Extended global symmetry of the Hubbard model on bipartite lattices

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The Hubbard model on bipartite lattices

$$\hat{H} = \hat{T} + \frac{U}{2}[N_a^D - \hat{Q}]$$

$$\hat{T} = -t \sum_{\langle \vec{r}_j \vec{r}_{j'} \rangle} \sum_{\sigma=\uparrow,\downarrow} [c^\dagger_{\vec{r}_j \sigma} c_{\vec{r}_{j'} \sigma} + h.c.]$$

$$\hat{Q} = \sum_{j=1}^{N_a^D} \sum_{\sigma=\uparrow,\downarrow} \hat{n}_{\vec{r}_j \sigma} (1 - \hat{n}_{\vec{r}_j -\sigma})$$

The global symmetry of the model is known to contain two SU(2) symmetries

OJ Heilmann, EH Lieb, Ann. NY Acad. Sci. 172, 583 (1973)

EH Lieb, Phys. Rev. Lett. 62, 1201 (1989)

A trivial result is that at U=0 the Hubbard model global symmetry is:

$$O(4) = SO(4) \times Z_2$$

Here the Z2 factor is associated with the Shiba particle-hole symmetry on a single spin under which the interacting Hamiltonian term is not invariant. In turn, the SO(4) symmetry survives at U>0.

It may be written as SO(4) = [SU(2)xSU(2)]/Z_2. Here the Z_2 denominator implies that, in contrast to the group SU(2)xSU(2), only state representations for which

$$(S_s + S_\eta)$$

is an integer number are allowed. The corresponding projections read:

$$S_s^z = -\frac{1}{2}(N_\uparrow - N_\downarrow)\,; \quad S_\eta^z = -\frac{1}{2}(N_a^D - N)$$

The question is then whether for U>0 the symmetry of the Hubbard model on bipartite lattices is SO(4) or higher. CN Yang and SC Zhang considered it to be only SO(4).

CN Yang, SC Zhang, Mod. Phys. B 4, 759 (1990)

SC Zhang, Phys. Rev. Lett. 65, 120 (1990)

The model is one of the most studied many-particle quantum problems, yet except in one-dimension it has no exact solution. Any new exact result for instance concerning its global symmetry is of interest for the further understanding of the effects of correlations in several systems. Indeed, symmetry plays an important role in physics and often can be used to extract useful information on unsolved non-perturbative quantum problems.

Here we briefly report the result of a recent study, according to which there is a hidden U(1) symmetry beyond SO(4), so that for U>0 the global symmetry of the Hubbard model on bipartite lattices is higher and given by:

$$[SO(4) \times U(1)]/Z_2 = SO(3) \times SO(3) \times U(1) = [SU(2) \times SU(2) \times U(1)]/Z_2^2$$

The $Z_2 x Z_2$ denominator in the latter expression implies that, in contrast to the group SU(2)xSU(2)xU(1), only state representations for which

$$(S_s + S_\eta + S_c)$$

is an integer number are allowed. Here Sc is a quantum number associated with the new found U(1) hidden symmetry whose physical meaning is given below. Its allowed values are zero and all positive half-odd integer and integer numbers.

The derivation of this extended global symmetry is presented in:

JMPC, Stellan Östlund, MJ Sampaio, Ann. Phys. 325, 1550 (2010)

The local SU(2)xSU(2)xU(1) gauge symmetry for U>0 and t=0

The new found global

$$[SO(4) \times U(1)]/Z_2 = SO(3) \times SO(3) \times U(1) = [SU(2) \times SU(2) \times U(1)]/Z_2^2$$

symmetry is closely related to the local SU(2)xSU(2)xU(1) gauge symmetry of the model for U>0 and t=0 reported in:

Stellan Östlund, Eugene Mele, Phys. Rev. B 44, 12413 (1991)

