Transport coefficients in Higher derivative gravity - II
A radial flow of Green's function

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Nordita

Hydrodynamic system

- *Long wavelength* effective description of strongly coupled field theory.
- It is formulated in the language of constituent equations.
- The simplest case: no global conserved currents

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \Rightarrow d+1 \textit{ equations}$$

- Need to reduce the number of independent elements of $T^{\mu\nu}$.

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- We consider the fluid is in local thermal equilibrium and fluctuations are of small energy.
- At any given time the system is described by the following local quantities

Temperature: $T(x)$ $\hookrightarrow 1$
Velocity vector: $u^{\mu}(x)$ $\hookrightarrow d+1$

- Velocity vectors are normalized : $u^{\mu}u_{\nu} = -1$
  - Total $d+1$ variables.
- In hydrodynamics we express $T^{\mu\nu}$ through $T(x)$ and $u^{\mu}(x)$ through the so-called constitutive equations.

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- As fluid dynamics is a long wavelength effective theory the constitutive relations are usually specified in a derivative expansion.
- At any given order, thermodynamics plus symmetries determine the form of this expansion up to a finite number of undetermined coefficients.
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0^{th} order

$$T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} + Pg^{\mu\nu}$$

$\epsilon \rightarrow$ energy density and $P \rightarrow$ pressure.

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1^{st} order

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}.$$

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \left[\eta \left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \nabla_\lambda u^\lambda\right) + \zeta g_{\alpha\beta} \nabla_\lambda u^\lambda\right].$$

$P^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu \to$ Projection operator.

$\eta \rightsquigarrow$ shear viscosity coefficient
$\zeta \rightsquigarrow$ bulk viscosity coefficients

For conformal fluid $\zeta = 0$.

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Hydrodynamic system

2^{nd} order

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \sigma^{\mu\nu} + \Theta^{\mu\nu}.$$

$$\begin{aligned}\Theta^{\mu\nu} = \eta\tau_\Pi &\left[{}^{\langle}D\sigma^{\mu\nu\rangle} + \frac{1}{d-1}\sigma^{\mu\nu}(\nabla\cdot u)\right] \\ &+ \kappa\left[R^{\langle\mu\nu\rangle} - (d-2)u_\alpha R^{\alpha\langle\mu\nu\rangle\beta}u_\beta\right] \\ &+ \lambda_1\sigma^{\langle\mu}{}_\lambda\sigma^{\nu\rangle\lambda} + \lambda_2\sigma^{\langle\mu}{}_\lambda\Omega^{\nu\rangle\lambda} + \lambda_3\Omega^{\langle\mu}{}_\lambda\Omega^{\nu\rangle\lambda}.\end{aligned}$$

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Kubo formula: Transport coefficients from thermal correlators

- Consider the response of the fluid to small and smooth metric perturbations: $g_{xy} = \eta_{xy} + h_{xy}(t,z)$

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu} - \sigma^{\mu\nu} + \Theta^{\mu\nu}$$
$$\Downarrow$$
$$T^{xy} = -Ph_{xy} - \eta\dot{h}_{xy} + \eta\tau_\Pi\ddot{h}_{xy} - \frac{\kappa}{2}[(d-3)\ddot{h}_{xy} + \partial_z^2 h_{xy}]\,.$$

- In linear response theory

$$\langle T_{xy}\rangle \sim G^R_{xy,xy}h_{xy}$$

$$G_R^{xy,xy}(\omega,k) = P - i\eta\omega + \eta\tau_\Pi\omega^2 - \frac{\kappa}{2}[(d-3)\omega^2 + k^2]\,.$$

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Holographic computation of Green's function

- We start we five dimensional action

$$S_{\rm EM} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\,(R+12)\,.$$

- The background has a black-brane solution as,

$$\begin{aligned} dS^2 &= -g_{tt}dt^2 + g_{rr}dr^2 + g_{ij}dx^2dx^j, \\ g_{tt} &= r^2(1-\frac{1}{r^4}), \quad g_{rr} = \frac{1}{g_{tt}} \\ g_{ij} &= r^2\delta_{ij}. \end{aligned}$$

- Solution is asymptotically AdS and boundary topology $\rightarrow R\times R^3$.
- The horizon is at $r\rightarrow 1$ and asymptotic boundary is at $r\rightarrow\infty$.

