# String Theory approach to non-equilibrium dynamics

# of strongly coupled systems

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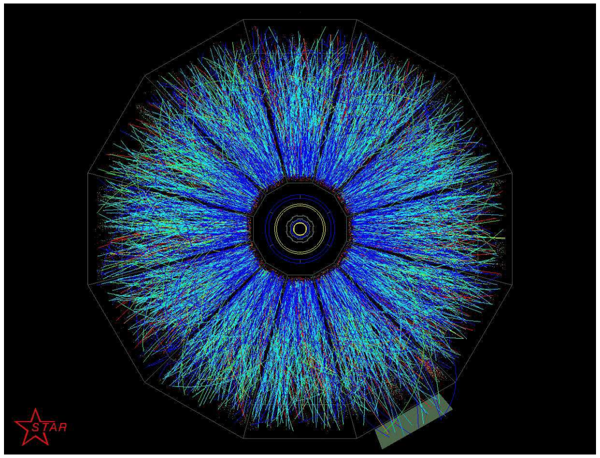
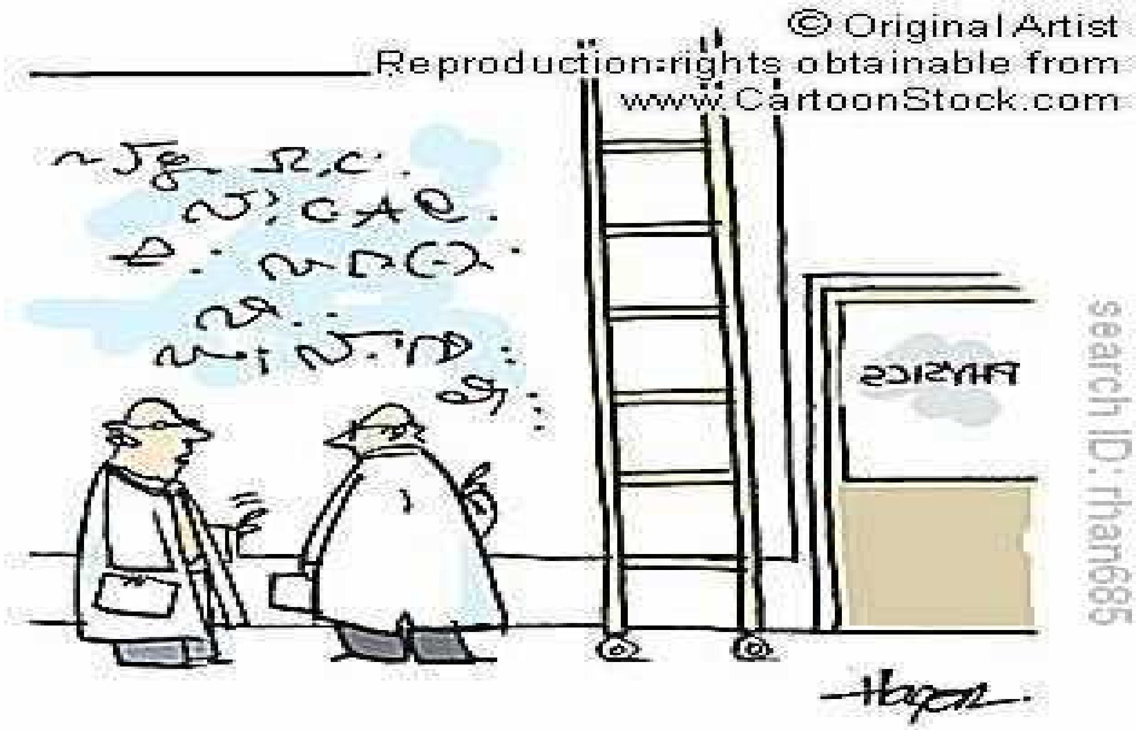



# Plan

- Low-energy effective description of near-equilibrium systems
  - “coupling constants” $\Leftrightarrow$ ”transport coefficients” (experiment)
  - ”operators” $\Leftrightarrow$ ”higher velocity gradient terms” (phenomenology)
- Gauge theory/string theory correspondence
- String Theory answers to strongly coupled systems (slightly) off-equilibrium:
  - the shear viscosity and its universal bound
  - causal relativistic hydrodynamics
- Further applications of gauge theory/gravity correspondence




OOO, I'VE THOUGHT OF A NEW ONE!
TWO SQUIGGLES AND A BACKWARDS G!

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$\Rightarrow$ Consider (relativistic) translationary invariant system in flat space time. We will assume that the system interacts with a thermal bath of temperature $T$; it might have a set of intrinsic scales $m_i$

$\Rightarrow$ We would like to construct a most general effective description of the system, valid on distances $\ell$ and time-scales $\tau$ much much larger than the intrinsic scales

$$\frac{1}{\ell} \equiv |\vec{k}| \ll \min\{T, m_i\}\,, \qquad \frac{1}{\tau} \equiv \omega \ll \min\{T, m_i\}$$

$\Rightarrow$ a theory providing such a description is **Hydrodynamics**

$\Rightarrow$ Hydrodynamics owes its existence to the presence of conserved quantities in the system

— the stress energy tensor $T_{\mu\nu}$,

— the conserved currents $J^\mu$ associated with the global $U(1)$ charges

it is a theory of slow and gradual variation of these quantities:

$$\nabla_\mu T^{\mu\nu} = 0\,, \qquad \nabla_\mu J^\mu = 0$$

How do we construct such an effective description?

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Sophisticated Landau approach

to be explained...

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 $\Leftrightarrow$ 

**Brute force**

For a system in equilibrium, in a local rest frame:

$$T_{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \qquad J^\mu = \rho\, \delta^\mu_0$$

where $\epsilon$ is the energy density, $P$ is a pressure and $\rho$ is a global $U(1)$ density. In any other reference frame, related to above by a Lorentz transformation with a (constant) time-like 4-velocity $u^\mu$, $u^\mu u_\mu = -1$:

