# International Conference on Quantum Information and Computation

Stockholm, 4-8 October, 2010



## Book of Abstracts

NORDITA

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ROYAL INSTITUTE OF TECHNOLOGY

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# International Conference on Quantum Information and Computation

Wenner-Gren Center, Stockholm, Sweden
October 4 - 8, 2010

Conference held in conjunction with the NORDITA Program on Quantum Information and in co-operation with the Institut Mittag-Leffler's scientific program on Quantum Information Theory.

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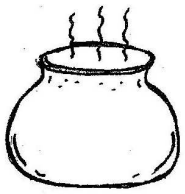

# Foreword

Let us begin by saying how pleased we are that you all wanted to come to this International Conference on Quantum Information and Computation at Stockholm. We hope that you will enjoy it, and learn new things during it.

Nordita, the Nordic Institute of Theoretical Physics which is the principal sponsor of this conference, has traditions going back to Niels Bohr, and has played a major role for physics in the Nordic countries ever since. We are grateful for all the support that Nordita has given, and we are also grateful for the support we have received from the Swedish Research Council (VR), from our home universities, Stockholm University and the Royal Institute of Technology (KTH), and from the Linnæus Center for Advanced Optics and Photonics (ADOPT).

Nordita moved to Stockholm quite recently, but in fact, Stockholm has proud traditions of its own. We believe that Stig Stenholm and Göran Lindblad are known to all of you, and that their presence is enough to prove our case. You may not know them personally as well as we do, so let us just mention that Stig is legendary for his ability to prolong seminars with an extra hour or thereabouts, because of his insistence that some philosophical point must be made absolutely clear. We will see what happens during the conference. You may also wonder why there is a picture of a pot on this page. Actually it is copied from the notes that one of us took during a lecture course that Göran gave. He told us that quantum mechanics is like a pot in which a soup is prepared: almost indestructible and very rigid, but very flexible as well, because you can choose any ingredient for your soup. We hope that this conference will prove him right.

It remains to thank a number of graduate students for the work we made them do. Perhaps, in the future, they will be as well known as Stig and Göran.

Ingemar Bengtsson  Gunnar Björk  Mohamed Bourennane

## Program Overview

International Conference on Quantum Information and Computation
Stockholm, October 4-8, 2010

| Time | Monday, Oct. 4 | Tuesday, Oct. 5 | Wednesday, Oct. 6 | Thursday, Oct. 7 | Friday, Oct. 8 |
|---|---|---|---|---|---|
| | | | | | |
| 8:45-9:00 | Welcome | | | | |
| | *Session Quantum Computation 1* | *Session Entanglement 1* | *Session Quantum Communication* | *Session Quantum Information 1* | *Session Foundations of QM 1* |
| 9:00-10:00 | Invited talk: I. Cirac | Invited talk: S. Haroche | Invited talk: P. Grangier | Invited talk: A. Holevo | Invited talk: R. Werner |
| 10:00-10:30 | Sciarrino | Barbosa | Hofmann | Invited talk: C. Bennett | Branciard |
| 10:30-11:00 | *Coffee* | *Coffee* | *Coffee* | | *Coffee* |
| 11:00-11:30 | Andersson | Pryde | Bény | *Coffee* | Brunner |
| 11:30-12:00 | Pomarico | Tóth | Leung | Verstraete | Kleinmann |
| 12:00-12:30 | Kendon | Osterloh | Lydersen | | Heinosaari |
| 12:30-14:00 | *Lunch* | *Lunch* | *Lunch* | *Lunch* | *Lunch* |
| | *Session Quantum Computation 2* | *Session Entanglement 2* | *Session Entanglement 3* | *Session Quantum Information 2* | *Session Foundations of QM 2* |
| 14:00-15:00 | Invited talk: R. Blatt | Invited talk: J. Martinis | Invited talk: J. Schmiedmayer | Hayden | Invited talk: A. Zeilinger |
| 15:00-15:20 | *Coffee* | *Coffee* | *Coffee* | *Coffee* | Concluding remarks/Coffe |
| 15:20-15:50 | Jungnitsch | Joshi | Siewert | Wolf | |
| 15:50-16:20 | Romero-Isart | Filipp | Marian | | |
| 16:20-16:50 | Tokunaga | Lamata | Ciccarello | Bengtsson | |
| 16:50-18:00 | | Poster session 1 | | Poster session 2 | |
| 18:00-19:00 | Welcome reception at | | Conference dinner | | |
| 19:00-20:00 | Albanova | | | | |
| 20:00-21:00 | | | | | |
| 21:00-22:00 | | | | | |

| | | | |
|---|---|---|---|
| Registration and information desk open at | AlbaNova main entrance | Sunday 17:00-21:00 | Roslagstullsbacken 21 |
| | WennerGren Center, -1 floor | Monday 8:00-12:00 | Sveavägen 166 |
| | WennerGren Center, -1 floor | Every day 8:30-9:00 | Sveavägen 166 |
| | WennerGren Center, -1 floor | During all coffee breaks | Sveavägen 166 |

# Conference program
# Monday, October 4, 2010

## 08:45-09:00 Welcome to ICQIC 2010

08:45-08:50 **"Conference opening"**

G. Björk

*Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden.*

08:50-09:00 **"Welcome address"**

Lárus Thorlacius

*Nordic Institute for Theoretical Physics (NORDITA), Roslagstullsbacken 23, 106 91 Stockholm, Sweden.*

## 09:00-12:30 Session O1: Quantum Computation 1

Session moderator: Ingemar Bengtsson

09:00-10:00 O1.1 **Invited talk: "Dissipation: a new tool for quantum information processing"**

Ignacio Cirac

*Max-Planck-Institut für Quantenoptik, Garching, Germany.*

10:00-10:30 O1.2 **"Quantum information processing by exploiting the photonic orbital angular momentum"**

Eleonora Nagali[1], Daniele Giovannini[1], Lorenzo Marrucci[2], and Fabio Sciarrino[1,3]

[1] *Dipartimento di Fisica dell'Università "La Sapienza", Roma 00185, Italy,* [2] *Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Compl. Univ. di Monte S. Angelo, 80126 Napoli, Italy,* [3] *Istituto Nazionale di Ottica, Firenze, Italy.*

10:30-11:00 Coffee break

11:00-11:30 O1.3 **"Photons walking the line – compact and scalable fibre loop quantum walk"**

Andreas Schreiber[1], Katiuscia N. Cassemiro[1], Vašek Potoček[2], Aurel Gábris[2], Peter Mosley[1], Erika Andersson[3], Igor Jex[2], and Christine Silberhorn[1]

[1] *Max Planck Institute for the Science of Light, Günther-Scharowsky-str. 1, Bau 24, 91058 Erlangen, Germany,* [2] *Department of Physics, FNSPE, Czech Technical University in Prague, Břehová 7, 115 19 Praha, Czech Republic,* [3] *SUPA, School of EPS, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom.*

11:30-12:00 O1.4 **"Quantum Cloning for absolute radiometry"**

Enrico Pomarico, Bruno Sanguinetti, Pavel Sekatski, Hugo Zbinden, and Nicolas Gisin

*Group of Applied Physics, University of Geneva, 20 rue de l' École-de-Médecine, Geneva, Switzerland.*

12:00-12:30 O1.5 **"Robust cluster state generation using ancilla-based systems"**

Viv Kendon[1], Katherine Louise Brown[1], Clare Horsman[2], and Bill Munro[3]

[1]*School of Physics and Astronomy, University of Leeds, LS2 9JT, United Kingdom,* [2]*Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK, H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, UK,* [3]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan, NTT Basic Research Laboratories, 3-1, Morinosato Wakamiya Atsugi-shi, Kanagawa 243-0198, Japan.*

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12:30-14:00 Lunch

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## 14:00-16:50 Session O2: Quantum Computation 2

Session moderator: Erik Sjöqvist

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14:00-15:00 O2.1 **Invited talk: "Quantum Information Science with Trapped $Ca^+$ Ions"**

Rainer Blatt

*Universtität Innsbruck, Innsbruck, Austria.*

15:00-15:20 Coffee break

15:20-15:50 O2.2 **"Increasing the statistical significance of entanglement detection in experiments"**

Bastian Jungnitsch[1], Sönke Niekamp[1], Matthias Kleinmann[1], Otfried Gühne[1], He Lu[2], Wei-Bo Gao[2], Yu-Ao Chen[2], Zeng-Bing Chen[2], and Jian-Wei Pan[2]

[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21A, A-6020 Innsbruck, Austria,* [2]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China.*

15:50-16:20 O2.3 **"Levitated Nano-dielectrics in the Quantum Regime"**

Oriol Romero-Isart, Anika C. Pflanzer, and J. Ignacio Cirac

*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748, Garching, Germany.*

16:20-16:50 O2.4 **"Experimental verification of quantum process in one-way quantum computing"**

Y. Tokunaga[1], S. Okamoto[2], R. Ikuta[2], R. Namiki[3], T. Yamamoto[2], M. Koashi[2], and N. Imoto[2]

[1]*NTT Information Sharing Platform Laboratories, NTT Corporation, Tokyo, Japan,* [2]*Division of Materials Physics, Graduate School of Engineering Science, Osaka University, Japan,* [3]*Department of Physics, Graduate School of Science, Kyoto University, Japan.*

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18:00-20:00 **Welcome reception** at AlbaNova Universitetscentrum, Roslagstullsbacken 21, Level 3

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# Tuesday, October 5, 2010

## 09:00-12:30 Session O3: Entanglement 1

Session moderator: Erika Andersson

09:00-10:00 O3.1 **Invited talk: "Manipulation and control of non-classical field states in a cavity by quantum non-demolition measurements and quantum Zeno dynamics"**

Serge Haroche

*École Normale Superieure, Paris, France.*

10:00-10:30 O3.2 **"Disentanglement for Finite Losses in Bipartite Systems"**

F. A. S. Barbosa[1], A. S. Coelho[1], A. J. de Faria[1], K. N. Cassemiro[2], A. S. Villar[2,3], P. Nussenzveig[1], and M. Martinelli[1]

[1] *Instituto de Física, Univ. de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil,* [2] *Max Planck Inst. for the Science of Light, Günther-Scharowsky-str. 1 / Bau 24, 91058 Erlangen, Germany,* [3] *Univ. of Erlangen-Nuremberg, Staudtstr. 7/B2, 91058 Erlangen, Germany.*

10:30-11:00 Coffee break

11:00-11:30 O3.3 **"Heralded Noiseless Linear Amplification and Distillation of Entanglement"**

G. Y. Xiang[1], T. C. Ralph[2], A. P. Lund[1], N. Walk[2], and G. J. Pryde[1]

[1] *Centre for Quantum Dynamics and Centre for Quantum Computer Technology, Griffith University, Brisbane, 4111, Australia,* [2] *Department of Physics, University of Queensland, Brisbane, 4072, Australia.*

11:30-12:00 O3.4 **"Permutationally Invariant Quantum Tomography"**

Géza Tóth[1,2,3], Witlef Wieczorek[4,5], David Gross[6], Roland Krischek[4,5], Christian Schwemmer[4,5], and Harald Weinfurter[4,5]

[1] *Department of Theoretical Physics, The University of the Basque Country, P.O. Box 644, E-48080 Bilbao, Spain,* [2] *IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain,* [3] *Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary,* [4] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany,* [5] *Fakultät für Physik, Ludwig-Maximilians-Universität, D-80797 München, Germany,* [6] *Institute for Theoretical Physics, Leibniz University Hannover, 30167 Hannover, Germany.*

12:00-12:30 O3.5 **"The invariant-comb approach and its relation to the balancedness of multipartite entangled states"**

Andreas Osterloh[1] and Jens Siewert[2,3,4]

[1] *Fakultät für Physik, Campus Duisburg, Universität Duisburg-Essen, Lotharstr. 1, 47048 Duisburg, Germany,* [2] *Departamento de Química Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Apdo. 644, 48080 Bilbao, Spain,* [3] *Ikerbasque, Basque Foundation for Science, Alameda Urquijo 36, 48011 Bilbao, Spain,* [4] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany.*

12:30-14:00 Lunch

## 14:00-16:50 Session O4: Entanglement 2

Session moderator: Per Delsing

14:00-15:00 O4.1 **Invited talk: "Deterministic Creation of NOON States in Two Superconducting Resonators"**

John Martinis

*University of California, Santa Barbara, USA.*

15:00-15:20 Coffee break

15:20-15:50 O4.2 **"Quantum superpositions of nanocantilevers"**

C. Joshi[1], A. Hutter[1,2], F. Zimmer[1,3], M. Jonson[1,4], E. Andersson[1], and P. Öhberg[1]

[1] *SUPA, Department of Physics, Heriot-Watt University, Edinburgh, EH14 4AS, UK,* [2] *Department of Physics and Astronomy, University of Würzburg, 97074 Würzburg, Germany,* [3] *Max Planck Institute for the Physics of Complex Systems, D-01187 Dresden, Germany,* [4] *Department of Physics, University of Gothenburg, SE-412 96 Göteborg, Sweden.*

15:50-16:20 O4.3 **"Entangled Quantum States of Photons and Superconducting Circuits"**

S. Filipp, M. Baur, P. Leek, and A. Wallraff

*Department of Physics, ETH Zurich, CH-8093 Zurich, Switzerland.*

16:20-16:50 O4.4 **"Towards electron-electron entanglement in Penning traps"**

L. Lamata[1], D. Porras[1], J. I. Cirac[1], J. Goldman[2], and G. Gabrielse[2]

[1] *Max-Planck-Institut für Quantenoptik, Garching, Germany,* [2] *Harvard University, Cambridge, MA, USA.*

# Tuesday, October 5, 2010

## 16:50-19:00 Poster session 1

P1.1 **"Driving universal quantum operations using a fixed interaction"**
Janet Anders, Daniel K. L. Oi, Elham Kashefi, Dan E. Browne, and Erika Andersson

P1.2 **"Universal dynamical decoupling from slow noise with minimal control"**
G. Bensky, E. Brion, F. Carlier, V. M. Akulin, and G. Kurizki

P1.3 **"Indirect Estimation of Quadratic Hamiltonians"**
Daniel Burgarth, Koji Maruyama, and Franco Nori

P1.4 **"Continuous variable entanglement in a time dependent two-particle system"**
Fernanda Raquel Pinheiro and A. F. R. de Toledo Piza

P1.5 **"Three-Level States in Superconducting Phase Circuits: Decoherence Characterization and Coupling to Two-Level Defects"**
Y. Shalibo, Y. Rofe, D. Shwa, F. Zeides, M. Neeley, J. M. Martinis, and N. Katz

P1.6 **"Quantify the robustness of optimal control signals in quantum control"**
Antonio Negretti, Tommaso Calarco, and Rosario Fazio

P1.7 **"Deterministic multi-mode photonic source and simulation of topological matter with collective encoding and Rydberg blockade in atomic ensembles"**
Anne E. B. Nielsen and Klaus Mølmer

P1.8 **"Molecular solution for the subset-sum problem on DNA-based quantum computing"**
M. Rezaei, Z. Azimi, and S. Ahadpour

P1.9 **"Experimental Realization of the Deutsch-Jozsa Algorithm with a Six-Qubit Cluster State"**
Giuseppe Vallone, Gaia Donati, Natalia Bruno, Andrea Chiuri, and Paolo Mataloni

P1.10 **"Developing scalable quantum computer hardware from naturally trapped ions"**
Ying Yan, Jenny Karlsson, Lars Rippe, and Stefan Kröll

P1.11 **"Heralded generation of entangled photon pairs"**
Stefanie Barz, Gunther Cronenberg, Anton Zeilinger, and Philip Walther

P1.12 **"Optimal and equilibrium solutions to quantum minority games"**
Puya Sharif and Hoshang Heydari

P1.13 **"Quantum Filtering equation with a coherent state filter"**
Anita Dąbrowska and Przemysław Staszewski

P1.14 **"Entanglement localization on a three-photon system"**
Miroslav Gavenda, Radim Filip, Eleonora Nagali, Fabio Sciarrino, and Francesco De Martini

P1.15 **"Non-Markovian dynamics of a driven qubit"**
Pinja Haikka and Sabrina Maniscalco

P1.16 **"Entanglement in Two-Mode Gaussian Open Quantum Systems"**
Aurelian Isar

P1.17 **"Investigation of magnetic field effect on thermal entanglement in a spin 1/2 chain"**
Safa Jami, Maryam Divsalar, and Mohsen Sarbishe

P1.18 **"Quantifying entanglement of two relativistic particles using optimal entanglement witnesses"**
M. A. Jafarizadeha1 and M. Mahdiana

P1.19 **"Measurement games against Nature in a world that minimizes changes in the number of black swan phenomena are quantum"**
Edward W. Piotrowski and Jan Sładkowski

P1.20 **"Optimal entanglement witnesses"**
Justyna Pytel and Dariusz Chruściński

P1.21 **"Geometric phases and invariants of four-qubit states"**
H. Cavit Sezer and Hoshang Heydari

P1.22 **"Entanglement in an Infinite Ising Spin Chain with a Time Dependent Coupling in an External Time-Varying Magnetic Field"**
Gehad Sadiek

P1.23 **"Optimal replication of Von Neuman measurements"**
Michal Sedlák, Alessandro Bisio, Paolo Perinotti, and Giacomo Mauro D'Ariano

P1.24 **"Quantum games that make sense"**
Edward W. Piotrowski and Jan Sładkowski

P1.25 **"Experimental demonstration of local expansion for W states"**
Toshiyuki Tashima, Tsuyoshi Kitano, Şahin Kaya Özdemir, Takashi Yamamoto, Masato Koashi, and Nobuyuki Imoto

P1.26 **"Non-Markovian dynamics of quantum discord in continuous variable systems"**
R. Vasile, P. Giorda, S. Olivares, M. G. A. Paris, and S. Maniscalco

P1.27 **"Efficient estimation of entanglement measures for large experimentally created graph states via simple measurements"**
Harald Wunderlich, Shashank Virmani, and M. B. Plenio

P1.28 **"Redundant imprinting of information in non-ideal environments: Quantum Darwinism via a noisy channel"**
Michael Zwolak, H. T. Quan, and Wojciech H. Zurek

P1.29 **"Single-shot information theory and statistical mechanics"**
Johan Åberg

P1.30 **"Simulation of static and random errors on Grover's search algorithm implemented in a Ising nuclear spin chain quantum computer with few qubits"**
Gustavo V. López, T. Gorin, and L. Lara

P1.31 **"Correlation induced non-Abelian quantum holonomies"**
Markus Johansson, Marie Ericsson, Kuldip Singh, Erik Sjöqvist, and Mark Williamson

P1.32 **"Ambiguity in the quantum trajectory model of open quantum systems"**
Erik Sjöqvist and Patrik Pawlus

P1.33 **"Bipartite entanglement of permutation symmetric states"**
Ludovic Arnaud and Nicolas J. Cerf

P1.34 **"Evolution of polynomial invariants of four-qubit systems controlled by local unitary operation"**
Duy Ngoc Hoang and Hoshang Heydari

P1.35 **"Sub shot-noise interferometry and multiparticle entanglement"**
Philipp Hyllus, Luca Pezzé, Augusto Smerzi, Roland Krischek, Christian Schwemmer, Wiesław Laskowski, Witlef Wieczorek, and Harald Weinfurter

P1.36 **"Bi-Partite Separability of Werner State in $2^{2p}$ Dimensions"**
Wei-Chi Su, Kuan-Peng Chen, Ming-Chung Tsai, and Zheng-Yao Su

P1.37 **"A Tripartite Entanglement Witness Independent of the Hilbert Space"**
Jean-Daniel Bancal, Nicolas Gisin, and Stefano Pironio

P1.38 **"Quantum non-locality is a non-additive resource"**
Antonio Acín, Mafalda Almeida, Daniel Cavalcanti, and Valerio Scarani

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# Wednesday, October 6, 2010

## 09:00-12:30 Session O5: Quantum Communication

Session moderator: Antonio Acín

09:00-10:00 O5.1 **Invited talk: "Quantum information processing with trapped individual atoms"**

Philippe Grangier

*Institut d'Optique, Palaiseau, France.*

10:00-10:30 O5.2 **"Quantum interference of photons emitted by two remotely trapped atoms"**

Michael Krug, Julian Hofmann, Florian Henkel, Christoph Kurz, Mathias Müller, Wenjamin Rosenfeld, Markus Weber, and Harald Weinfurter

*Fakultät für Physik, Ludwig-Maximilians-Universität München, D-80799 München, Germany.*

10:30-11:00 Coffee break

11:00-11:30 O5.3 **"Approximate reversal of quantum channels"**

Cédric Bény[1] and Ognyan Oreshkov[2]

[1] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543,* [2] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria.*

11:30-12:00 O5.4 **"Improving zero-error classical communication capacity with entanglement"**

Toby S. Cubitt[1], Debbie W. Leung[2], William Matthews[2], and Andreas Winter[1]

[1] *Department of Mathematics, University of Bristol, Bristol, United Kingdom,* [2] *Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, Canada.*

12:00-12:30 O5.5 **"Quantum hacking: Detector control on gated avalanche photodiodes"**

