Advanced School on Data Assimilation

CMCC, Bologna

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Three-dimensional variational data assimilation system in the Mediterranean Sea

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Formulation

3DVAR finds the minimum of a cost function linearized around the background state:

$$J = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}\left[\mathbf{H}\delta\mathbf{x} - \mathbf{d})\right]^T\mathbf{R}^{-1}\left[\mathbf{H}\delta\mathbf{x} - \mathbf{d})\right]$$

Increments: $\delta\mathbf{x} = \mathbf{x} - \mathbf{x}_b$

Misfits: $\mathbf{d} = \left[H(\mathbf{x}_b) - \mathbf{y}\right]$

Small and can be neglected

Linearisation: $H(\mathbf{x}) = H(\mathbf{x}_b) + \mathbf{H}\delta\mathbf{x} + O(\delta\mathbf{x}^2)$

Control space

Preconditioning using a control vector **v** defined by:

$$\mathbf{v} = \mathbf{V}^{+}\delta\mathbf{x} \qquad \text{and} \qquad \mathbf{B} = \mathbf{V}\mathbf{V}^{T}$$

$$J = \frac{1}{2}\mathbf{v}^{T}\mathbf{v} + \frac{1}{2}\left[\mathbf{HVv} - \mathbf{d})\right]^{T}\mathbf{R}^{-1}\left[\mathbf{HVv} - \mathbf{d})\right]$$

$$\nabla J = \mathbf{v} + \mathbf{V}^{T}\mathbf{H}^{T}\mathbf{R}^{-1}\left[\mathbf{HVv} - \mathbf{d})\right]$$

$$\delta\mathbf{x} = \mathbf{V}\mathbf{v}$$

Model of B

$\mathbf{V}$ is modelled as a sequence of linear operators:

$$\mathbf{V} = \mathbf{V}_D \mathbf{V}_{uv} \mathbf{V}_\eta \mathbf{V}_H \mathbf{V}_V .$$

$\mathbf{V}_V$ - Vertical EOFs to physical space.

$\mathbf{V}_H$ - Horizontal covariances.

$\mathbf{V}_\eta$ - Barotropic model.

$\mathbf{V}_{uv}$ - Diagnose u and v.

$\mathbf{V}_D$ - Divergence damping filter.

Numerical implementation

The code contains inner and outer loops:

• inner loops iterate the minimizer

• outer loop applies the multi-grid method (the minimum is found first on a low resolution grid and the solution is used as the first guess for the higher resolution grid)

The minimum is found using a minimizer based on the quasi-Newton method with the limited memory (free code L-BFGS)

