Holographic Hydrodynamics from 5d Dilaton–Gravity

Liuba Mazzanti
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based on:

U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:1006.3261]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0903.2859]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0812.0792]
U. Gursoy, E. Kiritsis, L. M., F. Nitti, [arXiv:0804.0899]

see also:

U. Gursoy, E. Kiritsis, G. Michalogiorgakis, F. Nitti, [arXiv:0906.1890]
U. Gursoy, E. Kiritsis, F. Nitti, [arXiv:0707.1349]
U. Gursoy, E. Kiritsis, [arXiv:0707.1324]

Nordita, June 18, 2010

Motivations and Goals: Gluon Plasma and Holography

1. **Phase diagram & hydrodynamics of the plasma**
   - QCD & QGP experiments
   - zero temperature: confinement
   - finite temperature: phase transition
   - deconfined phase: hydrodynamics

2. **5d holography as bottom–up approach**
   - gauge/gravity: gauge theory at strong coupling
   - setup: confinement & running coupling
   - thermodynamics & hydrodynamics
   - logarithmically–asymptotically–AdS "hairy" black–holes

Outline

Gauge/gravity context

5d dilaton–gravity setup: zero T and finite T (TG & BH)

Phase transition and thermodynamics

- **Drag force from worldsheet background**
- **Retarded correlators from worldsheet fluctuations**
- **Diffusion constants and "jet quenching parameters"**

Numerics

Concluding remarks and open issues

Gauge/Gravity Maldacena'97,Witten'98,Gubser-Klebanov-Polyakov'98

- Holography:

Gravity background in $(d+1)$d: $g_{\mu\nu}, \phi, F_n, \ldots$

$\Updownarrow$ gauge/gravity

Gauge theory at large-N_c in dd on the boundary: $T^{\mu\nu}, \mathrm{tr}F^2, \mathcal{O}_\Delta, \ldots$

- Field/operator:

$$\mathrm{e}^{-W_{\mathrm{gauge}}} = \langle \mathrm{e}^{\int \mathrm{d}^d \phi_0 \mathcal{O}} \rangle = Z_{\mathrm{gravity}}[\phi_0]$$

  - each gravity field associated to a gauge invariant operator
  - source and vev from boundary behavior

$$\phi(r) \simeq \phi_0 r^{d-\Delta} + \frac{\langle \mathcal{O} \rangle}{2\Delta - d} r^\Delta$$

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Thermodynamics and Hydrodynamics

- Finite temperature: Hawking-Page'83,Witten'98

Gravity solutions with periodic time: thermal gas and black hole

⇕ gauge/gravity

Gauge theory at finite temperature

- Langevin Diffusion: Casalderrey-Teany'06, Gubser'06,deBoer-Hubeny-Rangamany-Shigemori'08, Son-Teaney,Giecold-Iancu-Müller'09

quark moving / drag force / diffusion ⇔ worldsheet / background / fluctuations

$$\frac{\mathrm{d}p}{\mathrm{d}t} + \frac{1}{\tau_D} p = \xi, \ \langle \xi(t)\xi(t')\rangle = \kappa\delta(t-t')$$

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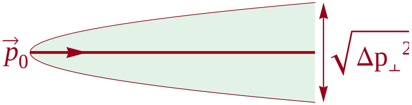
quark moving		worldsheet
drag force	⇔	background
diffusion		fluctuations



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5d Dilaton–Gravity Holography: Fields Gursoy-Kiritsis'07

Field content: minimal approach

"NSNS":

- metric $g_{\mu\nu}$ $\Leftrightarrow$ $T_{\mu\nu}$
- dilaton ϕ $\Leftrightarrow$ $\mathrm{tr}F^2$

"RR":

- D3–branes $F_5 = {}^*F_0$ $\Leftrightarrow$ $SU(N_c)$ color gauge group

Gursoy-Kiritsis-Nitti'07,Gursoy-Kiritsis-Mazzanti-Nitti'08
Casero-Kiritsis-Paredes'05,Gursoy-Kiritsis-Nitti'07

