Power Spectrum Analysis: The foreground influence.

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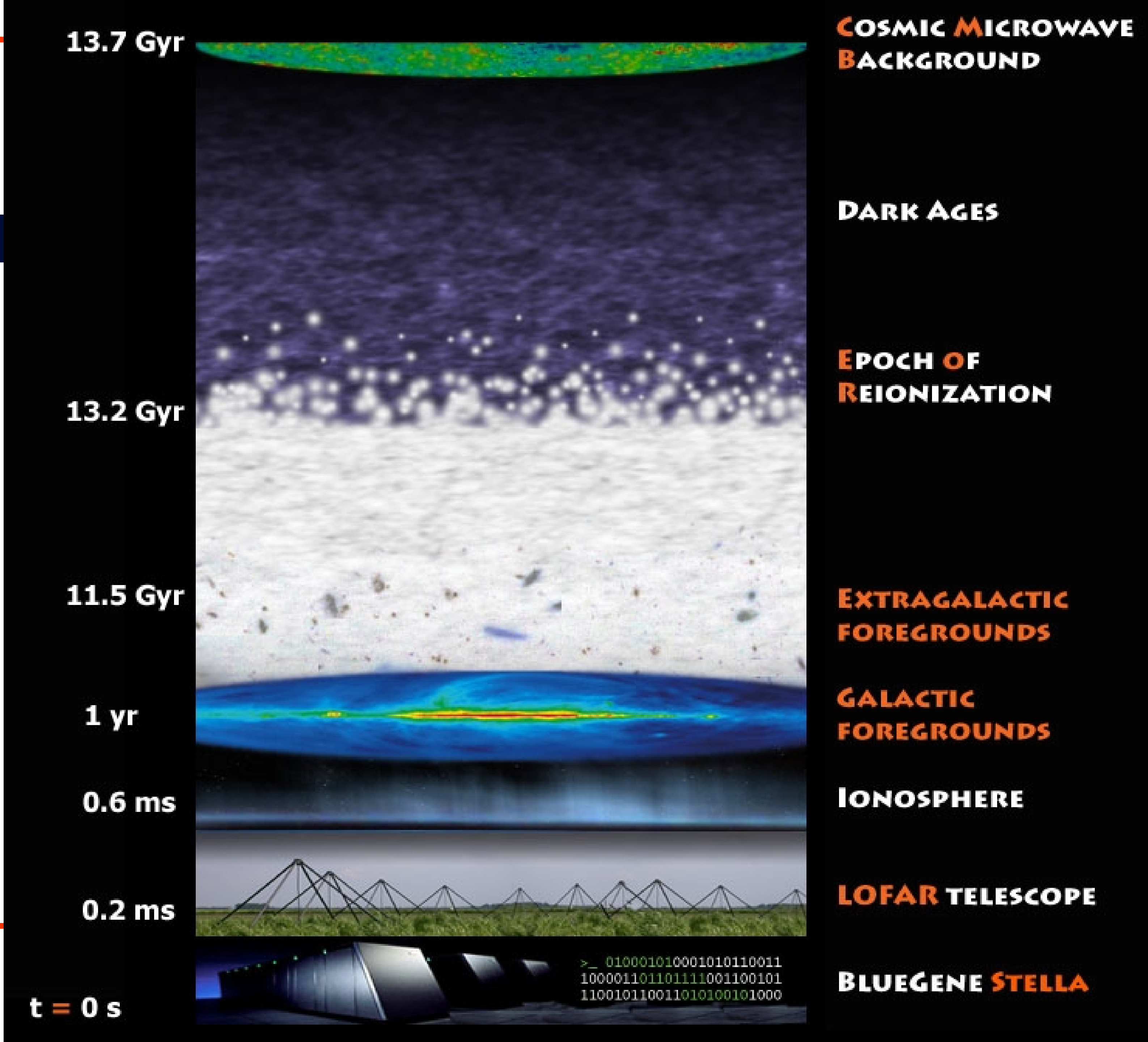
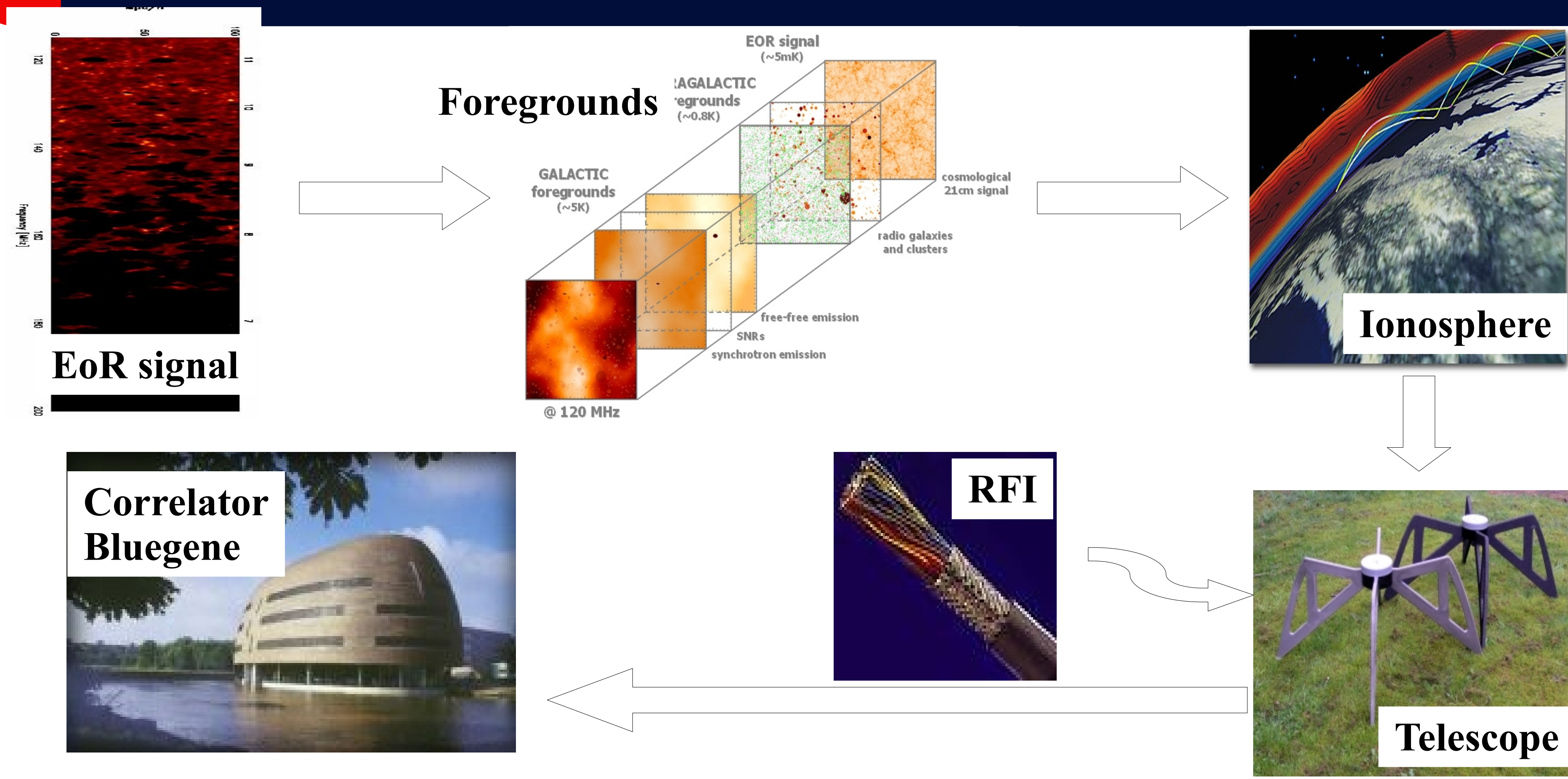
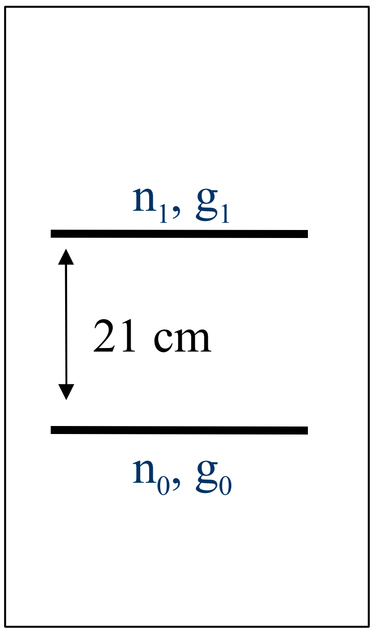

