A Bayesian Hierarchical Modelling approach to quantify wind forcing uncertainties in ocean ensemble forecasting

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Outline

- The forecast uncertainty
- Bayesian Hierarchical modelling to quantify the Surface Vector Wind (SVW) uncertainty and distributions
- A new method of Ocean Ensemble Forecasting using the BHM-SVW

How do we quantify the uncertainty in the winds? We use Bayes' Theorem

- *Y* is a physical variable of interest
- *X* is a set of observations relevant to *Y*
- [y|x] is the *posterior* distribution of Y

$$[y|x] = \frac{[x|y][y]}{\int [x|y][y]\, dy}$$

- [x|y] is the *measurement error* model or *Data Stage*
- [y] is the *prior* or *Process Model Stage*
- $\int [x|y][y]dy$ is the *normalization* so the *posterior distribution* integrates to *1*

Bayesian Hierarchical Modelling

- defines θ, η *parameters* necessary to define *Data* and *Process Models*

$$[y,\theta,\eta|x] = \frac{[x|y,\theta][y|\eta][\theta][\eta]}{\int [x|y,\theta][y|\eta][\theta][\eta]\, d\theta\, d\eta\, dy}$$

***Review*: Bayesian Hierarchical Models (BHM)**

BHM Building Blocks:

<u>Data Stage Distribution</u> (likelihood) quantifies uncertainty in relevant observations, through relevant parameter errors

<u>Process Model Stage Distribution</u> (prior) quantifies uncertainty in knowledge of the Physical process connected to the field to estimate

<u>Parameter Distributions</u> from Data Stage and Process Models (i.e. $[\theta_d]$, $[\theta_p]$) issues of identifiability, uncertainty, model validation

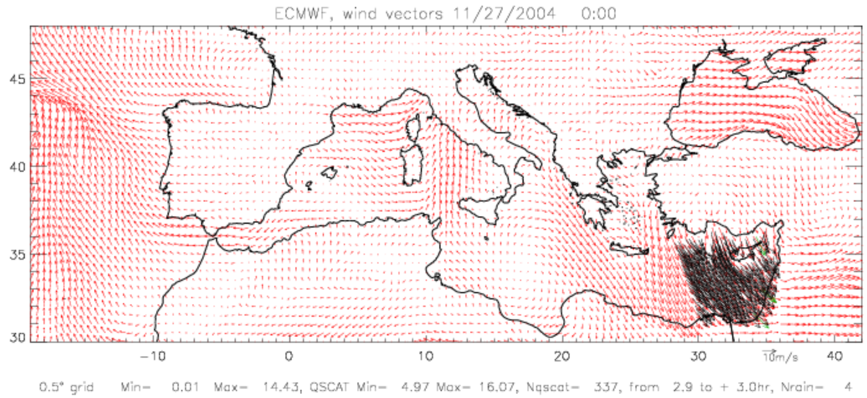
The end product of BHM is the Posterior Distribution of the field of interest

Estimates of posterior distributions are obtained via Gibbs sampler Algorithm and Markov Chain Monte Carlo methods

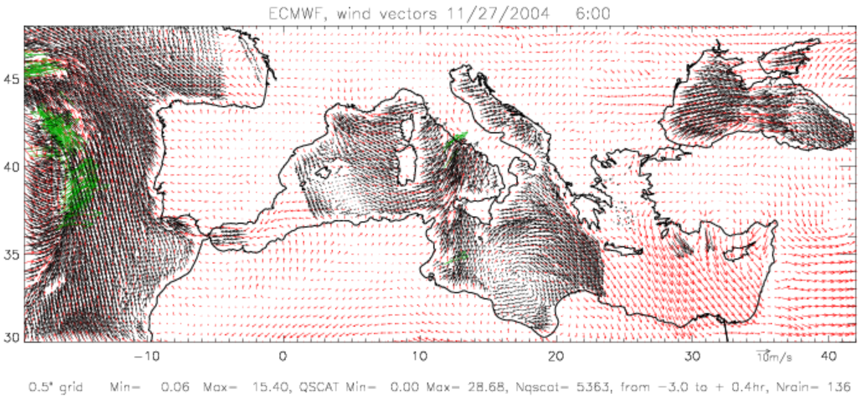
Posterior mean is <u>summary</u>

Standard deviation of posterior is an estimate of the <u>spread</u>

Our data stage: QuikSCAT and ECMWF Surface Wind Estimates for the Mediterranean Sea



0:00 UTC



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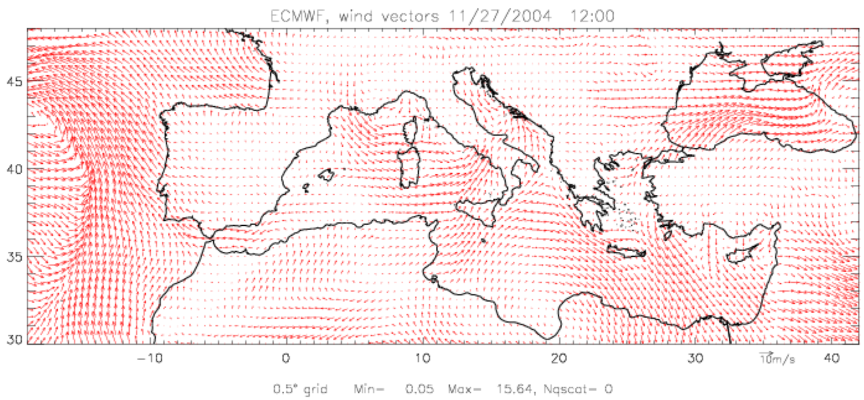

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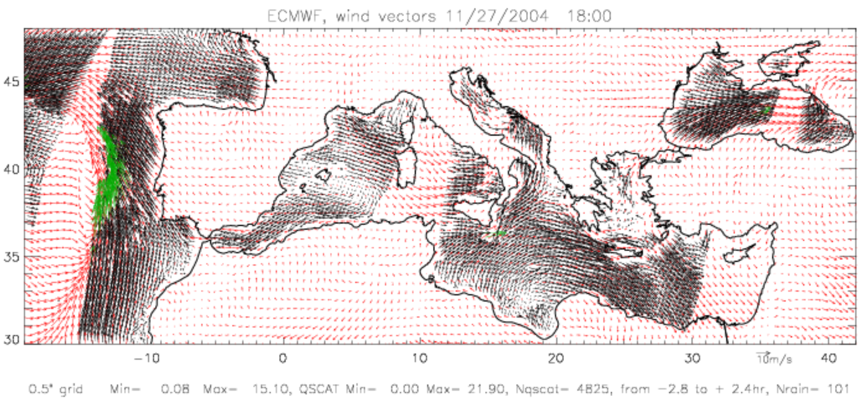

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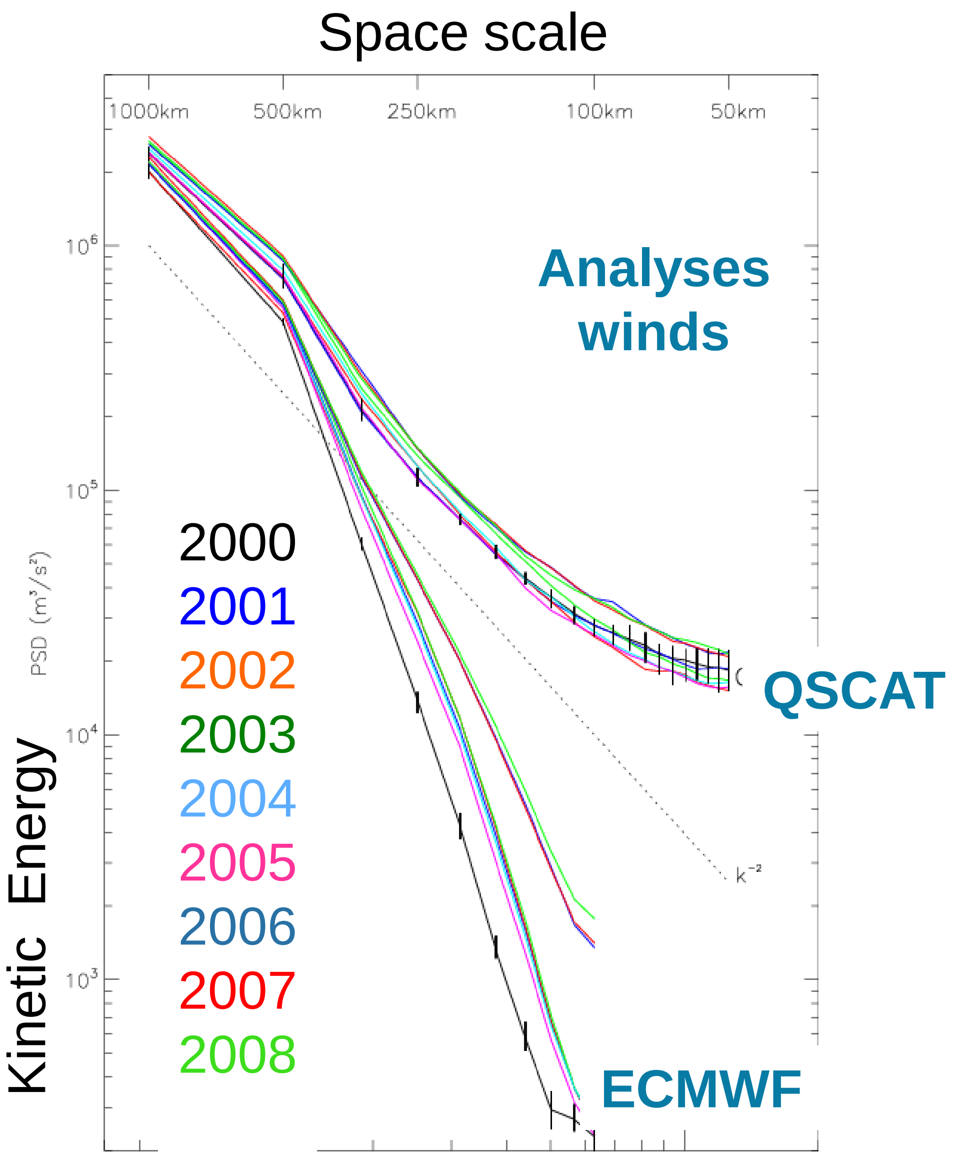
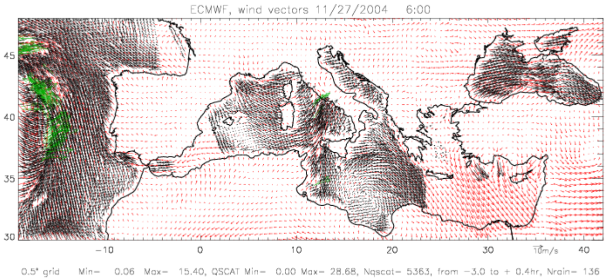
ECMWF NWP surface winds uncertainty over the Med Sea