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- We study the graviton's fluctuation in this background,

$$g_{xy} = g^{(0)}_{xy} + h_{xy}(r,x) = g^{(0)}_{xy}[1 + \epsilon\Phi(r,x)].$$

- Plugging it in the action and keeping terms to order ϵ^2,

$$\begin{aligned} S &= \int \frac{d^4k}{(2\pi)^4} dr(\mathcal{A}_1(r,k)\phi'(r,k)\phi'(r,-k) \\ &\qquad + \mathcal{A}_0(r,k)\phi(r,k)\phi(r,-k)) \end{aligned}$$

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- The coefficients $\mathcal{A}_1(r,k)$ and $\mathcal{A}_0(r,k)$ are given by

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\begin{aligned}
\mathcal{A}_1(r,k) &= -\frac{\frac{1}{2}g^{rr}\sqrt{-g}}{16\pi G_5}, \\
\mathcal{A}_0(r,k) &= -\frac{\frac{1}{2}\sqrt{-g}\,g^{\mu\nu}k_\mu k_\nu}{16\pi G_5}.
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Holographic computation of Green's function

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- Conjugate momentum of the transverse graviton

$$\Pi(r,k_\mu) = \frac{\partial S}{\partial \phi'(r,k)} = 2\mathcal{A}_1(r,k)\phi'(r,k)$$

- The equation of motion

$$\Pi'(r,k_\mu) - 2\,\mathcal{A}_0(r,k)\phi(r,k) = 0\ .$$

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Holographic computation of Green's function

- Computing the gravitons action on-shell it reduces to the following surface term

$$S = \int \frac{d^4k}{(2\pi)^4} (\mathcal{A}_1(r,k)\phi'(r,k)\phi(r,-k))\Big|_1^\infty .$$

- Following the Minkowskian prescription (Son-Starinets), the boundary retarded Green's function is given as,

$$G_R(k_\mu) = \lim_{r\to\infty} -\frac{2\mathcal{A}_1(r,k)\phi'(r,k)\phi(r,-k)}{\phi_0(k)\phi_0(-k)}$$

- $\phi_0(k_\mu)$ is the value of the graviton fluctuation at boundary.

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- We can rewrite the boundary retarded Green's function as,

$$G_R(k_\mu) \;=\; \lim_{r\to\infty} -\frac{\Pi(r,k_\mu)}{\phi(r,k_\mu)}.$$

- Calculate $\phi(r,k)$ solving a second order differential equation order by order in k_μ.
- Impose in-falling wave boundary condition at horizon and Dirichlet boundary condition at asymptotic boundary.
- Compute G^R order by order in k_μ

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Holographic computation of Green's function

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A response function

- Define a response function of the boundary theory

$$\bar{\chi}(k_\mu, r) = \frac{\Pi(r, k_\mu)}{i\omega\phi(r, k_\mu)} \qquad \omega = k_0$$

- This function is defined for all r and k_μ.
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Radial evolution of the response function

- Differentiate the response function w.r.t. r

$$\partial_r \bar{\chi}(r, k_\mu) = \frac{1}{i\omega}\left[\frac{\Pi'(r,k)}{\phi(r,k)} - \frac{\Pi(r,k)\phi'(r,k)}{\phi(r,k)^2}\right]$$

- Using the definition of $\Pi(r,k)$ and field equation of motion

$$\Pi(r, k_\mu) = 2\mathcal{A}_1(r,k)\phi'(r,k) \quad , \quad \Pi'(r,k_\mu) - 2\,\mathcal{A}_0(r,k)\phi(r,k) = 0\ .$$

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Liu+Iqbal

- Exact in k_μ.
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$$\begin{aligned}\Sigma(r) &= -2\mathcal{A}_1(r,k_\mu)\sqrt{-\frac{g_{rr}}{g_{tt}}} \\ \Upsilon(r) &= 2\mathcal{A}_0(r,k_\mu)\sqrt{-\frac{g_{tt}}{g_{rr}}}.\end{aligned}$$