13.7 Gyr
Cosmic Microwave Background
Dark Ages
Epoch of Reionization
13.2 Gyr
11.5 Gyr
Extragalactic foregrounds
1 yr
Galactic foregrounds
0.6 ms
Ionosphere
0.2 ms
LOFAR telescope
>_ 010001010001010110011
100001101101111001100101
110010110011010100101000
BlueGene Stella
t = 0 s

The Observation



The 21 cm transition



- The 21 cm hyperfine transition is a forbidden transition between the two $1^2s_{1/2}$ ground level states of hydrogen.
- The relative population of the two states is given, $n_1/n_0=g_1/g_0 \exp(-T_*/T_s)$ with T_s (the spin temp.) and $T_*=0.068$ k
- The value of the T_s is given by:

$$T_S = \frac{T_{CMB} + y_\alpha T_k + y_c T_k}{1 + y_\alpha + y_c}$$

Field 1958
Madau et al 98
Ciardi&Madau 2003

The brightness temperature: The measured quantity

- The quantity that is measured with radio telescopes along a given line of sight and is given by:

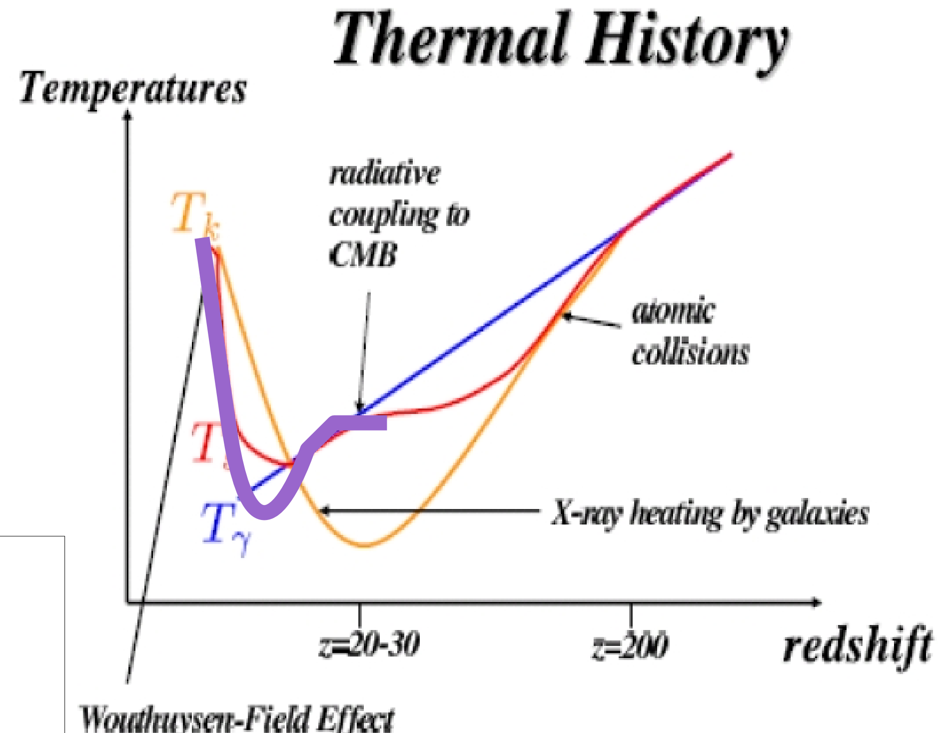
$$\delta T_b \approx 28\mathrm{mK}\,(1+\delta)x_{HI}\frac{T_s - T_{CMB}}{T_s}\frac{\Omega_b h^2}{0.02}\left[\frac{0.24}{\Omega_m}\left(\frac{1+z}{10}\right)\right]^{\frac{1}{2}}$$

- The Interpretation might be very complicated

The Global evolution of the Spin Temperature

At z~10 T_s is tightly coupled to T_{CMB}. In order to observe the 21 cm radiation decoupling must occur.

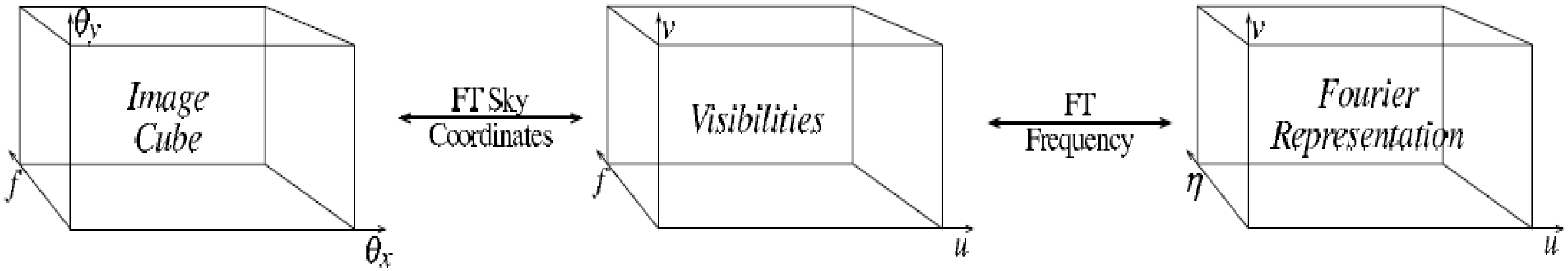
Heating much above the CMB temp. and decoupling do not necessarily occur together.



Loeb & Zaldarriaga 04

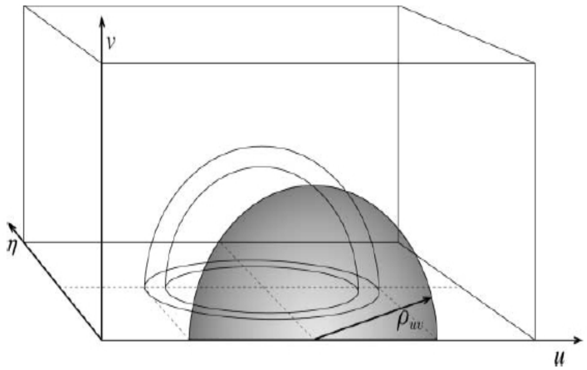
Noise Issues

Sensitivity & S/N



Morales & Hewitt 2004

Sensitivity & S/N



Not relevant for FG fitting

# of beams

# of cells in an annulus

System Temp.

$$\left[\overline{C^{N}_{ij}(|\mathbf{u}|)}\right]^{\frac{1}{2}}_{rms} \approx 2N_{beam}^{-\frac{1}{2}} N_{cell}^{-\frac{1}{4}} \left(\frac{2k_B T_{sys}}{\epsilon d A d\eta}\right)\left(\frac{1}{B\bar{n}(|\mathbf{u}|)t}\right)^{\frac{1}{2}}$$

