Langevin dynamics of heavy quarks in non-conformal 5D holography

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June 18 2010

Work with E. Kiritsis, U. Gursoy, L. Mazzanti, arXiv:1006.3261

Introduction and Motivation

Fast heavy quarks: important probes of deconfined phase at RHIC.
This talk: Holographic description of Langevin Diffusion of a heavy quark. In particular, *transverse momentum broadening* :

$$p_\perp(t_0) = 0; \qquad \langle p_\perp^2(t)\rangle \simeq 2\kappa_\perp\,(t - t_0)$$

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Goal: Langevin diffusion from a holographic prespective.
Setup: generic 5D theory admitting asymptotically AdS back holes.
(Along the lines of earlier work in the $AdS \leftrightarrow \mathcal{N} = 4$ SYM case, by Gubser '06, Casalderrey-Solana and Teaney '06, Son and Teaney '09, Iancu-Giecold-Mueller '09)

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- Holographic computation of Langevin correlators
  - AdS/CFT prescription for real-time correlators
  - Langevin coefficients
  - An explicit model: comparison with RHIC analysis

The Langevin Equation for a Heavy quark

Langevin equation: dissipative stochastic process.
For a single particle with momentum $\vec{p}(t)$:

$$\frac{d\vec{p}}{dt} = -\eta_D \, \vec{p}(t) + \vec{\xi}(t)$$

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Two forces on the r.h.s:

- η_D: “average” viscous friction force
- $\xi(t)$: Stochastic force with white noise

$$\langle \xi^i(t) \rangle = 0, \qquad \langle \xi^i(t) \xi^j(t') \rangle = \kappa^{ij} \delta(t - t')$$

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Physically, both forces have the same origin: the integrated effect of stochastic interactions with a medium. We will have in mind the motion of a probe quark through the QGP.

Solution of the Langevin equation

Go to 1D for simplicity. Assume η_D $p-$independent

$$\dot{p} = -\eta_D \, p(t) + \xi(t), \qquad \langle \xi(t)\xi(t') \rangle = \kappa \delta(t - t')$$

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$$\Rightarrow \quad p(t) = p_0 e^{-\eta_D t} + \int_0^t dt'\, e^{\eta_D (t'-t)}\xi(t')$$

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$$\langle p^2(t)\rangle - \langle p(t)\rangle^2 = \int_0^t dt'\, e^{\eta_D (t'-t)} \int_0^t dt''\, e^{\eta_D (t''-t)} \langle \xi(t')\xi(t'')\rangle$$

$$= \frac{\kappa}{2\eta_D}\left(1 - e^{-2\eta_D t}\right)$$

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In case $\eta_D = \eta_D(p)$, the short-time analysis still holds.

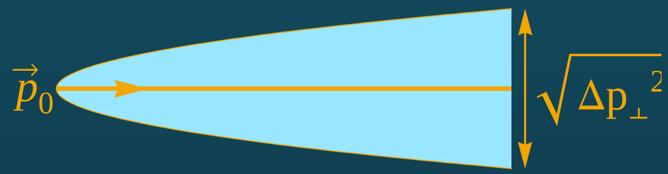
Transverse Momentum Broadening

Take a relativistic quark moving with initial velocity $\vec{v}$ through the plasma.

$$\vec{p} = p^{\|} + p^{\perp}, \qquad \vec{v} \cdot p^{\perp} = 0$$

The transverse momentum obeys a Langevin process with $\langle p^{\perp} \rangle = 0$, but with an increasing dispersion of $p^{\perp}$:

$$\langle (p^{\perp})^2 \rangle \sim 2\kappa^{\perp}\, t$$



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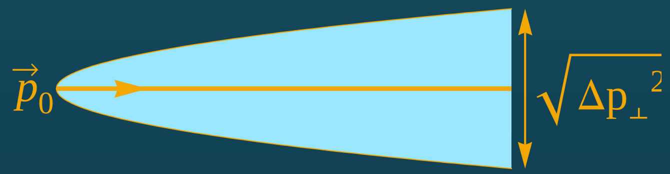