The seven local generators of such a SU(2)xSU(2)xU(1) gauge symmetry read:

$$\begin{aligned}
\hat{S}^z_{\eta,j} &= -\frac{1}{2}(1 - \hat{n}_{\vec{r}_j\uparrow} - \hat{n}_{\vec{r}_j\downarrow})\,; \quad \hat{S}^z_{s,j} = -\frac{1}{2}(\hat{n}_{\vec{r}_j\uparrow} - \hat{n}_{\vec{r}_j\downarrow}) \\
\hat{S}^\dagger_{\eta,j} &= e^{i\vec{\pi}\cdot\vec{r}_j}\, c^\dagger_{\vec{r}_j\downarrow}\, c^\dagger_{\vec{r}_j\uparrow}\,; \quad \hat{S}_{\eta,j} = e^{-i\vec{\pi}\cdot\vec{r}_j}\, c_{\vec{r}_j\uparrow}\, c_{\vec{r}_j\downarrow} \\
\hat{S}^\dagger_{s,j} &= c^\dagger_{\vec{r}_j\downarrow}\, c_{\vec{r}_j\uparrow}\,; \quad \hat{S}_{s,j} = c^\dagger_{\vec{r}_j\uparrow}\, c_{j,\downarrow} \\
\hat{S}_{c,j} &= \frac{1}{2}\sum_{\sigma=\uparrow,\downarrow} \hat{n}_{\vec{r}_j\sigma}\,(1 - \hat{n}_{\vec{r}_j-\sigma})\,, \quad j = 1, ..., N_a^D
\end{aligned}$$

The six generators of the global SO(4)=[SU(2)xSU(2)]/Z2 symmetry are simply obtained from those of the corresponding local SU(2)xSU(2) gauge symmetry by summing over the number of lattice sites:

$$\hat{S}^z_\eta = \sum_{j=1}^{N_a^D} \hat{S}^z_{\eta,j} = -\frac{1}{2}[N_a^D - \hat{N}]\,; \quad \hat{S}^z_s = \sum_{j=1}^{N_a^D} \hat{S}^z_{s,j} = -\frac{1}{2}[\hat{N}_\uparrow - \hat{N}_\downarrow]$$

$$\hat{S}^\dagger_\eta = \sum_{j=1}^{N_a^D} \hat{S}^\dagger_{\eta,j} = \sum_{j=1}^{N_a^D} e^{i\vec{\pi}\cdot\vec{r}_j}\, c^\dagger_{\vec{r}_j\downarrow}\, c^\dagger_{\vec{r}_j\uparrow}\,; \quad \hat{S}_\eta = \sum_{j=1}^{N_a^D} \hat{S}_{\eta,j} = \sum_{j=1}^{N_a^D} e^{-i\vec{\pi}\cdot\vec{r}_j}\, c_{\vec{r}_j\uparrow}\, c_{\vec{r}_j\downarrow}$$

$$\hat{S}^\dagger_s = \sum_{j=1}^{N_a^D} \hat{S}^\dagger_{s,j} = \sum_{j=1}^{N_a^D} c^\dagger_{\vec{r}_j\downarrow}\, c_{\vec{r}_j\uparrow}\,; \quad \hat{S}_{s,} = \sum_{j=1}^{N_a^D} \hat{S}_{s,j} = \sum_{j=1}^{N_a^D} c^\dagger_{\vec{r}_j\uparrow}\, c_{\vec{r}_j\downarrow}$$

However, for finite U/t values the operator obtained by summing over the number of lattice sites the local U(1) gauge symmetry generator,

$$\frac{1}{2}\hat{Q} = \sum_{j=1}^{N_a^D} \hat{S}_{c,j} = \frac{1}{2}\sum_{j=1}^{N_a^D}\sum_{\sigma=\uparrow,\downarrow} \hat{n}_{\vec{r}_j\sigma}\,(1-\hat{n}_{\vec{r}_j-\sigma})$$

does not commute with the Hamiltonian.

Does this then imply that the global symmetry is indeed SO(4)? Not necessarily.

