## Dynamo theory and its experimental validation

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#### Outline

- The idea of the self-exciting dynamo and elements of dynamo theory
- The Riga experiment
- The Karlsruhe experiment
- The Cadarache experiment
- Experiments under preparation
- What did we learn from dynamo experiments and what can we learn in future?

### Effects of the geomagnetic field known for long ...

- Evidence for a simple compass in Mexico about 3000 years ago
- Compass in China in the first century BC

- Petrus Peregrinus 1269: "Epistola de magnete"
- William Gilbert 1600: "De magnete" -- The Earth is a big loadstone!
- Gellibrand 1635: Westward drift of the declination
- Carl Friedrich Gauss 1836: "Allgemeine Theorie des Erdmagnetismus"
- David and Brunhes 1904/05: Reversals of the geomagnetic field

### In 1908 G. E. Hale discovered strong magnetic fields (~ 0.1 T) in sunspots

- There is also a (weaker) general solar magnetic field.

 The 2 x 11 years sunspot cycle coincides with the cycle of the general oscillatory solar magnetic field. The cosmic dynamo

Sir Joseph Larmor 1919

How could a rotating body such as the Sun become a magnet?

Magnetic field due to electric currents generated and maintained after the pattern of a self-exciting dynamo !

### The self-exciting dynamo

#### Werner von Siemens 1867





#### Charles Wheatstone 1867

Anianus Jedlick 1851/53 Sören Hjorth 1854 Samuel Alfred Varley 1866





$$L \frac{\mathrm{d}I}{\mathrm{d}t} + R I = \frac{\omega}{2\pi} \phi, \qquad \phi = L' I$$
$$L \frac{\mathrm{d}I}{\mathrm{d}t} + (R - \Omega^*) I = 0, \qquad \Omega^* = \frac{\omega}{2\pi} L'$$

$$\Rightarrow I(t) = I(0) \exp\left(-\frac{R - \Omega^*}{L}t\right)$$

 $\begin{array}{ll} R > \Omega^{*} & \mbox{decay} \\ R \leq \Omega^{*} & \mbox{dynamo} \\ & \omega_{\rm crit} = 2\pi \frac{R}{L'} \end{array}$ 

### Kinematic dynamo theory

The magnetic field B has to satisfy the induction equation

$$\partial_t B - \mathbf{\nabla} imes (U imes B) - \eta \mathbf{\nabla}^2 B = 0, \quad \mathbf{\nabla} \cdot B = 0$$
  
 $\eta = 1/\mu \sigma$ 

inside the fluid body

and proper initial and boundary conditions.



### (Anti-)Dynamo theorems

Concerning the geometry of the magnetic field

There is no dynamo with an axisymmetric magnetic field.

Cowling 1934, ...

Concerning the geometry of the fluid motion

There is no (spherical) dynamo with a completely toroidal fluid motion (if the conductivity is constant or depends on radius only).

Elsasser 1946, Bullard and Gellman 1954, ...

Various related theorems ...

### Examples of working dynamos

#### Herzenberg 1958



# 

Roberts 1972

T

#### Ponomarenko 1973



### Mean-field dynamo theory

Situations with complex behaviors of the fluid motion und magnetic field with respect to space and/or time.

Split velocity and magnetic fields into mean and "fluctuating" parts,

$$U = \overline{U} + u$$
,  $B = \overline{B} + b$ ,

with means fields defined by some averaging procedure that satisfies the Reynolds averaging rules.

Mean-field induction equation

$$\partial_t \overline{B} - \nabla \times (\overline{U} \times \overline{B} + \mathcal{E}) - \eta \nabla^2 \overline{B} = 0, \quad \nabla \cdot \overline{B} = 0,$$

contains the mean electromotive force

$$\mathcal{E}=\overline{u imes b}$$
 .

$$\partial_t \overline{B} - \nabla \times (\overline{U} \times \overline{B} + \mathcal{E}) - \eta \nabla^2 \overline{B} = 0, \quad \nabla \cdot \overline{B} = 0,$$

In the case of helical fluctuating motions

has a part parallel or antiparallel to the mean magnetic field -- $the \alpha-effect,$ 

$$\mathcal{E} = \alpha \overline{B} + \cdots$$

 $\mathcal{E} = \overline{u \times b}$ 

The  $\alpha$ -effect allows mean-field dynamo models which reflect essential features of real dynamos.

