AC transport during a holographic Hall transition

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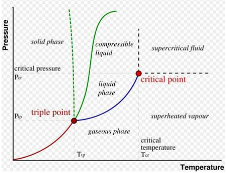
ref. J. Alanen, E.K-V, P. Kraus, V. Suur-Uski, JHEP11(2009)014
J. Alanen et. al., in progress

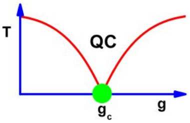
Classical Phase Transition

- Driven by thermal fluctuations at finite T
- System constituents reorganize to a new form with different macroscopic properties
- Free energy (or its derivatives) becomes discontinuous
- Critical points at finite T
- At criticality, scale invariance in space dimensions

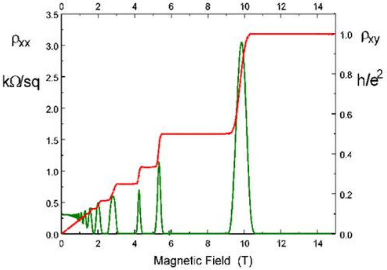


Quantum Phase Transition

- Even at $T = 0$ quantum fluctuations can drive a transition
- Ground state of the system changes as a transition parameter varied
- Transition at a quantum critical point
- At criticality, scale invariance in space-time dimensions
- Influence felt in a transition region at finite T

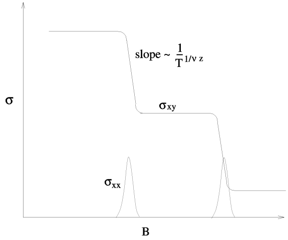


- Quantum Hall effect: plateaus for conductance at special filling factors



Quantum Hall plateau transition:

- Interesting universal behavior at transition between plateaus



- $\frac{\partial \sigma_{xy}}{\partial B} \sim \frac{1}{T^{1/\nu z}}$, steeper at low T. Experimentally (?) $\nu z \approx 3/7$.

Related transition model: 2+1 free fermions

- Consider free massive 2+1 Dirac fermions. Mass term parity odd.
- Nonzero transverse conductance[1] with $\sigma_{xy} = -\frac{\mathrm{sgn}(m)}{2}\frac{e^2}{h}$. Take N fermions, flip the mass of one of them to get $\sigma_{xy} \sim N$ transitioning to $\sigma_{xy} \sim N - 2$.
- Add interactions, finite temperature to smoothen the transition.
- Quantum critical point at $T = 0$?
- $\frac{\partial \sigma_{xy}}{\partial B} \sim \frac{1}{(T-T_c)^{1/\nu z}}$
- Try to model this with intersecting D-branes

[1] A.Ludwig et al. PRB50, 7526 (1994).

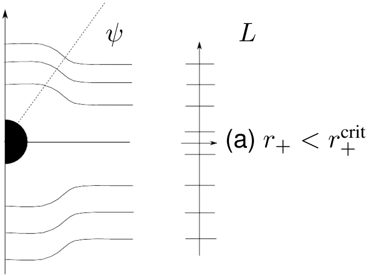
- N_7 D7-branes intersecting N_3 D3-branes over $2+1$ dimensions[2]

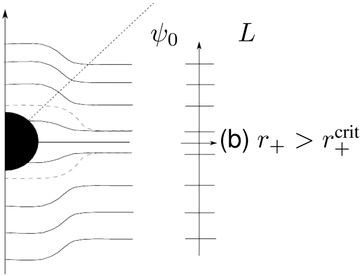
$$
\begin{array}{lcccccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D3: & \times & \times & \times & \times & & & & & & \\
D7: & \times & \times & \times & & \times & \times & \times & \times & \times &
\end{array}
$$

- D3-branes at $x^9 = 0$, D7-branes at $x^9 = L$: parameterizes the plateau-plateau transition, ranging from neg to pos values
- At $L = 0$ the massless spectrum consists of $N = N_3 N_7$ complex two-component spinors coming from the Ramond sector. The NS sector has massive states.
- No massless scalars. Fermion interactions suppressed by powers of the string scale.
- At strong coupling move to the gravitational dual.

[2] J.Davis, P.Kraus, A.Shah, JHEP 0811:020,2008; S.-J.Rey, STRINGS 2007.

- Gravitational dual tractable in the probe approximation $N_7 \ll N_3$. (Additionally use $N_3, gN_3 \gg 1$->)
- D7-branes probes in the $AdS_5 \times S^5$ geometry produced by the D3-branes. Wrap D7 -> worldvolume $AdS_4 \times S^4$. $S^4 \in S^5$: unstable
- Turn on finite T -> black brane background. Low T: Minkowski embeddings, High T: BH embeddings. Vary L, probe moves through BH to other side.





