


# How does the string in flat space-time know about curved space-time?

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Stockholm, 14.06.10

# Foreword

- ▶ This talk is based on the work done together with  Giuseppe D'Appollonio, Rodolfo Russo and Gabriele Veneziano, to appear.

# Plan of the talk

- 1 Introduction
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# Introduction

- ▶ String theory, as originally formulated, is a theory **in flat Minkowski space-time**.
- ▶ It contains in its spectrum a massless spin 2 particle that has all the properties of a graviton.
- ▶ It turns out that the low-energy string effective action that one derives from scattering of strings is a **(super)gravity theory with string corrections**.
- ▶ **Curved space-time is not put by hand, as in GR, but emerges from string scattering amplitudes.**
- ▶ At the perturbative level ( $g_s \sim 0$ ) string theory is only a theory of strings.
- ▶ If we take into account non-perturbative effects string theory contains additional  $p$ -dimensional states called D(irichlet) $p$  branes.

- ▶ On the one hand, they are classical solutions of the low-energy 10-dim string effective action:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[ R - \frac{1}{2} (\nabla\phi)^2 - \frac{1}{2(\rho+2)!} e^{-a\phi} (F_{\rho+2})^2 \right]$$

coupled to graviton, dilaton and RR  $(\rho+1)$ -form potential given by:

$$ds^2 = [H(r)]^{2A} (\eta_{\alpha\beta} dx^\alpha dx^\beta) + [H(r)]^{2B} (\delta_{ij} dx^i dx^j)$$

with  $r^2 \equiv \delta_{ij} x^i x^j$  and

$$e^{-\phi(x)} = [H(r)]^{\frac{\rho-3}{8}} \quad , \quad C_{01\dots\rho}(x) = \left( [H(r)]^{-1} - 1 \right)$$

- ▶ A, B and are equal to

$$A = -\frac{7-\rho}{16} \quad , \quad B = \frac{\rho+1}{16}$$

- ▶  $H(r)$  is an harmonic function given by

$$H(r) = 1 + \left(\frac{R}{r}\right)^{7-p} ; R_p^{7-p} = \frac{2\kappa_{10} T_p N}{(7-p)\Omega_{8-p}} = \frac{g_s N (2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}$$

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 ; T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}} ; \Omega_q = \frac{2\pi^{\frac{q+1}{2}}}{\Gamma(\frac{q+1}{2})}$$

- ▶ Mass per unit volume and RR charge

$$M_p = \frac{T_p}{\kappa_{10}} N = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} N ; \mu_p = \sqrt{2} T_p N$$

Non-perturbative in  $g_s$ .

- ▶ The low-energy string effective action is an action in curved space-time and therefore the classical solution will inherit all the properties of a theory of (super)gravity.

- ▶ On the other hand, in string theory the Dp branes are characterized by having open strings attached to their  $(p + 1)$ -dim world-volume.
  - ▶ Therefore, the open strings satisfy Dirichlet boundary conditions along the directions transverse to the world-volume of the Dp brane.
  - ▶ In string theory a Dp brane is described by a closed string state, called the boundary state.
  - ▶ Dp branes interact by exchanging open strings and therefore the lowest order interaction is given by the annulus diagram.
- [Polchinski, 1995]
- ▶ By open/closed string duality the annulus diagram satisfies the following identity:

$$-\text{Tr} \log(L_0 - a) = \int_0^\infty \frac{d\tau}{\tau} \text{Tr} \left( e^{-2\pi\tau(L_0 - a)} \right) = \langle B|D|B \rangle$$

that determines the boundary state

$$|B\rangle \equiv \frac{T_p}{2} N |B_X\rangle |B_\psi\rangle ; \quad D = \frac{\alpha'}{4\pi} \int_{|z|<1} \frac{d^2z}{|z|^2} Z^{L_0 - a} \bar{Z}^{\tilde{L}_0 - a}$$

- ▶ The bosonic part of the boundary state is equal to

$$|B_X\rangle = \delta^{d-p-1}(\hat{q}^i - y^i) \left( \prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n}S\cdot\tilde{\alpha}_{-n}} \right) |0\rangle_{\alpha}|0\rangle_{\tilde{\alpha}}|p=0\rangle$$

$$S \equiv (\eta_{\alpha\beta}; -\delta_{ij})$$

- ▶ Using the boundary state and the vertex operators for open and closed strings one can compute any amplitude involving scattering of strings on the D branes.
- ▶ In particular, these amplitudes determine the structure of the Born-Infeld action.
- ▶ All calculations **are done in flat space** where we are able to quantize the string.
- ▶ On the other hand, the D branes are classical solutions of the low-energy string effective action in curved space.
- ▶ How are we going to recover the curved space properties of the Dp branes computing scattering of strings in flat space?
- ▶ This is what I am going to show in some example in this seminar.



