Probing Holographic Superfluids with Solitons

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AdS / CM

- There has been much recent effort to construct exotic phases of matter using the AdS/CFT correspondence
    - superfluidity
    - superconductivity (high T_c 'ish behavior)
- These ground states could be result of bosonic and/or fermionic degrees of freedom
    - to model real systems we should understand what type of operator is condenses to give these phases as well as the properties of low lying fluctuations
- Solitons probe the structure of the ground state
    - the cores probe small scale effects and tails probe large scale effects

Outline

- Bosonic and Fermionic Superfluids in the real world
  - solitons in bosonic and fermionic superfluids are different
- Hairy Black Holes and Holographic Superfluids
- Holographic Solitons
  - holographic superfluids have features of both bosonic and fermionic superfluids
- Outlook

Superfluidity

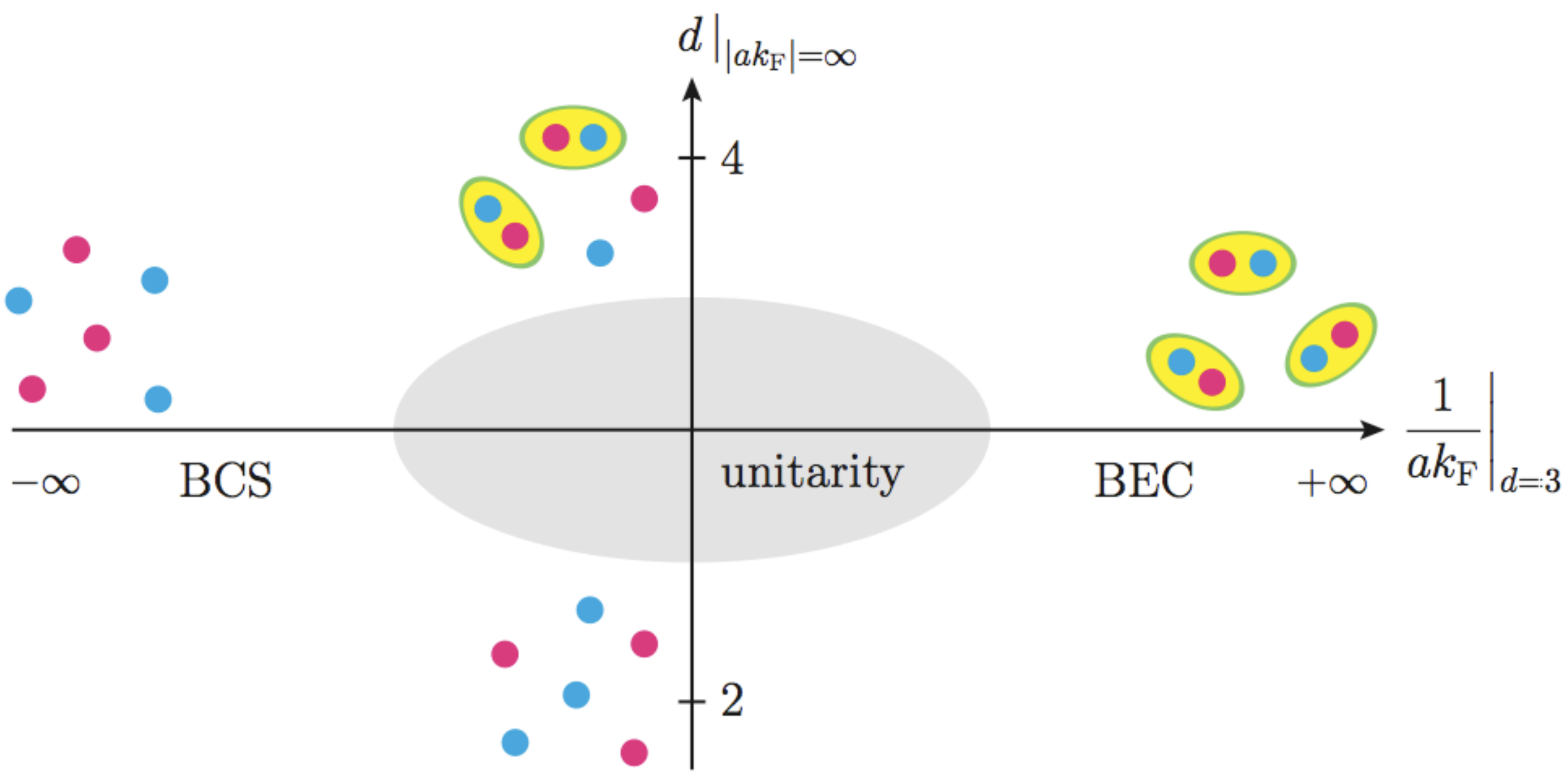
- Need spontaneously broken global U(1)
  - a charged order parameter takes a nonzero v.e.v.

$$\langle \mathcal{O} \rangle \neq 0$$

  - could be a fundamental d.o.f. (BEC) or complicated bound state d.o.f. (BCS)
- As many body system, has many particles in ground state
  - there is a gap which blocks scattering at low energies
  - the state becomes rigid against changes -- no viscocity

Lessons From (non-Rel.) Fermionic Superfluids

(from Nishida and Son [1004.3597])



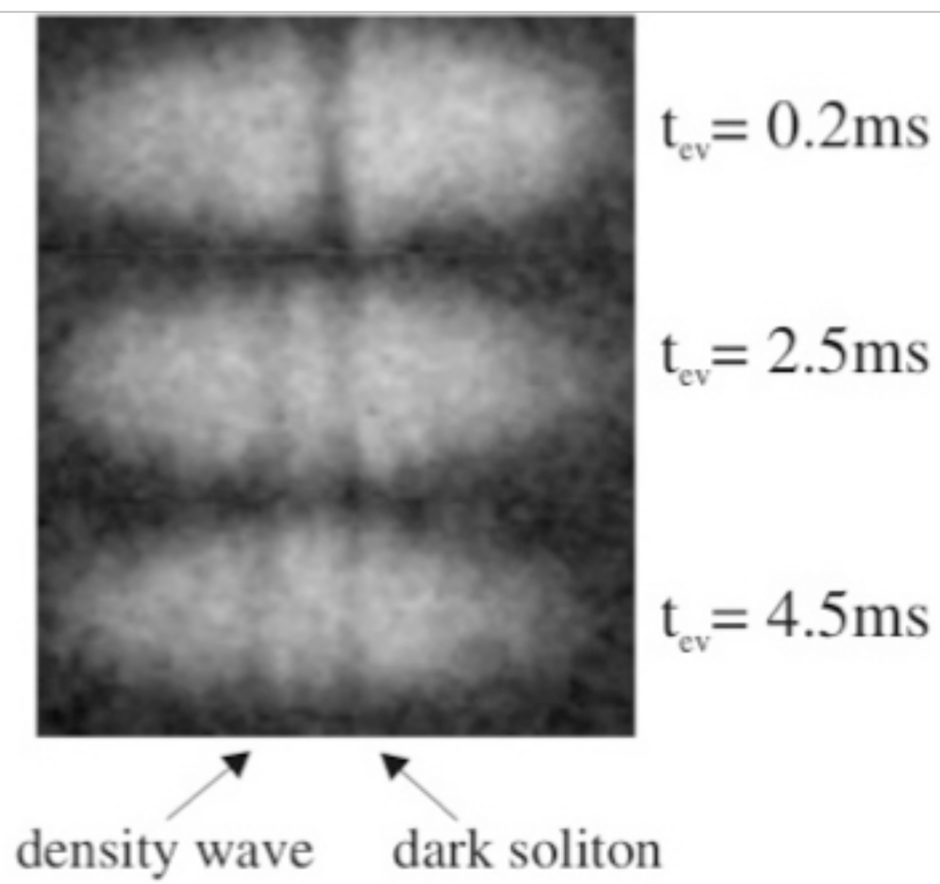
- this is the picture that arises for a non-relativistic 4-Fermi interaction
  - at weak coupling, changing the dimension changes the scaling dimension of the condensing operator (among other things)
- solitons in these systems give hints to the microscopic structure

