Numerical study of quasiholes at $\nu = \frac{5}{2}$ and the Majorana Fermion: more reasons to be hopeful

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Acknowledgments

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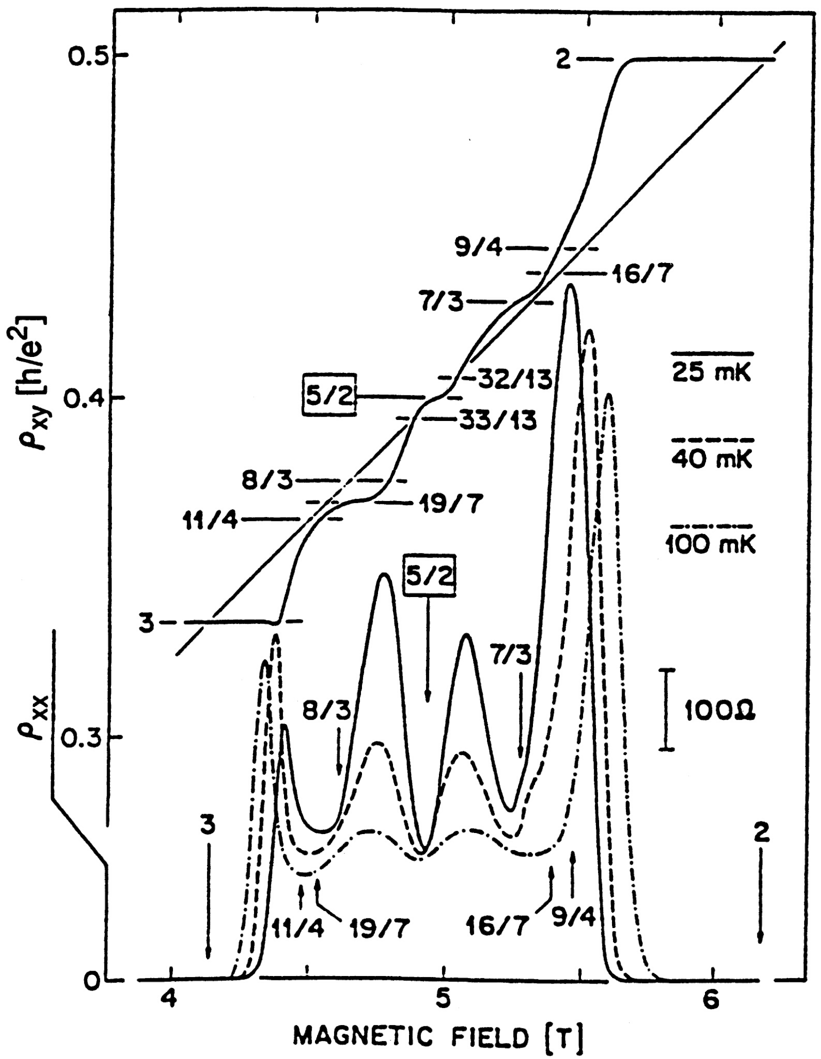
- what is known about spin-polarization at $\nu = 5/2$: very old and most recent results
- adiabatic continuity between Coulomb GS and Pfaffian for systems with $N \leq 18$ electrons
- phase diagram in pseudopotential plane: Gapped phase coincides with Pfaffian phase
- how to localize quasiholes
- adiabatic continuity between Coulomb "GS" and Pfaffian for 2 quasihole system
- adiabatic continuity between Coulomb "GS" and Pfaffian for 4 quasihole system
- conclusions
- open questions

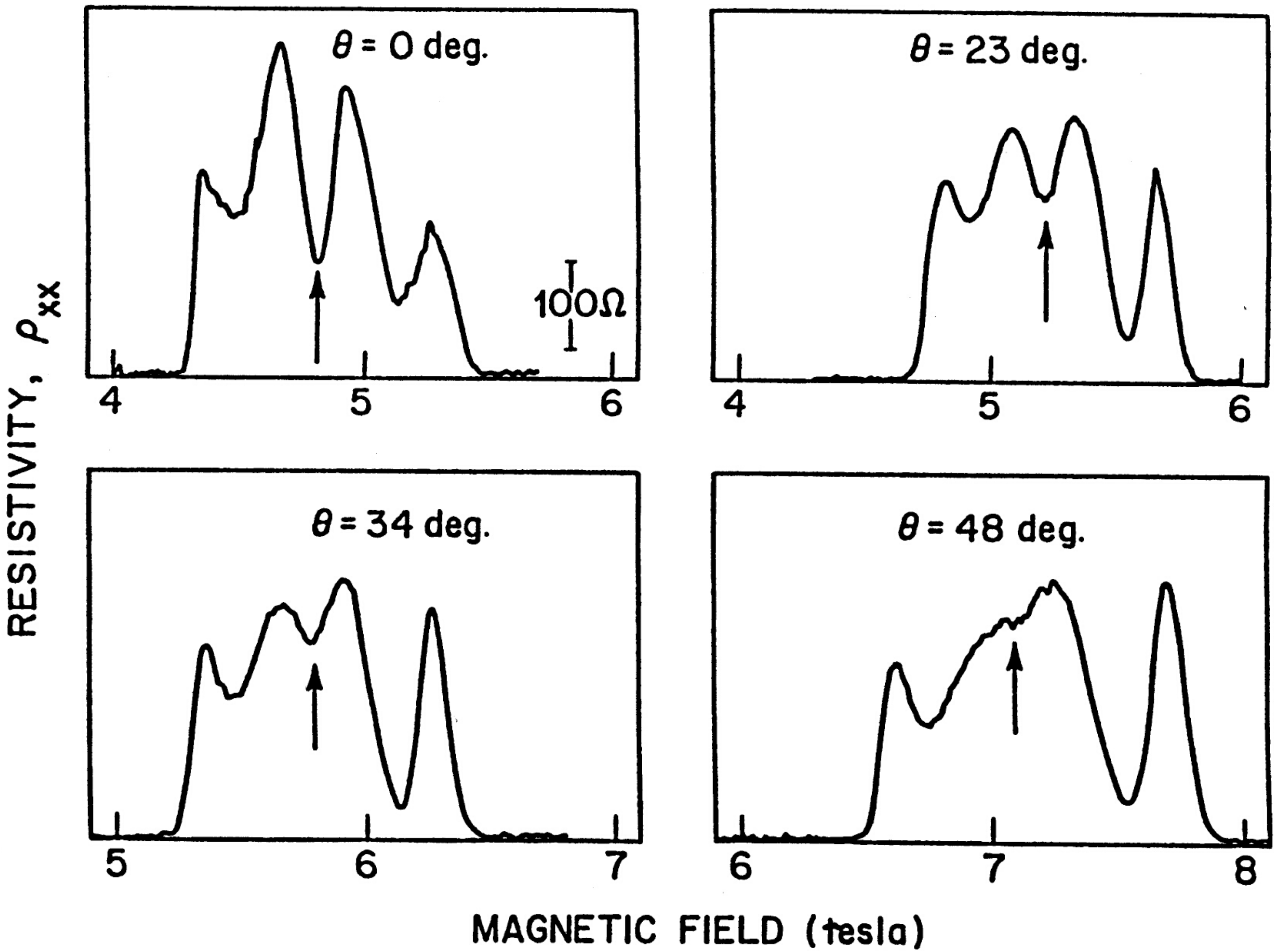
a short overview of the early history of $\nu = 5/2$

See references concerning $\nu = 5/2$ in Nayak, Simon, Stern, Freedman and Das Sarma, RMP 80, 1083 (2008)

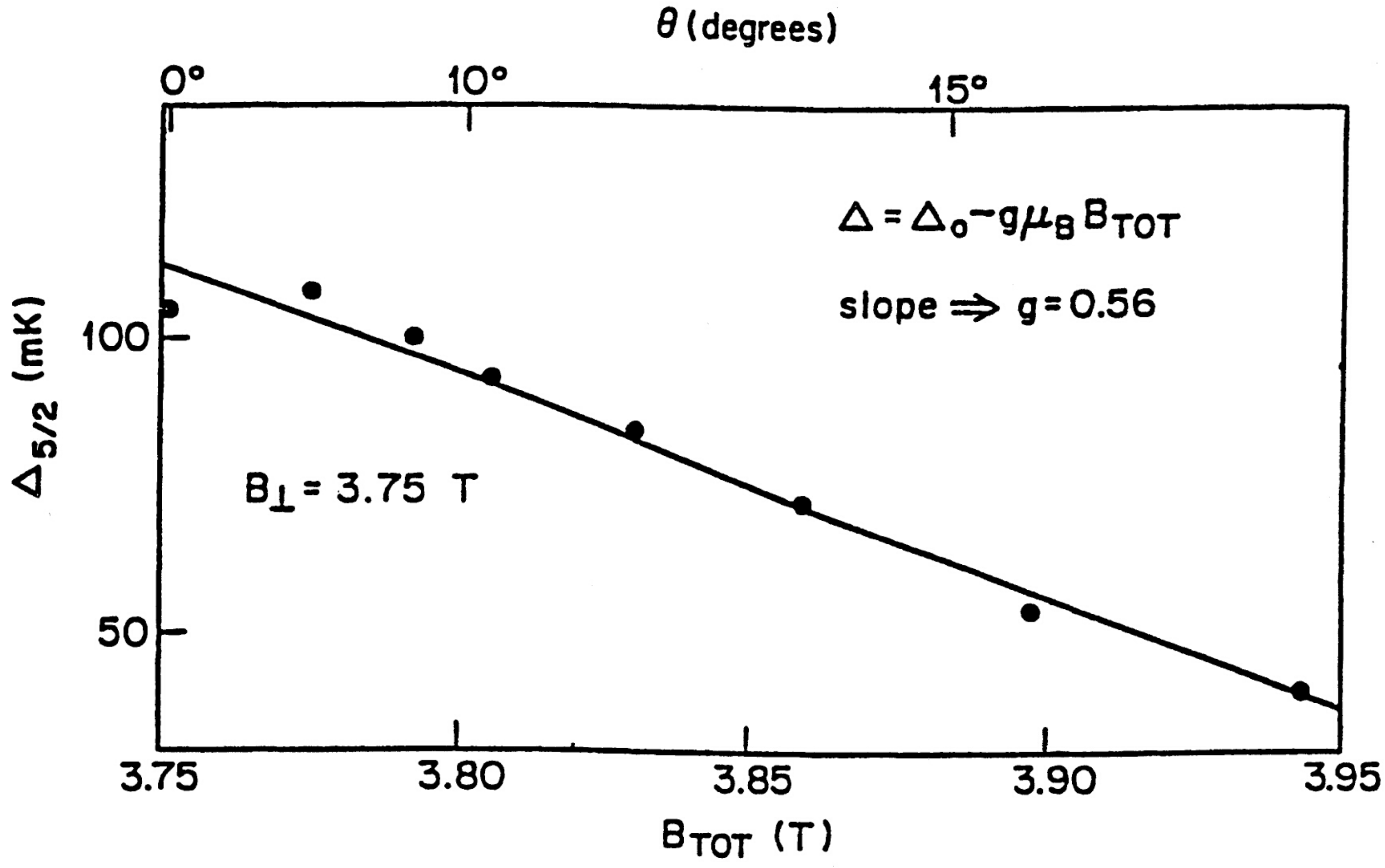
First observation of FQH state at $\nu = 5/2$: Willett et al. PRL 59, 1776 (1987)



Collapse of $\nu = 5/2$ state in tilted field:
Eisenstein et al. PRL 61, 997 (1988)



Activation energy of ρ_{xx} in a tilted field: Eisenstein et al., Surf. Sci. 229, 31 (1990)



Conclusions from Experiment

FQH-plateau at $\nu = 5/2$

Gap decreases in tilted field – gap reduction $\propto B_{tot}$

Transition to compressible state for $B_{tot} \geq B_{tot}^{c}$

Simplest scenario - generally believed for 10 years until 1998

- FQH state at most partially polarized or fully unpolarized (cf. $\nu = 8/5$)
- lowest energy excitations involve spin flip - gain in Zeeman energy
- Transition to gapless fully polarized state at $B = B_c$ induced by Zeeman energy

FQH states in half-filled Landau levels
Theoretical ideas

Halperin (1983): generalization of Laughlin wf (bilayers or spin)

-

$$\Psi_{mmn} = \prod_{i<k}^{N/2} (z_i - z_k)^m (w_i - w_k)^m \times \prod_{i,k}^{N/2} (z_i - w_k)^n \times \text{Gaussian}$$

$$\text{Fill Factor } \nu = \frac{2}{m+n}$$

- $\nu = 2/5$: spin-singlet state Ψ_{332} (observed by Eisenstein et al. 1988)
- $\nu = 1/2$: for bilayer systems Ψ_{331}: z_i electrons in layer 1, w_i electrons in layer 2

Haldane and Rezayi (1988) spin-singlet state (s-wave paired state).

Let $z_i = z_i^{\uparrow}$ and $w_i = z_i^{\downarrow}$

$$\Psi_{HR} = \Psi_{331} \times \text{permanent} \frac{1}{z_i - w_k} \quad \equiv \Psi_2 \times det \frac{1}{(z_i^{\uparrow} - z_k^{\downarrow})^2}$$

Moore and Read (1991) cf. also **Greiter, Wen and Wilczek (1991)**: spin polarized p-wave paired wave function

$$\Psi_{MR} = A\ (\Psi_{331}) = \Psi_2 \times \mathrm{Pf}\frac{1}{z_i - z_k}$$

Pfaffian (antisymmetric function of all variables) defined by

$$\mathrm{Pf}\frac{1}{z_i - z_k} = \sum_{P \in S_N} (-1)^{\sigma_P} \prod_{i=1}^{N/2} \frac{1}{z_{P[i]} - z_{P[i+N/2]}}$$

is exact ground state (zero-energy state) for special 3-body interaction

$$V_{3body} = \prod_{i<k<m}^{N} \mathcal{S}_{ikm}\left(\ \nabla_k^2 \nabla_m^4 \delta(z_i - z_k)\delta(z_i - z_m)\ \right)$$

Note $\Psi_{MR} \equiv A\Psi_{331}$ on disk and sphere, A is the antisymmetrizer. More complicated on torus.

