Epidemic thresholds for a static and dynamic small world network

J. K. Ochab and P. F. Góra

Marian Smoluchowski Institute of Physics
Jagiellonian University

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Motivations and Aims

- concerns: people, computers, livestock, plantations
- diseases: rhizomania, foot-and-mouth disease, swine/avian flu, SARS, syphilis, HIV . . .
- network models: small-world, scale-free, adaptive, weighted, (un)directed . . .
- data: real frequency distribution of contacts, networks of contacts

A Comment on other Research

- examined: development in time, maximal epidemic's size, costs, epidemic thresholds
- aim: effective methods for prediction, prevention and stopping epidemic's development

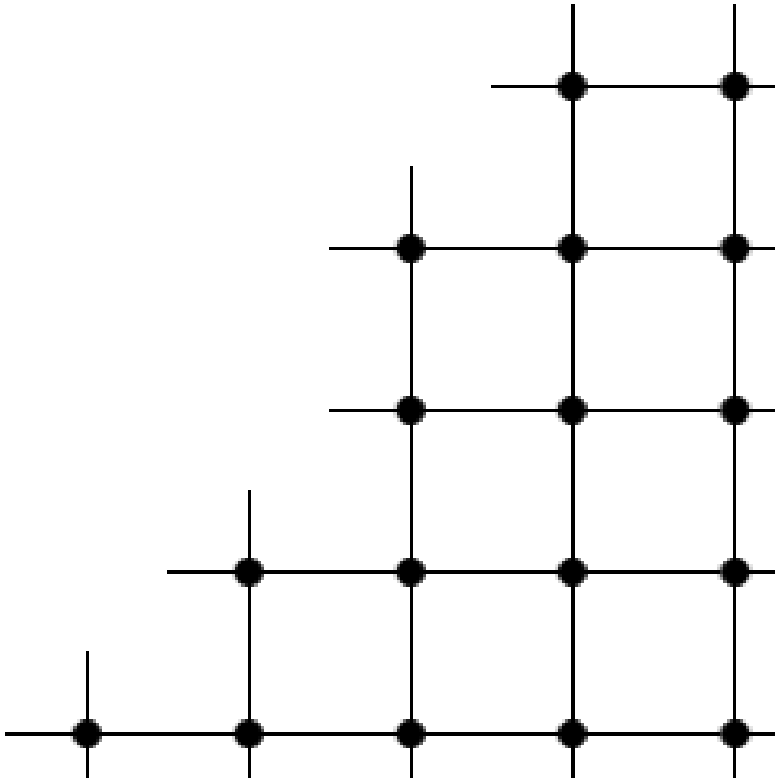
What is the influence of network dynamics on the epidemic outbreak?

Dybiec, Kleczkowski, and Gilligan, J. R. Soc. Interface 6 (2009) 941-950
Saramaki, Kaski, J. Theor. Biol. 234 (2005), 413-421
Volz, Meyers, Proc. R. Soc. B 274 (2007), 2925-2934

Model

The network:

- small-world network (Newman and Watts)
- periodic boundary conditions (toroidal)
- addition of "'shortcuts"':$2d\phi N$
- dynamics: shortcut drawing every turn



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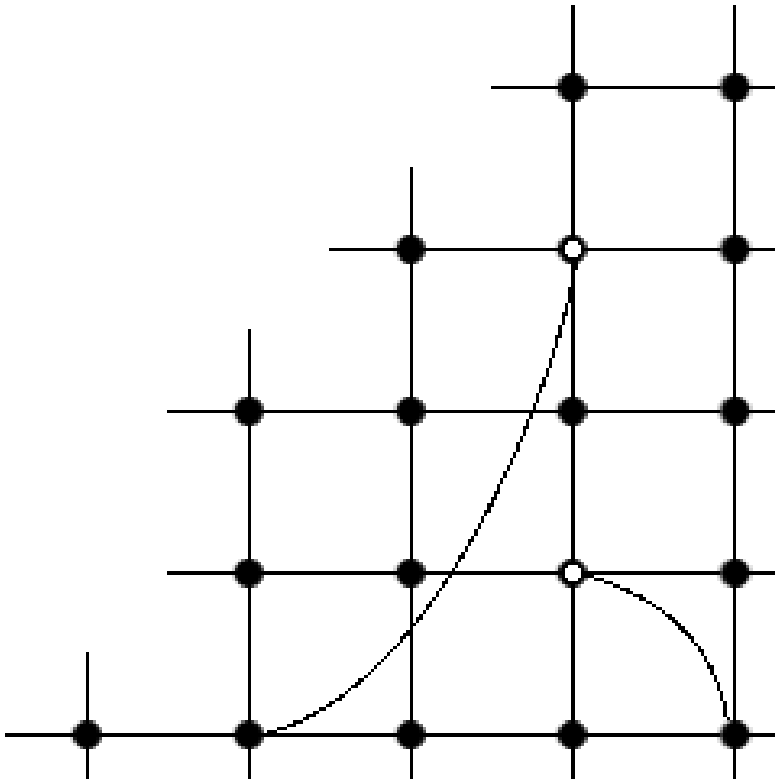