Solitons in the Crossover

- Non-Relativistic BCS - BEC crossover
  - different "fundamental" degree of freedom
  - display "dark soliton" solutions
    - unstable -- but observable
    - variable depletion fraction, bos. superfluid is near 100%, ferm. superfluid is much smaller

Dark Solitons

Experimental Imaging



Dark soliton: Experimental Imaging
(from Bongs, et. al. [cond-mat/0301069v1])

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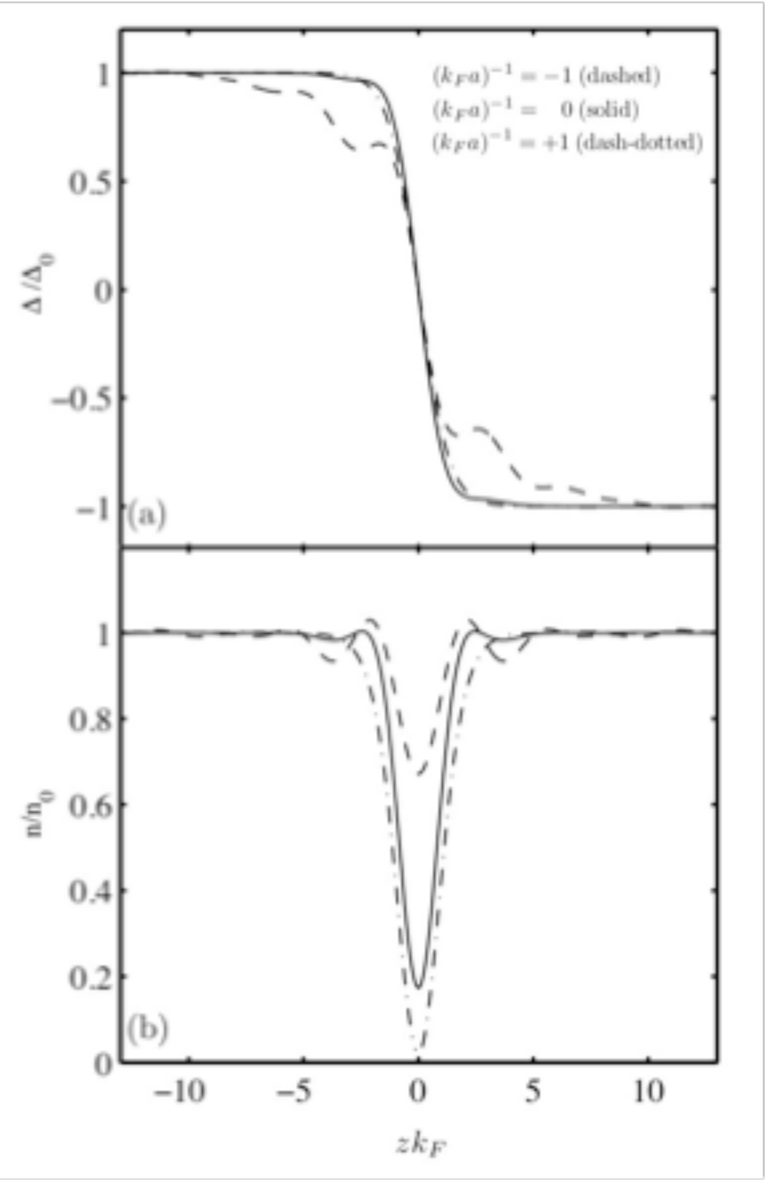
Dark Solitons

Soliton solution to gap equation



Dark soliton: BEC (dot-dashed), unitarity (cont.), and BCS (dashed) (from Antezza, et. al. [0706.0601])

Solitons in the Crossover

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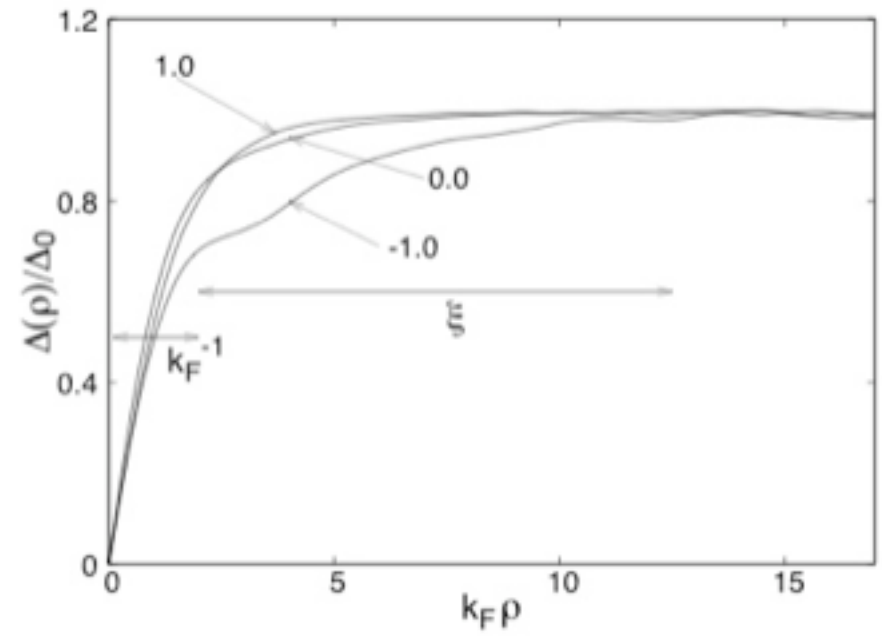
  - display vortex solutions
    - variable depletion fraction
    - same core and tail scales in bos. case, different for ferm.

Vortices



Vortex Profiles: BEC (“1”) and BCS (“-1”)
(from Sensarma, et. al. [cond-mat/0510761])

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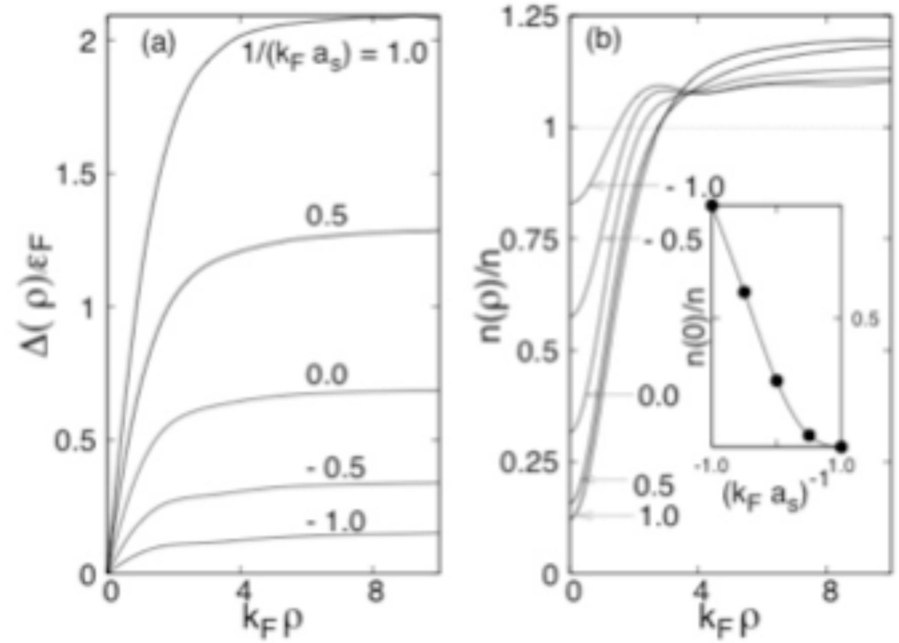