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, \Delta^{\mu\nu}\,, \qquad J^\mu = \rho\, u^\mu$$

with

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$$

being a symmetric transverse tensor —

$$u_\mu \Delta^{\mu\nu} = 0$$

Suppose that a system is *slightly* off-equilibrium:

$$\epsilon = \epsilon(t, \vec{x})\,, \qquad P = P(t, \vec{x})\,, \qquad \cdots$$

*Slightly* means that we still have local equilibrium, and thus a familiar thermodynamic relations

$$\epsilon + P = sT + \mu\rho\,, \qquad d\epsilon = T ds + \mu d\rho$$

but the equilibrium is not yet reached globally — specifically,

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P\, \Delta^{\mu\nu} + \delta T^{\mu\nu}\,, \qquad J^\mu = \rho\, u^\mu + \delta J^\mu$$

where

$$\delta T^{\mu\nu} = \delta T^{\mu\nu}\{u^\mu\,, T\,, \mu\,; \nabla u^\mu\,, \nabla^2 u^\mu\,, \cdots\}\,, \qquad \delta J^\mu = \delta J^\mu\{\cdots\}$$

with $u^\mu = u^\mu(t, \vec{x})$. We use

$$T\,, \qquad \mu\,, \qquad u^\mu$$

as an independent variables — fields of the effective description

There is a freedom of choosing the local reference frame, and we can do so that the corrections $\delta T^{\mu\nu}$ and $\delta J^{\mu}$ are transverse

$$u_\mu \delta T^{\mu\nu} = u_\mu \delta J^\mu = 0$$

If the fundamental variables are slowly varying in space-time we can — following the logic of the low-energy effective field theory — organize $\delta T^{\mu\nu}$ and $\delta J^{\mu}$ into gradient expansion of independent variables (fields). The most general transverse expansion to leading order in the derivatives

$$\delta T^{\mu\nu} = -\eta \left[\Delta^{\mu\lambda}\left(\nabla_\lambda u^\nu + \nabla^\nu u_\lambda - \frac{2}{3}\delta^\nu_\lambda \nabla_\alpha u^\alpha\right)\right] - \zeta \left[\Delta^{\mu\nu}\, \nabla_\alpha u^\alpha\right] + \cdots$$

$$\delta J^\mu = \sigma_Q \left[\mu \Delta^{\mu\nu}\, \nabla_\nu \ln \frac{T}{\mu}\right] + \cdots$$

where we choose a conventional parametrization. Thus to leading order in the derivative expansion we have 3 independent terms ($\equiv$ operators) and 3 transport coefficients ($\equiv$ couplings):

$$\eta\,, \qquad \zeta\,, \qquad \sigma_Q$$

the shear viscosity, the bulk viscosity, the charge conductivity

$\cdots$ we can proceed with constructing higher order corrections $\cdots$

**Landau approach**

$\Rightarrow$ The basic idea is to construct an *entropy current*, $\mathcal{S}^\mu$, which is conserved at equilibrium, but increases in -all- processes away from equilibrium.

At equilibrium, the obvious candidate:

$$\mathcal{S}^\mu\Big|_{equilibrium} = s\, u^\mu$$

$\Rightarrow$ Using conservation laws for the full stress energy tensor and the full global current we can derive (to first-order in the derivative expansion)

$$\nabla_\mu\left(su^\mu - \frac{\mu}{T}\,\delta J^\mu\right) = \delta J^\mu \times \frac{\mu}{T}\left[\nabla_\mu \ln\frac{T}{\mu}\right] - \frac{\delta T^{\mu\nu}}{T} \times \nabla_\mu u_\nu$$

If we interpret the entropy current away from equilibrium as

$$\mathcal{S}^\mu \equiv su^\mu - \frac{\mu}{T}\,\delta J^\mu$$

then, the Landau idea that the entropy is produced (increases) in the approach to equilibrium

$$\nabla_\mu S^\mu \geq 0$$

leads to

$$\delta J^{\mu} \propto \Delta^{\mu\nu}\frac{\mu}{T}\left[\nabla_{\nu}\ln\frac{T}{\mu}\right]$$

$$\delta T^{\mu\nu} \propto -\Delta^{\mu\lambda}\left(\nabla_{\lambda}u^{\nu}+\nabla^{\nu}u_{\lambda}-\frac{2}{3}\delta^{\nu}_{\lambda}\nabla_{\alpha}u^{\alpha}\right)-\propto\Delta^{\mu\nu}\nabla_{\alpha}u^{\alpha}$$

The **positive** proportionality coefficients are precisely the transport coefficients introduced previously:

- the charge conductivity $\sigma_Q$
- the shear viscosity $\eta$
- the bulk viscosity $\zeta$

Summarize: to leading order in the derivative expansion both the “brute force” and the “Landau method” produce the same effective description; we further understand peculiar organization of terms in the ”brute force” method as that due to the entropy production in the approach to equilibrium

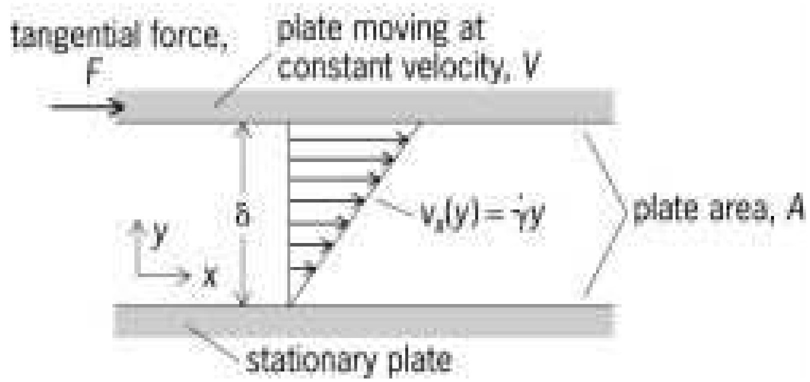
Before we proceed with discussion of differences in two methods lets consider in more details the remarkable hydrodynamic equations

$$\nabla_\mu \left( T^{\mu\nu}_{(0)} + \delta T^{\mu\nu}_{(1)} \right) = 0 \,, \qquad \nabla_\mu \left( J^\mu_{(0)} + \delta J^\nu_{(1)} \right) = 0$$

$\Rightarrow$ in the non-relativistic limit these are precisely the Navier-Stokes equations!

$\Rightarrow$ Probably the most famous transport coefficient in Navier-Stokes equations is the shear viscosity

$$\eta$$




Shear viscosity was introduced by Claude Navier in 1822 to explain the force between the moving planes:

$$F = \eta \, A \, \frac{\partial V_x}{\partial y}$$

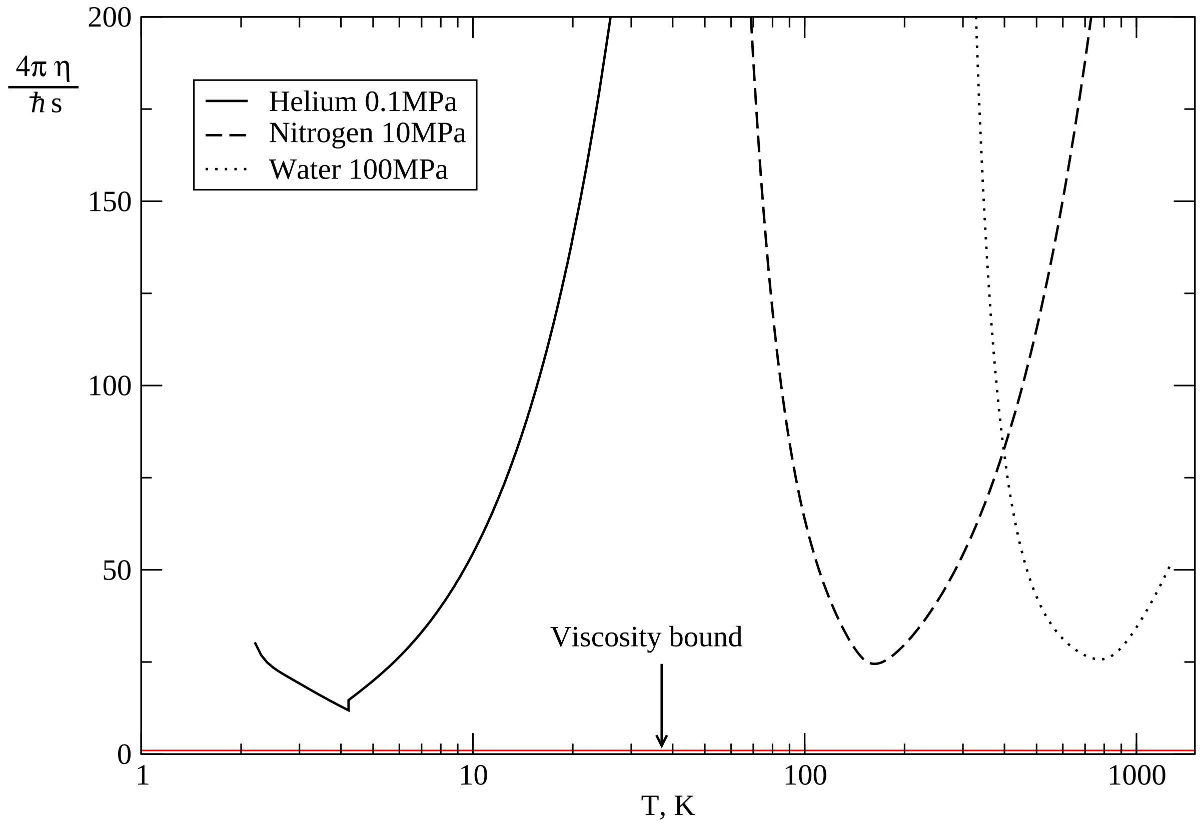
$\Rightarrow$ It is a fairly easy quantity to measure (at least for everyday fluids), but it is rather difficult to explain the results.

$\Rightarrow$ Observed viscosity in fluids spans a wide range

$$\eta \sim \left(10^{-2} \cdots 10^{15}\right) \eta_{water}$$

However....

There appears to be a well defined lower bound on the (dimensionless in units $\hbar/k_B$) ratio $\frac{\eta}{s}$




(from Kovtun, Son and Starinets [2004])

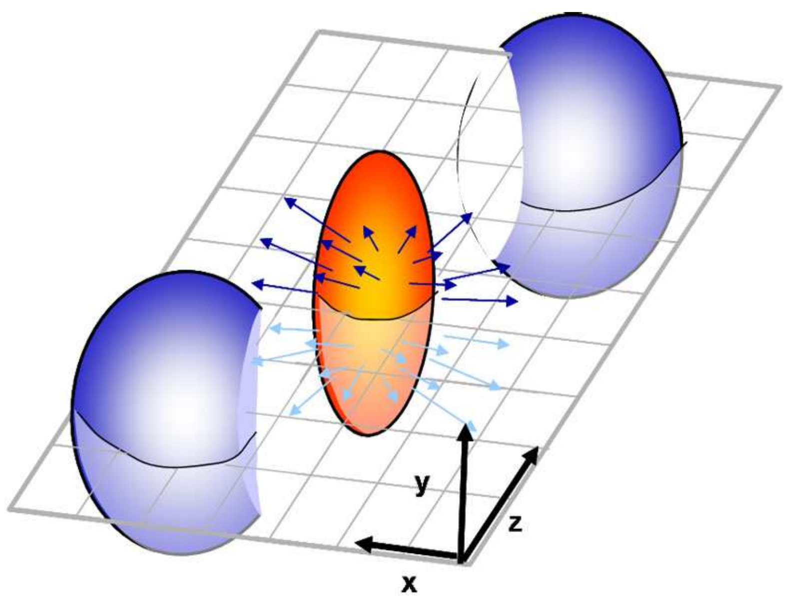
$\Rightarrow$ The question of small $\eta/s$ is not purely academic — though the existence of some bound on viscosity fascinated the physicist for over the centuries.

$\Rightarrow$ But it came back recently in the story of Relativistic Heavy Ion Collider (RHIC)

$\Rightarrow$ At RHIC gold beams collide with energy 200 GeV per pair of nucleons

$\Rightarrow$ "...In 2005, RHIC research captured worldwide attention with an astonishing surprise: instead of behaving like a gas, the matter created in RHICs energetic gold-gold collisions appears to be more like a ”perfect” liquid with virtually no resistance to flow..."
(from www.bnl.gov)

$\Rightarrow$ A generic (and interesting for our purposes) nucleon collisions are those slightly off-center:




[show movie]

$\Rightarrow$ An orange "almond" region created in collisions with non-zero impact parameter expands non-isotropically ( with higher pressure gradients along the short section) which is reflected in the axially asymmetric momentum distribution of the observed particles. This is known as **elliptic flow**.

$\Rightarrow$ Observation of the elliptic flow was one of the first results obtained at RHIC, which prompted the claims for a discovery of "near-perfect" fluid.

$\Rightarrow$ The Quark-Gluon Plasma (QGP) created in the collisions **must** behave as almost ideal (viscousless) fluid since hydrodynamic simulations show (Luzum, Romatschke (2008)) that for

$$\frac{\eta}{s} \gtrsim [\text{a few}] \times \frac{\hbar}{4\pi k_B}$$

the elliptic flow is destroyed.

$\Rightarrow$ Such a low ratio of shear viscosity to the entropy density is hard to explain with conventional methods:

- In general viscosity is very large at weak coupling; in perturbative gauge theories, $g^2 \ll 1$, is was computed to be

$$\frac{\eta}{s} \sim \frac{1}{g^4 \ln \frac{1}{g^2}} \gg 1$$

- Lattice techniques can be used to compute static properties of gauge theories at strong coupling, which is relevant to QGP fluid, but they are difficult to adopt to computation of real-time quantities such as $\eta$.