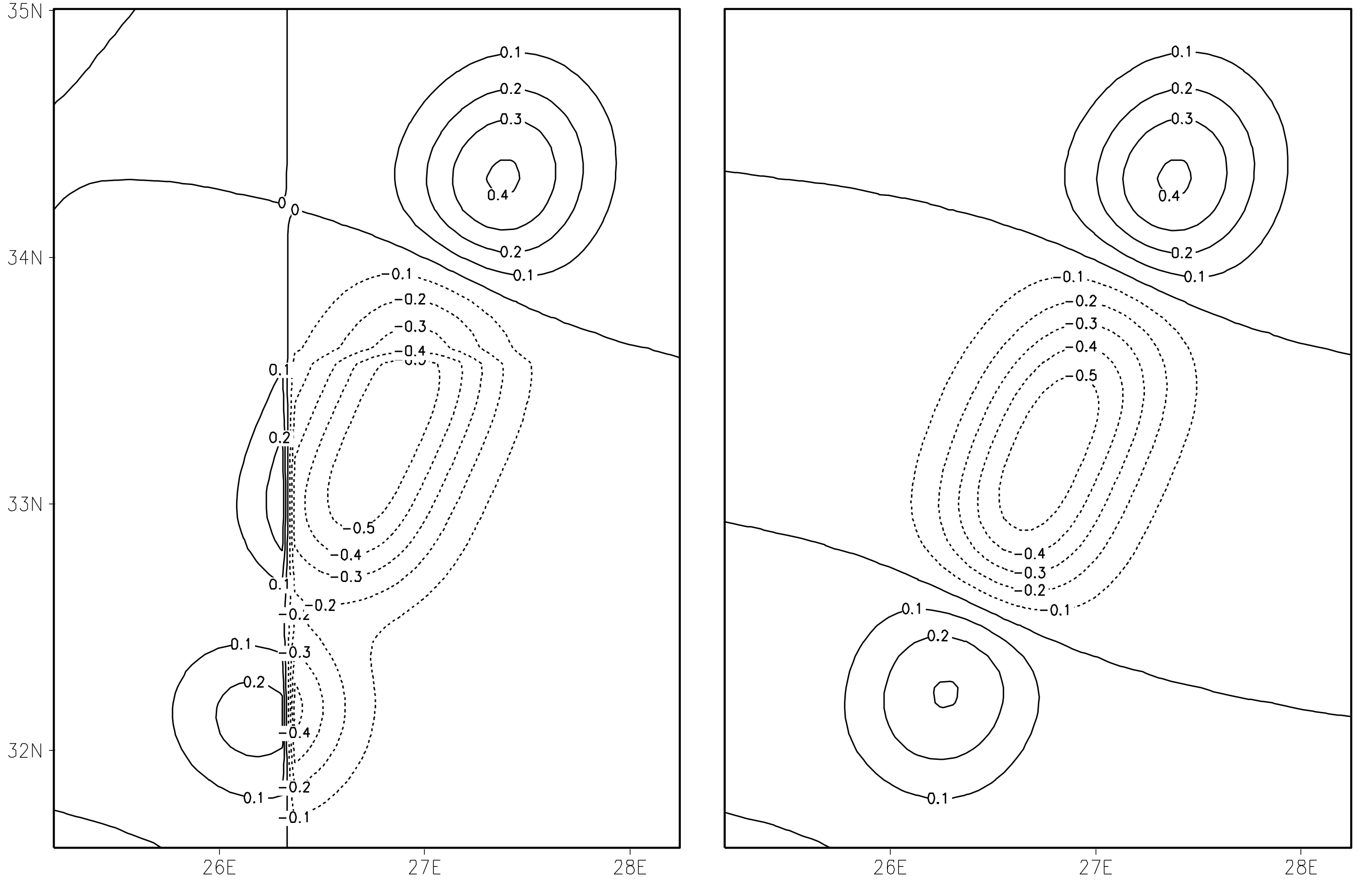
Transformation operators: vertical EOFs

$$\mathbf{V}_V = \mathbf{S}^T \mathbf{\Lambda}^{1/2}$$

Notice that, in order to eliminate jumps between regions, spatial covariances $\mathbf{V}_s$ are modeled by first applying vertical and then horizontal operators (computationally more expensive)

Transformation operators: vertical EOFs

$$\mathbf{V}_S = \mathbf{V}_V \mathbf{V}_H \qquad \mathbf{V}_S = \mathbf{V}_H \mathbf{V}_V$$



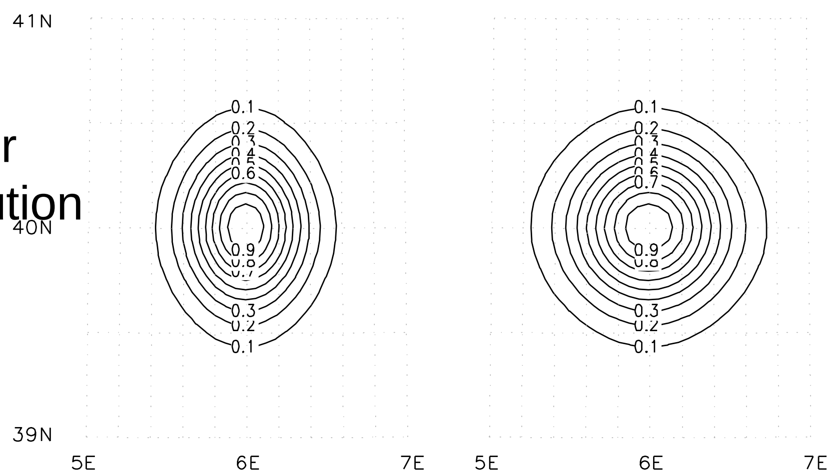
Transformation operators: horizontal

$$\mathbf{V}_H = \mathbf{W}_y \mathbf{G}_y \mathbf{W}_x \mathbf{G}_x$$

G – 1-d recursive filter, **W** – diagonal matrix with normalization factors

Parameters of the filter and normalization factors depend on the horizontal resolution and may be estimated for each model point from a look-up table