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Holographic Superfluids

- many recent attempts to use gauge/gravity duality to study 2+1 condensed matter system

Hartnoll, et. al. [0803.3295 and 0810.1563]

- focus on spontaneous breaking of an Abelian symmetry
- AdS4 Einstein-Maxwell-Higgs

$$\sqrt{-g}\mathcal{L} \sim \sqrt{-g}\left((R-12\Lambda)/2 - \frac{\kappa_4^2}{q^2}\left(F^2/4 + |D\Psi|^2 + m^2|\Psi|^2\right)\right)$$

  - probe limit: $\frac{\kappa_4}{q} \to 0$ (Schw-AdS b.h.)

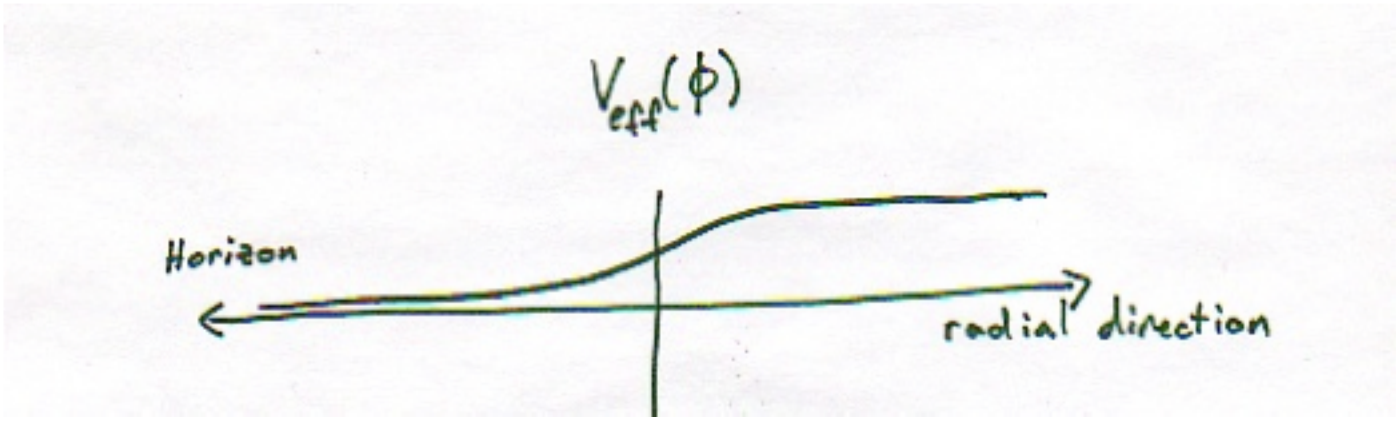
$$ds^2 = L^2\left(-\frac{f dt^2}{z^2} + \frac{dz^2}{f z^2} + \frac{d\vec{x}^2}{z^2}\right),\ f(z) = 1 - z^3$$

Hairy Black Holes

- There are robust theorems stating that black holes in asymptotically Minkowski space cannot support scalar hair

  Bekenstein [gr-qc/9605059]

    - idea:



- The negative curvature of AdS changes this to allow for possibility of hair

  Gubser [0801.2977]

    - can be used to study spontaneous symmetry breaking

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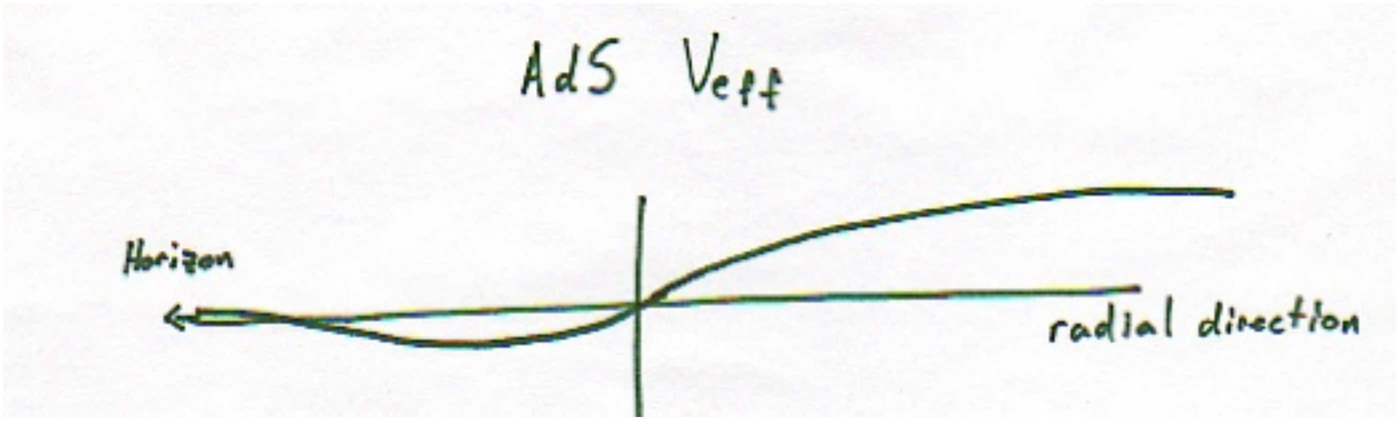

Gubser [0801.2977]

- The negative curvature of AdS changes this to allow for possibility of hair
  - can be used to study spontaneous symmetry breaking

- for $m^2 = -2/L^2$, gauge fixed e.o.m.

$$fR'' + f'R' - zR + \partial_x^2 R + \frac{A_t^2}{f} R = 0$$
$$fA_t'' + \partial_x^2 A_t - R^2 A_t = 0$$

- use rescaled fields ($R \equiv \sqrt{2}\frac{\Psi}{z}$)

- asymptotic solution:

$$R \sim R^{(1)}(x) + zR^{(2)}(x) + ...$$
$$A_t \sim A_t^{(0)}(x) + zA_t^{(1)}(x) + ...$$

- usually leading term sets the "source" and the subleading term fixes the v.e.v.'s

$$\varphi(z,x) \sim \varphi^{leading}(x) + z^{\#}\varphi^{subleading}(x) \quad \rightarrow \; J \sim \varphi^{leading}, \quad < O > \sim \varphi^{subleading}$$

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- for this "weird" mass two choices of quantization (Neumann or Dirichlet boundary conditions) :

$$< O_1 >\sim R^{(1)}, \quad 0 = R^{(2)} \qquad \text{or} \qquad 0 = R^{(1)}, \quad < O_2 >\sim R^{(2)}$$
$$\mu \sim A_t^{(0)}, \quad \rho \sim A_t^{(1)} \qquad\qquad \mu \sim A_t^{(0)}, \quad \rho \sim A_t^{(1)}$$

- both quantizations display spontaneous symmetry breaking for $\mu \geq \mu_c$ at $T = 1.$ (really varying μ/T)
  - charged operator has an expectation value as all charged external sources are removed
- Linear response coefficients show a gapped behavior ~ $e^{-\Delta/T}$
  - many papers working out details of the response theory and hydrodynamical description