# The large distance behaviour of the classical solution

[M. Frau, A. Lerda, I. Pesando, R. Russo, S. Sciuto and PDV, 1997]

- ▶ Given the boundary state one can compute the large distance behaviour of the various fields of the classical solution as follows:

$$\langle \psi | D | B \rangle \quad ; \quad D = \frac{\alpha'}{4\pi} \int \frac{d^2 z}{|z|^2} z^{L_0 - a} \bar{z}^{\tilde{L}_0 - a}$$

$\langle \psi |$  is the string state corresponding to the field of the classical solution.

- ▶ Let us start by computing the expression for the generic NS-NS massless field which is given by

$$J^{\mu\nu} \equiv {}_{-1}\langle \tilde{0}, \frac{k_{\perp}}{2} | {}_{-1}\langle 0, \frac{k_{\perp}}{2} | \psi_{1/2}^{\nu} \tilde{\psi}_{1/2}^{\mu} | D | B \rangle_{NS} = -\frac{T_p}{2k_{\perp}^2} V_{p+1} S^{\nu\mu}$$

- ▶ Specifying the different polarizations corresponding to the various fields we get for the dilaton

$$\delta\phi = \frac{1}{\sqrt{8}} (\eta^{\mu\nu} - k^{\mu}\ell^{\nu} - k^{\nu}\ell^{\mu}) J_{\mu\nu} = \frac{3-p}{4\sqrt{2}} T_p \frac{V_{p+1}}{k_{\perp}^2}$$

- ▶ For the graviton we get:

$$\begin{aligned}\delta h_{\mu\nu}(k) &= \frac{1}{2} (J_{\mu\nu} + J_{\nu\mu}) - \frac{\delta\phi}{\sqrt{8}} \eta_{\mu\nu} \\ &= T_p \frac{V_{p+1}}{k_{\perp}^2} \text{diag}(-A, A \dots A, B \dots B) ,\end{aligned}$$

where  $A = -\frac{7-p}{16}$  and  $B = \frac{p+1}{16}$

- ▶ For the Kalb-Ramond field we get:

$$\delta B_{\mu\nu}(k) = \frac{1}{\sqrt{2}} (J_{\mu\nu} - J_{\nu\mu}) = 0$$

- ▶ In the R-R sector we get instead

$$\delta C_{01\dots p}(k) \equiv \langle P_{01\dots p}^{(C)} | D | B \rangle_R = \mp \mu_p \frac{V_{p+1}}{k_{\perp}^2} ; \quad T_p = \sqrt{2} \mu_p$$

- ▶ We can express the previous fields in configuration space using the following Fourier transform valid for  $p < 7$

$$\int d^{(p+1)}x d^{(9-p)}x \frac{e^{ik_{\perp} \cdot x_{\perp}}}{(7-p) r^{7-p} \Omega_{8-p}} = \frac{V_{p+1}}{k_{\perp}^2},$$

- ▶ We must rescale the various fields according to

$$\varphi = \sqrt{2\kappa_{10}}\phi, \quad \tilde{h}_{\mu\nu} = 2\kappa_{10}h_{\mu\nu}, \quad C_{01\dots p} = \sqrt{2\kappa_{10}}C_{01\dots p}$$

- ▶ We get the following large distance behaviour for the dilaton

$$\delta\varphi(r) = \frac{3-p}{4} \left(\frac{R_p}{r}\right)^{7-p}$$

- ▶ For the graviton we get

$$\delta\tilde{h}_{\mu\nu}(r) = 2 \left(\frac{R_p}{r}\right)^{7-p} \text{diag}(-A, \dots, A, B \dots B) \quad ,$$

- ▶ For the RR field we get

$$\delta\mathcal{C}_{01\dots p} = \left(\frac{R_p}{r}\right)^{7-p}$$

- ▶ The previous equations reproduce exactly the behavior for  $r \rightarrow \infty$  of the metric, the dilaton and the R-R potential of the solution.
- ▶ The next to the leading behaviour is expected to come from the one-point function with two boundary states:

$$\langle B | \int d^2z_1 W(z_1, \bar{z}_1) D | B \rangle$$

$W$  is the vertex operator corresponding to the massless closed string.

