Deep Inelastic Scattering in Conformal QCD

Miguel S. Costa

Faculdade de Ciências da Universidade do Porto

0911.0043 [hep-th], 1001.1157 [hep-ph]

Work with L. Cornalba and J. Penedones

NORDITA - AdS/CFT Program, June 2010

Motivation

- QCD is approximately conformal at high energies, for very small probes with

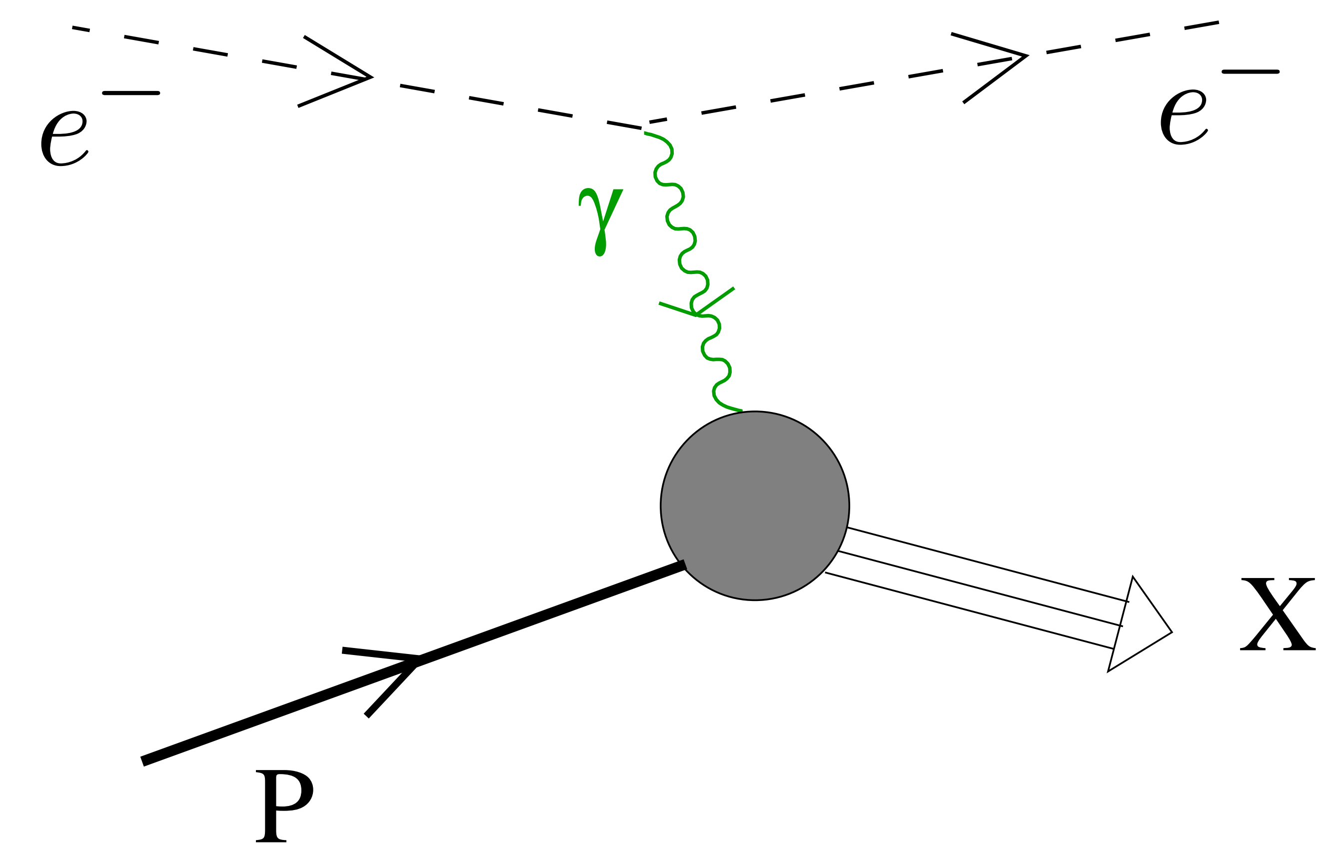
$$r_{probe}\Lambda_{QCD} \ll 1$$

- Regge limit of high center of mass energy with other kinematic invariants fixed ($s \gg -t, \Lambda^2_{QCD}$) corresponds to low Bjorken - x in DIS.

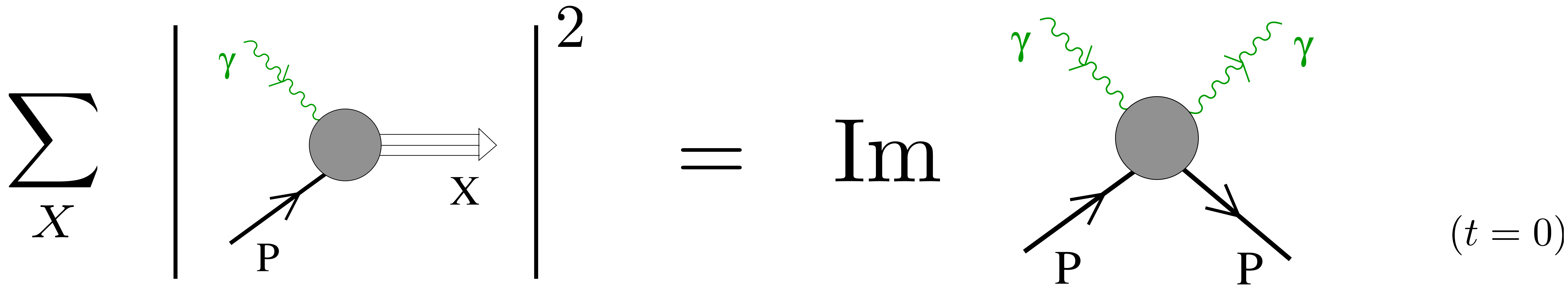
- Use conformal symmetry and AdS/CFT to make general predictions for conformal limit of DIS in QCD.

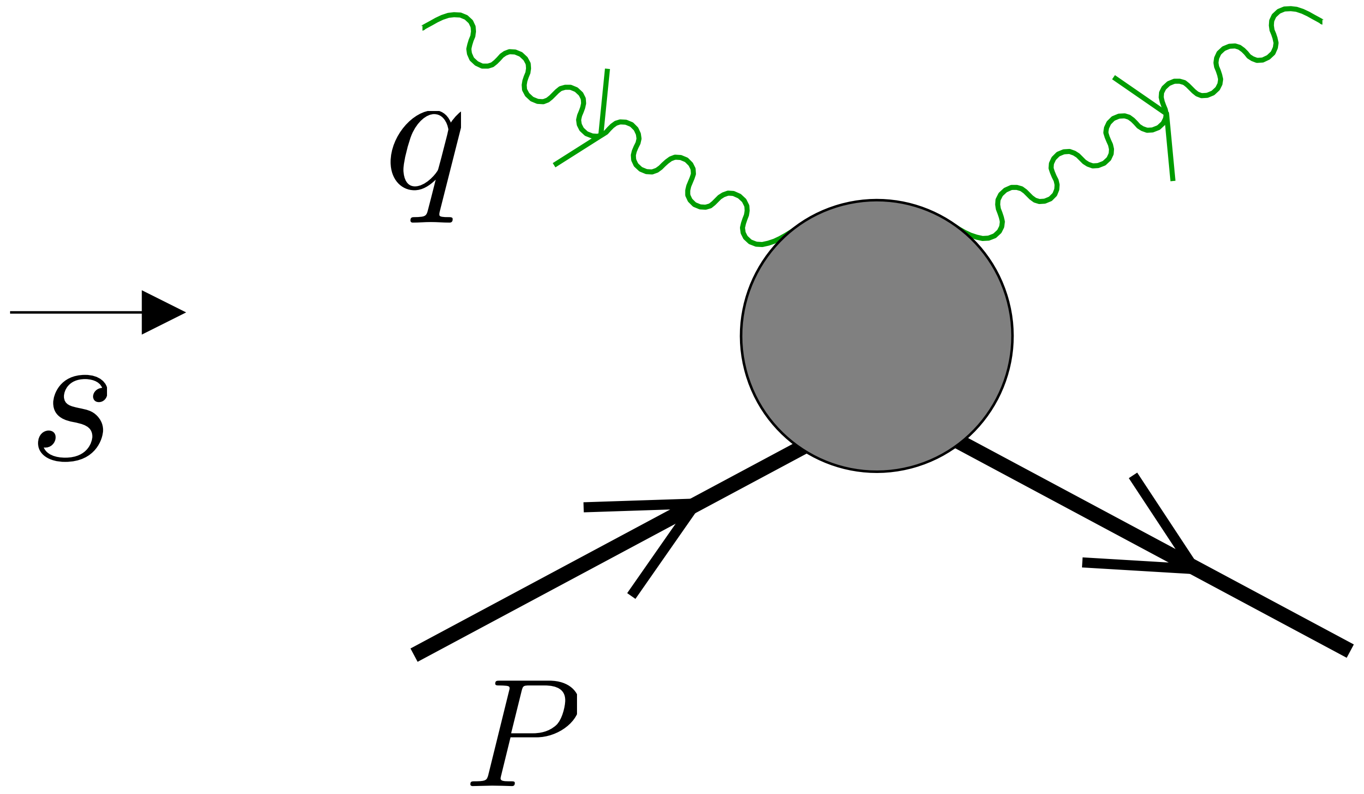
Deep Inelastic Scattering

- Electron interacts with proton via exchange of off-shell photon γ



- Optical theorem

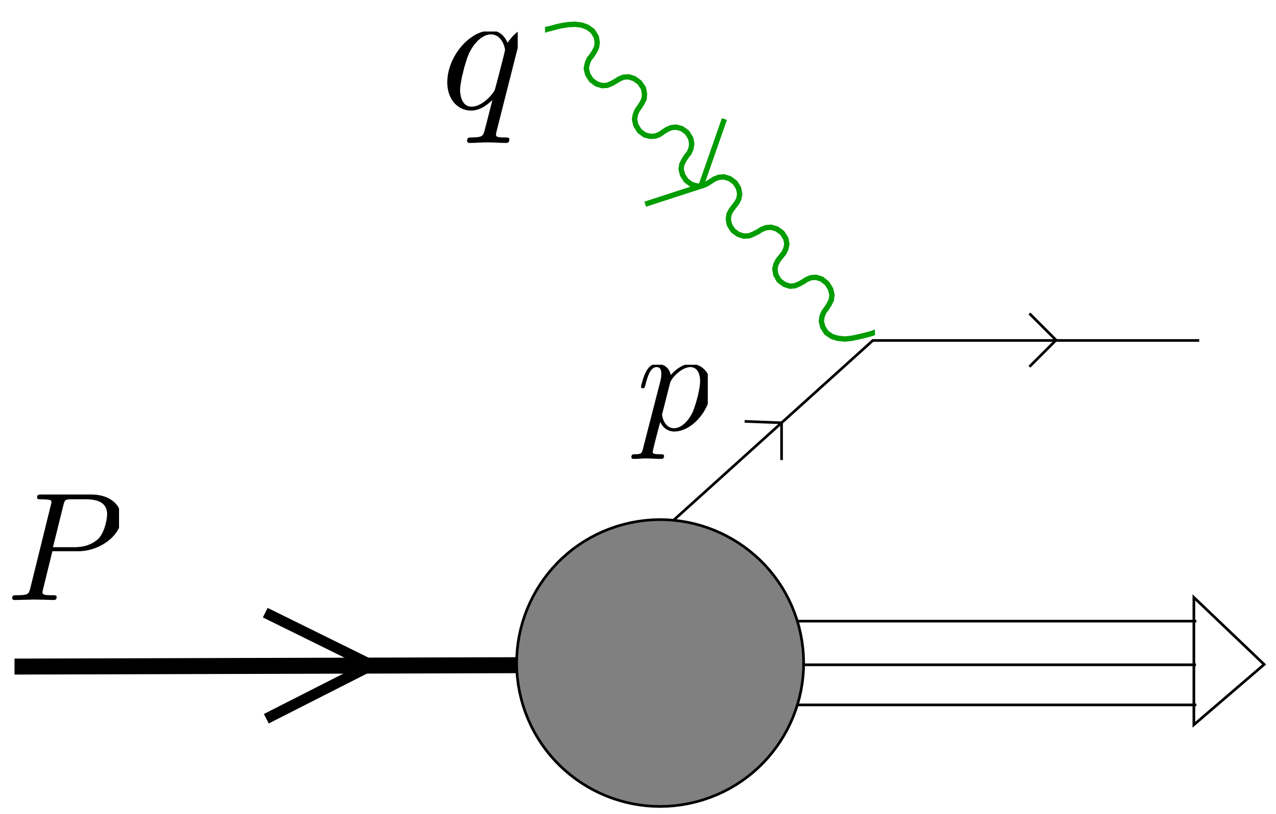




$$s = -\left(q + P\right)^2$$

$$Q^2 = q^2$$

- Bjorken x

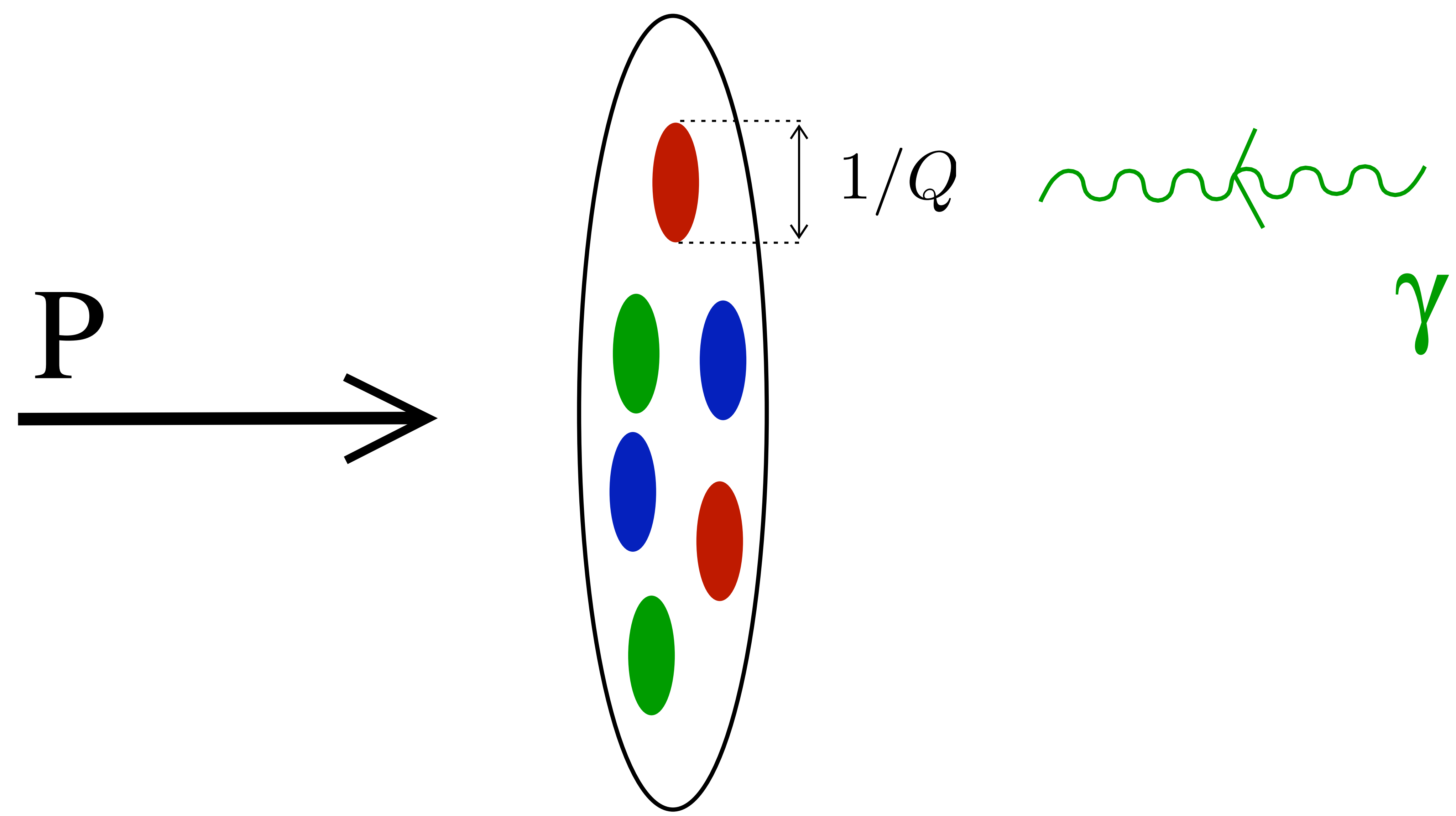


$p = xP$

$s \simeq \dfrac{Q^2}{x}$

large s $\Rightarrow$ small x

- Transverse resolution $1/Q$



- Hadronic tensor

structure functions

$$F_i\left(\mathrm{x}, Q^2\right) = \frac{1}{2\pi}\,\mathrm{Im}\,\Pi_i$$

$$T^{ab} = i\int d^4y\, e^{iq\cdot y}\langle P|\, T\left\{j^a(y_1)\, j^b(y_3)\right\}|P\rangle$$

$$T^{ab} = \left(\eta^{ab} - \frac{q^a q^b}{q^2}\right)\Pi_1(\mathrm{x}, Q^2) - \frac{2\mathrm{x}}{Q^2}\left(p^a + \frac{q^a}{2\mathrm{x}}\right)\left(p^b + \frac{q^b}{2\mathrm{x}}\right)\Pi_2(\mathrm{x}, Q^2)$$