F_5 is non dynamical $\Rightarrow$ can be integrated out

5d Dilaton–Gravity Holography: Action Gursoy-Kiritsis'07

$$S_{st} = -M^3 \int \mathrm{d}^5x\sqrt{-g}\left[\mathrm{e}^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{5}{\ell_s^2}\right) - \frac{F_5^2}{2\cdot 5!} - \frac{F_1^2}{2} - \frac{N_f}{\ell_s^2}\mathrm{e}^{-\phi}\right]$$

$$\Downarrow$$

$$S_E = -\frac{M^3 N_c^2}{2}\int \mathrm{d}^5x\sqrt{-g}\left[R - \frac{4}{3}(\partial\phi)^2 + V(\phi)\right]$$

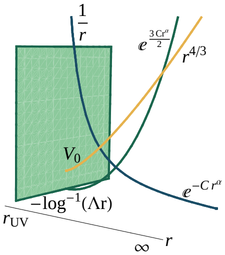
- dilaton potential determined phenomenologically: **bottom-up** $(\lambda \equiv \mathrm{e}^{\phi})$

$$V(\lambda) \simeq V_0 + V_1\lambda + V_2\lambda^2 + \cdots + \lambda^{2Q}\left(\tilde{V}_0 + \tilde{V}_1\lambda + \log^P + \ldots\right)$$

5d Dilaton–Gravity Holography: Metric Gursoy-Kiritsis'07,Gursoy-Kiritsis-Nitti'07

Gravity in asymptotic AdS + dilaton
$(N_f = 0)$

$$ds^2 = b(r)^2 \left(dr^2 - dt^2 + dx^2\right)$$
$$\phi = \phi(r), \quad V = V(\phi)$$



Dictionary: coupling and scales

- **Energy** $\Rightarrow E = b/\ell$ $\quad T^{\mu\nu} \Leftrightarrow g_{\mu\nu}$
- **'t Hooft coupling** $\Rightarrow \lambda_t \propto \lambda = N_c e^{\phi}$ $\quad \mathrm{tr}F^2 \Leftrightarrow \phi$

Asymptotic Freedom and Confinement

UV geometry ⇔ YM β–function $\left(V = \frac{4}{3}W_{o,\phi}^2 - \frac{64}{27}W_o^2\right)$ $(r \to 0)$

$$\beta(\lambda) = -b_0\lambda^2 - b_1\lambda^3 + \ldots = -\frac{9}{4}\lambda^2\,\partial_\lambda \log W_o \quad \text{as } \lambda \to 0$$

- b_0 free parameter due to $\lambda \propto \lambda_t$ ⇒ doesn't change the physics
- $b \equiv b_1/b_0^2$ fixed from YM
- AdS in UV $\lambda = 0$ ($\phi \to -\infty$) with $V'(\lambda) \neq 0 \Rightarrow \ell^2 = 12/V(0)$

IR geometry ⇔ confinement via Wilson loop $(r \to \infty)$

$$W_o \sim \lambda^Q (\log \lambda)^{P/2}$$

- singularity at ∞ ($Q = \frac{2}{3}$) ⇒ $P \geq 0$ ($P = 1/2$ for linear confinement)
- magnetic charge screening for all confining backgrounds
- discrete and gapped glueball spectrum for confining backgrounds

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Uniqueness

- Three first–order *e.o.m.* $\Rightarrow$ **three integration constants**

$$b(r_{\mathrm{UV}}) \equiv b_{\mathrm{UV}} \qquad \lambda(r_{\mathrm{UV}}) \equiv \lambda_{\mathrm{UV}}$$

1. only a combination of the two specifies the whole solution
2. other changes amount to constant rescaling of dimension–full quantities
3. one is set by requiring a "good" singularity

$$\Lambda_{QCD} = \frac{b_{\mathrm{UV}}}{\ell} \mathrm{e}^{-\frac{1}{b_0 \lambda_{\mathrm{UV}}}} (b_0 \lambda_{\mathrm{UV}})^{-b_1/b_0^2}$$

- ℓ sets the **energy units** (from lowest 0^{++} glueball mass)

Universal Asymptotics

Universal behavior in all confining solutions with good singularity at ∞

	UV	IR
$V(\lambda) \simeq$	$12\left[1+\frac{8}{9}b_0\lambda+\frac{4}{9}\left(b+\frac{23}{36}\right)b_0^2\lambda^2+\dots\right]$	$\lambda^{4/3}(\log\lambda)^P$
$W_o(\lambda) \simeq$	$\frac{9}{4}+b_0\lambda+\frac{2}{9}\left(1+\frac{9}{4}b\right)b_0^2\lambda^2+\dots$	$\lambda^{2/3}(\log\lambda)^{P/2}$
$b_o(r) \simeq$	$\frac{1}{r}\left(1+\frac{4}{9\log\Lambda r}+\dots\right)$	$e^{-\left(\frac{r}{L}\right)^{\frac{1}{1-P}}}$
$\lambda_o(r) \simeq$	$-\frac{1}{b_0\log\Lambda r}+\dots$	$e^{\left(\frac{3}{2}\right)\left(\frac{r}{L}\right)^{\frac{1}{1-P}}}\left(\frac{r}{L}\right)^{\frac{3P}{4(1-P)}}$