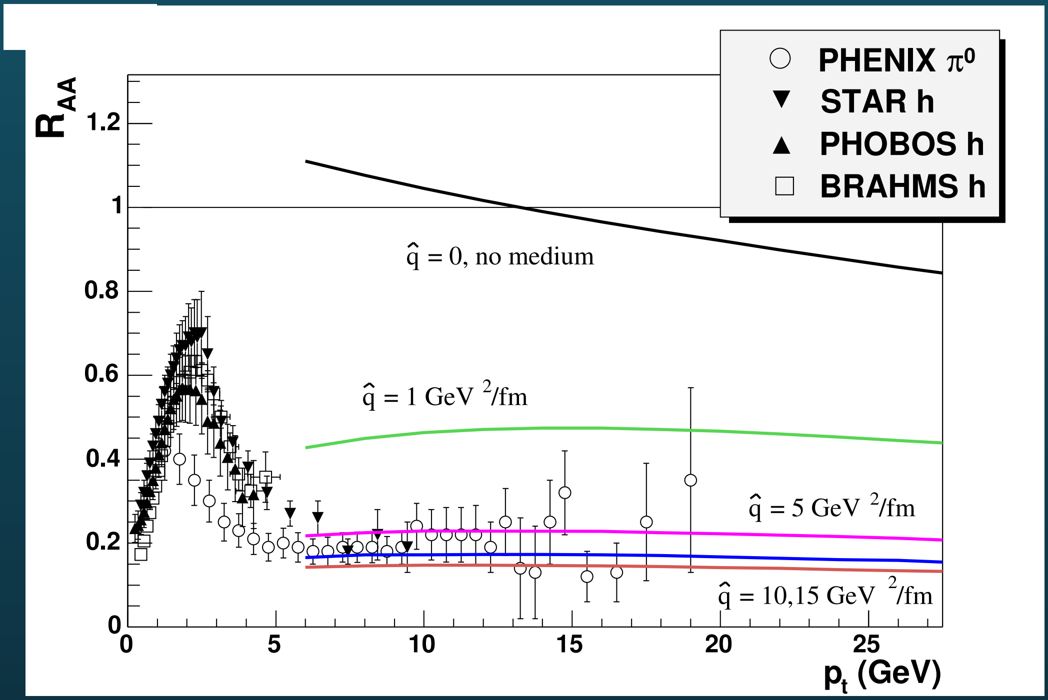
Define the *jet quenching parameter*

$$\hat{q} \equiv \frac{\langle (p^{\perp})^2 \rangle}{mean\, free\, path} = \frac{(p^{\perp})^2}{v\, t} = 2\frac{\kappa^{\perp}}{v}$$

RHIC Analysis

From the analysis of the RHIC data for hard probes production, the preferred values of $\hat{q}$ are in the range:

$$\hat{q} = 5 \sim 15 Gev^2/fm \qquad for \qquad |p_0| \sim 10 GeV, T \sim 200 MeV$$



Nuclear modification factor for light meson suppression.

analysis from Eskola *et al.*, ’05

Langevin coefficients and Holography

The diffusion coefficients are obtained as the zero-frequency limit of correlation functions of certain field theory operators, that can be computed holographically.

One way to see this is by deriving the Langevin equation "microscopically," via the Double Field Formalism.

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$$\rho(Q_f, Q'_f, t) = U(Q_f, Q_0, t, t_0)\, \rho(Q_0, Q'_0, t_0)\, U^\dagger(Q'_f, Q'_0, t, t_0)$$

the evolution operator is the path integral:

$$U(Q, Q_0, t, t_0) = \int \mathcal{D}Q\, e^{iS[Q]} = \int \mathcal{D}Q\, e^{i \int_{t_0}^{t_f} L(Q,\dot{Q})} \qquad \begin{array}{c} Q(t_0) = Q_0 \\ Q(t) = Q_f \end{array}$$

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The density matrix at time t is a double path-integral:

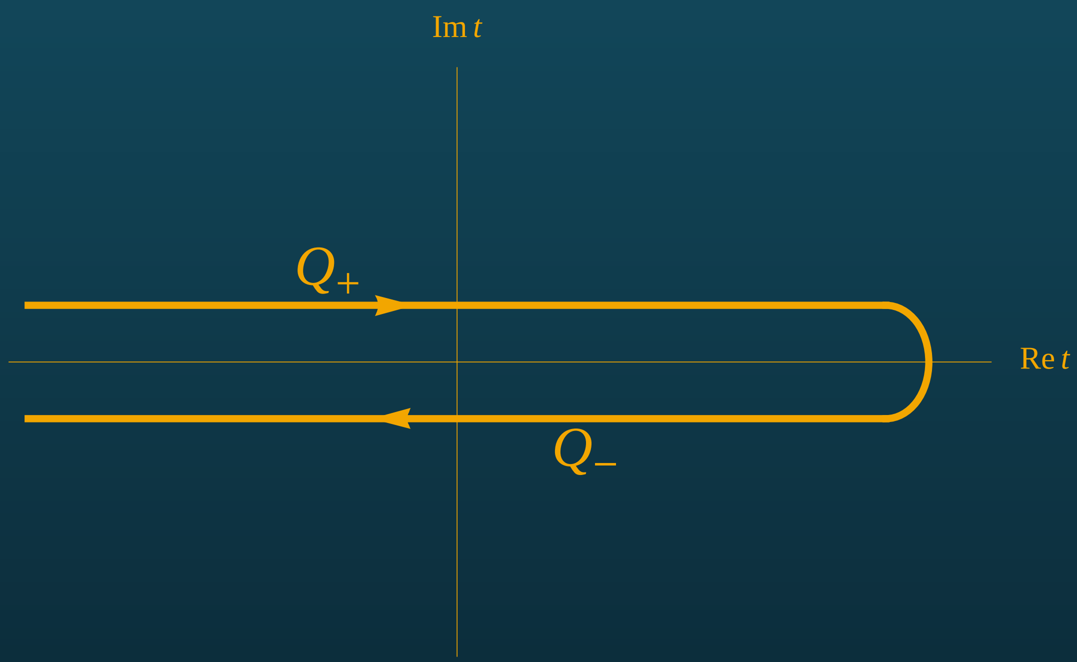
$$\rho(Q_f, Q'_f, t) = \int \mathcal{D}Q \int \mathcal{D}Q'\, e^{(iS[Q]-iS[Q'])}\, \rho(Q_0, Q'_0, t_0)$$

Double Field Formalism

It is natural to "double" the fields in the path integral:

$$\rho(Q_+, Q_-, t) = \int \mathcal{D}Q_+ \int \mathcal{D}Q_- e^{i(S[Q_+]-S[Q_-])} \rho(Q_0, Q_0', t_0)$$

think of $Q_+ \equiv Q$, $Q_- \equiv Q'$ as independent field variables, or as a single field defined on the contour:



Particle coupled to a bath

Now consider a system formed by:

- a single particle $X(t)$ (the probe quark)
- a QFT in a statistical ensemble (the QGP) with d.o.f. $\Phi(x)$

$$S = S_0[X(t)] + S_{QGP}[\Phi(x,t)] + S_{int}[X, \Phi(x)]$$

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- Suppose the particle starts at position x_i at $t_i = -\infty$, then

$$\rho_i = \delta(X - x_i)\delta(X' - x_i) \otimes \rho_i^{bath}(\Phi, \Phi')$$

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We are interested in the *reduced density matrix* at time t:

$$\rho(X, X', t) = Tr_{\Phi(x)}\left[\rho(X, X'; \Phi, \Phi'; t)\right]$$

Reduced density matrix

$$\rho(X, X', t) = \int \mathcal{D}X_+ \int \mathcal{D}X_- e^{(iS_0[X_+]-iS_0[X_-])} \times$$

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$$\rho(X, X', t) = \int \mathcal{D}X_+ \int \mathcal{D}X_- e^{(iS_0[X_+] - iS_0[X_-])} \times$$

$$\int_{\Phi_{+f}=\Phi_{-f}} \mathcal{D}\left[\Phi_\pm(x)\right] e^{i\left(S[\Phi_+] - S[\Phi_-] + \int dt\, X_+(t)\mathcal{F}_+(t) - \int dt\, X_-(t)\mathcal{F}_-(t)\right)} \rho_i^{bath}$$

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The reduced density matrix evolves according to the effective action:

$$S_{eff}[X_+, X_-] = S_0[X_+] - S_0[X_-] + S_{IF}[X_+, X_-]$$

$$e^{iS_{IF}[X_+,X_-]} = \left\langle e^{i\int dt\, X_+(t)\mathcal{F}_+(t)-i\int dt\, X_-(t)\mathcal{F}_-(t)} \right\rangle_{bath}$$

Feynman and Vernon, ’63

Effective Action (Influence Functional)

Expand the exponential to quadratic order in $X\mathcal{F}$.

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$$\left\langle e^{i\int dt\, X_+(t)\mathcal{F}_+(t) - i\int dt\, X_-(t)\mathcal{F}_-(t)} \right\rangle_{bath} \simeq 1 + i \int dt\, \langle \mathcal{F}(t) \rangle (X_+ - X_-)$$

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$$-\frac{i}{2}\int dtdt' \Big[- X_+(t) i\langle \mathcal{F}_+(t)\mathcal{F}_+(t')\rangle X_+(t') + X_-(t) i\langle \mathcal{F}_-(t)\mathcal{F}_+(t')\rangle X_+(t')$$

$$+X_+(t) i\langle \mathcal{F}_+(t)\mathcal{F}_-(t')\rangle X_-(t') - X_-(t) i\langle \mathcal{F}_-(t)\mathcal{F}_-(t')\rangle X_-(t') \Big]$$

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$$-\frac{i}{2}\int dt dt'\Big[-X_+(t)i\langle\mathcal{F}_+(t)\mathcal{F}_+(t')\rangle X_+(t') + X_-(t)i\langle\mathcal{F}_-(t)\mathcal{F}_+(t')\rangle X_+(t')$$

$$+X_+(t)i\langle\mathcal{F}_+(t)\mathcal{F}_-(t')\rangle X_-(t') - X_-(t)i\langle\mathcal{F}_-(t)\mathcal{F}_-(t')\rangle X_-(t')\Big]$$

$$\simeq \exp\left[i\int dt\,\langle\mathcal{F}\rangle(X_+ - X_-) - \frac{i}{2}X_a(t)G_{ab}(t,t')X_b(t')\right]$$

$$G_{ab}(t,t') = \langle\mathcal{P}\mathcal{F}_a(t)\mathcal{F}_b(t')\rangle \qquad a,b = +,-$$

$\mathcal{P}$: Path Ordering along the Keldysh contour.

Path Ordering

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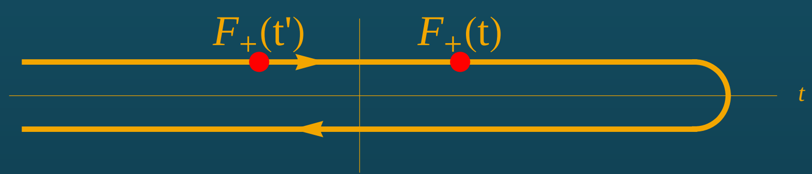
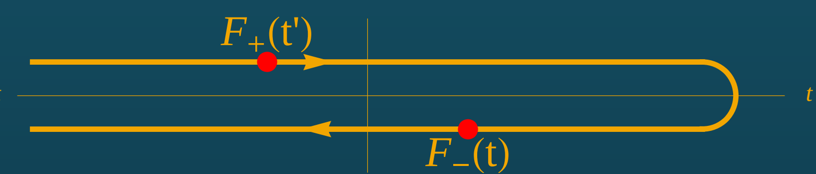
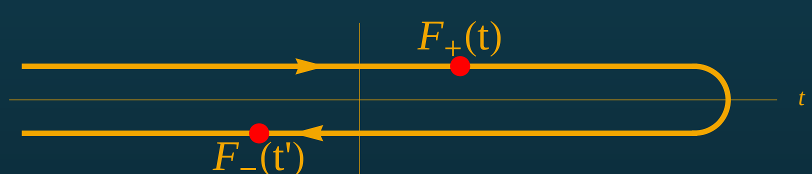
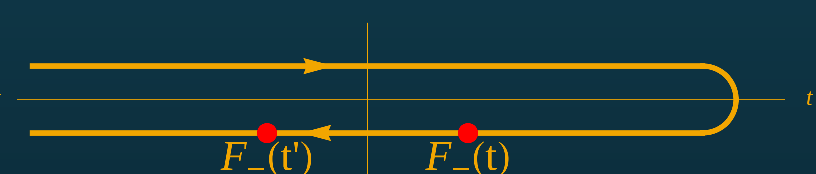
$$G_{++}(t,t') = -i \left\langle T\, \mathcal{F}_+(t)\mathcal{F}_+(t') \right\rangle \qquad G_{-+}(t,t') = -i \left\langle \mathcal{F}_-(t)\mathcal{F}_+(t') \right\rangle$$





$$G_{+-}(t,t') = -i \left\langle \mathcal{F}_-(t')\mathcal{F}_+(t) \right\rangle \qquad G_{--}(t,t') = -i \left\langle \overline{T}\, \mathcal{F}_-(t)\mathcal{F}_-(t') \right\rangle$$