L. Lydersen[1,2], C. Wiechers[3,4,5], C. Wittmann[3,4], D. Elser[3,4], J. Skaar[1,2] and V. Makarov[1]

[1] *Department of Electronics and Telecommunications, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway,* [2] *University Graduate Center, NO-2027 Kjeller, Norway,* [3] *Max Planck Institute for the Science of Light, Günther-Scharowsky-Str. 1/Bau 24, 91058 Erlangen, Germany,* [4] *Institut für Optik, Information und Photonik, University of Erlangen-Nuremberg, Staudtstraße 7/B2, 91058, Erlangen, Germany,* [5] *Departamento de Física, Universidad de Guanajuato, Lomas del Bosque 103, Fraccionamiento Lomas del Campestre, 37150, Léon, Guanajuato, México.*

12:30-14:00 Lunch

## 14:00-16:50 Session O6: Entanglement 3

Session moderator: Jon Magne Leinaas

14:00-15:00 O6.1 **Invited talk: "Hybrid quantum systems"**

Jörg Schmiedmeyer

*Atominstitut der Österreichischen Universitäten, Wien, Austria.*

15:00-15:20 Coffee break

15:20-15:50 O6.2 **"Discrimination and classification of four-qubit entanglement by means of polynomial invariants"**

Oliver Viehmann[1], Christopher Eltschka[2], Andreas Osterloh[3], and Jens Siewert[4,5]

[1]*Physik Department, Ludwig-Maximilians-Universität, Theresienstr. 37, D-80333 München, Germany,* [2]*Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany,* [3]*Fakultät für Physik, Campus Duisburg, Universität Duisburg-Essen, Lotharstr. 1, 47048 Duisburg, Germany,* [4]*Departamento de Química Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Apdo. 644, 48080 Bilbao, Spain,* [5]*IKERBASQUE, Basque Foundation for Science, Alameda Urquijo 36, 48011 Bilbao, Spain.*

15:50-16:20 O6.3 **"Geometric measure of nonclassicality"**

Paulina Marian and Tudor A. Marian

*Centre for Advanced Quantum Physics, University of Bucharest, P.O.Box MG-11, R-077125 Bucharest-Măgurele, Romania.*

16:20-16:50 O6.4 **"Optimal and scalable telecloning in a limited-control scenario"**

F. Ciccarello[1], M. Paternostro[2], S. Bose[3], D. E. Browne[3], G. M. Palma[4], and M. Zarcone[1]

[1]*CNISM and Dipartimento di Fisica e Tecnologie Relative, Universita' degli Studi di Palermo, Viale delle Scienze, Edificio 18, I-90128 Palermo, Italy,* [2]*School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom,* [3]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom,* [4]*NEST Istituto di Nanoscienze-CNR and Dipartimento di Scienze Fisiche ed Astronomiche, Universita' degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy.*

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18:00- **Conference dinner and boule tournament**

Boulebar Surbrunn, Surbrunnsgatan 46, Stockholm

# Thursday, October 7, 2010

This conference day is arranged in co-operation with Institut Mittag-Leffler's concurrent program on Quantum Information Theory

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## 09:00-12:30 Session O7: Quantum Information 1

Session moderator: Beth Ruskai

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09:00-10:00 O7.1 **Invited talk: "The entropy gain and the Choi-Jamiolkowski correspondence for infinite-dimensional quantum evolutions"**

Alexander Holevo

*Steklov Mathematical Institute, Moscow, Russia.*

10:00-11:00 O7.2 **Invited talk: "Quantum Computation and Cryptography in the presence of Closed Timelike Curves"**

Charles Bennett

*IBM Research, Yorktown Heights, USA.*

11:00-11:30 Coffee break

11:30-12:30 O7.3 **"Applying the Variational Principle to quantum field theories"**

Frank Verstraete

*University of Vienna, Faculty of Physics, Boltzmanngasse 5, 1090 Wien, Austria.*

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12:30-14:00 Lunch

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## 14:00-16:50 Session O8: Quantum Information 2

Session moderator: Debbie Leung

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14:00-15:00 O8.1 **"Explicit information locking from low distortion embeddings"**

Patrick Hayden

*School of Computer Science, McGill University, Montreal, Quebec, H3A 2A7, Canada.*

15:00-15:20 Coffee break

15:20-16:20 O8.2 **"Quantum evolution from partial information"**

Michael M. Wolf

*Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark.*

16:20-16:50 O8.3 **"Göran Lindblad and Stig Stenholm—a tribute"**

Ingemar Bengtsson

*Department of Physics, Stockholm University, SE-109 61 Stockholm, Sweden.*

Professor Göran Lindblad

Professor Stig Stenholm

# Thursday, October 7, 2010

## 16:50-19:00 Poster session 2

P2.1 **"A short and efficient error correcting code for polarization coded photonic qubits in a dissipative channel"**
J. Almlöf and G. Björk

P2.2 **"Reduced density matrix after long time walking"**
Mostafa Annabestani, Seyed Javad Akhtarshenas, Seyed Masoud Amini, and Mohamad Reza Abolhassani

P2.3 **"Superadditive quantum coding in atomic systems"**
Adetunmise Dada, Erika Andersson, Mark Everitt, Martin Jones, and Vivien Kendon

P2.4 **"Isotropic index for unitary quantum errors"**
André L. Fonseca de Oliveira, Efrain Buksman, and Jesús García-López

P2.5 **"Limiting distributions and statistical measures of quantum walks under weak measurements and weak values regimes"**
Debabrata Ghoshal, Marco Lanzagorta, and Salvador E. Venegas-Andraca

P2.6 **"Adaptive versus non-adaptive strategies for quantum channel discrimination"**
Aram W. Harrow, Avinatan Hassidim, Debbie W. Leung, and John Watrous

P2.7 **"Photonic Communications in Biological Systems"**
S. Mayburo

P2.8 **"Information Transfer Constraints and Their Role in Quantum Measurements"**
S. Mayburov

P2.9 **"Limit Distributions of Discrete Time Quantum Walks with Environment"**
Yutaka Shikano

P2.10 **"Entanglement-annihilating channels"**
Mário Ziman and Lenka Moravčíková

P2.11 **"Extended phase space and higher order correlation"**
Hilal Al Hadhrami

P2.12 **"Quantum key distribution robust against photon number splitting attacks"**
Steve Brierley

P2.13 **"Assessing the Hilbert space dimension of arbitrary quantum systems"**
Rodrigo Gallego, Nicolas Brunner, Christopher Hadley, and Antonio Acín

P2.14 **"Bell inequality violations with bipartite-entangled photonic qudits"**
Adetunmise Dada, Erika Andersson, Gerald Buller, Jonathan Leach, and Miles Padget

P2.15 **"SIC-POVM: Star Product and Relation to Other Probability Representations"**
Sergey N. Filippov and Vladimir I. Man'ko

P2.16 **"Quantum polarization of coherent and thermal states"**
Iulia Ghiu, Gunnar Björk, Paulina Marian, and Tudor A. Marian

P2.17 **"Graph states, algebraic curves and Mutually Unbiased bases"**
A. B. Klimov

P2.18 **"A Lot of Fringes: Super-Resolving Phase Measurements Without Entanglement or Joint Detection"**
Christian Kothe, Gunnar Björk, and Mohamed Bourennane

P2.19 **"The wave function collapse as an effect of field quantization"**
K. Lewin

P2.20 **"All-versus-nothing proofs with $n$ qubits distributed between $m$ parties"**
Adán Cabello and Pilar Moreno

P2.21 **"Semiclassical analysis for quantum systems involving two bosonic fields via a generalized Wigner function"**
José Luis Romero and Andrei B. Klimov

P2.22 **"Non-orthogonal mutually unbiased bases and quantum tomography"**
Isabel Sainz, Andrei Klimov, and Luis Roa

P2.23 **"Bell Inequalities from Multilinear Contractions"**
Alejo Salles, Daniel Cavalcanti, Antonio Acín, David Pérez-García, and Michael M. Wolf

P2.24 **"Quantum Degrees of Polarization and Stokes Measurements"**
Jonas Söderholm, Gunnar Björk, Luis L. Sánchez-Soto, and Andrei B. Klimov

P2.25 **"Controllable decoherence of single photon polarization qubits"**
A. Shaham and H. S. Eisenberg

P2.26 **"Measure for the non-Markovianity of quantum processes"**
Elsi-Mari Laine, Jyrki Piilo, and Heinz-Peter Breu

P2.27 **"Effects of Decoherence in the One-Way Model of Quantum Computation: The role of classical and quantum correlations"**
Rafael Chaves and Fernando de Melo

P2.28 **"Programmable discrimination for mixed states"**
Ramon Muñoz-Tapia, Gael Sentís, John Calsamiglia, and Emilio Bagan

P2.29 **"Limitations to the phase-sensitivity in practically realizable quantum systems"**
Oleg Żero and Gunnar Björk

P2.30 **"How asymmetric and correlated errors affect the performance of quantum codes"**
Carlo Cafaro and Stefano Mancini

P2.31 **"Transitions in the communication capacity of dissipative qubit channels"**
D. Daems

P2.32 **"Necessary and sufficient condition for non-zero quantum discord"**
Borivoje Dakić, Vlatko Vedral, and Časlav Brukner

P2.33 **"Searching the Optimal Quantum Circuits"**
Ming-Chung Tsai, Wei-Chi Su, Kuan-Peng Chen, and Zheng-Yao Su

P2.34 **"Classifying Quantum Error Correction Codes"**
Kuan-Peng Chen, Wei-Chi Su, Ming-Chung Tsai, and Zheng-Yao Su

P2.35 **"Single NV center quantum phase gate"**
J.-H. Schönfeldt, J. Twamley, and S Rebić

P2.36 **"Sequential measurement-based quantum computation with memories"**
R. Roncaglia, L. Aolita, A. Ferraro, and A. Acín

P2.37 **"Observation of tunable Popescu-Rohrlich correlations through post-selection of a Gaussian state"**
D. S. Tasca, S. P. Walborn, F. Toscano, and P. H. Souto Ribeiro

P2.38 **"Multipartite entanglement and phase transitions in circuit QED"**
Jonas Larson

P2.39 **"Rare earth doped crystals for quantum memories and slow light"**
Atia Amari, Felix Beaudoin, Stefan Kröll, Huang Maomao, Lin Nan, Lars Rippe, Mahmood Sabooni, Andreas Walther, Mikael Afzelius, Imam Usmani, Björn Lauritzen, Christoph Simon, Nicolas Sangouard, Jirí Minár, Hugues de Riedmatten, and Nicolas Gisin

P2.40 **"Relative-states approach to correlations in quantum systems"**
Pierre Rudolfsson

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# Friday, October 8, 2010

## 09:00-12:30 Session O9: Foundations of quantum mechanics 1

Session moderator: Adán Cabello

09:00-10:00 O9.1 **Invited talk: "Unfrustrated ground states: an algebraic perspective"**

Reinhard Werner

*Technische Universität Braunschweig, Braunschweig, Germany.*

10:00-10:30 O9.2 **"Characterizing the nonlocal correlations created in entanglement swapping experiments"**

Cyril Branciard[1], Nicolas Gisin[2], and Stefano Pironio[3]

[1] *Centre for Quantum Computer Technology, University of Queensland, Australia,* [2] *Group of Applied Physics, University of Geneva, Switzerland,* [3] *Laboratoire d'Information Quantique, Université Libre de Bruxelles, Belgium.*

10:30-11:00 Coffee break

11:00-11:30 O9.3 **"Measuring small longitudinal effects: weak measurements or interferometry?"**

Nicolas Brunner[1] and Christoph Simon[2]

[1] *H. H. Wills Physics Laboratory, University of Bristol, United Kingdom,* [2] *Institute for Quantum Information Science and Department of Physics and Astronomy, University of Calgary, Canada.*

11:30-12:00 O9.4 **"Memory cost of quantum contextuality"**

Matthias Kleinmann[1], Otfried Gühne[2], José R. Portillo[3], Jan-Åke Larsson[4], and Adán Cabello[5]

[1] *Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstr. 21, A-6020 Innsbruck, Austria,* [2] *Institut für Theoretische Physik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria,* [3] *Departamento de Mathemática Aplicada I, Universidad de Sevilla, E-41012 Sevilla, Spain,* [4] *Institutionen för Systemteknik och Matematiska Institutionen, Linköpings Universitet, SE-581 83 Linköping, Sweden,* [5] *Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain.*

12:00-12:30 O9.5 **"Non-disturbing quantum measurements"**

Teiko Heinosaari and Michael M. Wolf

*Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark.*

12:30-14:00 Lunch

## 14:00-16:50 Session O10: Foundations of quantum mechanics 2

Session moderator: Andrei Klimov

14:00-15:00 O10.1 **Invited talk: "Recent Quantum Tests with Photon Pairs"**

Anton Zeilinger

*Universität Wien, Wien, Austria.*

15:00-15:15 O10.2 **"Concluding remarks"**

Mohamed Bourennane

*Department of Physics, Stockholm University, SE-109 61 Stockholm, Sweden.*

15:15- Coffee break

# Author index

# Dissipation: a new tool for quantum information processing

Ignacio Cirac[1]
[1] *Max-Planck-Institut für Quantenoptik, Garching, Germany*

Quantum Entanglement, the most striking feature of Quantum Mechanics, is also the basic ingredient in most applications in the field of Quantum Information. Unfortunately, it is very fragile: in all experiments so far the coupling of the systems to the environment has leads to dissipation which either destroys entanglement or prevents its generation.

Here we propose, analyze, and demonstrate a new method to entangle two distant macroscopic atomic ensembles by purely dissipative means. This counterintuitive effect is achieved by engineering the coupling of our systems to the environment, and leads to a more robust and therefore longer lived entanglement. We will also mention some other applications of dissipative processes in quantum information science.

# Quantum information processing by exploiting the photonic orbital angular momentum

Eleonora Nagali,[1] Daniele Giovannini,[1] Lorenzo Marrucci,[2] and Fabio Sciarrino[1,3,*]

[1] *Dipartimento di Fisica dell'Università "La Sapienza", Roma 00185, Italy*
[2] *Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Compl. Univ. di Monte S. Angelo, 80126 Napoli, Italy*
[3] *Istituto Nazionale di Ottica, Firenze, Italy*

In quantum information processing based on optical techniques, single photons offers a variety of degrees of freedom in which information can be encoded. By exploiting these resources, it is possible to implement high-dimensional quantum states, or *qudits*, which enable higher security in quantum cryptographic protocols, as well as implications in fundamental quantum mechanics theory. In the last few years, many improvements have been achieved for qudit states with $d = 3$ (*qutrits*) and $d = 4$ (*ququarts*). In particular the generation of photonic qutrit and ququart states exploits either many-photon systems or different degrees of freedom such as linear momentum, arrival time of twin photons, and orbital angular momentum.

In this framework, the orbital angular momentum of photons, being defined in an infinitely dimensional Hilbert space, offers a promising resource for high-dimensional optical quantum information protocols. Recently we introduced and tested experimentally a series of optical schemes for the coherent transfer of quantum information from the polarization to the orbital angular momentum (OAM) of single photons and vice versa [1, 2]. All our schemes exploit a newly developed optical device, the so-called "q-plate", a suitably patterned non-uniform birefringent plate, which enables the manipulation of the photon orbital angular momentum driven by the polarization degree of freedom. Our experiments prove that these schemes are reliable, efficient and have a high fidelity. The optical "spin-orbit" coupling occurring in the q-plate also allows entangling the polarization of a single photon with its orbital angular momentum [1]. By making use of these spin-OAM information transfer tools, we latter reported the first observation of the HongOuMandel coalescence of two incoming photons having non-zero OAM into the same outgoing mode of a beamsplitter [3]. Such an effect has then been used to carry out the $1 \rightarrow 2$ universal optimal quantum cloning of OAM-encoded qubits, using the symmetrization technique already developed for polarization. By adopting a similar approach, we realized a hybrid polarization-orbital angular momentum entangled states by adopting a spontaneous parametric down conversion source of polarization entangled states and a polarization-OAM transferrer [4].

As following step, the experimental generation of a hybrid ququart encoded in the polarization and orbital angular momentum of a single photon has been recently achieved [5]. We will report the experimental realization of optimal quantum cloning for ququart states, i.e. quantum states with $d = 4$, encoded in the polarization and orbital angular momentum degrees of freedom [6].

Finally we will discuss the perspectives of this research: the generation of cluster states, hyper-entangled states of polarization and orbital angular momentum and their applications for quantum information processing and for non-locality tests.

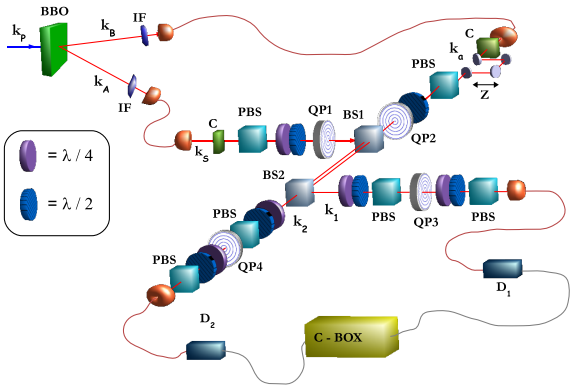


FIG. 1: Schematic representation of the $1 \rightarrow M$ cloning process of a qudit state through the symmetrization technique.

* Electronic address: fabio.sciarrino@uniroma1.it; URL: http://quantumoptics.phys.uniroma1.it

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# Photons walking the line – compact and scalable fibre loop quantum walk

Andreas Schreiber,[1] Katiuscia N. Cassemiro,[1] Vašek Potoček,[2] Aurel Gábris,[2,*] Peter Mosley,[1,†] Erika Andersson,[3] Igor Jex,[2] and Christine Silberhorn[1]

[1]*Max Planck Institute for the Science of Light, Günther-Scharowsky-str. 1 / Bau 24, 91058 Erlangen, Germany.*
[2]*Department of Physics, FNSPE, Czech Technical University in Prague, Břehová 7, 115 19 Praha, Czech Republic.*
[3]*SUPA, School of EPS, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom.*[‡]

The classical random walk is a fundamental model in the natural sciences, used to describe phenomena ranging from material transport in media to the evolution of stock market shares. For a quantum walk, a superposition of different paths or positions leads to quantum interference. Quantum walks have been shown to speed up search problems [1] and to play a role as a computational primitive [2]. Initial realisations of quantum walks exist [3], but it remains important to extend the number of walk steps and the flexibility of the setups.

We show how to explore the physics of quantum walks using a robust, flexible and simple optical implementation [4]. A straightforward setup would use a cascade of beam-splitters (Fig.1a). Unfortunately such a complex network suffers from mechanical instabilities and requires a fast increasing number of optical components to realize more steps of the walk. We avoid such complications by a realisation using optical fibres, employing a compact network loop as illustrated in Fig.1b. The polarization of the photon is manipulated by the quantum coin. The photon takes a longer path if the polarization is horizontal (H) and a shorter path if it is vertical (V), meaning that the quantum walk takes place in the time domain. A single half-wave plate (HWP) is used to prepare a symmetric superposition of $|H\rangle$ and $|V\rangle$. After that, the photon is sent through a polarization maintaining fiber, which delays the vertical component by 5ns with respect to the horizontal component. The fiber output is a photon wavepacket that is a superposition of two pulses. The walk continues by sending the photon again through the HWP (coin) and fiber (step), resulting in quantum interference of pulses that exit at the same time. After each step there is a 50% probability that the photon leaves the walk, in which case it can trigger an APD.

By realizing an ensemble of measurements one obtains the probability distribution for the arrival times (Fig.1c). The distribution after five steps is shown in Fig.1d. The obtained data clearly deviates from the expected classical distribution, instead agreeing with what would be expected for a quantum walk. The analysis of our results show that our implementation is free of degrading effects coming from phase drifts, and that the major source of errors comes from back reflections from the fiber. By optimising the setup, this source of error can still be reduced, and a quantum walk of 100 steps could be feasible.

Since this implementation enables to us easily operate with different coins, and since one may individually address time pulses by using commercially available fast electro-optical modulators (EOM), our work opens exciting new possibilities for the realization of quantum information protocols. The present experimental setup constitutes a starting point for implementing a one-dimensional quantum-walk-based search algorithm and for investigating time-dependent effects like Anderson localization.

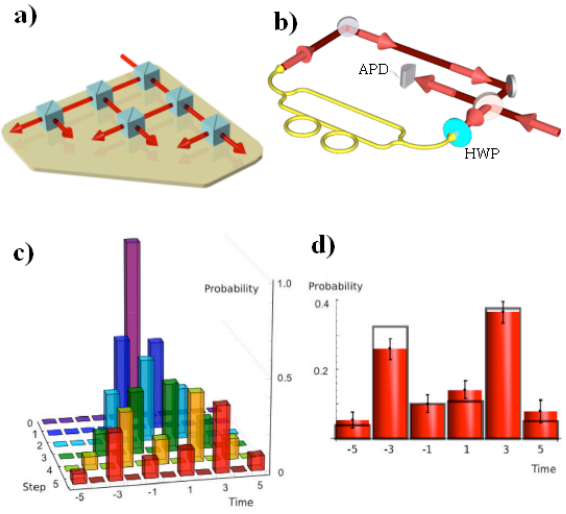


FIG. 1: a) Beam-splitter cascade. b) Sketch of scalable setup. c) Measured probability distribution from the beginning of the walk to the 5th step. d) Detail of the 5th step. Filled bars: measurement; Black frames: theory.

* Secondary address: Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, H-1525 Budapest, P. O. Box 49, Hungary.
† Present address: Centre for Photonics and Photonic Materials, Departmen of Physics, University of Bath, Claverton Down, Bath, BA2 7AY, United Kingdom.
‡ Electronic address: E.Andersson@hw.ac.uk

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[3] H. Schmitz et al., Phys. Rev. Lett. 103, 090504 (2009); M. Karski et al., Science 325, 174 (2009). F. Zähringer et al., *Phys. Rev. Lett.* **104**, 100503 (2010); D. Bouwmeester et al., *Phys. Rev. A* **61**, 013410 (1999).
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# Quantum Cloning for absolute radiometry

Enrico Pomarico,[1,*] Bruno Sanguinetti,[1] Pavel Sekatski,[1] Hugo Zbinden,[1] and Nicolas Gisin[1]

[1] *Group of Applied Physics, University of Geneva, 20 rue de l'École-de-Médecine, Geneva, Switzerland.*

Radiometry and quantum mechanics have always had deep ties. Radiometric measurements of the spectral distribution of heated black bodies [1] led Planck in 1900 to lay the foundations of the quantum theory. Nowadays, on the other hand, quantum effects can be exploited to provide conceptually new radiometric primary standards, for instance the correlations in photon pairs produced by Spontaneous Parametric Down Conversion (SPDC) can be used for the absolute calibration of single photon detectors [2].

Quantum information science can offer a new approach for absolute radiometry by exploiting a particular aspect of the quantum to classical transition. Optimal quantum cloning machines [3] produce copies whose fidelity increases gradually to 1 for a growing number of identical input qubits, i.e. the quality of cloning becomes perfect as the initial information becomes classical. Optimal cloning can be realized by stimulating PDC in bulk nonlinear crystals or by stimulating the emission in atomic systems [4].

In this abstract we present a novel radiometer based on stimulated emission in a $Er^{3+}$ doped fiber, whereby the optimal universal quantum cloning of polarization photonic qubits can be implemented. The working principle of this radiometer is schematically shown in Fig. 1. A certain amount of luminous power is sent to an $Er^{3+}$ doped fiber pumped by a diode laser, which provides the amplification. Cloning optimality has been demonstrated to be achieved even in a non-ideal amplifier [5]. Then, the fidelity $\mathcal{F}$ of the output polarization state is determined via a relative measurement as $\mathcal{F} = \frac{P_{||}}{P_{||}+P_{\perp}}$, where $P_{||}$ and $P_{\perp}$ are the output powers in the polarizations parallel and perpendicular to the polarization of the input light. The number of photons per mode, i.e. the spectral radiance, of the input radiation $\mu_{in}$ is finally obtained by a relation that for high amplification gains can be approximated by $\mu_{in} \simeq \frac{2\mathcal{F}-1}{1-\mathcal{F}}$.

We test the accuracy of this method by comparing the value of $\mu_{in}$ with the one obtained by using a calibrated reference power meter: $\mu^*_{in}$. The amplification gain is determined by directly comparing $\mu^*_{in}$ with $\mu^*_{out}$, the number of photons per mode at the output of the amplifier (inset of Fig. 2). Fig. 2 shows the fidelity values $\mathcal{F}$ versus $\mu^*_{in}$. By fitting the data, the discrepancy between our measurement of $\mu_{in}$ and the value obtained from the reference powermeter can be estimated. By considering systematic errors, we achieve an accuracy of approximately 4%.

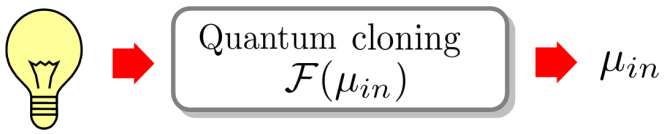


FIG. 1. Principle of the quantum cloning radiometer.

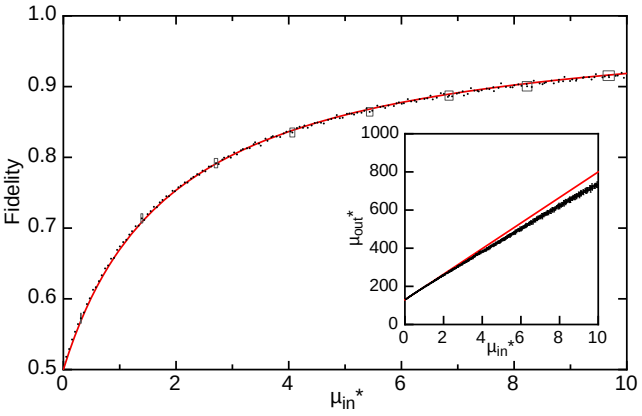


FIG. 2. Fidelity versus $\mu^*_{in}$ with fitting curve. Inset: $\mu^*_{out}$ versus $\mu^*_{in}$ with fitting line on the first data points ($\mu^*_{in} < 1$) in order to estimate the amplification gain.

Different aspects make this scheme attractive: it is insensitive to losses, since the fidelity is given by a relative measurement; it easily achieves high amplification gains through the atomic stimulated emission; it can measure power levels ranging from single photon up to several tens of nW; it is completely implemented in fibre: this ensures the presence of a single spatial mode and makes the system readily applicable to the current telecom technology.

We demonstrated that quantum cloning can be used for absolute power measurements by performing a proof-of-principle all-fibre experiment at telecom wavelengths, achieving a very good accuracy.

* Enrico.Pomarico@unige.ch

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# Robust cluster state generation using ancilla-based systems

Viv kendon,[1,*] Katherine Louise Brown,[1] Clare Horsman,[2] and Bill Munro[3]

[1]*School of Physics and Astronomy, University of Leeds, LS2 9JT, United Kingdom.*
[2]*Department of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, UK*
*H. H. Wills Physics Laboratory, University of Bristol, Tyndall Avenue, Bristol, BS8 1TL, UK*
[3]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*
*NTT Basic Research Laboratories, 3-1, Morinosato Wakamiya Atsugi-shi, Kanagawa 243-0198, Japan*

Efficient generation of cluster states is crucial for engineering large-scale measurement-based quantum computers. Hybrid matter-optical systems offer a robust, scalable path to this goal. Such systems have an ancilla which acts as a bus connecting the qubits. Ancilla-driven schemes are important for chip-based quantum computing architectures, where the flying ancilla mediates between static qubits. Hybrid architectures form a natural substrate for measurement-based quantum computing (MBQC) [1], one type of which (the topological model) has the best error threshold for quantum computing [2]. In this type of processing a highly-entangled *cluster state* is generated, and then computation performed by sequential qubit measurements. The quantum processing task is to generate the cluster state, after which it becomes a matter of measurement and classical communication. In physically-realizable implementations the cluster is prepared dynamically, a few layers at a time.

As the cluster state is the fundamental quantum resource of a measurement-based computation, it becomes extremely important to make it as error-free as possible. Errors in constructing the cluster can propagate rapidly through a computation because of the highly-entangled nature of the state, leading to failure of the computation. Hybrid systems are susceptible to specific error types that other systems are not, because of the use of the mediating ancilla. In cases where the ancilla is not destroyed after each gate there is a nonzero probability of errors propagating through ancilla reuse.