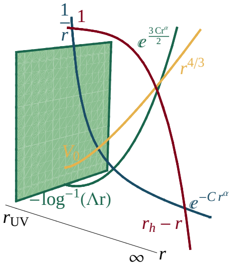
- linear confinement $\Rightarrow P=1/2$

Black Hole Solution for 5d Dilaton–Gravity Gursoy-Kiritsis-Mazzanti-Nitti'08

Asymptotic AdS BH + dilaton

$$ds^2 = b(r)^2 \left[\frac{dr^2}{f(r)} - f(r) dt^2 + dx^2 \right]$$

$$\lambda = \lambda(r), \quad V = V(\lambda)$$



Uniqueness

2 more w.r.t. zero–T ⇒

1. $\lambda_h \Leftrightarrow T = |\dot{f}(r_h)|/4\pi$ sets the temperature
2. UV normalization ⇒ $f(0) = 1$

UV and IR Horizon

UV: same log expansion as the thermal–gas $(r_h \to 0)$

$$b(r) = b_o(r)\left[1 + \mathcal{G}(T) r^4 + \dots\right], \; \lambda(r) = \lambda_o(r)\left[1 + \tfrac{45}{8}\mathcal{G}(T) r^4 \log \Lambda r + \dots\right]$$
$$f(r) = 1 - \tfrac{1}{4}\mathcal{B}(T) r^4 + \dots$$

Holography

- $\mathcal{G}(T)$: vev of the $\Delta = 4$ operator $\Rightarrow$ gluon condensate $\mathcal{G}(T) \sim \frac{T^4}{\log^2 T}$
- $\mathcal{B}(T)$: thermodynamic quantity $\mathcal{B}(T) = T\mathcal{S}$

IR: **good singularity** singled out $(r_h \to \infty)$

$$b(r) \sim b_o(r) \quad , \quad \lambda(r) \sim \lambda_o(r)$$
$$f(r) \simeq 1 - \frac{r_h}{r} \exp\left[\frac{r^2 - r_h^2}{L^2}\right]$$

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The Free Energy

Match the two solutions at finite–T on the boundary:

Asymptotic thermal AdS | Asymptotic AdS black hole

$\Downarrow$

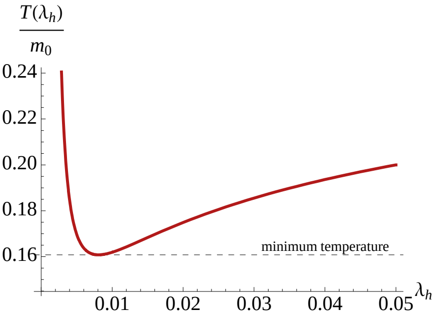
- **Free energy** from on–shell *regulated* actions $\Rightarrow$ cutoff $r = \epsilon$ $\quad (M_P \equiv M N_c^{\frac{2}{3}})$

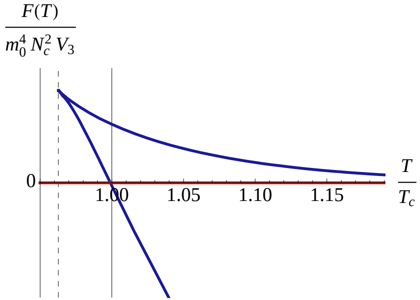
$$\frac{\mathcal{F}}{M_P^3 V_3} = \frac{S - S_o}{\beta M_P^3 V_3} = 15\mathcal{G}(T) - \frac{\mathcal{B}(T)}{4}$$

$\updownarrow$ gluon condensate $\quad$ $\updownarrow$ entropy $\cdot T$

Phase Transition

First order phase transition ⇔ confining backgrounds





Holographic Thermodynamics:

Trace Anomaly vs. Gluon Condensate: the Trace Anomaly Equation

From dilaton fluctuation and field/operator correspondence

$$\frac{1}{4}\frac{\beta(\lambda)}{\lambda^2}\langle \mathrm{tr} F^2 \rangle = e - 3p = 60\mathcal{G}(T)M^3 N_c^2$$

- **Trace**: $e - 3p \propto$ condensate $\Rightarrow$ latent heat $\sim N_c^2$
- **Pressure, Energy, Entropy**: $p, e, s \sim N_c^2$ for $T > T_c \Rightarrow$ deconfinement
- **Sound speed**: $c_s^2 \rightarrow 1/3$ at high–T, small at T_c

Ansatz for the Potential

Gursoy-Kiritsis-Mazzanti-Nitti'09

$$V(\lambda) = \frac{12}{\ell^2}\left\{1 + V_0\lambda + V_1\lambda^{4/3}\left[\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)\right]^{1/2}\right\}$$

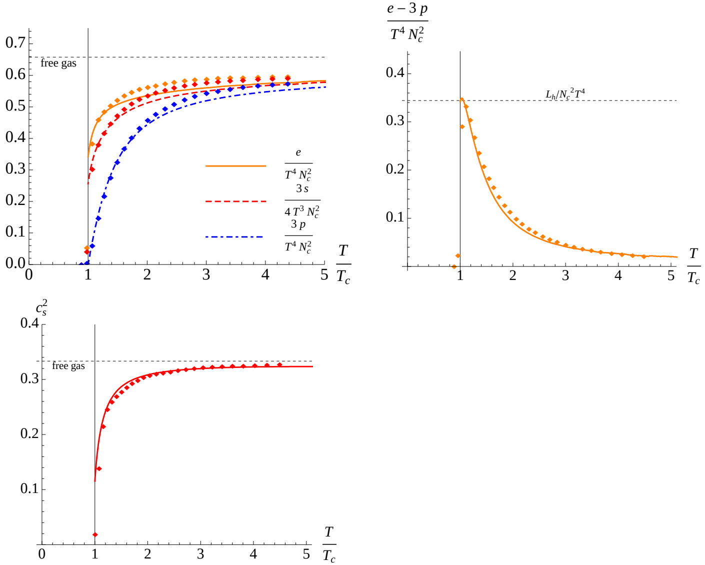
1. Monotonic
2. Asymptotic freedom and confinement
3. Linear Regge trajectories
4. YM for V_0, V_2

$$\Downarrow$$

- $V_1 = 14$ $\quad p/T^4$ at $T = 2T_c$
- $V_3 = 170$ $\quad e/T^4$ at $T = T_c$ (latent heat)

Thermodynamic Result

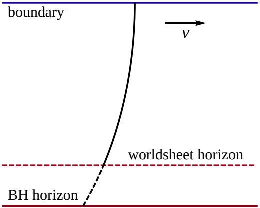
Boyd-et al.'05 ($SU(3)$), Lucini-Teper-Wenger'05 (large–N_c), Meyer (viscosity)



Trailing String Gursoy-Kiritsis-Mazzanti-Nitti'10

- Worldsheet background:

$$X^1 = vt + x(r), \quad X^2 = X^3 = 0$$
$$S_{ws} = -\frac{1}{2\pi\ell_s^2}\int \mathrm{d}r\mathrm{d}t\, b^2\sqrt{1 - \frac{v^2}{f} + f\dot{x}^2}$$



Worldsheet Horizon

The induced metric has a horizon at r_s with temperature: $f(r_s) = v^2$

$$4\pi T_s = \sqrt{f\dot{f}}\sqrt{4\frac{\dot{b}}{b} + \frac{\dot{f}}{f}}\Bigg|_{r_s}$$

$$g_{\alpha\beta} = b^2\begin{pmatrix} v^2 - f & v\dot{x} \\ v\dot{x} & f^{-1} + \dot{x}^2 \end{pmatrix}$$

Worldsheet Background: the Drag Force

- Drag force ⇒ momentum lost by the moving string to the horizon

$$F_d = \pi_x = -\frac{v b^2(r_s)}{2\pi \ell_s^2}$$

- Diffusion time ⇒ attenuation time for the momentum $\gamma \equiv 1/\sqrt{1-v^2}$

$$\tau_D \equiv -\frac{M v \gamma}{F_d} = \gamma M \frac{2\pi \ell_s^2}{b^2(r_s)}$$