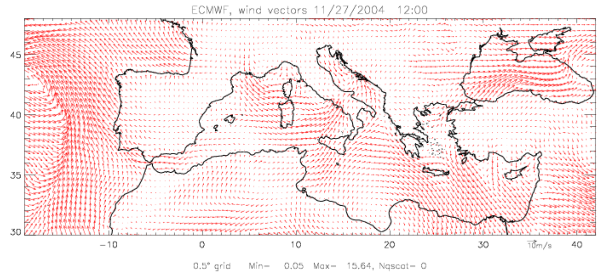
Building the wind distributions using Bayesian Hierarchical Modelling (BHM-SVW)

Conceptual and implementation blocks:

Data Stage: 2 types of data
QSCAT winds and
ECMWF analyses/forecasts



QSCAT



ECMWF

Process model stage:
Raylegh friction surface model translated into a stochastic finite difference equation

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y}$$

$$\begin{aligned} U_t &= \theta_{uy} D_y P_t + \theta_{ux} D_x P_t + \epsilon_u \\ V_t &= \theta_{vx} D_x P_t + \theta_{vy} D_y P_t + \epsilon_v \end{aligned}$$

Process model (prior) for the SVW :

$$\frac{\partial u}{\partial t} - f\ v = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} - \gamma\ u$$
$$\frac{\partial v}{\partial t} + f\ u = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - \gamma\ v$$

Choice: begin with RFE
linear, includes surface friction
dependent variables match well with “obs”
include a “friction” process w/o spec form

$$\frac{1}{f}\left[\frac{\partial^2}{\partial t^2} + \left(f^2+\gamma^2\right)\right]u + 2\frac{\gamma}{f}\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} - \frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial x \partial t} - \frac{\gamma}{\rho_0 f}\frac{\partial p}{\partial x}$$

$$\frac{1}{f}\left[\frac{\partial^2}{\partial t^2} + \left(f^2+\gamma^2\right)\right]v + 2\frac{\gamma}{f}\frac{\partial v}{\partial t} = \frac{1}{\rho_0}\frac{\partial p}{\partial x} - \frac{1}{\rho_0 f}\frac{\partial^2 p}{\partial y \partial t} - \frac{\gamma}{\rho_0 f}\frac{\partial p}{\partial y}$$

Rewriting the RFE separately for u and v Components

Model A1:

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y}$$

Choice: approximate models (“leading order”)
reduce no. terms,
concentrate data stage inputs to reduce uncertainty

Model A2:

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - 2\frac{\gamma}{\left(f^2+\gamma^2\right)}\frac{\partial u}{\partial t} - \frac{1}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial^2 p}{\partial x \partial t}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - 2\frac{\gamma}{\left(f^2+\gamma^2\right)}\frac{\partial v}{\partial t} - \frac{1}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial^2 p}{\partial y \partial t}$$

SVW Model A1: from Deterministic to Stochastic

1. Discretize RFE approximation (geostrophic-ageostrophic model)

$$u = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x}$$

$$v = \frac{f}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial x} - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)}\frac{\partial p}{\partial y}$$

$$U_t = -\frac{f}{\rho_0\left(f^2+\gamma^2\right)} D_y P_t - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)} D_x P_t$$

$$V_t = \frac{f}{\rho_0\left(f^2+\gamma^2\right)} D_x P_t - \frac{\gamma}{\rho_0\left(f^2+\gamma^2\right)} D_y P_t$$

2. Make the equations stochastic introducing parameters and model error

$$\begin{aligned} U_t &= \theta_{uy} D_y P_t + \theta_{ux} D_x P_t + \epsilon_u \\ V_t &= \theta_{vx} D_x P_t + \theta_{vy} D_y P_t + \epsilon_v \end{aligned}$$

$$\begin{aligned} \theta_{uy} &\sim N(-\frac{f}{\rho_o(f^2+\gamma^2)}, \sigma_{uy}^2) \\ \theta_{ux} &\sim N(-\frac{\gamma}{\rho_o(f^2+\gamma^2)}, \sigma_{ux}^2) \\ \theta_{vx} &\sim N(\frac{f}{\rho_o(f^2+\gamma^2)}, \sigma_{vx}^2) \\ \theta_{vy} &\sim N(-\frac{\gamma}{\rho_o(f^2+\gamma^2)}, \sigma_{vy}^2) \end{aligned}$$

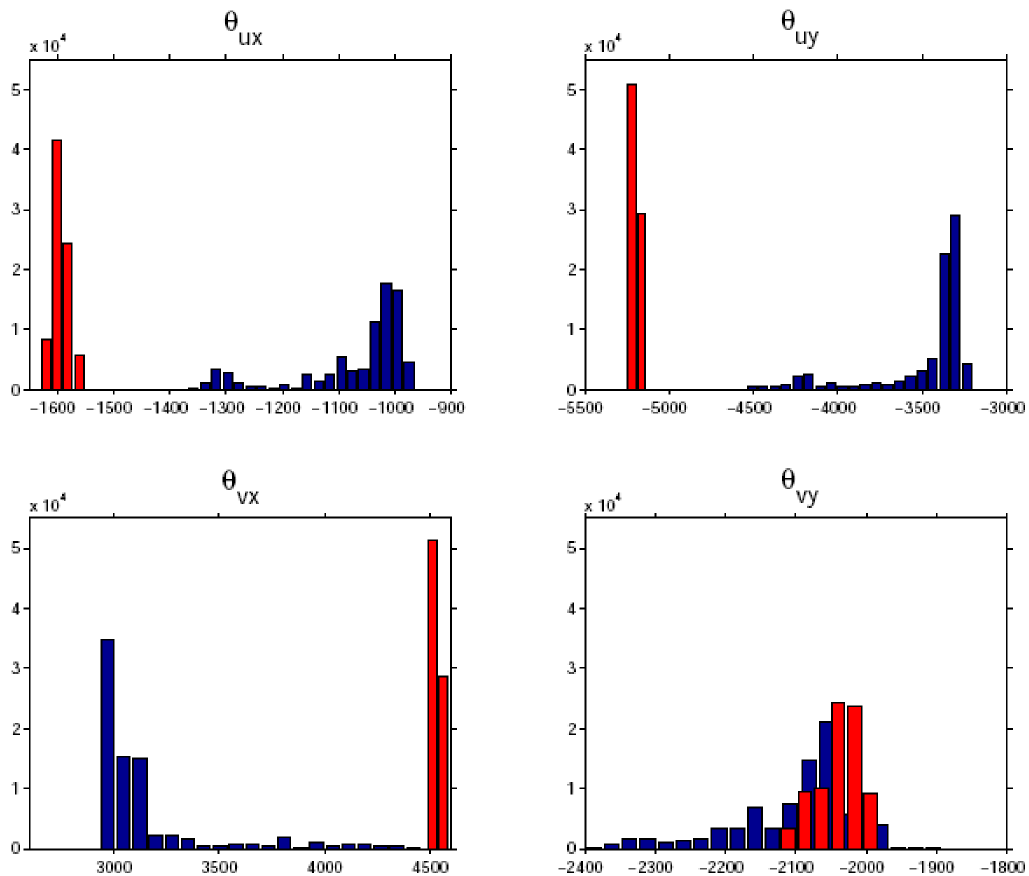
θ_i are <u>random</u> parameters distributed as gaussian

$$p(x,y,t) = \mu + \sum_{k=1} a_k(t)\,\phi_k(x,y).$$

a are <u>random</u> parameters distributed as gaussian and <u>'phi' are 20 spatial EOFs</u>

Posterior Distributions of the Parameters:

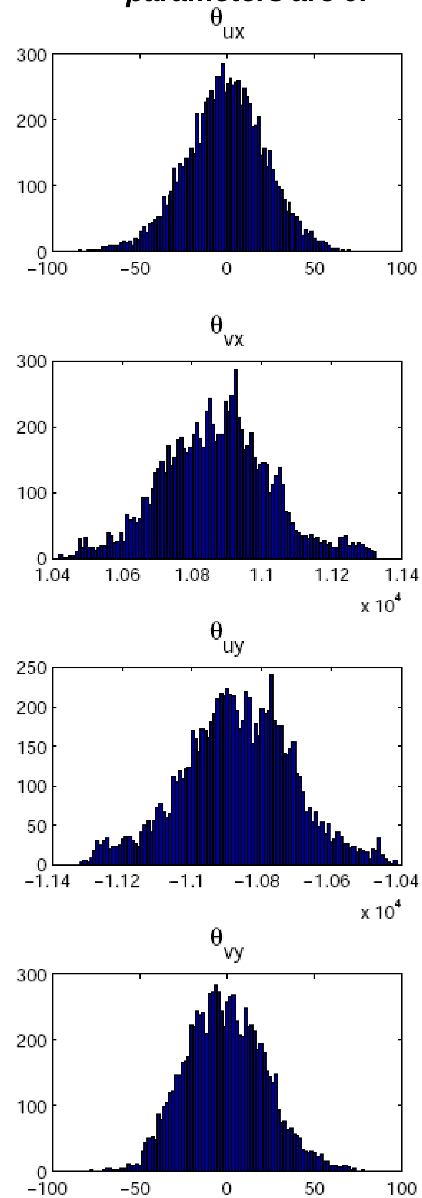
With and *Without* *QuikSCAT*



QuikSCAT data increase spread (non-Gaussian), and shift distributions farther from Geostrophy.

Geostrophic Validation

Modes for posterior distributions of parameters for geostrophic terms are +/- inverse Coriolis; ageostrophic parameters are 0.