Keldysh Propagators

In operator formalism:

$$G_R(t) = -i\theta(t)\left\langle \left[\mathcal{F}(t), \mathcal{F}(0)\right] \right\rangle, \quad G_A(t) = i\theta(-t)\left\langle \left[\mathcal{F}(t), \mathcal{F}(0)\right] \right\rangle$$

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$$G_{sym}(t) = -\frac{i}{2}\left\langle\, \left\{\mathcal{F}(t), \mathcal{F}(0)\right\}\,\right\rangle, \quad G_{asym}(t) = -\frac{i}{2}\left\langle\, \left[\mathcal{F}(t), \mathcal{F}(0)\right]\,\right\rangle$$

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$$\langle T\mathcal{F}(t)\mathcal{F}(0)\rangle = \theta(t)\langle\mathcal{F}(t)\mathcal{F}(0)\rangle + \theta(-t)\langle\mathcal{F}(0)\mathcal{F}(t)\rangle = G_{sym} + \frac{1}{2}(G_R + G_A)$$

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$$G_R(t) = -i\theta(t)\left\langle\,\left[\mathcal{F}(t),\mathcal{F}(0)\right]\,\right\rangle, \qquad G_A(t) = i\theta(-t)\left\langle\,\left[\mathcal{F}(t),\mathcal{F}(0)\right]\,\right\rangle$$

$$G_{sym}(t) = -\tfrac{i}{2}\left\langle\,\left\{\mathcal{F}(t),\mathcal{F}(0)\right\}\,\right\rangle, \qquad G_{asym}(t) = -\tfrac{i}{2}\left\langle\,\left[\mathcal{F}(t),\mathcal{F}(0)\right]\,\right\rangle$$

$$G_{asym}(t) = G_R(t) - G_A(t), \qquad G_R(-t) = G_A(t)$$

$$\langle T\mathcal{F}(t)\mathcal{F}(0)\rangle = \theta(t)\langle\mathcal{F}(t)\mathcal{F}(0)\rangle + \theta(-t)\langle\mathcal{F}(0)\mathcal{F}(t)\rangle = G_{sym} + \frac{1}{2}(G_R + G_A)$$

$$G_{++} = G_{sym} + \frac{1}{2}(G_R + G_A), \qquad G_{--} = G_{sym} - \frac{1}{2}(G_R + G_A)$$

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$$G_{++} + G_{--} - G_{+-} - G_{-+} = 0$$

Effective Action

Putting it all together:

$$S_{eff} = S_0[X_+] - S_0[X_-]$$

$$+ \int (X_+ - X_-) G_R (X_+ + X_-) + \frac{1}{2} \int (X_+ - X_-) G_{sym} (X_+ - X_-)$$

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Define $X_{cl} = (X_+ + X_-)/2, \qquad y \equiv X_+ - X_-$.
In the semiclassical limit, $y \ll X_{cl}$, and we can expand:

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$$S_{eff}[X_{cl}, y] \simeq \int dt \; y(t) \left(\frac{\delta S_0}{\delta X_{cl}(t)} + \int dt' \, G_R(t,t') X_{cl}(t') \right)$$
$$+ \frac{1}{2} \int dt dt' \; y(t) G_{sym}(t,t') y(t')$$

Enter the Noise

The path integral for X has become:

$$Z = \int \mathcal{D}X_{cl} \int \mathcal{D}y \, exp\left[i \int dt \, y(t) \left(\frac{\delta S_0}{\delta X_{cl}(t)} + \int dt' \, G_R(t,t') X_{cl}(t') \right) + \frac{1}{2} \int dt dt' \, y(t) G_{sym}(t,t') y(t') \right]$$

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"Integrate in" a Gaussian variable $\xi(t)$ with variance $G_{sym}(t,t')$:

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Now integrate over y to get a δ-functional in X_{cl}

$$Z = \int \mathcal{D}\xi \int \mathcal{D}X_{cl} \delta \left(\frac{\delta S_0}{\delta X_{cl}} + G_R X_{cl} - \xi \right) e^{-\frac{1}{2}\xi G_{sym}^{-1} \xi}$$

Generalized Langevin Equation

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The path integral is concentrated on the solution of the classical equation with a stochastic force:

$$\frac{\delta S_0}{\delta X_{cl}(t)} + \int_{-\infty}^{t} dt'\, G_R(t-t') X_{cl}(t') = \xi(t) \qquad \langle \xi(t)\xi(t')\rangle = G_{sym}(t-t')$$

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Generalized Langevin Equation

For a free particle, $\delta S_0/\delta X_{cl} = \dot{P}$. Integrate second term by parts:

$$\dot{P} + \int_0^{+\infty} dt'\, \gamma(t')\dot{X}(t-t') = \xi(t') \qquad \frac{d}{dt}\gamma(t) = G_{asym}(t)$$

Recovering Local Langevin Equation

$$\dot{P} + \int_0^{+\infty} dt'\, \gamma(t') \dot{X}(t-t') = \xi(t') \qquad \langle \xi(t)\xi(t') \rangle = G_{sym}(t,t')$$