Comparison to AdS_5 (λ_{AdS} fixed)

- $T_{s,AdS} = T/\sqrt{\gamma}$
- $\tau_{D,AdS} = \frac{2M}{\pi\sqrt{\lambda_{AdS}}T^2}$ momentum–independent

Worldsheet Fluctuations

- Worldsheet onshell action to second order

$$X^1 = vt + x(r) + \delta X^1, \quad X^2 = \delta X^2, \quad X^3 = \delta X^3$$

$$S_{ws}^{(2)} = -\frac{1}{2\pi\ell_s^2}\int \mathrm{d}r\mathrm{d}t\; G^{\alpha\beta}\left[\frac{1}{2}\partial_\alpha\delta X^1\,\partial_\beta\delta X^1 + \frac{Z^2}{2}\sum_{i=2}^{3}\partial_\alpha\delta X^i\,\partial_\beta\delta X^i\right]$$

$$Z \equiv b^2\sqrt{f-v^2}/\sqrt{b^4 f - b_s^4 v^2},\; G^{\alpha\beta} = \frac{b^2}{Z^3}\begin{pmatrix} -\frac{Z^2 f + v^2}{f^2} & v\dot{x} \\ v\dot{x} & f - v^2 \end{pmatrix}$$

Leading Asymptotics from *eom* $\quad I = \perp, \parallel$

- **boundary**:

$$\delta X^I \sim c^I_{sour} + c^I_{vev} r^3$$

- **horizon**:

$$\delta X^I \sim c^I_{in}\,(r_s - r)^{-\frac{i\omega}{4\pi T_s}} + c^I_{out}\,(r_s - r)^{\frac{i\omega}{4\pi T_s}}$$

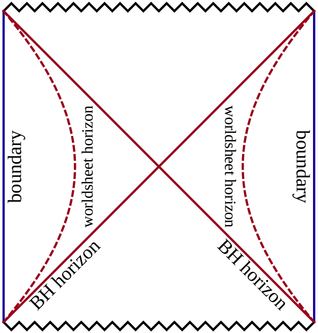
Retarded Correlators

Son-Starinets'02, Gubser'06, Son-Teaney, Giecold-Iancu-Müller'09

1. The retarded solution $\Psi_R^I \equiv \delta X^I e^{-i\omega t}$
   - ingoing at **horizon**: $c_{out} = 0$
   - non normalizable at **boundary**: $c_{sour} = 1$
   - $\Psi_R(\omega) = \Psi_R^*(-\omega)$
2. Analyticity in Kruskal plane
3. Shwinger-Keldysh Green function

$$G_R = -\frac{1}{2\pi\ell_s} G^{r\alpha} \Psi_R^* \partial_\alpha \Psi_R \Big|_{\text{bound.}}$$

$$G_{sym} = -\coth\left(\frac{\omega}{2T_s}\right) \text{Im} G_R$$



Diffusion Constants and Jet Quenching Gursoy-Kiritsis-Mazzanti-Nitti'10

$$\kappa = \lim_{\omega \longrightarrow 0} G_{sym} = \lim_{\omega \longrightarrow 0} \coth\left(\frac{\omega}{2T_s}\right) J^r$$

- J^r is a conserved current (number current): $J^\alpha \equiv -\frac{1}{2\mathrm{i}} G^{\alpha\beta} \Psi_R \overleftrightarrow{\partial}_\beta \Psi_R$
- κ can be evaluated at any r
- $\hat{q} \equiv \langle \Delta p^2 \rangle / L$ at strong coupling

$$\Downarrow$$

$$\kappa_\perp = \frac{1}{2} v \hat{q}_\perp = \frac{1}{\pi \ell_s^2} b_s^2 T_s, \qquad \kappa_\parallel = v \hat{q}_\parallel = \frac{16\pi}{\ell_s^2} \frac{b_s^2}{\dot{f}_s^2} T_s^3$$

Generalized Einstein relation to diffusion time:

$$\tau_D \kappa_\perp = 2\gamma M T_s$$

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Extreme Limits

- **Non–relativistic**: $(r_s \longrightarrow r_h)$
  - both $\kappa_\perp, \kappa_\parallel$ go to the same limit $\Rightarrow$ isotropic thermal fluctuations

$$\frac{\kappa_\perp}{\kappa_{\perp,AdS}}, \frac{\kappa_\parallel}{\kappa_{\parallel,AdS}} \longrightarrow \frac{2\ell^2}{\pi^2 \ell_s^2} \left(\frac{45\pi s}{4N_c^2 T^3} \right)^{\frac{2}{3}} \frac{\lambda_h^{\frac{4}{3}}}{\sqrt{\lambda_{AdS}}}$$