- ▶ The explicit calculation gave zero after the sum over the spin structures [R. Marotta, I. Pesando, PDV (1998), unpublished].

- ▶ Two alternative ways of getting the classical solution.
- ▶ One is by solving the classical supergravity equations of motion with the Dp brane as a source.
- ▶ The other is by computing the one-point function for the closed string fields in an action that contains their interaction in the bulk and their interaction with the D brane:

$$\langle \Phi(x) e^{iS_{bulk} + iS_{boundary}} \rangle$$

where

$$S_{boundary} = \int d^{p+1}x T_p \left[ -\eta_{\alpha\beta} h^{\alpha\beta} + \frac{3-p}{2\sqrt{2}} \phi + \sqrt{2} C_{01\dots p} \right]$$

- ▶ By explicit calculation one can reproduce the leading and the next to the leading behaviour of the classical solution.
- ▶ It is still not clear why the stringy calculation does not work as in field theory. **One may need an off-shell extrapolation.**

# The approach of Amati, Ciafaloni and Veneziano(ACV) [ACV, 1987; Sundborg, 1987]

- ▶ The starting point of the ACV approach is the four graviton scattering amplitude on the sphere in the Regge limit ( $s = 4E^2 \rightarrow \infty$  and small  $t$ ):

$$\frac{a_{Tree}}{s} = \frac{32\pi G_N}{\alpha' s} \frac{\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1 + \frac{\alpha'}{4}t)} \left(\frac{\alpha'}{4}s\right)^{2 + \frac{\alpha'}{2}t} e^{-i\pi\frac{\alpha'}{4}t} \implies 8\pi G_N \frac{s}{(-t)}$$

- ▶ At high energy its Fourier transform in the space of impact parameter exponentiates and one can compute the classical deflection angle for large impact parameter:

$$\Theta = \frac{\sqrt{\pi} \Gamma(\frac{D}{2})}{\Gamma(\frac{D-1}{2})} \left(\frac{r_s}{b}\right)^{D-3} ; \quad r_s^{D-3} = \frac{16\pi G_N \sqrt{s}}{(D-2)\Omega_{D-2}}$$

$D$  is the number of non-compact directions and  $r_s$  is the Schwarzschild radius.

- ▶ Then there are classical next to the leading corrections in the large impact parameter and string corrections.

- ▶ It corresponds to the deflection angle of a particle moving in the Aichelburg-Sexl metric that can be seen as the boosted Schwarzschild metric.
- ▶ It is the metric created by a fast moving particle as seen from the other particle.
- ▶ In the case of a scattering on a  $D_p$  brane, there is a background metric, namely the metric created by the  $D_p$  brane.
- ▶ How can we see the effect of this metric in the scattering of a massless closed string on a  $D_p$  brane?

# The classical deflection angle in brane background

- ▶ At the classical level we can compute the deflection angle of a massless probe moving in the metric created by a Dp brane.
- ▶ Consider a general metric of the kind:

$$ds^2 \equiv g_{\mu\nu}(x) dx^\mu dx^\nu = -\alpha(r) dt^2 + \beta(r) (dr^2 + r^2 d\theta^2)$$

where we have neglected coordinates that are not involved for a geodesic in which only  $t$ ,  $r$  and  $\theta$  vary.

- ▶ The geodesic equations can be best derived from the action of a massless point-particle in this metric:

$$S = \frac{1}{2} \int \frac{d\tau}{e} \dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x) = \frac{1}{2} \int \frac{d\tau}{e} \left( -\dot{t}^2 \alpha(r) + \beta(r) (\dot{r}^2 + r^2 \dot{\theta}^2) \right)$$

where  $e$  is the einbein to take care of the reparametrization invariance of the world line coordinate  $\tau$ .