Hartnoll, et. al. [0803.3295 and 0810.1563]
Herzog, et. al. [0809.4870]

- This is a fluid (presumably) comprised of bosonic and fermionic degrees of freedom. How can you determine the type of superfluid? $\Rightarrow$ Look at dark solitons and vortices

Holographic Dark Solitons

- Try to find "dark soliton" solutions to gravity equations

$$fR'' + f'R' - zR + \partial_x^2 R + \frac{A_t^2}{f} R = 0$$
$$fA_t'' + \partial_x^2 A_t - R^2 A_t = 0$$

  - want domain wall scalar field profiles external to black hole
    - tricky PDE's (must be solved numerically)
      - no obvious limit where reduces to ODE problem
      - use relaxation techniques on a lattice
    - change boundary conditions study different condensates
    - as a check, we expect to asymptote to the homogeneous solutions found previously

Aside on Numerical Methods

- Outside the horizon, the equations of motion are elliptic
  - like Poisson equation

$$-\nabla^2 \phi_0 = \rho$$

- "Relaxation": extend to a diffusion equation with new variable t $\in [0,\infty)$

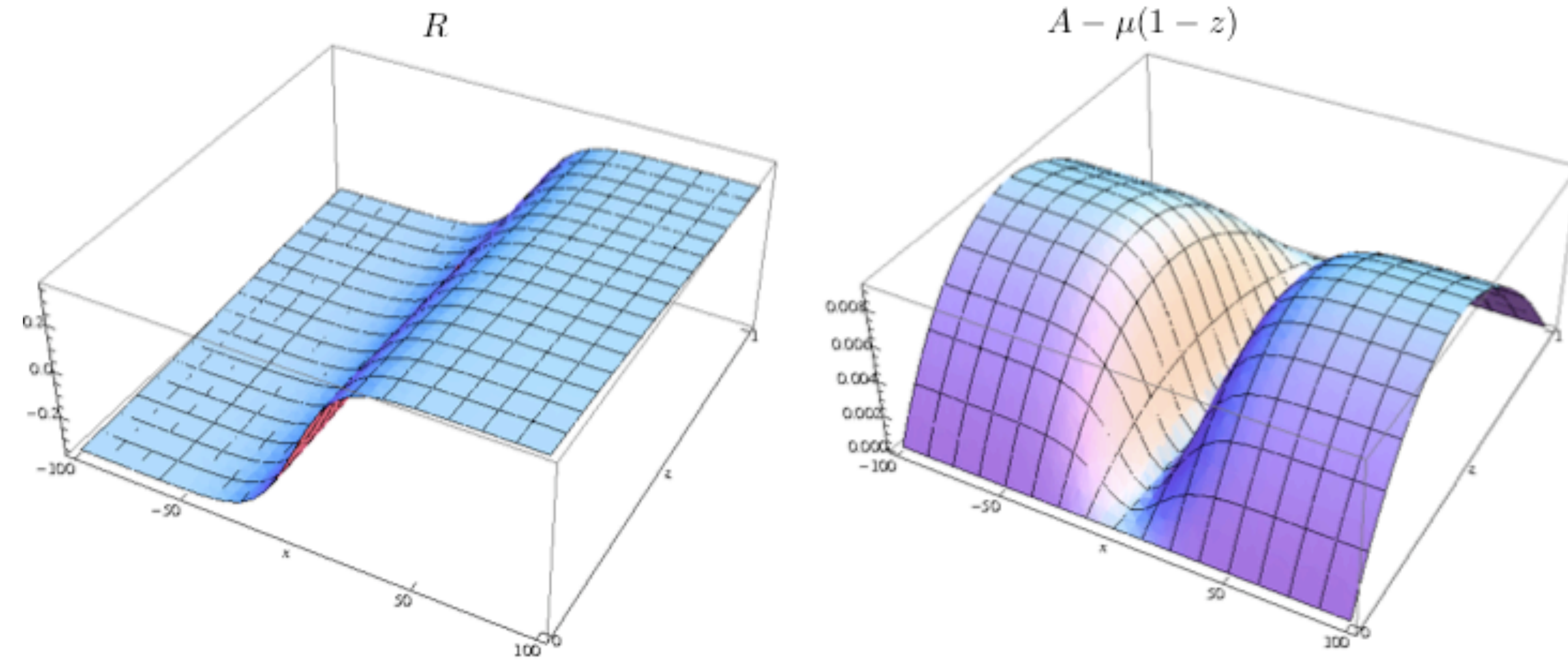
$$(\partial_t - \nabla^2)\phi(t) = \rho$$

  - now have a simple initial value problem: there is a flow to solution of Poisson equation
    - the error decreases in time

$$\mathcal{E}(t) \sim \phi_0 - \phi(t) \sim e^{-tk^2}$$

  - the "relaxation" scheme may be implemented on a lattice
  - topological information is contained in flow's initial conditions

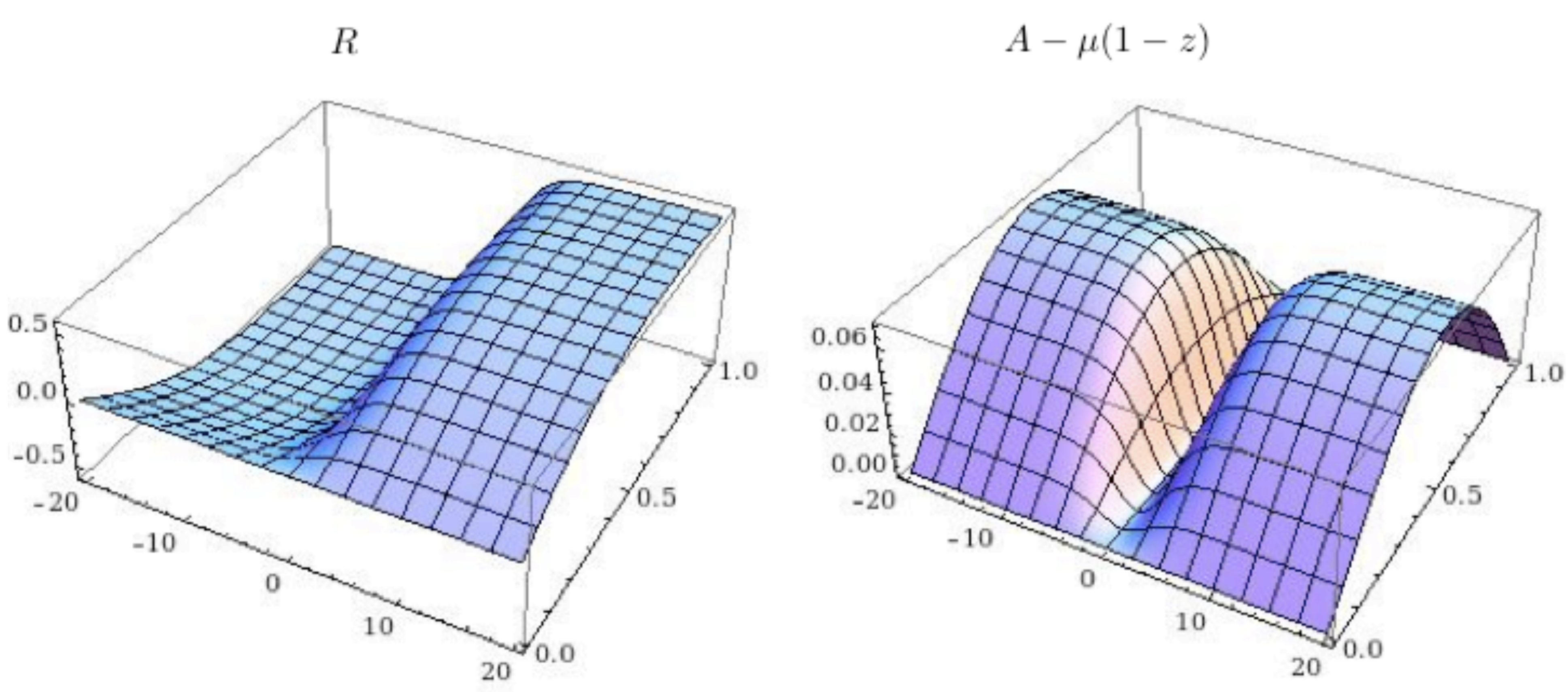
Holographic Dark Solitons



$< O_1 >$ condensate

- asymptote to homogeneous solution
- can show that the error systematically decreases with lattice size