The electron – rotated-electron unitary transformations

There is an infinite number of unitary transformations that map electrons onto rotated electrons, such that for rotated electrons double and single occupancy are good quantum numbers for U>0:

$$\tilde{c}^\dagger_{\vec{r}_j,\sigma} = \hat{V}^\dagger \, c^\dagger_{\vec{r}_j,\sigma} \, \hat{V} \, ; \qquad \tilde{c}_{\vec{r}_j,\sigma} = \hat{V}^\dagger \, c_{\vec{r}_j,\sigma} \, \hat{V}$$

J Stein, J. Stat. Phys. 88, 487 (1997)

An important property is that for any of such transformations the unitary operator expression only involves the following three kinetic operators,

$$\hat{T}_0 = - \sum_{\langle \vec{r}_j \vec{r}_{j'} \rangle} \sum_{\sigma} \hat{n}_{\vec{r}_j,-\sigma} \, c^\dagger_{\vec{r}_j,\sigma} \, c_{\vec{r}_{j'},\sigma} \, \hat{n}_{\vec{r}_{j'},-\sigma} + (1 - \hat{n}_{\vec{r}_j,-\sigma}) \, c^\dagger_{\vec{r}_j,\sigma} \, c_{\vec{r}_{j'},\sigma} \, (1 - \hat{n}_{\vec{r}_{j'},-\sigma})$$

$$\hat{T}_{+1} = - \sum_{\langle \vec{r}_j \vec{r}_{j'} \rangle} \sum_{\sigma} \hat{n}_{\vec{r}_j,-\sigma} \, c^\dagger_{\vec{r}_j,\sigma} \, c_{\vec{r}_{j'},\sigma} \, (1 - \hat{n}_{\vec{r}_{j'},-\sigma})$$

$$\hat{T}_{-1} = - \sum_{\langle \vec{r}_j \vec{r}_{j'} \rangle} \sum_{\sigma} (1 - \hat{n}_{\vec{r}_j,-\sigma}) \, c^\dagger_{\vec{r}_j,\sigma} \, c_{\vec{r}_{j'},\sigma} \, \hat{n}_{\vec{r}_{j'},-\sigma}$$

in terms of which the kinetic operator may be written as $\hat{T} = t \, (\hat{T}_0 + \hat{T}_{+1} + \hat{T}_{-1})$

Any operator can be written in terms of rotated-electron creation and annihilation operators as,

$$\begin{aligned}\hat{O} &= \hat{V}\,\tilde{O}\,\hat{V}^\dagger = \tilde{O} + [\tilde{O},\,\hat{S}\,] + \frac{1}{2}\,[[\tilde{O},\,\hat{S}\,],\,\hat{S}\,] + \ldots \\ &= \tilde{V}\,\tilde{O}\,\tilde{V}^\dagger = \tilde{O} + [\tilde{O},\,\tilde{S}\,] + \frac{1}{2}\,[[\tilde{O},\,\tilde{S}\,],\,\tilde{S}\,] + \ldots\end{aligned}$$

where

$$\hat{V}^\dagger = e^{\hat{S}}\,; \quad \hat{V} = e^{-\hat{S}}\,; \quad \hat{V} = \tilde{V}\,; \quad \hat{S} = \tilde{S}$$

To leading order in t/U, the unitary operator associated with any of the transformations under consideration has a universal form such that:

$$\hat{S} = -\frac{t}{U}\left[\hat{T}_{+1} - \hat{T}_{-1}\right] + \mathcal{O}(t^2/U^2) = \tilde{S} = -\frac{t}{U}\left[\tilde{T}_{+1} - \tilde{T}_{-1}\right] + \mathcal{O}(t^2/U^2)$$

The higher order terms are different for each unitary transformation. However, the corresponding rotated-electron operators are connected by an additional unitary transformation whose generator is given explicitly up to fourth order in t/U in:

AL Chernyshev, D Galanakis, P Philips, AV Rozhkov, AMS Tremblay, Phys. Rev. B 70, 235111 (2004)

An important property is that independently of the unitary transformation and thus of the chosen rotated-electron operators, the following expression refers to the same operator:

$$\tilde{S}_c = \frac{1}{2}\,\hat{V}^\dagger\,\hat{Q}\,\hat{V} = \frac{1}{2}\sum_{j=1}^{N_a^D}\sum_{\sigma=\uparrow,\downarrow} \tilde{n}_{\vec{r}_j\sigma}\,(1-\tilde{n}_{\vec{r}_j-\sigma})$$

Indeed, this operator counts the number of rotated-electron single occupied sides, which is the same for any choice of the unitary transformation.