- **Ultra–relativistic**: $(r_s \longrightarrow 0)$
  - $\kappa_\perp, \kappa_\parallel$ are logarithmically divergent
  - vanishing when normalized w.r.t. AdS
  - again, both *normalized* $\kappa_\perp, \kappa_\parallel$ go to the same limit

$$\frac{\kappa_\perp}{\kappa_{\perp,AdS}}, \frac{\kappa_\parallel}{\kappa_{\parallel,AdS}} \longrightarrow \frac{\ell^2}{\sqrt{2}\pi^3 \ell_s^2} \left(\frac{45\pi^2 s}{N_c^2 T^3} \right)^{\frac{3}{4}} \frac{\left(\frac{b_0}{2} \log \gamma \right)^{-\frac{4}{3}}}{\sqrt{\lambda_{AdS}}}$$

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WKB Approximation for Large Frequencies

1. WKB formula for large ω: $(\omega r_s \gg 1)$

$$\Psi_R \sim C_1 \cos\left[\int \frac{\omega Z}{f - v^2} + \theta_1\right] + C_2 \sin\left[\int \frac{\omega Z}{f - v^2} + \theta_2\right]$$

2. **horizon** $\Rightarrow$ $C_1 = -\mathrm{i} C_2 = b_s \mathrm{e}^{-\mathrm{i}\theta_i} c_{in}, \ \theta_1 = \theta_2$
3. **boundary** $\Rightarrow$ determines all coefficients ($\lambda_{tp} = \lambda$ at turning point)

$$c_{vev} = -\mathrm{i}\omega\ell\gamma^{\frac{3}{2}}, \quad c_{in} = -\mathrm{i}\frac{\omega\ell\gamma}{b_s}\lambda_{tp}^{\frac{2}{3}}, \quad \theta_i = 0$$

Spectral Densities at Large Frequencies $\rho \equiv -\frac{1}{\pi}\mathrm{Im}G_R$

$$\rho_{\|} \simeq \gamma^2 \rho_{\perp} \simeq \frac{\ell^2\gamma^2}{\pi^2\ell_s^2}\omega^3\lambda_{tp}^{\frac{4}{3}}$$

WKB Approximation for Large Frequencies

1. WKB formula for large ω: $(\omega r_s \gg 1)$

$$\Psi_R \sim C_1 \cos\left[\int \frac{\omega Z}{f - v^2} + \theta_1\right] + C_2 \sin\left[\int \frac{\omega Z}{f - v^2} + \theta_2\right]$$

2. **horizon** $\Rightarrow$ $C_1 = -\mathrm{i}C_2 = b_s \mathrm{e}^{-\mathrm{i}\theta_i} c_{in}, \ \theta_1 = \theta_2$
3. **boundary** $\Rightarrow$ determines all coefficients ($\lambda_{tp} = \lambda$ at turning point)

$$c_{vev} = -\mathrm{i}\omega\ell\gamma^{\frac{3}{2}}, \quad c_{in} = -\mathrm{i}\frac{\omega\ell\gamma}{b_s}\lambda_{tp}^{\frac{2}{3}}, \quad \theta_i = 0$$

Spectral Densities at Large Frequencies $\rho \equiv -\frac{1}{\pi}\mathrm{Im}G_R$

$$\rho_{\parallel} \simeq \gamma^2 \rho_{\perp} \simeq \frac{\ell^2\gamma^2}{\pi^2\ell_s^2}\omega^3\lambda_{tp}^{\frac{4}{3}}$$

Finite Mass Quarks

- Introduces a UV cutoff $r_Q \sim 1/M$
- Diffusion constants and jet quenching independent of M
- **Large frequency** behavior modified $\Rightarrow$ linear in ω

- **low** velocities: $(\gamma\omega r_Q \ll 1)$

$$\rho_\perp \simeq \frac{\omega^3}{\pi^2}\frac{\ell^2}{\ell_s^2}\,\gamma^2\,\lambda_{tp}^{\frac{4}{3}}\left[1+(\gamma\omega r_Q)^2+\left(\mathcal{U}_Q^2-1\right)\left(\sin(\gamma\omega r_Q)-\gamma\omega r_Q\cos(\gamma\omega r_Q)\right)^2\right]^{-1}$$