$\Rightarrow$ Running ahead, the first-ever reliable computation of shear viscosity of strongly coupled plasma was done using string theory methods:

G. Policastro, D. T. Son and A. O. Starinets, "The shear viscosity of strongly coupled N = 4 supersymmetric Yang-Mills plasma," Phys. Rev. Lett. **87**, 081601 (2001)

**Brute force vs Landau method towards effective hydrodynamics**

$\Rightarrow$ The Navier-Stokes (first-order in the velocity gradients) hydrodynamics we discussed earlier is acausal, and leads to propagation of superluminal signals. Formally, this is due to the fact that N.-S. equations are not hyperbolic.

$\Rightarrow$ The positivity of the entropy production (Landau) method was used to formulate second-order in the velocity gradients (causal) hydrodynamics, most notably by Müller (1976) and Israel and Stewart (1976). In this case, one introduces add second-order term, with the coupling $\tau_\Pi$ — the relaxation time:

$$T^{\mu\nu}_{(2)}\Big|_{M-IS} = T^{\mu\nu}_{(2)}(\tau_\Pi)$$

$\Rightarrow$ The brute force analysis (Baier et.al (2007)) show that

$$T^{\mu\nu}_{(2)}\Big|_{\text{brute force,CFT}} = T^{\mu\nu}_{(2)}(\tau_\Pi;\kappa,\lambda_1,\lambda_2,\lambda_3)$$

In generic (non-CFT) hydrodynamics there are 8 additional second order transport coefficients!

It is not that Landau approach is incorrect, the construction of the entropy current is simply ambiguous....

We would like to call onto String Theory to help us understand:

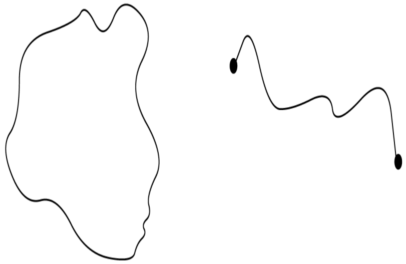
- (observed) small viscosities of strongly coupled plasma
- help us identify the correct theory of causal relativistic hydrodynamics

**Our tool:**

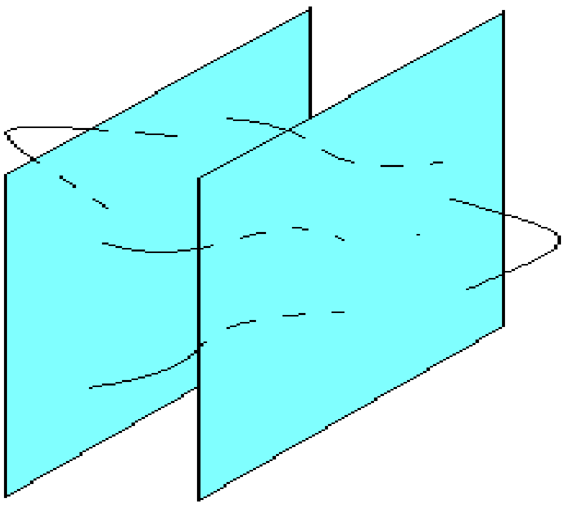
Gauge theory/String theory correspondence

(Maldacena, (1997))

In String Theory we have:



Closed strings propagate freely in the ambient space, while the ends of the open strings are 'stuck' to the solitonic objects of String Theory —- D-branes:



$\Rightarrow$ A Dp-brane is a $(p+1)$-dimensional hypersurface in 10-dimensional space-time in which strings propagate, with a tension

$$\tau_p \sim \frac{1}{g_s}$$

where $g_s$ is the string coupling constant

$\Rightarrow$ The fluctuations of the open strings attached to Dp-branes are described by (supersymmetric) Yang-Mills theory [ala-theory of strong interactions, Quantum Chromodynamics] in $(p+1)$ dimensional space-time.

$\Rightarrow$ To make contact with the real world we will be interested in a sector of string theory with $p = 3$ branes — in this case we have a QCD-like model

$\Rightarrow$ In 1997 Juan Maldacena observed that there are two regimes in a string theory with $N$ D3-branes:

- first,

$$g_s N \ll 1$$

In this case the massive hypersurfaces do not gravitate much: the gravitational backreaction on a flat space time is

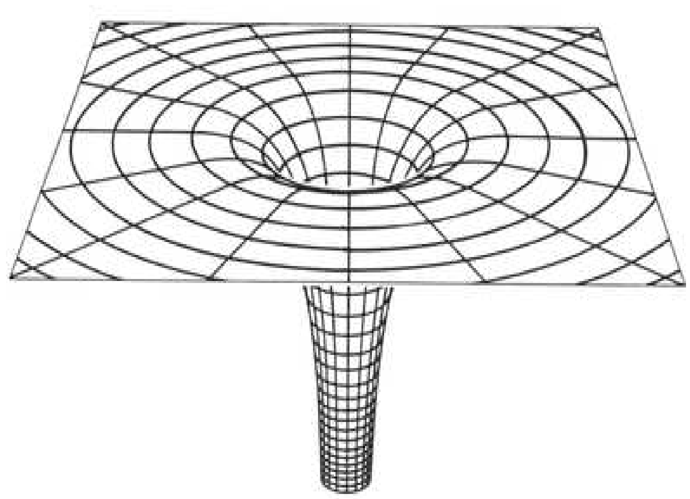
$$\propto G_{10} \times (N\tau_3) \propto g_s^2 \times \left(N \frac{1}{g_s}\right) \propto N g_s \ll 1$$

where $G_{10}$ is a 10-dimensional Newton's constant. We have a theory of **free closed strings** and a weakly coupled **gauge theory living on D3-branes**

- second,

$$g_s N \gg 1$$

The D3 branes gravitate a lot — they 'dissolve' into geometry producing a highly warped region, but with an asymptotically flat space. There are only closed strings in this regime!



The highly warped region produced by gravitating D3 brane is the so-called $AdS$ (anti-de-Sitter) space-time

$\Rightarrow$ Although there are only closed strings propagating in this warped space-time, there are 2 kinds of low energy excitations of these strings:

- closed strings “living” deep in the AdS throat
- closed strings ”living” in asymptotically flat space-time

These 2 kinds of strings almost do not interact!

$\Rightarrow$ Both for $g_s N \ll 1$ and $g_s N \gg 1$ we have a non-interacting sector of closed strings, thus following Maldacena, we identify the remaining parts, i.e,

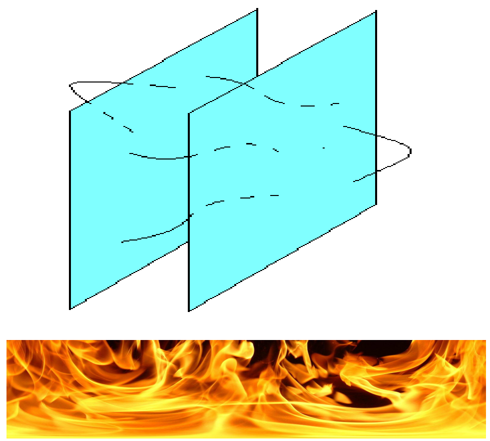
$$\mathcal{N} = 4\, SU(N)\, \mathrm{SYM} \qquad \Longleftrightarrow \qquad \text{String Theory on } AdS_5 \times S^5$$

$\Rightarrow$ The great power of this correspondence (often referred to as a duality) in that when the gauge theory is strongly coupled — large quantum corrections, the String Theory reduces to weakly coupled — classical — higher-dimensional gravity!