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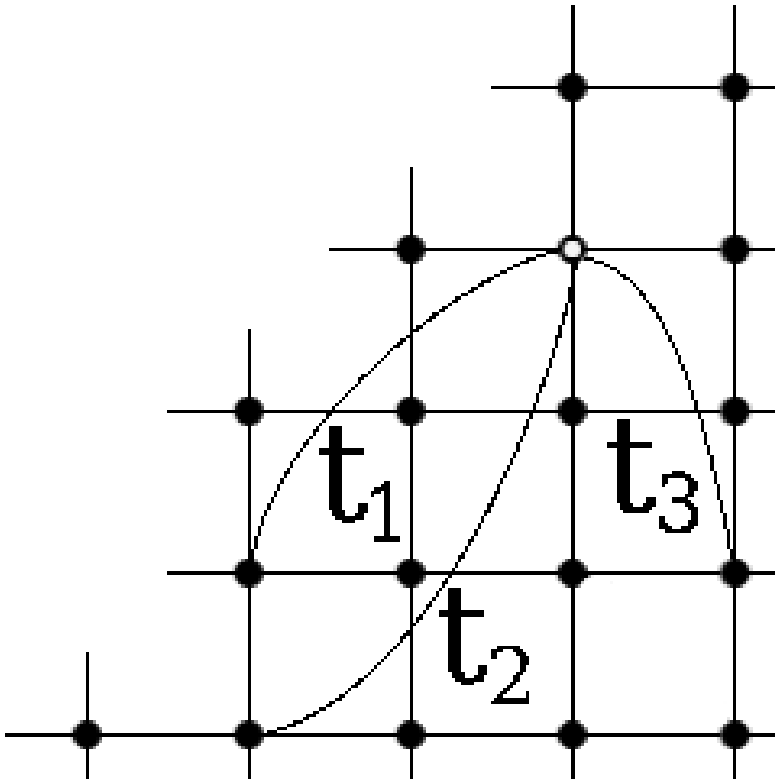
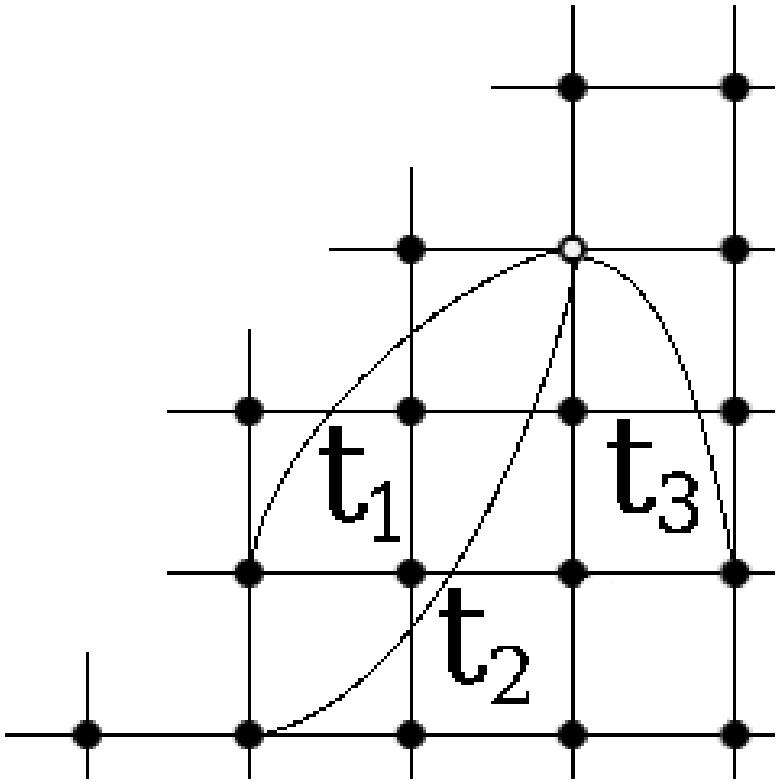


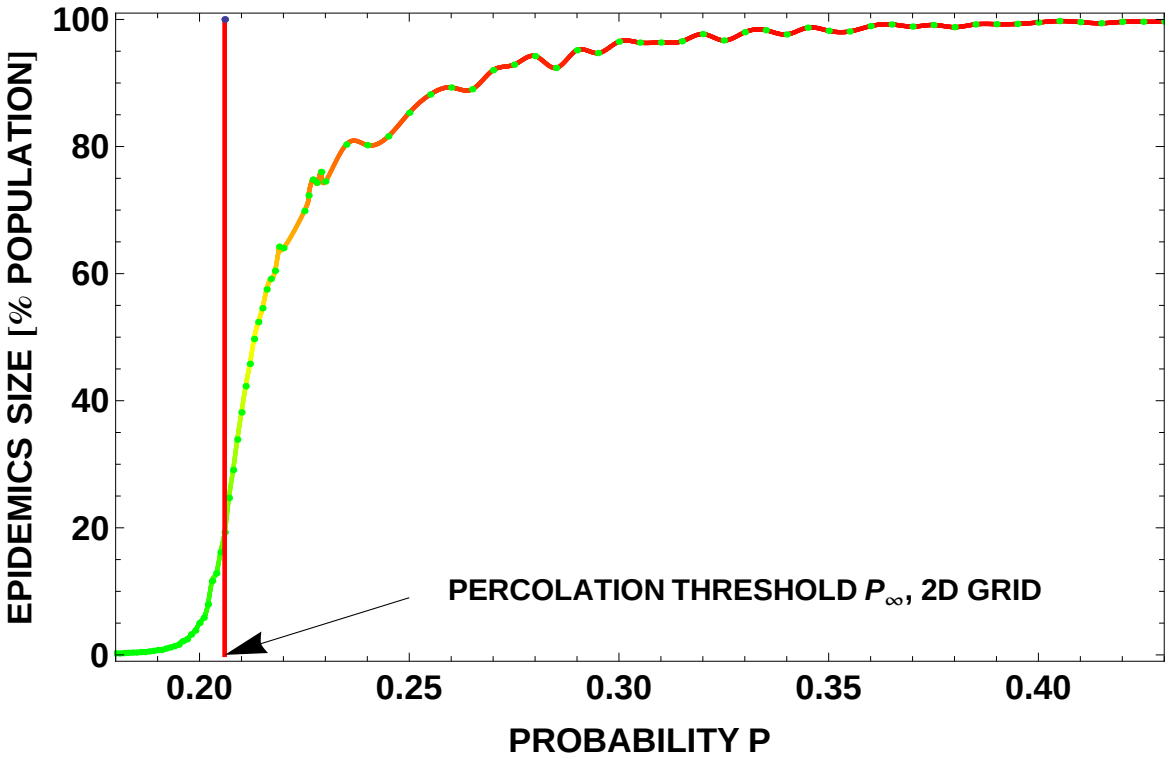
Model

The epidemic:

- SIR (Susceptible-Infectious-Removed)
- discrete time
- infection probability p (of a given neighbour in a given time step)
- latency of the disease $l = 3$



Epidemic's developement



$$T = \sum_{t=1}^{l} p(1-p)^{t-1} = 1-(1-p)^{l}$$

(bond percolation)

P. Grassberger, Math. Biosci. **63**, 157 (1983)

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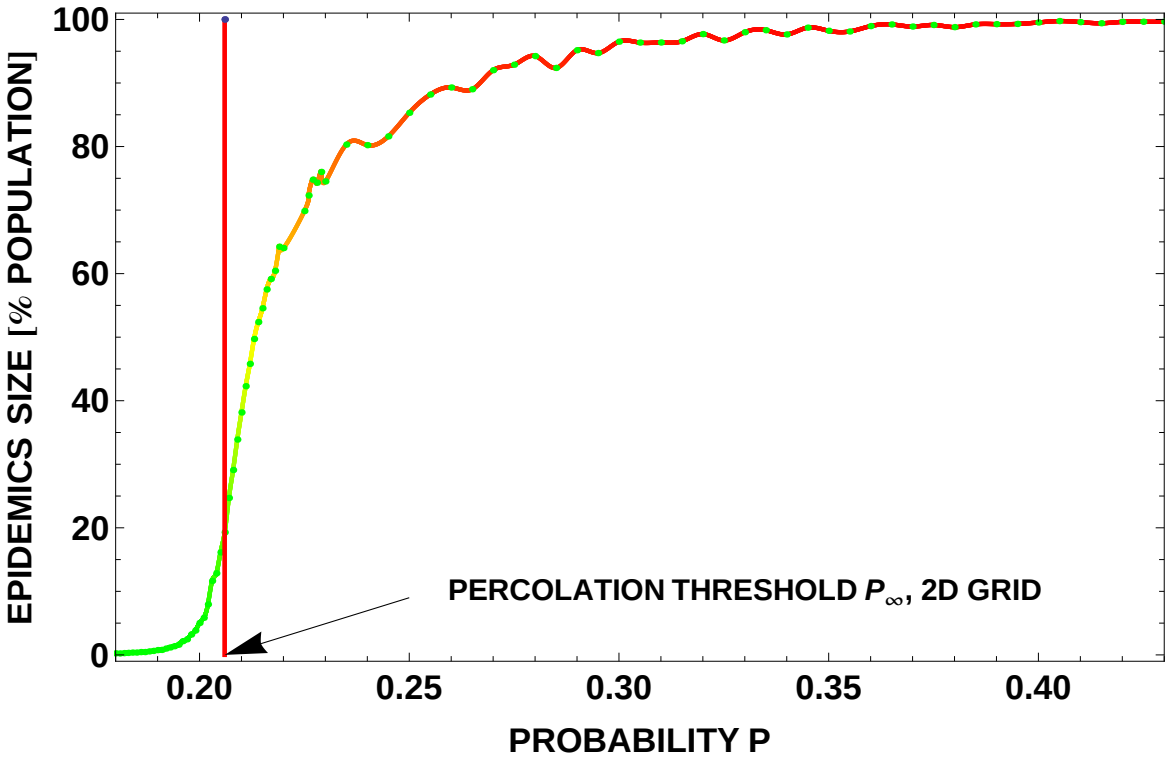

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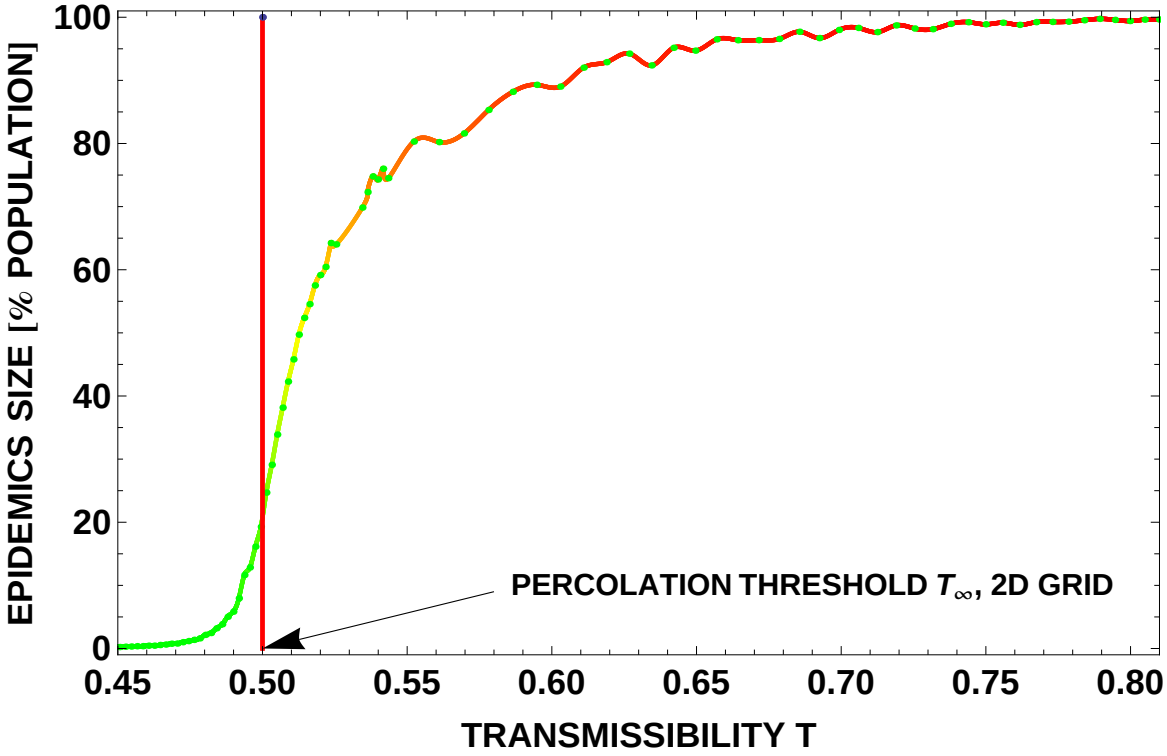

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Theory - MEJ Newman

$H(z) = \sum_{n=1}^{\infty} P(n) z^n$
$P(n)$ - probability of a node belonging to n-cluster on SWN

$H_0(z) = \sum_{n=1}^{\infty} P_0(n) z^n$
$P_0(n)$ - prob. of a node belonging to n-cluster on regular network

$P(m|n)$ - prob. of m shortcuts emanating from an n-cluster

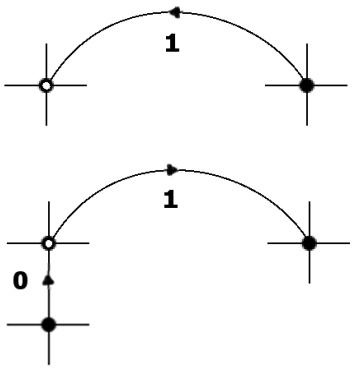
$$H(z) = \sum_{n=1}^{\infty} P_0(n) z^n \sum_m P(m|n) [H(z)]^m$$

$$H(z) = H_0\{z \exp[2d\phi T(H(z) - 1)]\}$$

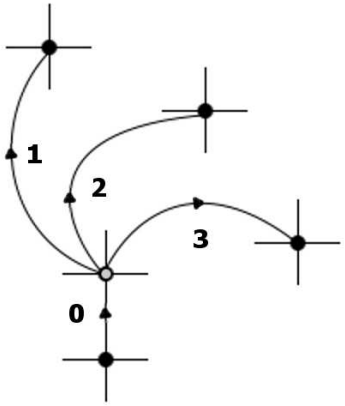
$\langle n \rangle = H_0'(1)/(1 - 2d\phi T H_0'(1))$ - average cluster size

Newman, Jensen, Ziff, *Phys. Rev. E* **65** (2002), 021904

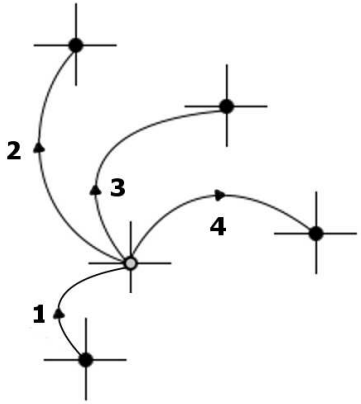
Theory



Symmetric infection on static net.



Infection from regular net.



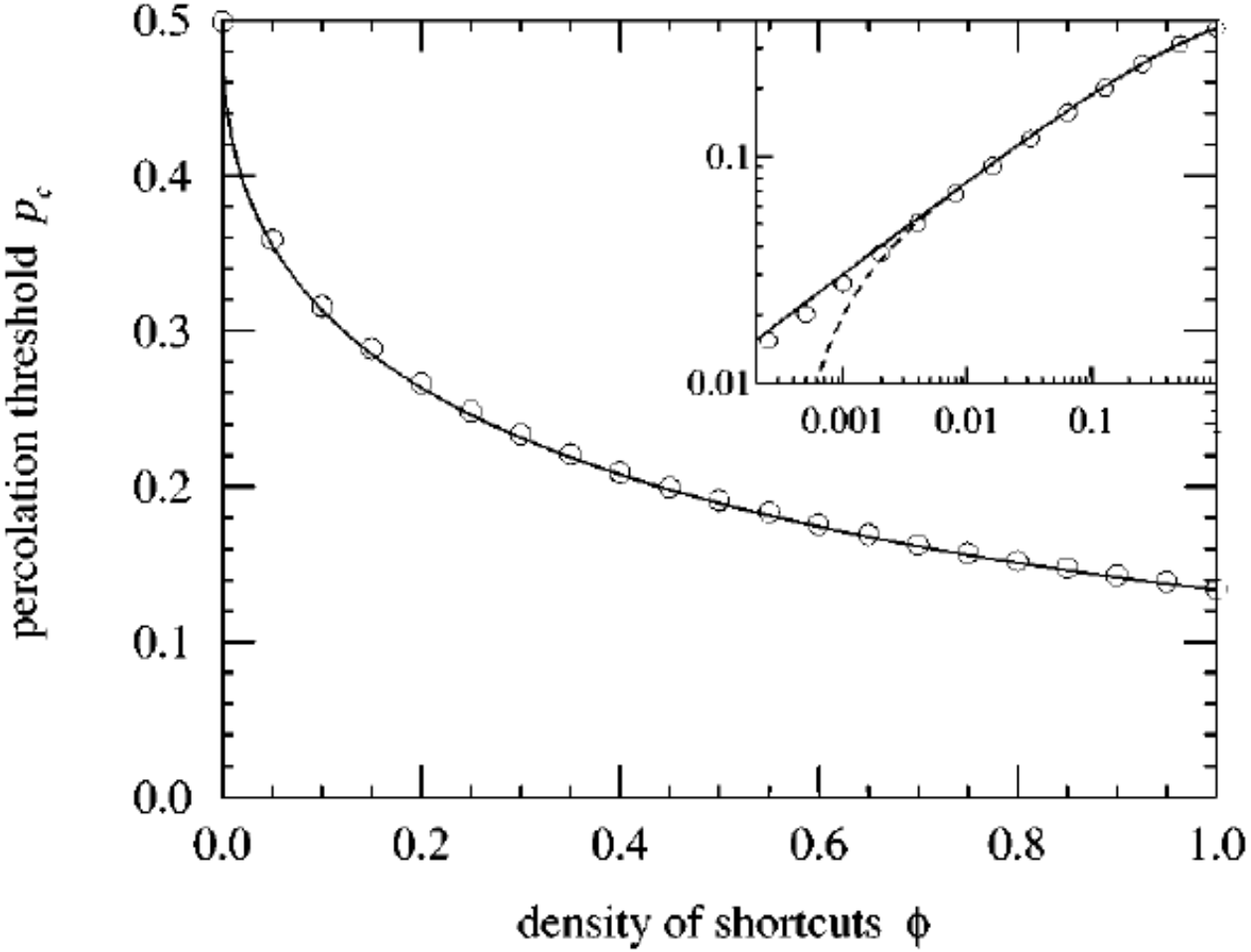
Infection through a shortcut.

$$N_{stat} = \phi_{stat} N \cdot T = \phi_{stat} N \cdot \sum_{t=1}^{l} p(1-p)^{t-1}$$

$$N_{dyn} = \phi_{dyn} N/2 \cdot lp + \phi_{dyn} N/2 \cdot (l+1)\, p$$

$$r(p,l) = \phi_{stat}/\phi_{dyn} = \frac{p\,(l+1/2)}{T} = \frac{p\,(l+1/2)}{1-(1-p)^{l}}$$

Result 1: shift of percolation thresholds



$$T_c(\phi)$$

Newman, Jensen, Ziff, *Phys. Rev. E* **65** (2002), 021904

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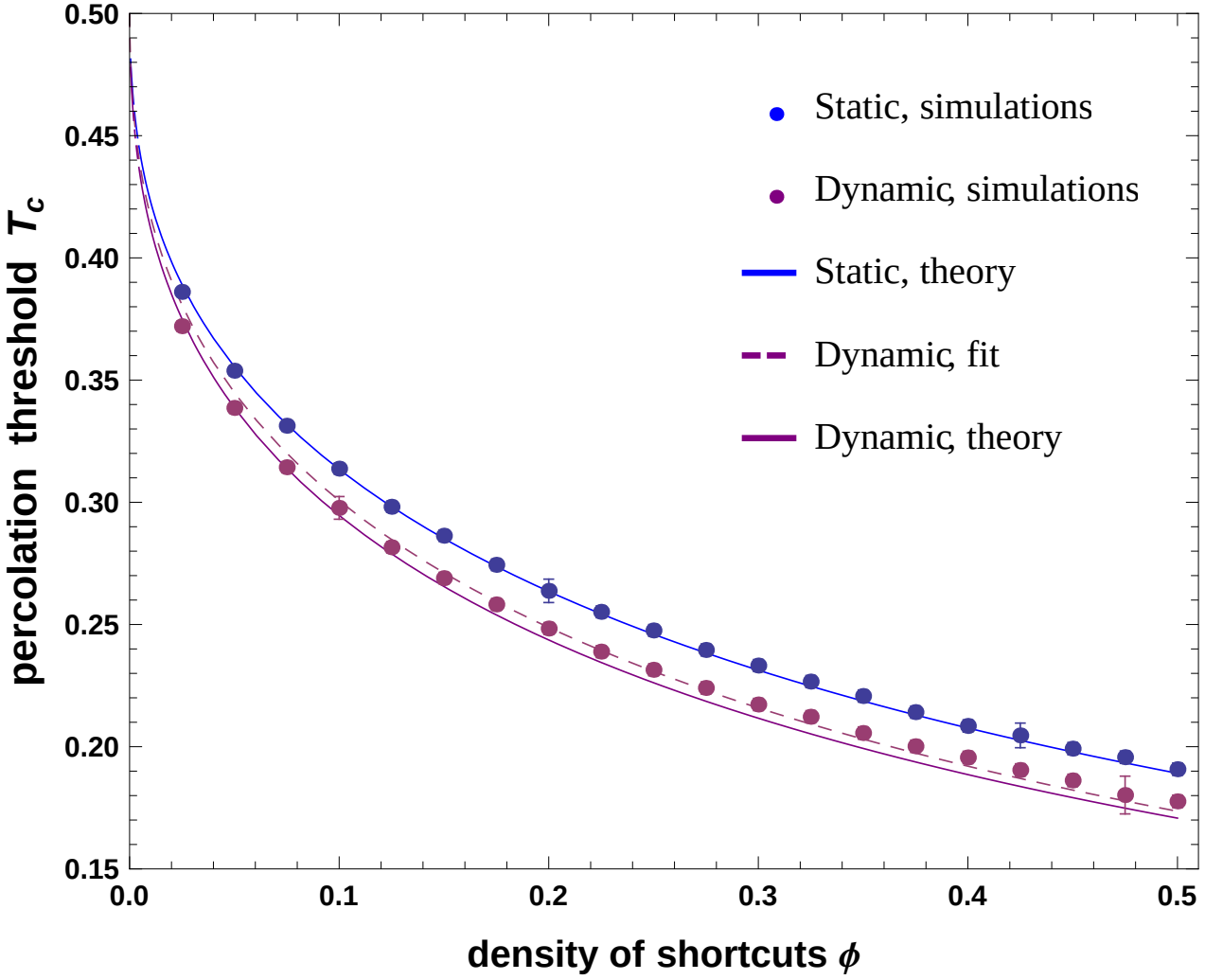
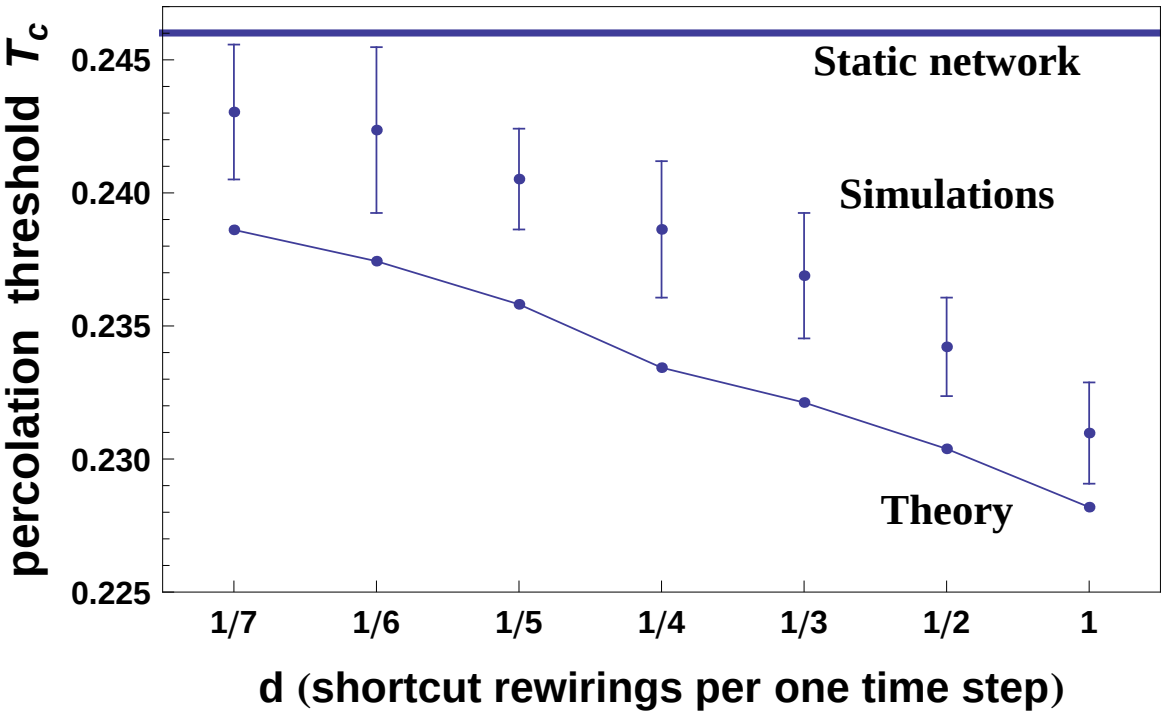