The dynamo is one of the basic phenomenae in the universe.

$$\partial_t \overline{B} - \nabla \times (\overline{U} \times \overline{B} + \mathcal{E}) - \eta \nabla^2 \overline{B} = 0, \quad \nabla \cdot \overline{B} = 0,$$

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Homogeneous isotropic non-mirrorsymmetric (helical) turbulence

$$\alpha = \int_0^\infty \int_\infty G(\boldsymbol{\xi}, \tau) \left\langle \boldsymbol{u}(\boldsymbol{x}, t) \cdot (\boldsymbol{\nabla} \times \boldsymbol{u}(\boldsymbol{x} - \boldsymbol{\xi}, t - \tau)) \right\rangle \mathrm{d}^3 \boldsymbol{\xi} \, \mathrm{d}\tau$$

### In the last six decades ....

- magnetic fields at A and other stars (~ 0.1 T)
- magnetic fields at most of the solar planets
- strong magnetic fields at some white dwarfs ( $\sim 10^{2}$  T)
- magnetic cycles at sun-like stars
- large-scale galactic and intergalactic magnetic fields ( $\sim 10^{-9}$  T)
- extremely strong magnetic fields at neutron stars (~ 10^11 T)

### The full dynamo problem E.g., geodynamo

$$\partial_t U + (U \cdot \nabla)U = -\frac{1}{\varrho} \nabla P + \nu \nabla^2 U + 2\Omega \times U + \frac{1}{\mu \varrho} (\nabla \times B) \times B - \alpha_T g \theta$$
  
 $\partial_t B - \nabla \times (U \times B) - \eta \nabla^2 B = 0$   
 $\partial_t \theta + U \cdot \nabla \theta - \kappa \Delta \theta = -U \cdot \nabla T_0$   
 $\nabla \cdot U = \nabla \cdot B = 0$ 

Dimensionless parameters  $E = \nu/\Omega D^2$  Ekman number  $Ra = \alpha_T g \Delta T D / \nu \Omega$  Rayleigh number  $Pr = \nu / \kappa$  Prandtl number  $Pm = \nu / \eta$  magnetic Prandtl number

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Earth's core

$$E = \nu / \Omega D^2 = O(10^{-15})$$
  

$$Ra = \alpha_T g \Delta T D / \nu \Omega = O(10^2)$$
  

$$Pr = \nu / \kappa = O(1)$$
  

$$Pm = \nu / \eta = O(10^{-6})$$

Note that *Re = Rm/Pm* 

#### Direct numerical simulations

Parameters far away from realistic ones for the Earth's core

Glatzmaier and Roberts 1995



considering the back-reaction of the magnetic field on the rotation rate ("eddy current braking")



$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = \frac{\omega}{2\pi}\phi, \qquad \phi = L'I$$
$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\Theta\omega^2 + LI^2) = D\omega - \frac{\Theta\omega^2}{\tau^*} - I^2R$$
$$I = 0, \ \omega \text{ steady: } \omega = \omega_0 = \frac{D\tau^*}{\Theta}$$

Back-reaction of the magnetic field on the rotation rate

Steady case





considering the back-reaction of the magnetic field on the rotation rate and an imposed magnetic field



$$L\frac{\mathrm{d}I}{\mathrm{d}t} + RI = \frac{\omega}{2\pi}\phi, \qquad \phi = \phi_0 + L'I$$
$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}(\Theta\omega^2 + LI^2) = D\omega - \frac{\Theta\omega^2}{\tau^*} - I^2R$$
$$\omega_0 = \frac{D\tau^*}{\Theta}$$