This operator commutes with the model Hamiltonian and is the generator of the hidden global U(1) symmetry of the Hubbard model on a bipartite lattice. Its eigenvalue is the above considered number Sc.

Since at finite U that generator does not commute with the unitary operator, its expression is involved in terms of electron creation and annihilation operators:

$$\tilde{S}_c = \frac{1}{2}\hat{V}^\dagger\,\hat{Q}\,\hat{V} = \left(\frac{1}{2}\hat{Q} + [\frac{1}{2}\hat{Q},\,\hat{S}^\dagger\,] + \frac{1}{2}\,[[\frac{1}{2}\hat{Q},\,\hat{S}^\dagger\,],\,\hat{S}^\dagger\,] + ...\right)$$

Consistently, for U finite this operator is different from that which counts the number of electron singly occupied sites.

For finite U also the Hamiltonian does not commute with the unitary operator. In contrast to the generator of the new found symmetry, its expression is simple in terms of electron creation and annihilation operators. Hence it is involved in terms of rotated-electron creation and annihilation operators:

$$\hat{H} = \hat{V}\,\tilde{H}\,\hat{V}^\dagger = \tilde{V}\,\tilde{H}\,\tilde{V}^\dagger = \tilde{H} + [\tilde{H},\,\tilde{S}\,] + \frac{1}{2}\,[[\tilde{H},\,\tilde{S}\,],\,\tilde{S}\,] + ...$$

Importantly, the following commutators, which involve the six generators of the SO(4) algebra and the momentum operator, vanish:

$$[\hat{S}^z_\alpha, \hat{T}_l] = [\hat{S}^\dagger_\alpha, \hat{T}_l] = [\hat{S}_\alpha, \hat{T}_l] = [\hat{P}, \hat{T}_l] = 0\,; \quad \alpha = \eta, s\,, \;\; l = 0, \pm 1$$

This implies that,

$$[\hat{S}^z_\alpha, \hat{V}] = [\hat{S}^\dagger_\alpha, \hat{V}] = [\hat{S}_\alpha, \hat{V}] = [\hat{P}, \hat{V}] = 0\,; \quad \alpha = \eta, s$$

and thus that,

$$[\hat{S}^z_\alpha, \hat{S}] = [\hat{S}^\dagger_\alpha, \hat{S}] = [\hat{S}_\alpha, \hat{S}] = [\hat{P}, \hat{S}] = 0\,; \quad \alpha = \eta, s$$

Hence the six generators of the SO(4) algebra and the momentum operator have the same expression in terms of electron and rotated-electron creation and annihilation operators.

In contrast, that for U finite one has that,

$$[\tilde{S}_c, \hat{V}] \neq 0\,; \quad \tilde{S}_c \neq \hat{S}_c$$

is the reason why the new found global U(1) symmetry remained hidden. Indeed, for finite U its generator can be written in terms of the corresponding generator of the local U(1) gauge symmetry, expressed in terms of rotated-electron operators rather than of electron operators,

$$\tilde{S}_c = \sum_{j=1}^{N_a^D} \tilde{S}_{c,j} = \frac{1}{2}\sum_{j=1}^{N_a^D}\sum_{\sigma=\uparrow,\downarrow} \tilde{n}_{\vec{r}_j\sigma}\,(1-\tilde{n}_{\vec{r}_j-\sigma})$$

$$\tilde{S}_{c,j} = \frac{1}{2}\sum_{\sigma=\uparrow,\downarrow} \tilde{n}_{\vec{r}_j\sigma}\,(1-\tilde{n}_{\vec{r}_j-\sigma})$$

Hence while the generators of the SO(4) symmetry, which is contained in [SO(4)xU(1)]/Z_2, have the same expression in terms of both electron and rotated-electron creation and annihilation operators, that of the new found hidden U(1) symmetry does not. Indeed, it does not commute with the electron - rotated-electron unitary operator.