- Probe action contains two terms

$$S = S_{DBI} + S_{CS} \ ,$$

where S_{DBI} is a Born-Infeld action, and S_{CS} is a Chern-Simons action (D7 gauge field WZ interacting with D3 flux). The former depends on metric:

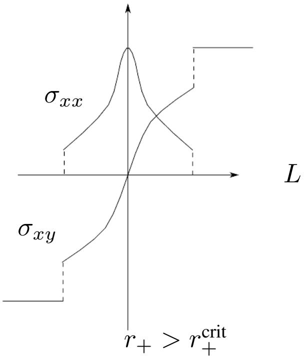
$$S_{DBI} = -\tau_0 \int d^4x \sin^4 \psi(L) \sqrt{-\det(g_4 + 2\pi\alpha' F)}$$

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + V(r)dx^i dx^i \ .$$

- Turning on a charge density ρ, magnetic field B creates a background field strength $F^{(0)}_{\mu\nu}$. Apply electric field, calculate current

$$j^x = \frac{\delta \mathcal{L}}{\delta(\partial_{\tilde{r}} a_x)} = \sigma_{xx} E^x + \sigma_{xy} E^y$$

- Result (Davis et al.) for conductivity during the plateau transition (conductivity during BH embeddings)

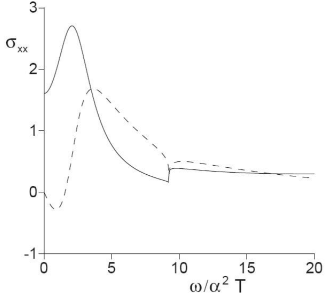


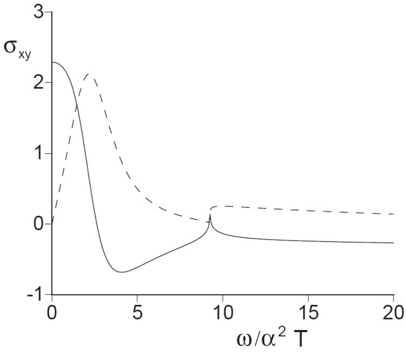
Current driven by oscillating E-field

- Two regimes depending on $\hat{\omega} = \hbar\omega/k_B T$:
  1. Collision-dominated low-freq $\hat{\omega} \ll 1$. Transport by interacting thermal excitations.
  2. Phase-coherent hi-freq $\hat{\omega} \gg 1$. Carriers excited by external field, no collisions.
- Dynamic conductivity becomes universal function of $\hat{\omega}$, $\sigma(\omega) = \sigma_0 \Sigma(\hat{\omega})$.
- $T \to 0$ or $\hat{\omega} \to \infty$: approach universal conductivity $\sigma_0 \Sigma(\infty)$, indep of microscopic details. Classifies qu critical points.

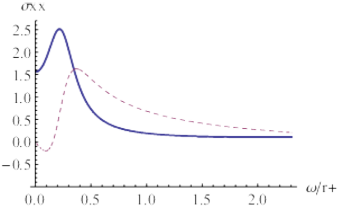
K.Damle, S.Sachdev, PRB56,8714 (1997)

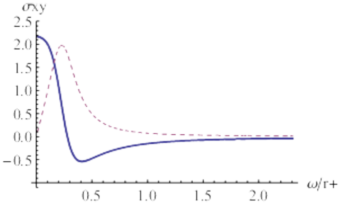
- A model by S. Sachdev for finite temperature AC conductivity
- Fermions interacting through a statistical $U(1)$ CS-field
- Combination of perturbation theory and collision effects, tricky analysis



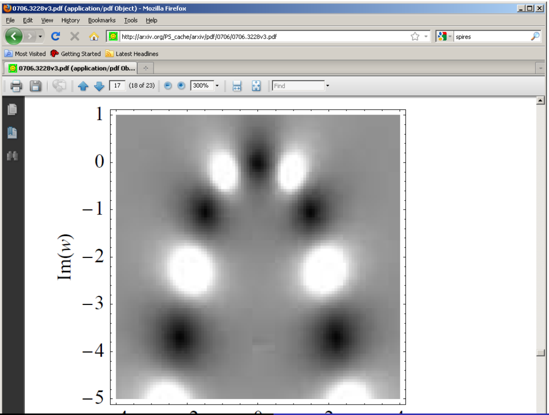


- Analyze the probe brane model
- Derive gauge field profiles in the time-dep case
- Can find surprisingly good resemblance with Sachdev's results (a part of J.Alanen, E.K-V., P.Kraus, V.Suur-Uski, JHEP '09)
- (Caveat: different symmetries)





- Study the analytic structure underlying the peaks: poles / quasinormal modes
- A well known case: AC conductivity in a classical Hall liquid (dual: dyonic black hole in AdS_4). From Hartnoll,Hertzog:



- First pole at relativistic cyclotrone frequency with damping, $\omega_* = \omega_c - i\gamma$.
- In agreement with prediction from magnetohydrodynamics.