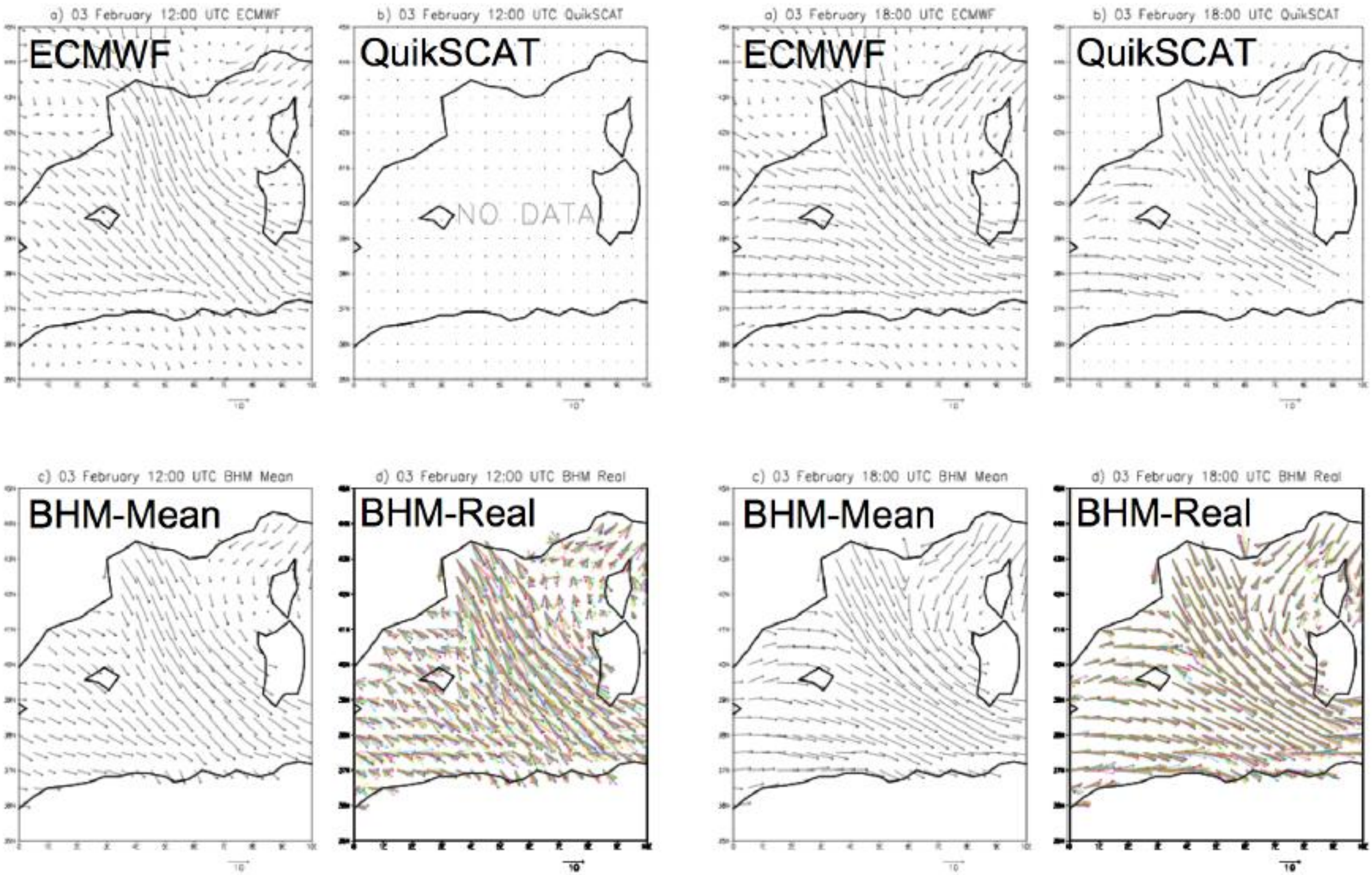


A1E3 s2a_corrected

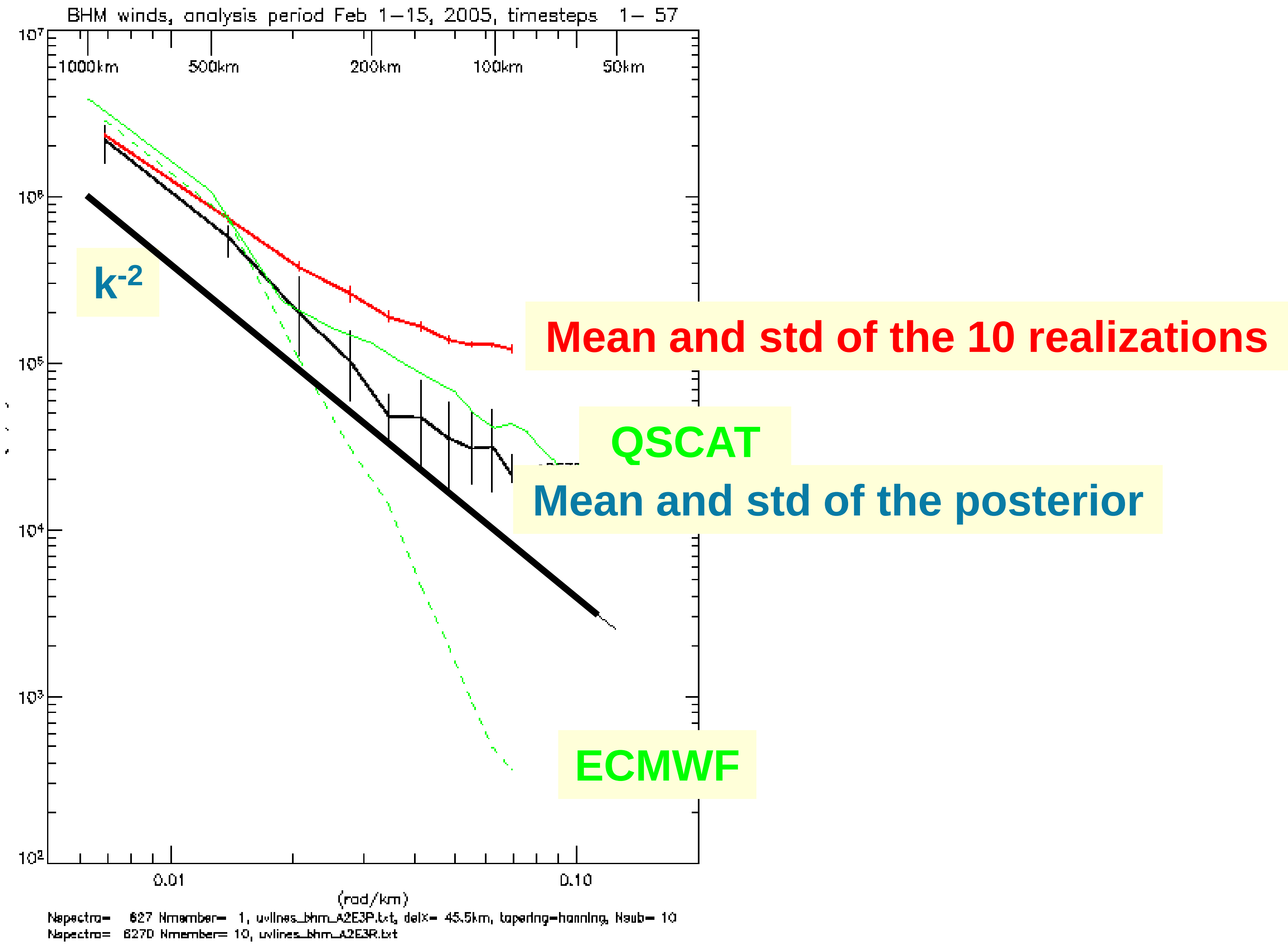
The wind posterior distribution

03 Feb 05 - 12 UTC

03 Feb 05 - 18 UTC

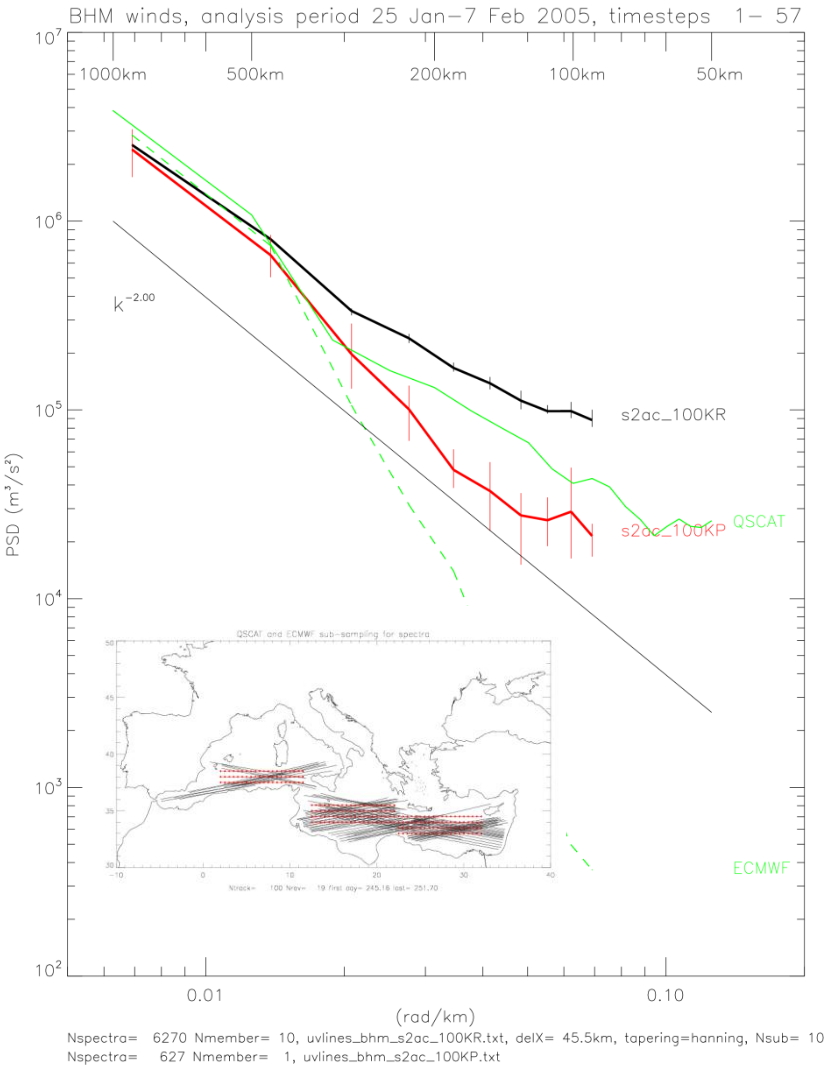


The Kinetic Energy Spectra of the posterior distribution

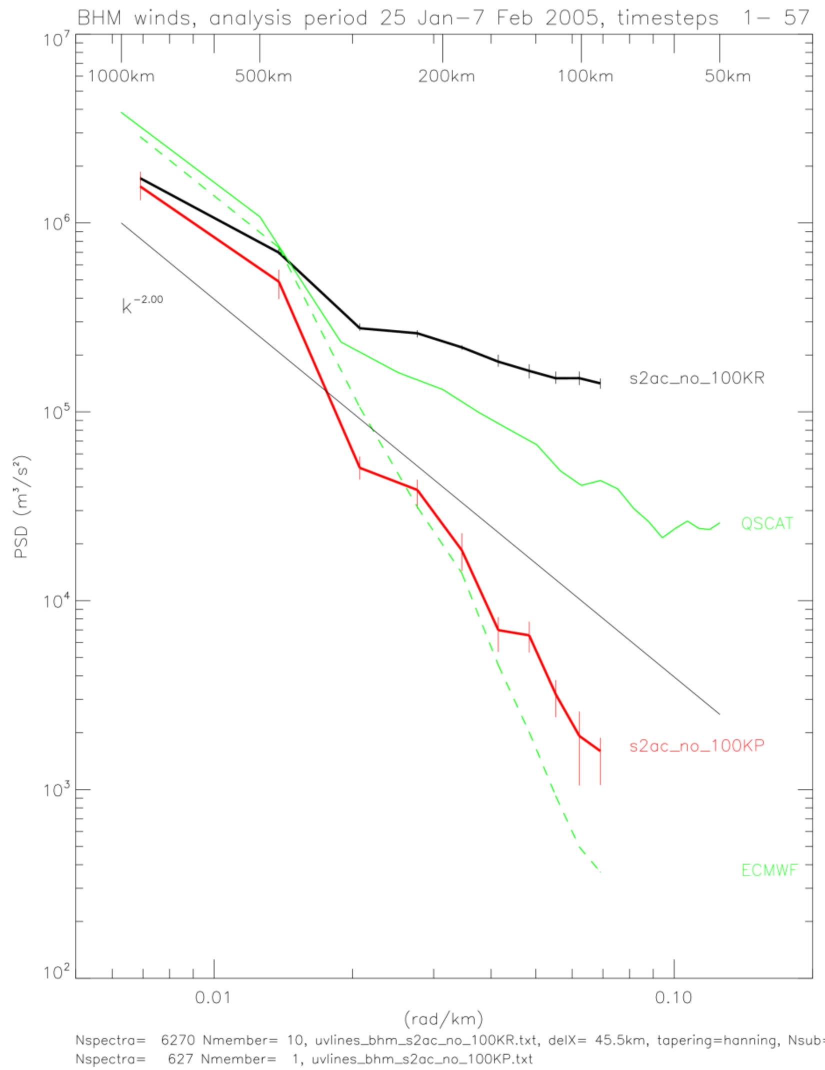