2 quasihole excitation:

$$\Psi_{MR+2qh} = \Psi_2 \times \mathrm{Pf}\,\frac{(z_i - w)(z_k - u) + (u \longleftrightarrow w)}{z_i - z_k}$$

4 quasihole excitation:

$$\Psi_{MR+4qh} = \Psi_2 \times \mathrm{Pf}\,\frac{(z_i - w_1)(z_i - u_1)(z_k - w_2)(z_k - u_2) + (u_\ell \longleftrightarrow w_\ell)}{z_i - z_k}$$

Note: There exists a second, linearly independent wf with 4 quasiholes at positions w_1, u_1, w_2, u_2: interchanging $u_1 \leftrightarrow w_2$

$$\Psi'_{MR+4qh} = \Psi_2 \times \mathrm{Pf}\,\frac{(z_i - w_1)(z_i - w_2)(z_k - u_1)(z_k - u_2) + (u_\ell \longleftrightarrow w_\ell)}{z_i - z_k}$$

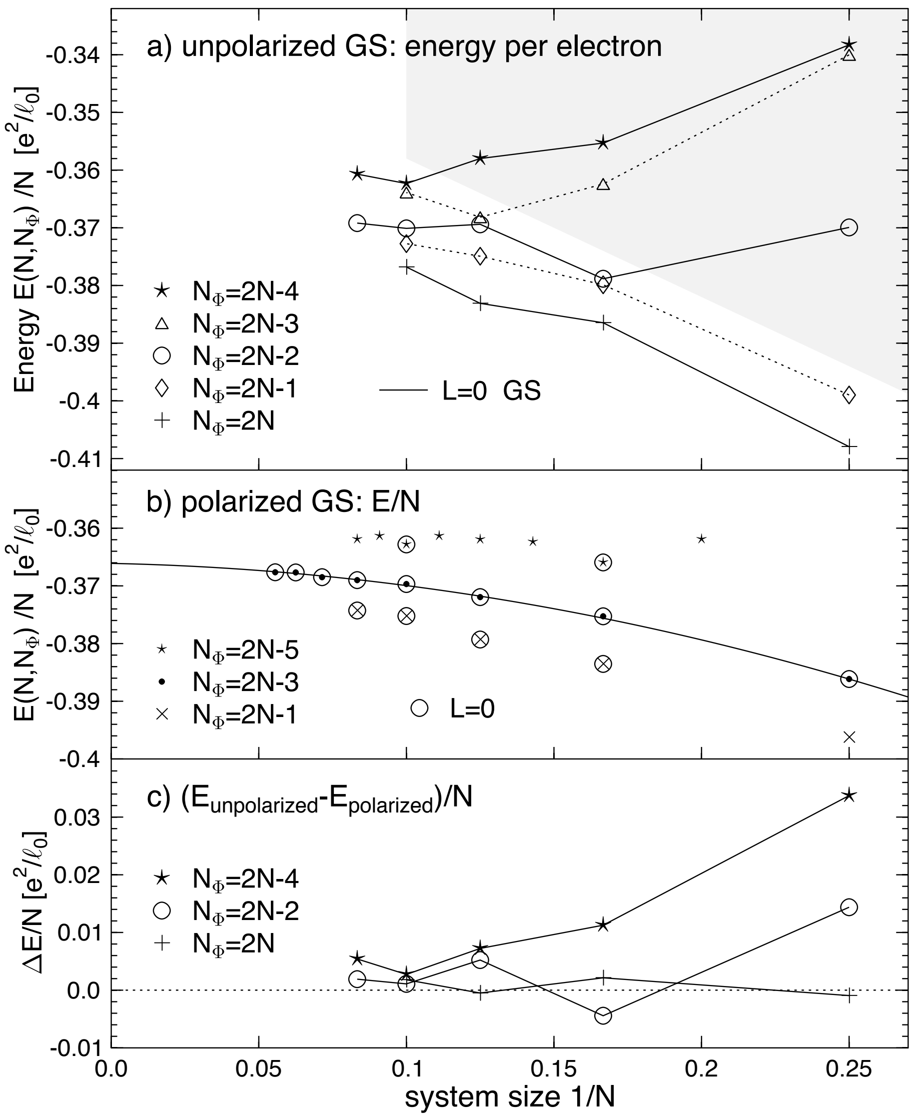
Nayak and Wilczek (1996), Milovanovic and Read (1996):

$2n$-quasiholes: 2^{n-1} fold degeneracy for Pfaff-interaction $\Longrightarrow$ non-abelian statistics

Moore Read Pfaffian: Ψ_{MR} characterized by non-abelian statistics ($q = 1/4$)

Numerical investigation Polarized vs. Spin-singlet

Unbiased numerical study (except fully polarized vs unpolarized): RM Phys. Rev. Lett. 80, 1505 (1998)



Unpolarized system $Spin = 0$

- very large finite size effects
- spin-singlet is GS only at $N = 6$, $N_\Phi = 10$ (for vanishing Zeeman energy, $g = 0$)
- no consistent energy gap values
- no local singlet: pair correlation function resembles that of polarized state with long-wavelength spinwave excitation to establish $Spin = 0$. Real GS would be polarized.

Polarized system

- $L = 0$ GS at $N_\Phi = 2N - 3$, $\quad S = 3$ for all even tested $(N \leq 18)$
- For all other values of the shift S we obtain GS with $L = O(N^0) = O(1)$, consistent with charged excitations in an incompressible background (2 or 4 quasiparticle states).

Quantized state at $\nu = 5/2$ is spin polarized!

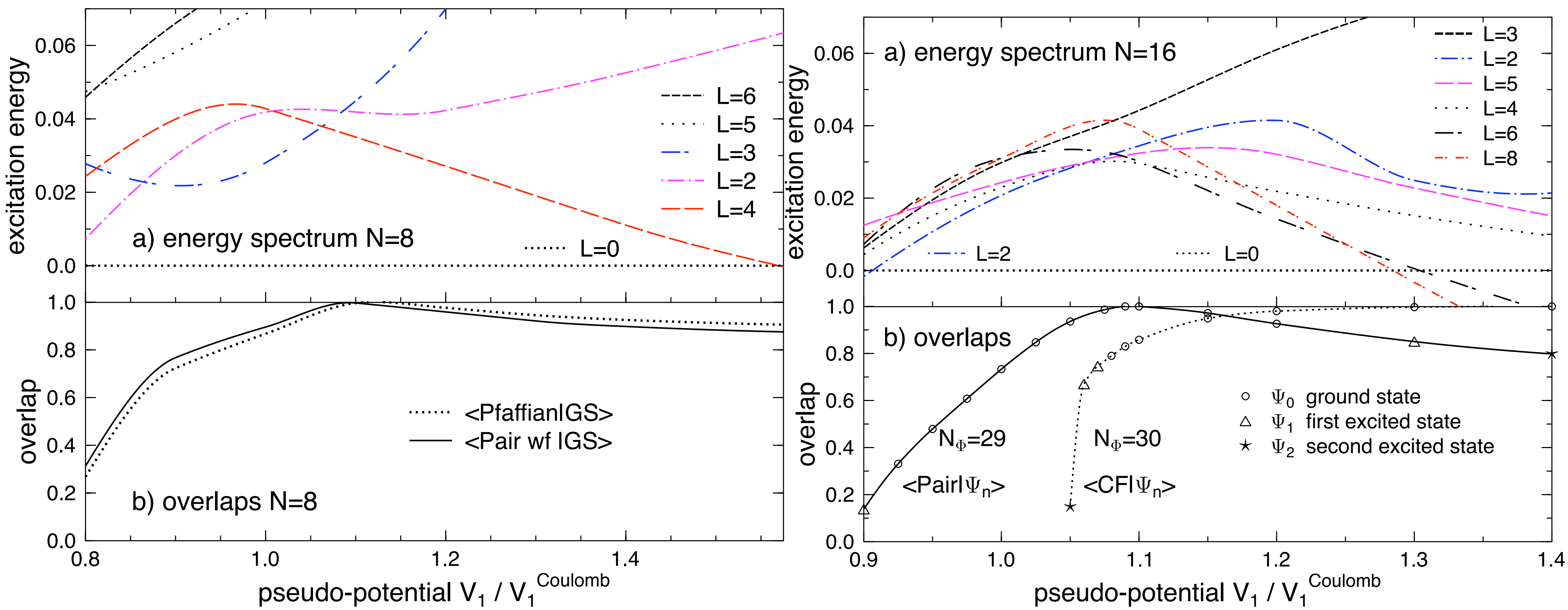
Recent References: Spin Polarization

Ivailo Dimov, Bertrand I. Halperin, Chetan Nayak, arXiv:0710.1921

A.E. Feiguin, E. Rezayi, Kun Yang, C. Nayak, S. Das Sarma, Phys.Rev.B79, 115322 (2009)

A. Wojs, G. Möller, S.H. Simon, N.R. Cooper, PRL 104, 086801 (2010)

Test system by varying V_1



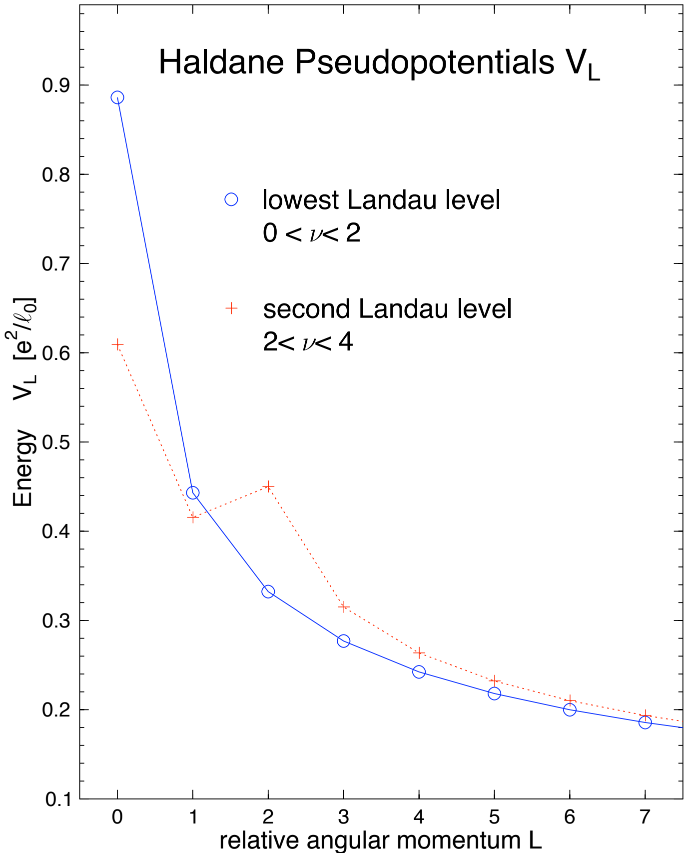
The Haldane pseudopotential

2 electrons in lowest Landau level with relative angular momentum L

$$\phi_{L,n}(z_1, z_2) = (z_1 - z_2)^L (z_1 + z_2)^n e^{-(|z_1|^2 + |z_2|^2)/4}$$

Haldane pseudoptential: energy of two-electron state

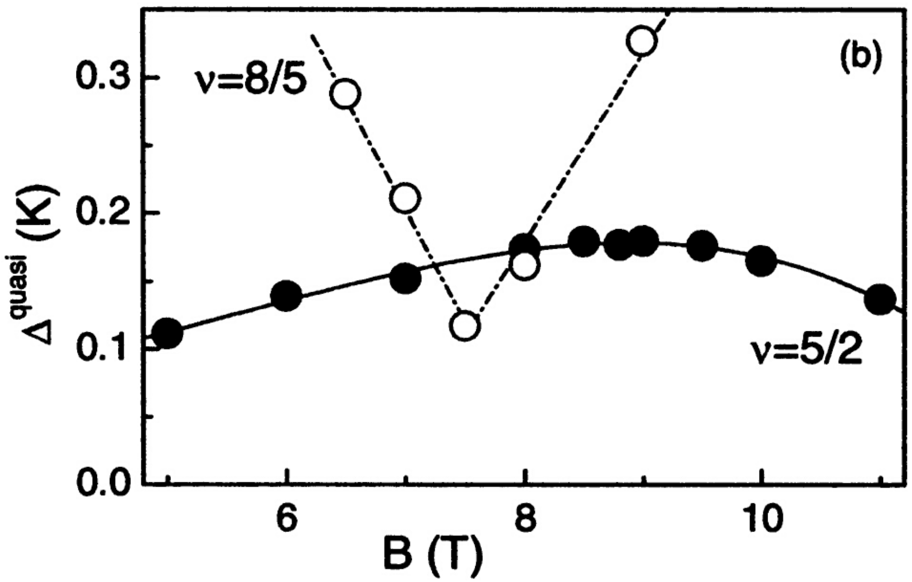
$$V_L = \frac{< \phi_{L,n}|V|\phi_{L,n} >}{< \phi_{L,n}|\phi_{L,n} >}$$



- Paired state stable in the window 0.95 $\lesssim V_1/V_1^{Coulomb} \lesssim 1.2$
- Gap $\Delta_{5/2} \approx 0.025 e^2/\epsilon\ell_0$ at $V_1 = V_1^{Coulomb}$
- Gap is maximum for V_1 which maximizes overlap of GS with Ψ_{MR} or pair wave function Ψ_{HM}
- For $V_1 \gtrsim 1.2$ transition to Composite Fermion liquid state (like in the lowest half-filled Landau level)
- For $V_1 < 0.9$ transition to symmetry broken state (at $L = 2$). Charge density wave state à la Fogler and Shklovskii.

Experimental results possibly confirming the spin-polarized nature of $\nu = 5/2$

W. Pan et al. Sol. St. Comm. 119, 641 (2001): $\nu = 5/2$ vs. $\nu = 8/5$ (unpolarized at low density)

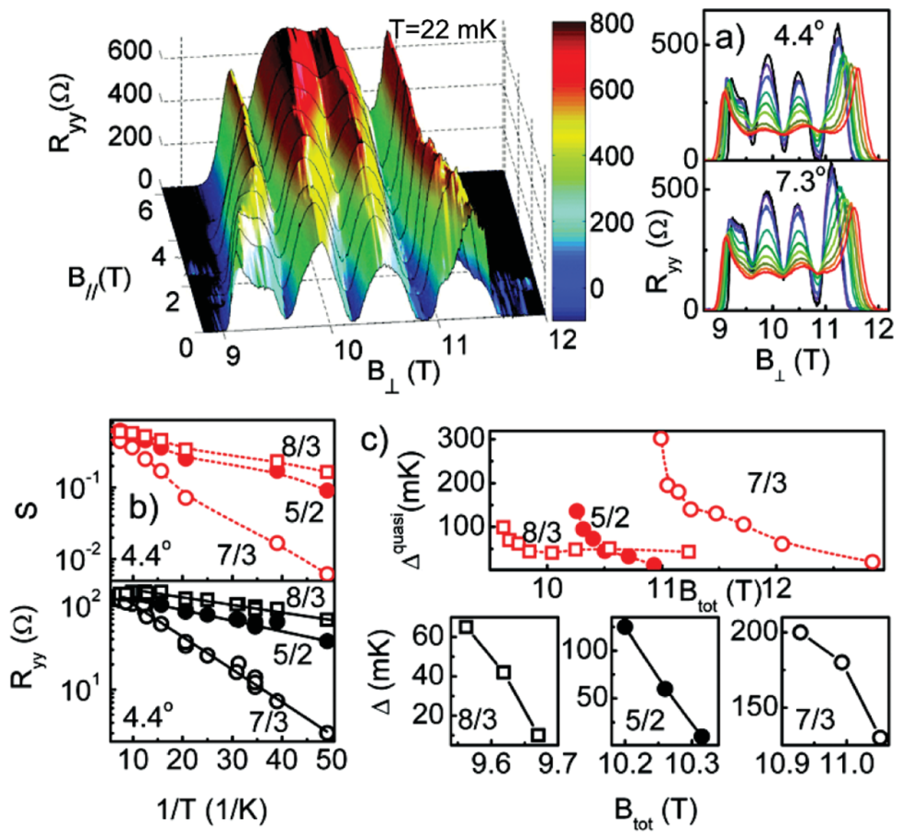


Smooth dependence of gap on magnetic field, no break in slope or sign of discontinuity, indicates that neither ground- nor excited state at $\nu = 5/2$ is changing its character, while Zeeman energy changes by ≈ 2K.

Thus, no phase transition in the spin sector appears to occur in this large range of fields. At the largest field, the system is likely polarized, implying that it should be <u>SPIN-POLARIZED</u> in the whole range of magnetic field shown.