B =Bandwidth
dη=inv. Bandwidth
n= mean # of baselines

efficiency

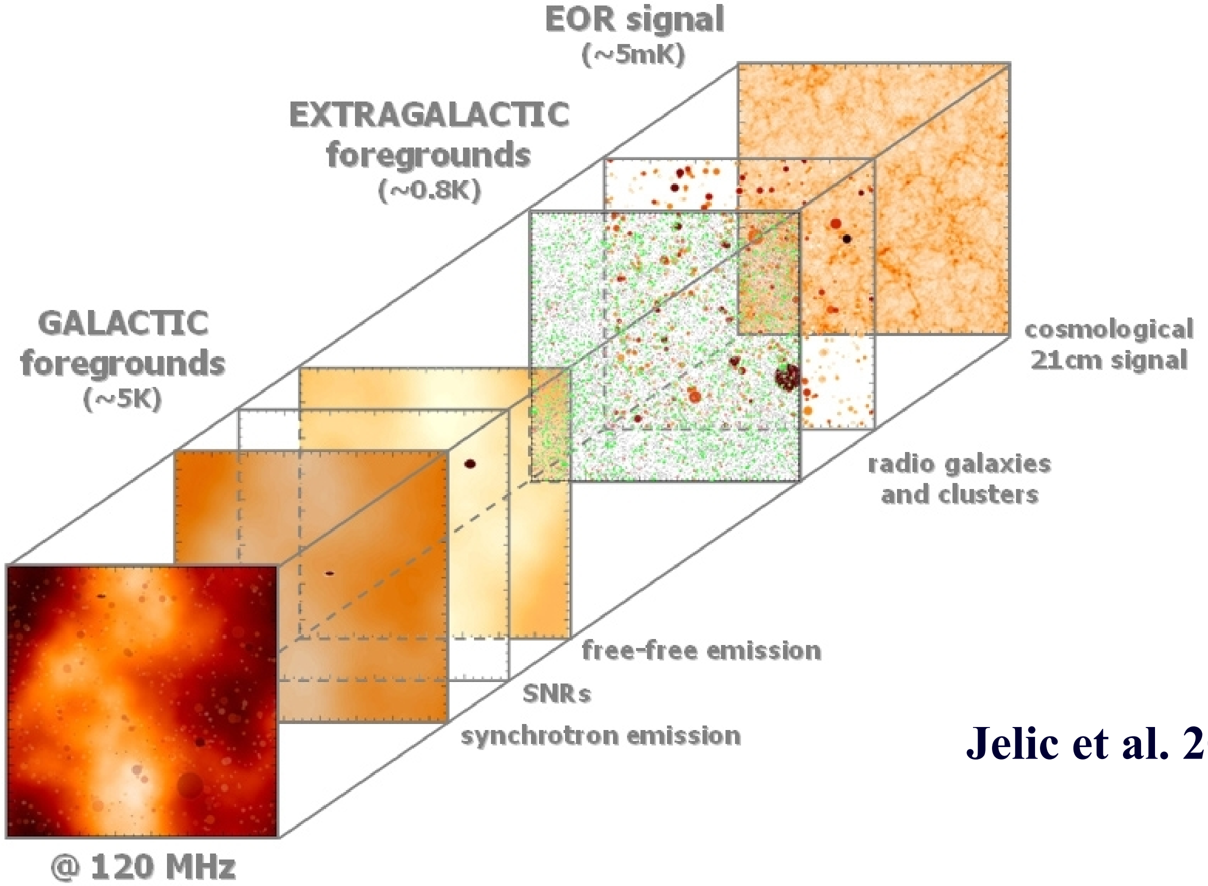
Station area

Integration time

Morales& Hewitt 2004

Extraction

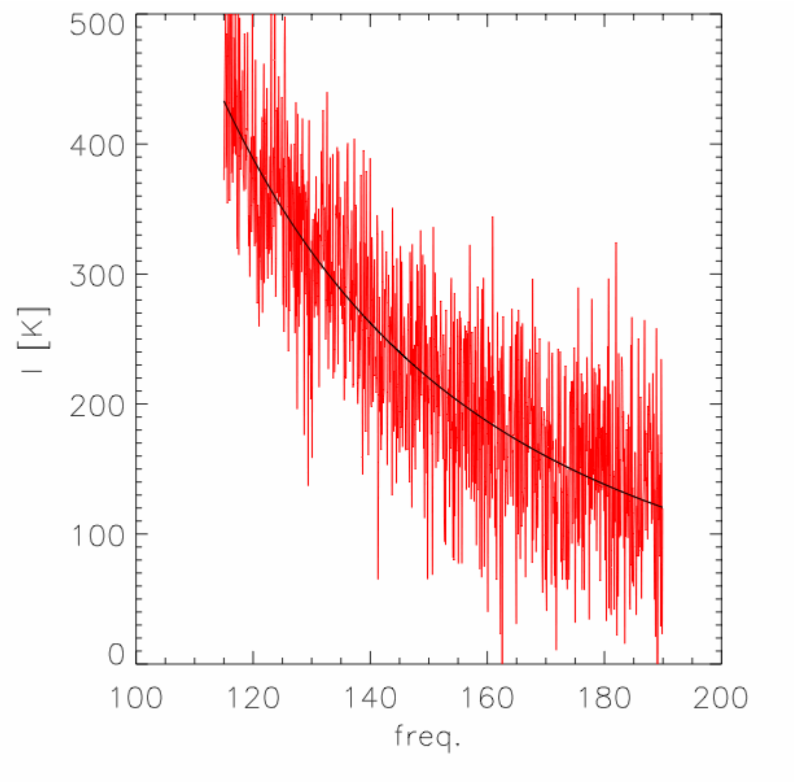
The signal + Foregrounds

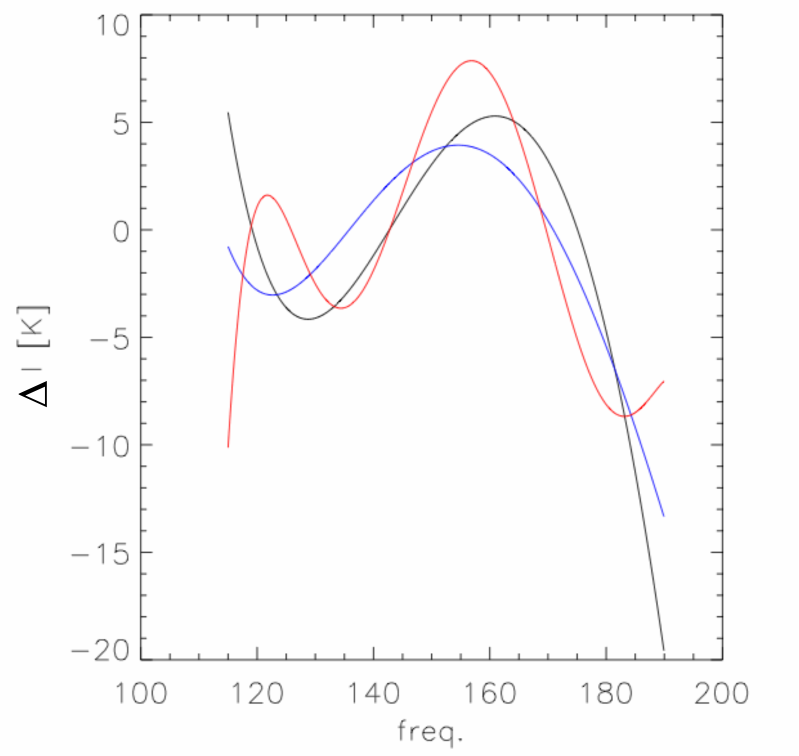


Jelic et al. 2009

Under- and Over-fitting

Real Space

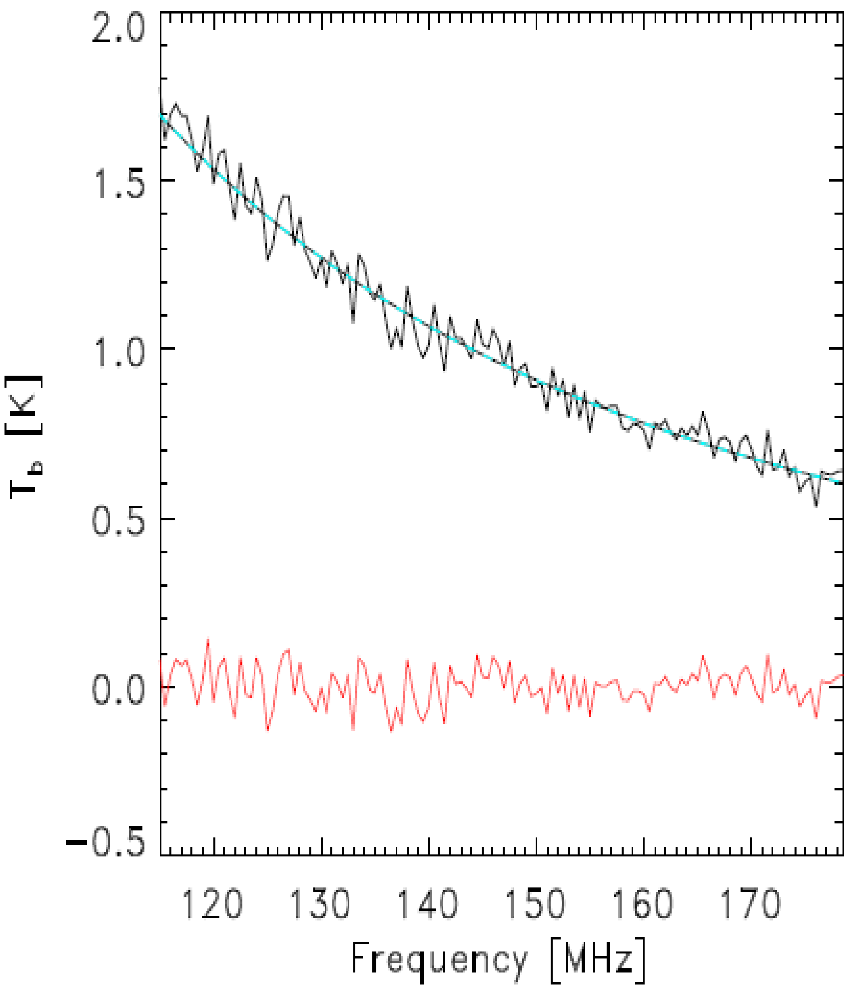


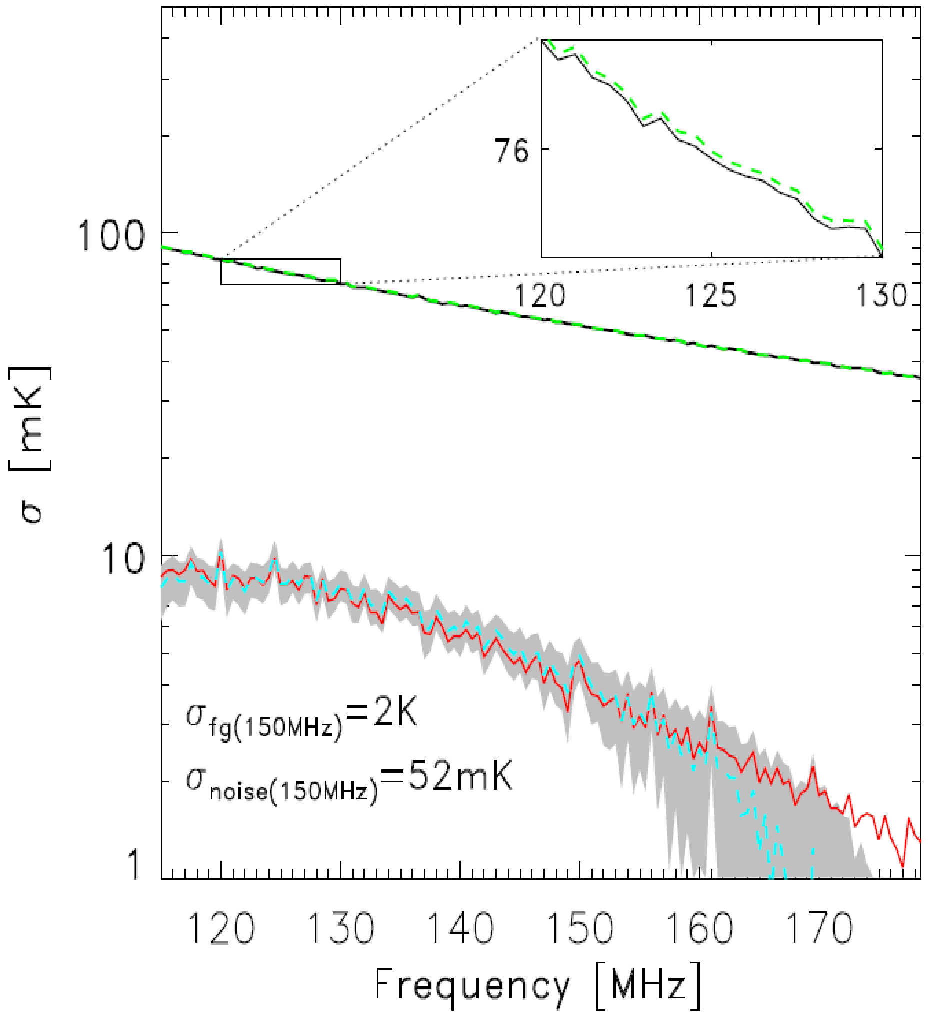


Wish list for a foreground fitting algorithm

- Algorithm should be accurate to better than 1/1000 per 1MHz.
- Should be Unbiased.
- Avoid under-fitting or over-fitting.
- Make minimal assumptions about the functional form of the foregrounds; i.e., exploit their smoothness directly.
- Speed (less important since fitting is done once)

Extraction with Polynomials





Statistical approach

- Model data points (x_i, y_i) by:

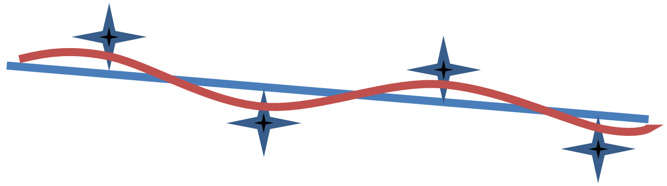
$$y_i = f(x_i) + \varepsilon_i, \;\; i = 1, \ldots, n$$

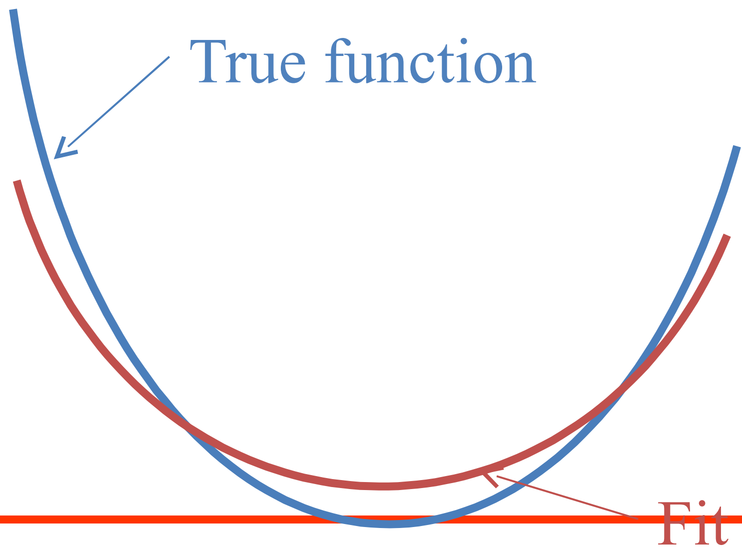
- Then we wish to solve the following problem:

$$\min_f \left\{ \underbrace{\sum_{i=1}^{n} \rho_i(y_i - f(x_i))}_{\text{"Least squares"}} + \underbrace{\lambda R[f]}_{\text{Roughness penalty}} \right\}$$

Choosing a roughness penalty *R*[*f*]

- Require a roughness penalty that stops the curve wiggling towards individual data points, but avoids the problem of attrition.
- 'Smoothing splines' use integrated curvature as the roughness penalty, but in Wp smoothing the integrated *change* of curvature is used instead.





Wp smoothing

- An approximation to the change of curvature, f'''/f'', blows up at the inflection points $f''=0$.
- $R[f]$ measures the change of curvature 'apart from the inflection points', w_i
- Perform the minimization with the position of the inflection points (and s_f) fixed.

$$R[f] = \int_{x_1}^{x_n} h_f'(t)\mathrm{d}t$$

$$f''(x) = p_{\mathbf{w}}(x)e^{h_f(x)}$$

$$\begin{aligned} p_{\mathbf{w}}(x) &= s_f(x-w_1)(x-w_2) \\ &\quad \times \ldots (x-w_{n_w}) \end{aligned}$$

Wp smoothing

- Mächler (1993,1995), who proposed the method, showed that the variational problem leads to the following differential equation:

$$h_f'' = p_{\mathbf{w}} e^{h_f} \left[-\frac{1}{2\lambda} \sum_{i=1}^{n} (x - x_i)_+ \psi_i(y_i - f(x_i)) \right]$$

