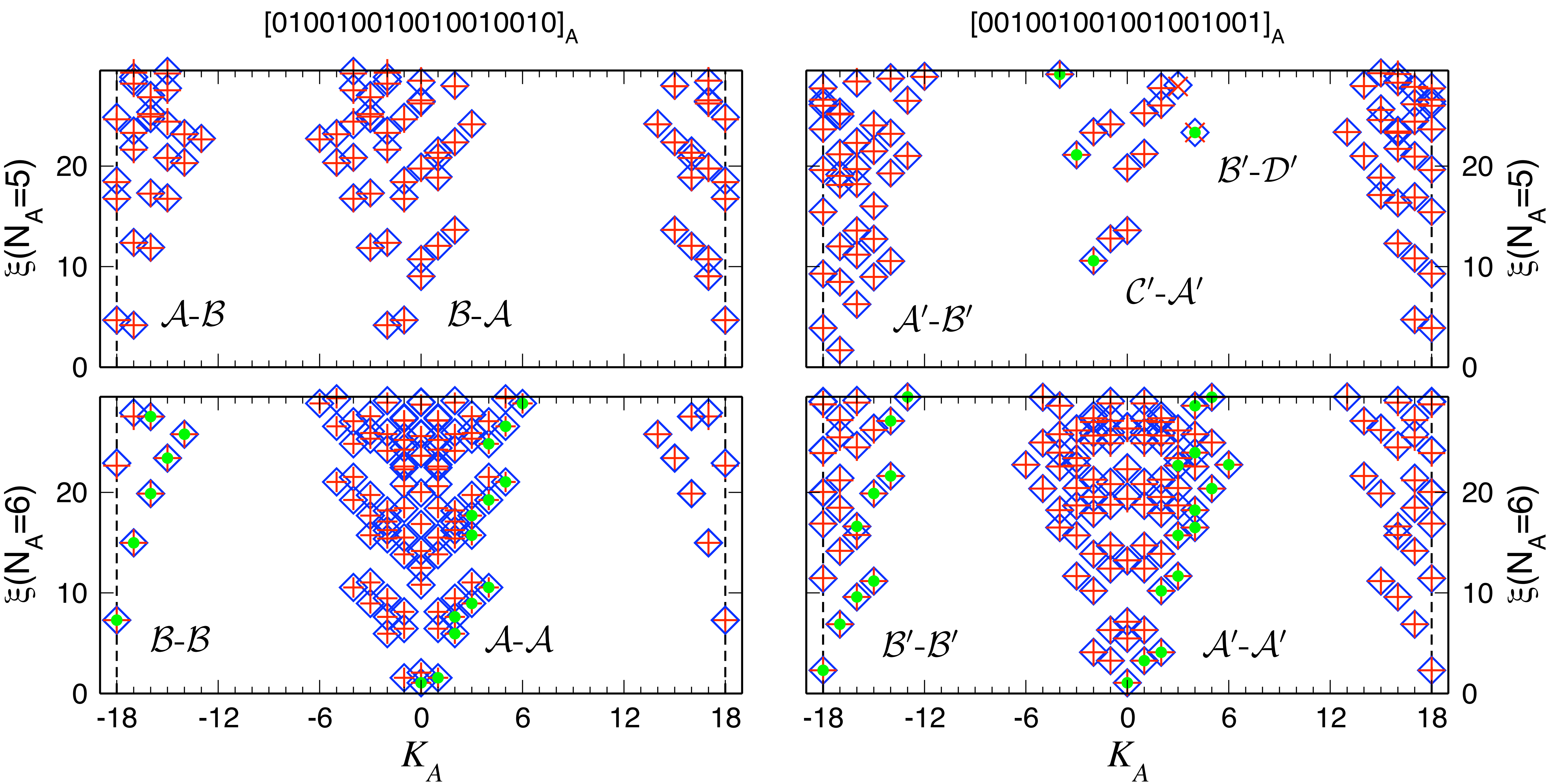
- Adiabatic evolution: perform perturbation theory for small L_1

The predictive power of the two edge hypothesis

- Based on some simple microscopic picture, one can predict the occurrence and the type of energetics of many towers, even with different N_A