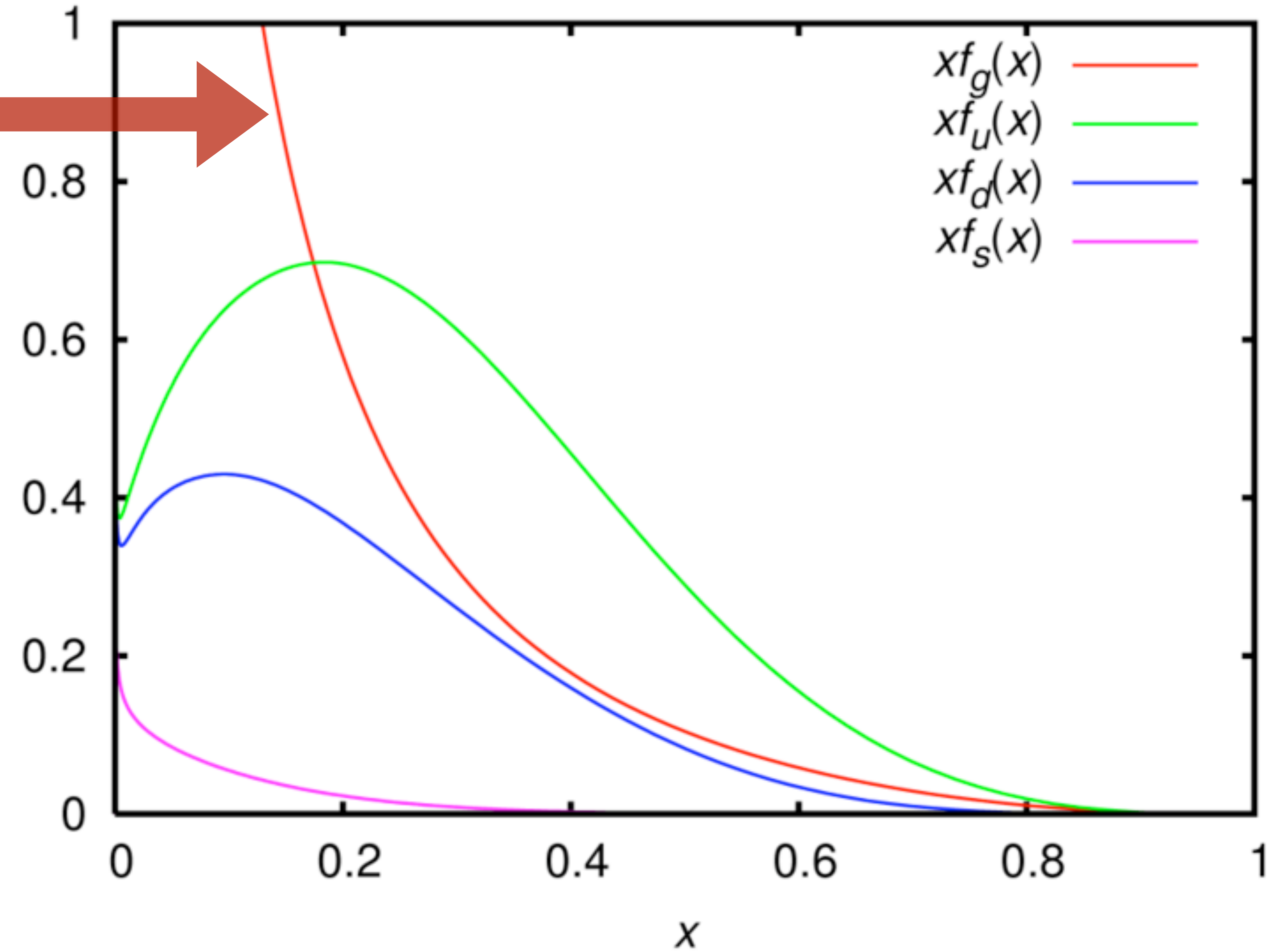
- Simulate proton with scalar operator $\mathcal{O}$ of dimension Δ and confinement with off-shellness $-p^2 = \bar{Q}^2 \sim \Lambda^2_{QCD}$

$$(2\pi)^d\,\delta\left(\sum k_j\right) T^{ab}(k_j) = \langle j^a(k_1)\,\mathcal{O}(k_2)\, j^b(k_3)\,\mathcal{O}(k_4)\rangle$$

j^a with dimension $\xi = 3$

- Parton distributions $f_i\left(x, Q^2\right)$

Gluons dominate at small x



$$F_L \simeq F_2 - 2xF_1 \sim Q^2 \sigma_L(x, Q^2)$$

$$\propto xg(x, Q^2)$$

Callan - Gross relation:
free quark model $F_2 = 2xF_1$

Regge Kinematics in CFTs

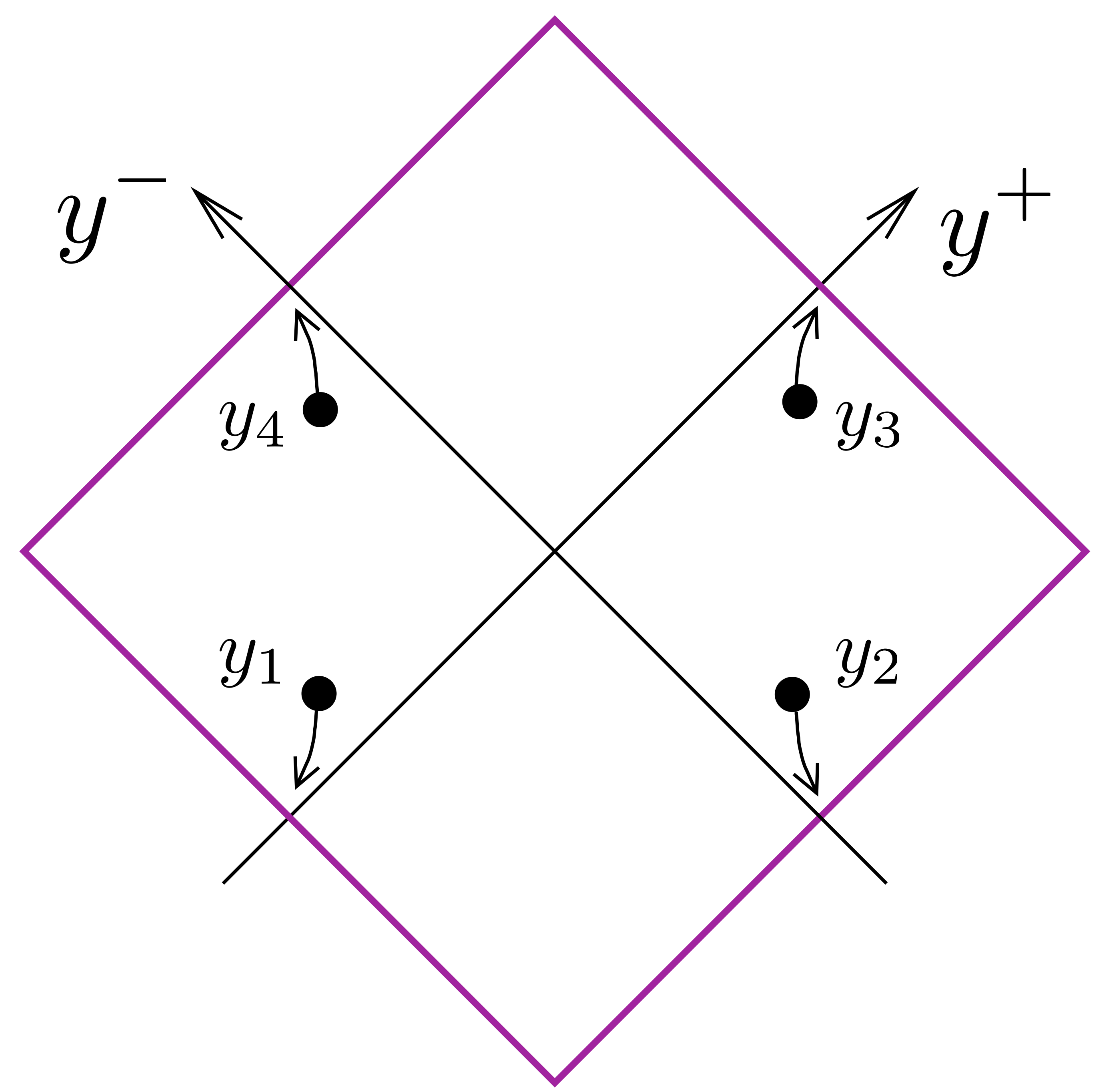
- Consider correlator with EMG current and scalar operators in position space

$$A^{ab}(y_i) = \langle j^a(y_1)\,\mathcal{O}(y_2)\,j^b(y_3)\,\mathcal{O}(y_4)\rangle$$

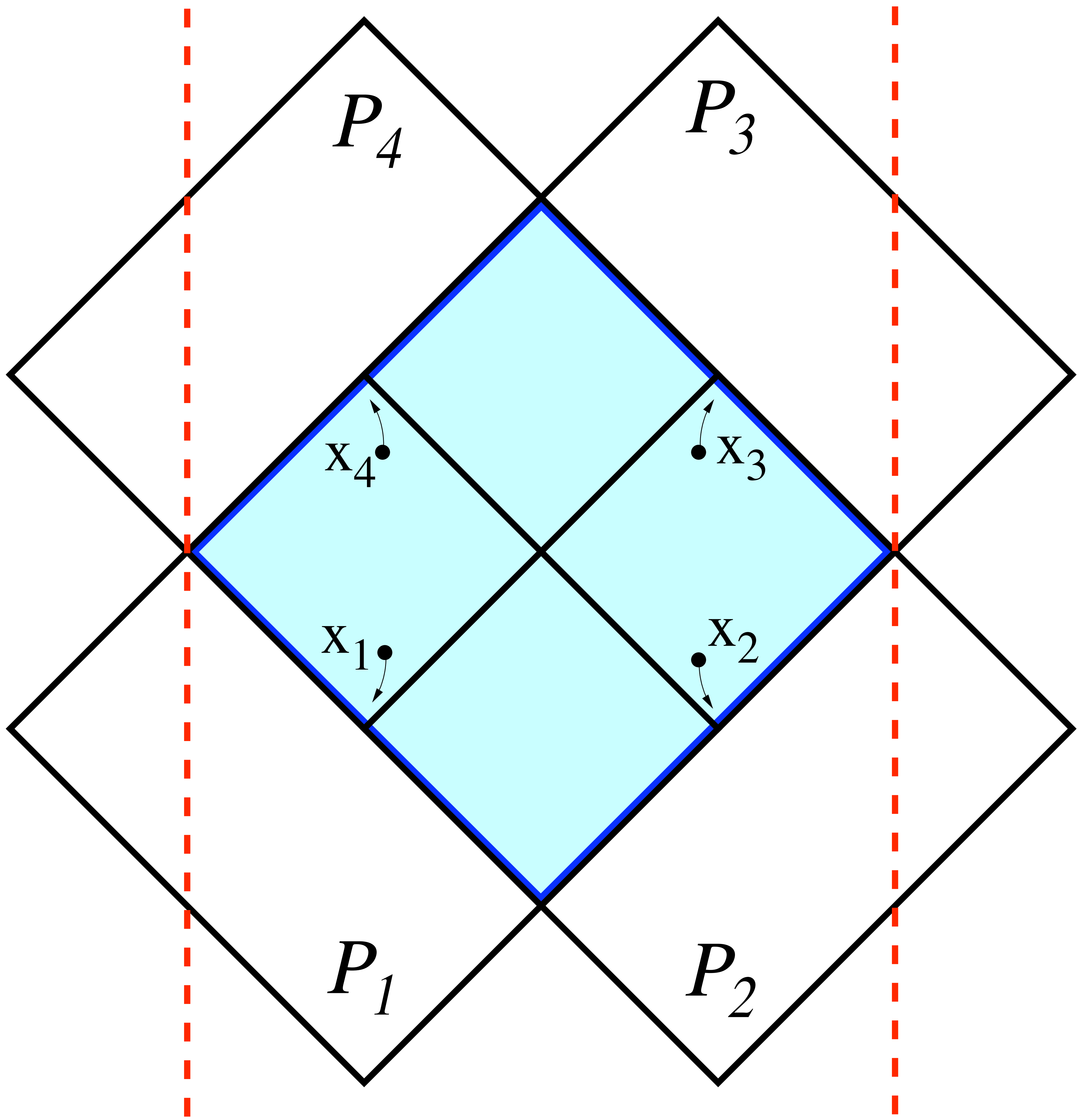
- Regge limit $\quad y = (y^+, y^-, y_\perp)$

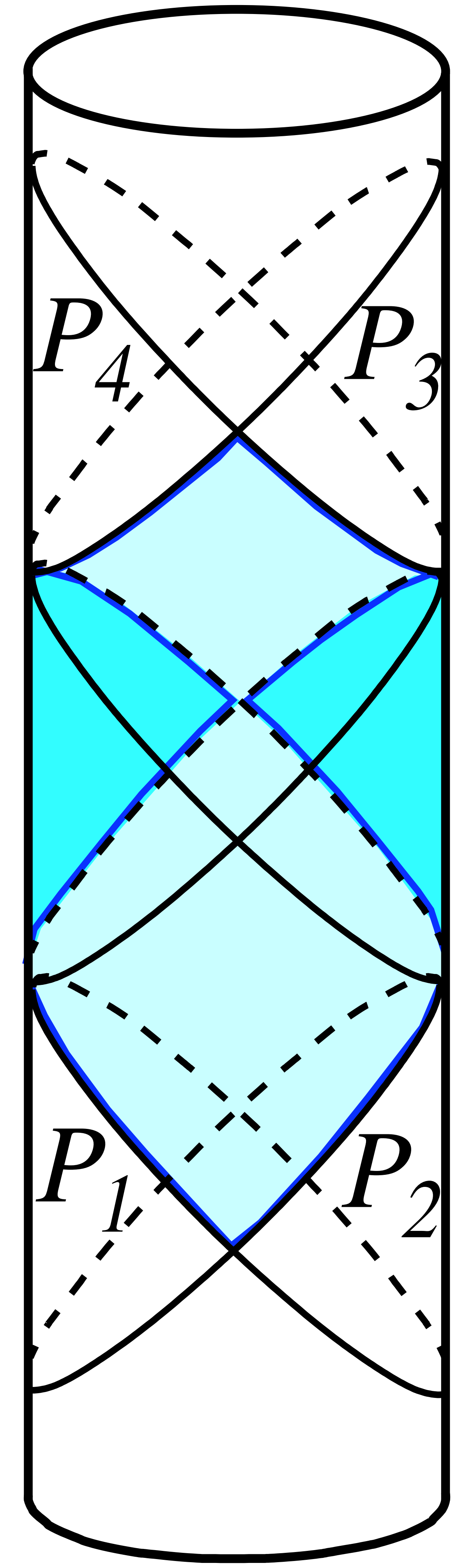
$$y_1^+ \to -\infty \qquad y_2^- \to -\infty$$
$$y_3^+ \to +\infty \qquad y_4^- \to +\infty$$

$y_i^2,\ y_{i\perp}^2$ fixed



- Use different Poincaré patches to cover each operator



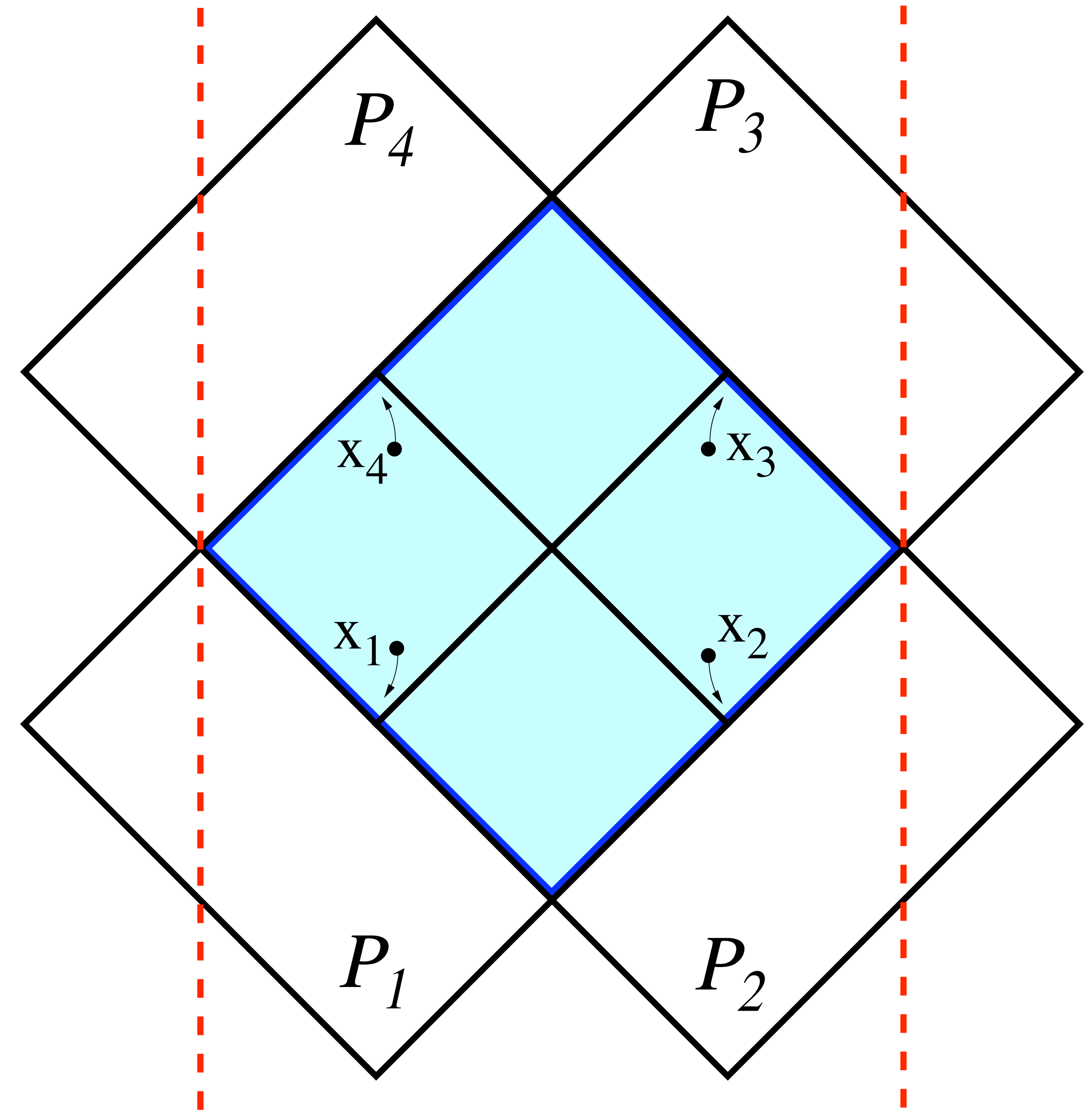


- Conformal transformation for each operator

$$x_i = \left(x_i^+, x_i^-, x_{i\perp}\right) = -\frac{1}{y_i^+}\left(1, y_i^2, y_{i\perp}\right) , \qquad i = 1,3$$

$$x_i = \left(x_i^+, x_i^-, x_{i\perp}\right) = -\frac{1}{y_i^-}\left(1, y_i^2, y_{i\perp}\right) , \qquad i = 2,4$$

$$-dy^+dy^- + dy_\perp^2 = \frac{1}{\left(x^+\right)^2}\left(-dx^+dx^- + dx_\perp^2\right)$$



- In CFT Regge limit useful to consider correlator

$$A^{mn}(x_i) = \langle j^m(x_1)\,\mathcal{O}(x_2)\,j^n(x_3)\,\mathcal{O}(x_4)\rangle$$

Regge limit $x_i \to 0$

$$j^a(y) = \left|\frac{\partial x}{\partial y}\right|^{\frac{\xi+1}{d}} \frac{\partial y^a}{\partial x^m}\, j^m(x)$$

- Cross ratios

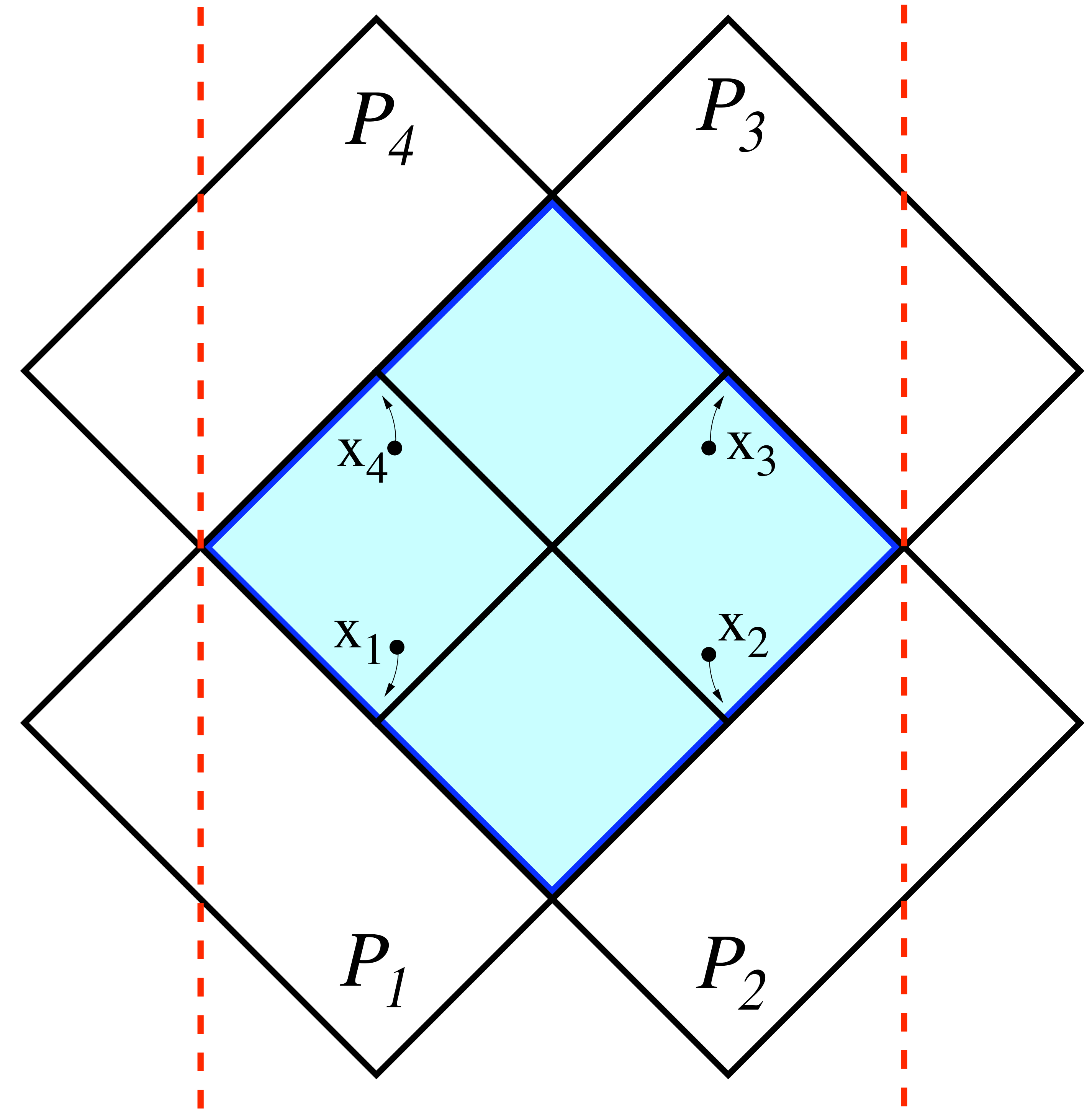
Translations in $P_1\,,P_3$ or Special Conformal in $P_2\,,P_4$

$$x_{1,3} \to x_{1,3} + a\,, \qquad x_{2,4} \to x_{2,4} + O(a^2)$$

Translations in $P_2\,,P_4$ or Special Conformal in $P_1\,,P_3$

$$x_{1,3} \to x_{1,3} + O(b^2)\,, \qquad x_{2,4} \to x_{2,4} + b$$

$$x \approx x_1 - x_3\,, \qquad \bar{x} \approx x_2 - x_4$$



Lorentz Transformations $x \to \Lambda x\,, \quad \bar{x} \to \Lambda \bar{x}$

Dilatations $x \to \lambda x\,, \quad \bar{x} \to \frac{1}{\lambda}\,\bar{x}$

Residual transverse conformal group $SO(1,1) \times SO(3,1)$

Invariants

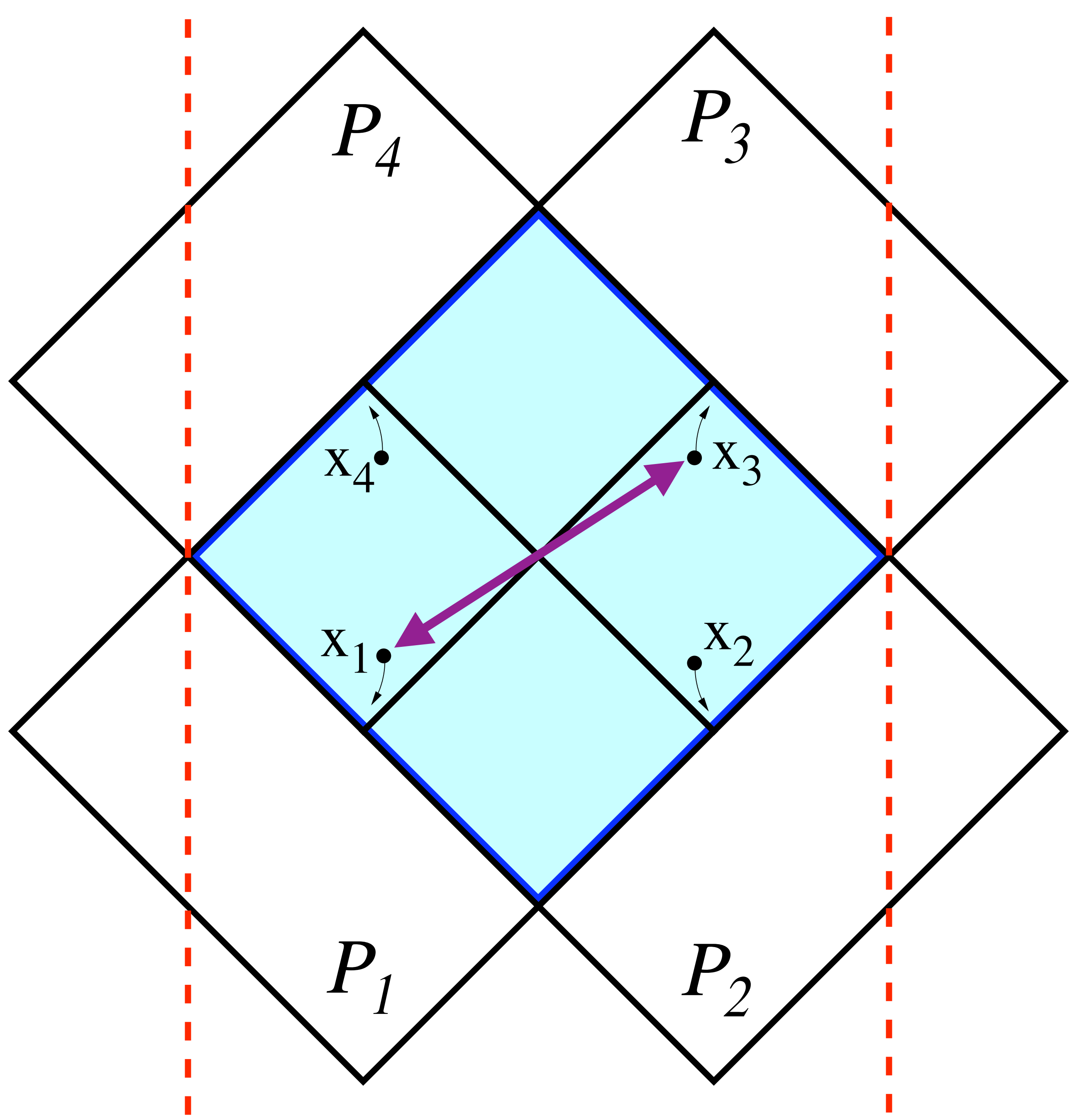
$$\sigma^2 = x^2 \bar{x}^2\,, \qquad \cosh\rho = -\frac{x\cdot\bar{x}}{|x||\bar{x}|}$$

Regge limit $\sigma \to 0$ fixed ρ

Regge Theory in CFTs

- Conformal symmetry restricts $A^{mn}(x_i)$ to have the form

$$A^{mn} = \frac{\mathcal{A}^{mn}(x,\bar{x})}{(x^2 - i\epsilon_x)^{\xi}(\bar{x}^2 - i\epsilon_{\bar{x}})^{\Delta}}\,, \qquad \mathcal{A}^{mn} = \sum_{k=1}^{4} f_k(\sigma,\rho)\, t_k^{mn}(x,\bar{x})$$

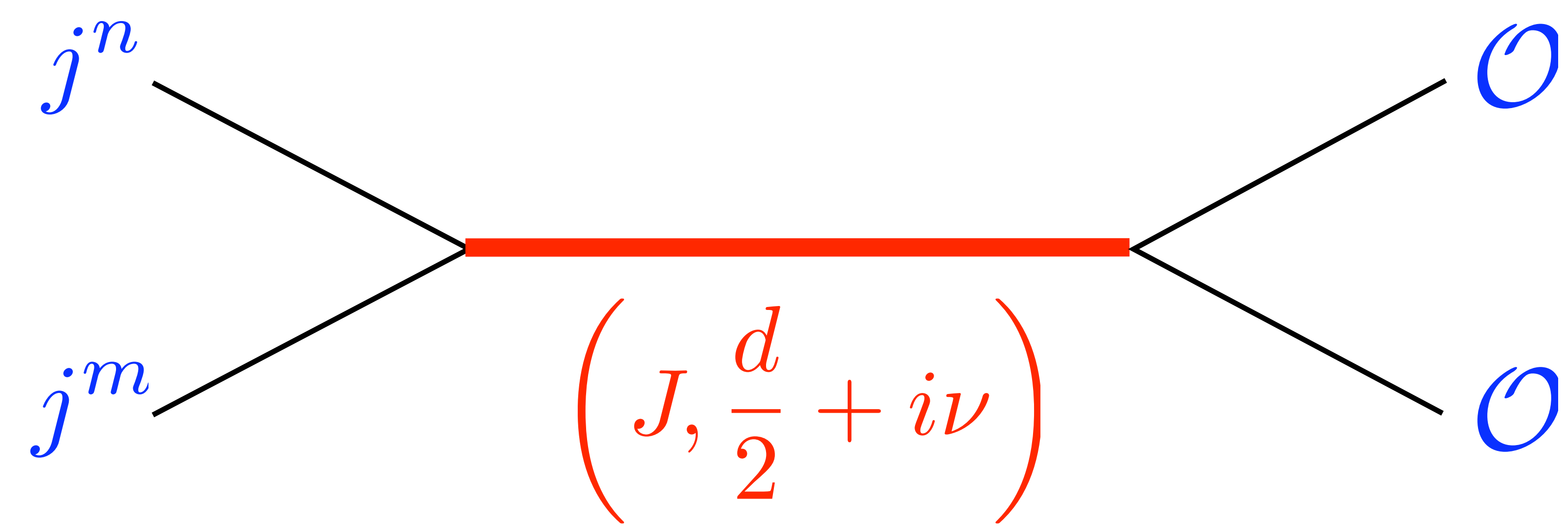


$$\langle j^m(x_1) j^n(x_3)\rangle \propto \frac{t_1^{mn}}{(x^2 - i\epsilon_x)^{\xi}}$$

$t_k^{mn}(x,\bar{x})$ invariant under scale invariance and $SO(3,1)$ tensors, e.g.

$$t_1^{mn} = \eta^{mn} - 2\,\frac{x^m x^n}{x^2}$$

- Single t-channel conformal partial wave gives in Regge limit



$$\mathcal{A}^{mn} \approx \sigma^{1-J} \left[E(\rho)\, \eta^{mn} + F(\rho)\, \frac{x^m x^n}{x^2} + G(\rho)\, \frac{\bar{x}^m \bar{x}^n}{\bar{x}^2} + H(\rho)\, \frac{x^m \bar{x}^n + \bar{x}^m x^n}{|x||\bar{x}|} \right]$$

- Sum over spins and dimensions gives for a Regge pole $j(\nu)$ [Cornalba 07]

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu\, \sigma^{1-j(\nu)} \alpha_k(\nu)\, \mathcal{D}_k^{mn}\, \Omega_{i\nu}(\rho)$$

$\Omega_{i\nu}(\rho)$ harmonic functions on H_3

$$\left(\Box_{H_3} + \nu^2 + 1\right)\, \Omega_{i\nu}(\rho) = 0$$

$$\Omega_{i\nu}(\rho) = \frac{\nu}{4\pi^2}\, \frac{\sin \nu\rho}{\sinh \rho}$$

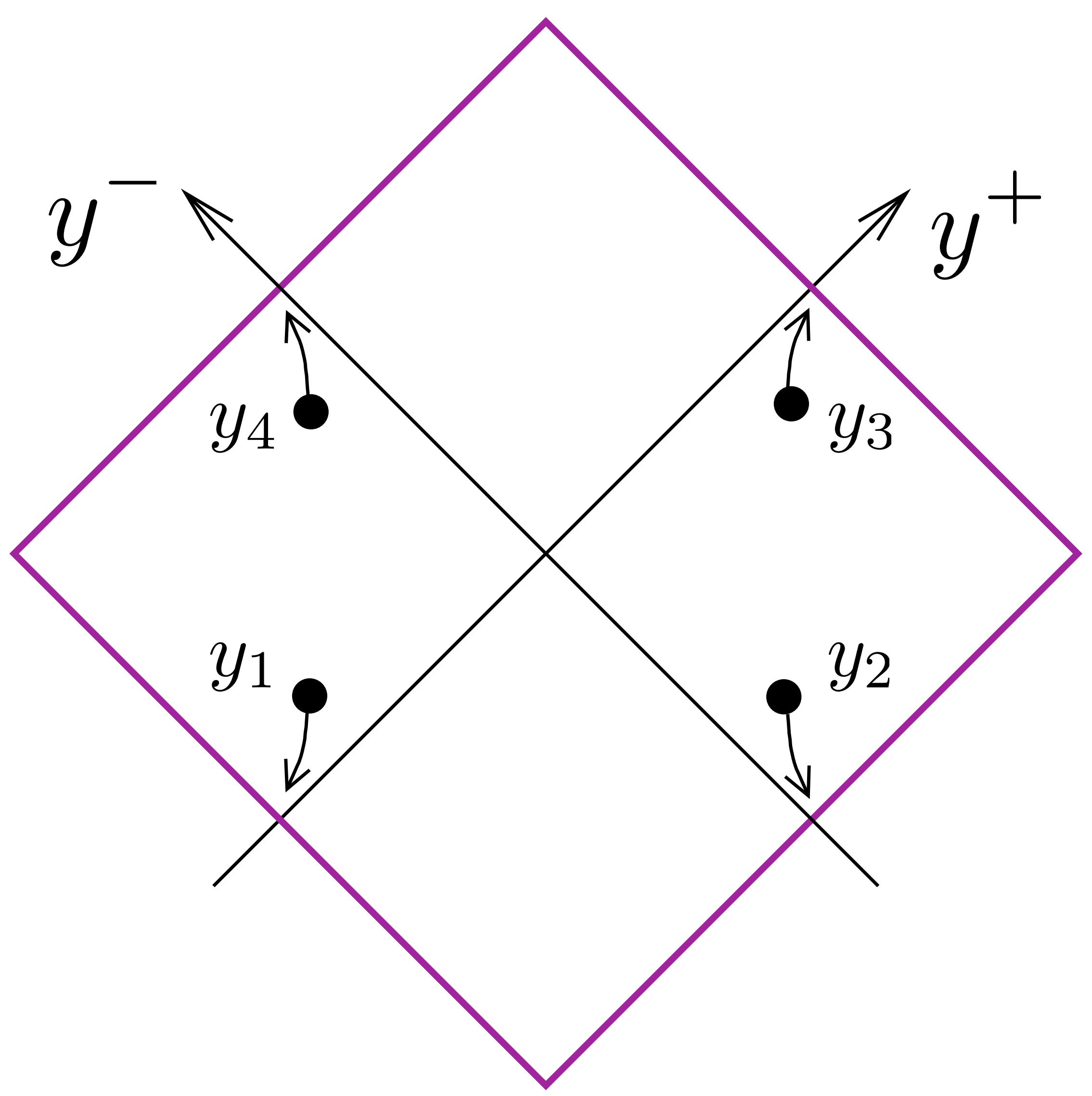
$$\mathcal{D}_1^{mn} = \eta^{mn} - \frac{x^m x^n}{x^2}$$