Suppose $t \gg \tau_c$ (autocorrelation time of the force)

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$$\dot{P} = -\eta_D P(t) + \xi(t) \qquad \eta_D \equiv \frac{\dot{X}}{P}\eta \left(= \frac{\eta}{\gamma M} \text{for a relativistic particle} \right)$$

Fourier Space

Go to Fourier space:

$$G_{sym}(\omega) = \int_{-\infty}^{+\infty} dt\, e^{i\omega t} G_{sym}(t),$$

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- Relation between $G_{sym}(\omega)$ and $G_R(\omega)$ is ensemble-dependent.
- For an equilibrium ensemble at temperature T:

$$G_{sym}(\omega) = -\coth\left(\frac{\omega}{2T}\right) Im G_R(\omega) \quad \Rightarrow \quad \kappa = 2T\eta = 2MT\eta_D$$

Einstein's Relation for a non relativistic particle

Holographic correlators

- The functions entering the generalized langevin equations for a point particle are given in terms of correlation functions of the QFT operator $\mathcal{F}$ that sees $X(t)$ as an external source

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  2. Solve its classical equations of motion
  3. Find the correlators from the boundary on-shell action, following the Son-Starinets prescription for real-time Green's functions

5D Holographic setup

We consider a 5D black hole background (in the string frame)

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

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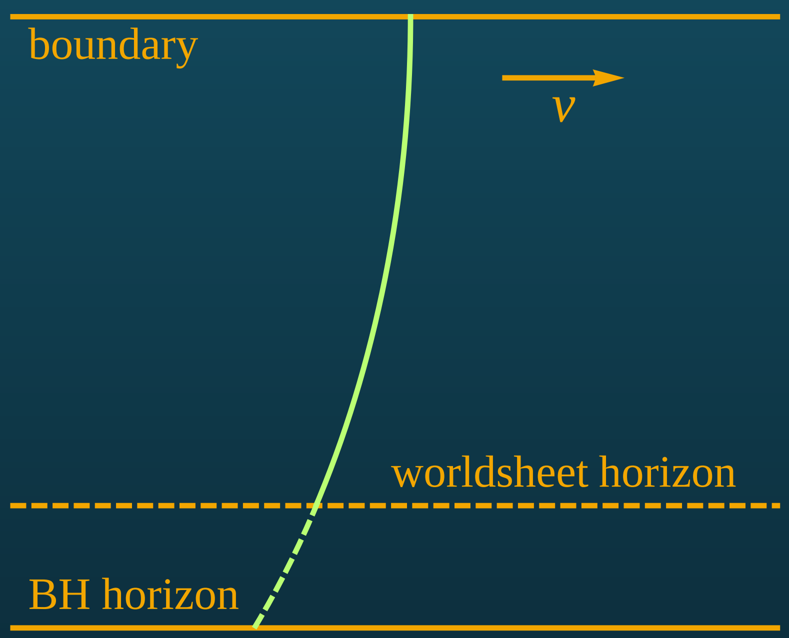
The dual gauge theory is in a deconfined phase.

Trailing string

Probe quark moving at velocity v on the boundary

$\Updownarrow$

Classical string attached at the boundary and extending in the interior.



Trailing string profile

$$S_{NG} = -\frac{1}{2\pi\ell_s^2}\int dtdr\sqrt{\det \hat{g}_{\alpha\beta}}, \quad \hat{g}_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu, \quad \alpha,\beta = t,r$$

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Induced worldsheet metric:

$$g_{\alpha\beta} = b^2(r)\begin{pmatrix} v^2 - f(r) & v\xi'(r) \\ v\xi'(r) & f(r)^{-1} + \xi'^2 \end{pmatrix},$$

$Horizon\,@\,f(r_s) = v^2$

$0 < r_s < r_h$

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Drag froce in the longitudinal direction:

$$\frac{dp^{\parallel}}{dt} = -\frac{b^2(r_s)}{2\pi\ell_s^2}v \equiv -\eta_D^{class}\,p^{\parallel} \qquad \eta_D^{class} = \frac{1}{M\gamma}\frac{b^2(r_s)}{2\pi\ell_s^2}$$

Gürsoy, Kiritsis, Michalogeorgakis, FN, ’09

The Worldsheet Black Hole

The 2D metric can be diagonalized by a diffeomorphism:

$$t = \tau + \zeta(r), \qquad \zeta' = \frac{v\xi'}{f - v^2}$$

and it becomes that of a 2D black hole, $\quad (b_s \equiv b(r_s)$, etc)

$$ds^2 = b^2 \left[-(f - v^2) d\tau^2 + \frac{b^4}{(b^4 f - C^4)} dr^2 \right], \qquad C = v b_s^2$$

with horizon $r = r_s$ where $f(r_s) = v^2$. $\qquad r_s < r_h$.