- ▶ The conjugate momenta are given by:

$$p_t \equiv \frac{\partial L}{\partial \dot{t}} = -\frac{\dot{t} \alpha}{e} ; \quad p_r \equiv \frac{\partial L}{\partial \dot{r}} = \frac{\beta(r) \dot{r}}{e} ; \quad p_\theta \equiv \frac{\partial L}{\partial \dot{\theta}} = \frac{\dot{\theta} r^2 \beta(r)}{e}$$



- ▶ The Eq. of motion for  $e$  gives:

$$\beta(r)\dot{r}^2 + \beta(r)r^2\dot{\theta}^2 = \alpha(r)\dot{t}^2$$

- ▶ Since the Lagrangian does not depend explicitly on either  $t$  or  $\theta$  there are two conserved quantities: the energy and the angular momentum

$$E = -\alpha(r)\dot{t} \quad ; \quad J = \beta(r)r^2\dot{\theta}$$

where a dot denotes derivative with respect to  $\tau$  and we have taken  $e = 1$ .

- ▶ Combining the three previous equations we get

$$\frac{\dot{\theta}}{\dot{r}} = \frac{J}{\beta r^2} \frac{1}{\sqrt{\frac{E^2}{\alpha\beta} - \frac{J^2}{\beta^2 r^2}}} = \frac{b}{r^2} \frac{1}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}}$$

where  $b \equiv J/E$  is the impact parameter.

- ▶ The deflection angle is therefore given by:

$$\Theta_p = 2 \int_{r_*}^{\infty} \frac{dr}{r^2} \frac{b}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}} - \pi$$

$r_*$  is the turning point i.e. the largest root of the equation

$$\frac{\beta}{\alpha} - \frac{b^2}{r^2} = 0.$$

- ▶ The result depends only on  $\alpha/\beta$ .
- ▶ It is therefore invariant under a  $r$ -dependent rescaling of the whole metric.
- ▶ Therefore, we can work alternatively in either the string or the Einstein frame.
- ▶ In our case we find, for a  $Dp$  brane:

$$\frac{\beta}{\alpha} = 1 + \left( \frac{R_p}{r} \right)^{7-p}$$

- ▶ Changing variable to  $u = \frac{b}{r}$  one gets:

$$\Theta_p = 2 \int_0^{u^*} \frac{du}{\sqrt{1 - u^2 + \left(\frac{R_p}{b}\right)^{7-p} u^{7-p}}} - \pi$$

where  $u^*$  is the smallest root of the equation:

$$1 - (u^*)^2 + \left(\frac{R_p}{b}\right)^{7-p} (u^*)^{7-p} = 0$$

- ▶ The integral can be done exactly for the cases  $p = 5, 6$ :

$$\tan \frac{\Theta_6}{2} = \frac{R_6}{2b} ; \quad \Theta_5 = \frac{\pi}{\sqrt{1 - \left(\frac{R_5}{b}\right)^2}} - \pi$$

- ▶ For the case  $p = 3$  we get instead:

$$\Theta_3 = 2\sqrt{1 + k^2}K(k) - \pi ; \quad K(k) = \int_0^1 \frac{dv}{\sqrt{(1 - v^2)(1 - k^2v^2)}}$$

$K$  is the complete elliptic integral of first kind.

- ▶ For general  $p$  we have not yet been able to write the deflection angle in closed form.
- ▶ We have computed the leading and the next to the leading behaviour for large impact parameter:

$$\Theta_p = \sqrt{\pi} \left[ \frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})} \left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2} \frac{\Gamma(\frac{15-2p}{2})}{\Gamma(6-p)} \left(\frac{R_p}{b}\right)^{2(7-p)} + \dots \right]$$

## Scattering of a closed string on a Dp brane: disk

- ▶ We consider the scattering of a massless closed string of the NS-NS sector on a Dp brane.
- ▶ The two closed strings have respectively momentum  $p_1$  and  $p_2$ .
- ▶ Along the directions of the world-volume of a Dp brane there is conservation of energy and momentum:

$$(p_1 + p_2)_{\parallel} = 0 \quad ; \quad p_1^2 = p_2^2 = 0$$

- ▶ The scattering is described by two Mandelstam variables:

$$t = -(p_{1\perp} + p_{2\perp})^2 = -4E^2 \cos^2 \frac{\theta}{2} \quad ; \quad s = E^2 = |p_{1\perp}|^2 = |p_{2\perp}|^2$$

$\theta$  = the angle between the d-dim ( $d \equiv 9 - p$ ) vectors  $p_{1\perp}$  and  $p_{2\perp}$ .