- The DBI action reads

$$S_{BI} = -\int dx^4 \tau(r) \sqrt{-\det(g+F)}$$

and ansatz for gauge field is

$$\begin{aligned} A_v &= A_v^{(0)}(r), \\ A_j &= B\epsilon_{ij}x^j, \\ A_r &= 0. \end{aligned}$$

- The background metric, in Eddington-Finkelstein coordinates, and the background gauge field are

$$\begin{aligned} ds^2 &= 2dvdr - U(r)dv^2 + V(r)dx^i dx^i, \\ F_{12}^{(0)} &= B, \\ F_{rv}^{(0)} &= \frac{\rho(r)/\tau(r)}{\sqrt{\rho(r)^2/\tau(r)^2 + B^2 + V(r)^2}}. \end{aligned}$$

- The fluctuations are assumed to depend only on r and v:

$$\begin{aligned} A_v &= A_v^{(0)}(r), \\ A_j &= B\epsilon_{ij}x^j + a_j(r,v), \\ A_r &= 0. \end{aligned}$$

- Action at second order in fluctuations reads

$$S_{BI} = -\int dx^4 \frac{\tau(r)}{2\sqrt{\Delta_0}} \left[2V f_{ri} f_{vi} + 2BF_{rv}^{(0)} \epsilon_{ij} f_{ri} f_{vj} + UV f_{ri} f_{ri} \right],$$

where

$$\sqrt{\Delta_0} = \frac{(B^2 + V(r)^2)^2}{\rho(r)^2/\tau(r)^2 + B^2 + V(r)^2}, \tag{1}$$

$$f_{ri} = \partial_r a_i(r,v), \tag{2}$$

$$f_{vj} = \partial_v a_i(r,v). \tag{3}$$

- The equation for $a_\pm$ is

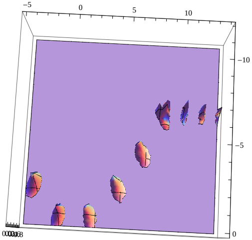
$$a''_\pm + \left(\frac{\mathcal{C}'}{\mathcal{C}} - 2i\omega\frac{\mathcal{A}}{\mathcal{C}}\right) a'_\pm - i\omega\left(\frac{\mathcal{A}'}{\mathcal{C}} \mp i\frac{\mathcal{B}'}{\mathcal{C}}\right) a_\pm = 0.$$

- Conductivity is calculated to be

$$\sigma_\pm(\omega) = \tau_\infty \left(1 + i\frac{r^2\partial_r a_\pm(\omega,r)}{\omega a_\pm(\omega,r)}\right)_{r\to\infty}. \tag{4}$$

- Study analytic structure

- At first poles resemble those in dyonic black hole background:



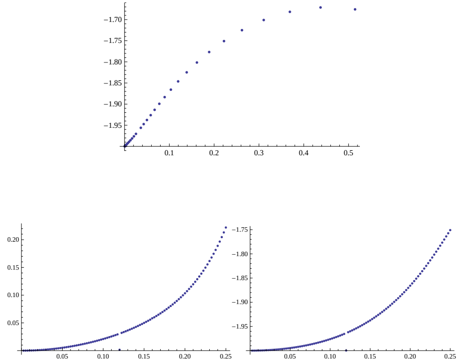
- Does the first pole have a hydrodynamic limit?
- No: flows to $\omega = -2i \equiv \omega_{Matsubara,n=-1}$ (restoring units).

(draw figure)

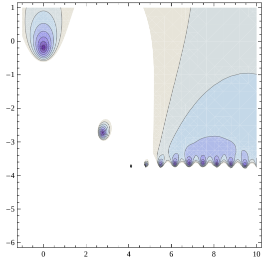
- Turn on momentum dependence. Now the equations do not decouple!

$$
\begin{aligned}
C\partial_r^2 a_x + \left[C' - 2i\omega A\right]\partial_r a_x - \left[i\omega A' + k_y^2 D(r)\right] a_x + \left[-i\omega B' + k_x k_y D(r)\right] a_y &= 0\,, \\
C\partial_r^2 a_y + \left[C' - 2i\omega A\right]\partial_r a_y - \left[i\omega A' + k_x^2 D(r)\right] a_y + \left[i\omega B' + k_x k_y D(r)\right] a_x &= 0\,.
\end{aligned}
$$

- The pole depends on k:



- Restore parameter values $\rho = 15.1, B = 1.4$ used in matching to Sachdev's conductivity:



- Dispersion relations for the first pole: