Superfluid instabilities in neutron star dynamics

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The ‘Neutron star lab’

- Neutron stars are cosmic laboratories of extreme physics. The main goal is to probe and understand the properties of ultradense matter.
- Theoretical input for building a ’minimal’ model to test against astrophysical observations:

– Supranuclear equation of state (hyperons, quarks etc.)

– General Relativity

– Rotation (known for pulsars, could lead to instabilities)

– Magnetic fields (exterior dipole field might be ‘known’, unknown interior field configuration and strength)

– Crust (‘tectonic’ activity may be taking place in AXPs and SGRs)

– Superfluidity/superconductivity (vortices, fluxtubes, multifluid hydrodynamics)

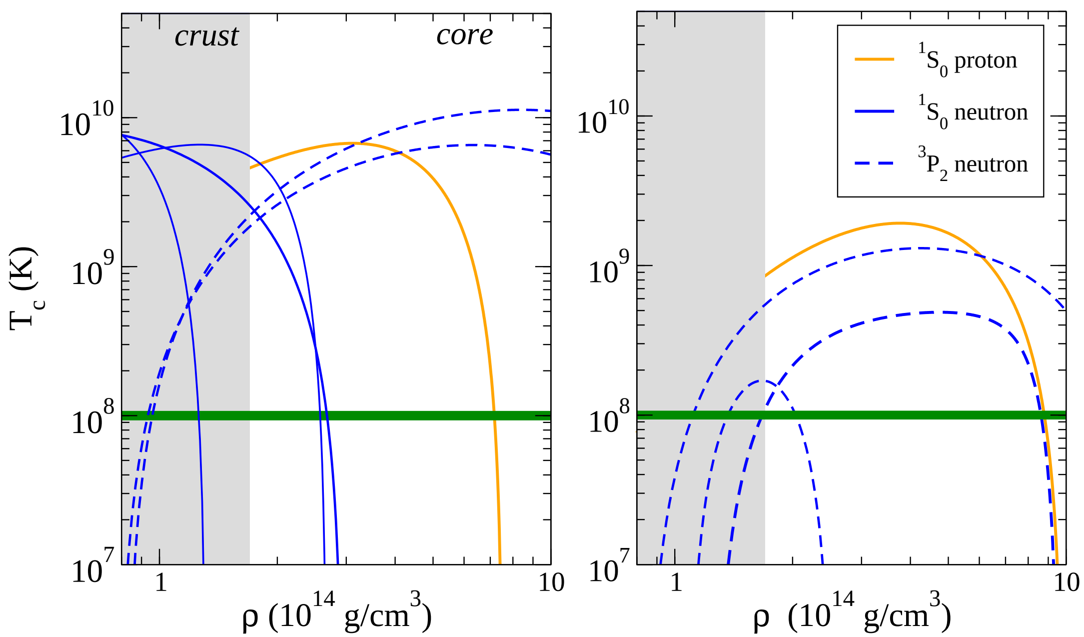
– Temperature evolution (cooling mechanisms)

Why superfluidity?

- Mature neutron stars are cold ($T \sim 10^8\,\mathrm{K} \ll T_{\mathrm{fermi}} \sim 10^{12}\,\mathrm{K}$), they should be either solid or superfluid.
- Critical temperature T_c for neutron and proton BCS pairing reached soon after birth.

- At least two distinct fluids in the interior (likely more in the presence of 'exotica' in the inner core).
- A "realistic" neutron star model should be based on multifluid hydrodynamics.

Critical temperatures for neutron and proton superfluidity



This talk

- We observe aspects of neutron star dynamics, like glitches and precession, that we do <u>not</u> really understand.

- The observed dynamics is very sensitive to the presence of superfluids in neutron star interiors $\rightarrow$ could be used to probe the properties of neutron star matter.

- Existing work on glitches/precession is based on phenomenological "rigid-body" models, and/or studying the <u>local</u> vortex dynamics (eg. vortex pinning/creep ...). This is an obvious restriction.

- What can we learn if we address these problems using <u>global</u> superfluid hydrodynamics ?

Two-fluid hydrodynamics

- "Minimal" model for a superfluid neutron star:

neutron fluid = superfluid neutrons, 'proton' fluid = superconducting protons and electrons.

- Hydrodynamics:

$$\left(\partial_t + v_{\mathrm{n}}^j \nabla_j\right)\left(v_{\mathrm{n}}^i + \epsilon_{\mathrm{n}} w^i\right) + \epsilon_{\mathrm{n}} w_j \nabla^i v_{\mathrm{n}}^j + \nabla^i(\tilde{\mu}_{\mathrm{n}} + \Phi) = f_{\mathrm{mf}}^i$$

$$\left(\partial_t + v_{\mathrm{p}}^j \nabla_j\right)\left(v_{\mathrm{p}}^i - \epsilon_{\mathrm{p}} w^i\right) - \epsilon_{\mathrm{p}} w_j \nabla^i v_{\mathrm{p}}^j + \nabla^i(\tilde{\mu}_{\mathrm{p}} + \Phi) = -(\rho_{\mathrm{n}}/\rho_{\mathrm{p}}) f_{\mathrm{mf}}^i$$

$$\partial_t \rho_{\mathrm{n,p}} + \nabla_i(\rho_{\mathrm{n,p}} v_{\mathrm{n,p}}^i) = 0, \qquad \nabla^2 \Phi = 4\pi G(\rho_{\mathrm{n}} + \rho_{\mathrm{p}})$$

where $w^i = v_{\mathrm{p}}^i - v_{\mathrm{n}}^i$.

- Couplings:

Entrainment parameters $\{\epsilon_{\mathrm{n}}, \epsilon_{\mathrm{p}}\}$, mutual friction force f_{mf}^i, chemical potentials $\tilde{\mu}_{\mathrm{n,p}}$ (through the EoS) and gravity.

Mutual friction

- Mutual friction is a vortex-mediated force:

$$\vec{f}_{\rm mf} = \frac{\mathcal{R}}{1+\mathcal{R}^2}\,\hat{\omega}_{\rm n}\times(\vec{\omega}_{\rm n}\times\vec{w}) + \frac{\mathcal{R}^2}{1+\mathcal{R}^2}\,(\vec{\omega}_{\rm n}\times\vec{w})\,, \qquad \vec{\omega}_{\rm n} = \nabla\times\vec{v}_{\rm n}$$

.

- The drag $\mathcal{R}$ makes contact with the dynamics of a single vortex:

$$\mathcal{R} \sim \frac{\text{drag force}}{\text{Magnus force}}$$

$\mathcal{R}\to 0$ zero coupling limit, $\mathcal{R}\to\infty$ perfect pinning limit.

- Diversity in drag forces:

– Weak drag $\mathcal{R}\ll 1$ (electron-vortex scattering).

– Strong drag $\mathcal{R}\gg 1$ (interaction with fluxtubes (core) or lattice (crust)).

Exploring some new dynamics

- Lets consider linear oscillations ($v^i_{\rm n} \to v^i_{\rm n} + \delta v^i_{\rm n}$ etc) including two key properties:

(1) In the background the fluids are in rigid-body rotation with different spin frequencies:

$$\vec{v}_{\rm n,p} = \vec{\Omega}_{\rm n,p} \times \vec{r}, \qquad \vec{\Omega}_{\rm n} \neq \vec{\Omega}_{\rm p}.$$

(2) Mutual friction is present.

- For simplicity we ignore entrainment and assume a uniform background ($\Rightarrow \nabla_j \delta v^j_{\rm n,p} = 0$).

- Main result: We find two new instabilities associated with inertial modes.

A new r-mode instability

- Global mode analysis: we obtain axial r-mode solutions (for $\ell = m$).
- One of the r-mode solutions becomes unstable provided:

$$\mathcal{R} \gg 1 \quad \text{and} \quad \Delta = \frac{\Omega_{\rm n} - \Omega_{\rm p}}{\Omega_{\rm p}} \neq 0$$

- The instability appears for:

$$m > m_c \approx 320 \left(\frac{0.05}{x_{\rm p}} \right)^{1/2} \left(\frac{10^{-4}}{\Delta} \right)^{1/2}$$

- The instability is dynamical. Growth timescale:

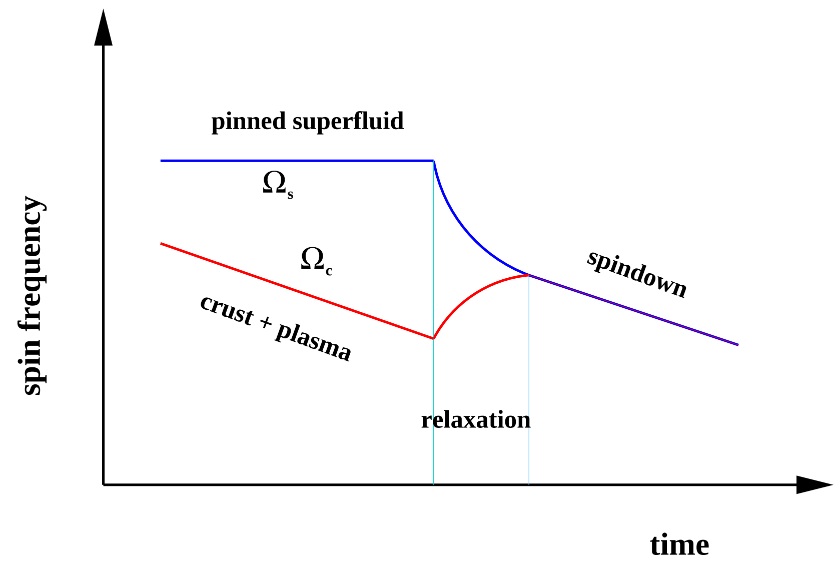
$$\frac{\tau_{\rm grow}}{P} \approx 2.5 \left(\frac{x_{\rm p}}{0.05} \right)^{1/2} \left(\frac{10^{-4}}{\Delta} \right)^{1/2}$$

Pulsar glitches

- Crust & proton fluid: spin-down together ($\Omega_{\rm p} = \Omega_{\rm c}$)

- Neutron fluid: unable to follow spin down if vortices are pinned ($\Omega_{\rm n} > \Omega_{\rm p}$).

- At a critical spin-lag vortex unpinning is forced, and angular momentum is transferred to the crust:



$$\Delta_g = \frac{\Omega_{\rm s} - \Omega_{\rm c}}{\Omega_{\rm c}} \approx \frac{I_{\rm c}}{I_{\rm s}} \frac{\Delta\Omega_{\rm c}}{\Omega_{\rm c}} \approx 5 \times 10^{-4}$$

- Data for Vela-like glitches: $\Delta\Omega_{\rm c}/\Omega_{\rm c} \sim 10^{-6}, \quad I_{\rm s}/I_{\rm c} \lesssim 0.02$

A glitch-trigger mechanism

- The r-mode instability needs to overcome viscous dissipation, (dominated by electron shear viscosity). Combining $\tau_{\rm grow} < \tau_{\rm sv}$ and $m > m_c$ leads to a critical Δ at which the instability first appears:

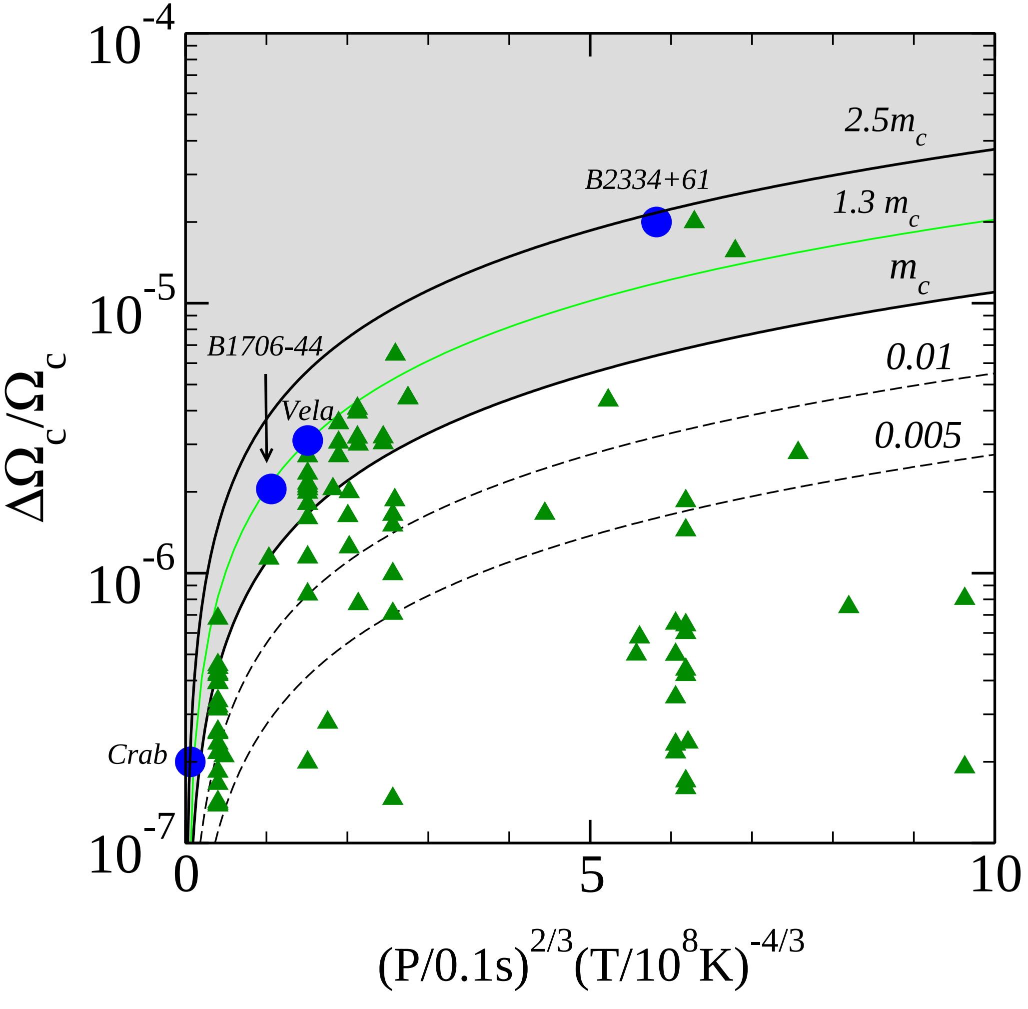
$$\Delta_c \approx 6 \times 10^{-5} \left(\frac{P}{0.1 \text{ s}} \right)^{2/3} \left(\frac{T}{10^8 \text{ K}} \right)^{-4/3}$$

- Estimate glitch size:

$$\Delta_g = \Delta \geq \Delta_c \quad \Rightarrow \quad \frac{\Delta\Omega_{\rm c}}{\Omega_{\rm c}} \gtrsim 10^{-6} \left(\frac{I_{\rm s}/I_{\rm c}}{0.02} \right) \left(\frac{P}{0.1 \text{ s}} \right)^{2/3} \left(\frac{T}{10^8 \text{ K}} \right)^{-4/3}$$

- Given an estimated ("observed") core temperature, and the moment of inertia ratio I_s/I_c we can confront observations.
- Remark: Expect a glitch to occur after the object has "evolved" into the instability region (i.e. $\tau_{\rm grow}$ minimum): $m \gtrsim 1.3 m_c \Rightarrow \Delta \gtrsim 1.7 \Delta_c$

Confronting glitch observations



- For systems without T data we use a standard URCA cooling law.
- Curves for $I_s/I_c = 0.02,\ 0.01,\ 0.005$

The “Glaberson-Donnelly” instability

- We carry out a local plane-wave analysis:

$$\delta v^i_{\mathrm{n,p}} = \mathcal{A}^i_{\mathrm{n,p}} e^{i\sigma t + i k_j x^j}, \qquad \mathcal{A}^i_{\mathrm{n,p}} = \text{constant}$$

- We find that inertial waves could go unstable when there is sufficient misalignment between $\vec{\Omega}_{\mathrm{n}}$, $\vec{\Omega}_{\mathrm{p}}$.

- Instability similar to the Glaberson-Donnelly instability in superfluid Helium (where it leads to superfluid turbulence).

- The instability naturally applies to precessing neutron stars where the precession modes are derived by assuming rigid body rotation and misaligned $\vec{\Omega}_{\mathrm{n}}$, $\vec{\Omega}_{\mathrm{p}}$.

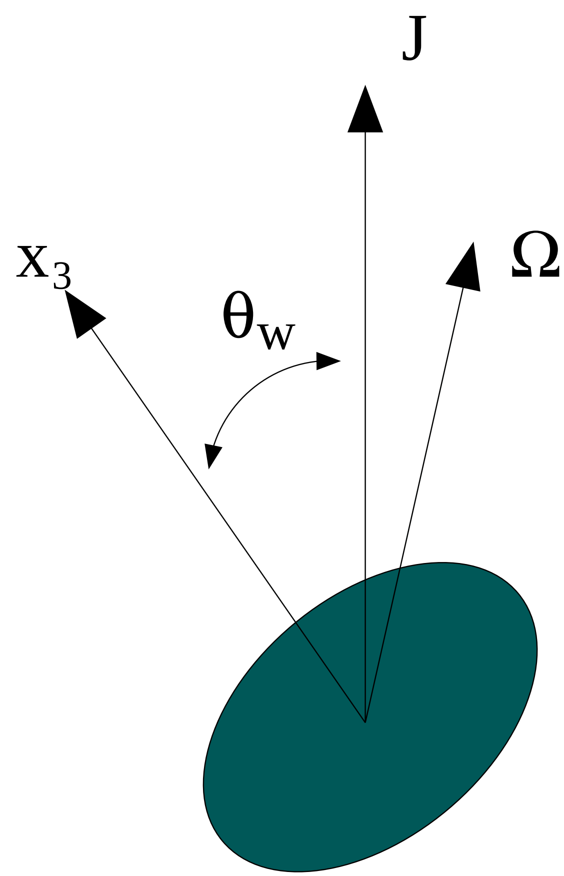
Neutron star precession

- Simplest model is the biaxial oblique rotator *without* any hydrodynamics:

$$J^i = I^{ij}\Omega_j, \quad I^{ij} = I_0(1,1,1+\epsilon)\delta^{ij}$$

- For small amplitude precession of nearly spherical body the *free precession period* is:
$P_{\rm pr} \approx P/\epsilon$

- Best candidate precessor is the pulsar PSR B1828-11:
$P_{\rm pr} \approx 511\,{\rm days}, \quad \theta_w \approx 3^\circ$



Precession of superfluid neutron stars

- Nature of precession modes decided by mutual friction $\mathcal{R}$.

- Weak drag precession:

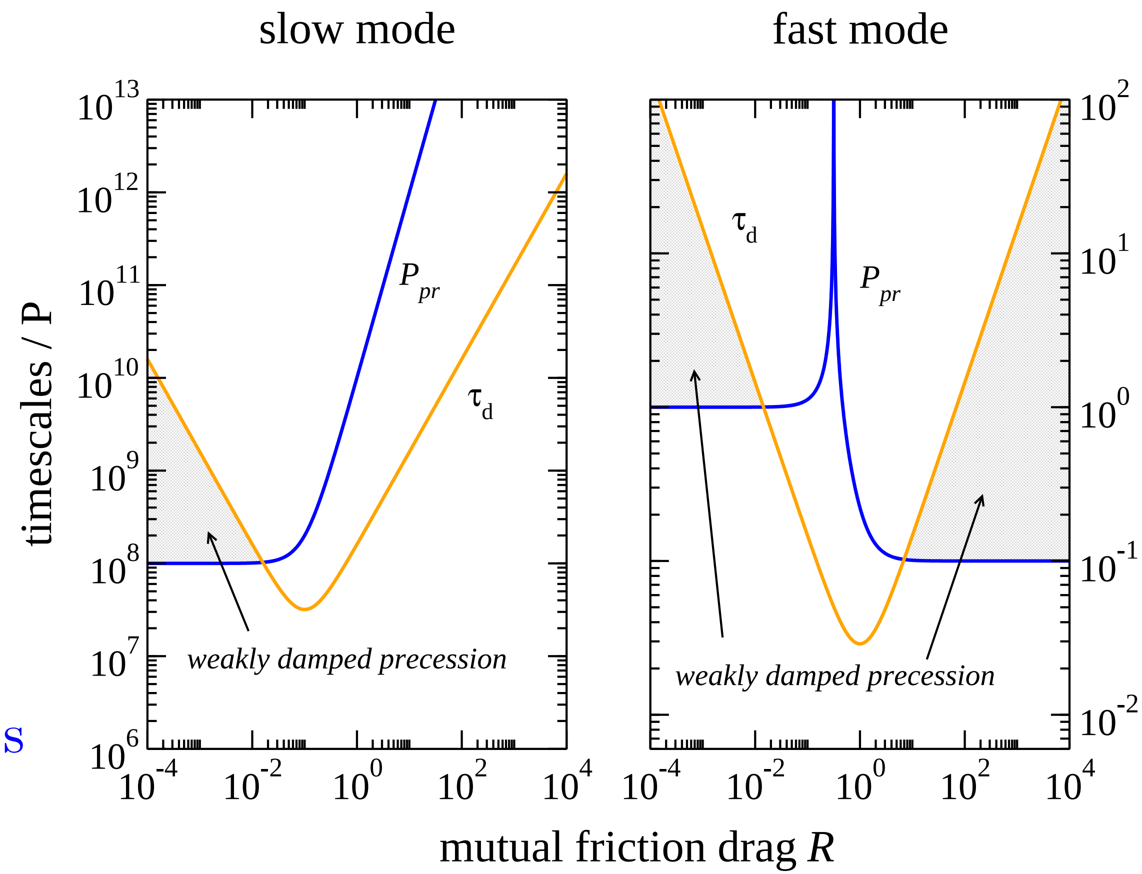
$$P_{\rm pr} \approx 3 \left(\frac{P}{1\,{\rm s}}\right)\left(\frac{10^{-8}}{\epsilon}\right)\,{\rm yr}$$

or

$$P_{\rm pr} \approx 10^5 \left(\frac{I_{\rm n}/I_{\rm p}}{10}\right)\left(\frac{P}{1\,{\rm s}}\right)\left(\frac{10^{-4}}{\mathcal{R}}\right)\,{\rm s}$$

- Strong drag precession:

$$P_{\rm pr} \approx 0.1 \left(\frac{10}{I_{\rm n}/I_{\rm p}}\right)\left(\frac{P}{1\,{\rm s}}\right)\,{\rm s}$$



The precession conundrum

- There is clearly a problem here:

  The observations seem to favour slow, weak mutual friction, precession. This is theoretically puzzling, particularly in the light of the glitch observations and our understanding of mutual friction microphysics, i.e. vortex-fluxtube interactions cause strong mutual friction coupling.

- One would be led to conclude:

  Either the two superfluids coexist nowhere in the stellar core, or the outer core is a type I superconductor rather than a type II.

The precession conundrum (cont.)

- This is an interesting, non-trivial conclusion to draw from the precession observations. *But is it secure?*

- Let's return at the beginning. The precession modes were derived by assuming *rigid body* rotation ($\vec{\Omega}_{\rm n}$, $\vec{\Omega}_{\rm p}$).
In doing so the fluid degrees of freedom were lost.

- What happens if we restore the supressed fluid degrees of freedom ?

- We can apply our previous local wave analysis.

Is *slow* precession unstable?

- Slow precession is only possible for weak mutual friction drag $\mathcal{R} \ll 1$.

- The instability is active above a critical wobble angle:

$$\frac{\theta_w}{1^\circ} > 1900 \left(\frac{P}{1\,\mathrm{s}}\right)^{1/2} \left(\frac{10^{-8}}{\epsilon}\right) \left(\frac{10\,\mathrm{km}}{R}\right)$$

- Conclusion:

– Long period precession is typically stable (e.g for a precessor like $\mathrm{PSR\,B1828-11}$).

– A notable exception: the instability could be relevant for millisecond-period neutron stars.

Is *fast* precession unstable?

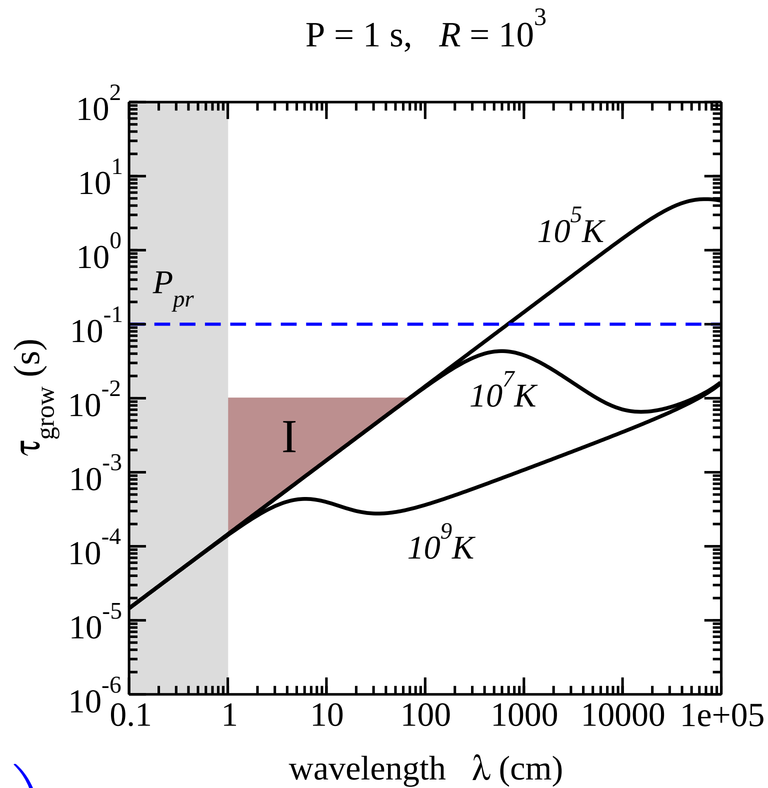
- Strong drag leads to fast "Shaham" precession.

- Wobble angle constraint:

$$\frac{\theta_w}{1^\circ} > 0.014 \left(\frac{P}{1\ \mathrm{s}}\right)^{1/2} \left(\frac{\mathcal{R}}{10^3}\right)$$

- Growth timescale:

$$\tau_{\mathrm{grow}} \approx 140 \left(\frac{\theta_w}{1^\circ}\right)^{-1} \left(\frac{P}{1\ \mathrm{s}}\right) \left(\frac{\mathcal{R}}{10^3}\right) \left(\frac{\lambda}{R}\right)\ \mathrm{s}$$



- Conclusion:

Fast rigid-body precession is generically <u>unstable</u>.

Yet another twist: the magnetic field

- Our model does not account for hydromagnetic stresses.

– This simplification should be obviously relaxed, since the magnetic restoring force may dominate small-wavelength perturbations.

- In general, the magnetic field will play a stabilising (?) role.

- Local analysis: can be easily modified to include the magnetic field (van Hoven & Levin '08). The constraint on fast precession becomes weaker, but the instability is still astrophysically relevant:

$$\frac{\theta_w}{1^\circ} > 3 \left(\frac{P}{1\,\mathrm{s}}\right) \left(\frac{B}{10^{12}\,\mathrm{G}}\right) \left(\frac{x_\mathrm{p}}{0.1}\right)$$

- Global analysis: not so easy to modify ... but there is no reason why our proposed glitch mechanism should not work in magnetised neutron stars.

Outlook

• The two fluid model for superfluid neutron stars predicts new dynamical instabilities provided there is relative motion in the background and mutual friction coupling.

• These instabilities may well be relevant for “real life” neutron star dynamics (glitches, precession ...).

• Obviously, the model is far from being “realistic”. In order to make more reliable predictions, one should worry about things like:

– Additional restoring forces (magnetic and crustal stresses)

– The phenomenology assumed in the mutual friction force

For example, vortex pinning in the core and the crust is poorly understood $\rightarrow$ need for a consistent description of vortex dynamics.