Consistency with the Hilbert space dimension

Addition of chemical-potential and magnetic-field operator terms to the Hubbard Hamiltonian lowers its symmetry. However, such terms commute with it. Hence, the number of state representations of its symmetry group must equal the Hilbert space dimension. In,

JMPC, Stellan Östlund, MJ Sampaio, Ann. Phys. 325, 1550 (2010)

it is shown that if the model global symmetry was SO(4), the corresponding number of state representations would be smaller than that dimension,

$$\sum_{S_c=0}^{N_a^D/2}\ \sum_{S_\eta=0}^{(N_a^D/2-S_c)}\ \sum_{S_s=0}^{S_c}\ \prod_{\alpha=\eta,s} \frac{[1+(-1)^{(2S_\alpha+2S_c)}]}{2}\,\mathcal{N}(S_\alpha,M_\alpha) < 4^{N_a^D}$$

Here,

$$\mathcal{N}(S_\alpha,M_\alpha) = (2S_\alpha+1)\left\{\binom{M_\alpha}{M_\alpha/2-S_\alpha} - \binom{M_\alpha}{M_\alpha/2-S_\alpha-1}\right\}, \quad \alpha = s,\eta$$

$$M_\eta = N_a^D - 2S_c\,; \quad M_s = 2S_c$$

In turn, in the case of the global SO(3)xSO(3)xU(1) symmetry, the number of state representations is found to exactly equal the Hilbert space dimension.

Consistency with the exact solution of the 1D Hubbard model

The Hubbard model has an exact solution on the one-dimensional lattice, which is a bipartite lattice. Is the new found hidden global U(1) symmetry consistent with that solution?

The model exact solution was first reached by the so called coordinate Bethe ansatz.

EH Lieb, FY Wu, Phys. Rev. Lett. 20, 1445 (1968)

An important result is that the imaginary part of the Bethe-ansatz complex rapidities simplifies for very large number of lattice sites.

M Takahashi, Prog. Theor. Phys. 47, 69 (1972)

The one-dimensional Hubbard model was solved by the inverse-scattering method thirty years after its solution by the coordinate Bethe-ansatz.

MJ Martins, PB Ramos, Nucl. Phys. B 522, 413 (1998)

Why was the solution of the problem by the algebraic scattering method achieved only thirty years after that of the coordinate Bethe ansatz?

The point is that it was expected that the charge and spin monodromy matrices had the same traditional ABCD form, found previously for the related one-dimensional isotropic Heisenberg model.

EK Sklyanin, LA Takhtadzhan, LD Faddeev, Theor. Math. Phyz. 40, 194 (1979)

Such an expectation was that consistent with a spin SU(2) symmetry and a charge SU(2) symmetry, associated with the model global SO(4) = [SU(2)xSU(2)]/Z_2 symmetry.

Fortunately, in their 1998 paper Martins and Ramos used an appropriate representation of the charge and spin monodromy matrices, which allows for possible hidden symmetries.

Our studies reveal that for U>0 the model charge and spin degrees of freedom are associated with U(2)=SU(2)xU(1) and SU(2) symmetries, rather than with two SU(2) symmetries, respectively.

This is behind the ABCDF and ABCD forms of the charge and spin monodromy matrices, respectively, found by Martins and Ramos to achieve the model exact solution by the inverse scattering method.

Physical consequences of the model extended global symmetry?

The search of the physical consequences of the new found extended symmetry of the Hubbard model on bipartite lattices in a scientifically interesting problem.

On the one-dimensional and square lattices the model is expected to describe the effects of correlations in several types of materials such as quasi-one-dimensional conductors and high-temperature superconductors, respectively. However, the model on the square lattice remains poorly understood.

The relation of the new found global U(1) symmetry with spontaneously broken symmetries in the case that in the thermodynamic limit the ground state of the Hubbard model on a given bipartite lattice goes superconducting is an issue that deserves investigations.

Another interesting problem is the role of symmetry in the possible connection of the Hubbard model on the bipartite honey-comb lattice to the correlation effects in graphene. Recently, Monte Carlo simulations by Muramatsu's group revealed that a quantum spin liquid emerges between the state described by massless Dirac fermions and an antiferromagnetic ordered Mott insulator.