Zonal Wind Kinetic Energy Spectra: *BHM Metric*

with QuikSCAT

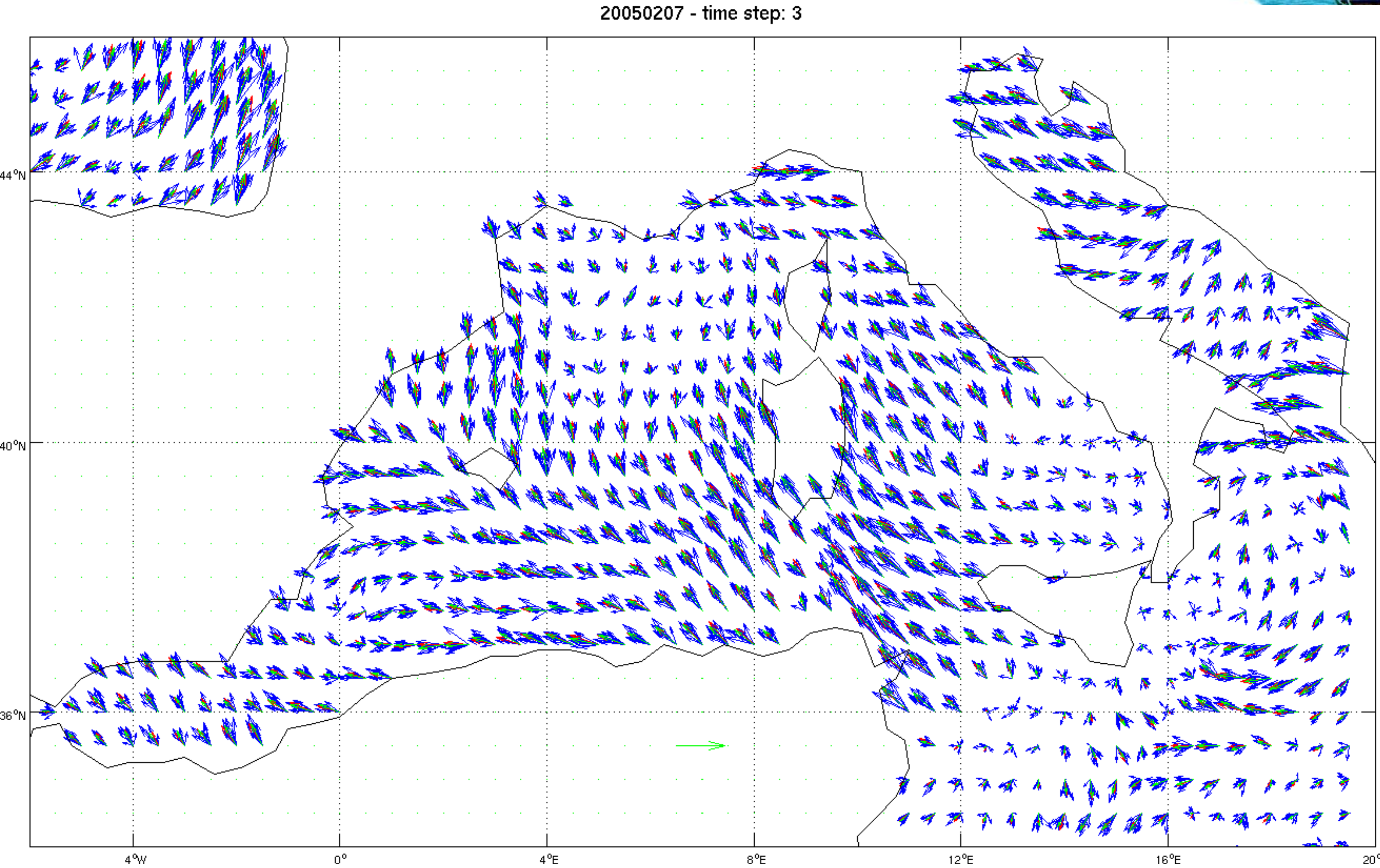


without QuikSCAT



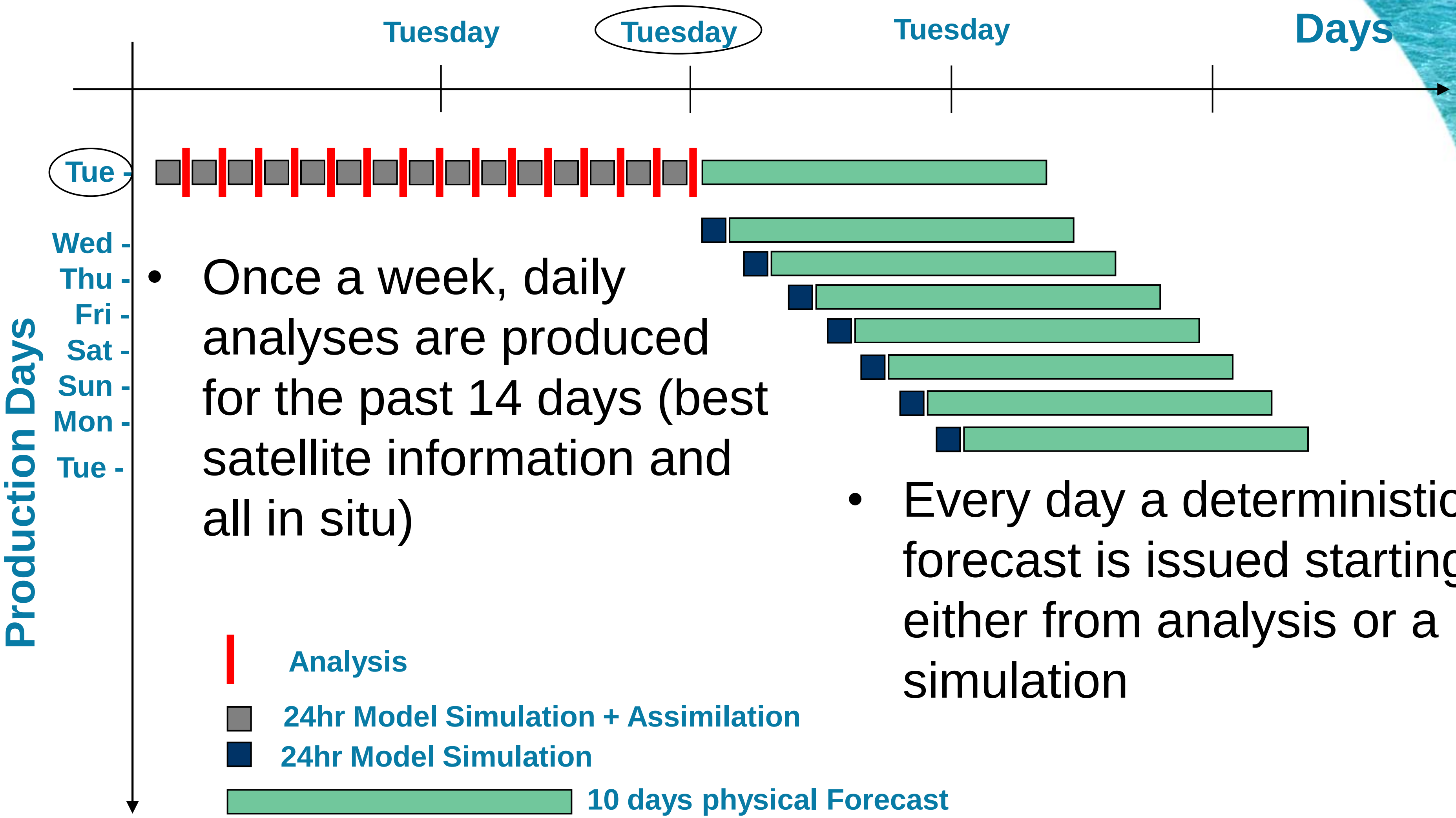
QuikSCAT data support (observed) power-law behavior at synoptic and mesoscales. Without QuikSCAT, high-wavenumber variance is noise.