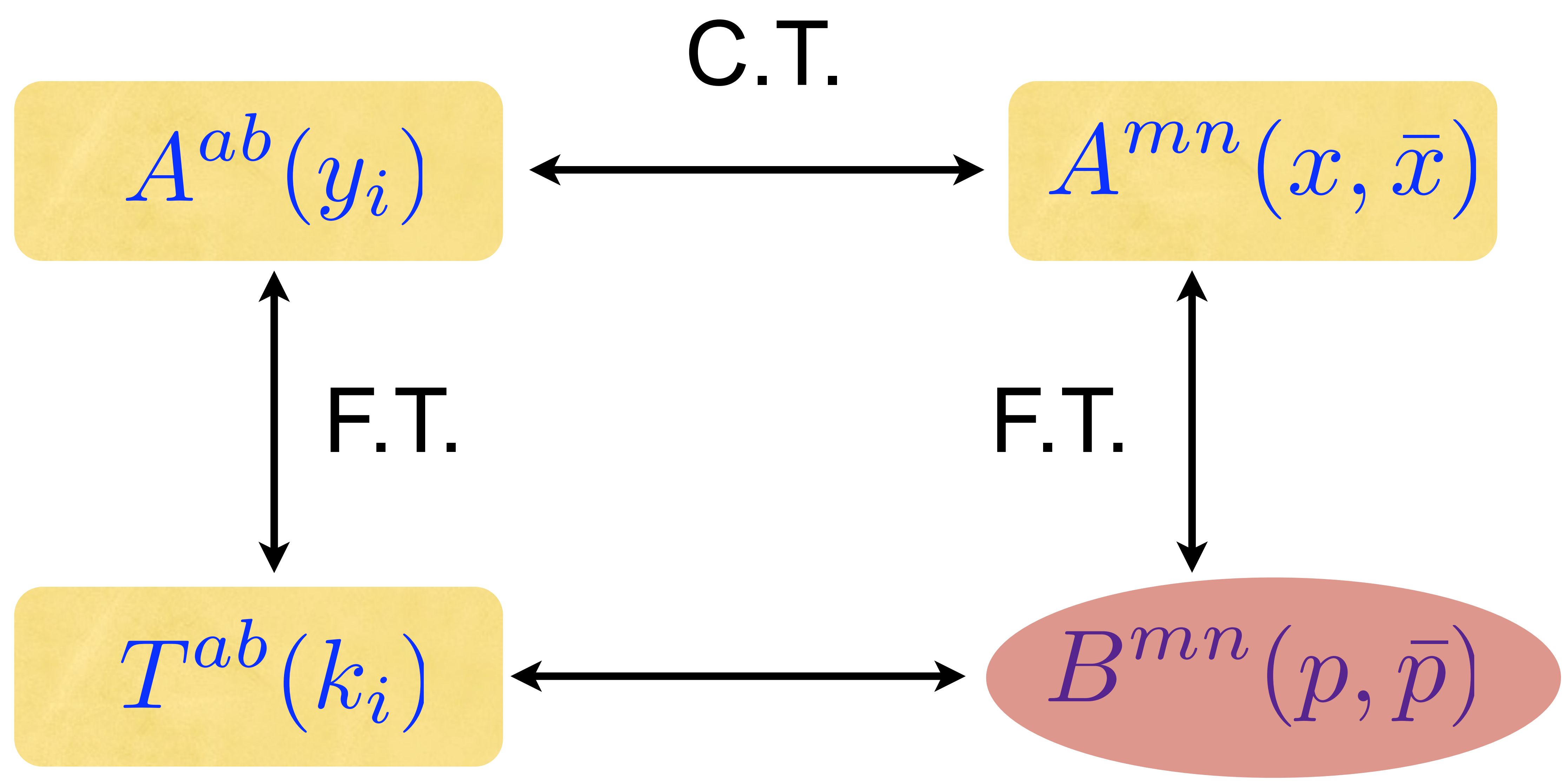
$$\mathcal{D}_2^{mn} = \frac{x^m x^n}{x^2}$$

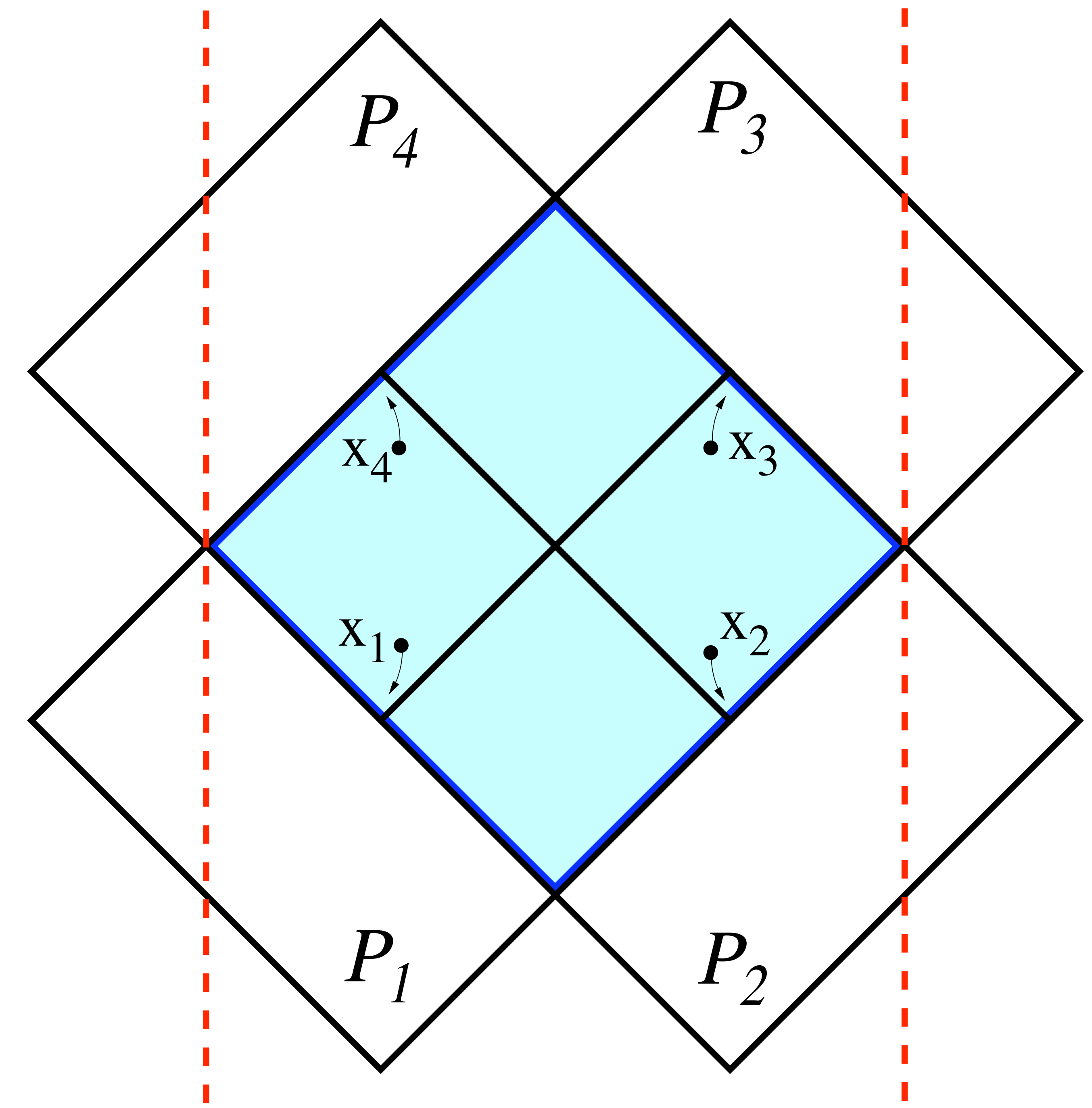
$$\mathcal{D}_3^{mn} = x^m \partial^n + x^n \partial^m$$

$$\mathcal{D}_4^{mn} = x^2 \partial^m \partial^n + (x^m \partial^n + x^n \partial^m) - \frac{1}{3}\left(\eta^{mn} - \frac{x^m x^n}{x^2}\right) x^2 \Box_x$$

Overview



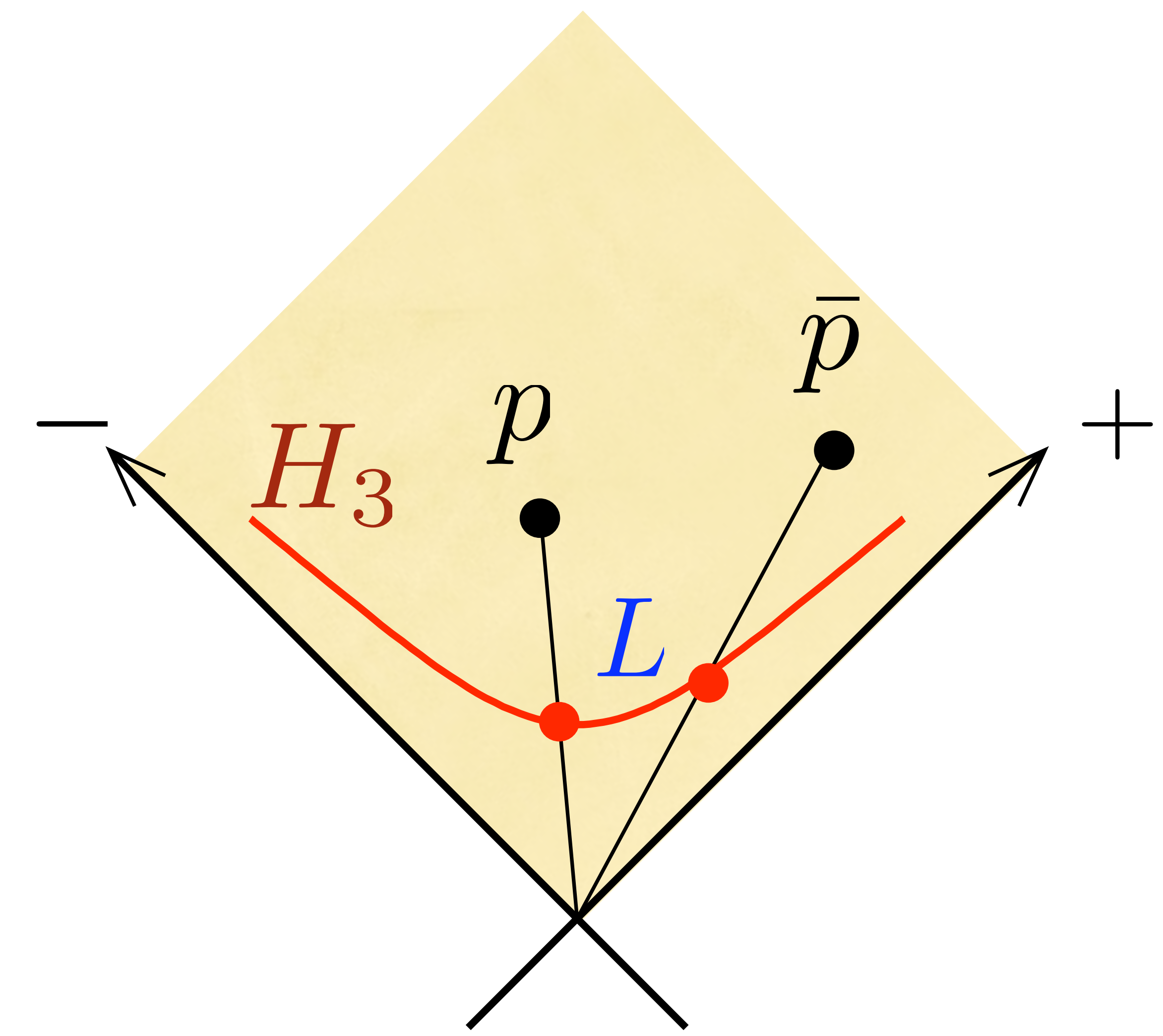




- Where is AdS?

AdS Phase-shift

$$A^{mn}(x,\bar{x}) = \int dp\, d\bar{p}\, e^{-2ip\cdot x - 2i\bar{p}\cdot\bar{x}} \frac{\mathcal{B}^{mn}(p,\bar{p})}{(-p^2)^{\frac{d}{2}-\xi}(-\bar{p}^2)^{\frac{d}{2}-\Delta}}$$



- Current conservation

$$p_m\, \mathcal{B}^{mn} = 0 \quad \Rightarrow \quad \mathcal{B}^i_{\ j} \text{ matrix on } H_3 \text{ polarization space}$$

$$S = 4|p||\bar{p}|$$

$$\cosh L = -\frac{p\cdot\bar{p}}{|p||\bar{p}|}$$

$$\mathcal{B}^i_{\ j} \approx 2\pi i \int d\nu\ S^{j(\nu)-1} \left[\beta_1(\nu)\,\delta^i_j + \beta_4(\nu)\left(\nabla^i\nabla_j - \frac{1}{3}\,\delta^i_j\right)\right]\Omega_{i\nu}(L)$$

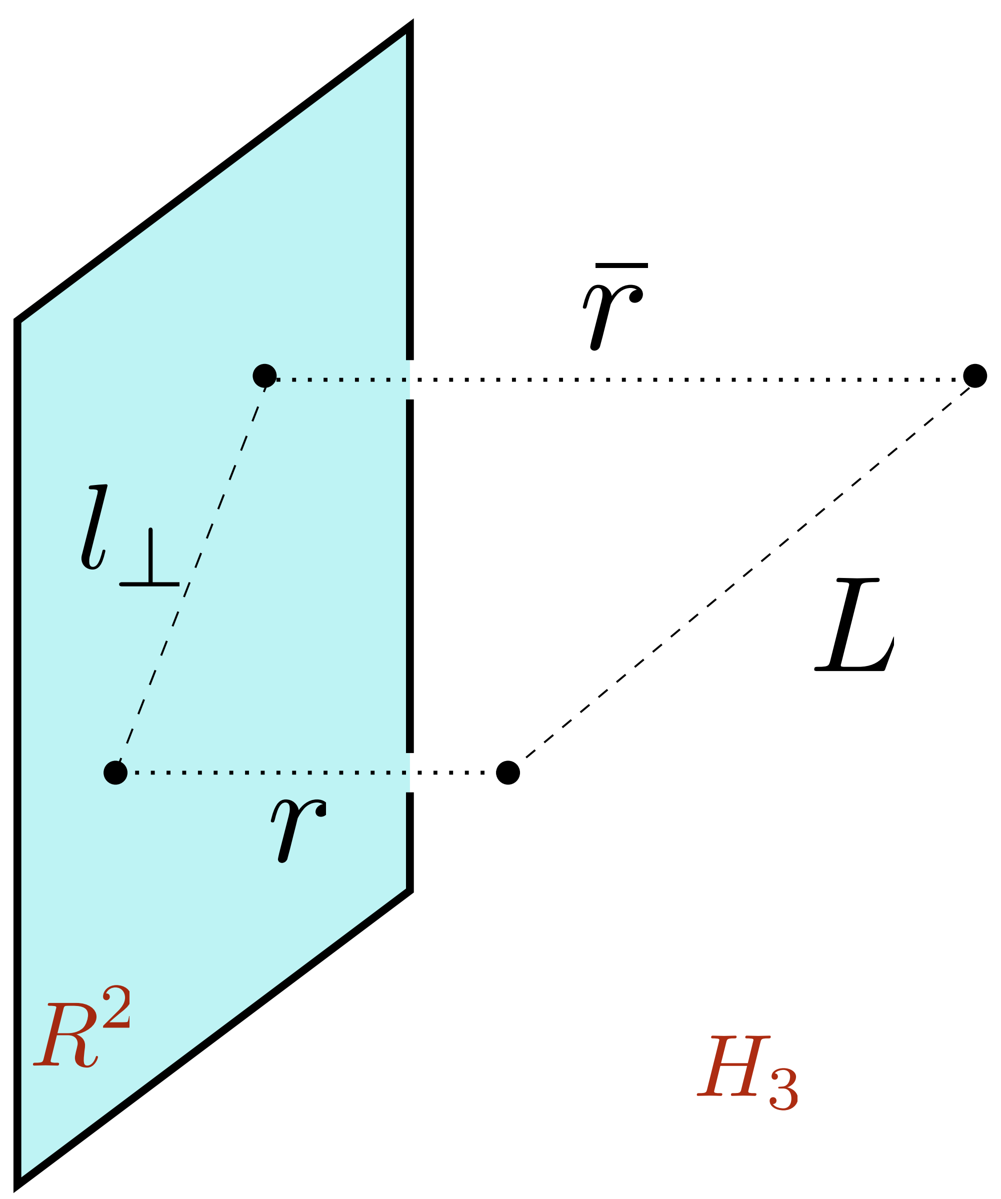
- Relation to amplitude in momentum space