$T_c((1+x)\phi))$ fitted for $x = 0.207 \pm 0.014$

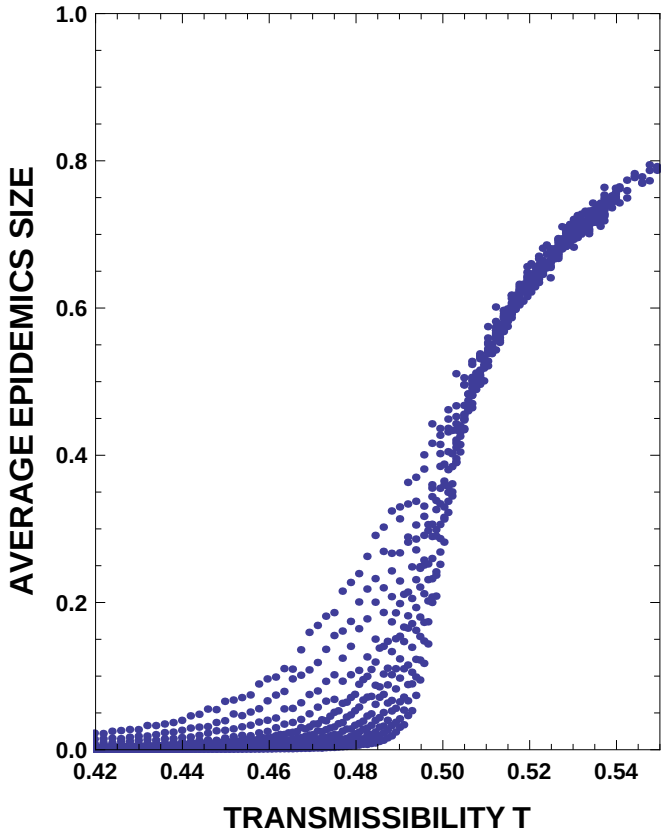
Result 2: network dynamics

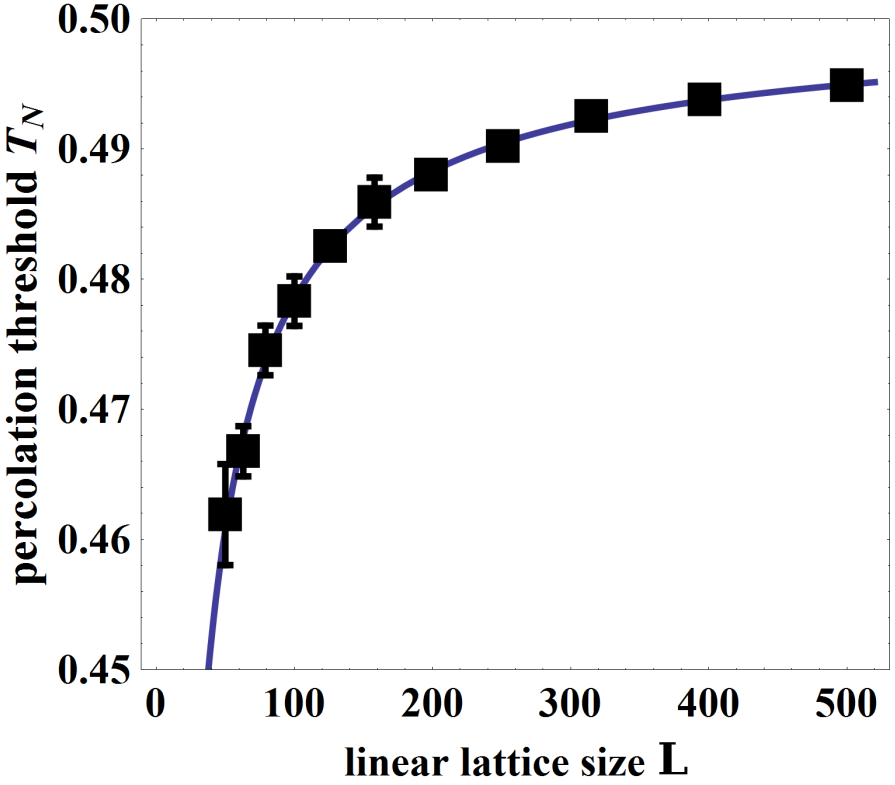
$$\phi = 0.25, l = 4$$



Two time scales: latency time and rewiring rate.