BHM-SVW realizations: example for February 7, 2005 at 18:00 GMT



The MFS deterministic forecast system

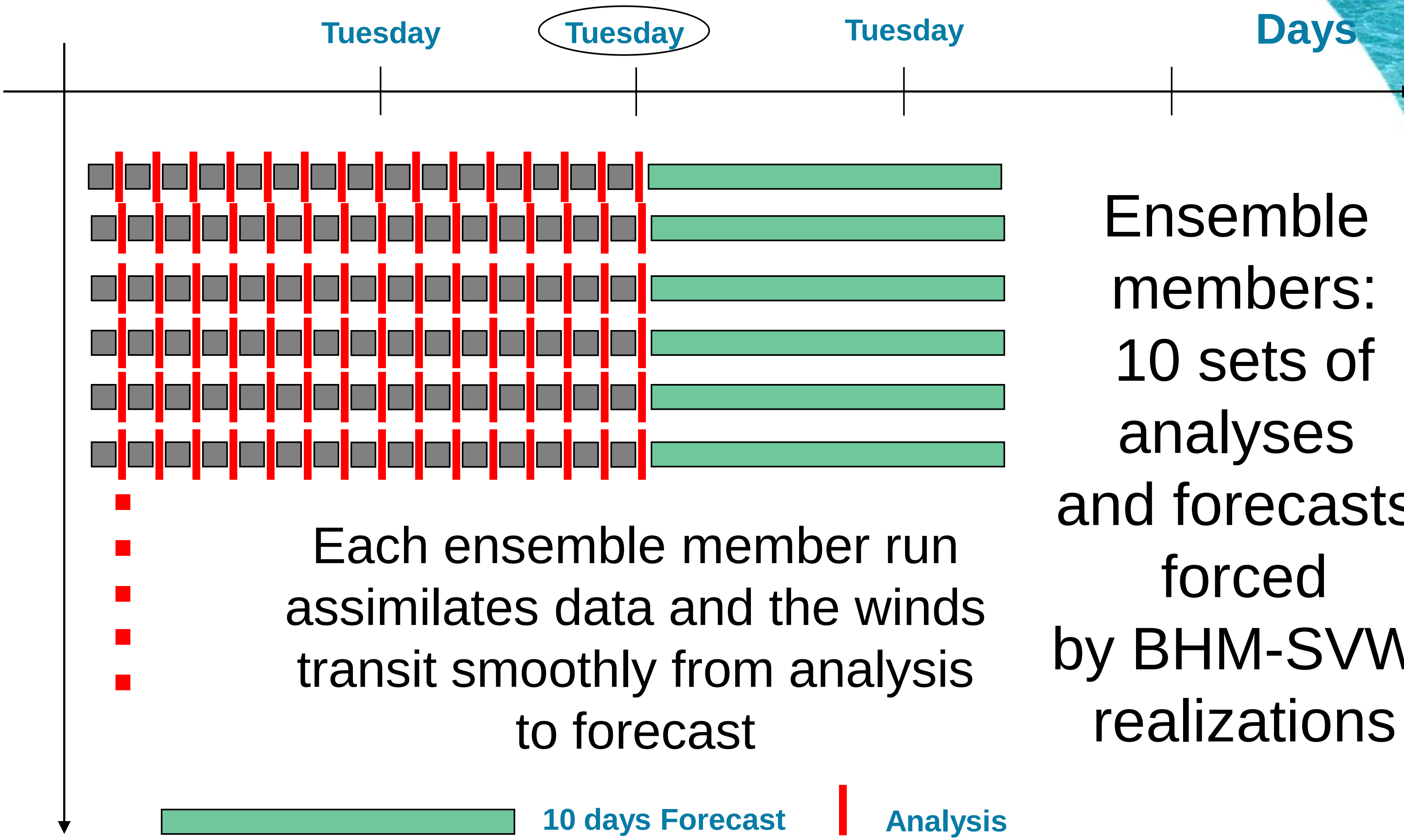
Forecast and analyses production cycle



- Once a week, daily analyses are produced for the past 14 days (best satellite information and all in situ)

- Every day a deterministic forecast is issued starting either from analysis or a simulation

The BHM-SVW Ocean Ensemble Forecast method

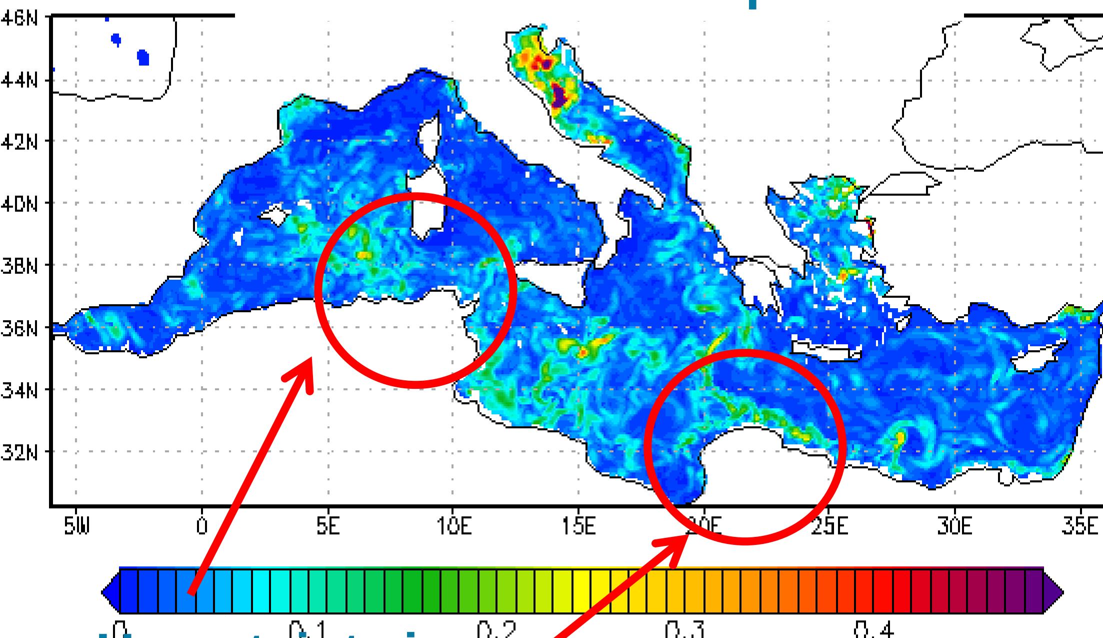


Each ensemble member run assimilates data and the winds transit smoothly from analysis to forecast

Ensemble members:
10 sets of analyses
and forecasts forced
by BHM-SVW realizations

BHM-SVW-OEF initial condition spread: amplification of the uncertainty due to winds where there is no data assimilated

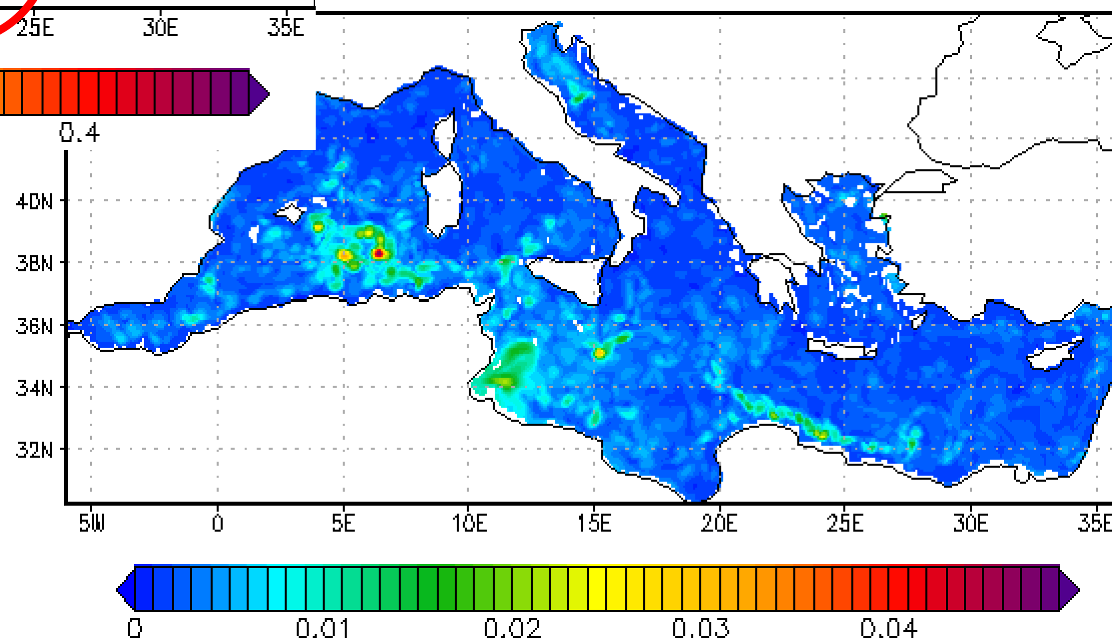
Initial condition spread



Sea Surface Temperature

Initial condition spread

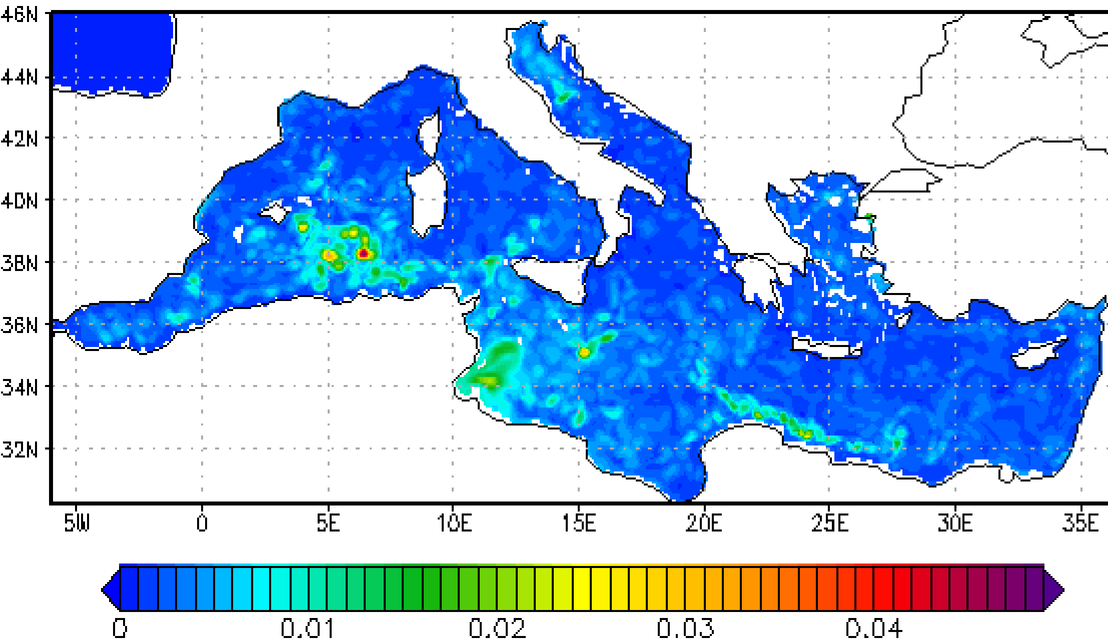
Uncertainty is concentrated at the mesoscales