- ▶ At high energy we consider the following kinematical configuration:

$$p_1 = (E, \underbrace{0 \dots 0}_d; E, \vec{p}_1) \quad ; \quad p_2 = (-E, \underbrace{0 \dots 0}_d; -E, \vec{p}_2)$$

$\vec{p}_1, \vec{p}_2$  are  $(d - 1)$ -dim vectors orthogonal to the  $(p + 1)$  direction.

- ▶ At high energy ( $s \rightarrow \infty$ ) only the coefficient of the term  $\text{Tr}(\epsilon_1 \epsilon_2^T)$  survives:

$$A_1 = - \left( \frac{\kappa_{10} T_p N}{2} \right) (\alpha' E^2)^2 \frac{\Gamma(-\alpha' E^2) \Gamma(-\frac{\alpha'}{4} t)}{\Gamma(1 - \alpha' E^2 - \frac{\alpha'}{4} t)}$$

where  $N$  is the number of D branes,

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 ; \quad T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}} ; \quad \frac{\kappa_{10} T_p N}{2} = \frac{R_p^{7-p} \pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})}$$

[ Ademollo et al, 1974, Klebanov and Thorlacius, 1995;  
Klebanov and Hashimoto, 1996, Garousi and Myers, 1996]

- ▶ The poles in the  $t$ -channel correspond to exchanges of closed strings, while those in the  $s$ -channel correspond to exchanges of open strings:

$$2 + \frac{\alpha'}{2} t = 2m ; \quad m = 2, 4, \dots ; \quad 1 + \alpha' E^2 = n ; \quad n = 1, 2, \dots$$

- ▶ Regge behaviour at high energy:

$$A_1 = \left( \frac{\kappa_{10} T_p N}{2} \right) e^{-i\frac{\alpha'}{4}t} (\sqrt{\alpha'} E)^{2+\frac{\alpha'}{2}t} \Gamma(-\frac{\alpha'}{4}t)$$

- ▶  $A_1$  diverges when  $E \rightarrow \infty$  and this creates problems with the unitarity of the S matrix.
- ▶ This problem is cured by higher orders in the perturbative expansion.
- ▶ They contribute with higher power of the energy in such a way that they can be summed to get an imaginary exponential: **eikonal approximation**  $\implies$  **no problems with unitarity**.
- ▶ The properly normalized S matrix is:

$$S = 1 + iT = 1 + i \frac{A}{\sqrt{2E_1} \sqrt{2E_2}} = 1 + i \frac{A}{2E} ; E_1 = E_2 \equiv E$$

- ▶ The quantity that exponentiates at high energy and small  $t$  is:

$$iT_1 \equiv \frac{A_1}{2E} \implies i \left( \frac{\kappa_{10} T_p N}{2} \right) e^{-i\frac{\alpha'}{4}t} \frac{(\sqrt{\alpha'} E)^{2+\frac{\alpha'}{2}t}}{2E} \Gamma\left(-\frac{\alpha'}{4}t\right)$$

- ▶ Assume that the amplitude is dominated by the graviton massless pole at  $t \sim 0$  ( $\alpha' \rightarrow 0$ ):

$$iT_1(t, E) = i \left( \frac{\kappa_{10} T_p N}{2} \right) \left( \frac{2E}{(-t)} + i\pi \frac{\alpha' E}{2} (\sqrt{\alpha'} E)^{\frac{\alpha'}{2}t} \right)$$

- ▶ The real part describes the scattering of the closed string on the Dp brane, while the imaginary part describes the absorption of the closed string by the Dp brane.
- ▶ Go to impact parameter space:

$$T_1^R(b, E) + iT_1^I(b, E) = \int \frac{d^{d-1} q_t}{(2\pi)^{d-1}} e^{-ib \cdot q_t} T_1(t = -q_t^2, E)$$



# Deflection angle from string theory

- ▶ For the real part one gets:

$$iT_1^R(b, E) \equiv 2i\delta(E, b) = i \frac{E \sqrt{\pi} R_p^{d-2}}{(d-3)b^{d-3}} \cdot \frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})} ; \quad d \equiv 9 - p$$

- ▶ Assuming that it exponentiates, we get:

$$S(E, b) \equiv e^{2i\delta(E, b)} = e^{i \frac{E \sqrt{\pi} R_p^{d-2}}{(d-3)b^{d-3}} \cdot \frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})}}$$