Largest density sample

PRL **104,** 166801 (2010) PHYSICAL RE



PRL **104,** 166801 (2010) PHYSICAL REVIEW LETTERS week ending 23 APRIL 2010

$\nu = 5/2$ Fractional Quantum Hall Effect at 10 T: Implications for the Pfaffian State

Chi Zhang,[1,2] T. Knuuttila,[1] Yanhua Dai,[1] R. R. Du,[1,2,*] L. N. Pfeiffer,[3,4] and K. W. West[3,4]

Decrease of gap on magnetic field cannot be explained by Zeeman effect with a reasonable g-factor

Thus, system is <u>SPIN-POLARIZED</u> in the whole range of magnetic field shown.

But.....

PRL **105,** 096801 (2010) | Selected for a Viewpoint in *Physics* | PHYSICAL REVIEW LETTERS | week ending 27 AUGUST 2010

Optical Probing of the Spin Polarization of the $\nu = 5/2$ Quantum Hall State

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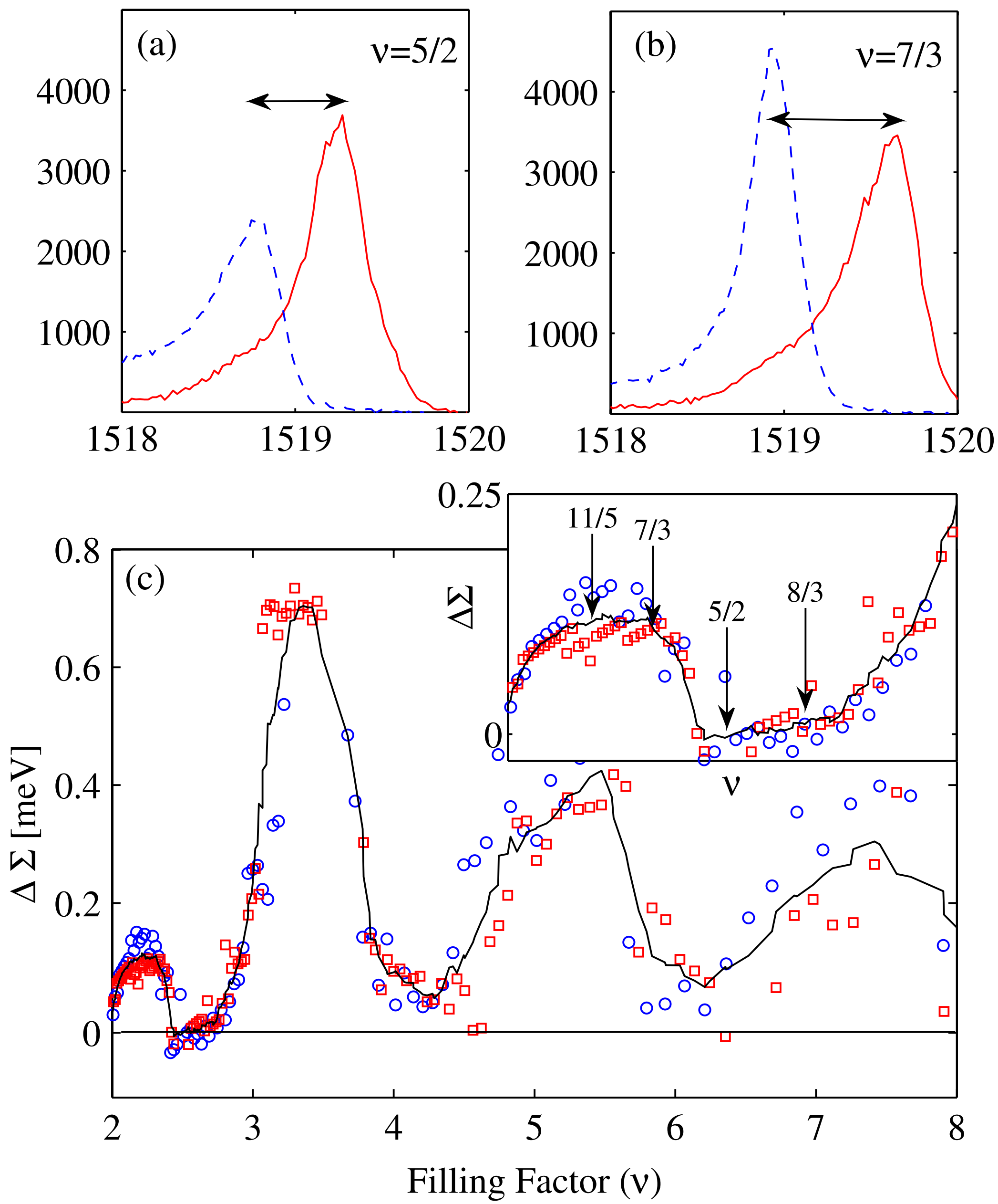
(Received 17 May 2010; published 23 August 2010)

We apply polarization resolved photoluminescence spectroscopy to measure the spin polarization of a two dimensional electron gas in perpendicular magnetic field. We find that the splitting between the σ^+ and σ^- polarizations exhibits a sharp drop at $\nu = 5/2$ and is equal to the bare Zeeman energy, which resembles the behavior at even filling factors. We show that this behavior is consistent with filling factor $\nu = 5/2$ being unpolarized.



this behavior is consistent with filling factor $\nu = 5/2$ being unpolarized

Adiabatic continuity between Pfaffian and Coulomb GS?

PRL **104**, 076803 (2010) PHYSICAL REVIEW LETTERS week ending 19 FEBRUARY 2010

Fractional Quantum Hall State at $\nu = \frac{5}{2}$ and the Moore-Read Pfaffian

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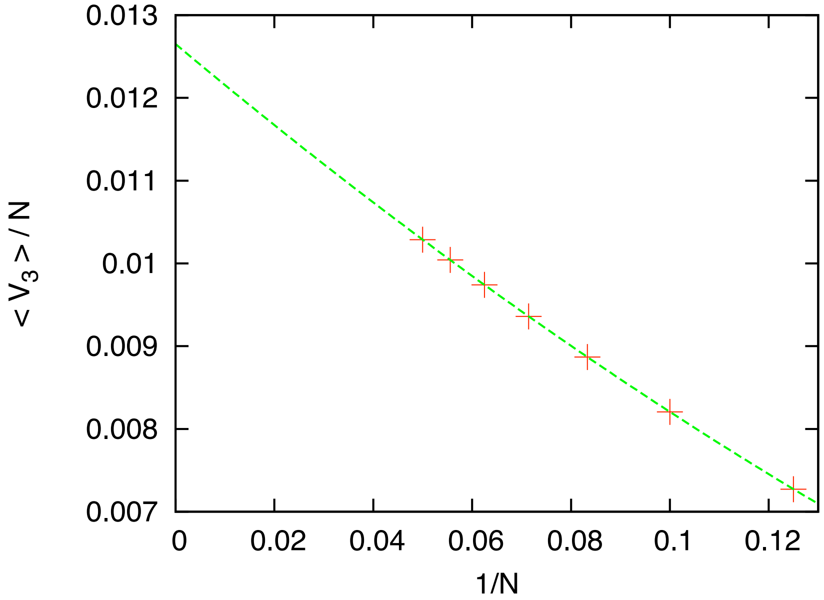
(Received 20 December 2008; published 18 February 2010)

Following Wen (1992): study system in the presence of a hypothetical interaction

$$V_{int} = (1-x)\,V_{Coulomb} + x\,\lambda V_{3body}\,, \qquad V_{3body} = N^{-5}\prod_{i<k<m}^{N} \mathcal{S}_{ikm}\left(\,\nabla_k^2\nabla_m^4\delta(z_i-z_k)\delta(z_i-z_m)\,\right)$$

V_{3body} is the interaction for which the Pfaffian ist the exact zero-energy GS.



$< V_{3body} > / N$ vs $(1/N)$ at $\nu = 1$.

V_{int} interpolates between Coulomb and this three-body interaction when x is varied from 0 to 1.

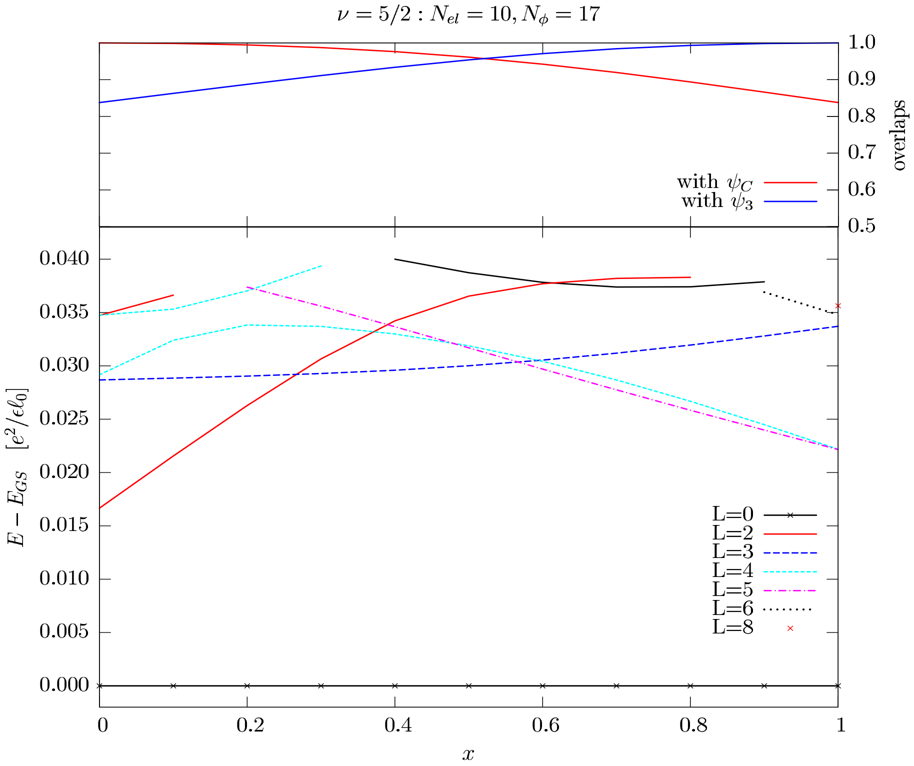
The parameter λ sets the energy scale of the 3-body interaction such that the gap at $x = 1$ coincides with the Coulomb gap in the second Landau level. It is independent of system size.

Adiabatic continuity between Pfaffian and Coulomb GS?

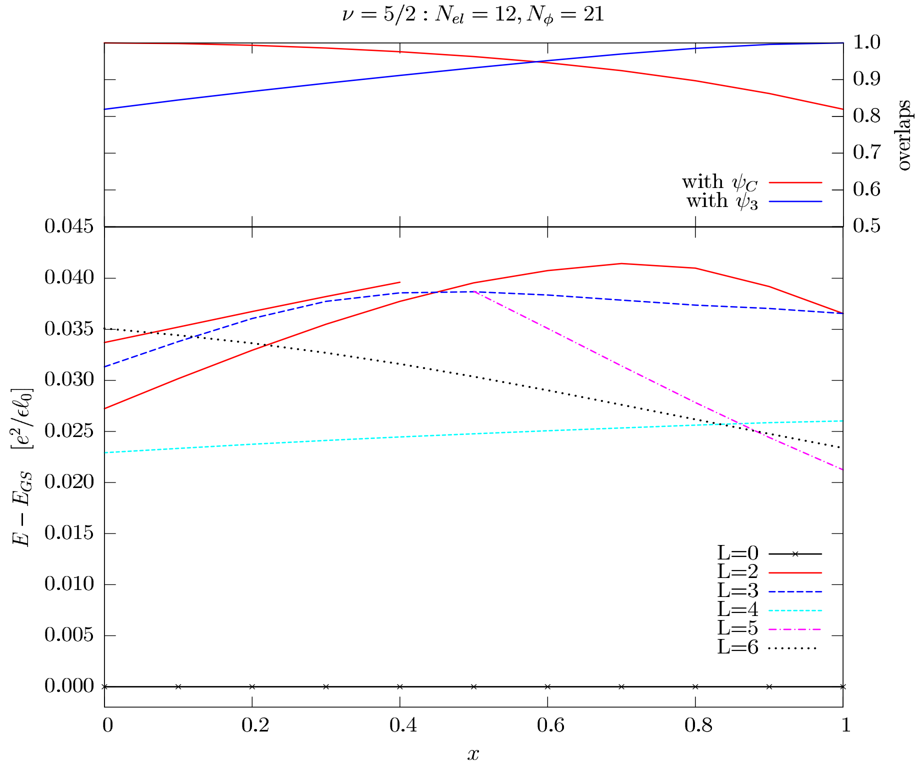
Study system in the presence of a hypothetical interaction

$$V_{int} = (1-x)\, V_{Coulomb} + x\, \lambda V_{3body}\,, \quad V_{3body} = N^{-5} \prod_{i<k<m}^{N} \mathcal{S}_{ikm}\left(\, \nabla_k^2 \nabla_m^4 \delta(z_i - z_k)\delta(z_i - z_m) \,\right)$$

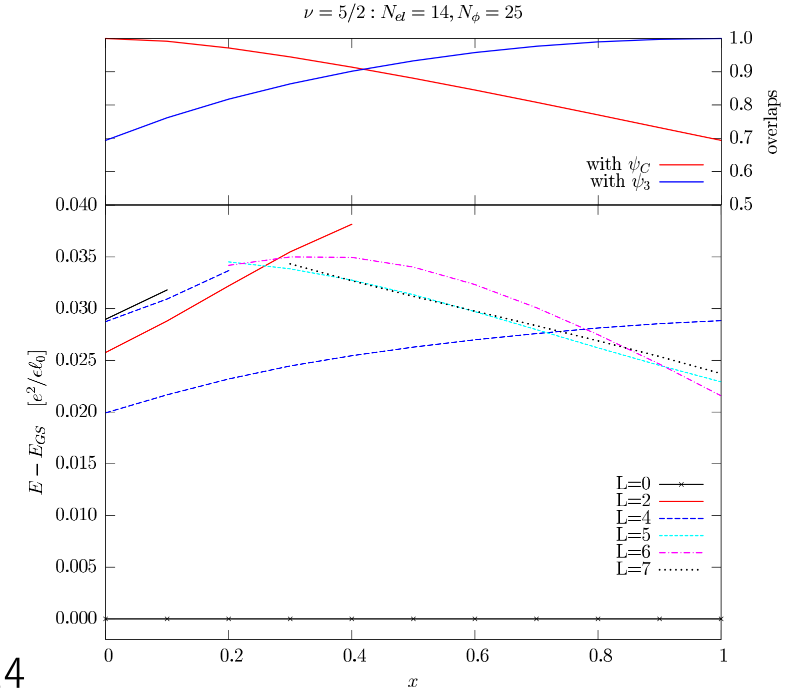
which interpolates between Coulomb and the three-body interaction when x is varied from 0 to 1. The parameter λ sets the energy scale of the 3-body interaction such that the gap at $x = 1$ coincides with the Coulomb gap in the second Landau level.