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Hawking temperature:

$$T_s = \frac{1}{4\pi} \sqrt{f_s f_s' \left[\frac{4 b_s'}{b_s} + \frac{f_s'}{f_s} \right]} \quad \begin{cases} \sim T & v \ll 1 \\ \sim T/\sqrt{\gamma} & v \sim 1 \end{cases}$$

Traling string fluctuations

Now we consider fluctuations around the trailing string solution:

$$\vec{X}(t,r) = \big(vt + \xi(r)\big)\frac{\vec{v}}{v} + \delta\vec{X}(r,t)$$

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$\delta\vec{X}(r,t)$ is the bulk field dual to the boundary operator $\vec{\mathcal{F}}$

$\Rightarrow$ According to the AdS/CFT prescription correlators of $\vec{\mathcal{F}}$ are obtained from the solutions of the wave equation for $\delta\vec{X}(r,t)$

Quadratic action

To obtain the linear equations for $\delta\vec{X}$, expand the string NG action to quadratic order around the traling string.
In the diagonal WS frame:

$$S_{NG}^{(2)} = -\frac{1}{2\pi\ell_s^2}\int d\tau dr\,\frac{1}{2}H^{\alpha\beta}\left[\frac{1}{Z^2}\partial_\alpha\delta X^{\|}\partial_\beta\delta X^{\|} + \sum_{i=2}^{3}\partial_\alpha\delta X_i^{\perp}\partial_\beta\delta X_i^{\perp}\right]$$

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$$H^{\alpha\beta} = \begin{pmatrix} -\frac{b^4}{\sqrt{(f-v^2)(b^4f-C^2)}} & 0 \\ 0 & \sqrt{(f-v^2)(b^4f-C^2)} \end{pmatrix},$$

$$Z \equiv b^2\sqrt{(f-v^2)/(b^4f-C^2)}$$

Asymptotic solutions

Look for harmonic solutions: $\delta\vec{X}(r,\tau) = e^{-i\omega\tau}\delta\vec{X}(r,\omega)$

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- Near the boundary $r \to 0$:

$$\partial_r^2\Psi(r,\omega) - \frac{2}{r}\partial_r\Psi(r,\omega) + \gamma^2\omega^2\Psi(r,\omega) = 0, \qquad \Psi \equiv \delta X^{\|,\perp}$$

Solutions are linear combinations of a normalizable and a non-normaliable mode, $\Psi(r,\omega) \sim C_s + C_v r^3$

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- Near the worldsheet horizon r_s:

$$\partial_r^2\Psi + \frac{1}{(r_s - r)}\partial_r\Psi + \left(\frac{\omega}{4\pi T_s(r_s - r)}\right)^2\Psi = 0$$

  Solutions are infalling and outgoing waves:

$$\Psi(r,\omega) \sim C_{out}(r_s - r)^{+\frac{i\omega}{4\pi T_s}} + C_{in}(r_s - r)^{-\frac{i\omega}{4\pi T_s}}$$

Retaded Correlator

$$S = \int dr\,dt\,\mathcal{H}^{\alpha\beta}\partial_\alpha\Psi\partial_\beta\Psi \qquad \mathcal{H}^{\alpha\beta} = \begin{cases} \frac{1}{2\pi\ell_s^2}H^{\alpha\beta} & \perp \\ \frac{1}{2\pi\ell_s^2}H^{\alpha\beta}/Z^2 & \parallel \end{cases}$$

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Son + Starinets prescription for the retarded propagator:

$$G_R(\omega) = \mathcal{H}^{r\alpha}\, \Psi_R^*(r,\omega)\, \partial_\alpha\, \Psi_R(r,\omega)\Big|_{Boundary}$$

Ψ_R is the solution with boundary conditions:

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Retaded Correlator

$$S = \int dr dt \, \mathcal{H}^{\alpha\beta} \partial_\alpha \Psi \partial_\beta \Psi \qquad \mathcal{H}^{\alpha\beta} = \begin{cases} \frac{1}{2\pi\ell_s^2} H^{\alpha\beta} & \perp \\ \frac{1}{2\pi\ell_s^2} H^{\alpha\beta}/Z^2 & \parallel \end{cases}$$

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This is not the thermal equilibrium relation for fluctuations in the plasma, since $T_s \neq T$.

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Langevin Friction terms

$$\dot{\vec{p}} = -\eta_D^{\parallel}(p) p^{\parallel} \hat{v} - \eta_D^{\perp}(p) \vec{p}^{\perp} + \vec{\xi}(t).$$

To connect with holographic calculation we must write the Langevin equations into equations for δX.

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Expand to first order $\Rightarrow$ equations for position fluctuations:

$$\gamma M\delta\ddot{X}^{\perp} = -\eta^{\perp}\delta\dot{X}^{\perp} + \xi^{\perp}(t),$$
$$\gamma^3 M\delta\ddot{X}^{\parallel} = -\eta^{\parallel}\delta\dot{X}^{\parallel}(t) + \xi^{\parallel}(t)$$

$$\eta^{\perp} = \frac{1}{\gamma M}\eta_D^{\perp} \qquad \eta^{\parallel} = \frac{1}{\gamma^3 M}\left[\eta_D^{\parallel} + \gamma M v\frac{\partial\eta_D^{\parallel}}{\partial p}\right]$$

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Consistency check: $\eta_D^{\|} = \eta_D^{\perp} = \eta_D^{class} = (2\pi\ell_s^2)^{-1} b(r_s)^2/M\gamma$
satisfiies both relations!

Validity of the local approximation

- *Local* Langevin equation: *long time* approximation,

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Consistency:

$$1/\eta_D \gg 1/T_s$$

If this fails, need the full non-local generalized Langevin dynamics

Einstein-Dilaton gravity

A simple 5D concrete realization.

$$S_E = -M_p^3 \int d^5x \sqrt{-g} \left[R - \frac{4}{3} (\partial\Phi)^2 - V(\Phi) \right]$$

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- Specific choice of $V(\Phi)$ gives good agreeement with Lattice thermo and spectra (cfr. L. Mazzanti's talk)

Qhat

We can compute the transverse momentum broadeining broadening numerically, given the solution for the background metric

$$\hat{q} = 2\frac{\kappa^{\perp}}{v} = \frac{2b^2(r_s)}{\pi \ell_s^2}\frac{T_s}{v}$$