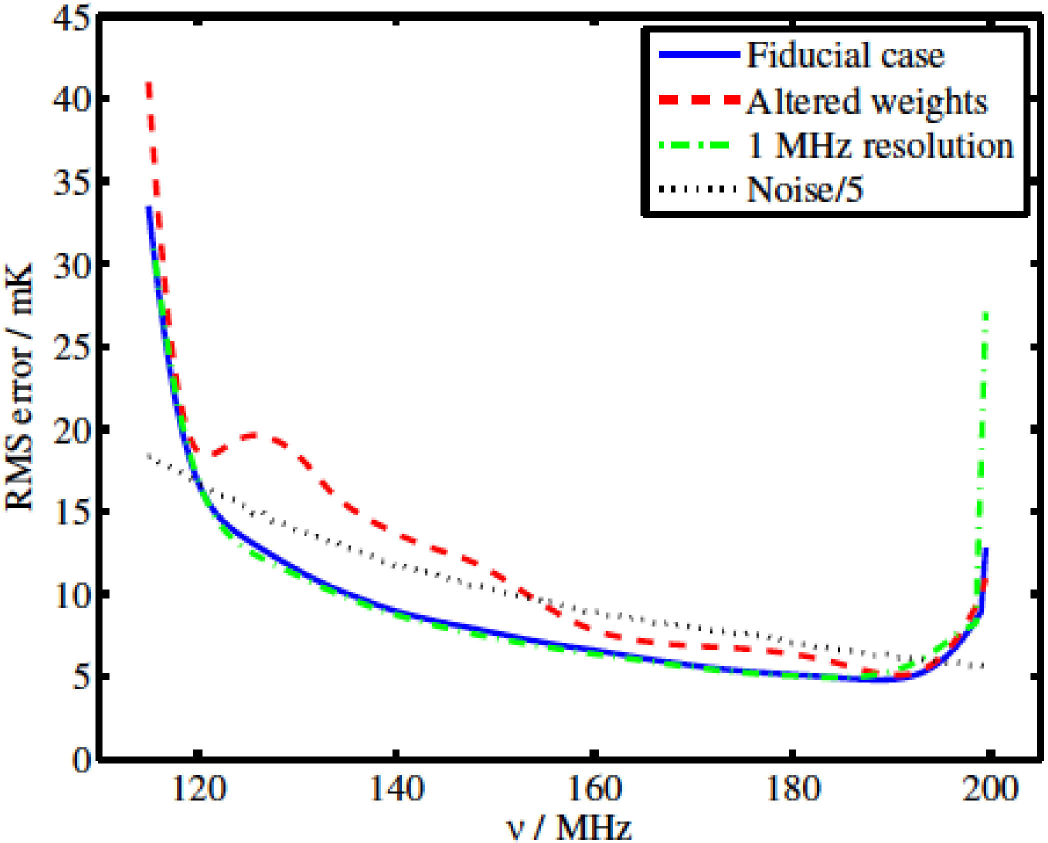
where a_+ =max(0,a), $\psi_i(\delta) = \frac{\mathrm{d}}{\mathrm{d}\delta}\rho_i(\delta)$, and the boundary conditions are

$$h_f'(x_1) = h_f'(x_n) = \sum_i \psi_i(y_i - f(x_i)) = \sum_i x_i \psi_i(y_i - f(x_i)) = 0$$

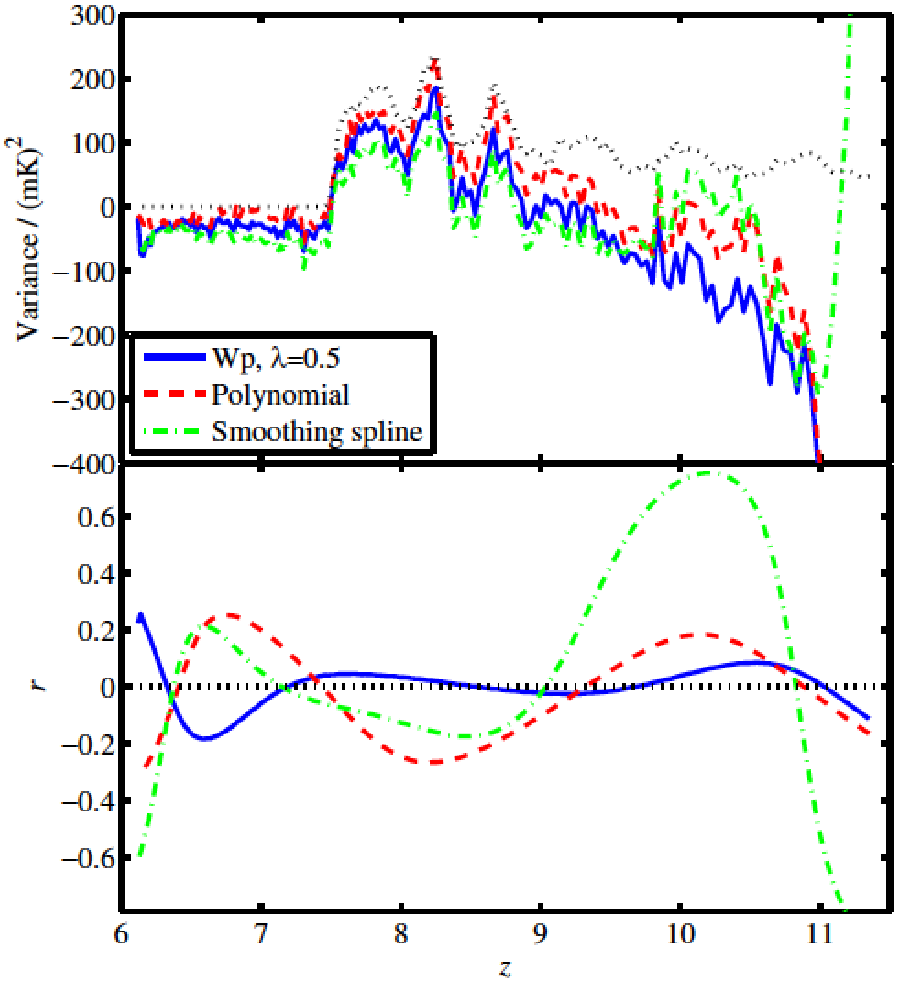
Implementation

- In general we need a method to find the number of inflection points, and need to perform a further minimization over their position.
- For the foreground fitting we find that it works well to have no inflection points (this would be the case anyway for a sum of negative-index power laws).
- The differential equation and the boundary conditions are in a nonstandard form:
  - Can rewrite as a system of $5n$-4 coupled first-order equations and use a standard BVP solver.
  - Alternatively, convert to a finite difference equation and perform a multidimensional function minimization (seems better so far).
- Either approach requires a reasonable initial guess for the solution; we fit a power law since this has no inflection points.