Sea Surface Height

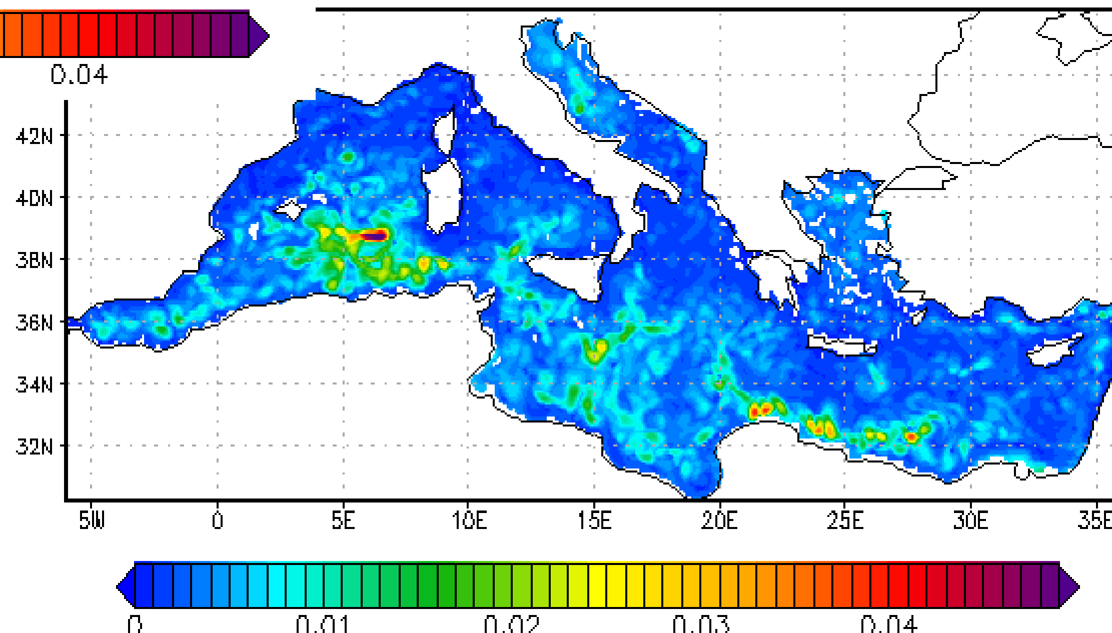
BHM-SVW-OEF last forecast day spread

Initial condition spread



Sea Surface Height

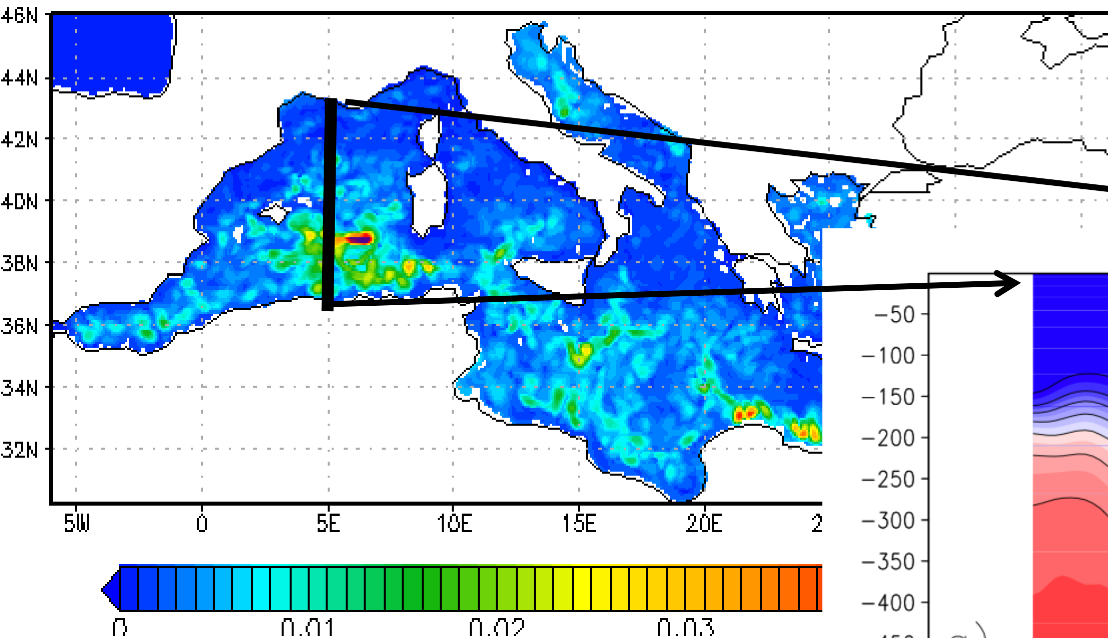
10-th fcst day spread

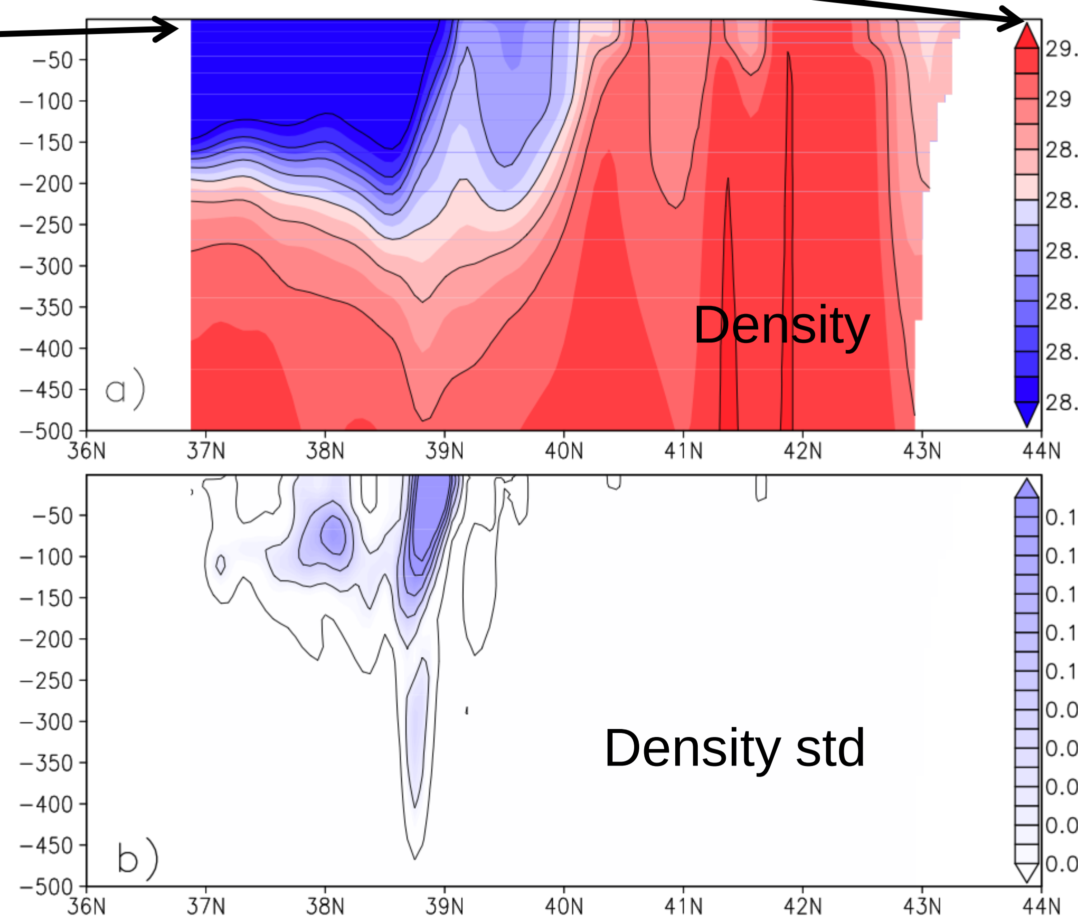


Initial condition ensemble spread has amplified at the 10 fcst day in mesoscale regions