- ▶ Going back to momentum space, we get:

$$\int d^{d-1} b e^{i(b \cdot q_t + 2\delta(E, b))}$$

- ▶ For large impact parameter we have the saddle point equation:

$$\vec{q}_t - \vec{b} b^{1-d} \frac{E \sqrt{\pi} R_p^{d-2}}{b^{d-3}} \cdot \frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})} = 0$$

- ▶ From which we compute the deflection angle:

$$\Theta_p = \frac{|q_t|}{E} = \sqrt{\pi} \left( \frac{R_p}{b} \right)^{7-p} \cdot \frac{\Gamma\left(\frac{8-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}$$

- ▶ It agrees with the classical calculation for large impact parameter!!
- ▶ Assuming that also the imaginary part exponentiates, we get the absorption amplitude:

$$S^{abs}(E, b) = e^{-g_s \sqrt{\alpha'} E} \frac{(2\pi)^{\frac{d-1}{2}}}{16} (\log(\sqrt{\alpha'} E))^{\frac{1-d}{2}} e^{-\frac{b^2}{2\alpha' \log(\sqrt{\alpha'} E)}}$$

that is a purely stringy effect.

- ▶ To check the exponentiation and to compute the next to the leading behaviour in the expansion for large impact parameter we need to compute the annulus diagram.

# The annulus diagram

- ▶ The annulus diagram is given by:

$$A = \mathcal{N} \int d^2 z_a d^2 z_b \langle B | W_a(z_a, \bar{z}_a) W_b(z_b, \bar{z}_b) D | B \rangle$$

$W_{a,b}(z_{a,b}, \bar{z}_{a,b})$  are the closed string vertices and  $\mathcal{N}$  is a normalization factor.

- ▶ The sum over the spin structures can be explicitly performed obtaining in practice only the contribution of the bosonic degrees of freedom without the bosonic partition function.
- ▶ The final result is rather explicit.  
[Pasquinucci, 1997 and Lee and Rey, 1997]

- ▶ In the closed string channel the coefficient of the term with  $\text{Tr}(\epsilon_1 \epsilon_2^T)$  (relevant at high energy) of annulus diagram is equal to:

$$\begin{aligned}
 A_2 &= \left(\frac{\kappa_{10}}{\pi}\right)^2 (8\pi^2 \alpha')^{-\frac{p+1}{2}} \frac{(2\pi)^4}{2} N^2 (\alpha' s)^2 \\
 &\times \int_0^\infty \frac{d\lambda}{\lambda} \lambda^{\frac{p+1}{2}} \int_0^{\frac{1}{2}} d\rho_a \int_0^{\frac{1}{2}} d\rho_b \int_0^{\frac{1}{\lambda}} d\omega_a \int_0^{\frac{1}{\lambda}} d\omega_b \\
 &\times e^{-\alpha' s V_s - \frac{\alpha'}{4} t V_t} ; \quad Z_{a,b} \equiv e^{2\pi i(\rho_{a,b} + i\omega_{a,b})}
 \end{aligned}$$

- ▶ where

$$V_s = -2\pi \lambda \rho_{ab}^2 + \log \frac{\Theta_1(i\lambda(\zeta + \rho_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - \rho_{ab})|i\lambda)}{\Theta_1(i\lambda(\zeta + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - i\omega_{ab})|i\lambda)}$$

and

$$V_t = 8\pi \lambda \rho_a \rho_b + \log \frac{\Theta_1(i\lambda(\rho_{ab} + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\rho_{ab} - i\omega_{ab})|i\lambda)}{\Theta_1(i\lambda(\zeta + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - i\omega_{ab})|i\lambda)}$$

$$\rho_{ab} \equiv \rho_a - \rho_b ; \quad \zeta = \rho_a + \rho_b ; \quad \omega_{ab} \equiv \omega_a - \omega_b$$

- ▶ The high energy behaviour ( $E \rightarrow \infty$ ) of the annulus diagram can be studied, by the saddle point technique, looking for points where  $V_s$  vanishes.
- ▶ This happens for  $\lambda \rightarrow \infty$  and  $\rho_{ab} \rightarrow 0$ .
- ▶ Performing the calculation one gets the leading term for  $E \rightarrow \infty$ :

$$A_2(E, t) \rightarrow \frac{i}{4E} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} A_1(E, t_1) A_1(E, t_2) V(t_1, t_2, t)$$

where

$$t_1 \equiv -\left(\frac{q}{2} + k\right)^2 ; \quad t_2 \equiv -\left(\frac{q}{2} - k\right)^2 ; \quad t = -q^2$$

and

$$V(t_1, t_2, t) = \frac{\Gamma\left(1 + \frac{\alpha'}{2} (t_1 + t_2 - t)\right)}{\Gamma^2\left(1 + \frac{\alpha'}{4} (t_1 + t_2 - t)\right)} \implies 1$$

in the field theory limit ( $\alpha' \rightarrow 0$ ).