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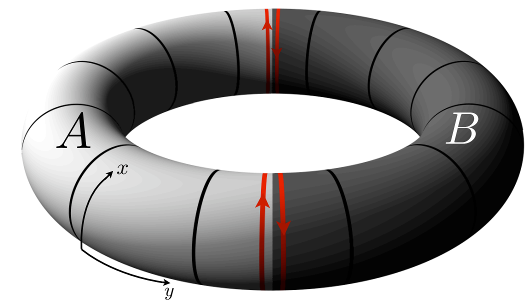
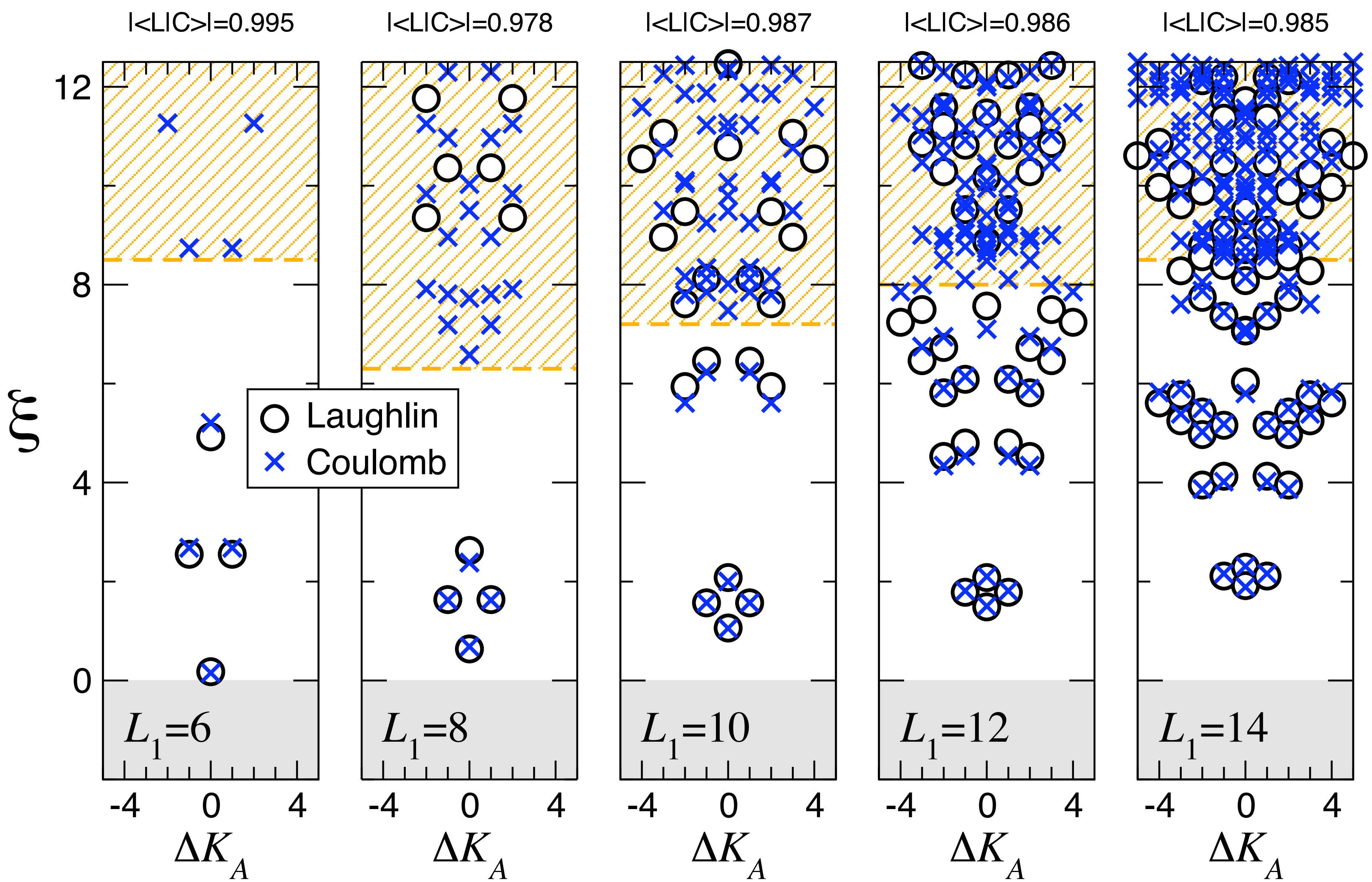
Coulomb vs Laughlin states at ν=1/3