BHM-SVW-OEF last forecast day spread

10-th fcst day spread



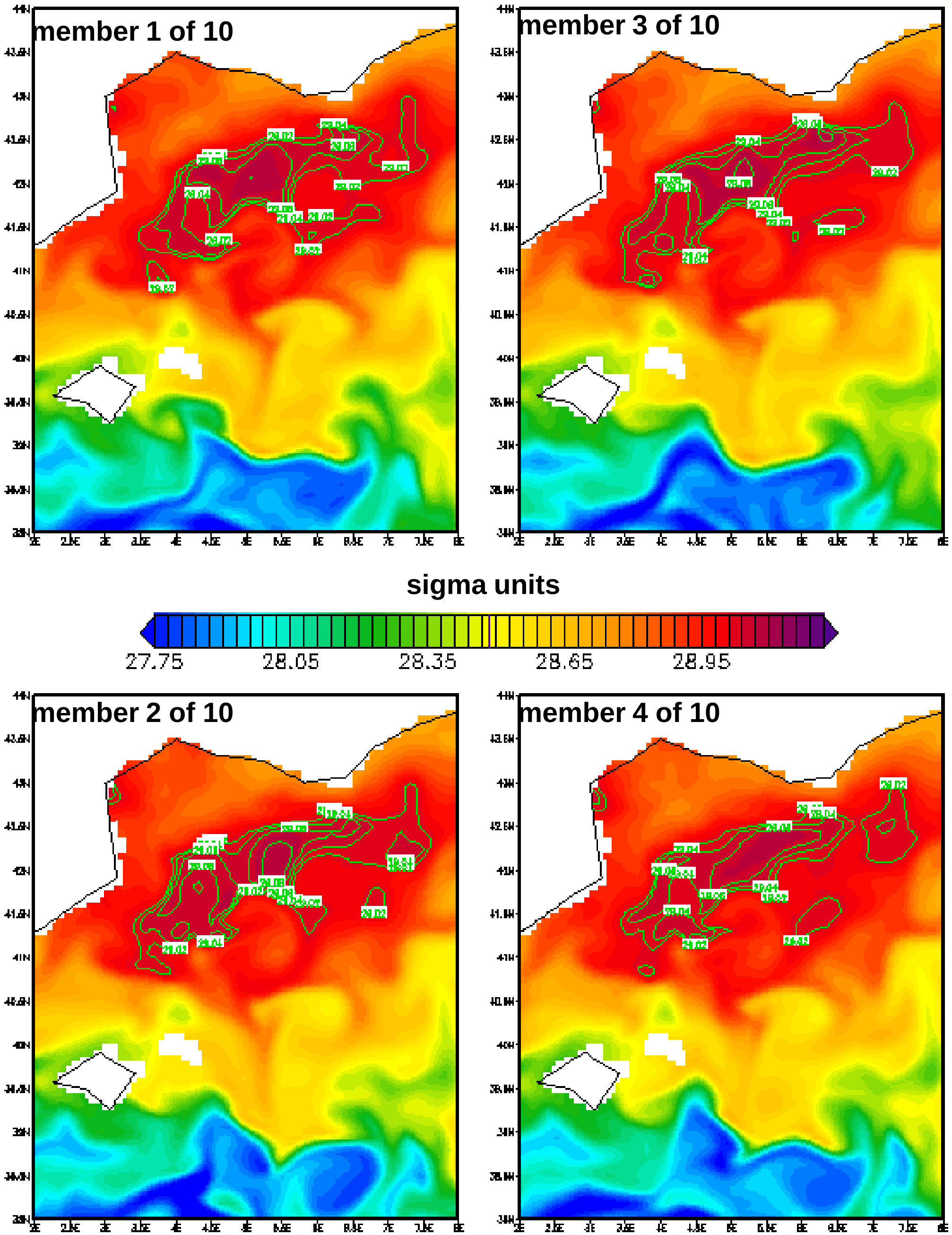


Ensemble Forecast spread is upper thermocline intensified

Ensemble Spread: Surface Density, Gulf of Lion

Gulf of Lions Gyre:
small, but climatically important spread

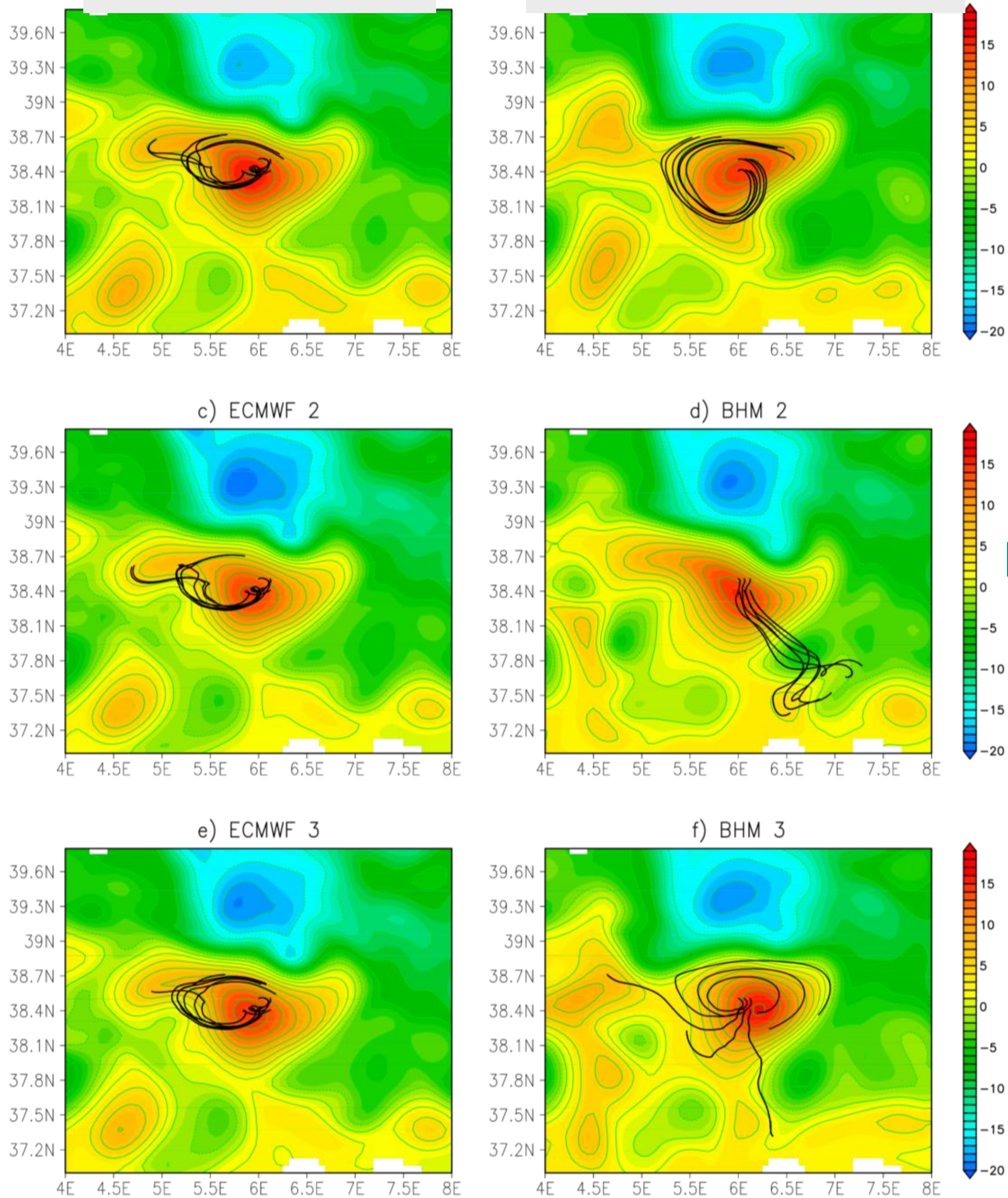
Algerian Current:
large spread



MFS-Wind-BHM
Forecast day 10F

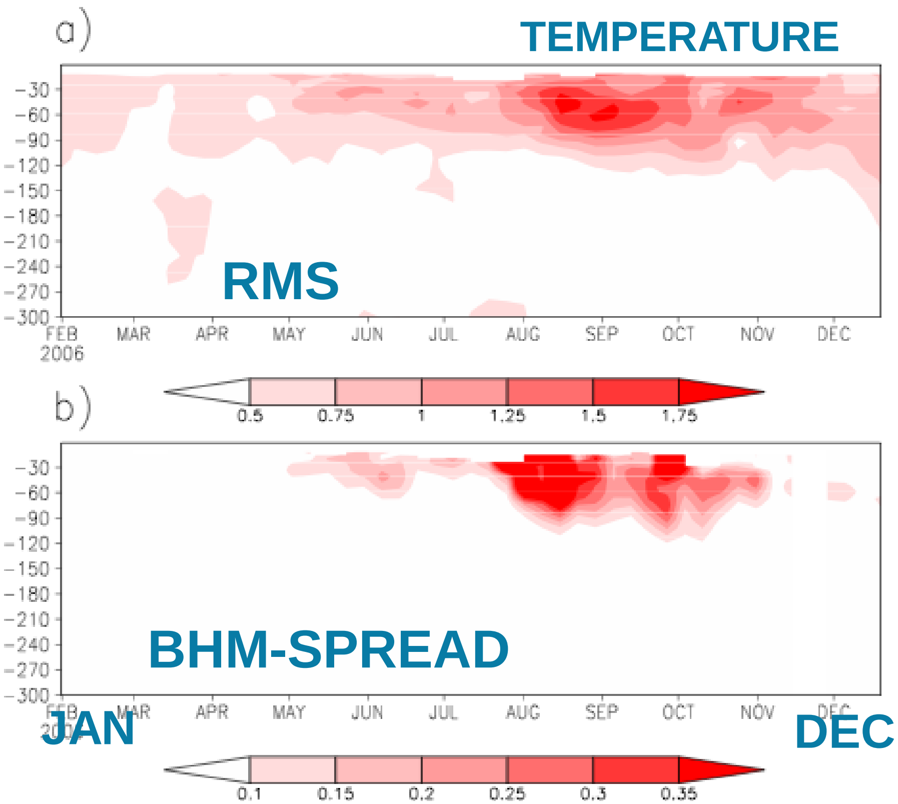
The forecast spread at 10F

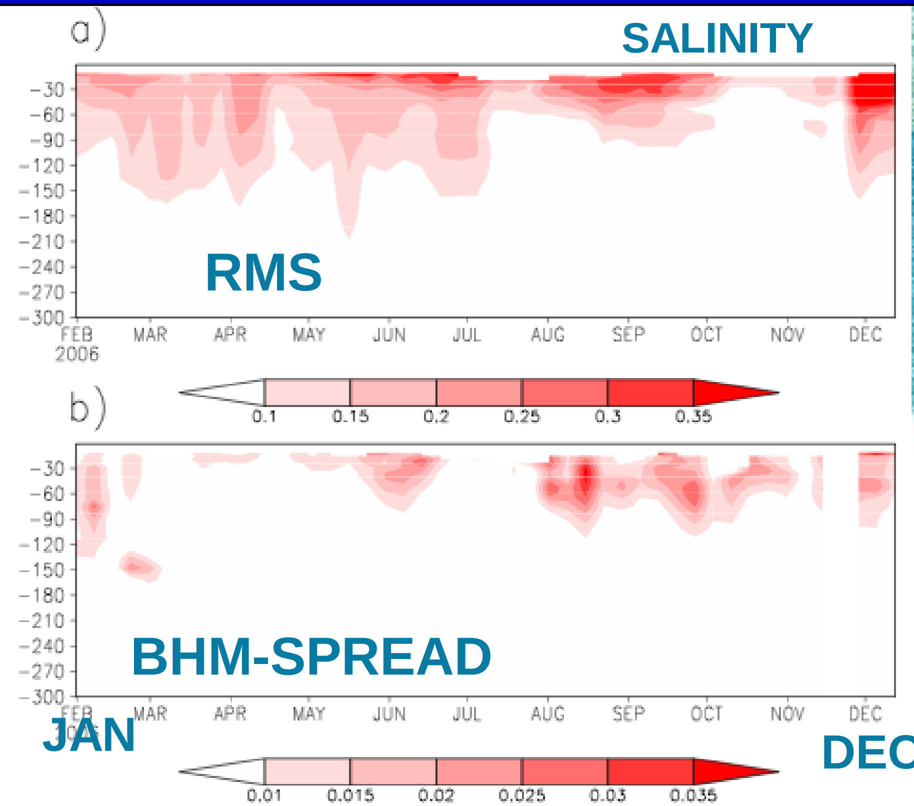
EEPS forced ensemble	BHM-SVW ensemble

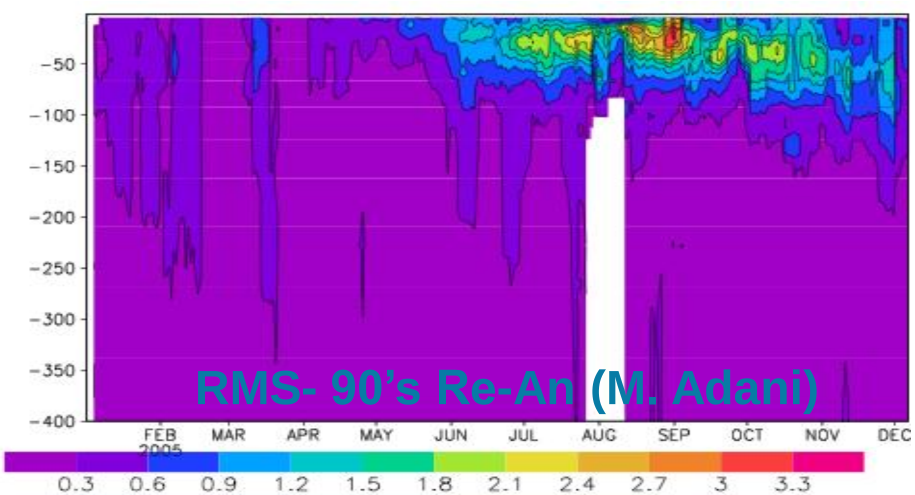


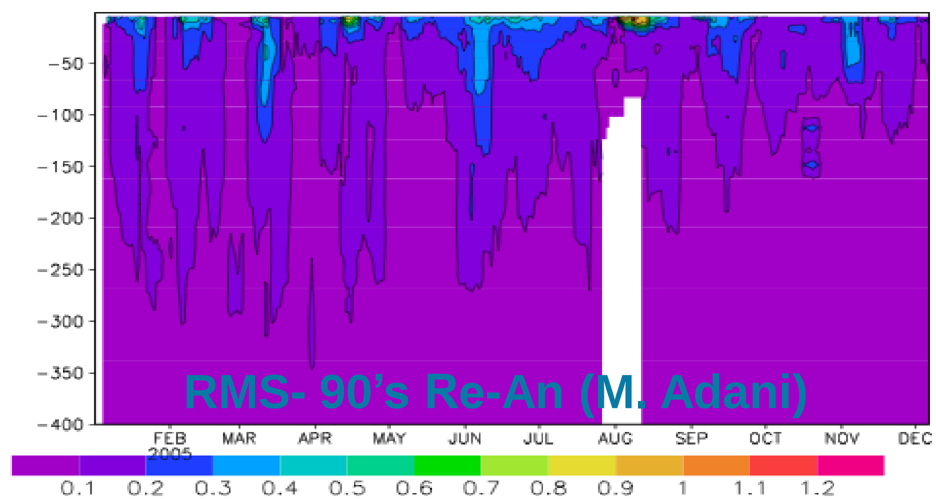
ECMWF EPS forcing is not effective to produce flow field changes at the mesoscales