- ▶ Going to impact parameter space:

$$A_2(E, b) = \int \frac{d^{d-1}q}{(2\pi)^{d-1}} e^{-ib \cdot q} A_2(E, -q^2) = \frac{i}{4E} (A_1(E, b))^2$$

- ▶ In terms of the matrix  $T \equiv \frac{A}{2E}$ :

$$T_2(E, b) = \frac{i}{2} (T_1(E, b))^2$$

- ▶ This implies that:

$$S(E, b) = 1 + iT_1 + iT_2 + \dots = 1 + iT_1 - \frac{1}{2} (T_1)^2 + \dots = e^{iT_1}$$

- ▶ At high energy the amplitude exponentiates: no problems with unitarity.
- ▶ We have extracted the leading behaviour at high energy:

$$T_1 \sim E \quad ; \quad T_2 \sim E^2$$

But there is also a next to the leading contribution to  $T_2^{nl} \sim E$ .

- ▶ It can be extracted from the annulus amplitude.
- ▶ It must also exponentiate.
- ▶ It gives the next to the leading correction to the deflection angle that agrees with the one obtained from the classical calculation.

# Conclusion and outlook

- ▶ We have seen how from string scattering in flat space-time we can recover properties of curved space-time.
- ▶ In particular, from the scattering of a massless closed string on a Dp brane at high energy and low transfer momentum we have computed the deflection angle of a probe particle moving in the metric of the Dp brane.
- ▶ The result reproduces the leading and the next to the leading contributions for large impact parameter computed from classical gravity in the metric of a Dp brane.
- ▶ String corrections to the field theory results can also be computed.
- ▶ We have not seen any effect from the dilaton and the RR field on the deflection angle.



# The absorption cross section from a D3 brane

[Klebanov, 1997]

- ▶ The low energy absorption cross section of a dilaton by a D3 brane can be calculated and one gets:

$$\sigma_{abs.D3} = \frac{\pi^4}{8} \omega^3 R^8 ; \quad H(r) = 1 + \frac{R^4}{r^4}$$

$\omega$  is the dilaton energy.

- ▶ This calculation is done using the curved space formalism in the metric of a D3 brane.
- ▶ On the other hand, the same quantity can be computed from the the scattering of a closed string on a D3 brane that generates two open strings (gluons).
- ▶ At low energy the coupling of the DBI action that is relevant is the one involving a dilaton and two gauge fields that gives the following amplitude:

$$\mathcal{A} = -\frac{\kappa_{10}}{\sqrt{2}} \cdot 2 \cdot \frac{p_1 \cdot p_2}{\sqrt{2\omega} \omega} = -\frac{\kappa_{10} \sqrt{\omega}}{\sqrt{2}} ; \quad E_1 = E_2 = \frac{\omega}{2}$$

- ▶ The absorption cross section is equal to:

$$\frac{1}{2} \int \frac{d^3 p_1}{(2\pi)^3} \int \frac{d^3 p_2}{(2\pi)^3} (2\pi)^4 \delta(E_1 + E_2 - \omega) \delta^3(\vec{p}_1 + \vec{p}_2) \mathcal{A}^2$$

Factor  $\frac{1}{2}$  because the two particles in the final state are identical.

- ▶ One gets:

$$\sigma_{abs.D3} = \frac{\kappa_{10}^2 N^2 \omega^3}{32\pi} = \frac{\pi^4}{8} \omega^3 R^8 \quad ; \quad R^4 = \frac{\kappa_{10} N}{2\pi^{\frac{5}{2}}}$$

- ▶ The same result from the coupling of a dilaton with two gauge fields that has a priori nothing to do with curved space-time.
- ▶ This calculation is at the origin of the Maldacena conjecture.