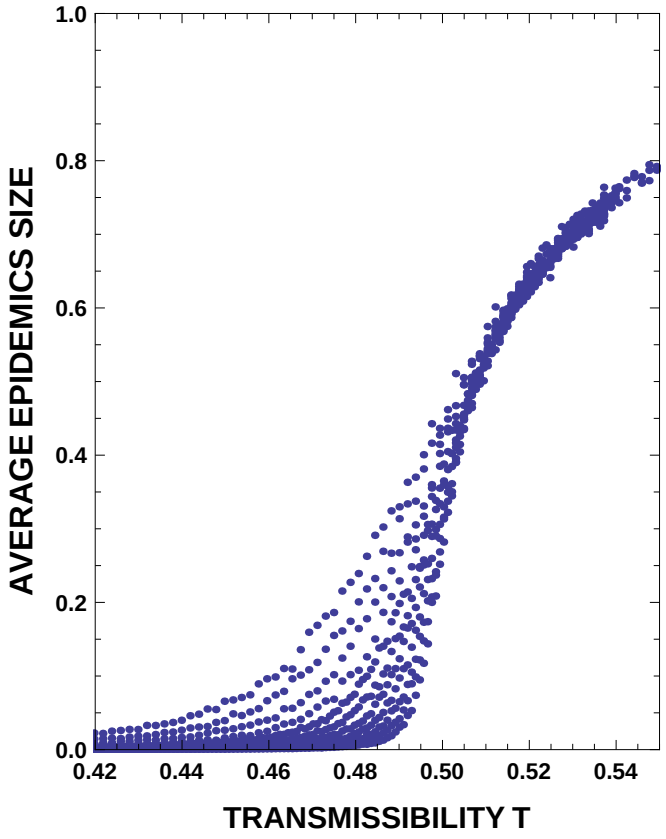
Result 3: scaling behaviour

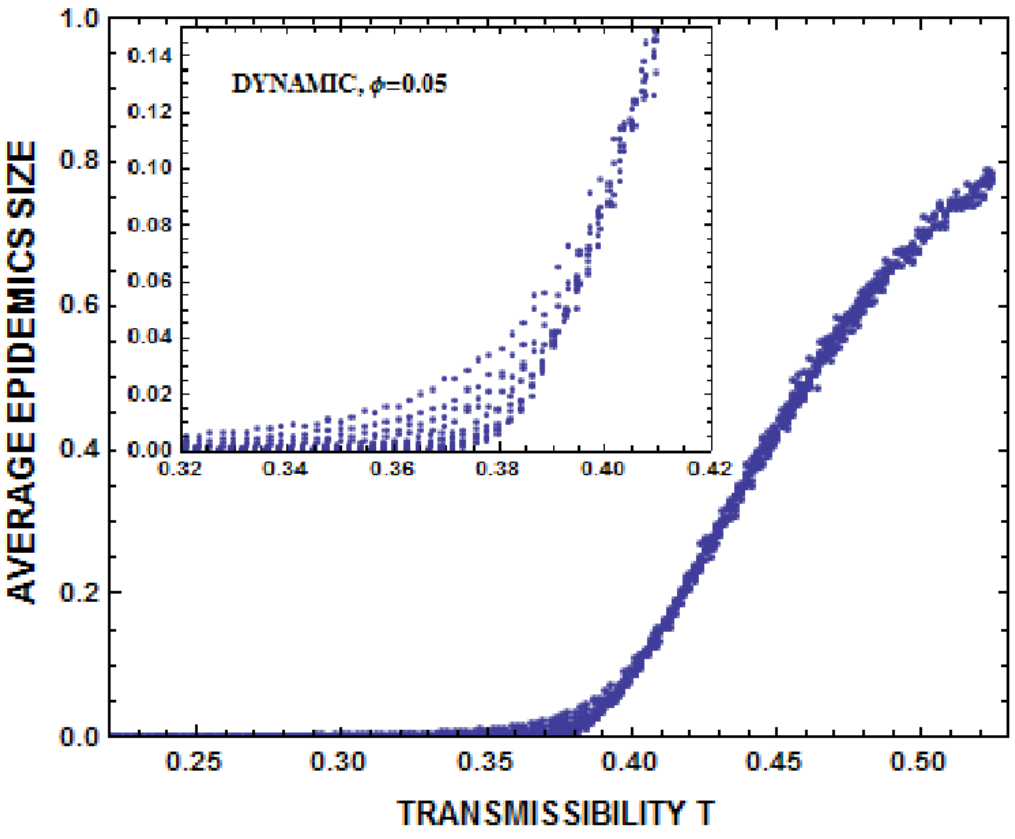




$$T_N = T_\infty - N^{-1/\nu} = T_\infty - L^{-2/\nu}$$

Result 3: scaling behaviour?





Summary

- Network dynamics significantly reduces percolation threshold for epidemic (simulations and theory).
- The ratio of network dynamics and disease latency must be used as a parameter.
- Other finite-size scaling effect for SWN than for regular one.