Boundary condition

- We need to impose one boundary condition to solve this flow equation.
- The boundary condition follows automatically.
- Demanding the solution to be regular at the horizon

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- For two derivative gravity

$$\bar{\chi}(k_\mu, 1) = \sqrt{\frac{\Sigma(1)\Upsilon(1)}{\omega^2}} = \frac{1}{16\pi G_5}$$

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1^{st} order transport coefficients from flow equation

- With this boundary condition, one can integrate out the differential equation from horizon to asymptotic boundary and obtain the AdS/CFT response for all momentum k_μ.
- Tt is trivial to see that at $(\omega, k_i) \to 0$ limit,

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1^{st} order transport coefficients from flow equation

Therefore we conclude that for two derivative gravity dual, the full momentum response at the horizon automatically corresponds to only the zero momentum limit of the boundary response.

$$\bar{\chi}(k_\mu, 1) = \bar{\chi}(k_\mu \to 0, \infty) = \eta$$

Liu+Iqbal

Flow from horizon to boundary

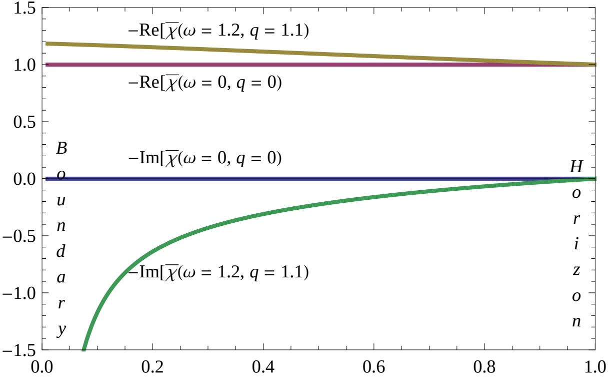

Figure: Flow of Green's function from horizon to boundary for two derivative gravity.

2^{nd} order transport coefficients from flow equation
N. Banerjee, SD

$$\partial_r \bar{\chi}(k_\mu, r) = i\omega \sqrt{-\frac{g_{rr}}{g_{tt}}} \left[\frac{\bar{\chi}(k_\mu, r)^2}{\Sigma(r)} - \frac{\Upsilon(r)}{\omega^2} \right]$$

- The right hand side of this equation is proportional to ω.
- To solve this equation up to order $\omega^2 \rightarrow$ replace the leading value for $\bar{\chi}$

$$\partial_r \bar{\chi}(k_\mu, r) = i\omega \sqrt{-\frac{g_{rr}}{g_{tt}}} \left[\frac{\eta^2}{\Sigma(r)} - \frac{\Upsilon(r)}{\omega^2} \right] + \mathcal{O}(\omega^2, k_i^2).$$

- We impose the regularity condition at the horizon.

$$\partial_r \bar{\chi}(k_\mu, r) = i\omega\sqrt{-\frac{g_{rr}}{g_{tt}}}\left[\frac{\bar{\chi}(k_\mu, r)^2}{\Sigma(r)} - \frac{\Upsilon(r)}{\omega^2}\right]$$

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2^{nd} order transport coefficients from flow equation N. Banerjee, SD

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✠ The solution

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\begin{aligned}
-i\omega\bar{\chi}(k_\mu,\infty) &= -i\omega\left(\frac{1}{16\pi G_5}\right) \\
&\quad +\omega^2\left[\frac{1}{2}(1-\ln 2)\left(\frac{1}{16\pi G_5}\right)\right] \\
&\quad -\frac{q^2}{2}\left(\frac{1}{16\pi G_5}\right) + Divergent\ \ piece.
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- Comparing it with the expression of Green's function $G_R^{xy,xy}(\omega,k) = -i\eta\omega + \eta\tau_\Pi\omega^2 - \frac{\kappa}{2}[(d-3)\omega^2 + q^2]\,.$

$$
\eta = \frac{T^3\pi^3}{16\pi G_5}, \quad \kappa = \frac{\eta}{\pi T}, \quad \tau_\pi = \frac{2-\ln 2}{2\pi T}\,.
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&\quad -\frac{q^2}{2}\left(\frac{1}{16\pi G_5}\right) + Divergent\ \ piece.
\end{aligned}
$$