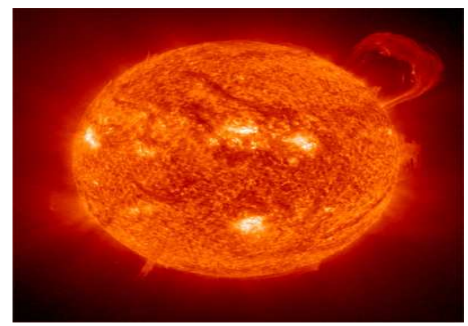
$\Rightarrow$ Though the correspondence is not proved in a mathematical sense, it survived many highly-nontrivial consistency checks; it has been generalized to gauge theories other than $\mathcal{N} = 4$ SYM, which more closely resemble real-world QCD.

We will use Maldacena duality to study the $\mathcal{N} = 4$ plasma

⟹ First, we need to turn the vacuum of a gauge theory into a plasma



⟹



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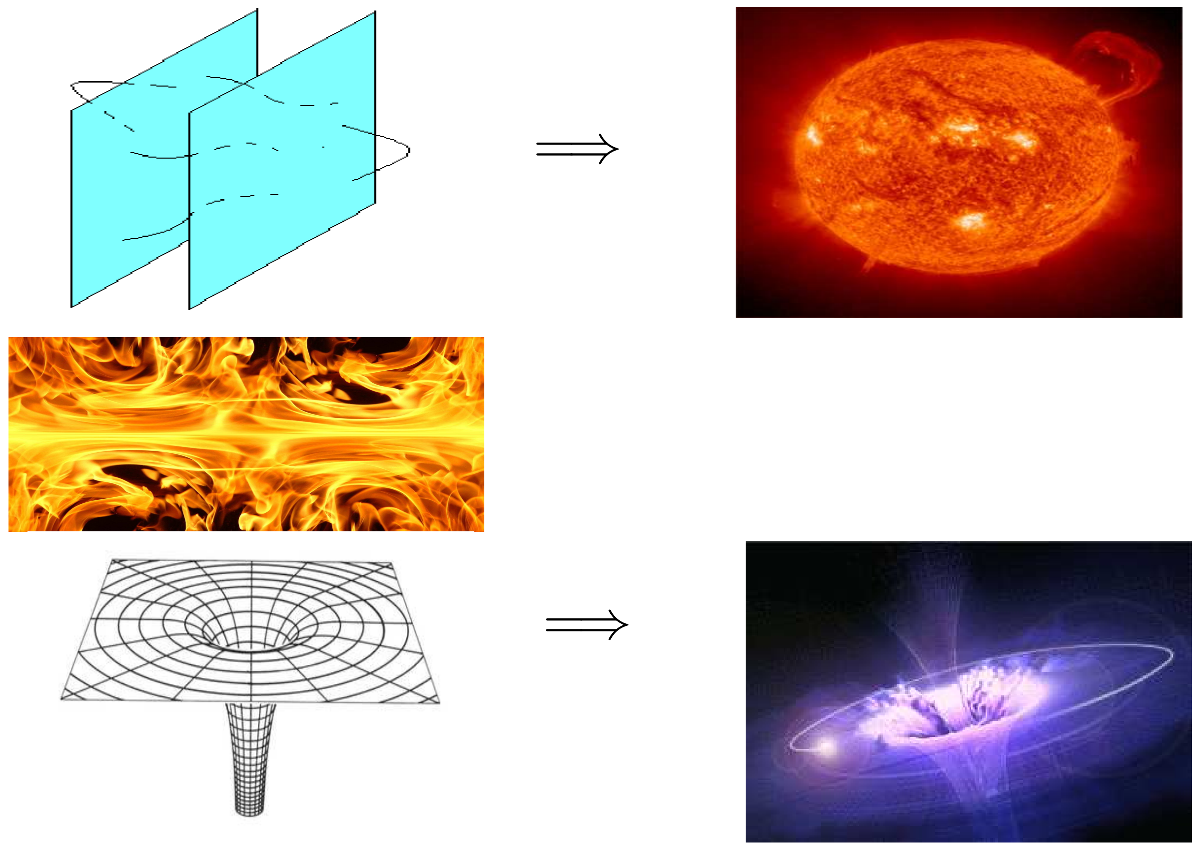

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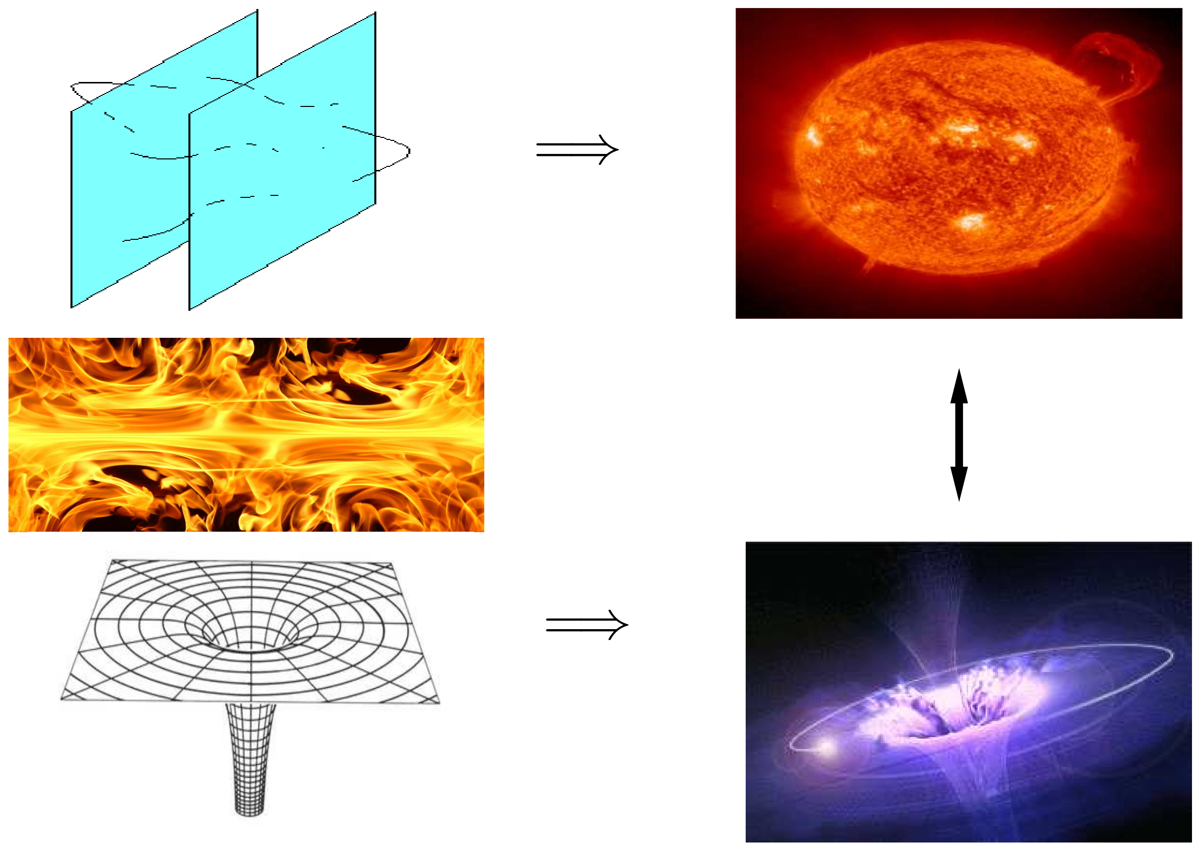