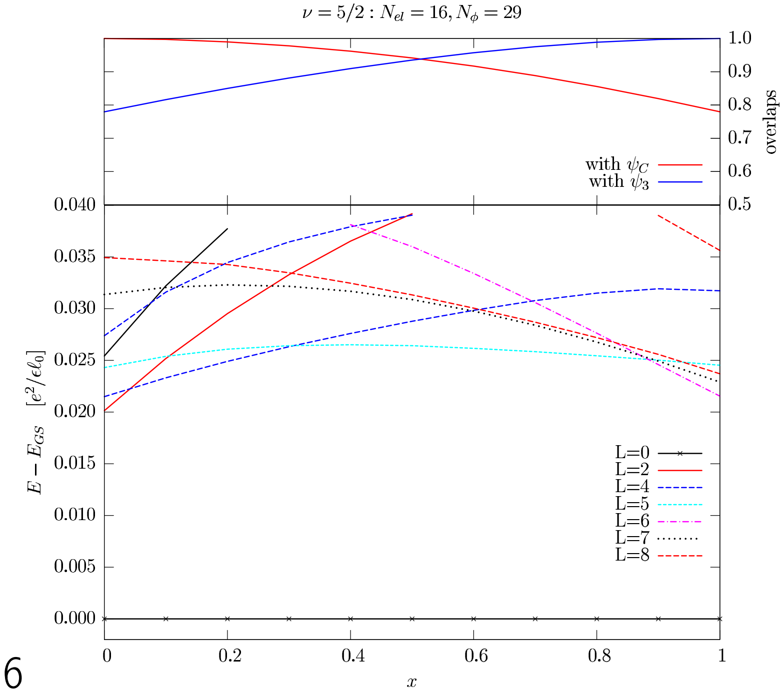
N=10



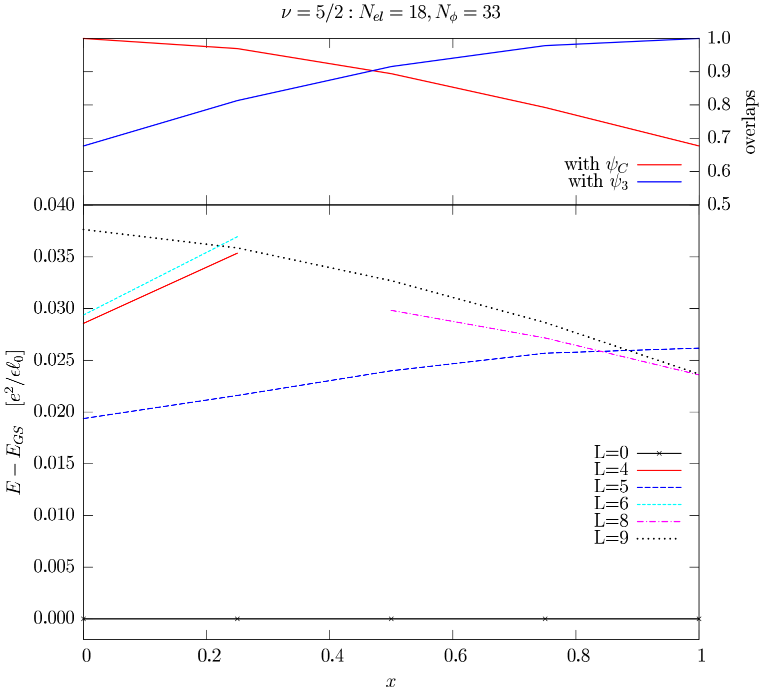
N=12



N=14

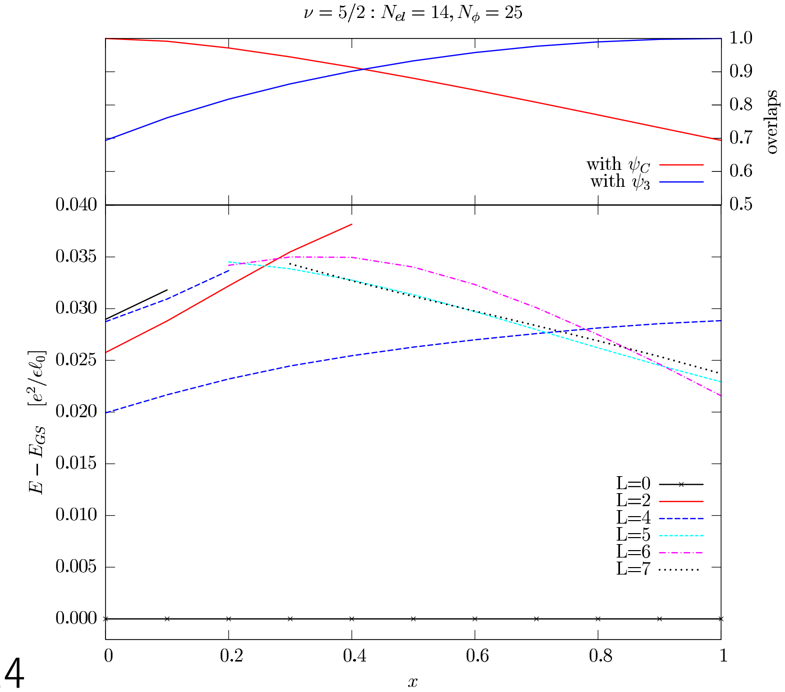


N=16

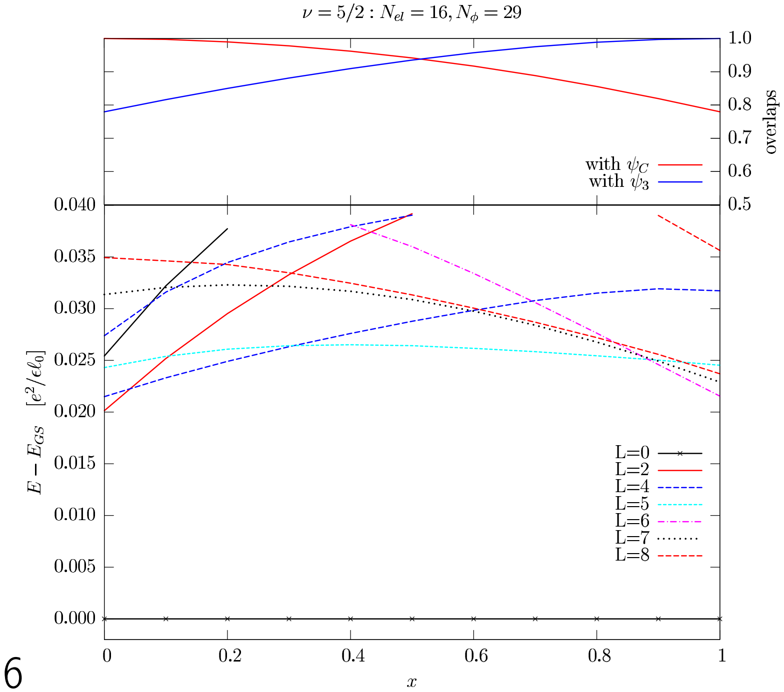


N=18

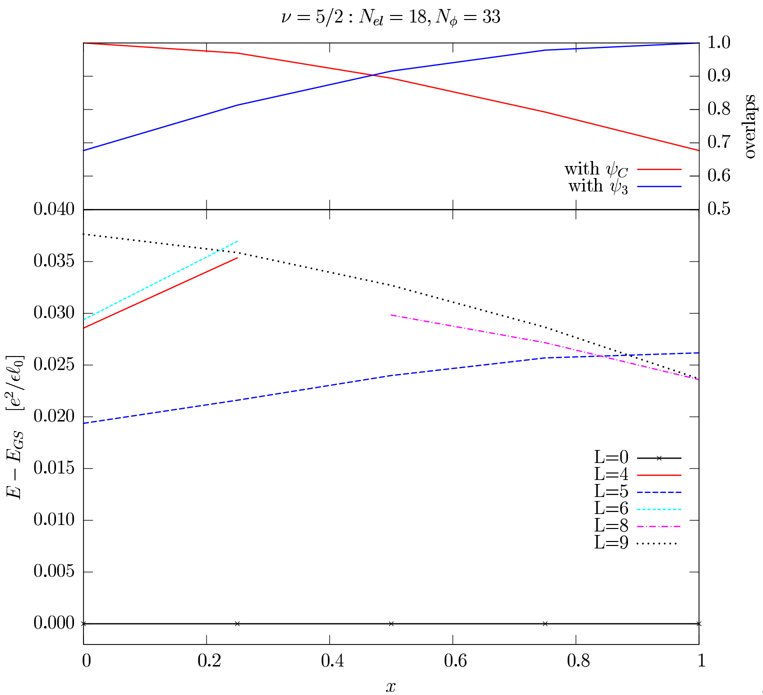
how does this compare to ν=1/3?



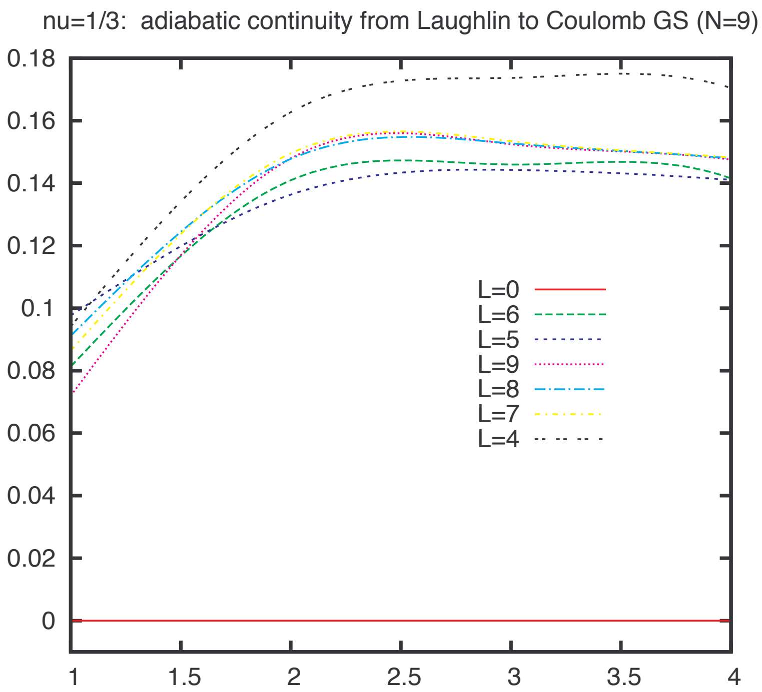
N=14



N=16

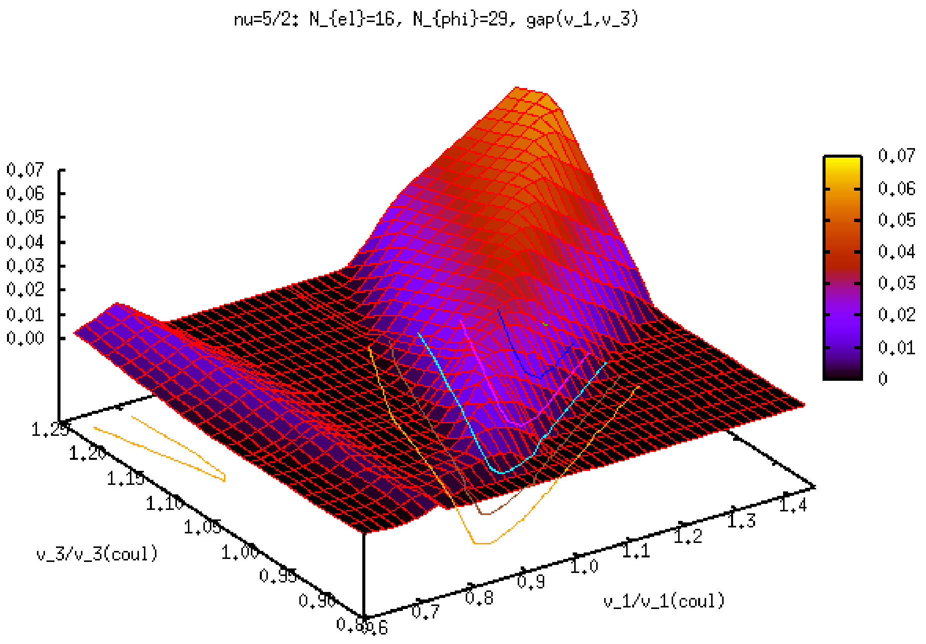


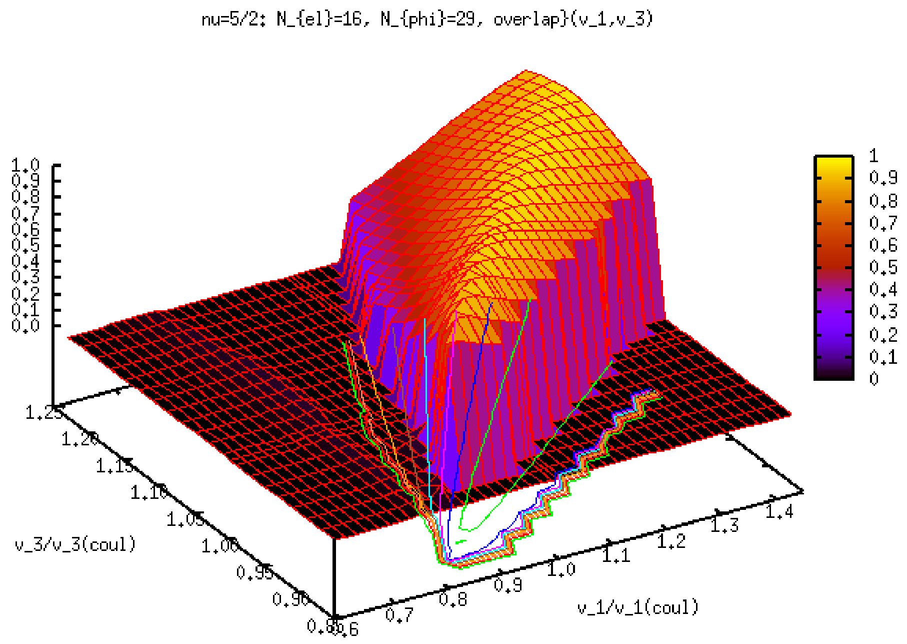
N=18



$\nu = 1/3$

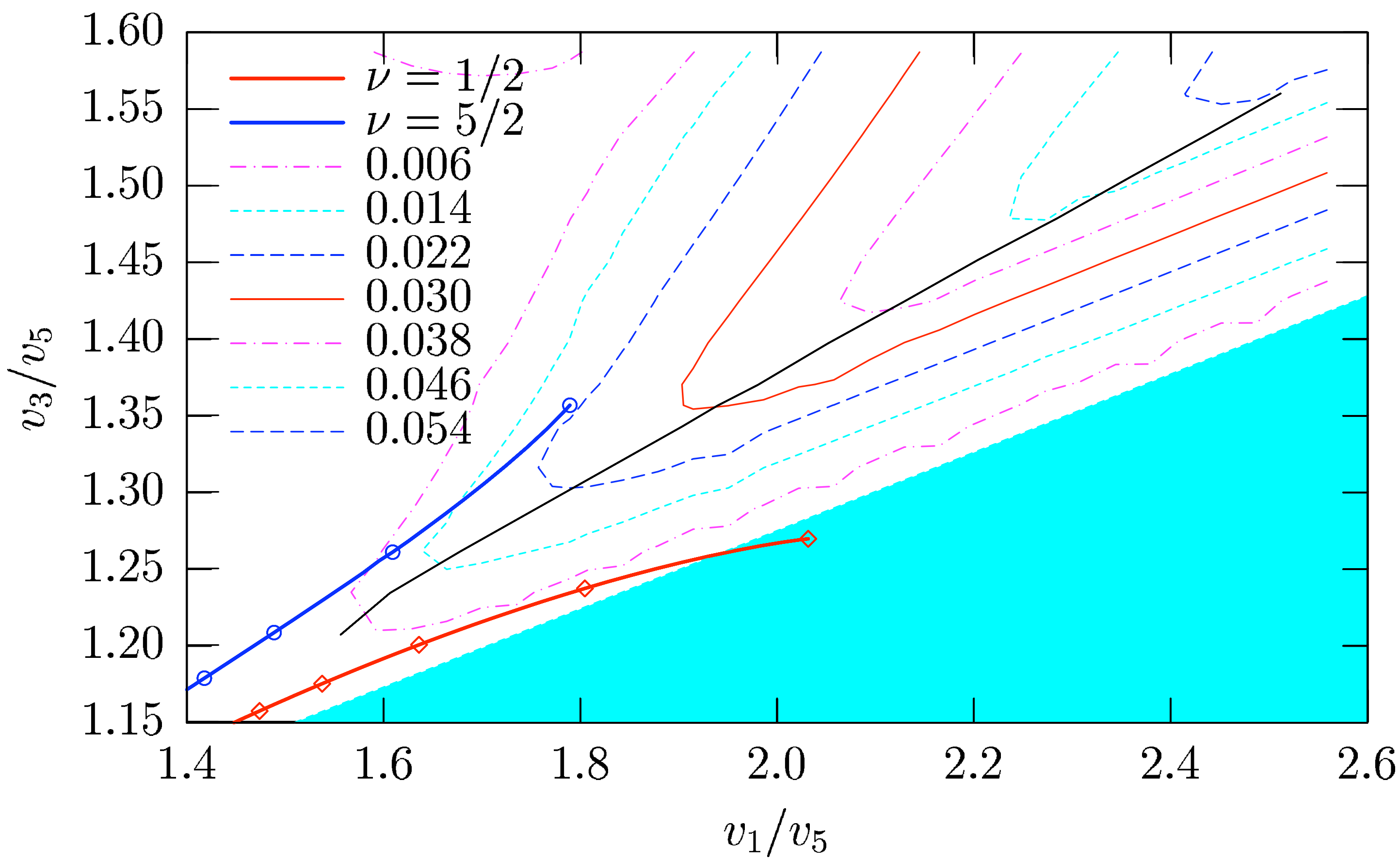
Phase diagram in the v_1, v_3-plane at N=16