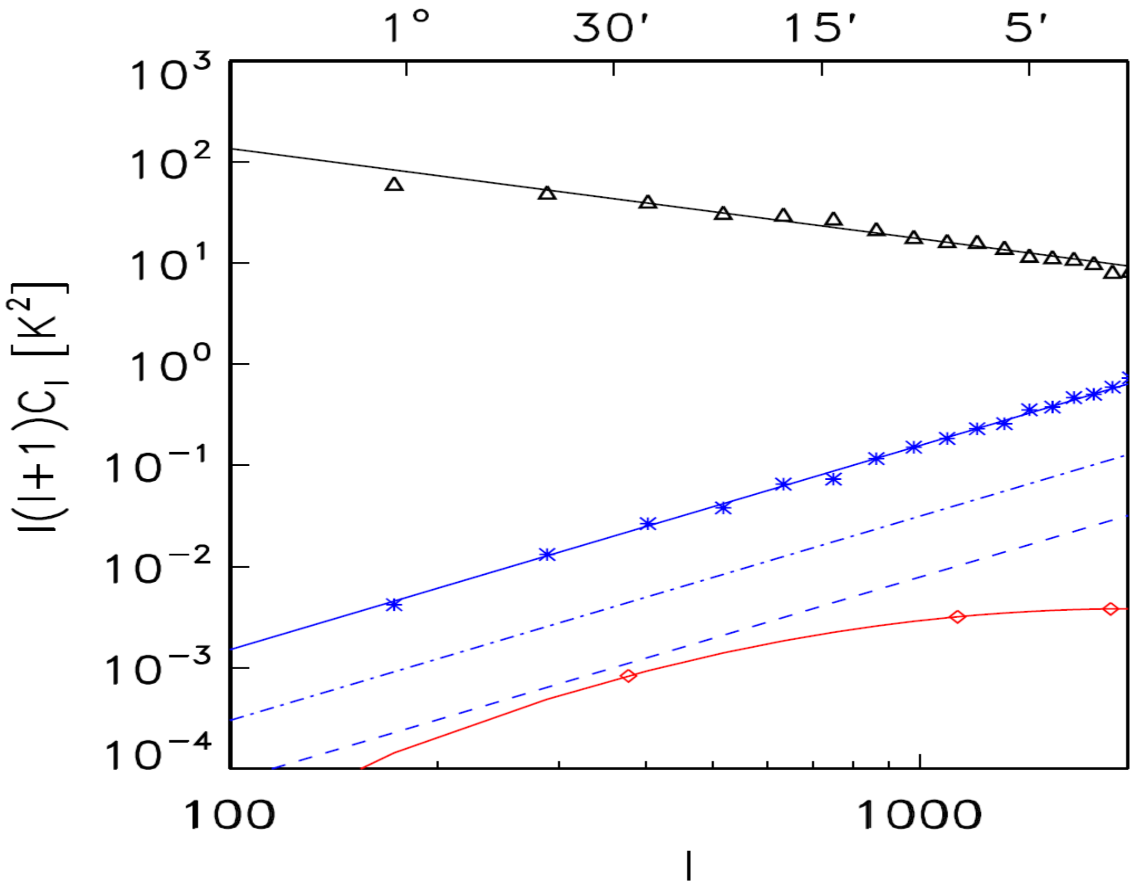
RMS fitting error



Cross-correlation of residuals with foregrounds



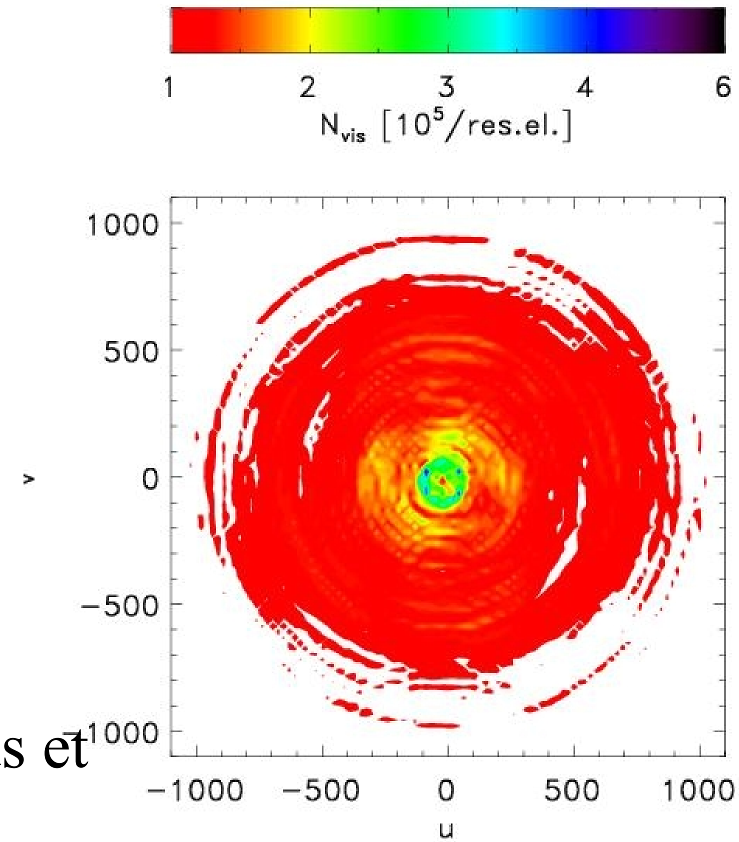
Power spectra of various contributions

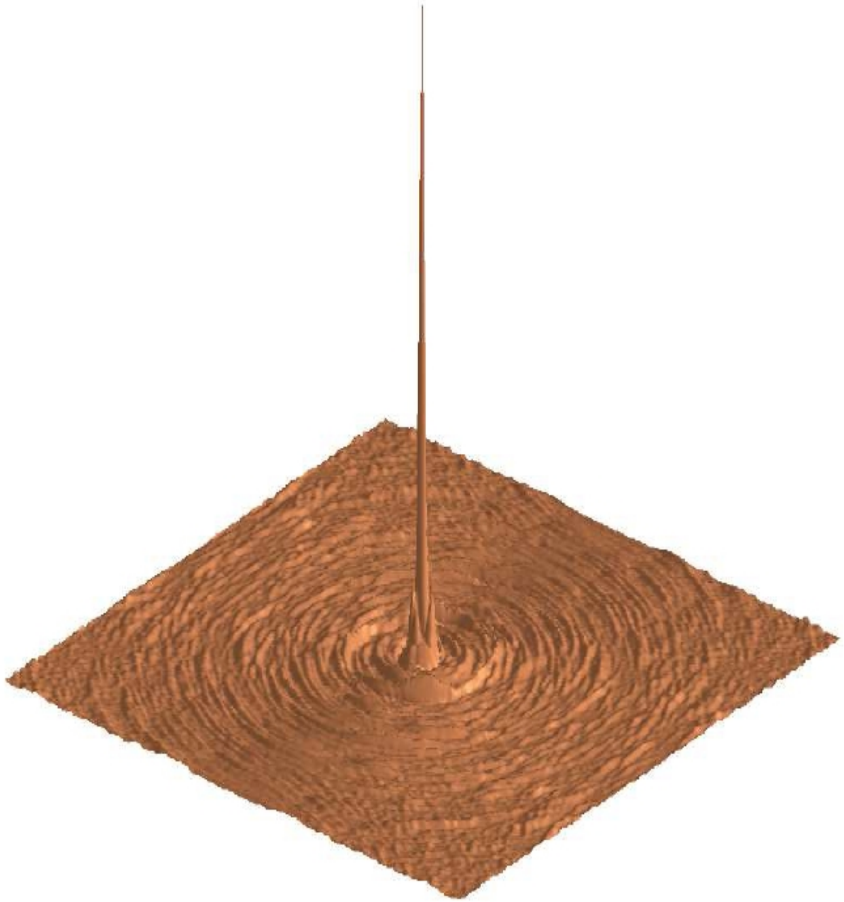


Jelic at el. 2009
Santos et al.2006

Fitting is expected to be worse at large scales (small *l*)

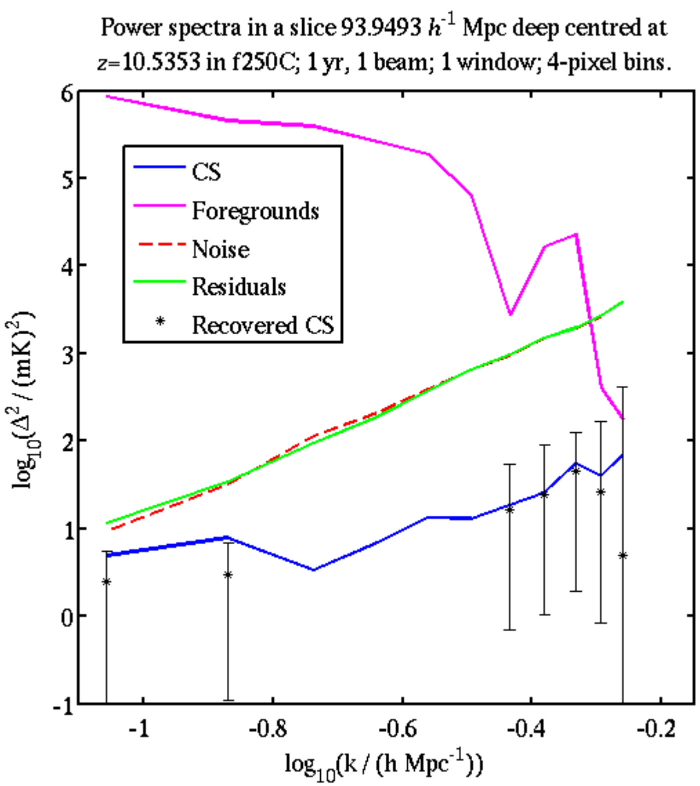
Create a Datacube with cosmic signal, FG and inst. model.