$$\left[1 - e^{i\delta(s,l_\perp)}\right]^{ab}$$

AdS scalar wave function

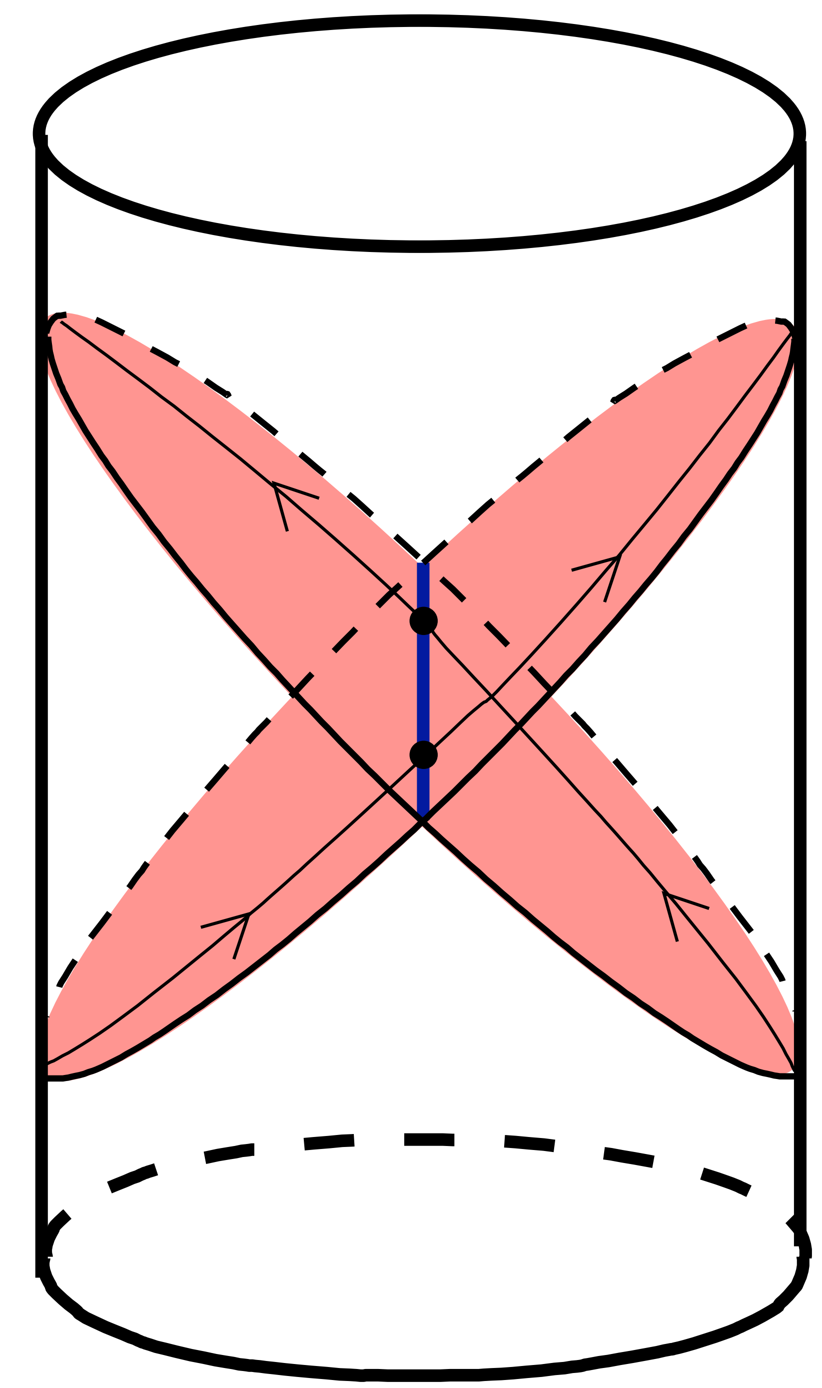
$$T^{ab}(k_j) \approx 2s\,i \int dl_\perp e^{iq_\perp \cdot l_\perp} \int \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3} F_1{}^a{}_i(r)\, F_3{}^{bj}(r)\, F_2(\bar{r})\, F_4(\bar{r})\, \mathcal{B}^i{}_j(S,L)$$

AdS gauge field wave function



$S = r\bar{r}s$,   AdS energy squared

$\cosh L = \dfrac{r^2 + \bar{r}^2 + l_\perp^2}{2r\bar{r}}$, impact parameter

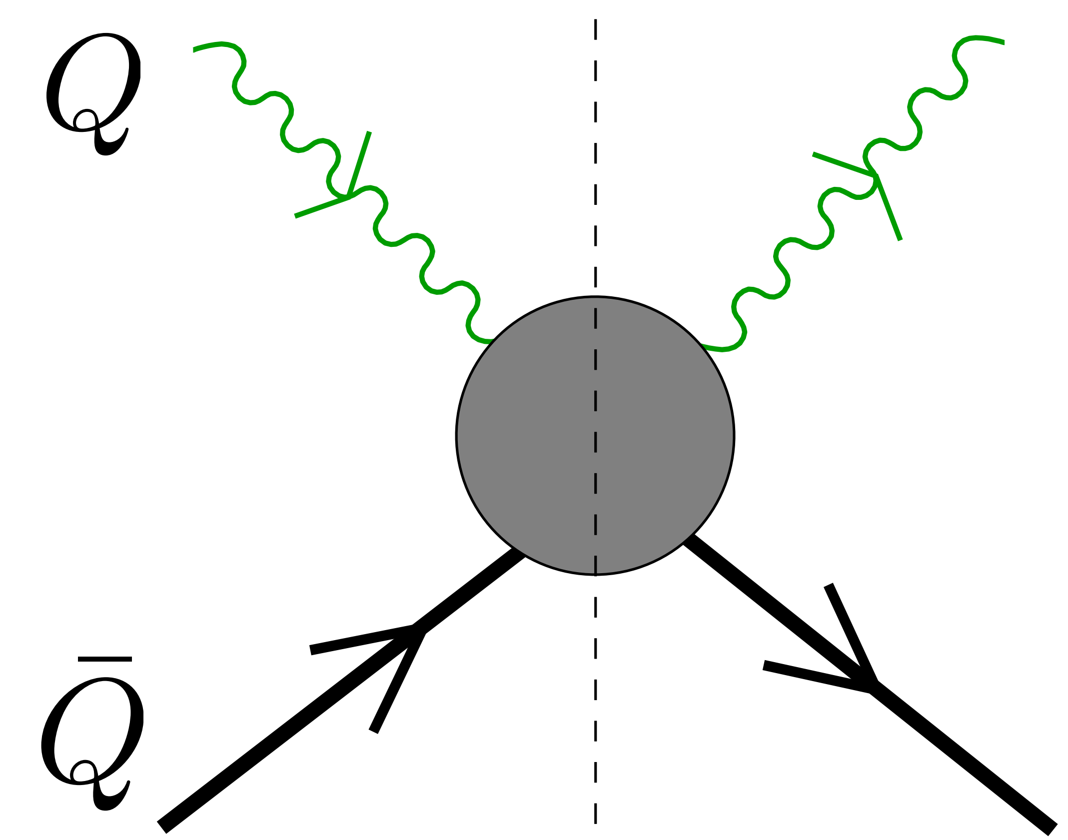


Structure functions

$$T^{ab} = \left(\eta^{ab} - \frac{q^a q^b}{q^2}\right)\Pi_1(\mathrm{x}, Q^2) - \frac{2\mathrm{x}}{Q^2}\left(p^a + \frac{q^a}{2\mathrm{x}}\right)\left(p^b + \frac{q^b}{2\mathrm{x}}\right)\Pi_2(\mathrm{x}, Q^2)$$

structure functions

$$F_i\left(\mathrm{x}, Q^2\right) = \frac{1}{2\pi}\,\mathrm{Im}\,\Pi_i$$



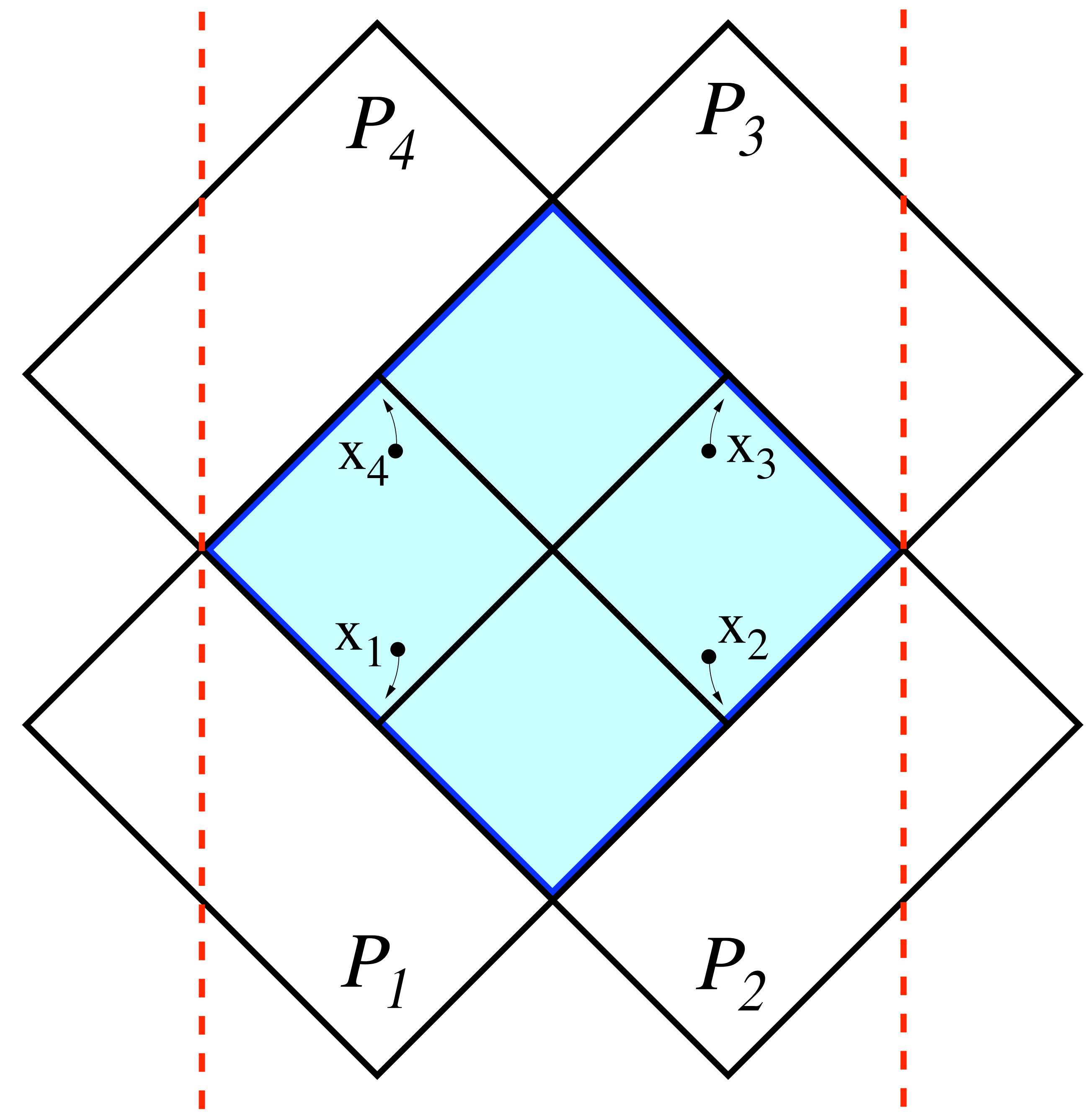
- Contribution from Regge pole

$$2\mathrm{x}\Pi_1 \approx \bar{Q}^{2\Delta-6}\int d\nu\,\gamma_1(\nu)\,\mathrm{x}^{1-j(\nu)}\left(\frac{Q}{\bar{Q}}\right)^{i\nu+j(\nu)}$$

$$\Pi_2 - 2\mathrm{x}\Pi_1 \approx \bar{Q}^{2\Delta-6}\int d\nu\,\gamma_2(\nu)\,\mathrm{x}^{1-j(\nu)}\left(\frac{Q}{\bar{Q}}\right)^{i\nu+j(\nu)}$$

Summary so far

Up to now, results are valid at any value of the ‘t Hooft coupling



CFT Regge pole

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu \, \sigma^{1-j(\nu)} \alpha_k(\nu) \, \mathcal{D}_k^{mn} \, \Omega_{i\nu}(\rho)$$

- CFT impact parameter representation (AdS phase shift)
- Regge representation of structure functions

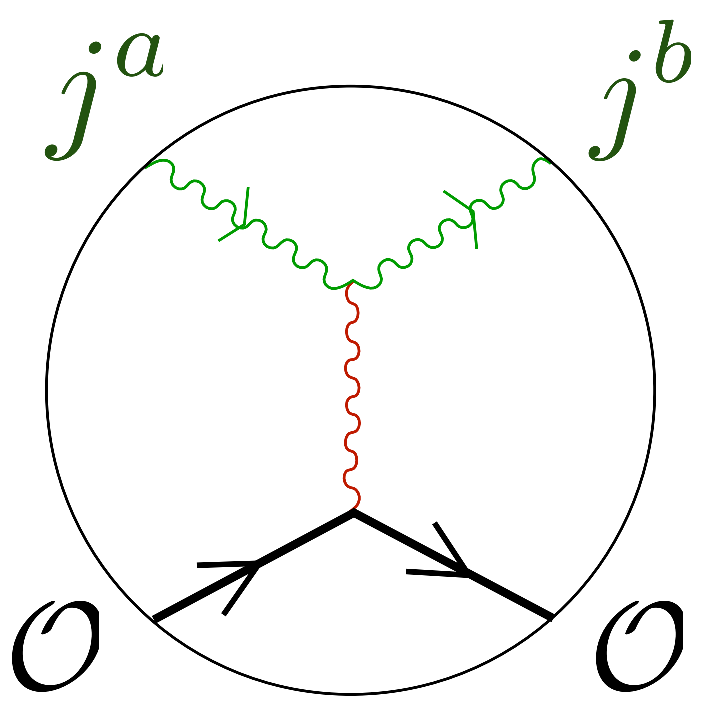
Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Trajectory depends on 't Hooft coupling $j = j(\nu, \bar{\alpha_s})$ $\alpha_k = \alpha_k(\nu, \bar{\alpha_s})$

- Strong coupling
(N=4 graviton trajectory)

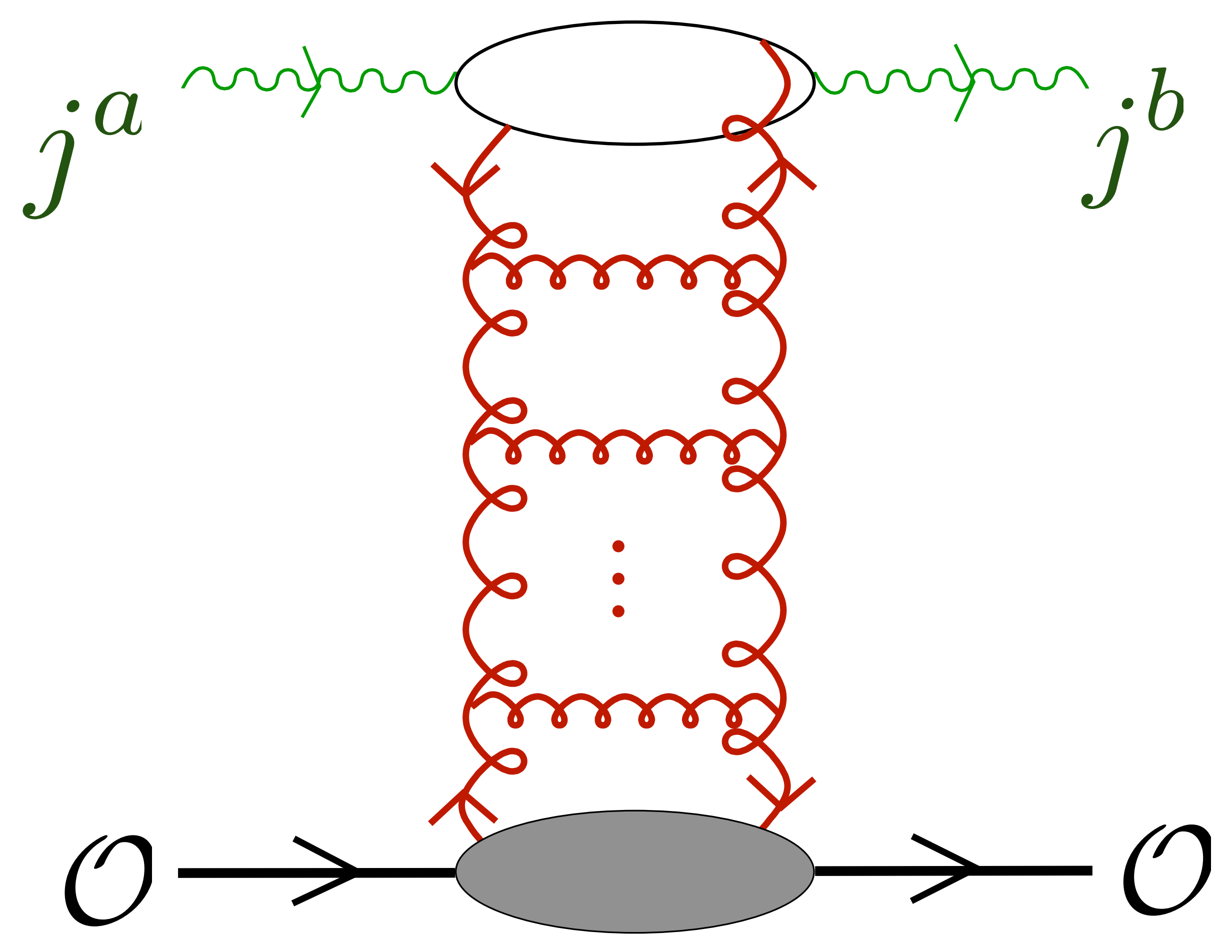
$$j \approx 2 - \frac{4+\nu^2}{4\pi\sqrt{\bar{\alpha}_s}}$$

α_k real



- Weak coupling
(BFKL Pomeron trajectory)

$$j \approx 1 + \bar{\alpha}_s\left(2\Psi(1) - \Psi\left(\frac{1+i\nu}{2}\right) - \Psi\left(\frac{1-i\nu}{2}\right)\right)$$