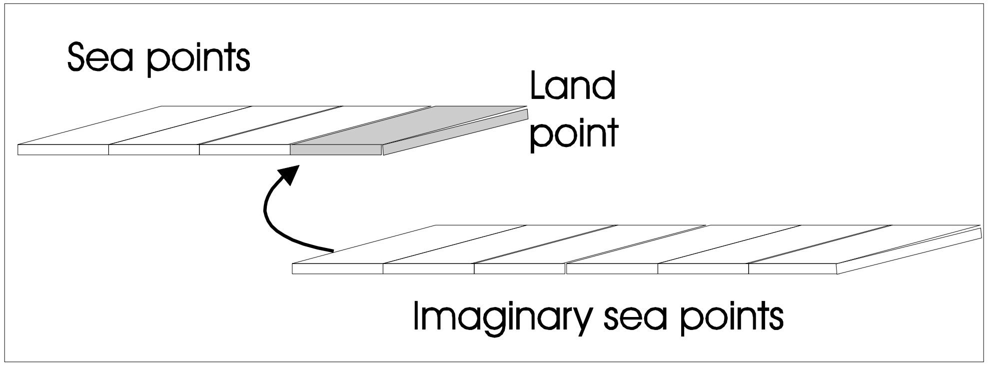
Covariance bubble without accounting for variable lat-lon resolution



Covariance bubble which accounts for variable lat-lon resolution (Lambert projection at 40N)

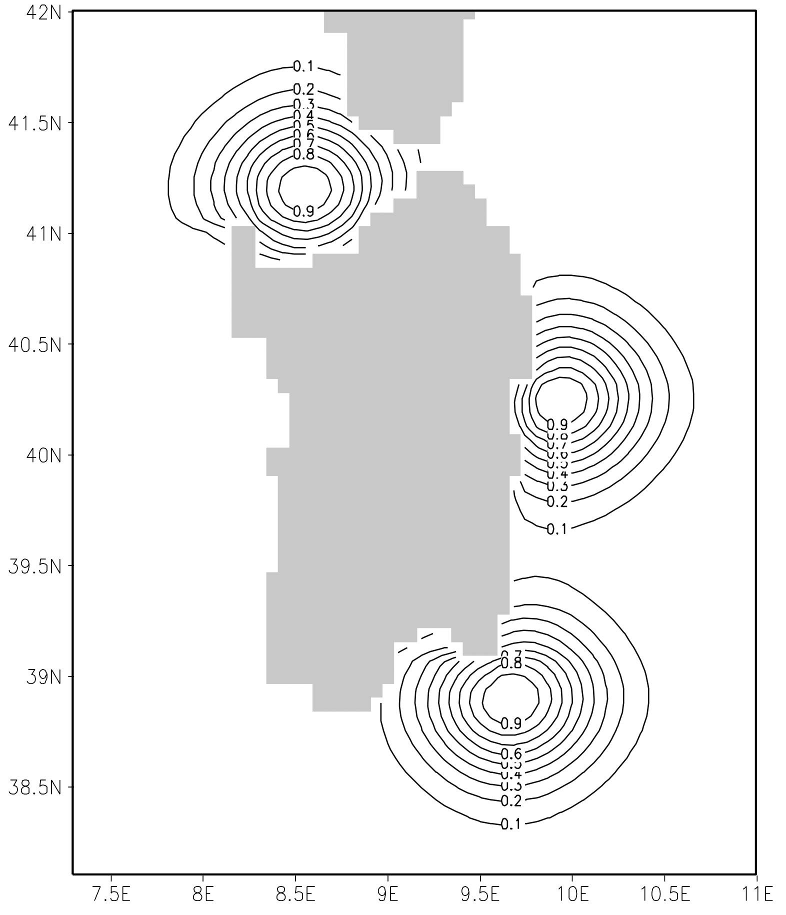
Transformation operators: horizontal

Coastal boundary conditions may be implemented by adding imaginary points at coastlines:



$$\mathbf{V}_H = \left(\mathbf{W}_y \mathbf{G}_y \mathbf{W}_x \mathbf{G}_x + \mathbf{W}_x \mathbf{G}_x \mathbf{W}_y \mathbf{G}_y\right)/\,2$$