Thermal state of the gauge theory is identified with a Black Hole in AdS space-time!

$\Rightarrow$ Second, we need to measure the shear viscosity of produced plasma.

- On a gauge theory side we imagine measuring the response of the system to an external shear stress;
- on a gravity side we measure absorption cross section for a graviton scattering off a black hole horizon (can be justified with detailed analysis of the duality)

$\Rightarrow$ Finally, further developing gauge theory/ string theory correspondence (Gubser, Polyakov, and Klebanov (1998); Witten (1998)) allows to compute **any** correlation function(s) of **any** gauge invariant operators. Specifically, we can directly compute the velocity-gradients expansion of the stress-energy tensor:

$$T^{\mu\nu}\Big|_{gravity} = \sum_{i=0}^{\infty} T^{\mu\nu}_{(i)}$$

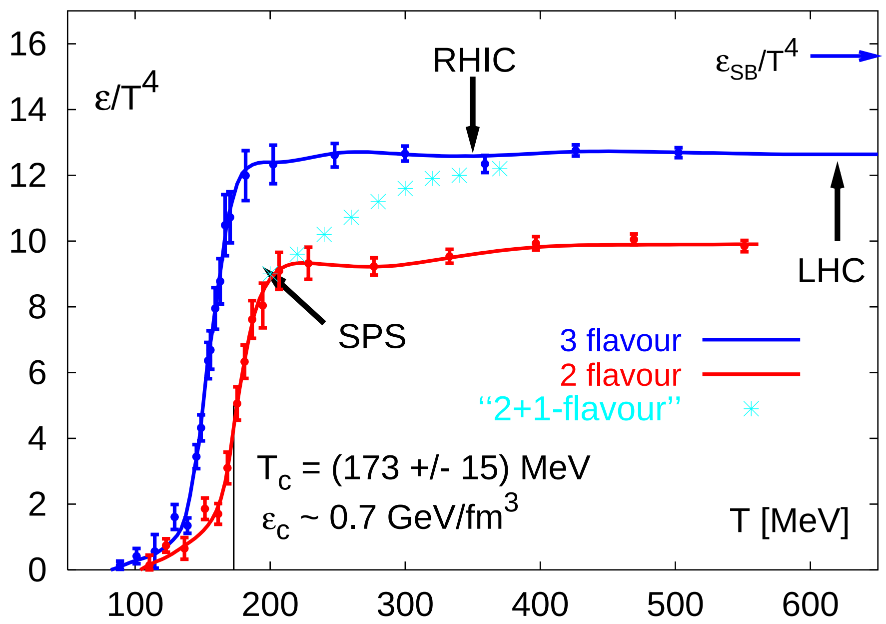
(Baier et.al (2007), Bhattacharyya et.al (2007))

# Results, and the reality check:

- Black Hole Thermodynamics
- shear viscosity from gravity
- causal relativistic hydrodynamics from gravity

$\Rightarrow$ Thermodynamics of 10-dimensional black hole in $AdS_5 \times S^5$ is equivalent to a thermodynamics of a scale-invariant 4-dimensional plasma!

$\Rightarrow$ In fact, it is even better: although $\mathcal{N} = 4$ SYM is not a QCD:

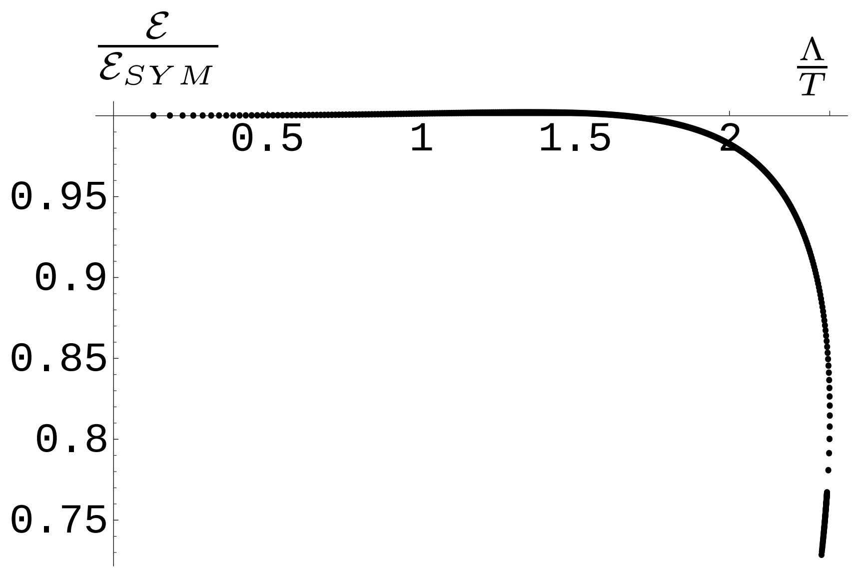



QCD thermodynamics from lattice; F.Karsch and E.Laermann, hep-lat/0305025.

$$\frac{\epsilon_{\text{QCD+3 flavors}}}{\epsilon_{\text{SB}}} \approx 0.8\,, \qquad \frac{\epsilon_{\text{Black hole}}}{\epsilon_{\text{SB}}} = \frac{3}{4} = 0.75$$

$\Rightarrow \mathcal{N} = 4$ SYM theory is scale invariant, so it's thermodynamics — $\frac{\epsilon}{T^4} = \mathrm{const}$ — does not exhibit a sharp transition, followed by a plateau (which is present)

$\Rightarrow$ we can look at other examples of gauge theory/ string theory correspondence where the gauge theory *undergoes a phase transition*:

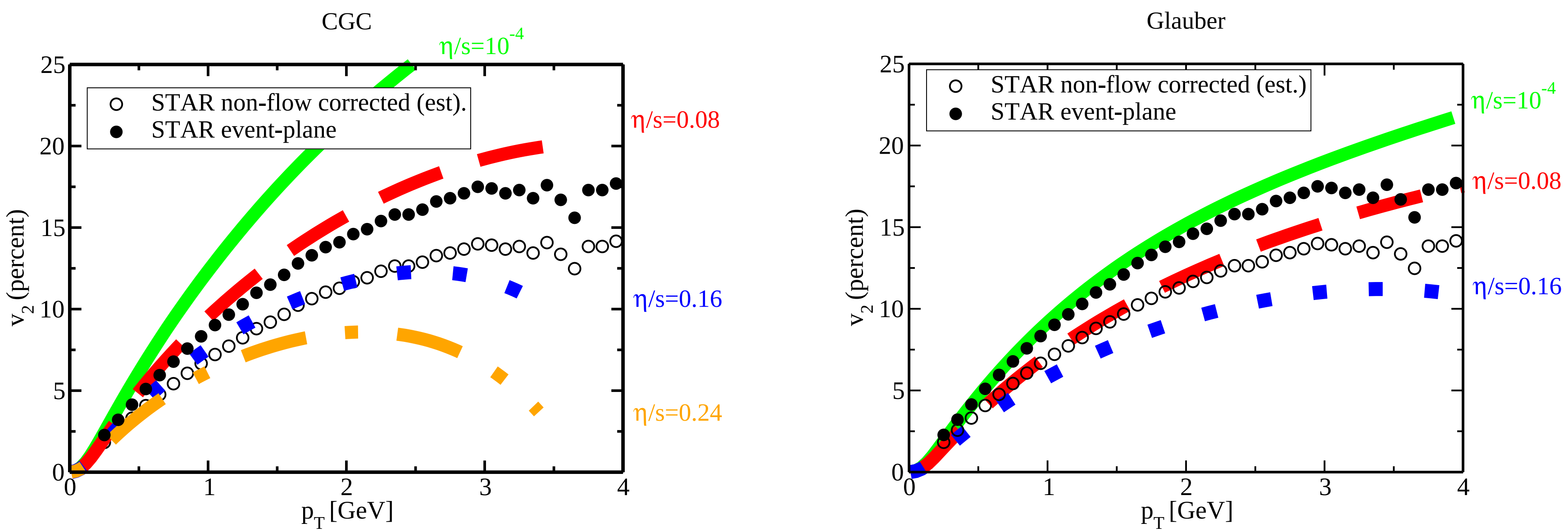



Equation of state of the mass deformed $\mathcal{N} = 4$ gauge theory plasma. At $T \sim \Lambda$ the deviation from the conformal thermodynamics is $\sim 2\%$. For the ideal gas approximation the deviation is about $40\%$. (Buchel el.al, (2007))

⇒ Policastro, Son, Starinets (2001) found

$$\left.\frac{\eta}{s}\right|_{SYM} = 1 \times \frac{\hbar}{4\pi k_B} \approx 0.08 \times \frac{\hbar}{k_B}$$

⇒ From hydro simulations, the elliptic flow data observed by RHIC experiments is recovered, provided




(Luzum, Romatschke (2008))

$$\left.\frac{\eta}{s}\right|_{QGP} = \Big[0.1 \pm 0.1(\text{theory}) \pm 0.08(\text{experiment})\Big] \times \frac{\hbar}{\mathrm{k_b}}$$

$\Rightarrow$ Despite similarities of SYM and QCD, why is

$$\left.\frac{\eta}{s}\right|_{SYM} \qquad \simeq \qquad \left.\frac{\eta}{s}\right|_{QGP}$$

$\Rightarrow$ The answer appears to lie in the following universality theorem:

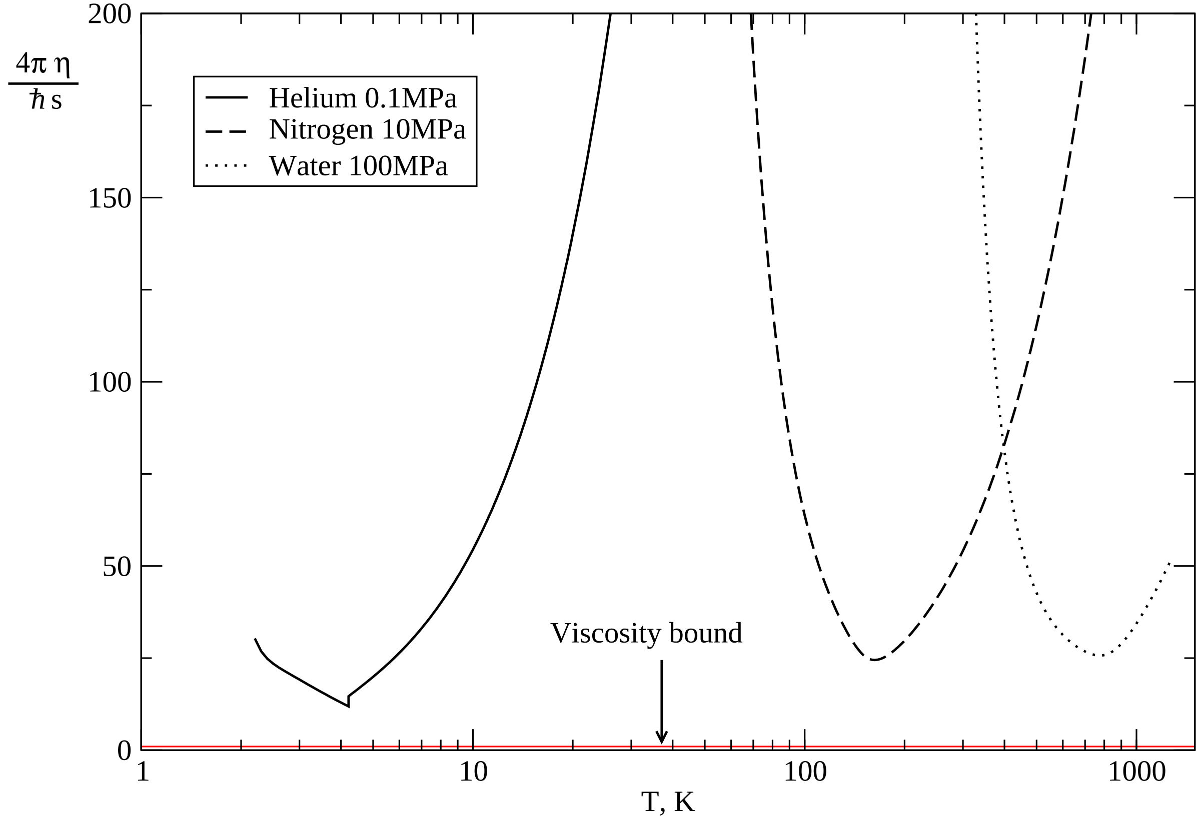
*For all gauge theories* **for which a string dual exists** *, with arbitrary gauge groups and matter content, at infinite strong coupling (under all conceivable conditions: finite chemical potentials, external fields, space-time dimension, etc.)*