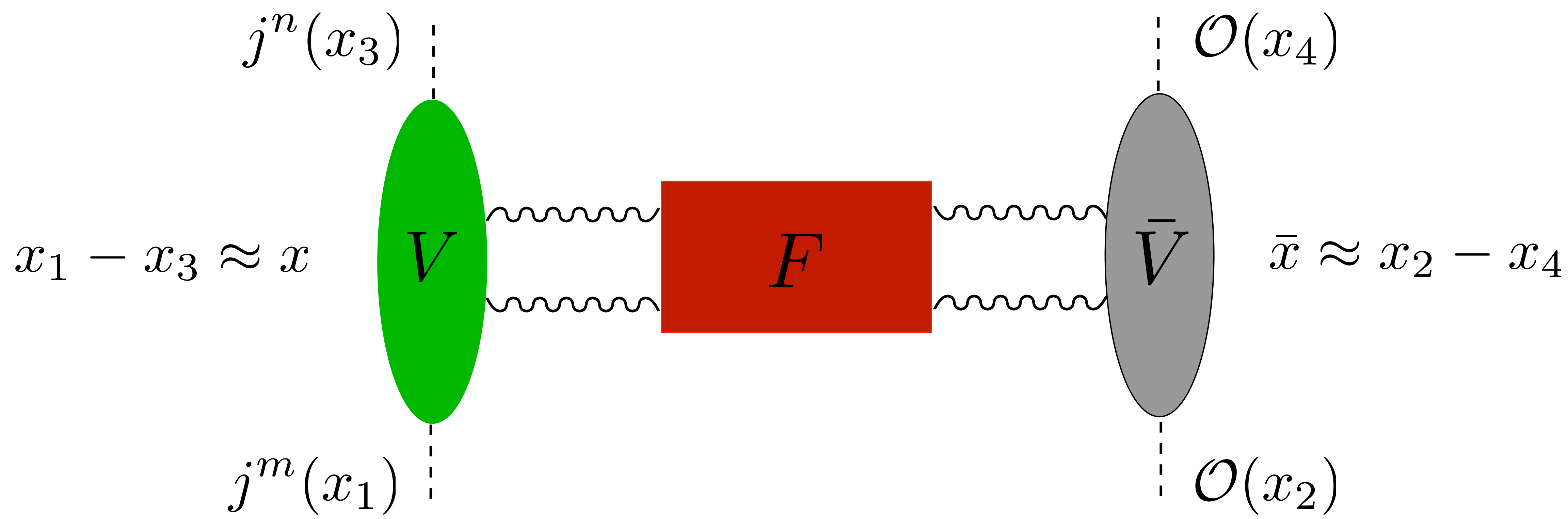


α_k imaginary

From now on weak coupling

Hard Pomeron & Transverse conformal symmetry [Balitsky, Fadin, Kuraev, Lipatov]

- Reduced amplitude can also be written in BFKL form



Invariants

$$\sigma^2 = x^2 \bar{x}^2$$

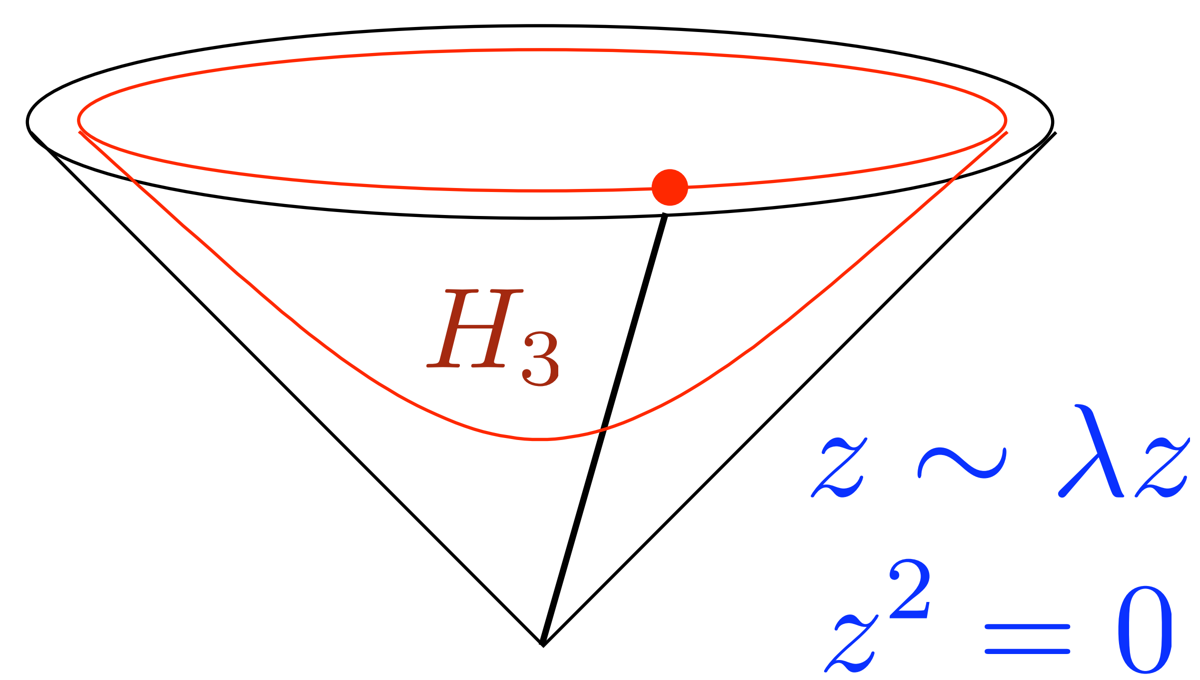
$$\cosh \rho = -\frac{x \cdot \bar{x}}{|x||\bar{x}|}$$

$$\mathcal{A}^{mn}(x,\bar{x}) \approx -\frac{1}{N^2} \int_{\partial H_3} \frac{dz_1 dz_3}{(z_{13})^2} \frac{dz_2 dz_4}{(z_{24})^2} \, V^{mn}(x,z_1,z_3)\, F(z_1,z_3,z_2,z_4)\, \bar{V}(\bar{x},z_2,z_4) \qquad (j=1)$$

- Nice story about impact factors and conformal symmetry

$$V^{mn} = \frac{2}{\pi}\,\bar{\alpha}_s\, u^3 \left(\eta^{mn} + 2\,\frac{z_1^m z_3^n + z_1^n z_3^m}{z_{13}} \right)$$

u is cross-ratio

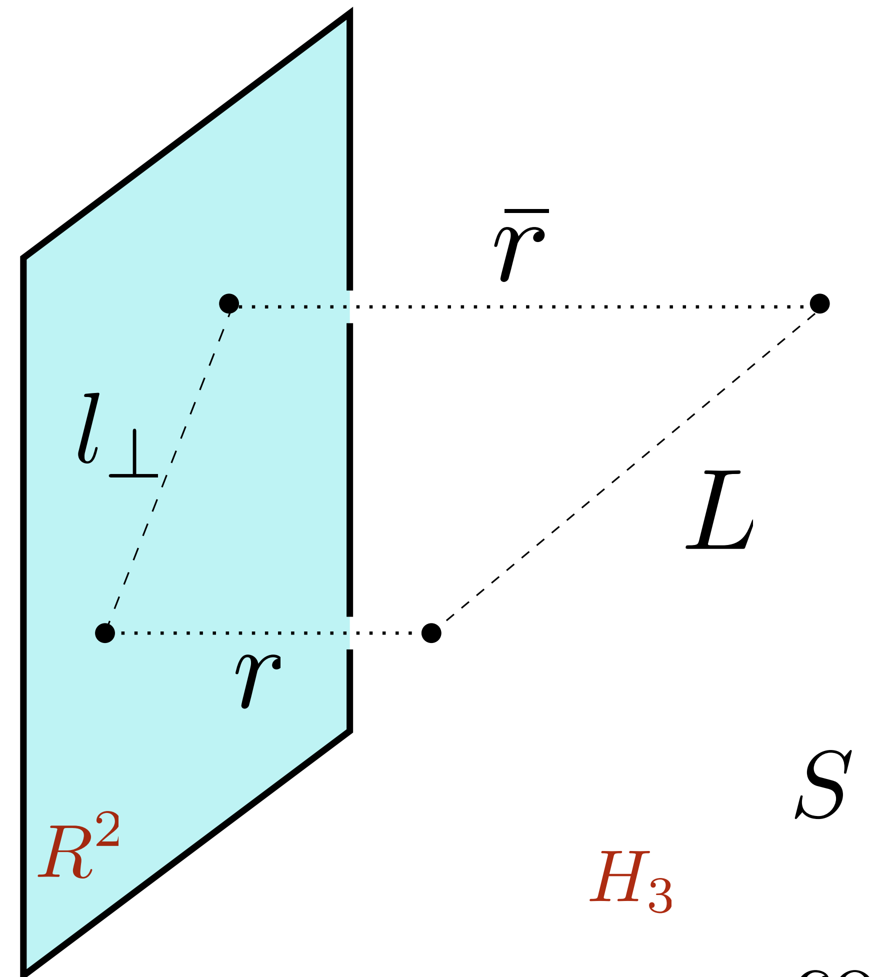


AdS black disk model for small-x DIS

$$T^{ab}(k_j) = 2is \int d^2 l_\perp \, e^{iq_\perp \cdot l_\perp} \left[1 - e^{i\delta(s,l_\perp)}\right]^{ab}$$

Showed in conformal limit

$$\left[1 - e^{i\delta(s,l_\perp)}\right]^{ab} = \int \frac{dr}{r^3}\frac{d\bar{r}}{\bar{r}^3} \Psi^{ab}{}_i{}^j(r) \, \Phi(\bar{r}) \left[1 - e^{i\chi(S,L)}\right]^i{}_j$$



$S = r\bar{r}s$

$$\cosh L = \frac{r^2 + \bar{r}^2 + l_\perp^2}{2r\bar{r}}$$

$\Psi^{ab}{}_i{}^j(r) = \psi^{ai}_{in}(r)\,\psi^{bj}_{out}(r)$ non-normalizable, localized around $0 < r \lesssim 1/Q$

$\Phi(\bar{r}) = |\phi(\bar{r})|^2$ normalizable, localized around $\bar{r} \sim 1/M$

$$S \sim \frac{Q}{x}$$

$$L \sim \ln \frac{Q}{\bar{Q}}$$

At zero momentum transfer write as integral in L

$$\int d^2 l_\perp \to 2\pi r\bar{r} \int_{|\ln \frac{\bar{r}}{r}|}^{\infty} dL \sinh L$$

conformal symmetry → $\chi = \chi(S, L)$

- At weak coupling

$$T^{ab} = 4\pi s \int \frac{dr}{r^3}\,\frac{d\bar{r}}{\bar{r}^3}\,\Psi^{ab}{}_i{}^j(r)\;\Phi(\bar{r})\int_{|\ln\frac{\bar{r}}{r}|}^{\infty} dL\,\sinh L\;\chi^i{}_j(S,L)$$

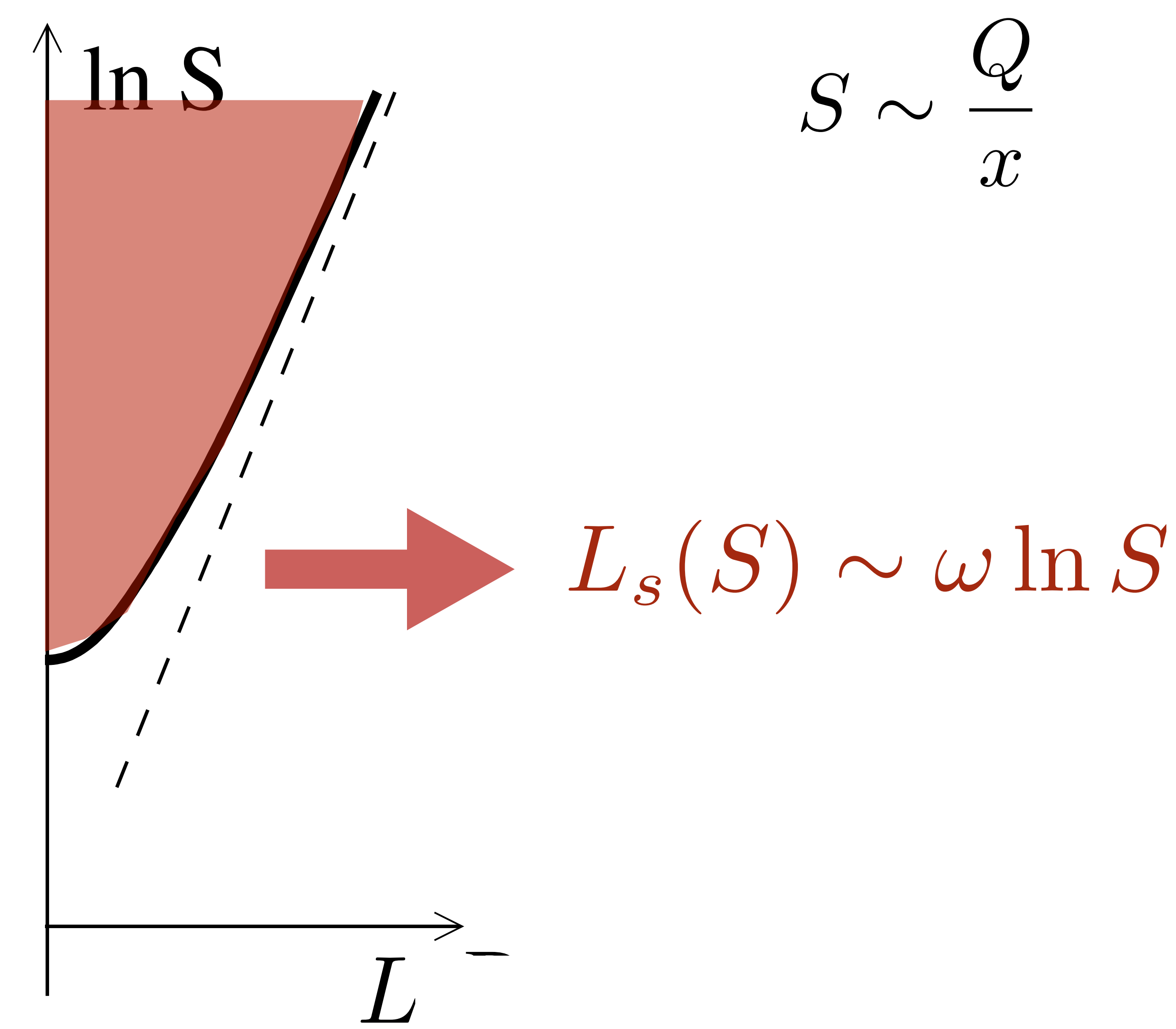
- Can estimate growth with linear pomeron exchange. Saddle point gives

$$\chi^i{}_j \sim \frac{1}{N^2}\int d\nu\,\beta(\nu)\,e^{(j(\nu)-1)\ln S-(1+i\nu)L} \quad\Rightarrow\quad j'(\nu)\ln S = iL$$

- Non-linear effects become important for $\mathrm{Im}\,\chi^i{}_j(S,L)\sim 1$

This happens when phase in saddle vanishes

$$L_s(S)\sim\omega\ln S$$



$S \sim \frac{Q}{x}$

Linear pomeron gives

$\omega = -ij'(\nu_s) = 2.44\bar{\alpha}_s$

From DIS geometric scaling $\omega \simeq 0.14$

AdS black disk

$$\mathrm{Im}\,\chi^i{}_j(S,L) \gg 1$$ → Black disk in AdS (or in conformal QCD)

$$\left[1 - e^{i\chi(S,L)}\right]^i{}_j = \Theta\big(L_s(S) - L\big)\delta^i{}_j$$

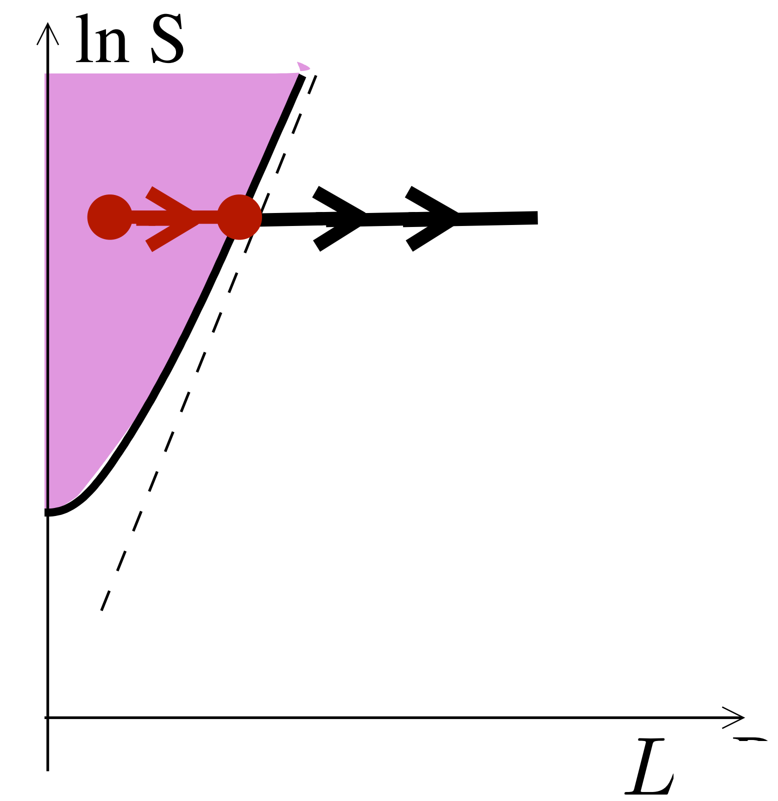
$$T^{ab} \approx 4\pi i s \int \frac{dr}{r^2}\,\frac{d\bar{r}}{\bar{r}^2}\,\Psi^{ab}{}_i{}^j(r)\;\Phi(\bar{r})\;\int_{|\ln\frac{\bar{r}}{r}|}^{L_s} dL \sinh L \cdot \delta^i{}_j$$