Transformation operators: horizontal



Example of filtered field corresponding to 3 unit perturbations close to the coast of Sardinia

Transformation operators: barotropic model

It finds stationary solution of free-surface equations forced by constant perturbations of salinity and temperature:

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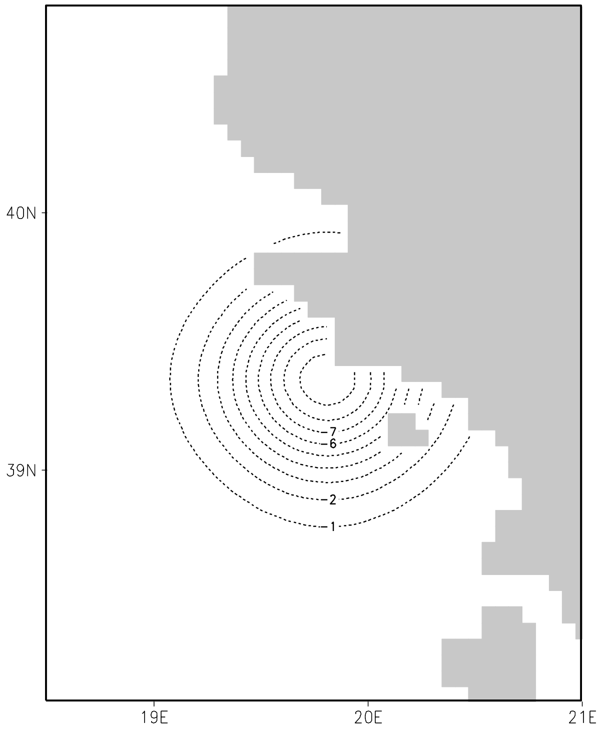
$$\frac{U^{n+1}-U^{n-1}}{\Delta t} - fV^{n} = -gH\frac{\partial \eta^{*}}{\partial x} - \int_{-H}^{0}\left[\int_{-z}^{0}\frac{\partial(\delta b)}{\partial x}dz'\right]dz + \gamma\nabla^{2}U^{n-1}$$

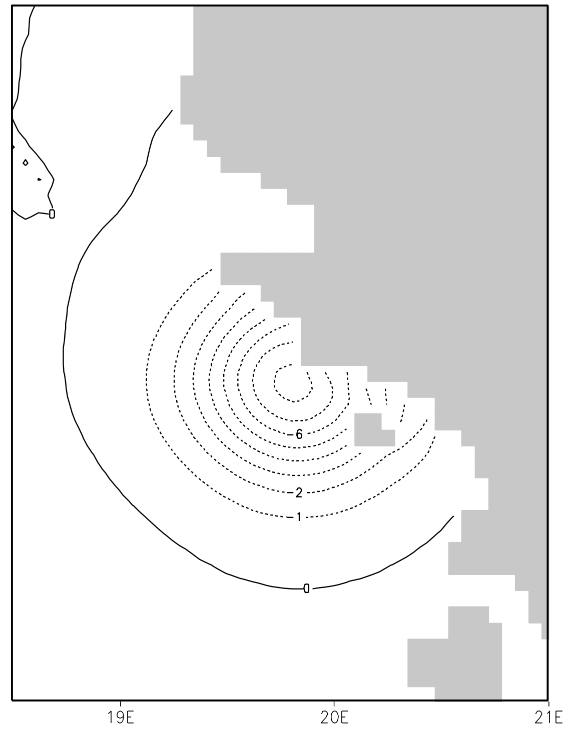
$$\frac{V^{n+1}-V^{n-1}}{\Delta t} + fU^{n} = -gH\frac{\partial \eta^{*}}{\partial y} - \int_{-H}^{0}\left[\int_{-z}^{0}\frac{\partial(\delta b)}{\partial y}dz'\right]dz + \gamma\nabla^{2}V^{n-1}$$

$$\frac{\eta^{n+1}-\eta^{n-1}}{\Delta t} + \left(\frac{\partial U^{*}}{\partial x} + \frac{\partial V^{*}}{\partial y}\right) = 0$$

$$\delta b = g(\delta\rho/\rho_0) \qquad \delta\rho = \alpha\delta T + \beta\delta S$$