- Comparing it with the expression of Green's function

$G_R^{xy,xy}(\omega,k) = -i\eta\omega + \eta\tau_{\Pi}\omega^2 - \frac{\kappa}{2}[(d-3)\omega^2 + q^2]\,.$

$$
\eta = \frac{T^3\pi^3}{16\pi G_5}, \quad \kappa = \frac{\eta}{\pi T}, \quad \tau_\pi = \frac{2-\ln 2}{2\pi T}\ .
$$

Generalized flow equation in HD gravity

N. Banerjee, SD

- We consider a gravity set-up with n derivative action.

$$\mathcal{I} = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R + 12 + \alpha' R^{(n)}\right]$$

- Not clear how to define the conjugate momentum and response function.
- Way out: Effective action.

$$S_{eff} = \frac{1}{16\pi G_5}\int \frac{d^4k}{(2\pi)^4}dr\Big[\mathcal{A}_1^{HD}(r,k)\phi'(r,k)\phi'(r,-k) + \mathcal{A}_0^{HD}(r,k)\phi(r,k)\phi(r,-k)\Big]$$

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§ Steps to write the flow equation in HD gravity

- Generalized momentum $\Pi^{HD}(r,k) = 2\mathcal{A}_1^{HD}(r,k)\phi'(r,k)$
- Boundary Green's function

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Boundary Condition

- The response function $\bar{\chi}^{HD}$ should be well-defined at horizon. This implies,

$$\bar{\chi}^{HD}(k_\mu, r)\Big|_{r=r_h} = \sqrt{\frac{\Sigma^{HD}(r)\Upsilon^{HD}(r)}{\omega^2}}\Bigg|_{r=r_h}$$

- A comparison with 2-derivative gravity. In 4 derivative theory : $\mathcal{L} \sim \beta_1 R^2 + \beta_2 Rim^2 + \beta_3 Ric^2$

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Flow from horizon to boundary in HD gravity

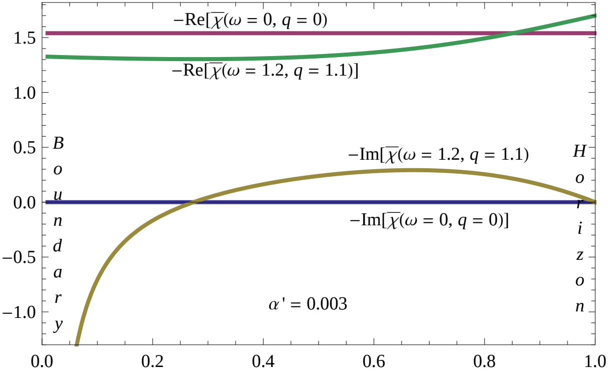

Figure: Flow of Green's function from horizon to boundary in higher derivative gravity.

Results

- *Weyl*^{4} term

$$\frac{\eta}{\pi^3 T^3} = 1 + 135\gamma + \mathcal{O}(\gamma^2)$$
$$\kappa = \frac{\eta}{\pi T}(1 - 145\gamma) + \mathcal{O}(\gamma^2)$$
$$\tau_\pi T = \frac{2 - \log(2)}{2\pi} + \frac{375\gamma}{4\pi} + \mathcal{O}(\gamma^2)\ .$$

Earlier found by [Buchel+Paulos]

- Four derivative gravity

$$\eta = \frac{1}{16\pi G_5}\left(1 - 8\,(5\beta_1 + \beta_3)\,\alpha'\right) + \mathcal{O}(\alpha'^2)$$
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\tau_\pi T &= \frac{1}{4\pi(-1+4\lambda_{gb})}\Bigg[-8\lambda_{gb}^2+12\sqrt{1-4\lambda_{gb}}\lambda_{gb} \\
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- To preserve causality of a conformal fluid there exists a bound [Buchel+Myers]

$$\tau_\pi T - 2\frac{\eta}{s} \geq 0\ .$$

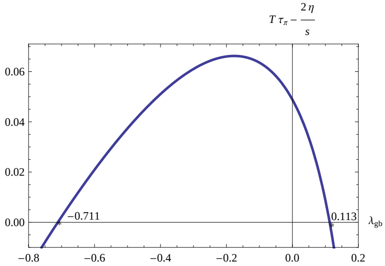

Figure: Bound on λ_{gb}

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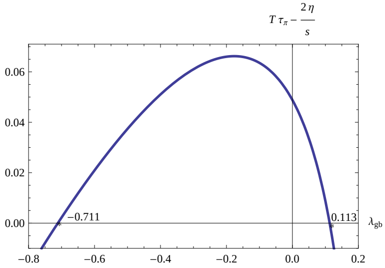