ZY Meng, TC Lang, S Wessel, FF Assad, A Muramatsu, Nature 464, 847 (2010)

A preliminary application of the new found extended symmetry: Objects whose occupancies generate its state representations

A preliminary application of the extended symmetry is the description of the Hubbard model on the square lattice in terms of operators associated with three types of quantum objects.

JMPC, Nucl. Phys. B 824, 452 (2010); Nucl. Phys. B 840, 553 (2010)

Such quantum object operators have simple expressions in terms of the rotated-electron creation and annihilation operators generated by a suitably chosen unitary transformation.

For very large U/t values they become the quasicharge, spin, and pseudospin operators considered in:

Stellan Östlund, Mats Granath, Phys. Rev. Lett 96, 066404 (2006)

The occupancy configurations of the three types of objects generate state representations of the two SU(2) symmetries and hidden U(1) symmetry, respectively, contained in the model extended global symmetry.

Specifically, the representation involves c fermions operators associated with the new found U(1) symmetry. In terms of rotated-electron operators, their operator expression reads,

$$f^\dagger_{\vec{r}_j,c} = \tilde{c}^\dagger_{\vec{r}_j,\uparrow}\,(1-\tilde{n}_{\vec{r}_j,\downarrow}) + e^{i\vec{\pi}\cdot\vec{r}_j}\,\tilde{c}_{\vec{r}_j,\uparrow}\,\tilde{n}_{\vec{r}_j,\downarrow}$$

Moreover, it involves spinon operators and η-spinon operators associated with the two corresponding SU(2) symmetries. Their operators are given by,

$$s^l_{\vec{r}_j} = n_{\vec{r}_j,c}\,q^l_{\vec{r}_j}\,; \qquad p^l_{\vec{r}_j} = (1-n_{\vec{r}_j,c})\,q^l_{\vec{r}_j}\,, \quad l = \pm, z$$

respectively. The rotated quasi-spin operators appearing here read in terms of rotated-electron creation and annihilation operators,

$$q^l_{\vec{r}_j} = s^l_{\vec{r}_j} + p^l_{\vec{r}_j}\,; \qquad l = +,-,z$$

$$q^+_{\vec{r}_j} = (\tilde{c}^\dagger_{\vec{r}_j,\uparrow} - e^{i\vec{\pi}\cdot\vec{r}_j}\,\tilde{c}_{\vec{r}_j,\uparrow})\,\tilde{c}_{\vec{r}_j,\downarrow}$$

$$q^-_{\vec{r}_j} = (q^+_{\vec{r}_j})^\dagger\,, \qquad q^z_{\vec{r}_j} = \frac{1}{2} - \tilde{n}_{\vec{r}_j,\downarrow}$$

Moreover,

$$n_{\vec{r}_j,c} = f^\dagger_{\vec{r}_j,c}\,f_{\vec{r}_j,c}$$

It turns out that in the half-filling ground state and its spin-triplet excited states there are only c fermions and s1 fermions. The latter are spin-neutral two-spinon composite objects. (In s1, 1 denotes the number of spinon pairs, which is one.) Indeed, the spinons that are not invariant under the electron - rotated-electron unitary transformation are confined within such composite objects. The corresponding confining order is an example of those considered in:

T Senthil, A Vishwanath, L Balents, S Sachdev, MPA Fisher, Science 303, 1490 (2004)

The spin-triplet excitation spectrum is generated by the emergence of two holes in the s1 fermion momentum band. It has a simple expression in terms of the corresponding s1 energy dispersion:

$$\Delta E(\vec{k}) = -\epsilon_{s1}(\vec{q}) - \epsilon_{s1}(\vec{q}\,')$$

$$\vec{k} = \vec{\pi} - \vec{q} - \vec{q}\,'$$

The spin-wave coherent spectral weight corresponds to processes where one of the s1 holes is created at the s1 boundary line and the other at a s1 band nodal direction. For such processes, the above general spectrum simplifies to:

$$\begin{aligned} \omega_{SW}(\vec{k}) &= [\mu_0/2]|\sin([k_{x1}+k_{x2}]/2)| \\ &+ W^0_{s1}|\sin([k_{x1}-k_{x2}]/2)| \end{aligned}$$