Black disk

$$\approx 2\pi i s \int \frac{dr}{r^2}\,\frac{d\bar{r}}{\bar{r}^2}\,\Psi^{ab}{}_i{}^j(r)\;\Phi(\bar{r})\;\left[(sr\bar{r})^{\omega} + (sr\bar{r})^{-\omega} - \frac{r}{\bar{r}} - \frac{\bar{r}}{r}\right]$$

dominant for low x

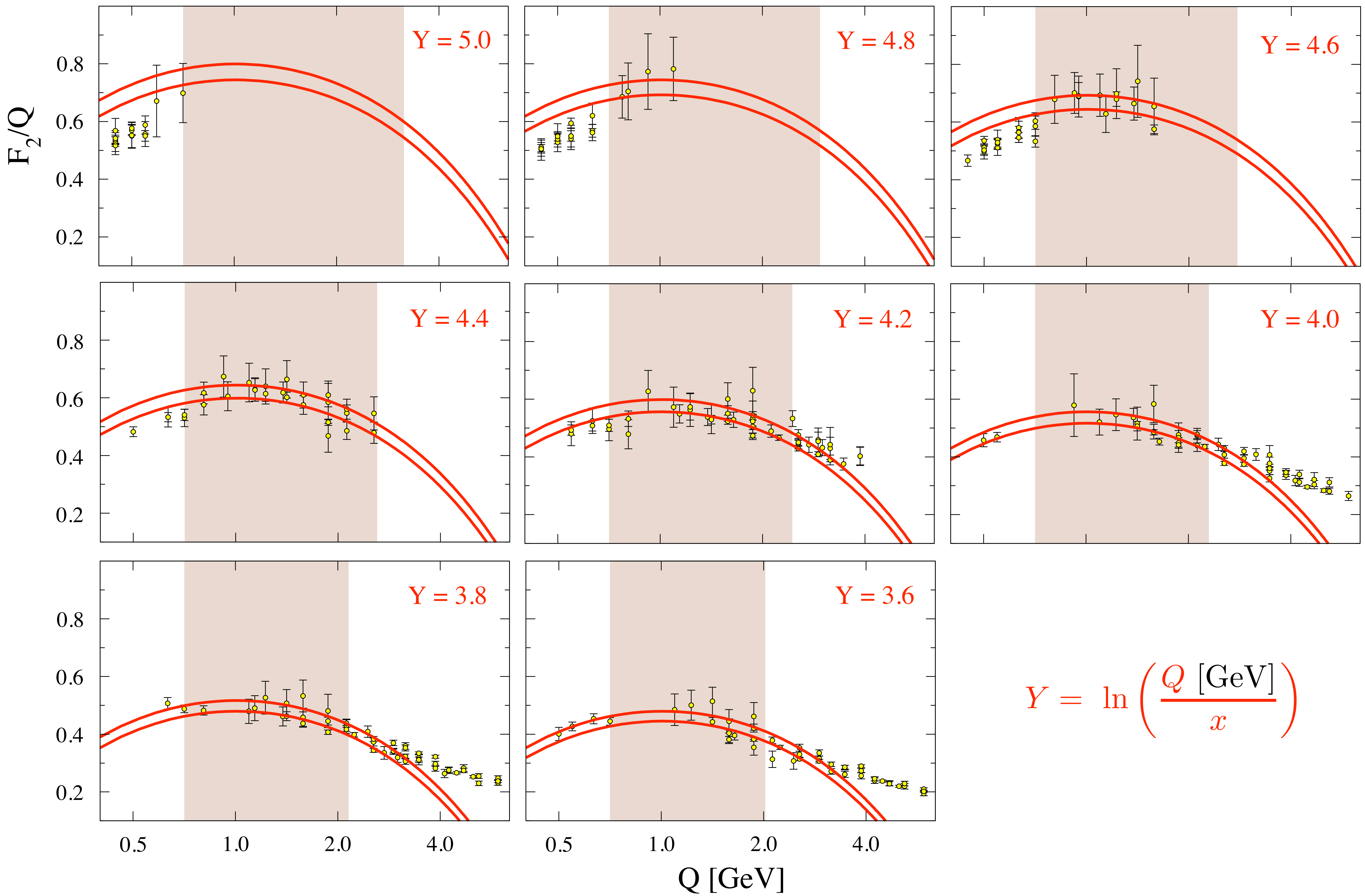
$S \sim \frac{Q}{x}$



- It is all AdS (or CFT) kinematics. Only dynamical information is the on-set of black disk region.
- Our ignorance is in $\bar{r}$ integrals

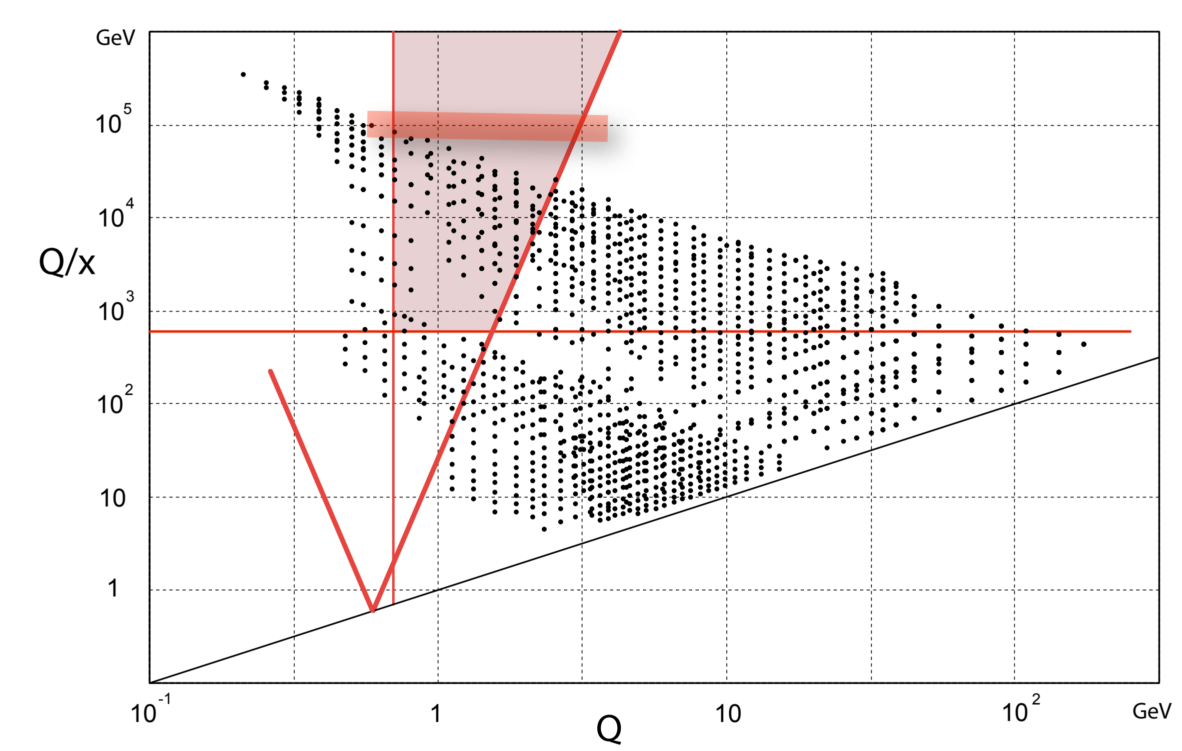
$$\int \frac{d\bar{r}}{\bar{r}^{2-\omega}}\,\Phi(\bar{r}) = \frac{h(\omega)}{M^{1+\omega}}$$

Compare with Data



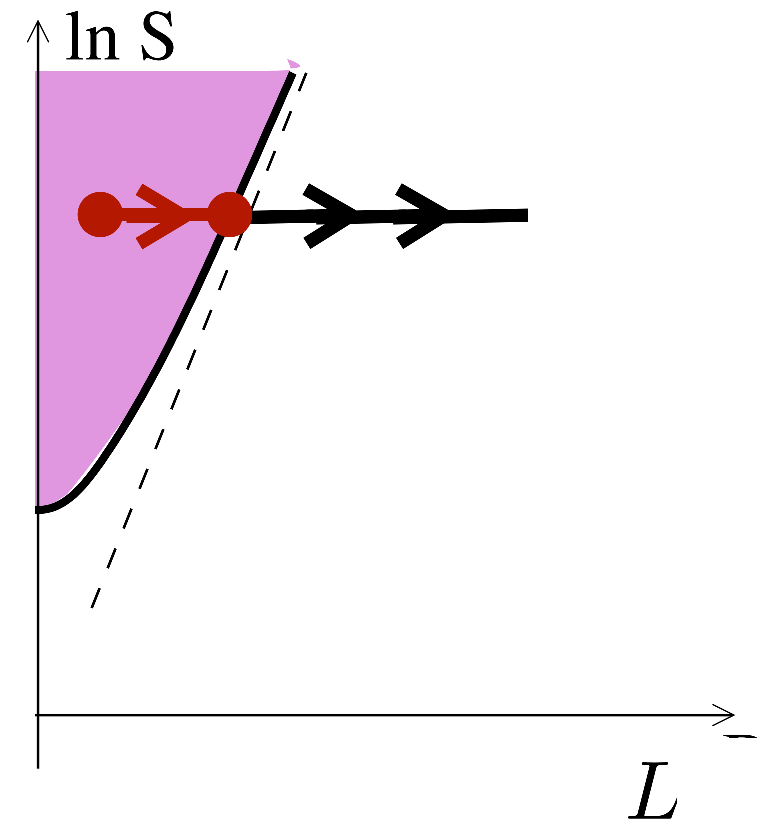
With 4 parameters can reproduce available data

$$Y = \ln\left(\frac{Q\ [\mathrm{GeV}]}{x}\right)$$



Universal ratio from kinematics deep inside disk

$S \sim \dfrac{Q}{x}$



$$T^{ab} \approx 2\pi i s^{1+\omega} \int \frac{dr}{r^{2-\omega}}\, \Psi^{ab}{}_i{}^i(r) \int \frac{d\bar{r}}{\bar{r}^{2-\omega}}\, \Phi(\bar{r})$$

$$T^{\hat{i}\hat{j}} = \delta^{\hat{i}\hat{j}} \Pi_1 \ , \qquad T^{++} \approx \frac{1}{2x^2} \left(\Pi_2 - 2x\Pi_1\right)$$

$$F_2 - 2xF_1 \approx x^{-\omega} (Q/M)^{1+\omega} \frac{\pi^{\frac{5}{2}} \Gamma^3\left(\frac{3+\omega}{2}\right) C\, h(\omega)}{3\Gamma\left(\frac{4+\omega}{2}\right)} \ ,$$

$$2xF_1 \approx x^{-\omega} (Q/M)^{1+\omega} \frac{\pi^{\frac{5}{2}} \Gamma\left(\frac{5+\omega}{2}\right) \Gamma\left(\frac{3+\omega}{2}\right) \Gamma\left(\frac{1+\omega}{2}\right) C\, h(\omega)}{12\, \Gamma\left(\frac{4+\omega}{2}\right)}$$

$$\frac{F_L}{F_T} \approx \frac{F_2 - 2xF_1}{2xF_1} \approx \frac{1+\omega}{3+\omega}$$

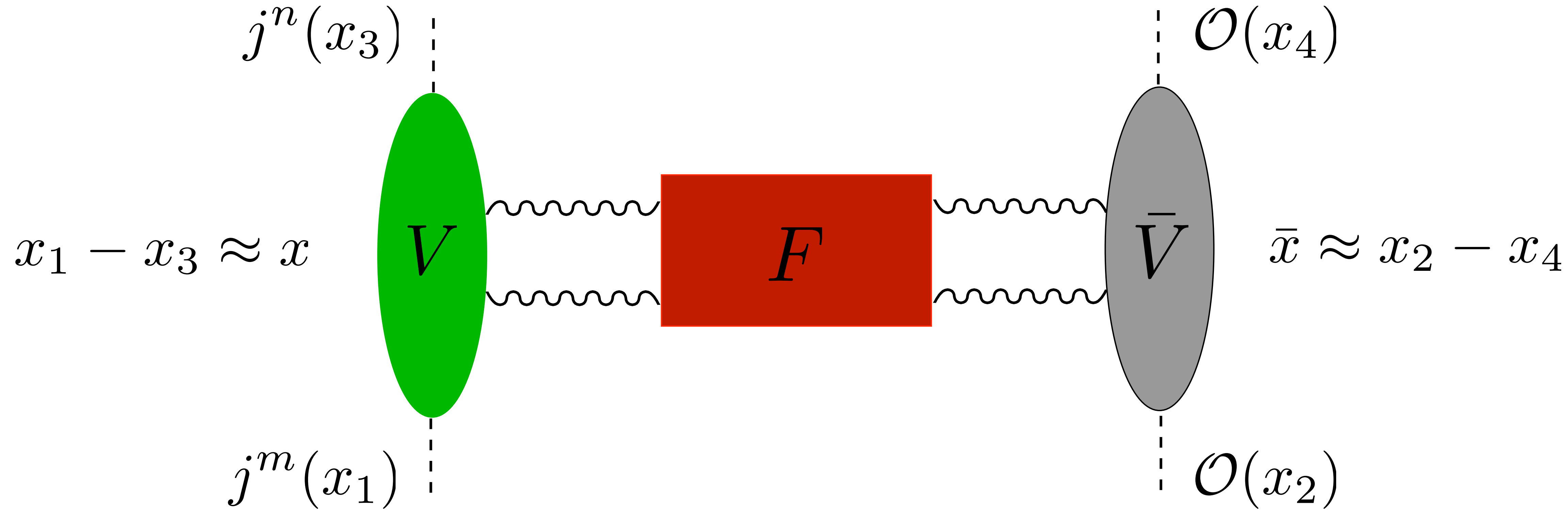
Final Comments

- Is deeply saturated DIS showing us a black disk in AdS (or in conformal QCD)?
- Non-conformal extension
- Understand AdS unitarization
- Next-to-leading order impact factors (and relation to integrability)

THANK YOU

Hard Pomeron & Transverse conformal symmetry [Balitsky, Fadin, Kuraev, Lipatov]

- Reduced amplitude can also be written in BFKL form



Invariants

$$\sigma^2 = x^2 \bar{x}^2$$

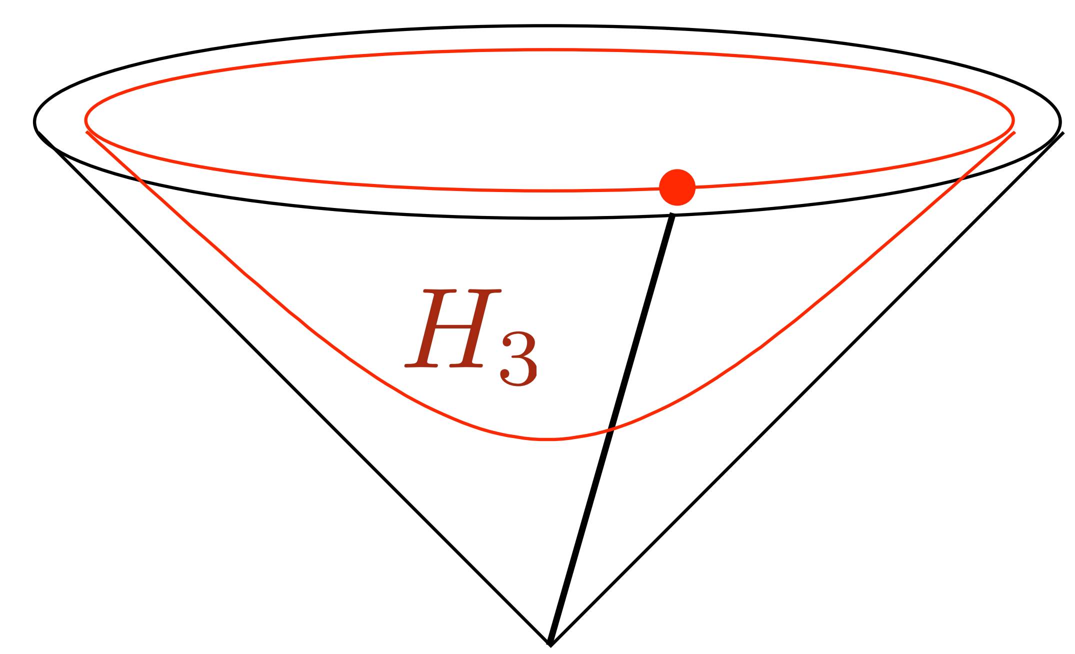
$$\cosh \rho = -\frac{x \cdot \bar{x}}{|x||\bar{x}|}$$

$$\mathcal{A}^{mn}(x,\bar{x}) \approx -\frac{1}{N^2} \int_{\partial H_3} \frac{dz_1 dz_3}{(z_{13})^2} \frac{dz_2 dz_4}{(z_{24})^2} V^{mn}(x, z_1, z_3)\, F(z_1, z_3, z_2, z_4)\, \bar{V}(\bar{x}, z_2, z_4) \qquad (j=1)$$

- Transverse conformal symmetry SO(3,1) manifest

$$z^m = (z^+, z^-, z_\perp) = (1, z_\perp^2, z_\perp) \qquad z \sim \lambda z$$

$$z_{ij} = -2 z_i \cdot z_j = (z_{i\perp} - z_{j\perp})^2 \qquad z^2 = 0$$



BFKL propagator

- $F(z_1, z_3, z_2, z_4)$ transforms as 4pt-function of scalar primaries of zero dimension.