Back-reaction of the magnetic field plus imposed magnetic field

#### Steady case



### Dynamos under laboratory

 $R_{\rm m} = \frac{U L}{\eta}$ 

Dynamo requires

 $R_{\rm m} \ge R_{\rm m\,crit} = O(1)$ 

E.g., 
$$U=1\,\mathrm{m/s}$$
 and  $L=1\,\mathrm{m}$ 

Earth's core $\eta = 3 \, \mathrm{m}^2/\mathrm{s}$  $R_{\mathrm{m}} = 0.33$ Mercury $\eta = 0.8 \, \mathrm{m}^2/\mathrm{s}$  $R_{\mathrm{m}} = 1.25$ Liquid sodium $\eta = 0.08 \, \mathrm{m}^2/\mathrm{s}$  $R_{\mathrm{m}} = 12.5$ 

No Bonsai version of a cosmic dynamo !

### Self-excitation of magnetic fields in fast-breeder reactors?

The danger of self-excitation of magnetic fields in large liquid-metal circuits of reactors has been pointed out in a memorandum of Max Steenbeck 1971 to the Soviet Academy of Sciences

→ Meeting in Obninsk near Moscow 1974

This possibility has been independently considered by Bevir 1973 and Pierson 1975

Measurements at the Belojarsk BN-600 reactor by Kirko 1985 ...

Investigations related to the French Super Phoenix by Plunian et al. 1995 / 99

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Steenbeck 1974 in a letter to the president of the Academy of Sciences of the GDR and several Soviet scientists:

Dynamo experiment with about 10 m<sup>3</sup> Na and volumetric flow rates of more than 10 m<sup>3</sup>/s

### The need for experimental investigations

In magnetohydrodynamics one should not believe the product of a long and complicated piece of mathematics if unsupported by observation.

Fermi (reported by Roberts 1993)

Laboratory experiments with dynamos are more than demonstrating or checking an important principle.

Natural dynamos

with complex interactions of motion and magnetic field are hardly accessible to direct numerical simulations, e.g., because of the smallness of realistic values of Pm and the complex turbulence phenomenae. Experiments should improve our knowledge in this field.

The interaction of theory and experiment pushes the understanding of dynamo processes forward.

Lehnert 1958

Cylindrical vessel with 58 I sodium

Differential rotation due to a rotating plate

Poloidal 

toroidal magnetic field



#### Herzenberg 1958

### Lowes and Wilkinson 1963/68



## Lowes and Wilkinson 1963/68



Riga **α**-box

Steenbeck, Kirko, Gailitis, Klawinia, Krause, Laumanis, Lielausis 1967





#### Riga **α**-box



#### The motivation: low excitation threshold

→ Ponomarenko dynamo Ponomarenko 1973

$$B = \Re(\hat{B}(r) \exp(i(m\varphi + kz) + \lambda t))$$

$$Rm_{\perp} = \frac{|\omega|a^2}{\eta}, \quad Rm_{\parallel} = \frac{|u|a}{\eta}$$
$$Rm = \sqrt{Rm_{\perp}^2 + Rm_{\parallel}^2}$$

Marginal mode ( $\lambda = 0$ ) with lowest Rm

$$Rm = 17.722$$
,  $Rm_{\perp}/Rm_{\parallel} = 0.7625$   
 $m = 1$ ,  $k/a = -0.3875$ 



The 1987 experiment Gailitis, Lielausis, ...







*Fig. 1.* Left hand side: the main parts of the Riga dynamo facility: 1 - propeller, 2 - helical flow region, 3 - back-flow region, 4 - sodium at rest, 5 - thermal insulation, F - Position of the flux-gate sensor and the induction coil, H1...H6 - positions of six aligned Hall sensors, H7/H8 - two Hall sensors at different azimuths. Right hand side: computed magnetic field energy iso-surface for the kinematic dynamo model.