Gapped phase coincides with (v_1, v_3)-domain of finite overlap between the GS(v_1, v_3) and the Pfaffian

Phase diagram in the v_1, v_3-plane at N=16



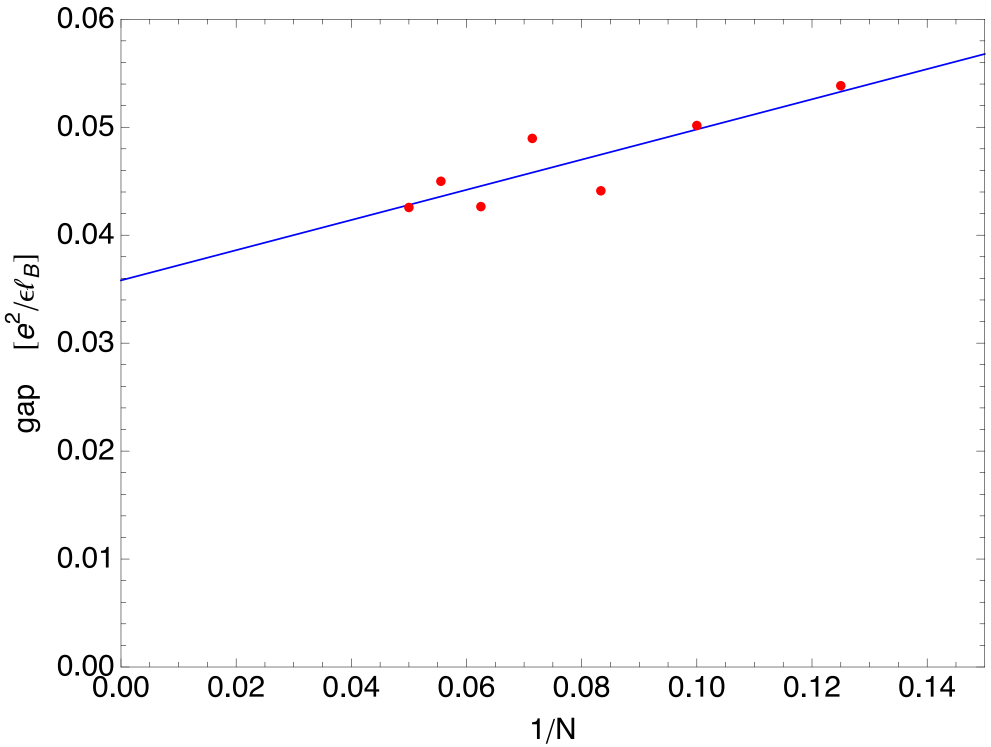
Blue (Red) curve denotes the physically accessible (v_1, v_3) points in lowest (second) Landau level when varying the finite width of the wf in the z-direction. The points refer to values $w/\ell_0 = 0, 1, 2, 3, 4$ starting right. The domain coloured in light blue is compressible.

Black solid line: maximum of overlap $< GS(v_1, v_3) \mid Pfaffian >$ is very close to maximum of gap

The red line referring to the (v_1, v_3)-values accessible at $\nu = 1/2$ are so close to the compressible domain that no definite conclusion can be reached on the existence of a Pfaffian phase at $\nu = 1/2$.

Energy gap at $\nu = 5/2$ in thermodynamic limit

computed from exciton with largest $L = N/2$

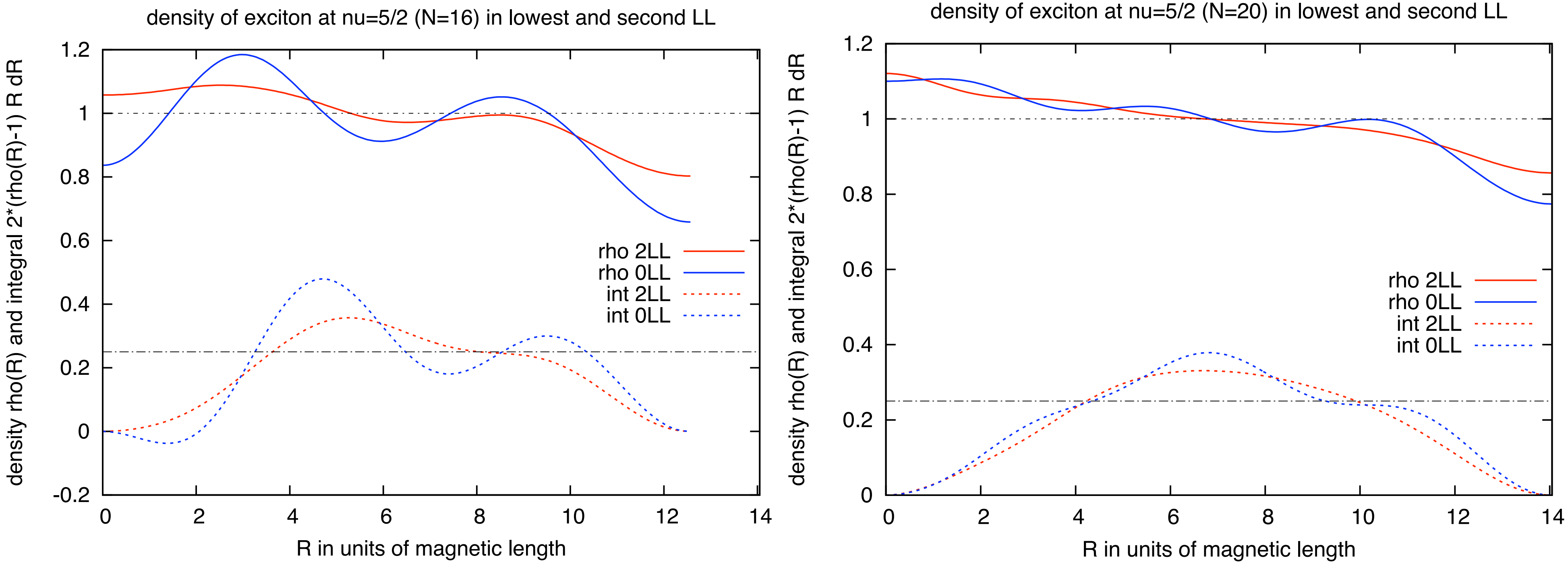


$$\Delta \approx 0.036 \pm\ 0.005\ e^2/\epsilon\ell_0$$

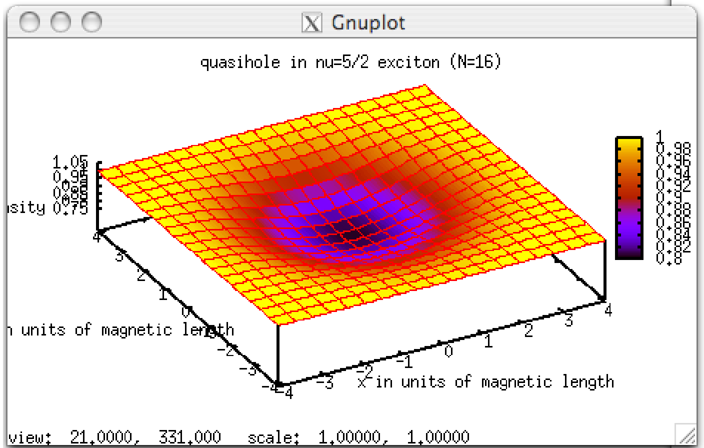
Charge density of exciton state

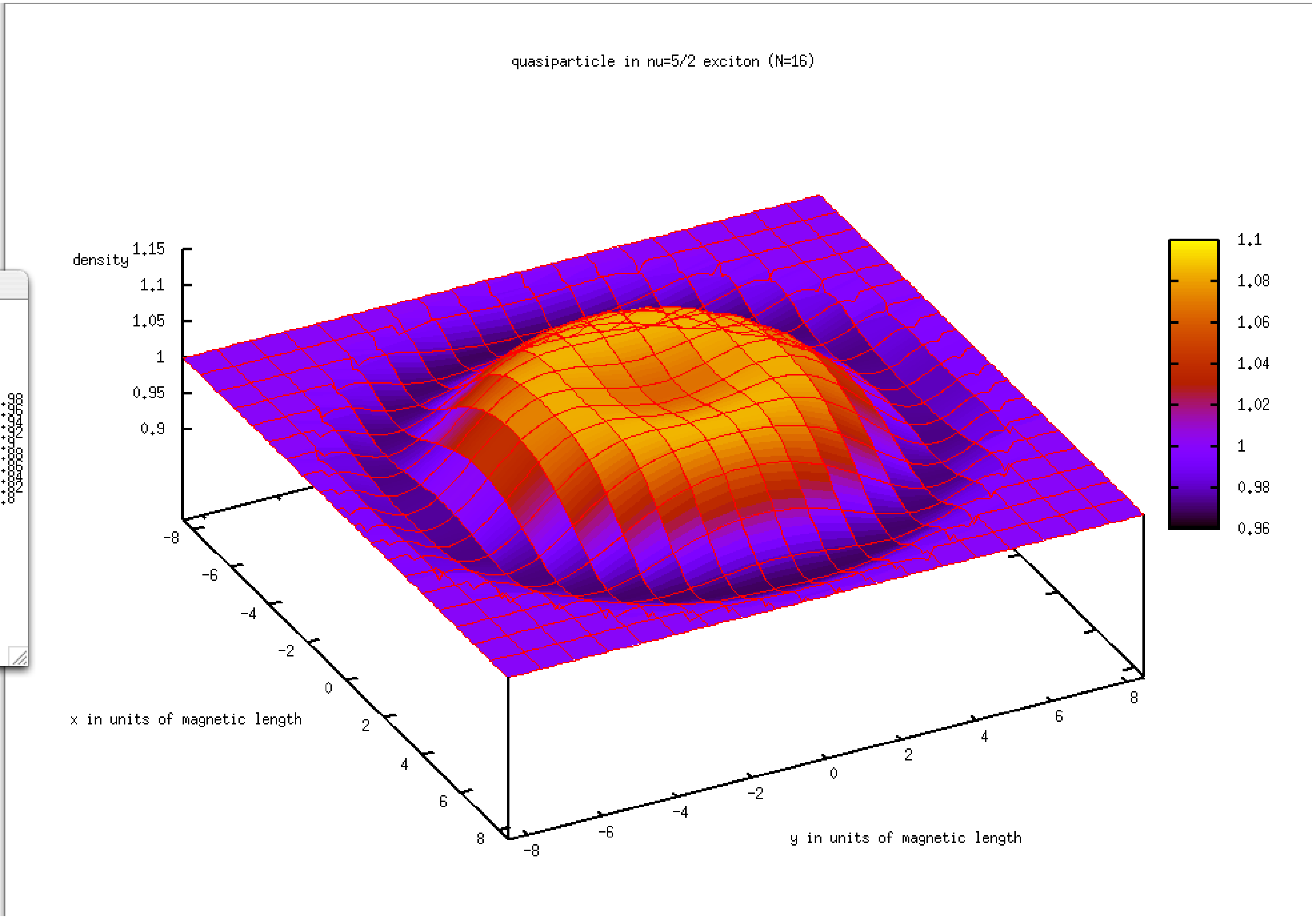
To localize qp and qh at the north and south pole, respectively:

Study exciton with maximum $L = N/2$ and $L_z = \pm L$



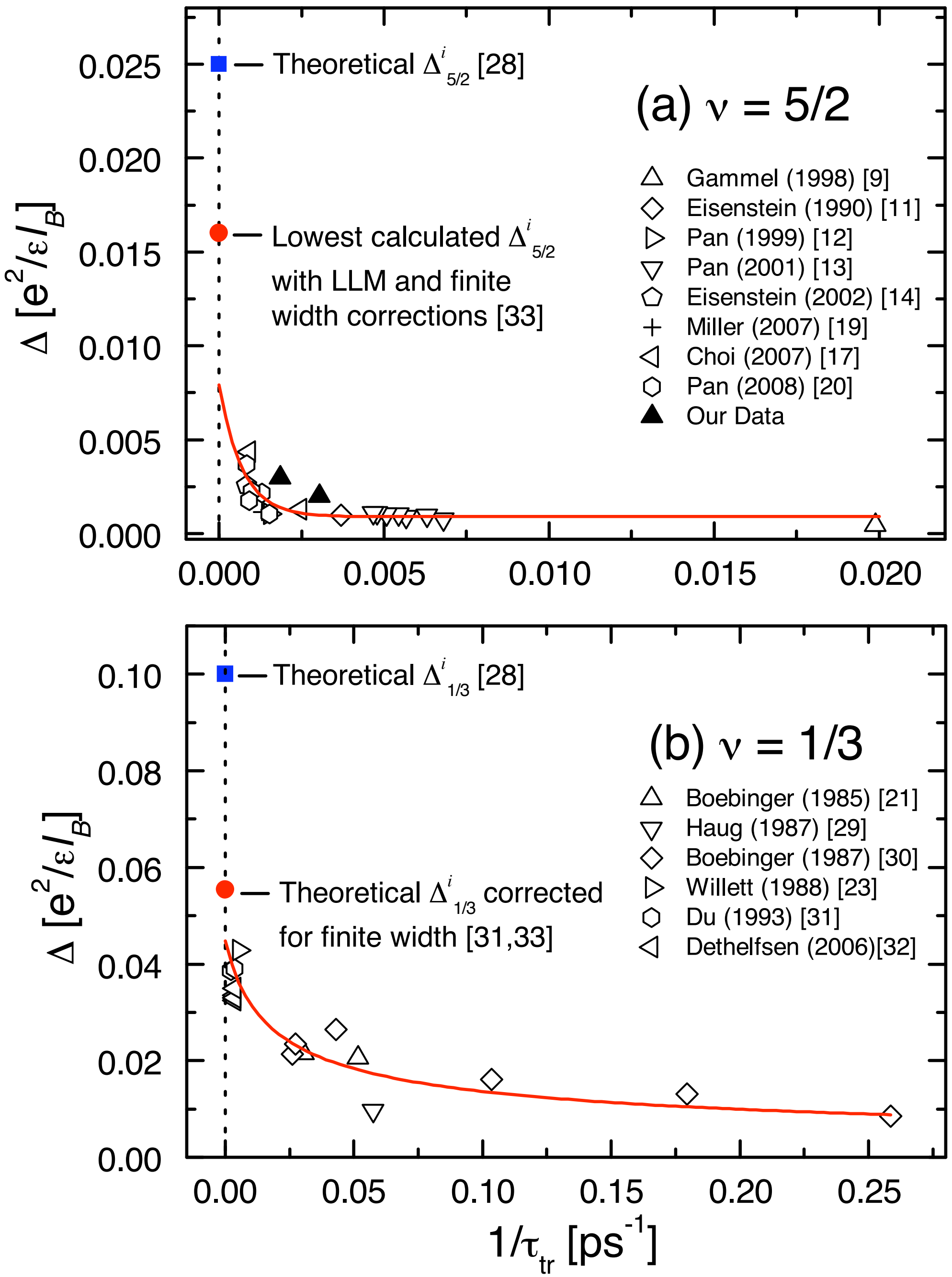
To get a better impression, let's look at them from "above"





Densities shown are the real ones in the second LL (not their lowest LL image).