For two gluons

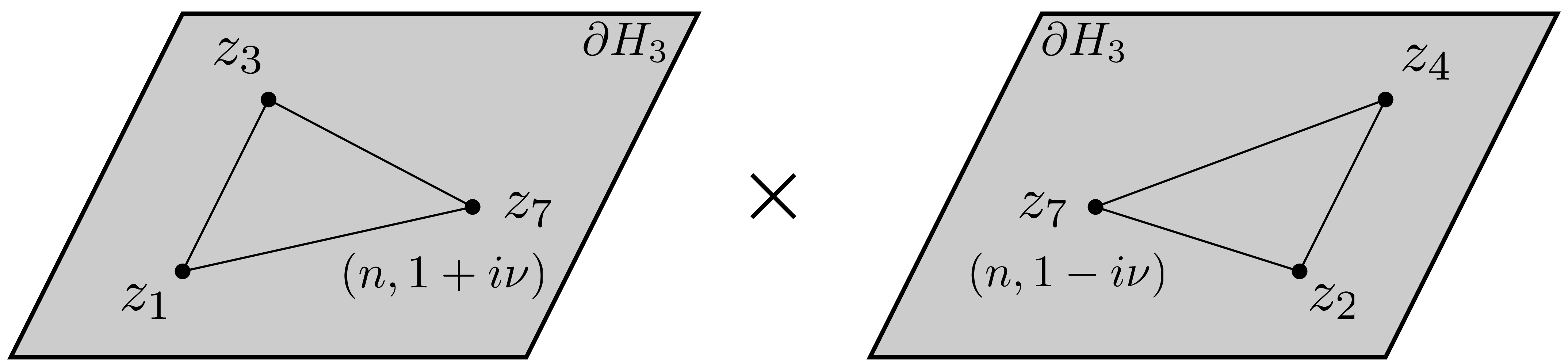
$$F(z_1, z_3, z_2, z_4) = \ln \frac{z_{13} z_{24}}{z_{12} z_{34}} \ \ln \frac{z_{14} z_{23}}{z_{12} z_{34}}$$

[Lipatov 86]

- Decomposition in transverse conformal partial waves of spin n and dimension $E = 1 + (n \pm 1)$

$$F(z_1, z_2, z_3, z_4) = \frac{4}{\pi^2} \sum_{n=0}^{\infty} 2^n \int d\nu \, \frac{\nu^2 + n^2}{\left(\nu^2 + (n-1)^2\right)\left(\nu^2 + (n+1)^2\right)} \times$$

$$\int dz_7$$



(spin,dimension)

Impact factors

- Conformal symmetry $\Rightarrow V^{mn}(x, z_1, z_3)$ homogeneous of weight 0

- A single conformal invariant cross ratio

$$u = \frac{(-x^2) z_{13}}{(-2x \cdot z_1)(-2x \cdot z_3)}$$

- Five possible tensor structures

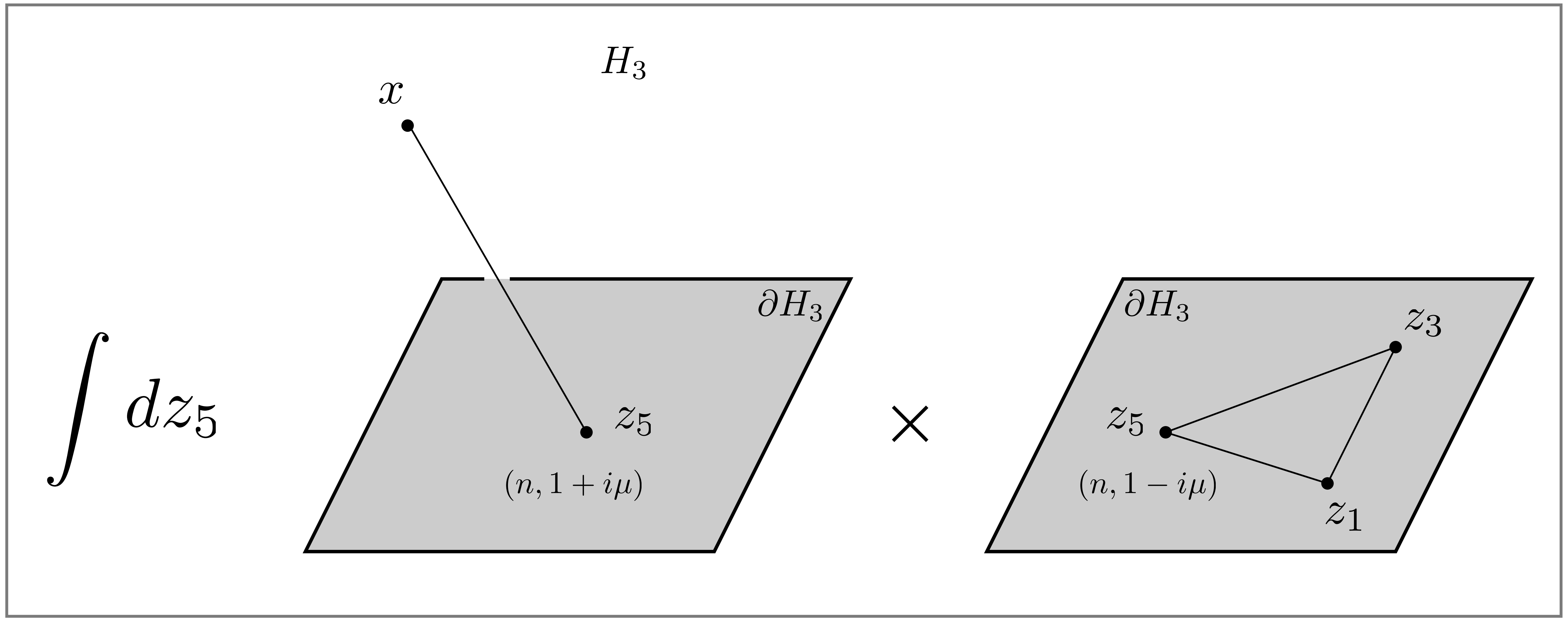
$$\begin{aligned}
\mathcal{I}_1^{mn} &= \eta^{mn} \\
\mathcal{I}_2^{mn} &= \frac{x^m x^n}{x^2} \\
\mathcal{I}_3^{mn} &= \frac{x^m z_1^n + x^n z_1^m}{-2x \cdot z_1} + \frac{x^m z_3^n + x^n z_3^m}{-2x \cdot z_3} \\
\mathcal{I}_4^{mn} &= \frac{z_1^m z_1^n (-x^2)}{(-2x \cdot z_1)^2} + \frac{z_3^m z_3^n (-x^2)}{(-2x \cdot z_3)^2} \\
\mathcal{I}_5^{mn} &= \frac{z_1^m z_3^n + z_3^m z_1^n}{z_{13}}
\end{aligned}$$

general form

$$V^{mn} = \sum_{k=1}^{5} h_k(u)\, \mathcal{I}_k^{mn}$$

- Basis with definite transverse spin (n=0 and n=2)

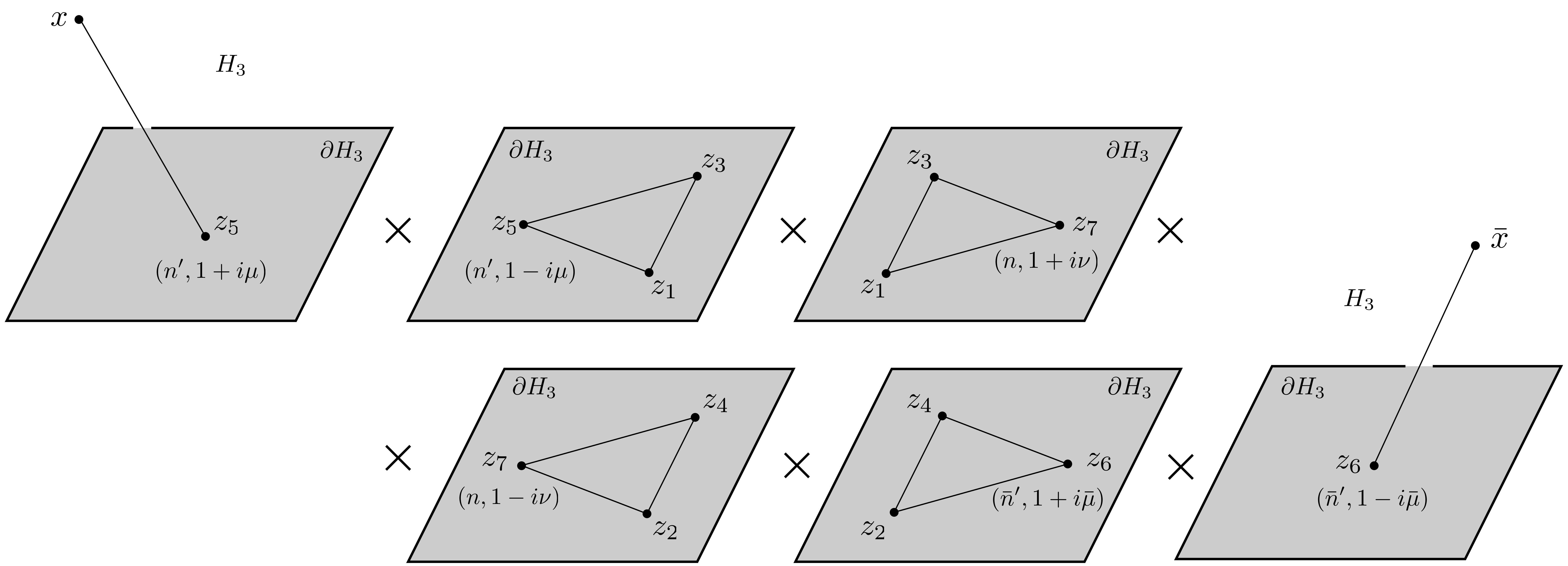
$$V^{mn}(x, z_1, z_3) = \int d\mu \left[T(\mu)\, \psi_\mu^{mn}(u) + \sum_{k=1}^{4} S_k(\mu)\, \mathcal{D}_k^{mn} \chi_\mu(u) \right]$$



$$= \begin{cases} \chi_\mu(u)\,, & n = 0 \\ \psi_\mu^{mn}(u)\,, & n = 2 \end{cases}$$

Back to Regge theory

$$\int \prod_{i=1}^{7} dz_i$$



Obtain general Regge form

$$\bar{n}' = 0 \quad \Rightarrow \quad n = 0 \quad \Rightarrow \quad n' = 0$$

$$\mathcal{A}^{mn} \approx 2\pi i \sum_{k=1}^{4} \int d\nu \, \sigma^{1-j(\nu)} \alpha_k(\nu) \, \mathcal{D}_k^{mn} \, \Omega_{i\nu}(\rho)$$

$$j(\nu) = 1 + \bar{\alpha}_s \, \chi(\nu) + \cdots$$

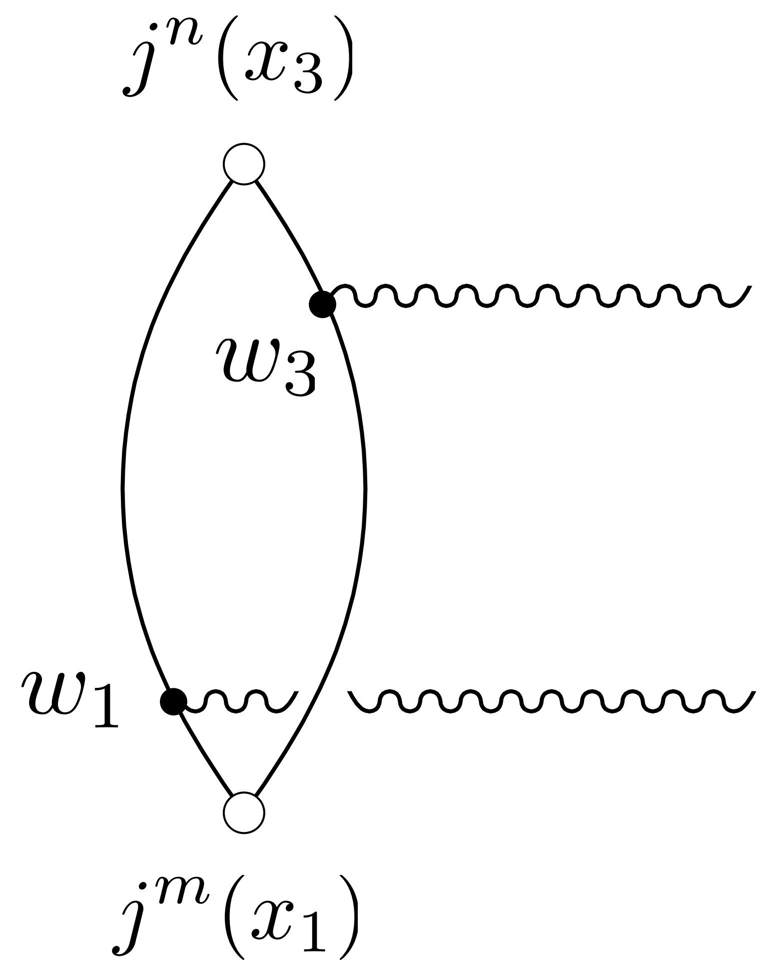
$$\alpha_k(\nu) = S_k(\nu) \, \frac{\tanh \frac{\pi\nu}{2}}{\nu} \, \bar{V}(\nu)$$

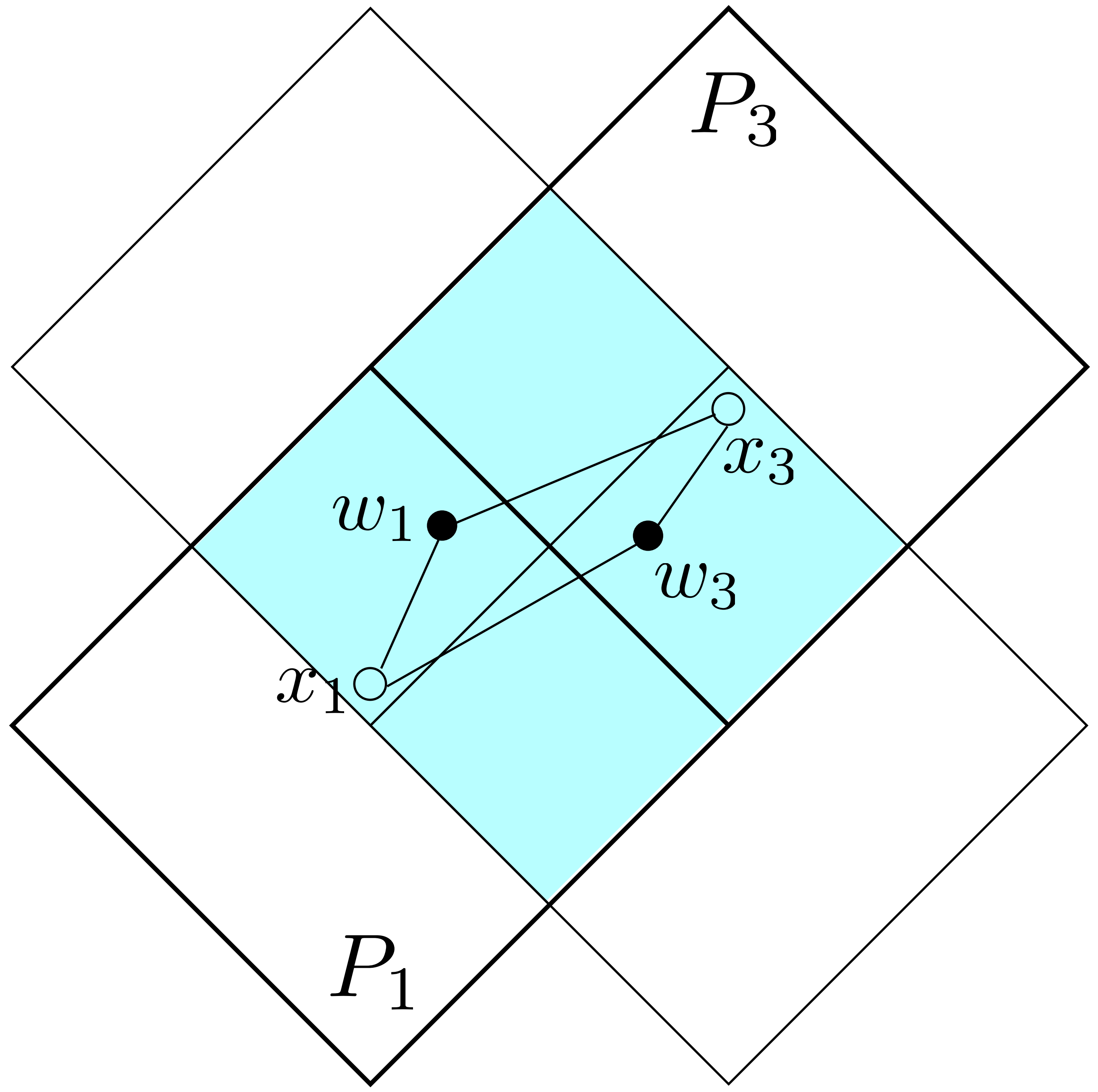
transverse spin n=2 drops out

Impact factor in QCD (massless quark)

$j^m = \bar{\psi}\gamma^m\psi$

$x_1 - x_3 \approx x$





parameterize w_i with

$$w_i = \sigma_i z_i + \lambda_i n$$

$$z_i = (1, z_{i\perp}^2, z_{i\perp})$$

$$n = (0, 1, 0)$$

Simple result

$$V^{mn} = \frac{2}{\pi}\,\bar{\alpha}_s\,u^3\left(\eta^{mn} + 2\,\frac{z_1^m z_3^n + z_1^n z_3^m}{z_{13}}\right)$$

$$u = \frac{(-x^2)z_{13}}{(-2x\cdot z_1)(-2x\cdot z_3)}$$

Transverse spin 0 and spin 2 - a conjecture for N=4 SYM

$$V^{mn}(x,z_1,z_3) = a\int \frac{d\mu}{\cosh\frac{\pi\mu}{2}}\left[t(\mu)\,\psi_\mu^{mn}(u) + \sum_{k=1}^{4} s_k(\mu)\,\mathcal{D}_k^{mn}\chi_\mu(u)\right]$$

Weyl fermion

$$\begin{aligned} s_1 &= \frac{\mu^2+25}{48} \\ s_2 &= \frac{\mu^2-7}{16} \\ s_3 &= \frac{1}{4} \\ s_4 &= -\frac{1}{32}\,\frac{\mu^2+1}{\mu^2+4} \\ t &= -\frac{\left(1+\mu^2\right)^2\left(9+\mu^2\right)}{256\left(4+\mu^2\right)} \end{aligned}$$

Complex scalar

$$\begin{aligned} s_1 &= \frac{\mu^2+13}{48} \\ s_2 &= -\frac{1}{4} \\ s_3 &= \frac{1}{16} \\ s_4 &= -\frac{1}{32}\,\frac{\mu^2+7}{\mu^2+4} \\ t &= \frac{\left(1+\mu^2\right)^2\left(9+\mu^2\right)}{256\left(4+\mu^2\right)} \end{aligned}$$

R-current spin 2 component vanishes

Conjecture: impact factor of N=4 chiral primaries only have transverse spin 0