- **high** velocities: $(\gamma\omega r_Q \gg 1)$

$$\rho_\perp \simeq \frac{\omega^3}{\pi^2\ell_s^2}\gamma^3 r_Q^2\mathcal{R}_Q\left[1+(\gamma\omega r_Q)^2\right]^{-1}$$

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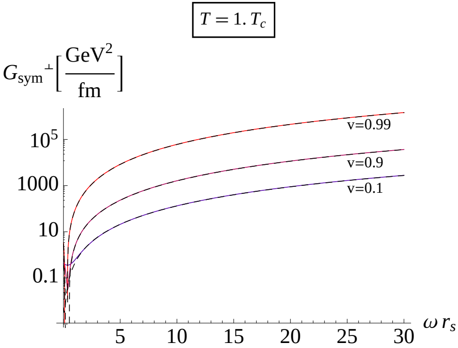
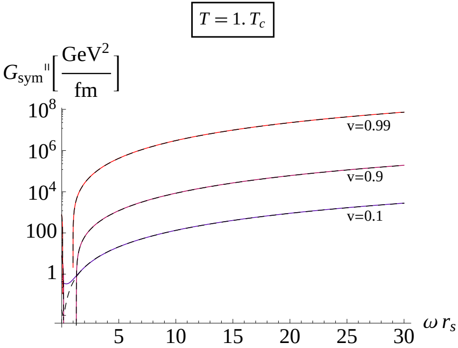
$$\mathcal{U}_Q \simeq \frac{\lambda_{tp}^{\frac{4}{3}}}{\lambda_Q^{\frac{4}{3}}}, \quad \mathcal{R}_Q \simeq \frac{b_Q^2\lambda_Q^{\frac{4}{3}}}{\gamma}$$

Spectral Densities — Infinitely Massive Quarks

Symmetric Correlator





Spectral Densities — Infinitely Massive Quarks

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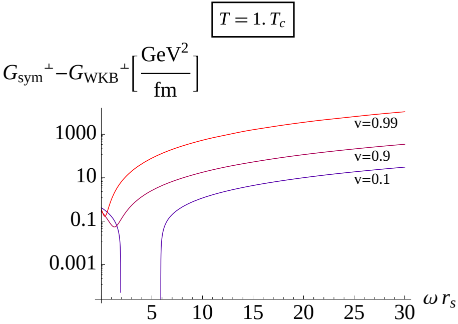
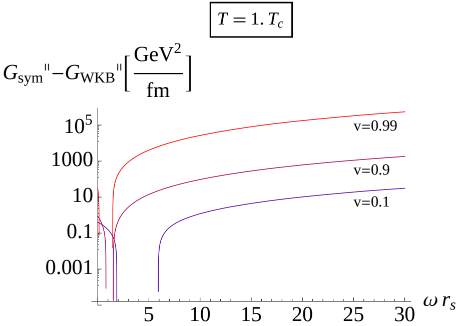
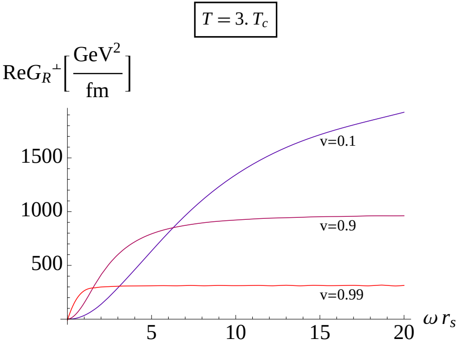
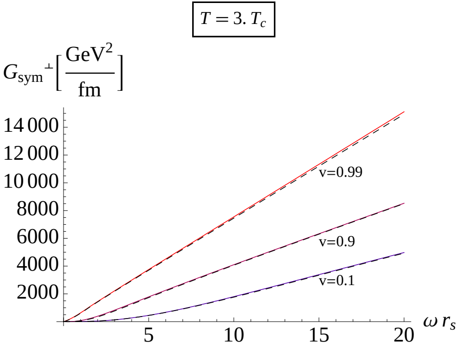