Gaps at 5/2 from transport experiments

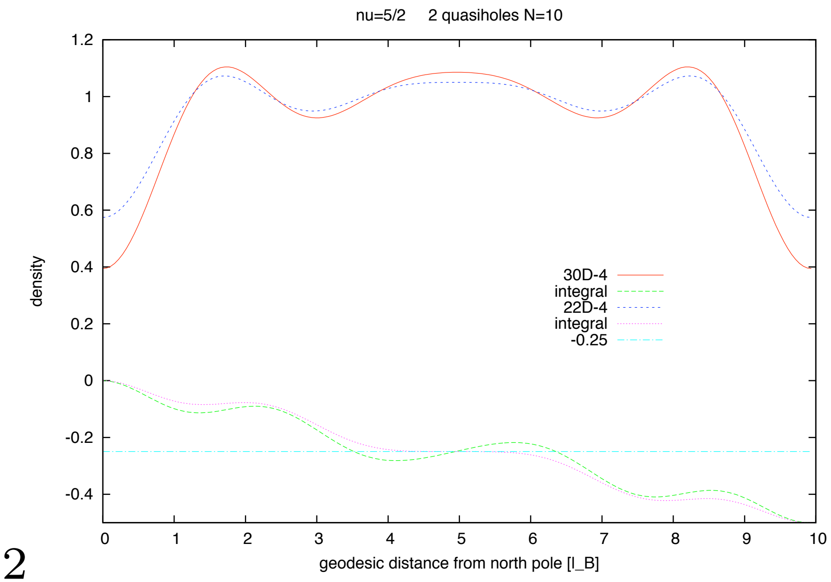


Note: Paper on Transport Gaps: N. d'Ambrumenil, B.I. Halperin, RHM, arXiv:1008.0969

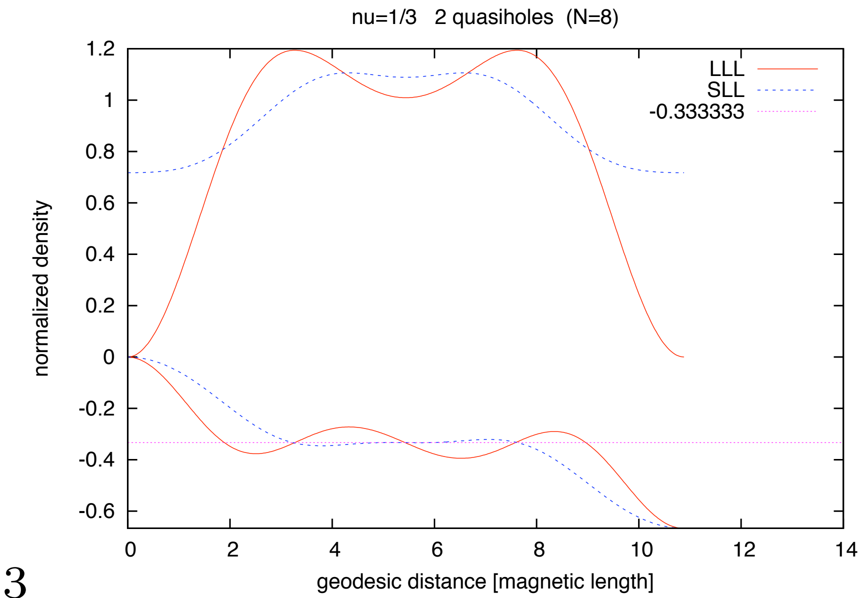
Charge density of 2 quasiholes at north and south pole

cf. Töke, Regnault and Jain, PRL 98, 036806 (2007)

Solution: Use δ-function localization potential which couples to charge and quadrupole moment of excitations



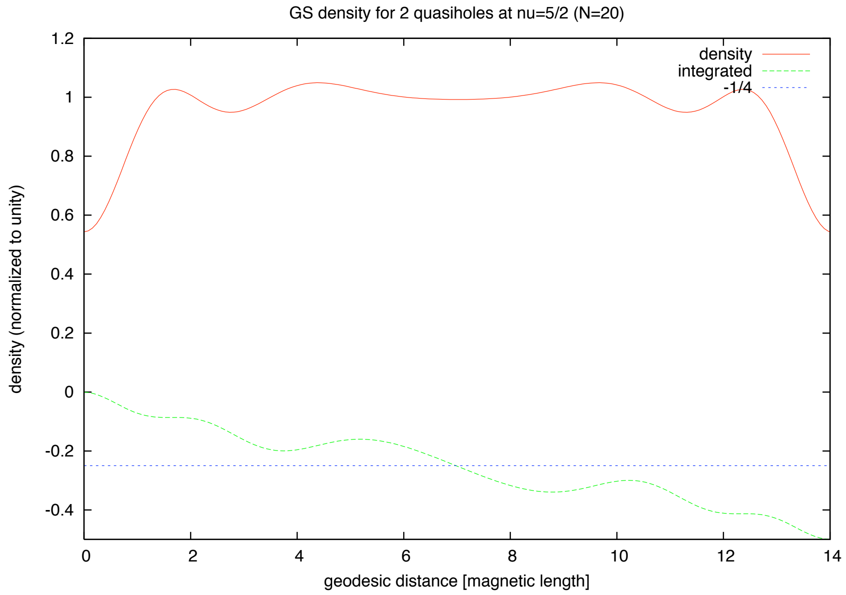
$\nu = 5/2$

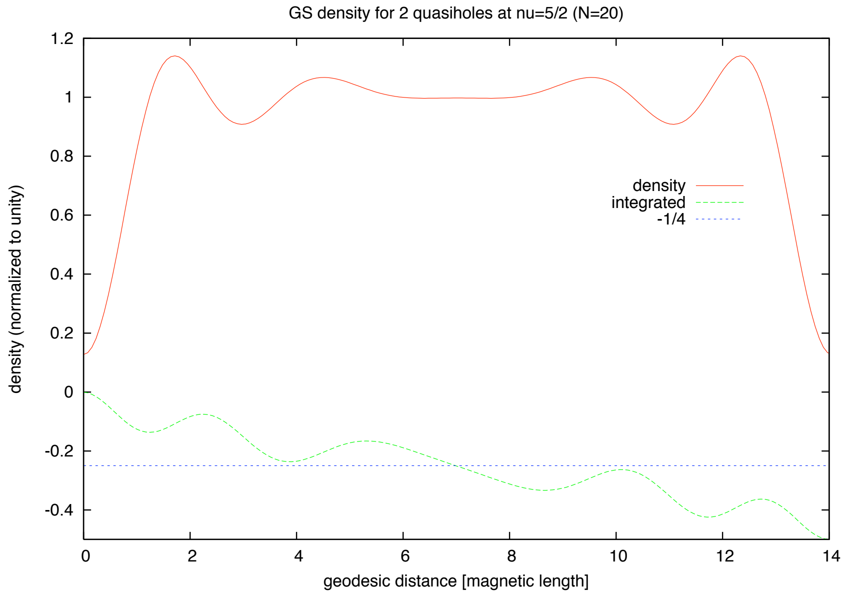


$\nu = 1/3$

In $\nu = 1/3$ system: a density 'plateau' forms where the missing charge 1/3 is observed

What about quasiholes in bigger systems?



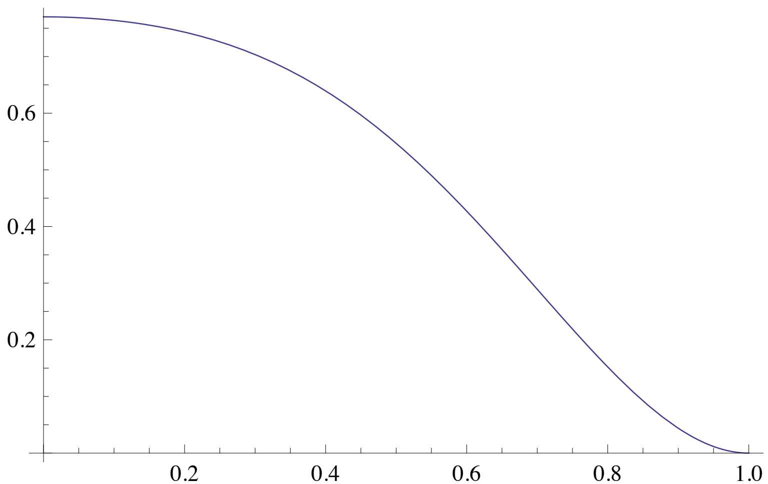


quasihole and quasiparticle are very large with area $A_{qh} \gtrapprox (5-8)\,2\pi\ell_0^2$ and $A_{qp} \gtrapprox (20-30)\,2\pi\ell_0^2$

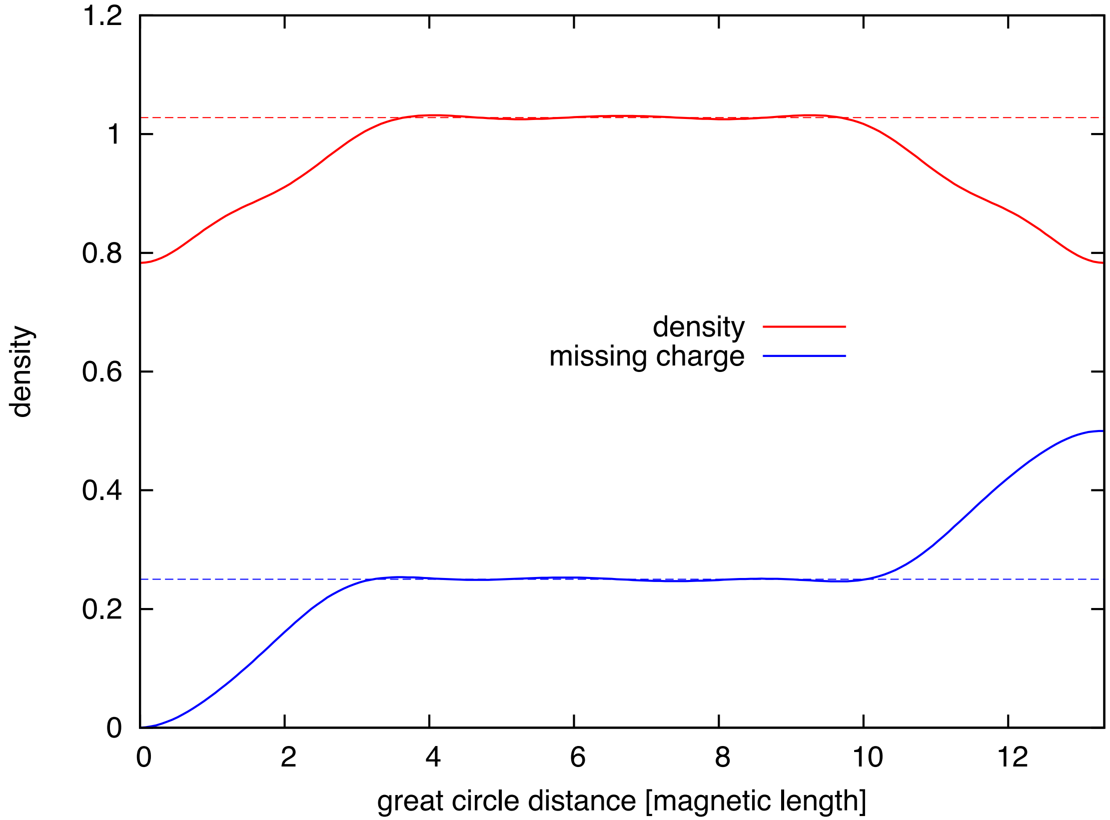
total area of sphere $A_{sphere} = (N/\nu)\,2\pi\ell_0^2 \implies$ for 2 quasiholes, we need at least $N \gtrsim 6$