Transformation operators: barotropic model





Sea level correction corresponding to an ARGO float observation without barotropic model

Sea level correction corresponding to an ARGO float observation with barotropic model

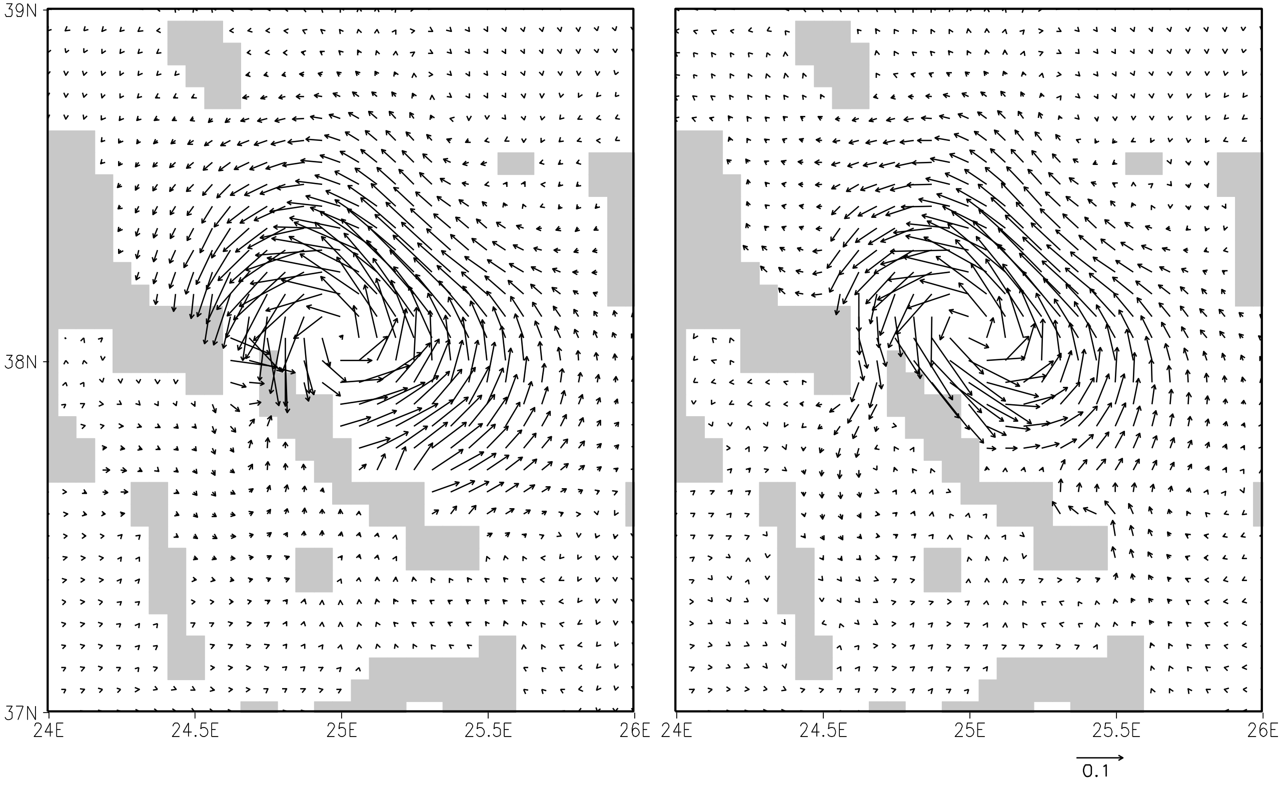
Transformation operators: divergence damping

Velocity close to coasts is adjusted by using the divergence damping filter:

$$\mathbf{v}^{n+1} = \mathbf{v}^{n} + \kappa \nabla D^{n}$$

Transformation operators: divergence damping

Unfiltered



Filtered

Coupling with NEMO