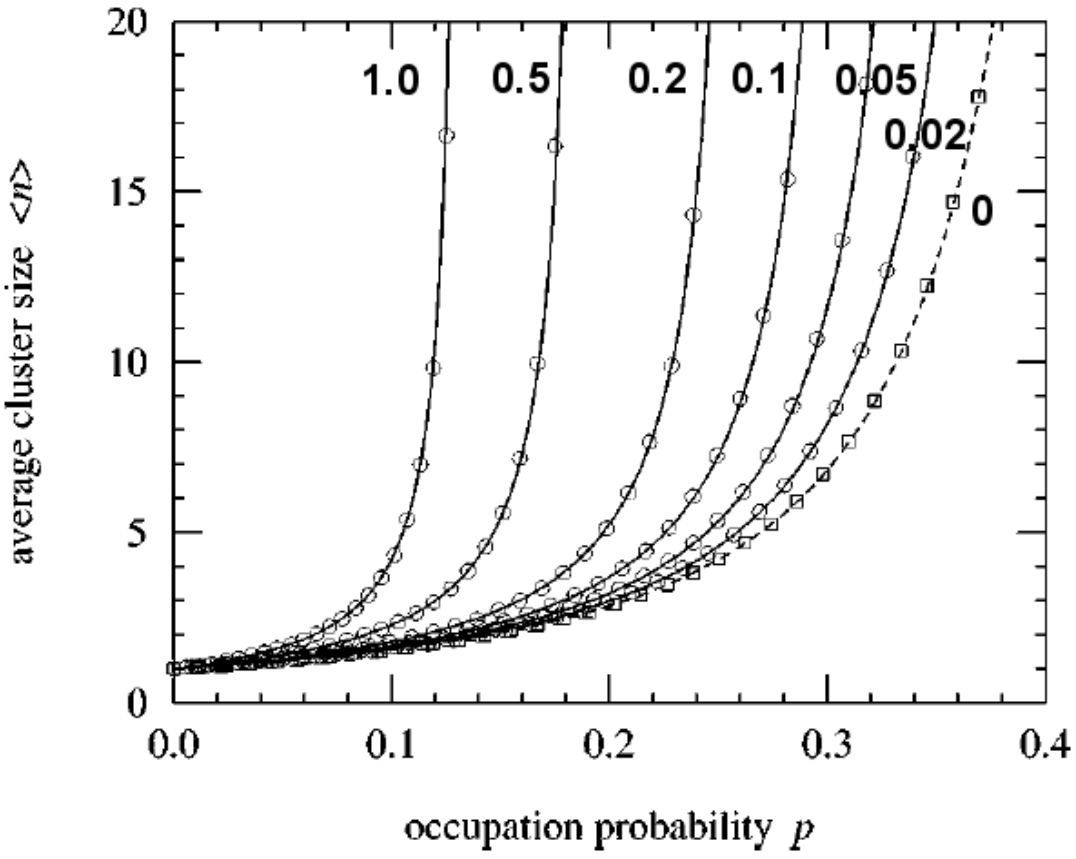
INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

FNP
Foundation for Polish Science

EUROPEAN UNION
EUROPEAN REGIONAL DEVELOPMENT FUND

Thank you

Checking and calibration



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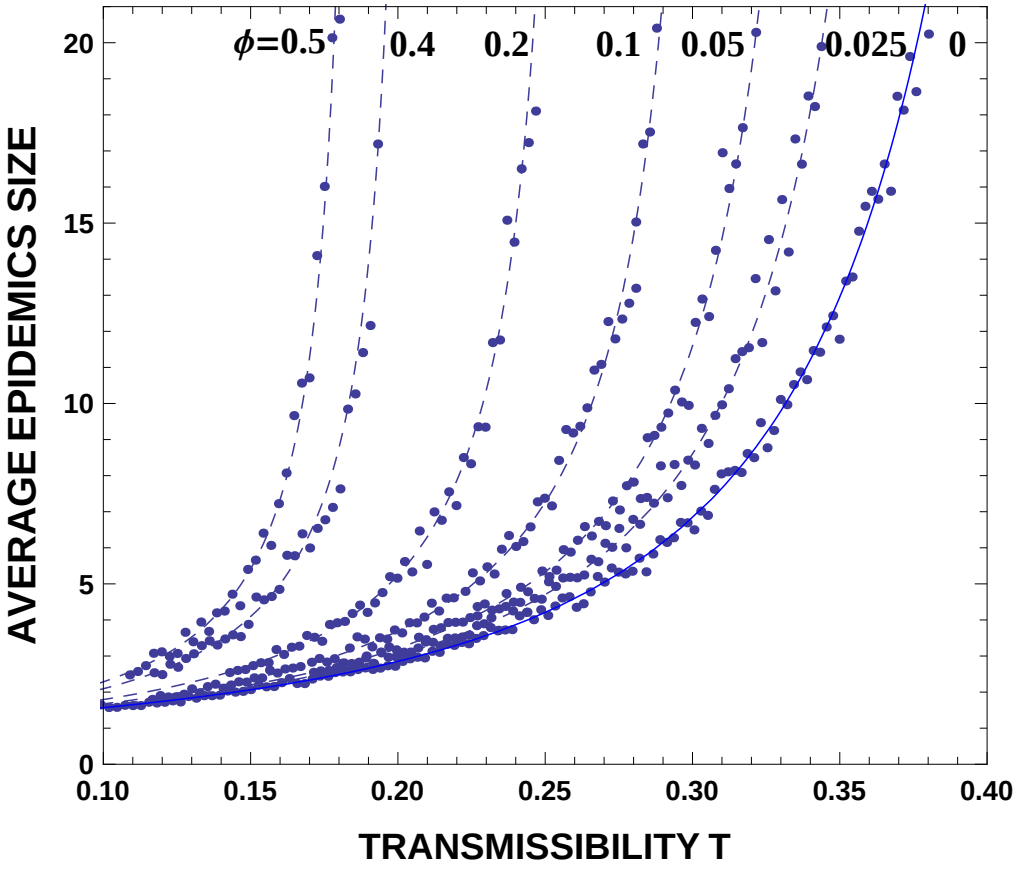

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