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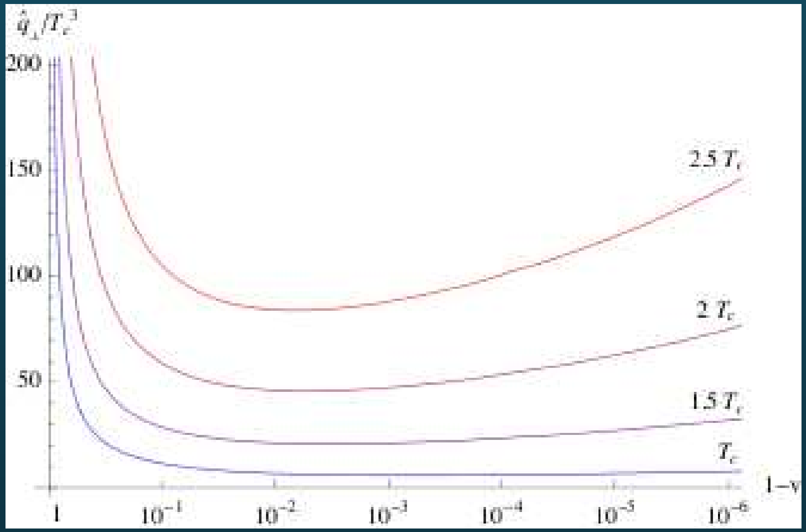

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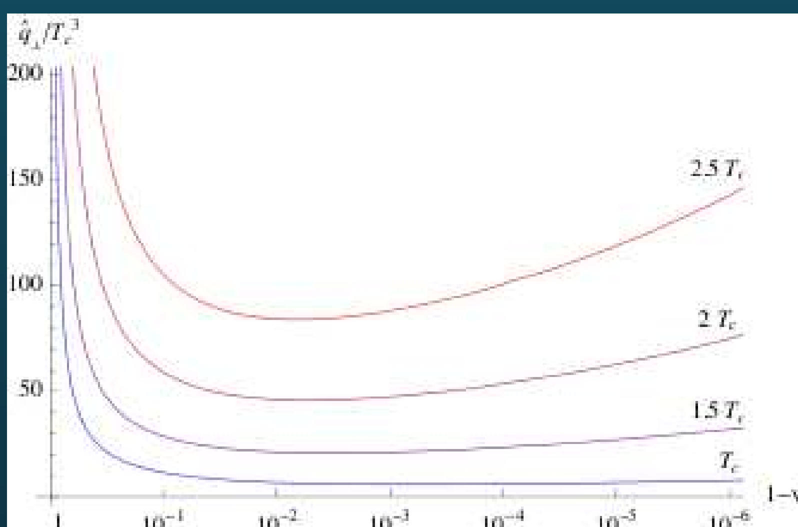

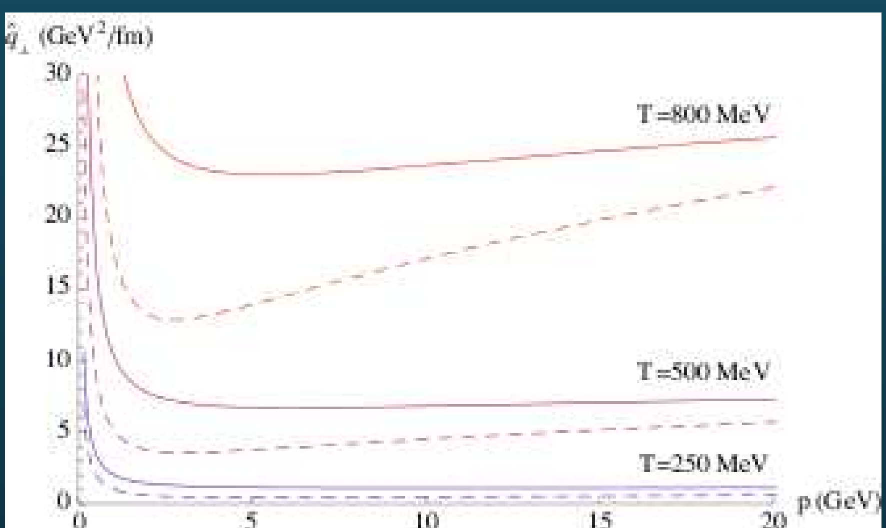

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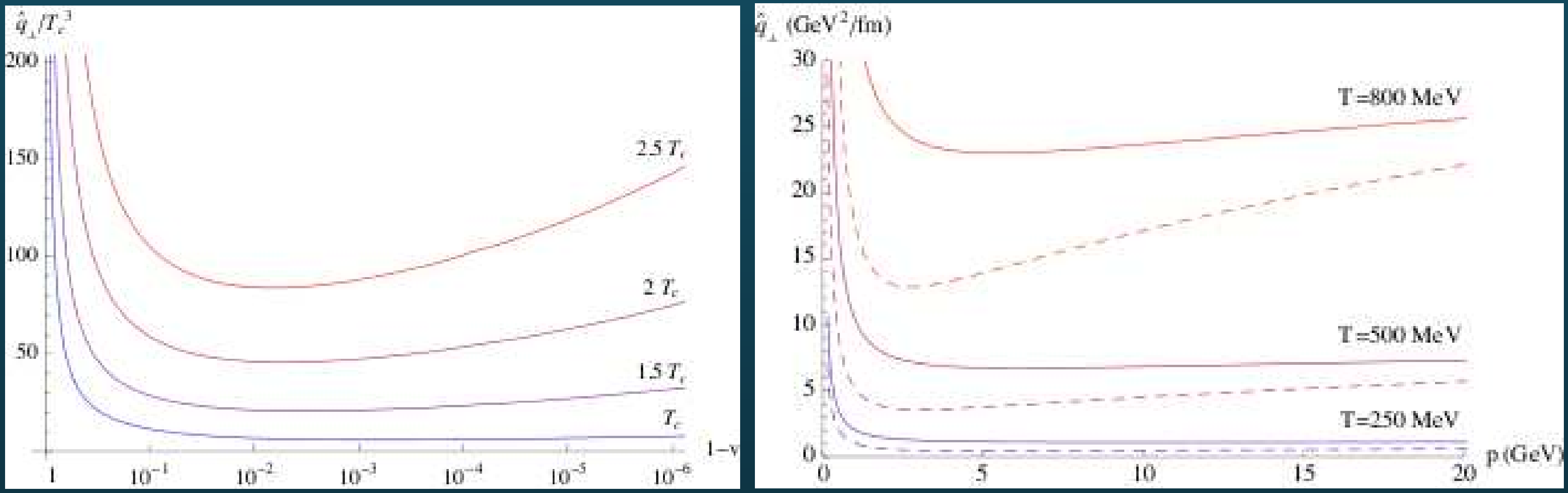