Spectral Densities — Finite Massive Quarks

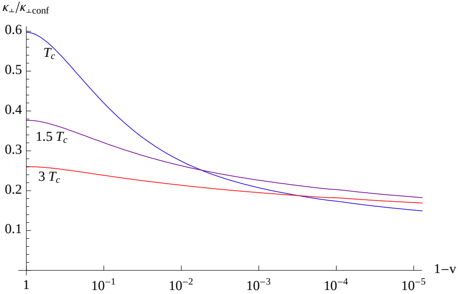
Retarded and Symmetric Correlator — Charm

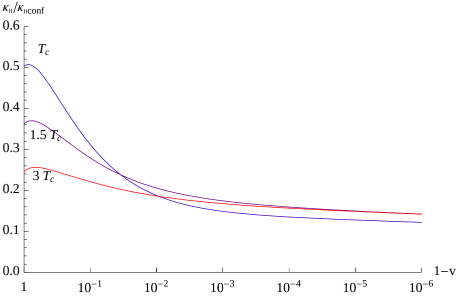




Diffusion Constants

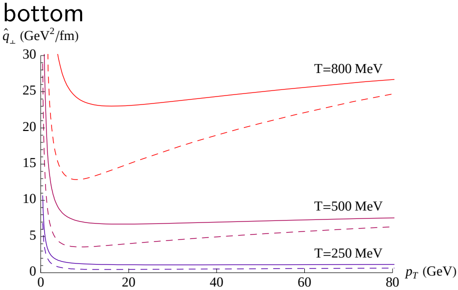
Ratio to AdS

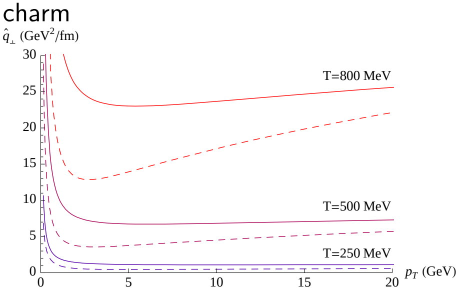




Jet Quenching Parameter

Bottom and Charm Quarks: $\hat{q}$ vs. momentum





Summary

5D dilaton–gravity $\Leftrightarrow$ 4D holographic large-N_c gauge theory

$\Downarrow$

Confinement with discrete spectrum in agreement with lattice at low–T

1. **Phase transition** first order Hawking–Page confinement/deconfinement
   - only for confining backgrounds
2. **Thermodynamics** in good agreement with lattice
3. **Hydrodynamics** compatible with phenomenological models and experiments

Work in Progress and Open Issues

- Potential from first principles ?

Further issues

1. Debye screening masses *in preparation*
2. Hydrodynamics: transport coefficients at higher orders *ongoing work*
3. Flavors via D4–branes (following Casero-Kiritsis-Paredes'06,Gursoy-Kiritsis-Nitti'07)
4. Finite chemical potentials and number densities

Thermodynamic Results

Summary of Parameters

	HQCD	$N_c = 3$	$N_c \to \infty$	Parameter
$m_{0^{++}}/\sqrt{\sigma}$	**3.37**	3.56 *	**3.37** **	$\ell_s/\ell = 0.15$
$\left[\frac{p}{(N_c^2 T^4)}\right]_{T\to\infty}$	$\pi^2/45$	$\pi^2/45$	$\pi^2/45$	$M\ell = [45(2\pi)^2]^{-1/3}$
$\left[\frac{p}{(N_c^2 T^4)}\right]_{T=2T_c}$	**1.2**	**1.2** ●●	-	$V1 = 14$
$\frac{L_h}{(N_c^2 T_c^4)}$	**0.31**	0.28 ●	**0.31** ●●	$V3 = 170$

Table: *=Chen-et al.'05, **=Lucini-Teper'01, ●=Boyd-et al.'96, ●●=Lucini-Teper-Wenger'05

Thermodynamic Results

Summary of Results

	HQCD	$N_c = 3$	$N_c \to \infty$
$m_{0^{*++}}/m_{0^{++}}$	1.61	1.56(11) *	1.90(17) **
$m_{2^{*++}}/m_{2^{++}}$	1.36	1.40(4) *	1.46(11) **
$T_c/m_{0^{++}}$	0.167	-	0.177(7) ••

Table: *=Chen-et al.'05,**=Lucini-Teper'01,•=Boyd-et al.'96,••=Lucini-Teper-Wenger'05