In the high symmetry directions connecting the momentum points,

$$M = (\pi, \pi)\,, \quad O = (\pi/2, \pi/2)\,, \quad \Gamma = (0,0)\,, \quad X = (\pi, 0)$$

the latter energy spectrum reads:

$$\begin{aligned}
\omega_{\Gamma O}(\vec{k}) &= \frac{\mu^0}{2}\sin(k_i) \\
\vec{k} &= [\pi, -\pi] - [\pi/2 - k_i, -\pi/2 - k_i] - [\pi/2, -\pi/2] \\
&= [k_i, k_i]\,; \qquad k_i = k_x = k_y \in (0, \pi/2)
\end{aligned}$$

$$\begin{aligned}
\omega_{MO}(\vec{k}) &= \frac{\mu^0}{2}\sin(k_i) \\
\vec{k} &= [\pi, \pi] - [\pi/2 - k_i, 3\pi/2 - k_i] - [\pi/2, -\pi/2] \\
&= [k_i, k_i]\,; \qquad k_i = k_x = k_y \in (\pi/2, \pi)
\end{aligned}$$

$$\begin{aligned}
\omega_{\Gamma X}(\vec{k}) &= \left[\frac{\mu^0}{2} + W^0_{s1}\right]\sin(k_x/2) \\
\vec{k} &= [\pi, -\pi] - [\pi/2 - k_x/2, -\pi/2 - k_x/2] \\
&- [\pi/2 - k_x/2, -\pi/2 + k_x/2] \\
&= [k_x, 0]\,; \qquad k_x \in (0, \pi)
\end{aligned}$$

$$\omega_{XM}(\vec{k}) = \left[\frac{\mu^0}{2} + W^0_{s1}\right]\cos(k_y/2)$$

$$\vec{k} = [\pi,\pi] - [-k_y/2, \pi - k_y/2] - [k_y/2, -k_y/2]$$
$$= [\pi, k_y]\,; \qquad k_y \in (0,\pi)$$

$$\omega_{XO}(\vec{k}) = \frac{\mu^0}{2} - W^0_{s1}\cos(k_x)$$
$$= \frac{\mu^0}{2} + W^0_{s1}\cos(k_y)$$
$$\vec{k} = [\pi,-\pi] - [0,-\pi] - [\pi - k_x, -\pi + k_x]$$
$$= [\pi,\pi] - [0,\pi] - [k_y, -k_y]$$
$$= [k_x, \pi - k_x]\,; \quad k_x \in (\pi/2, \pi)$$
$$= [\pi - k_y, k_y]\,; \quad k_y \in (0, \pi/2)$$