Holographic Dark Solitons

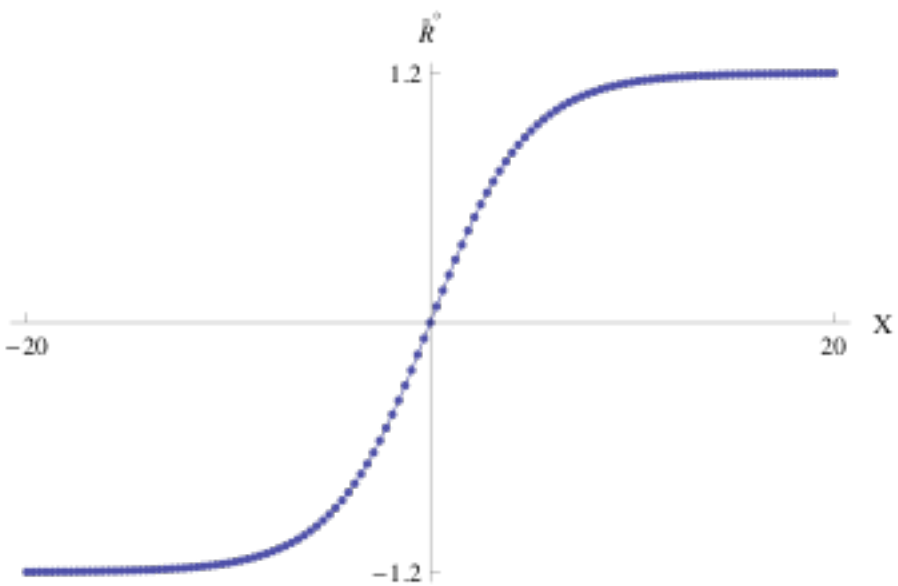


$< O_2 >$ condensate

- asymptote to homogeneous solution
- can show that the error systematically decreases with lattice size