Use tuned pinning potential F which couples to the electron density in the second Landau level for quasiholes at positions $\vec{R}_k$

$$F = \sum_{i,k} f(|\vec{r}_i - \vec{R}_k|)$$

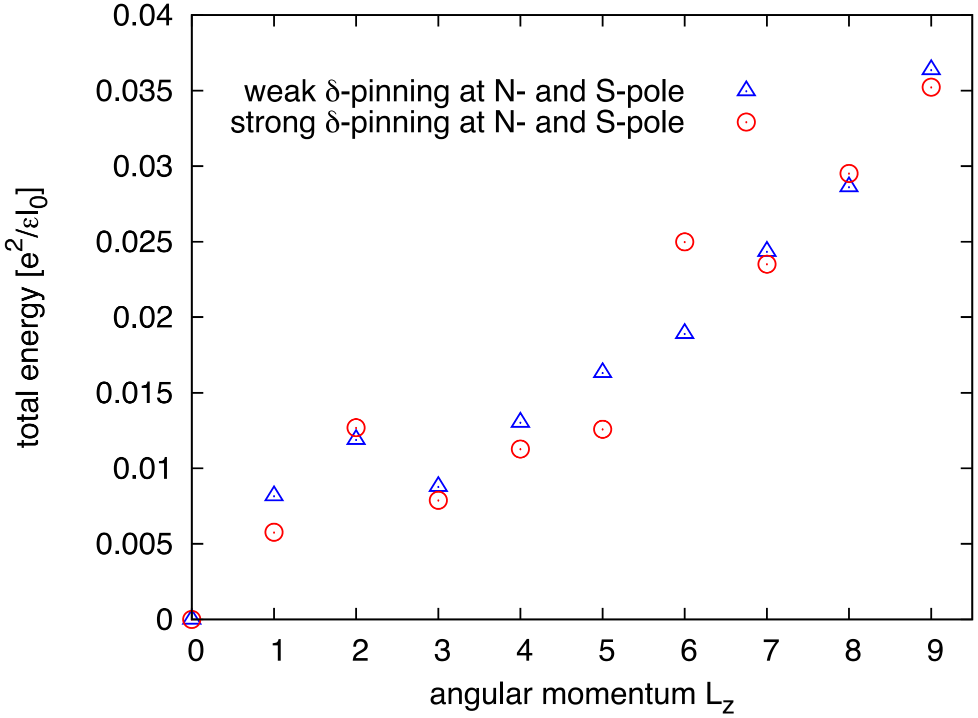


Pinning potential

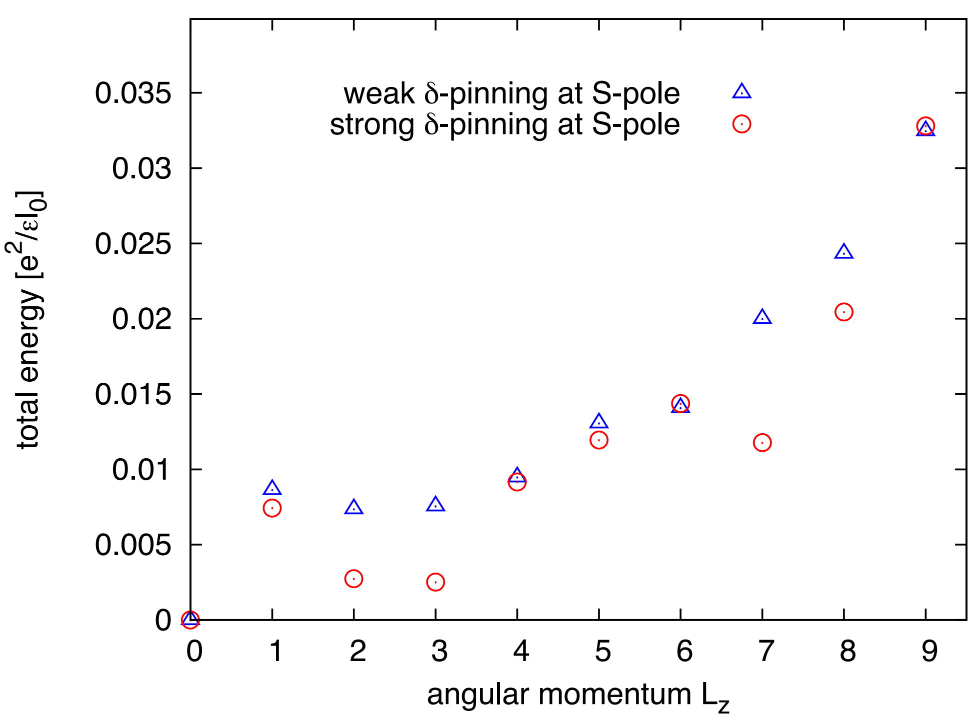


N=18 electrons

Energy spectrum of system with 2 quasiholes (N=16 electrons)



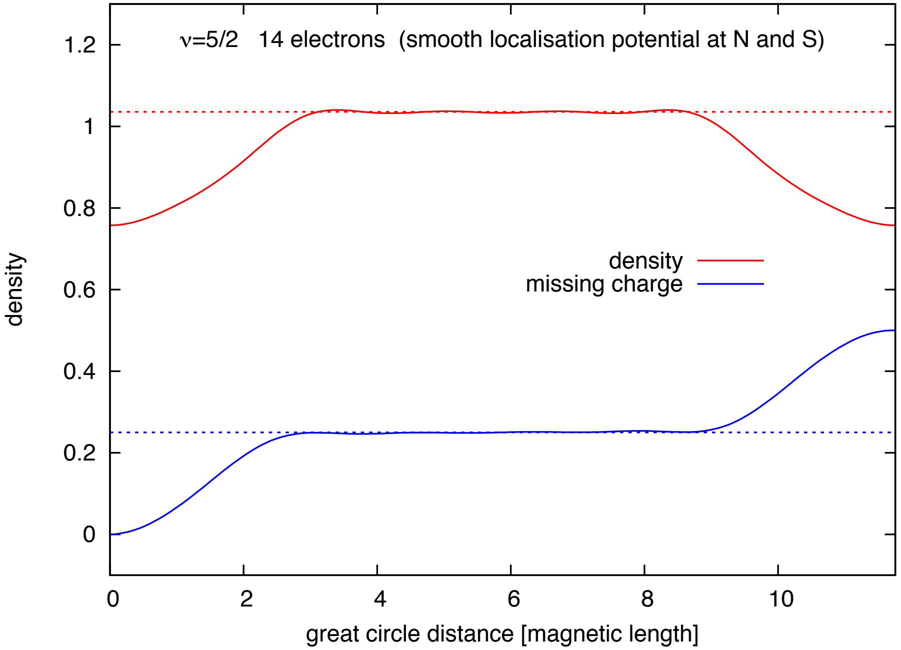
pinning at N- and S-pole

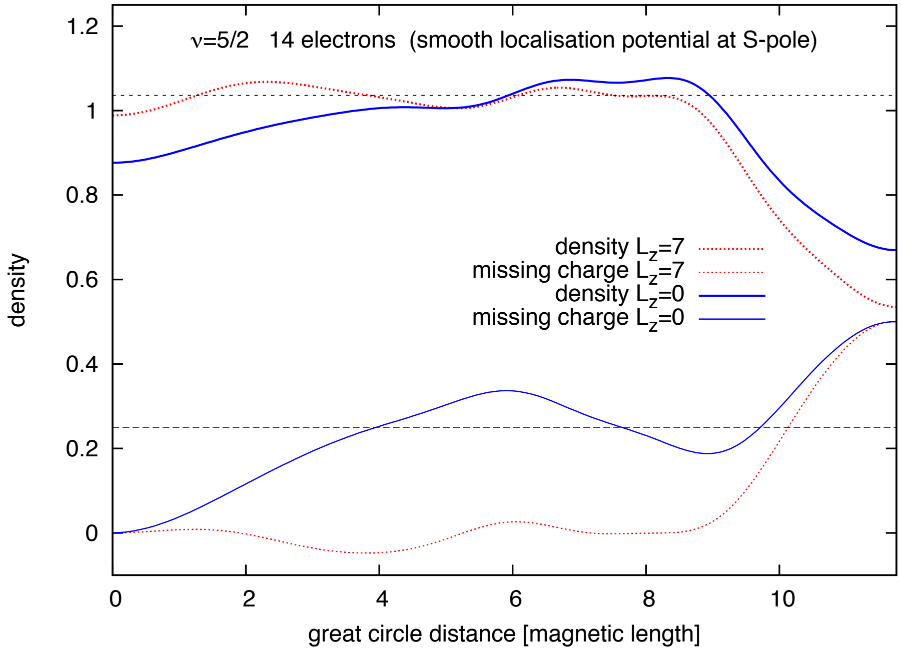


pinning only at S-pole

I all cases studied, we find that the GS occurs at $L_z = 0$ even if there is only a pinning at one pole. We do expect that for very large pinning potential at only one pole, there will be a transition to a GS with $L_z \neq 0$, see right panel.

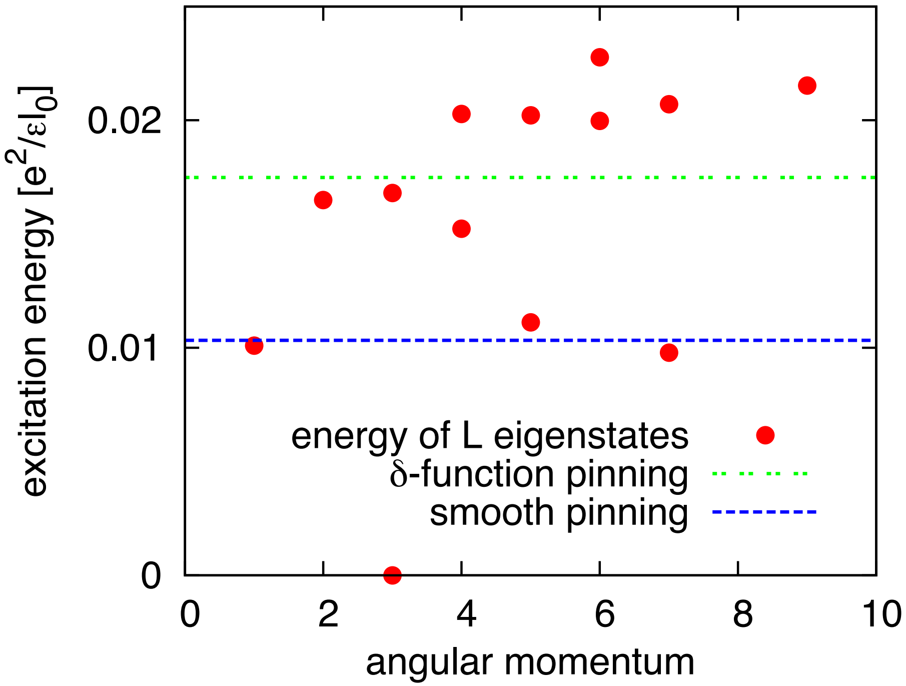
density of system of 2 pinned quasiholes