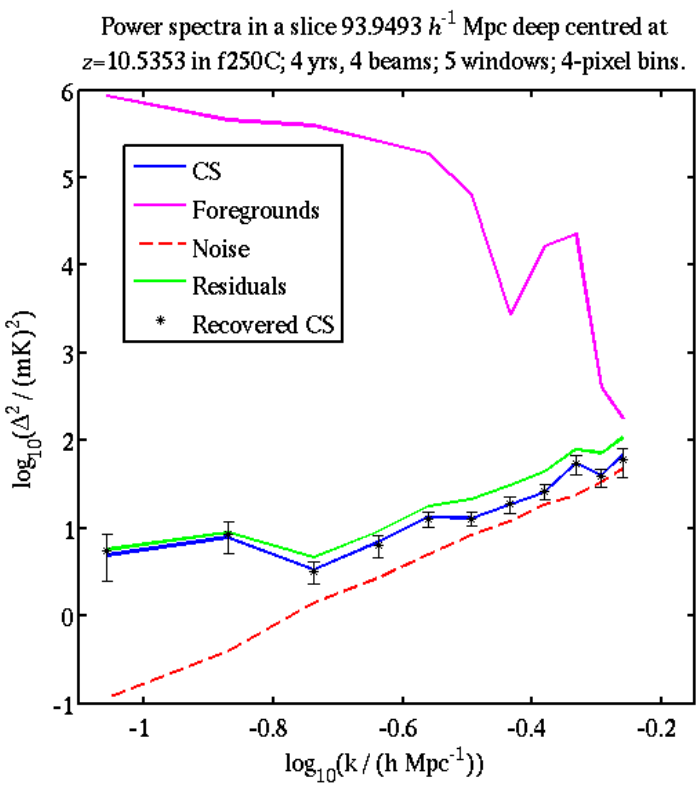




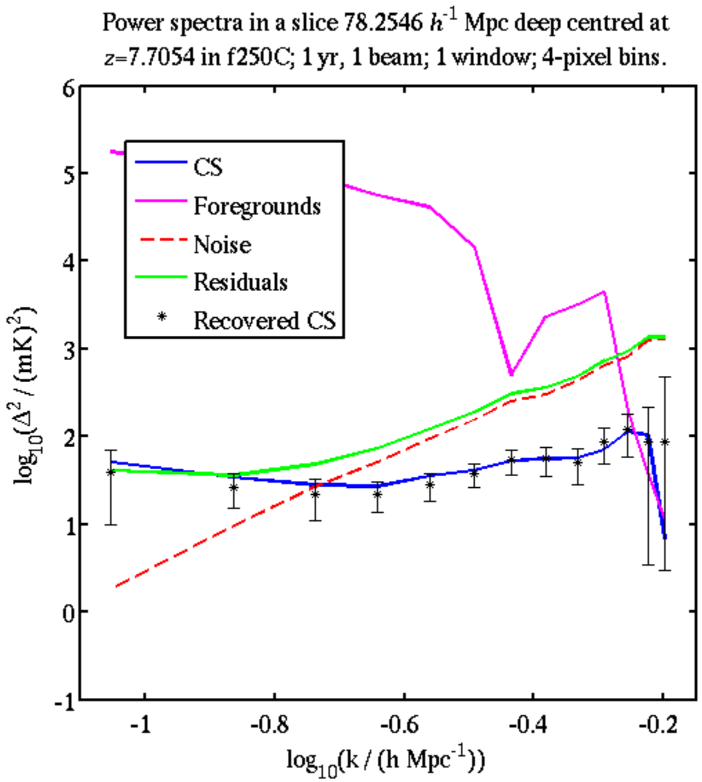
Labropulous et al, 2009

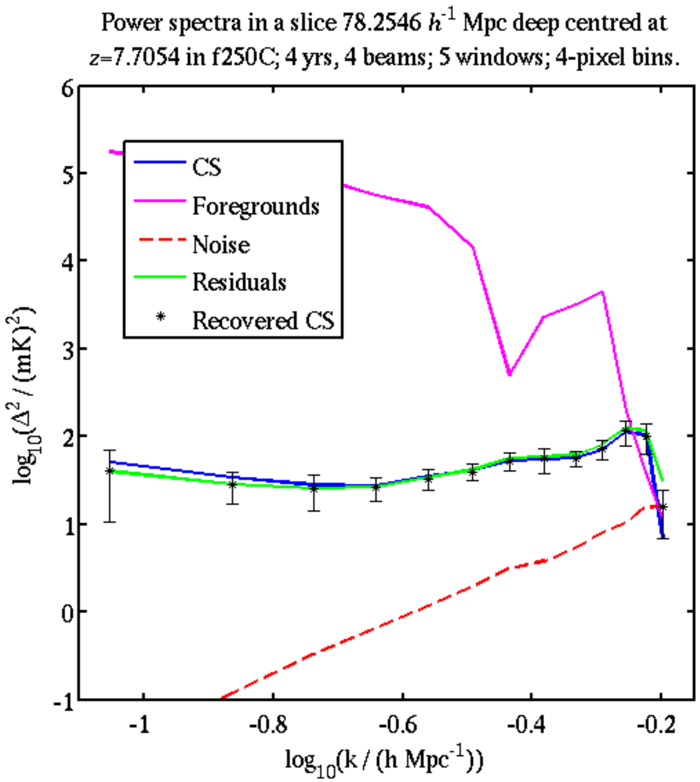
Power Spectrum Recovery 100 Mpc/h simulation



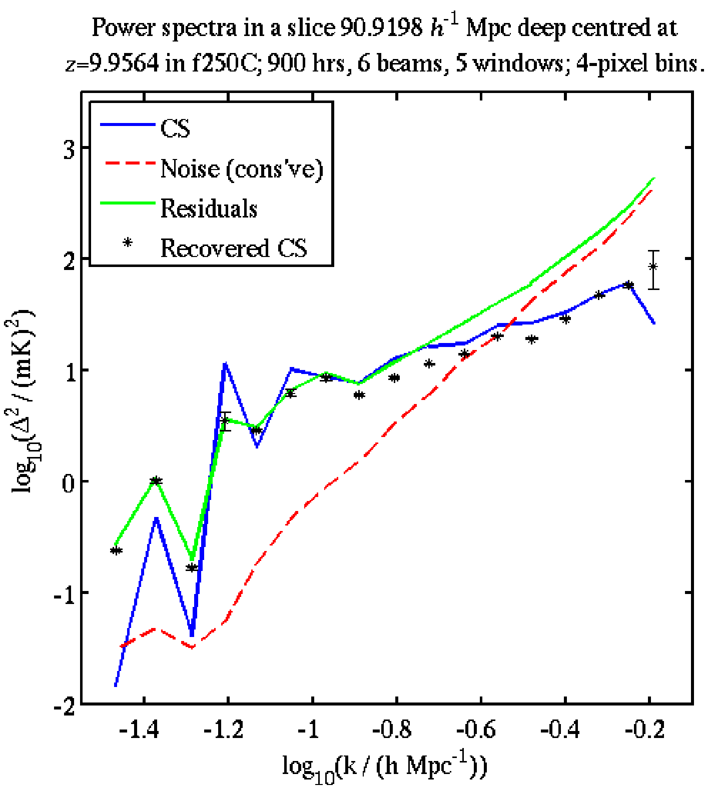


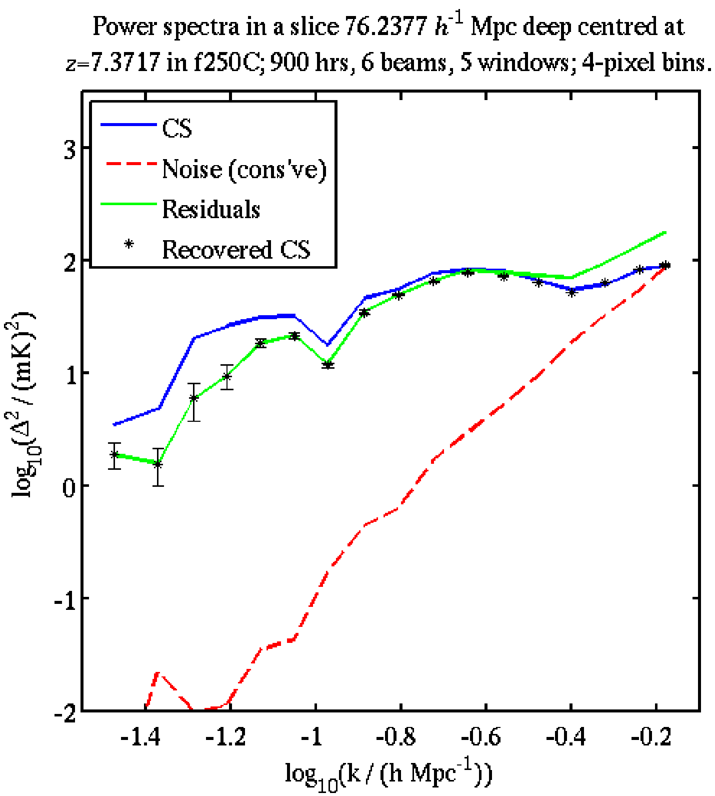
z=7.7


Power spectra in a slice 78.2546 h^-1 Mpc deep centred at
z=7.7054 in f250C; 1 yr, 1 beam; 1 window; 4-pixel bins.
CS
Foregrounds
Noise
Residuals
Recovered CS
log10(Δ^2 / (mK)^2)
log10(k / (h Mpc^-1))
6
5
4
3
2
1
0
-1
-0.8
-0.6
-0.4
-0.2