Background Error versus Ensemble Spread









Ocean Ensemble Forecast Experiments

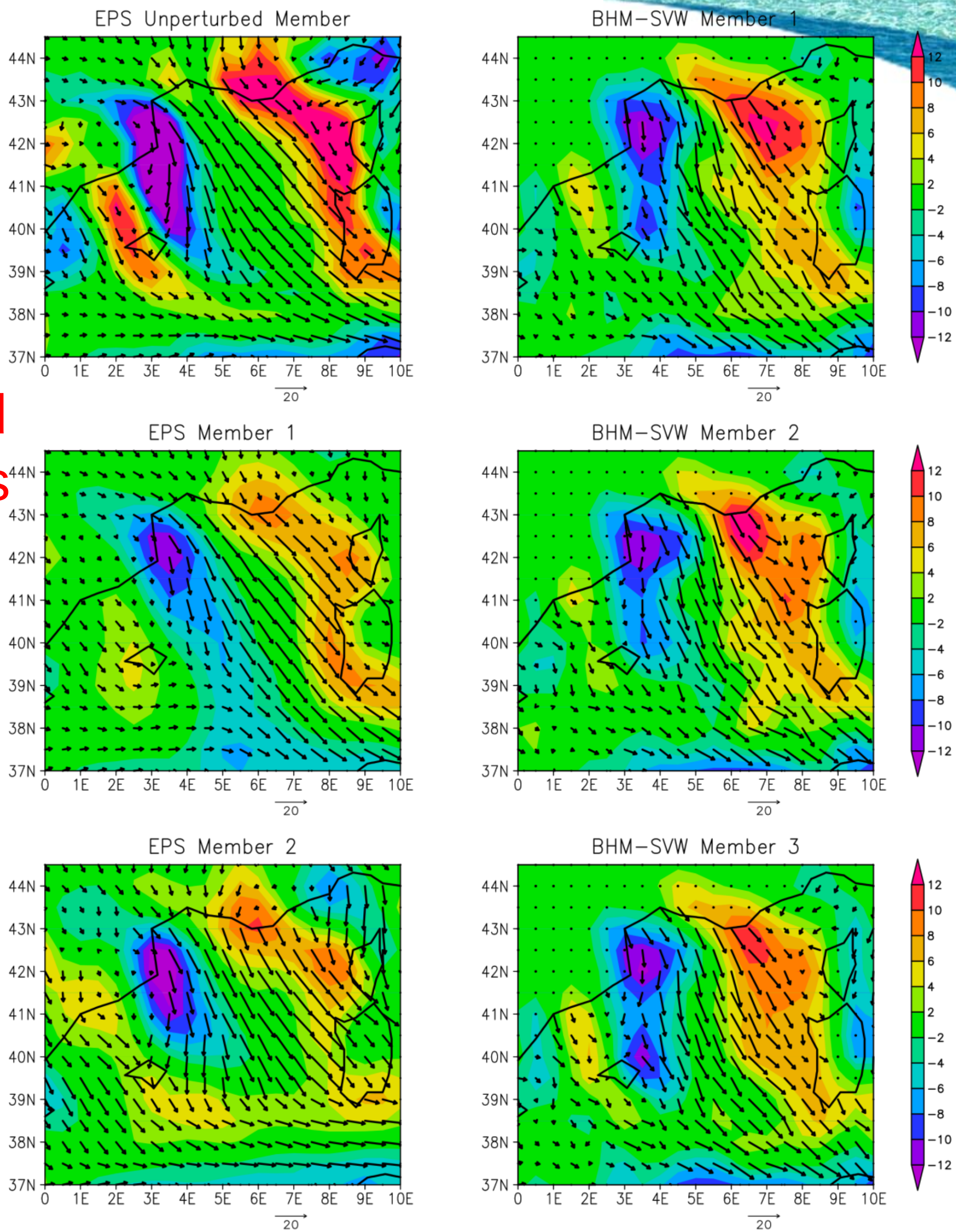
NAME OF THE EXPERIMENT	ENSEMBLE METHOD
BHM-SVW-OEF-16	MEMBERS GENERATED BY REALIZATIONS OF BHM-SVW with full resolution model (1/16)
EEPS-OEF	MEMBERS GENERATED BY ECMWF EPS WINDS, SAME INITIAL CONDITION
TIRP-OEF	MEMBERS GENERATED BY PERTURBED INITIAL CONDITIONS
BHM-SVW-OEF-4	MEMBERS GENERATED BY REALIZATIONS OF BHM-SVW with low resolution model (1/4)

Member 1

EEPS wind realizations

Member 2

Member 3



BHM wind realizations

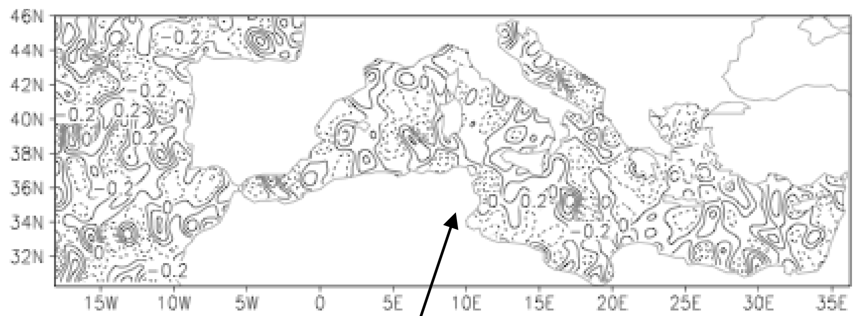
Member 1

Member 2

Member 3

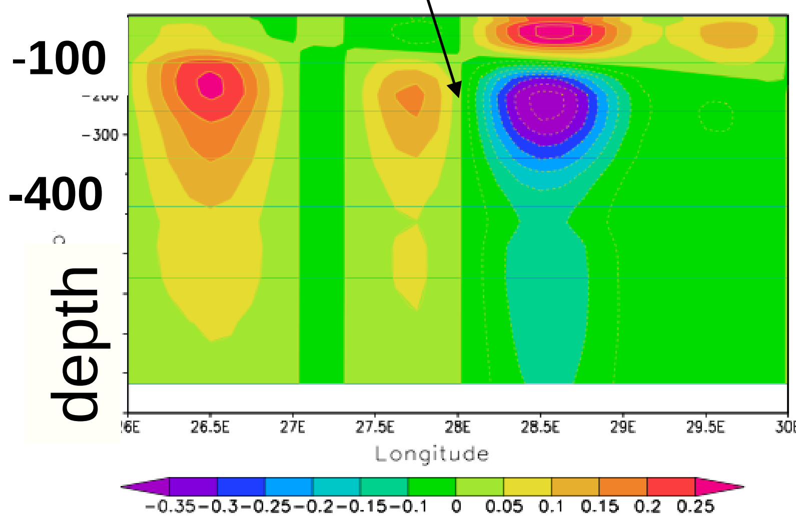
Alternative ensemble methods: the TIRP-OEF

THERMOCLINE INTENSIFIED
RANDOM PERTURBATION



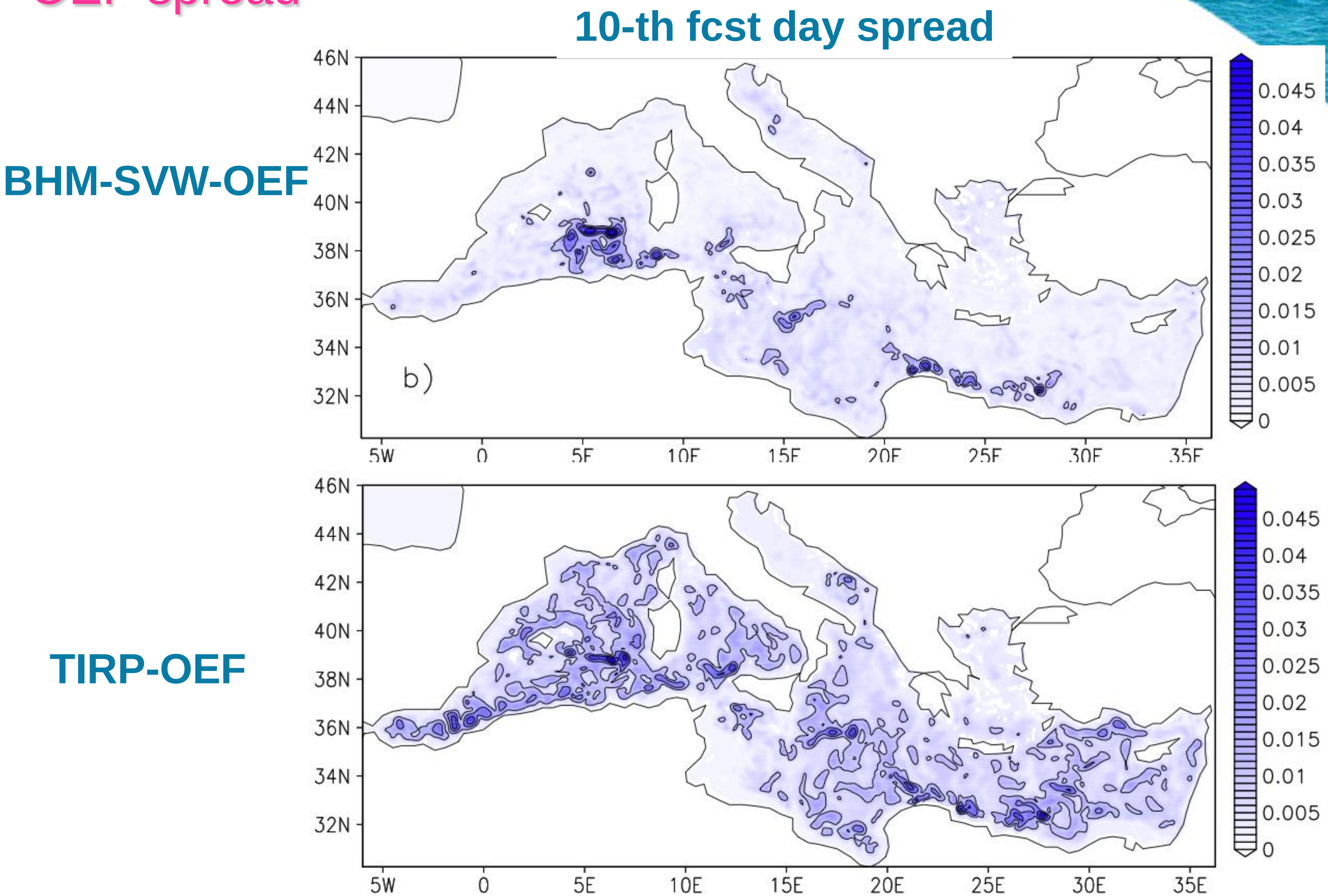
The horizontal structure is random,

the vertical is thermocline intensified



Longitude -->

Comparison TIRP-OEF and BHM-SVW-OEF spread



TIRP Perturbations vertical structure has been chosen ad-hoc

Final considerations

- A new method to produce realistic distributions of surface winds (SVW) from QSCAT and NWP analyses and forecasts has been developed (Milliff et al., 2009, submitted)
- BHM-SVW distributions are used to design a new ocean ensemble forecasting method: BHM-SVW-OEF (Bonazzi et al., 2009, submitted)
- The BHM-SVW-OEF produces 10 days forecast spread at the mesoscales and in the upper thermocline
- Ad-hoc I.C. perturbations can produce similar results while large scale NWP ensemble prediction winds are not effective
- BHM-SVW-OEF coupled to IC condition perturbation methods promises in the future to contribute to the understanding of the ‘uncertainty conundrum’