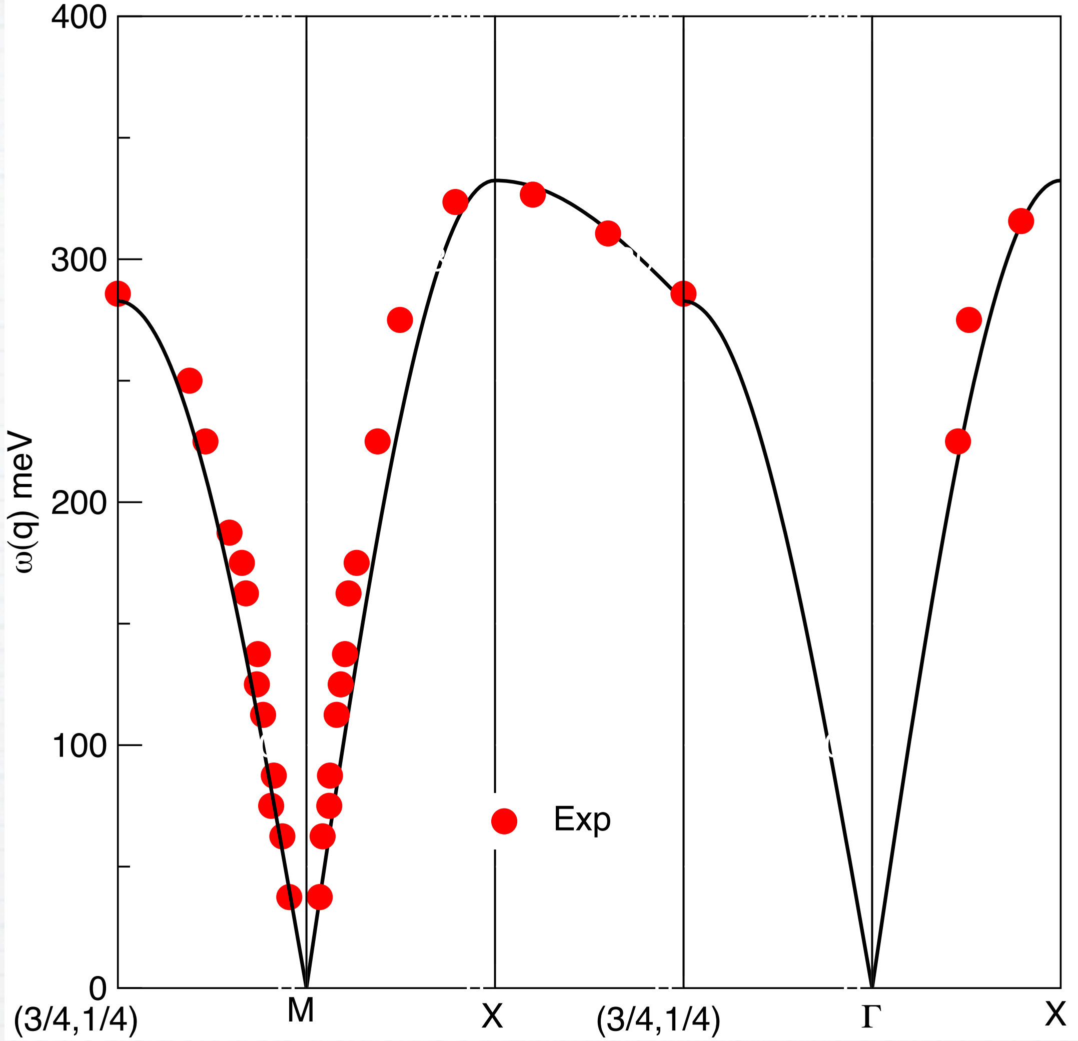
Here:

$\mu^0 \approx 565.6$ meV and $W^0_{s1} \approx 49.6$ meV for $U/4t = 1.525$ and $t = 0.295$ meV

Such a theoretical spectrum for the high symmetry directions of the Brillouin zone is plotted here. It is very similar to that obtained by the standard formalism of many-body physics for the same U and t values, which involves summing up an infinite number of ladder diagrams.

NMR Peres, MAN Araújo, Phys. Rev. B 65, 132404 (2002)

The use of this object operator description becomes more complex away from half filling. The study of that problem is in progress.



The theoretical spin spectra for the high symmetry directions (solid lines), U/4t= 1.525 eV, and t= 0.295 eV and experimental data (circles) of the parent compound La(2)CuO(4) in meV measured by inelastic neutron scattering. Experimental points from:

R Coldea, SM Hayden, G Aeppli, TG Perring TE Manson, SW Cheong, Z Fisk, Phys. Rev. Lett 86, 5377 (2001)

Conclusions

- The global symmetry of the Hubbard model on a bipartite lattice is larger than SO(4) and given by SO(3)xSO(3)xU(1).

- What are the physical consequences of such an extended global symmetry is an interesting scientific problem, which deserves further studies about the Hubbard model on several bipartite lattices.

- Whether the ground state of the model on a given bipartite lattice is for some parameter-space region superconducting and what is the relation of the new found global U(1) symmetry to the spontaneously broken symmetries is an issue to be investigated.

- A preliminary application of the extended global symmetry to the Hubbard model on the square lattice includes a representation in terms of c fermions, spinons, and η-spinons, whose occupancy configurations generate state representations of the U(1), spin SU(2), and η-spin SU(2) symmetries, respectively.