- More and more U(1) structure emerging in the Coulomb state with increasing L_1



- Difference at small L_1 understood in perturbation picture

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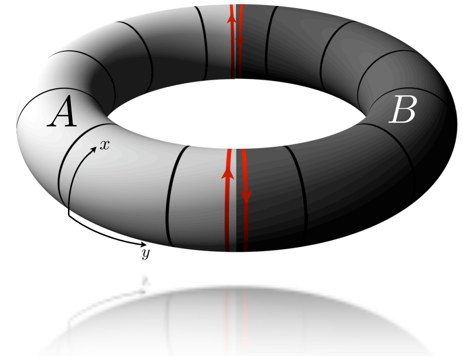
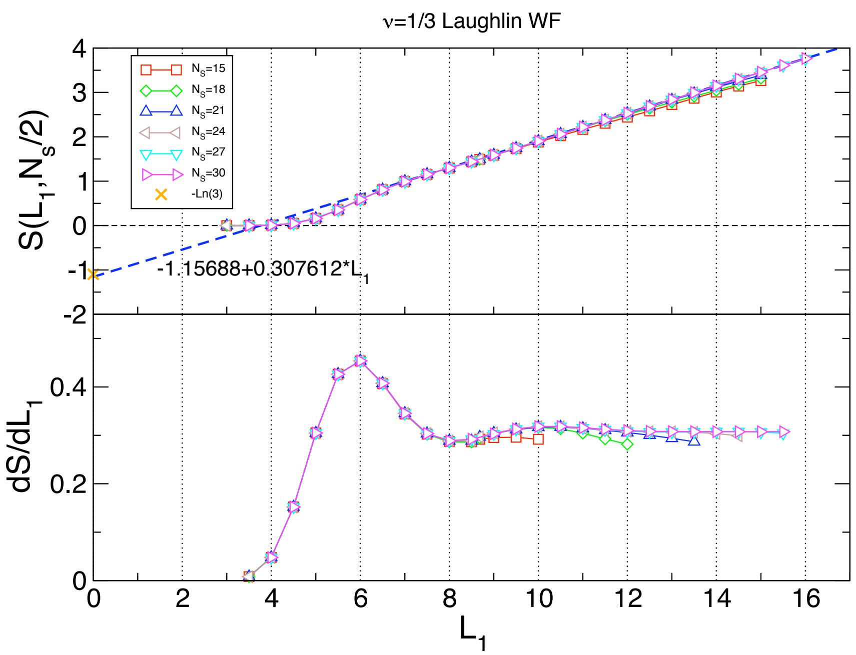
Conclusions



Topological Entanglement Entropy



- Exploiting the advantage of the torus to continuously change the circumference allows to get a significantly better estimates for the topological entanglement entropy.

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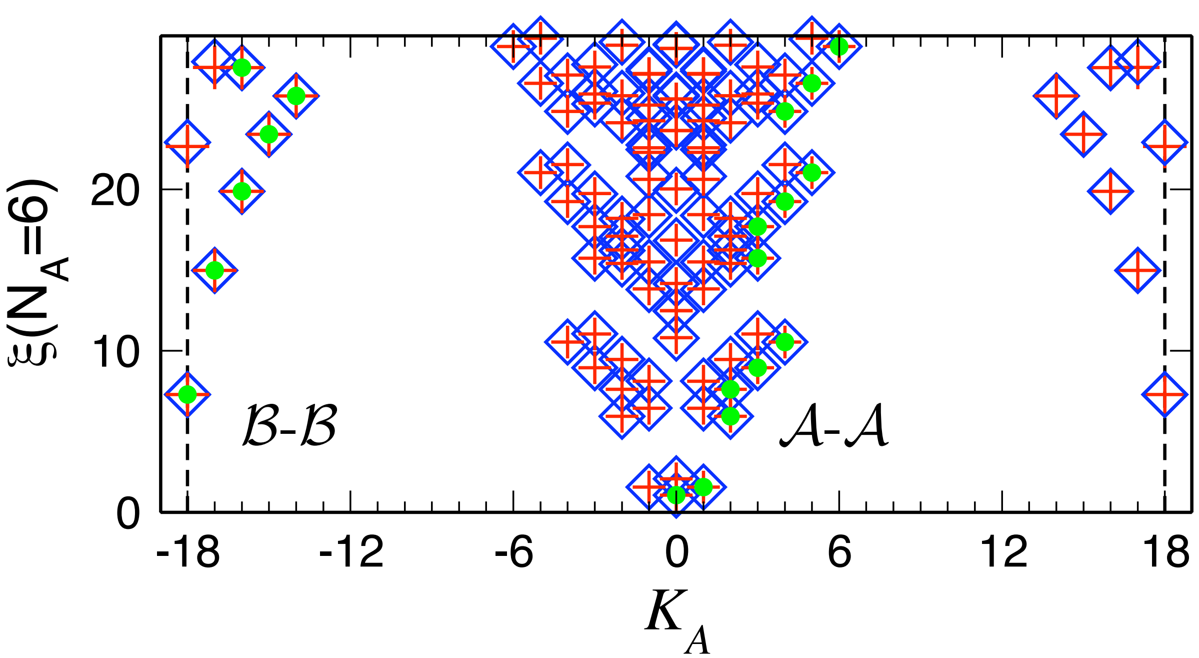
Entanglement Spectrum



- Combination of two spatially separated edges to form conformal towers with correct Virasoro count. Microscopics understood from the TT limit.

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