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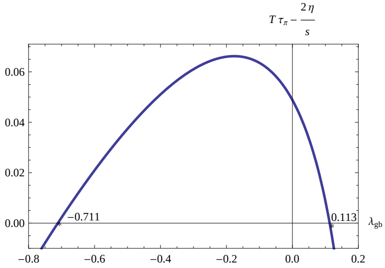

Figure: Bound on λ_{gb}

Flow of retarded Green's function of boundary R current

- Retarded Green's function of boundary R-current

$$G^R_{i,j}(k) = -i \int dt d^3x e^{ik\cdot x} \langle [J_i(x), J_j(0)] \rangle$$

$J_\mu(x)$ is the CFT current dual to a bulk gauge field A_μ.

- In hydrodynamic approximation

$$J_\nu = -\tilde{\kappa} P^\alpha_\nu \partial_\alpha \frac{\mu}{T} + \Omega l_\nu + \mathcal{O}(\partial^2)$$

where, $\tilde{\kappa}$ and $\Omega \mapsto$ first order transport coefficients, $\mu \mapsto$ chemical potential, $T \mapsto$ temperature and

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$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right).$$

- Solution is given by

$$ds^2 = -\frac{r_0^2 U(r)}{r}dt^2 + \frac{dr^2}{4r^2U(r)} + \frac{r_0^2}{r}(d\vec{x}^2)$$
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- Temperature

$$T = \frac{r_0}{\pi}\left(1 - \frac{Q^2}{2r_0^6}\right)$$

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Flow of retarded Green's function of boundary R current

- We start with Einstein-Maxwell action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left(R + 12 - \frac{1}{4}F^2\right).$$

- Solution is given by

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- We turn on small fluctuations for x component of gauge fields.
- Since the A_t component of the bulk vector is non-vanishing in this background, the perturbations A_x can couple to the tx component of graviton.
- Therefore we also need to consider small metric fluctuations for components g_{tx}. Writing them in momentum space

$$A_x(r,x) = \int \frac{d^4k}{(2\pi)^4} e^{ik.x} A_1(r,k)$$
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$$S_A = \frac{1}{16\pi G_5}\int \frac{d^4k}{(2\pi)^4}\Big[-r_0^2 U(r) A_x'^2(r,k) + \left(\frac{\omega^2}{4rU(r)} - \frac{q^2}{4r}\right) A_x^2\Big].$$

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$$J_x(r,k) = \frac{\delta S_A}{\delta A_x'(r,k)} = -2r_0^2 U(r) A_x'(r,k).$$

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- The equation of motion for $A_x(r,k)$ is given by,

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$$\sigma(1,k)^2 = 1.$$

- Low frequency limit is not *trivial*.
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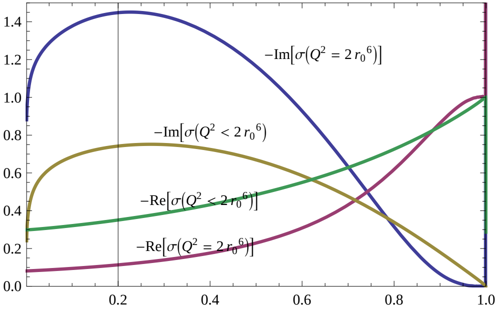

Figure: Flow of σ for non-extremal and extremal black holes.

Summary and Outlook

- We study the flow of retarded Green's function of energy momentum tensor in presence of generic higher derivative gravity.
- The flow equation is valid for any momentum: $k_\mu \rightarrow$ Any higher order transport coefficient can be computed from this flow equation.
- It would be interesting to understand the connection between low frequency behavior of membrane fluid and boundary fluid using this flow equation. *Any UV/IR relation*??

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