The order of magnitude is correct. Comparison to data is hard due to momentum dependence.

Validity of the local approximation, pt. 2

Parametrization in terms of $\hat{q}$ justified if:

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This translates to a bound on quark momentum:

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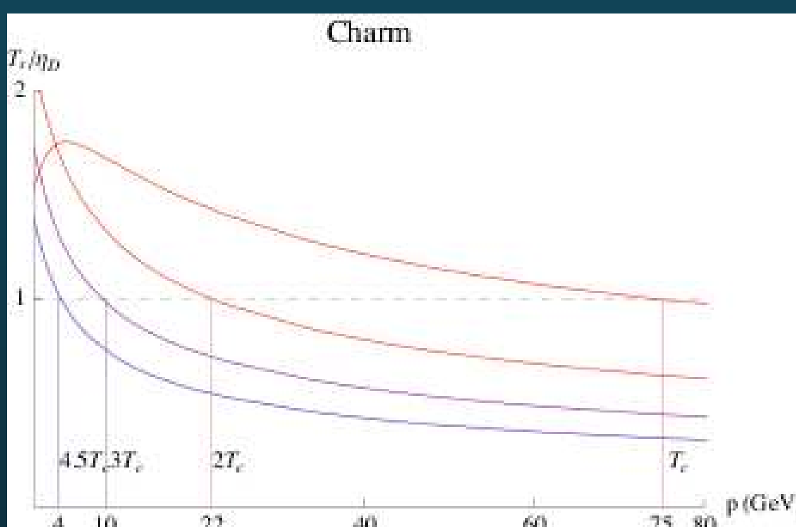

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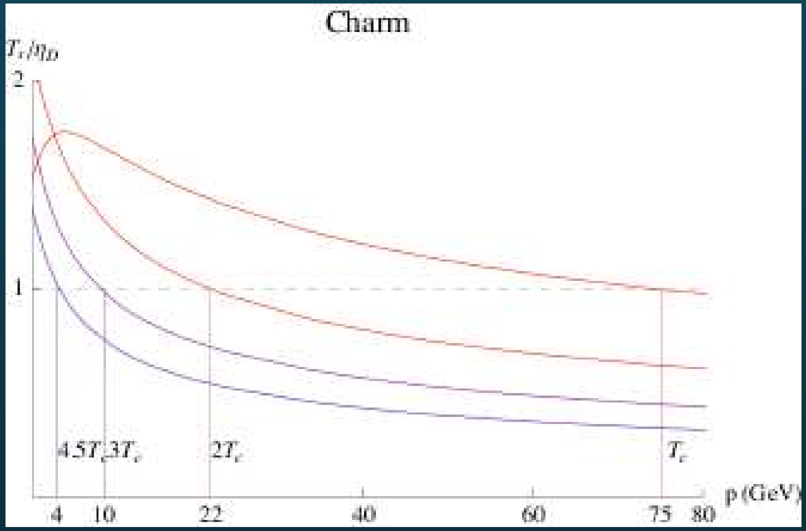

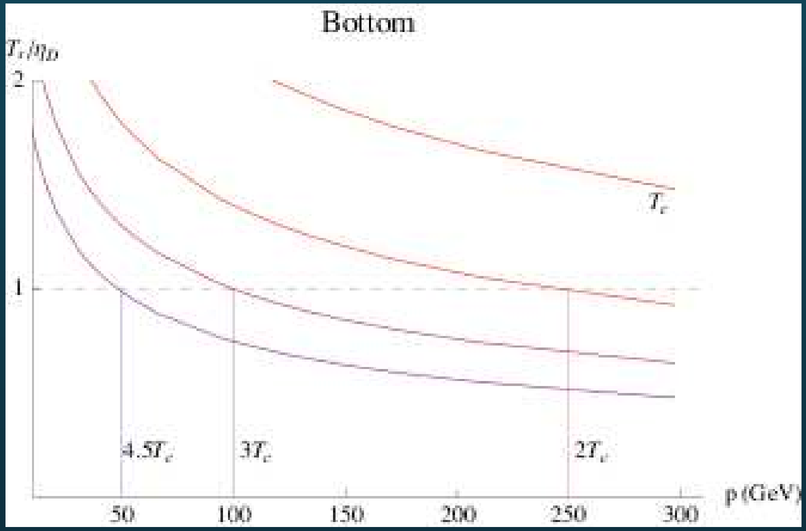

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- Compare with direct results for b/c suppression (not much available at the moment, wait for LHC)