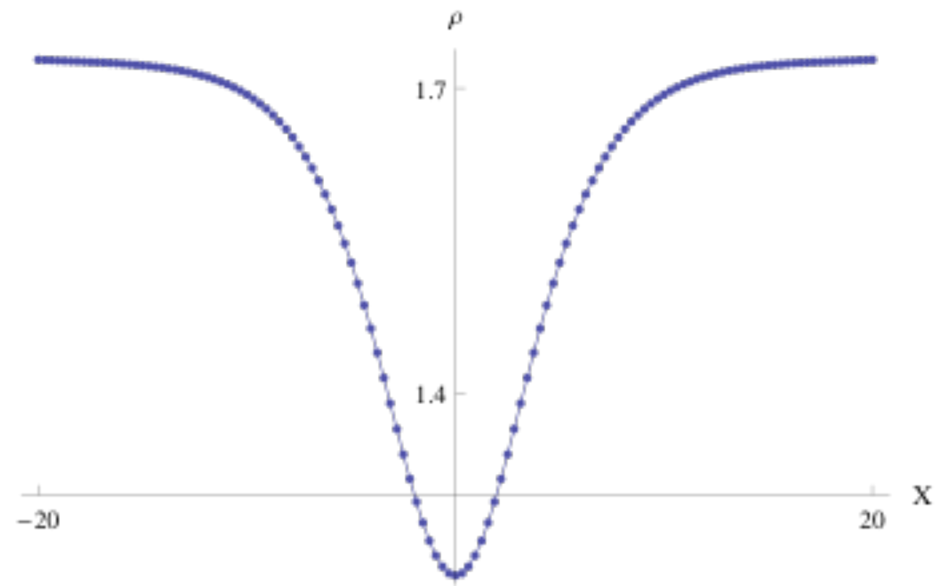
Holographic Dark Solitons

$< O_{1,2} >$



$\sim A \tanh(x/\xi)$

$< \rho > \sim \partial_z A$

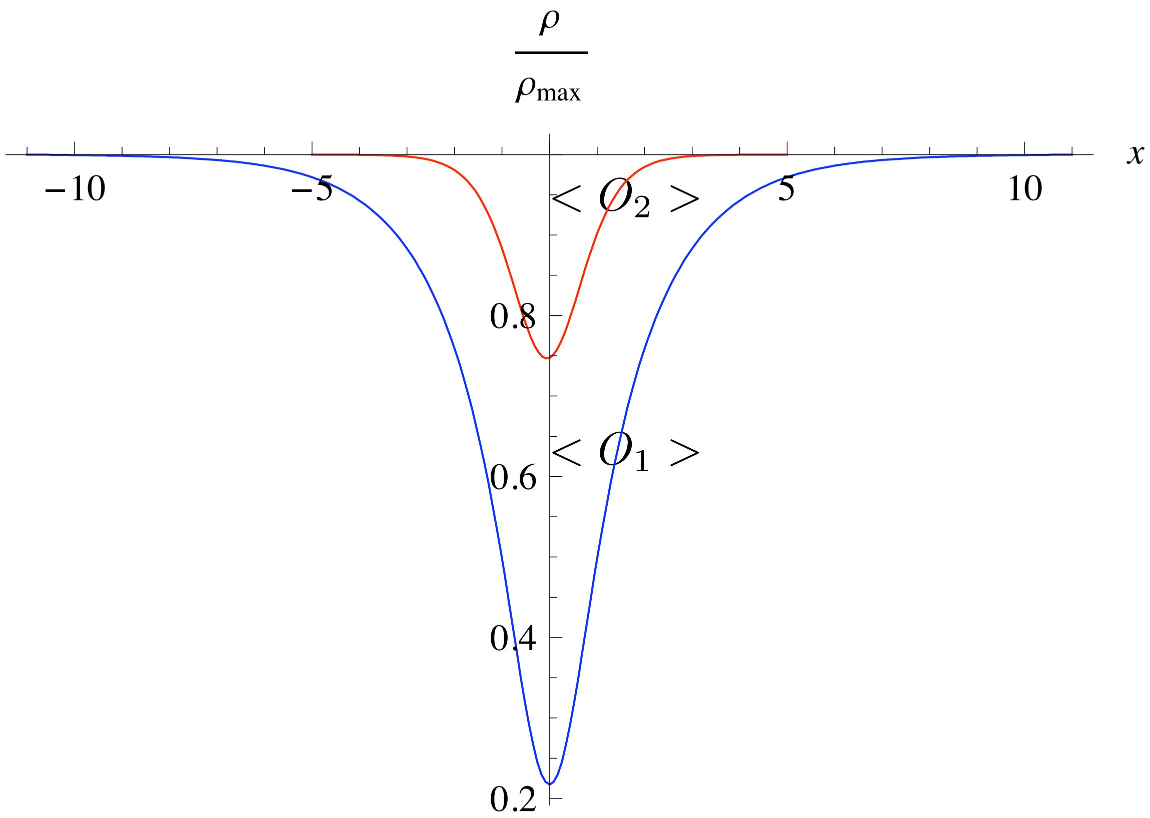


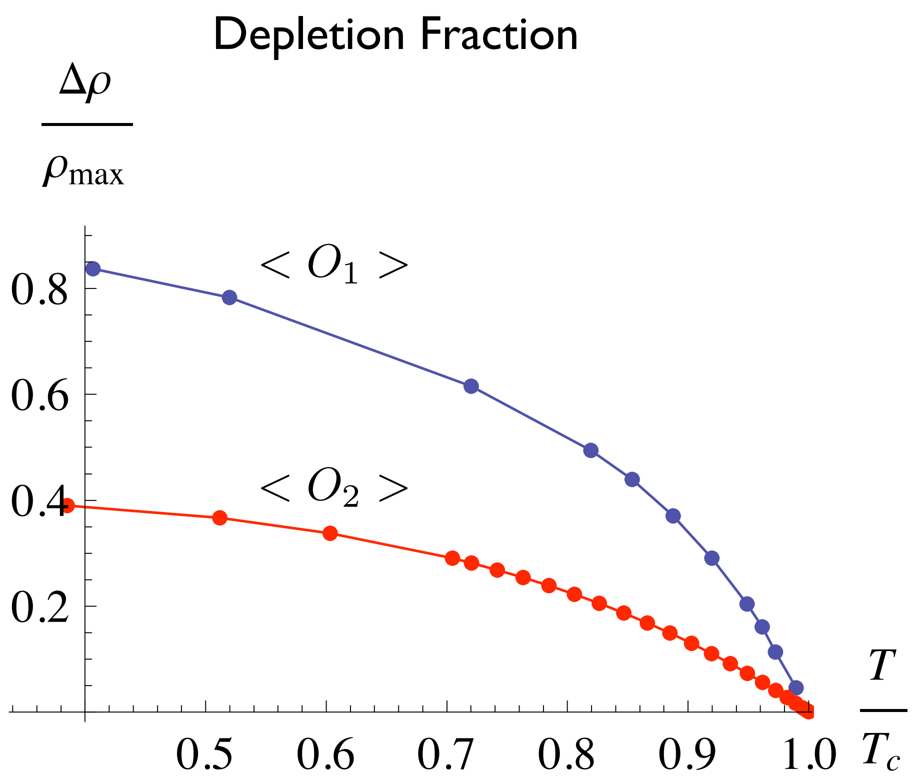
$\sim A + B\ \mathrm{sech}^2(x/\xi_q)$

- asymptote to homogeneous solution
- can show that the error systematically decreases with lattice size

Dark Soliton Features

- two very different depletion fractions (as with the non-rel. example)
  - suggests that $< O_1 >$ consists of pointlike boson while $< O_2 >$ is more BCS-like





  - also suggested in earlier works because $< O_2 >$ has larger scaling dim

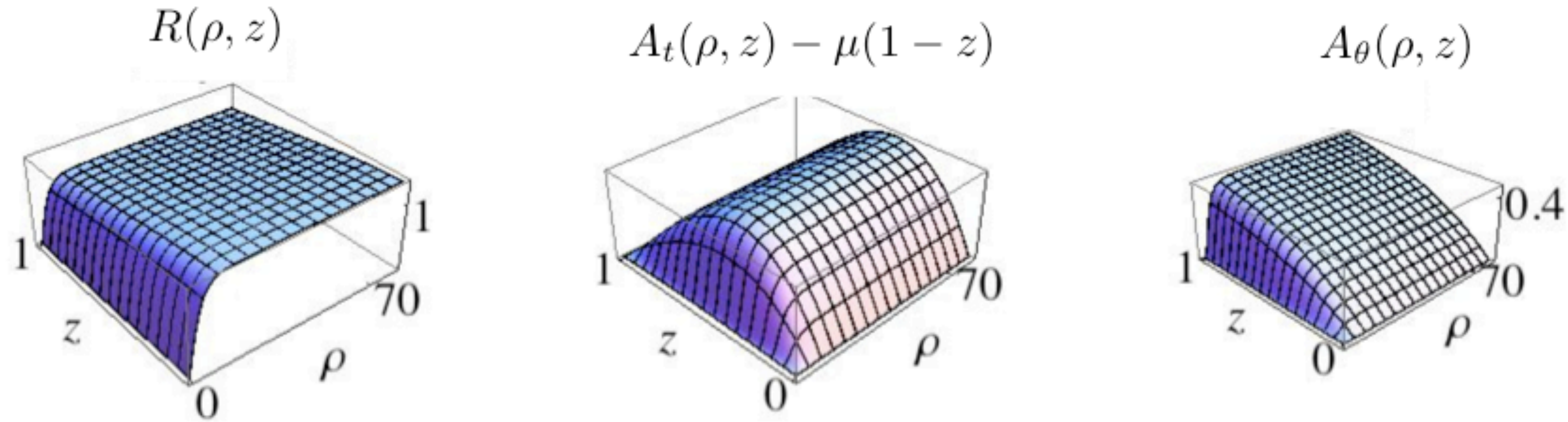
Holographic Vortices

- Seek vortex solutions which carry nontrivial winding
  - must include angular component of gauge field A_θ
    - dual to the superfluid current density $\langle j_s \rangle$
    - now have 3 coupled nonlinear PDE's (harder than Landau-Ginzberg but easier than a gap equation)

$$
\begin{aligned}
0 &= f\partial_z^2 R + \partial_z f \partial_z R - zR + \frac{1}{\rho}\partial_\rho(\rho\partial_\rho R) - R(-\frac{1}{f}A_t^2 + \frac{(A_\theta - n)^2}{\rho^2}) \\
0 &= f\partial_z^2 A_t + \frac{1}{\rho}\partial_\rho(\rho\partial_\rho A_t) - R^2 A_t \\
0 &= \partial_z(f\partial_z A_\theta) + \rho\partial_\rho(\frac{1}{\rho}\partial_\rho A_\theta) - R^2(A_\theta - n)
\end{aligned}
$$

- surprisingly, does reduce to ODE's in asymptotic tail

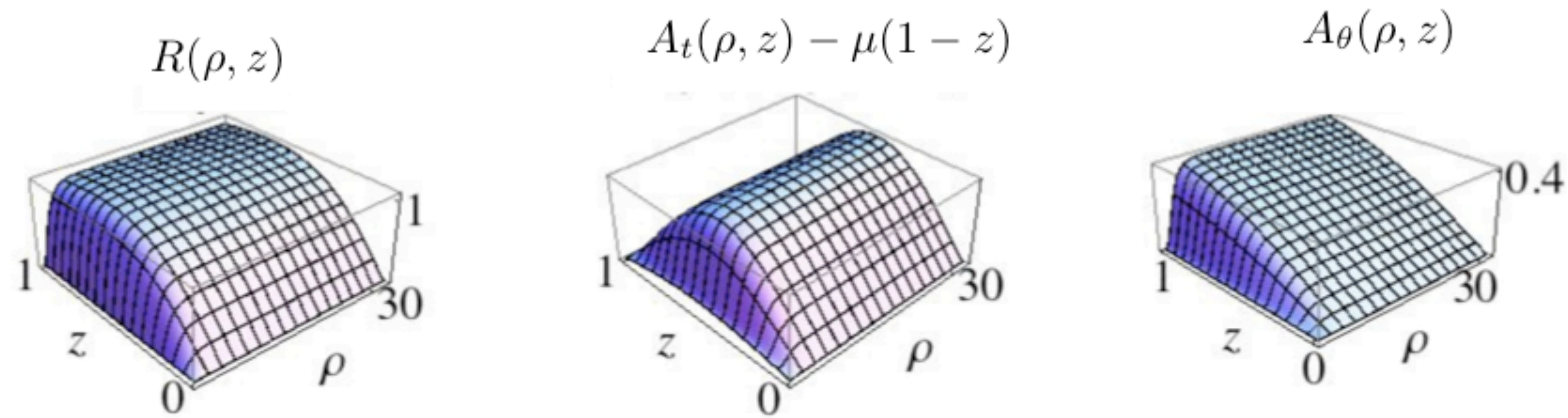
Holographic Vortices



$< O_1 >$ condensate

- Can verify the tail behavior using numerical packages (Mathematica,...)