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{SYM}$$

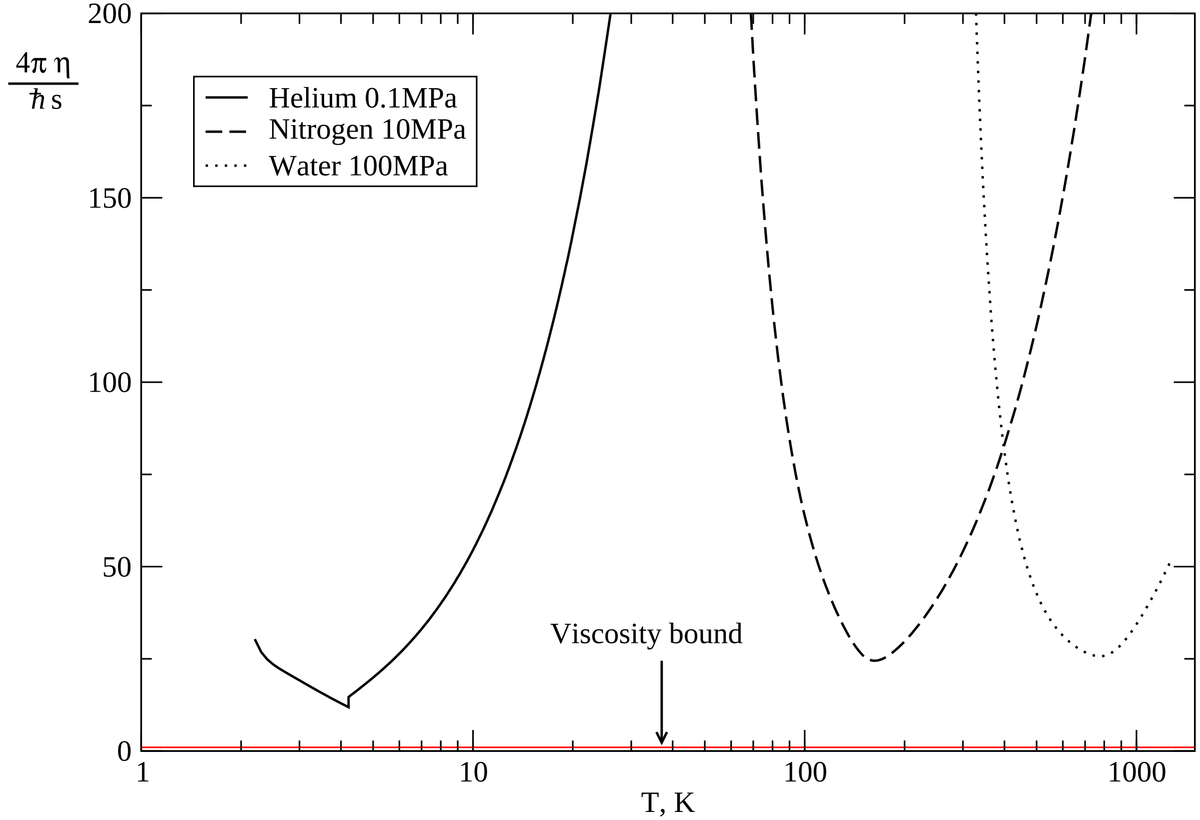
Buchel and Liu (2003); Kovtun, Son and Starinets (2003); $\cdots$

$\Rightarrow$ There has been a lot of studies of shear viscosity in gravitational dual, with the main focus being the question:

$$\text{Does a universal bound on } \frac{\eta}{s} \text{ exist?}$$




(from Kovtun, Son and Starinets [2004])




(from Kovtun, Son and Starinets [2004])

$\Rightarrow$ The bound is clearly satisfied in all known substances

$\Rightarrow$ But.... it is violated for some specific gauge theories (Kats and Petrov (2007))!

$\Rightarrow$ Interestingly, causality of hydrodynamics allows to violate the bound, but does not allow to push $\frac{\eta}{s}$ all the way to zero (Brigante et.al, (2008))

⇒ What are the lessons of gravity for the framework near-equilibrium phenomena?

⇒ For the relativistic conformal hydrodynamics:

- with Müller-Israel-Stewart entropy current:

$$\tau_\Pi = \frac{\mathrm{constant}}{T}\,, \qquad \kappa = \lambda_1 = \lambda_2 = \lambda_3 = 0$$

- explicit computations in $\mathcal{N} = 4$ SYM plasma (Baier et.al (2007), Bhattacharyya et.al (2007))

$$\tau_\Pi = \frac{2 - \ln 2}{\pi T}\,, \qquad \kappa = \frac{\eta}{\pi T}$$

$$\lambda_1 = \frac{2\eta}{\pi T}\,, \qquad \lambda_2 = \frac{2\eta \ln 2}{\pi T}\,, \quad \lambda_3 = 0$$

⇒ It is possible to definite a local entropy current, which would incorporate Landau idea in the context of (almost) most general $\mathcal{N} = 4$ hydrodynamics! (Bhattacharyya et.al (2008))

$\Rightarrow$ Recently, gauge theory/ string theory correspondence obtained a *new life* — it was reinterpreted as used as a generic dual framework, in which quantum effects in strongly coupled systems are represented by classical effects in asymptotically Anti-de-Sitter gravity.

**Other developments:**

- Theory of continuous critical phenomena at finite temperature (static/dynamic)
- Quantum phase transitions
- non-Fermi liquids
- charge transport, superconductivity, superfluidity
- Turbulence
- non-relativistic CFTs
- $\cdots$

## Conclusions

- I hope I convinced you that hydrodynamics is just another low-energy effective theory
- String Theory has been very useful in deriving low-energy effective description from first principles; thus, it is probably not a surprise that it made an impact on hydrodynamic description of strongly couple systems; to name some successful applications:
  - provided a reliable framework to compute transport properties
  - the best model to explain RHIC hydrodynamic data
  - precise formulation of the causal relativistic hydrodynamics
- More to come!