Fig. 5 The Riga dynamo experiment and its eigenfield. (a) Sketch of the facility. M - Motors. B - Belts. D - Central dynamo module. T- Sodium tank. (b) Sketch of the central module. 1 - Guiding blades. 2 - Propeller. 3 - Helical flow region without any flow-guides, flow rotation is maintained by inertia only. 4 - Back-flow region. 5 - Sodium at rest. 6 - Guiding blades. 7 - Flow bending region. (c) Simulated magnetic eigenfield. The gray scale indicates the vertical components of the field.
### November 10/11, 1999



*Fig. 5.* Magnetic field signal measured at 2150 rpm at the flux gate sensor F and fitting curve (left). Decomposition of the fitting curve into two curves with different frequencies (right). The growth rate of the 1.326 Hz signal was  $p = +0.03s^{-1}$ .

### Riga dynamo experiment

# The Karlsruhe dynamo experiment The motivation



Earth's core Busse 1970, ...

Experiment

Müller, Stieglitz, ... 1999



An experiment as later carried out in Karlsruhe has been prosed by Busse 1975

#### Gailitis' idea 1967





R = 0.85 m a = 0.21 m





Karlsruhe dynamo experimentRoberts dynamoG. O. Roberts 1972Fluid velocity, e.g.,
$$u_x = -u_{\perp} \frac{\pi}{2} \sin(\frac{\pi}{a} x) \cos(\frac{\pi}{a} y)$$
 $u_y = u_{\perp} \frac{\pi}{2} \cos(\frac{\pi}{a} x) \sin(\frac{\pi}{a} y)$  $u_z = -u_{\parallel} \frac{\pi}{2} \sin(\frac{\pi}{a} x) \sin(\frac{\pi}{a} y)$ Non-decaying magnetic fields $B = \Re(\hat{B}(x, y) \exp(ikz + pt))$ if $Rm_{\perp} Rm_{\parallel} \phi(Rm_{\perp}, k) \ge \frac{16}{\pi^2} ak$  $Rm_{\perp} = u_{\perp}a/2\eta$ ,  $Rm_{\parallel} = u_{\parallel}a/\eta$ .

# Roberts dynamo

Non-decaying magnetic fields

$$B = \Re(\hat{B}(x, y) \exp(ikz + pt))$$

*if* 

$$Rm_{\perp} Rm_{\parallel} \phi(Rm_{\perp}, k) \ge \frac{16}{\pi^2} a k$$
$$Rm_{\perp} = u_{\perp} a/2\eta, \quad Rm_{\parallel} = u_{\parallel} a/\eta.$$

Most easily excitable *B* modes contain parts independent of *x* and *y*.

(limit of small k)



Roberts dynamo allows mean-field description:

$$\partial_t \overline{B} - \nabla \times (\overline{U} \times \overline{B} + \mathcal{E}) - \eta \nabla^2 \overline{B} = 0, \quad \nabla \cdot \overline{B} = 0,$$

with mean fields defined by averaging over x and y.

$$\mathcal{E} = \alpha \cdot \overline{B} + \cdots$$
$$\alpha = \begin{pmatrix} \alpha_{\perp} & 0 & 0 \\ 0 & \alpha_{\perp} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

# anisotropic alpha-effect

(limit of small k)

$$\alpha_{\perp} = \frac{\pi^2 \eta}{16 a} Rm_{\perp} Rm_{\parallel} \phi(Rm_{\perp})$$

### Karlsruhe experiment

Two branches in the theory of the experiment

Direct numerical simulations

Busse, Tilgner 1996...2004

Mean-field theory

Apstein, Brandenburg, Fuchs, Schüler, Rädler, Rheinhardt 1996...2002



m=0



m=1

m≠1

m=1







# Self-excitation if

$$C_{\alpha} = \frac{\alpha_{\perp} R}{\eta} = F(V_C, V_H)$$

### exceeds a critical value.

Vc volumetric flow rate through central channel Vн ...