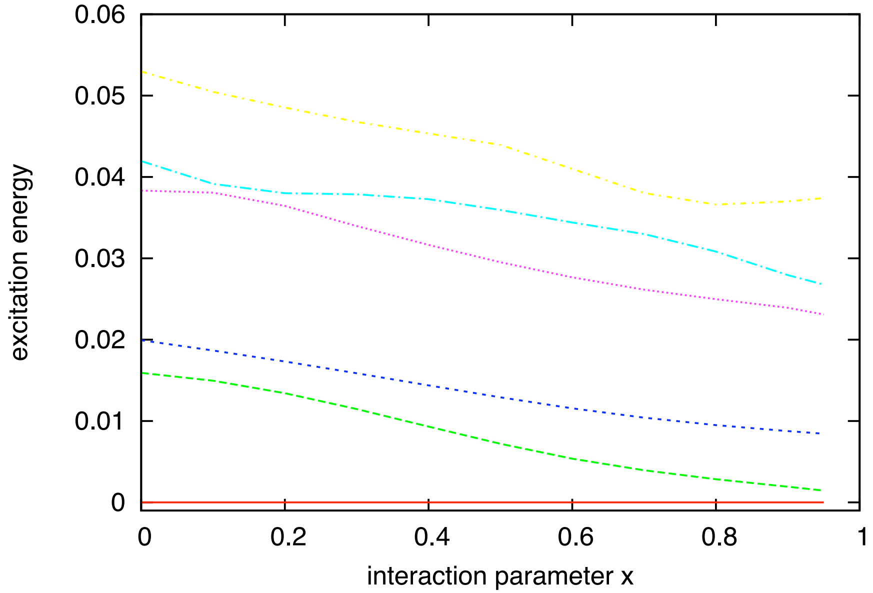
Single pinning potential at S-pole

Energy of L-Eigenstates of 2 quasiholes vs Coulomb energy of 2 pinned quasiholes

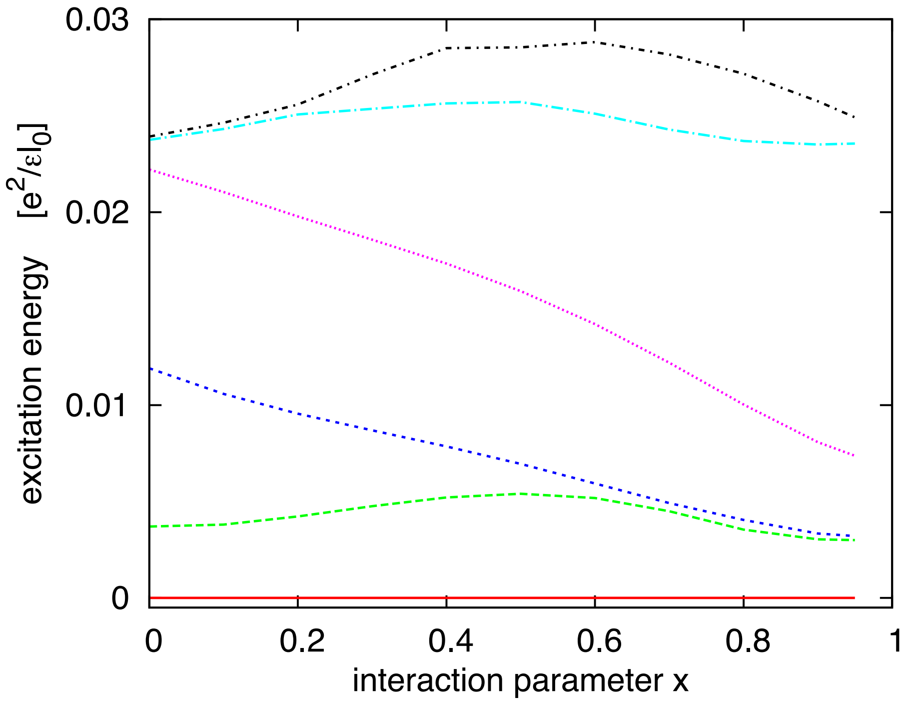


N=16 electrons with 2 quasiholes pinned at N- and S-pole

Adiabatic continuity of GS with 2 Quasiholes localized at north and south pole



N=8

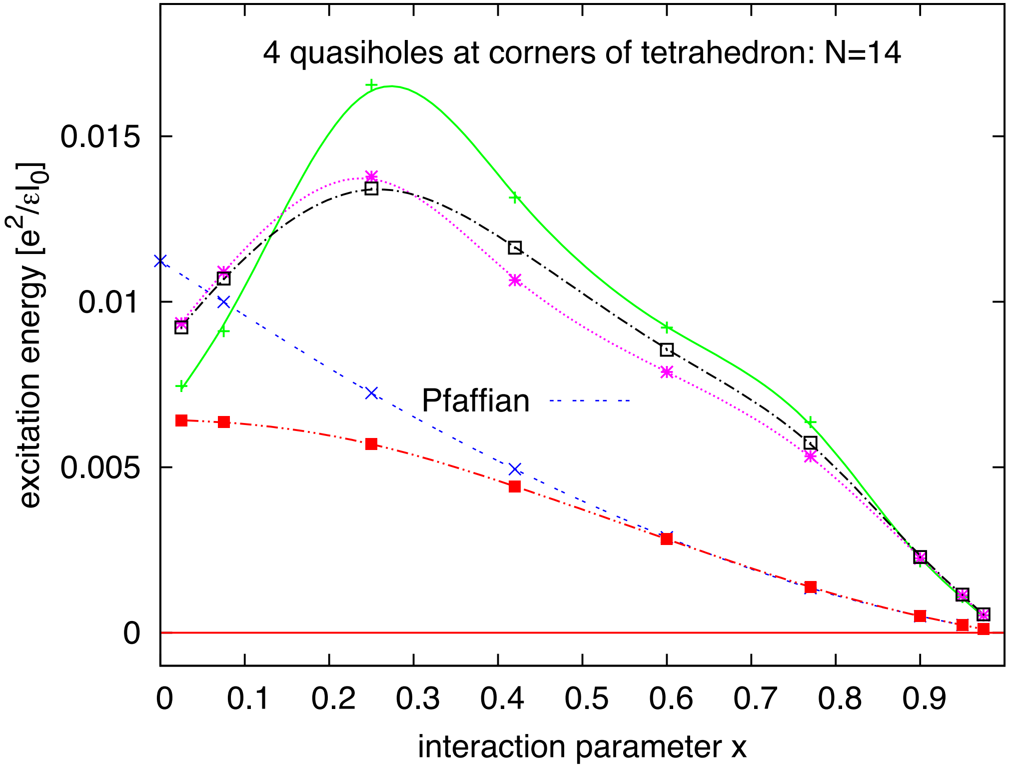


N=12

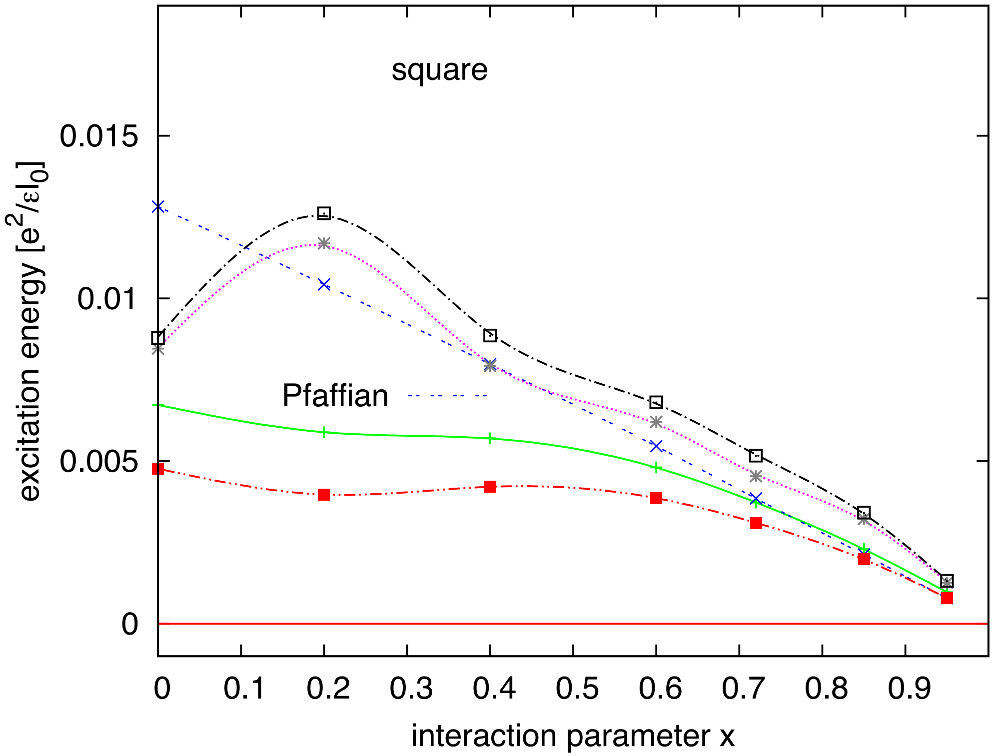
Ground state energy defines zero-energy reference. GS is protected by a small gap for $0 \leq x \leq 1$.

Adiabatic continuity of "non-abelian doublet" in system with 4 pinned Qholes

Note: To avoid the formation of double quasiholes with charge $\frac{e}{2}$ we use scaling of pinning potential like Coulomb interaction: $(1-x) \times F$



quasiholes on tetrahedral positions



quasiholes at corners of square

In both geometries, we observe that the lowest energy doublet (red lines) remains lowest for $0 < x < 1$.

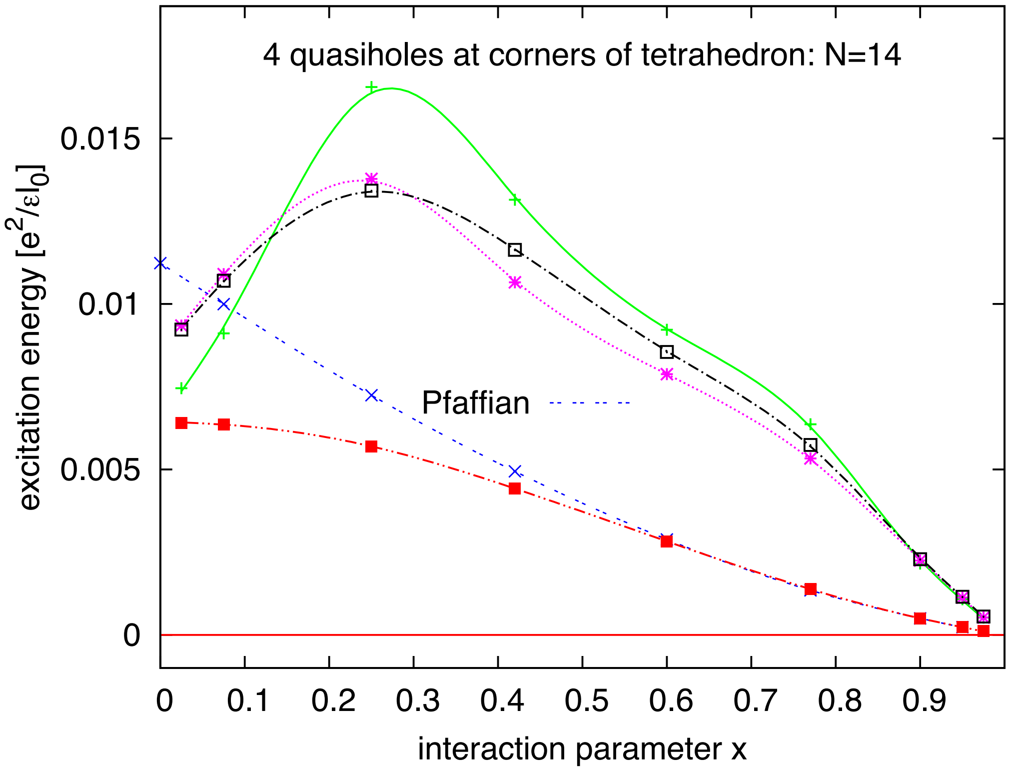
The label "Pfaffian" denotes the variational results for the splitting of the non-abelian doublet using MR-limit states as trial states (cf. Baraban et al. PRL 103, 076801 (2009)).

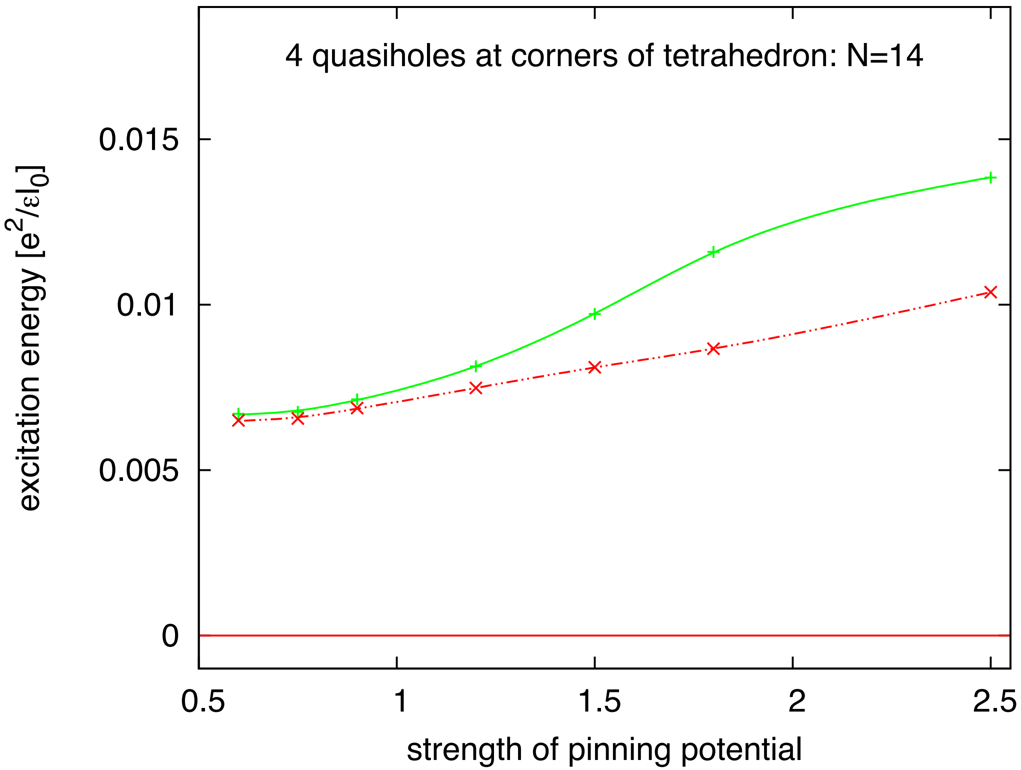
Adiabatic continuity of "non-abelian doublet" in system with 4 pinned Qholes

Note: Breaking rotation invariance leads to vastly larger Hilbert space dimension D and number of 3-body interaction matrix element

$$N = 14 \quad D \approx 4 \times 10^7 \quad N_c \approx 10^{11}$$

$$N = 16 \quad D \approx 6 \times 10^8 \quad N_c \approx 3 \times 10^{12}$$





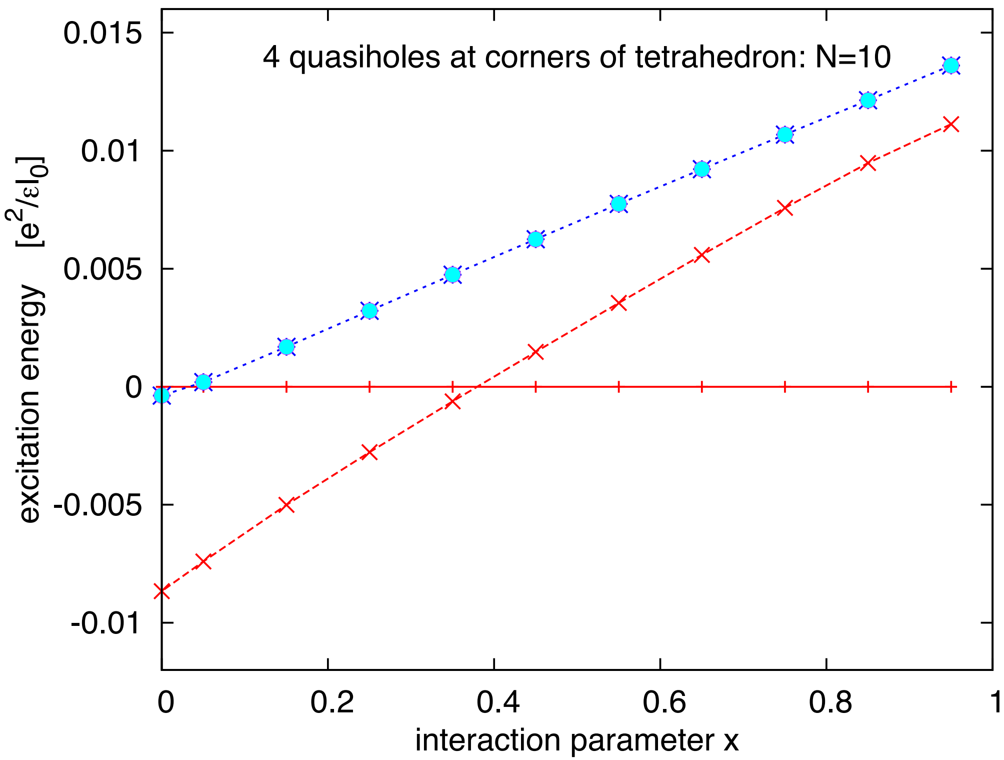
quasiholes on tetrahedral positions

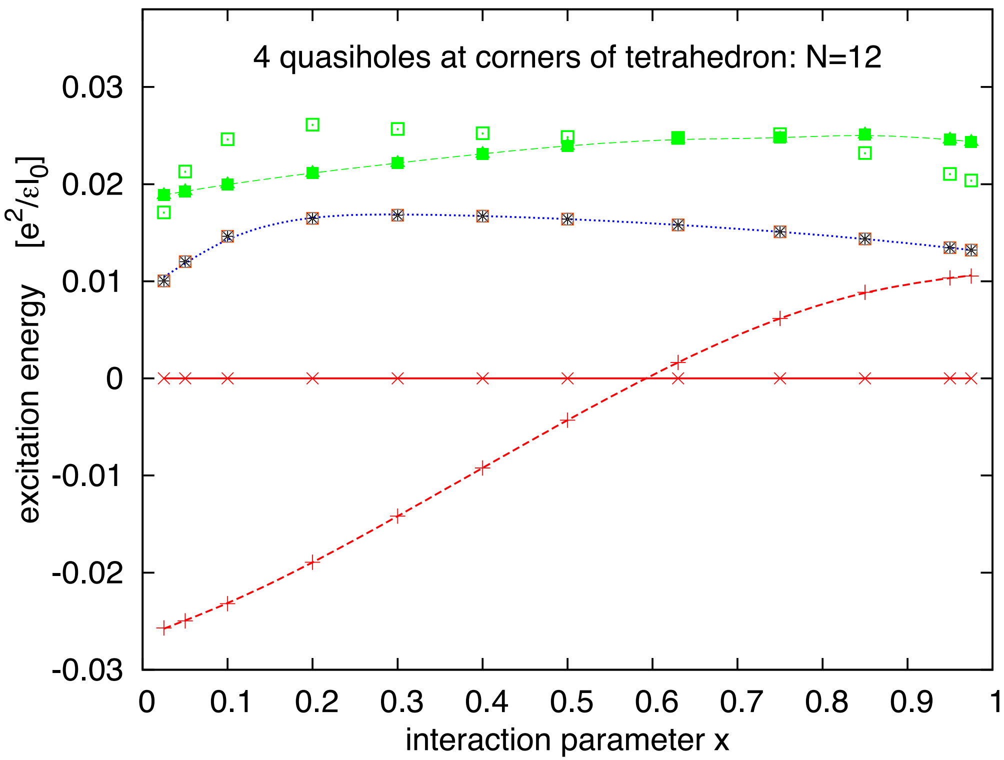
energy splittings vs. pinning strength for Coulomb interaction

How to approach MR-limit at $x = 1$ for finite pinning potential

Duncan Haldane: Use STM-tip with a quarter electron at position $\vec{R}_k$ as pinning potential:

$$F_H = c \times \frac{1}{4} \frac{e^2}{\epsilon \ell_0} \sum_{i,k} \frac{1}{|\vec{r}_i - \vec{R}_k|}$$





Low-lying spectrum for pinning potential F_H with $c = 1$. The energy of the state that becomes the GS at $x = 1$ is the reference zero energy. The red lines depict the lowest energy doublet. It has a finite splitting in the MR-limit because the pinning potential F_H mixes in states with non-zero energy in this limit. The third state (2nd excited state) is 3-fold degenerate and corresponds to an excitation in the pinning potential. We find that its energy separation from the doublet depends sensitively on the strength c of F_H (not shown).

Conclusions

This makes us hopeful:

- Adiabatic continuity at $\nu = 5/2$ between Pfaffian and Coulomb GS for all sizes studied ($N \leq$18).
- The gapped phase at $\nu = 5/2$ observed in the plane of pseudopontials v_1, v_3 coincides with the domain of non-zero overlap between the overlap of the GS(v_1, v_3) with the Pfaffian state.
- Maximum overlap between GS and Pfaffian essentially coincides with gap maximum when varying v_1 and keeping v_3 fixed.
- Adiabatic continuity at $\nu = 5/2$ between Pfaffian and Coulomb GS for 2 quasiholes.
- "Non-abelian doublet" in the MR-limit for 4 quasiholes remains lowest energy doublet for Coulomb interaction.

Open questions

- Pfaffian or Antipfaffian?
- where is the strong-paired phase?
- spin- and subband-mixing effects?
- tilted fields
- correlation length induced by Majorana fermion