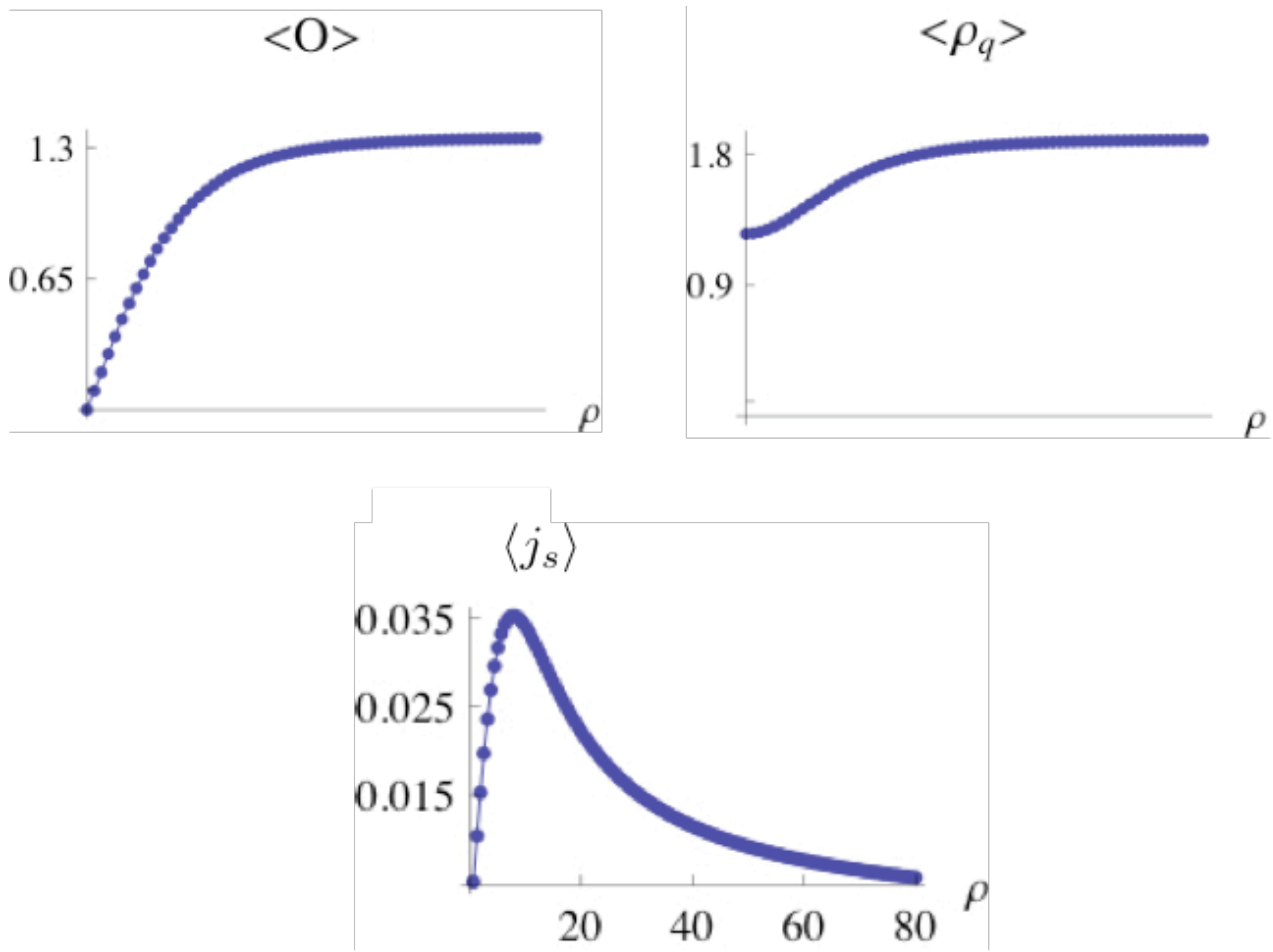
Holographic Vortices



$< O_2 >$ condensate

- Can verify the tail behavior using numerical packages (Mathematica,...)