Power spectra in a slice 78.2546 h^-1 Mpc deep centred at
z=7.7054 in f250C; 4 yrs, 4 beams; 5 windows; 4-pixel bins.
CS
Foregrounds
Noise
Residuals
Recovered CS
log10(Δ^2 / (mK)^2)
log10(k / (h Mpc^-1))
6
5
4
3
2
1
0
-1
-0.8
-0.6
-0.4
-0.2

200 Mpc/h
Preliminary Results



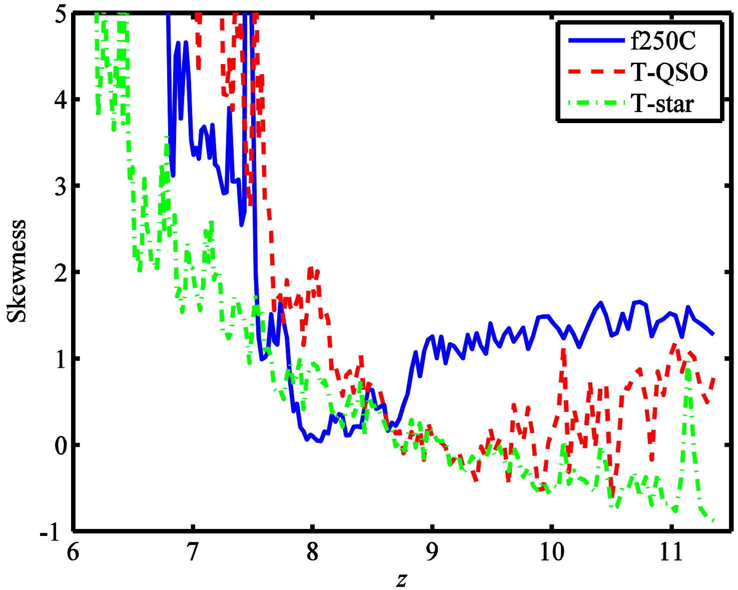


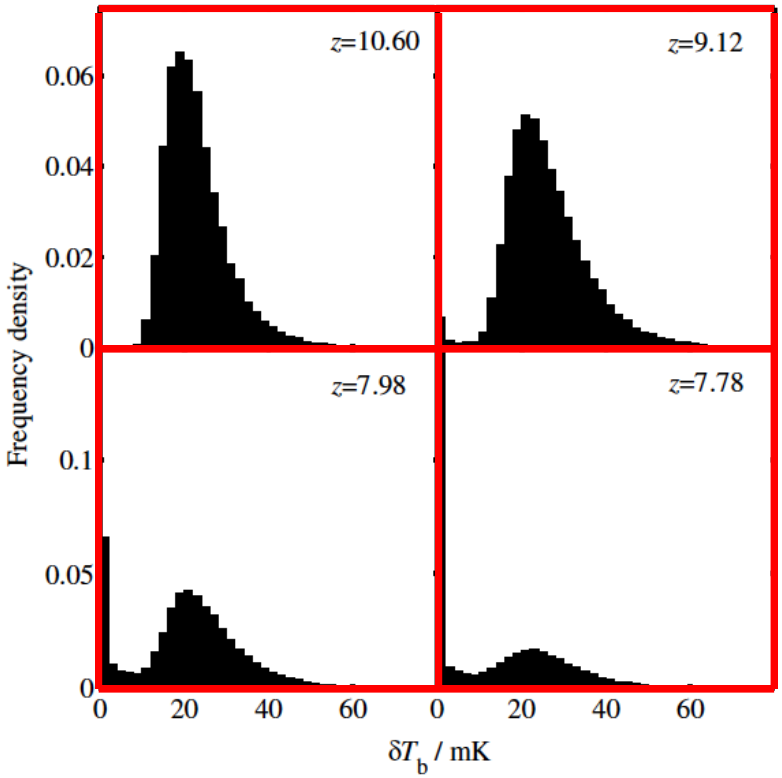
Harker et al. In prep.

High Order Statistics

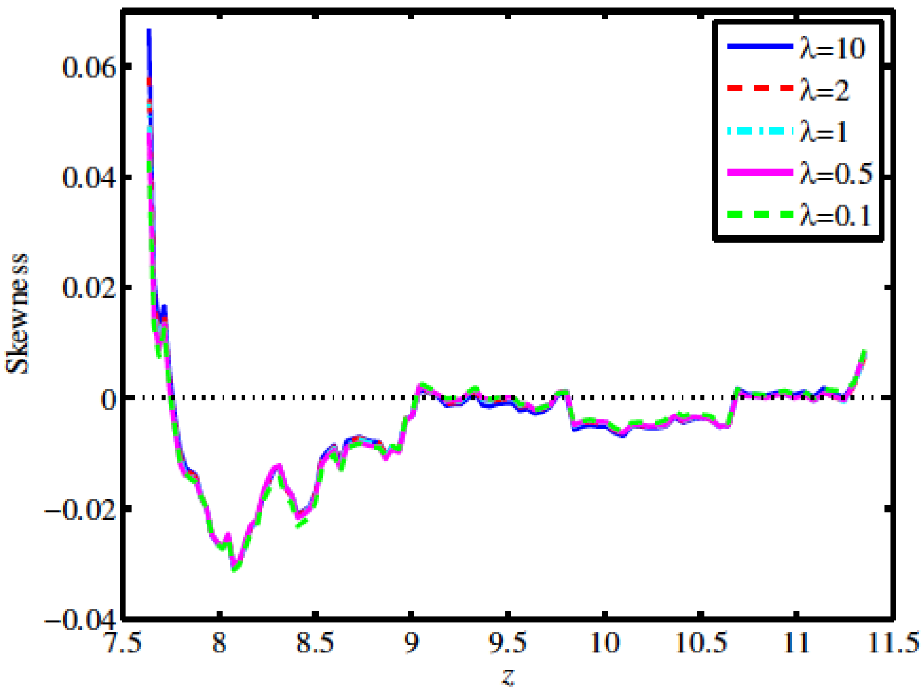
The Skewness

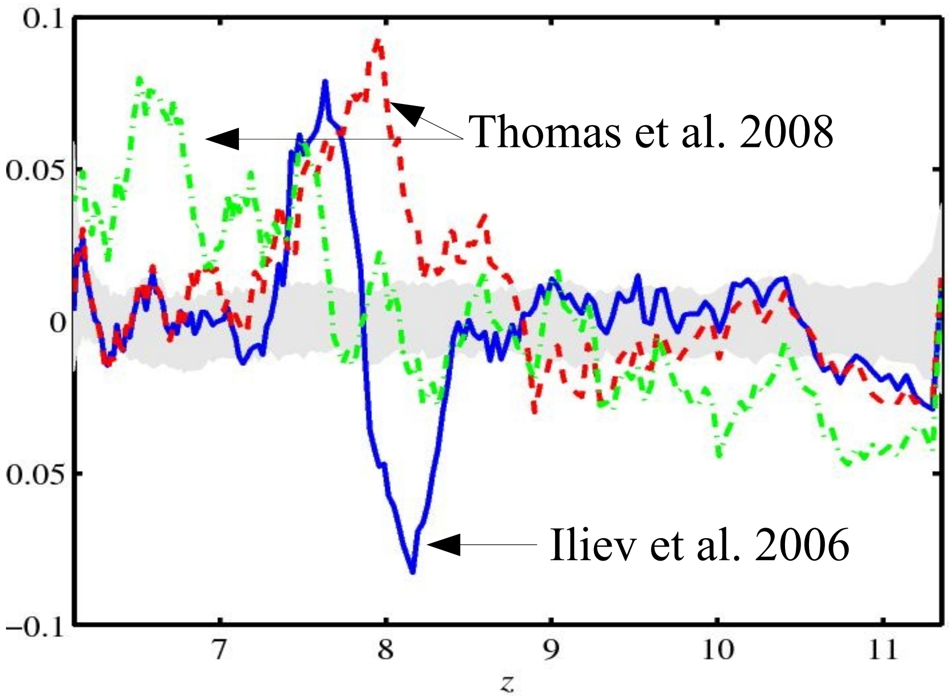
Original simulations





Extraction through the skewness





Harker et al. 2009

Conclusions

- Interpretation of the Power Spectrum measurements might be complicated due to the T_s.
- Simple fitting procedure can bias the results.
- Accurate and unbiased foreground fitting is a crucial part of our signal extraction.
- Non-parametric methods are promising but more approaches should be applied.
- Recovering the PS on very large scales might be biased due to the FG power on these scales.