through helical channel of a spin generator

#### Isolines of $C\alpha$



### Spingenerator flow





### Isolines of $C\alpha$





Fig. 8. A bifurcation diagram (solid and dashed lines, used as in Fig. 5) for  $B^{\text{exp}}$  with  $C^* = 9.189$ ,  $V_{\rm C} = 112.5 \,\mathrm{m}^3/\mathrm{h}, B_c = 0.285 \,\mathrm{G}$  and  $\epsilon' = 1.47 \times 10^{-7}$  that reproduces in some approximation a set of measured data (squares).



# Disc dynamo

Back-reaction of the magnetic field plus imposed magnetic field



The saturation of the dynamo is accompanied by developing a piston-like flow profile.

The temperature dependence of the excitation condition is mainly due to the temperature dependence of the electric conductivity.

The onset of dynamo action is shifted to lower mean axial flow rates if this flow varies periodically in time with periods of the order of the magnetic diffusion time.

# The Cadarache (VKS) dynamo experiment



### Pinton et al.



- Grenoble
- Paris
- Madison
- Maryland
- New-Mexico
- Perm

Magnetostrophic motions

 Grenoble --- in a differentially rotating spherical shell under the influence of an imposed magnetic field

- Paris
- Madison

Nataf, Schaeffer, ...

- Maryland
- New-Mexico
- Perm



- Grenoble
- Paris ------
- Madison
- Maryland
- New-Mexico
- Perm



## Leorat et al.

- Grenoble
- Paris
- Madison -------
- Maryland
- New-Mexico
- Perm



Forest, ...



- Grenoble
- Paris
- Madison
- Maryland ---
- New-Mexico
- Perm



Lathrop, ...

heat flux sensors

-T/C and 3D hall probe

60 cm

stationary hall probes

coolant in

## First Maryland experiment

### Lathrop and coworkers 2000





# Lathrop and coworkers 2000

# Maryland



# Maryland

### 3 m diameter sphere



- Grenoble
- Madison
- Maryland
- New-Mexico ------
- Perm



*aw* dynamo similar to such in astrophysical objects Colgate, ...

- Grenoble
- Madison
- Maryland
- New-Mexico
- Perm ------



## non-stationary dynamo

Frick et al.

# The Perm dynamo experiment

(under preparation)

Proposed by Denisov, Noskov, Sokoloff, Frick, Khripchenko 1999

Background: Ponomarenko 1973



non-stationary dynamo

divertor



# The Perm dynamo experiment



Large radius 0.40 m small radius 0.12 m

115 kg Na 50 rps 140 m/s braking time 0.1 s

Dynamo action if

$$\begin{array}{ll} \mathsf{Rm} > 40 \\ \longrightarrow & \mathsf{Re} > 4 \cdot 10^6 & \longrightarrow \textbf{turbulence} \end{array}$$
## Water experiments



Large radii 0.103 m and 0.154 m

Helical features in large and small scales





## The gallium experiment



Large radius of the torus 88 mm small radius 23 mm

Noskov, Denisov, Frick, Khripchenko, Sokoloff and Stepanov Magnetic field rotation in the screw gallium flow

EPJ 2004

## What did we learn from the experiments?

- The dynamo is a robust phenomenon
- The kinematic dynamo theory describes the onset of dynamos well
- Mean-field theory of the Karlsruhe dynamo experiment works well
- In both the Riga and the Karlsruhe cases the dynamos saturate due to deformation of the flow profiles by the magnetic field. Turbulence plays a subordinate role.
- In other cases (in particular Cadarache und Madison) fluid motions on small scales seem to be important.

## Which problems could be addressed by future experiments ?

- The role of smaller scales of motion in less constrained flows
- Saturation mechanisms
- Dynamos due to magneto-rotational instability
- Reversals of magnetic fields