Holographic Vortices



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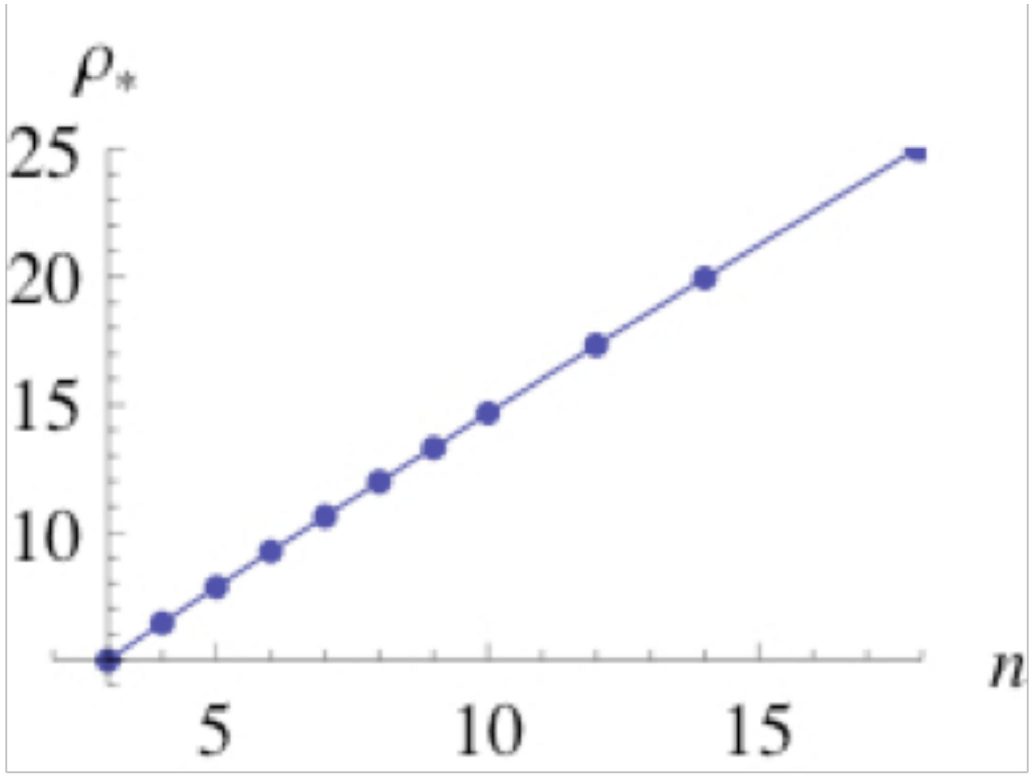
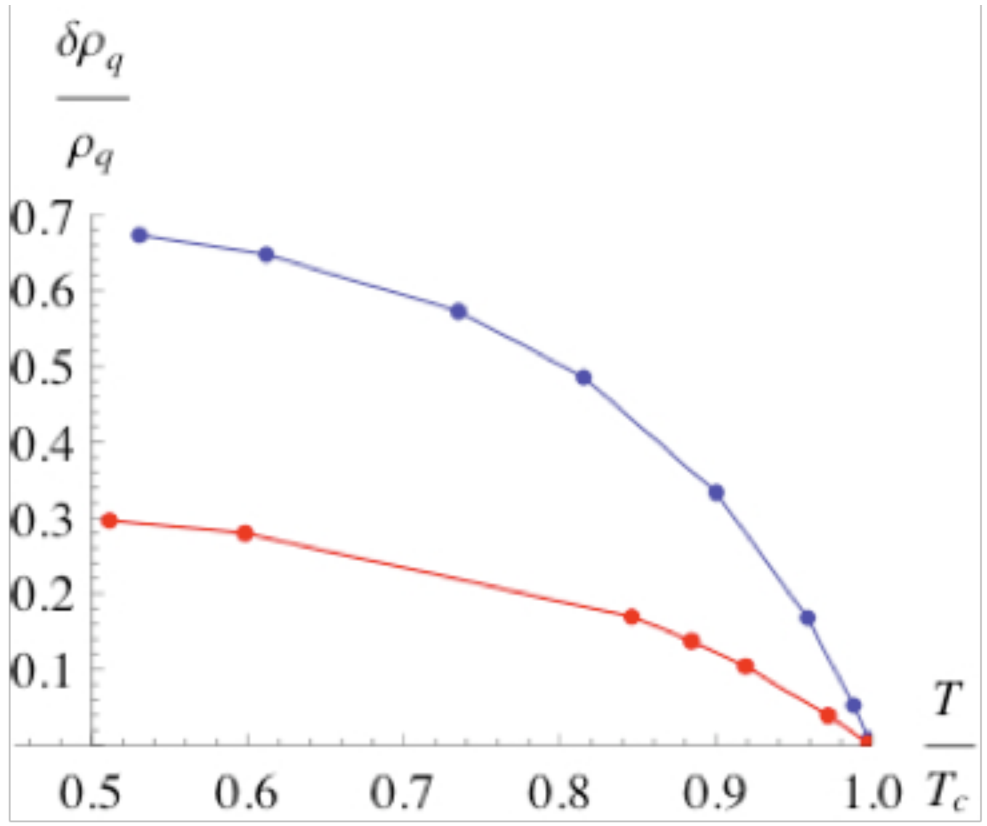
Vortex Features



Linear dependence of vortex's critical velocity with winding

- correct dependence on winding number
- same pattern of core depletion fractions
- consistent with one length scale in core and tail for $< O_1 >$ and two for $< O_2 >$

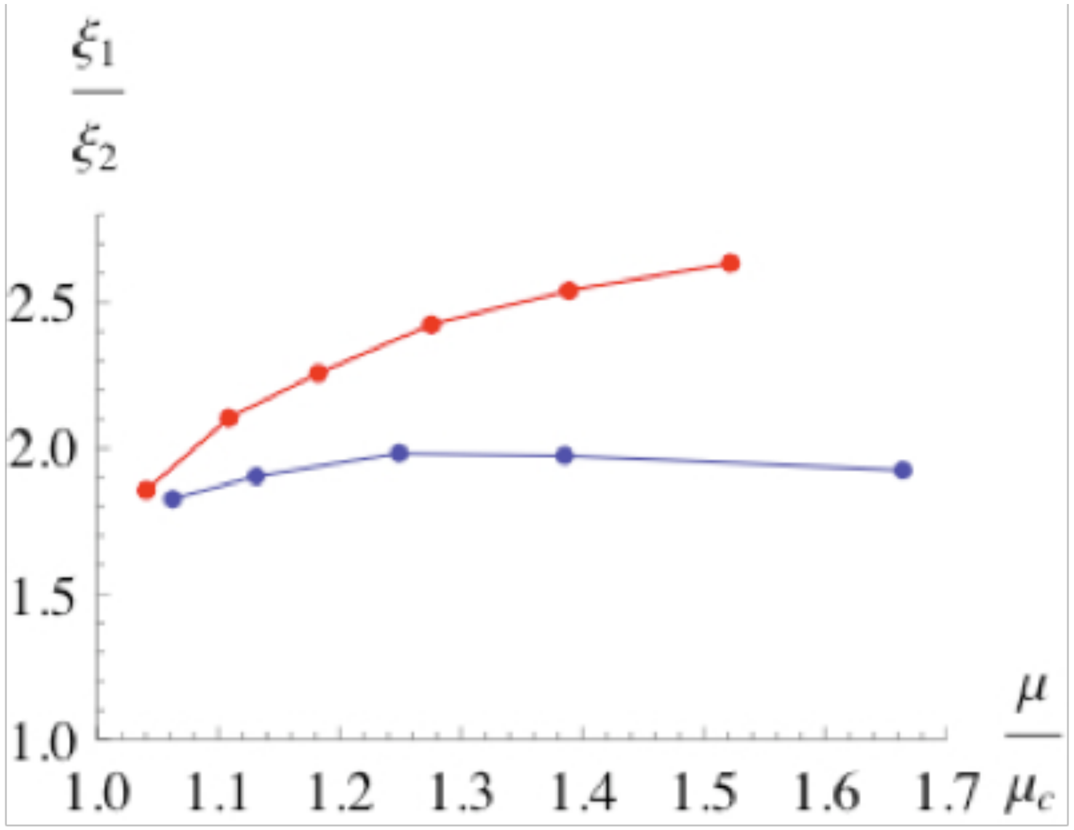
Vortex Features



Density depletion suggests that $< O_1 >$ is BEC-like and that $< O_2 >$ is BCS-like.

- correct dependence on winding number
- same pattern of core depletion fractions
- consistent with one length scale in core and tail for $< O_1 >$ and two for $< O_2 >$

Vortex Features



Ratio of core to tail length scales: $< O_1 >$ in blue and $< O_2 >$ in Red.

- correct dependence on winding number
- same pattern of core depletion fractions
- consistent with one length scale in core and tail for $< O_1 >$ and two for $< O_2 >$

Vortex Features

In the context of hol. superconductors, a logarithmic dependence was also noted in Montull et. al. [0906.2396]

- The energy cost of a vortex is logarithmic

$$\frac{1}{2}\int d^3x \rho_s v_s^2 \sim \frac{n^2 \rho_s(\infty)}{\mu} \log\left(\frac{\Lambda}{r_{core}}\right)$$

  - vanishes near Tc

- The entropy is logarithmic

$$S \sim 2\log\left(\frac{\Lambda}{r_{core}}\right)$$

- Helmholtz free energy changes sign as one approaches Tc from below

$$F = E - TS$$

- Suggests the possibility of BKT transition
  - What is the gravity dual to a vortex gas phase?

Summary and Outlook

- reviewed solitons of bosonic and fermionic superfluids
- presented dark soliton and vortex solutions in holographic superfluid
- the core density depletion was very deep for $< O_1 >$ (boson-like) and relatively shallow for $< O_2 >$ (fermion-like)
- structure of vortex cores are different for the two superfluids
  - both features suggest that we might vary the condensate type by changing the value of m^2 --- Can we study relativistic crossover physics? What is "unitarity"?
- many generalizations: gravitational backreaction, varying m^2, transport properties, instabilities, non-relativistic symmetries...