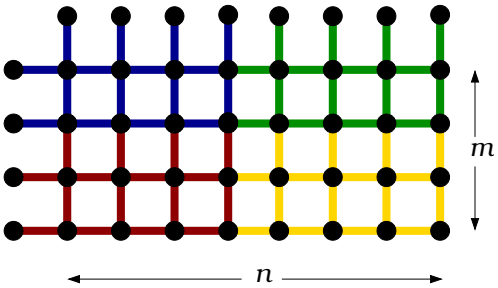


FIG. 1: A cluster of 4 bricks, each made using a different bus.

We present the optimal scheme for dynamic fault-tolerant 2-D cluster state generation in hybrid systems where the mediating system can be used for more than a single gate operation without being reset. We divide the cluster state into "Lego bricks", each of which uses a single bus. We give the optimal method for constructing the bricks, reducing the number of system-bus entanglements. We then show how to determine the block size based on the error threshold of the system being used. We find that, even when the probability of error in the system is high, this scheme can still deliver significant efficiency savings through bus reuse, enabling a larger cluster to be generated. By reducing the time required to prepare sections of the cluster, bus reuse more than doubles the size of the computational workspace that can be used before decoherence effects dominate [3].

A simple example of the bricks is shown in figure 1. They are always the same height ($m$ direction), but have length $b$ ($n$ direction) determined by [3]

$$\frac{1}{2}\left(1-\exp[-(6b+4)\gamma\tau-16b\eta\beta^2]\right)\leq\varepsilon. \qquad (1)$$

For a given set of experimental parameters $\gamma$, $\tau$, $\eta$ and $\beta$, and desired dephasing limit to $\varepsilon$, this determines $b$. If we use one bus per CPhase gate to generate a brick, we have

$$\frac{1}{2}\left(1-\exp[-16b\gamma\tau-4\eta\beta^2]\right)\leq\varepsilon. \qquad (2)$$

Comparing equations (1) and (2), we find our Lego scheme produces less qubit dephasing than using one bus per CPhase gate provided $\eta\beta^2\lesssim\gamma\tau/2$.

Compared with $8mn-4(m+n)$ bus operations with no bus reuse, for large clusters, the Lego scheme uses less than half for $b>2$. And even for $b=1$, the reduction is $O(5mn)$ compared to $O(8mn)$, equivalent to the method of [4] for five qubits per bus. This will thus be the method of choice for any ancilla-based cluster generation that allows bus reuse.

* Electronic address: V.Kendon@leeds.ac.uk; URL: http://quantum.leeds.ac.uk/~viv

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# Quantum Information Science with Trapped $Ca^+$ Ions

Rainer Blatt[1]

[1] *Universtität Innsbruck, Innsbruck, Austria*

Trapped strings of cold ions provide an ideal system for quantum information processing. The quantum information can be stored in individual ions and these qubits can be individually prepared; the corresponding quantum states can be manipulated and measured with nearly 100% detection efficiency. With a small ion-trap quantum computer based on up to fourteen trapped $Ca^+$ ions as qubits we have generated genuine quantum states in a pre-programmed way. In particular, we have generated GHZ and W states in a fast and scalable way and we have demonstrated, for the first time, a Toffoli gate with trapped ions which is analyzed via state and process tomography. High fidelity CNOT-gate operations are investigated towards fault-tolerant quantum computing and decoherence of multi-qubit GHZ states was investigated. First experiments implementing repetitive quantum error correction will be reported. As applications of quantum information processing, quantum simulations of the Dirac equation were implemented and a quantum walk with a trapped ion was realized.

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# Increasing the statistical significance of entanglement detection in experiments

Bastian Jungnitsch,[1,*] Sönke Niekamp,[1] Matthias Kleinmann,[1] Otfried Gühne,[1,†] He Lu,[2] Wei-Bo Gao,[2] Yu-Ao Chen,[2,‡] Zeng-Bing Chen,[2] and Jian-Wei Pan[2,‡]

[1] *Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21A, A-6020 Innsbruck, Austria*

[2] *Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*

Entanglement is often verified by a violation of an inequality like a Bell inequality or an entanglement witness. Considerable effort has been devoted to the optimization of such inequalities in order to obtain a high violation [1]. We demonstrate theoretically and experimentally that such an optimization does not necessarily lead to a better entanglement test, if the statistical error is taken into account [2].
To this end, we consider the statistical significance

$$\mathcal{S} = \frac{\mathcal{V}}{\mathcal{E}}\,. \tag{1}$$

of a Bell inequality or an entanglement witness. Here, $\mathcal{V}$ refers to the violation of a Bell inequality or, in the case of an entanglement witness, to the negative expectation value of the witness. $\mathcal{E}$ denotes the statistical error. Theoretically, we then show for different error models that reducing the violation can improve the significance. This effect occurs both in a simple error model in which we take the square root of the variance as statistical error and in the standard photonic error model. The latter assumes a Poisson distribution of the experimental count numbers and applies Gaussian error propagation.

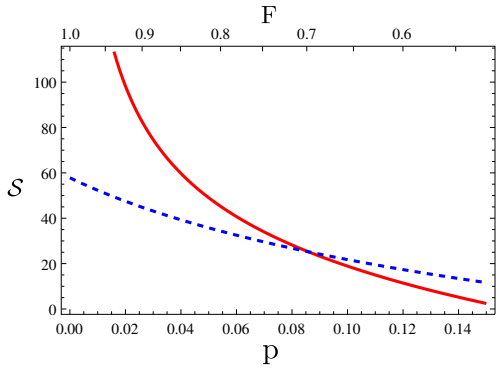


FIG. 1: Theoretical statistical significances $\mathcal{S}$ for Mermin (red, solid) and Ardehali (blue, dashed) inequality of a noisy four-qubit GHZ state. The perfect GHZ state is affected by bit-flip noise with flip probability p (and has thus fidelity F with respect to the perfect state).

We then compare the Mermin [3] and the Ardehali [4] inequality, the latter of which allows for a higher violation of the local hidden variable bound than the former. For a four-qubit GHZ state whose qubits are individually affected by bit-flip noise with bit-flip probability p, we obtain Figure 1 for the statistical significances of these two entanglement tests. We find that for a small amount of bit-flip noise, the Bell inequality with a lower violation, namely the Mermin inequality, nevertheless provides a statistically stronger test.
Moreover, we test our findings in a photonic four-qubit experiment for different levels of bit-flip noise. In this way, we can verify the high statistical significance of the Mermin inequality for states with a high fidelity with respect to the perfect GHZ state experimentally.
Finally, we trace this observation back to the fact that the Mermin inequality contains only stabilizer terms of the GHZ state. Thus, we provide a way to obtain entanglement tests with a high statistical significance.

* Electronic address: `bastian.jungnitsch@oeaw.ac.at`
† Also at Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
‡ Also at Physikalisches Institut, Ruprecht-Karls-Universität Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany

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# Levitated Nano-dielectrics in the Quantum Regime

Oriol Romero-Isart,[1,*] Anika C. Pflanzer,[1] and J. Ignacio Cirac[1]

[1] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748, Garching, Germany.*

We propose to optically levitate a dielectric nano-dielectric inside a high-finesse optical cavity in order to couple its mechanical center-of-mass motion to the optical cavity field [1, 2]. This opto-mechanical system has the unique feature of not having thermal contact to other objects. We analyze the main sources of decoherence: scattering of the sorrounding gas, scattering of light, and coupling to the other modes, and show that for sufficiently small objects these sources of noise are also negligible. Therefore, the center-of-mass is effectively isolated.

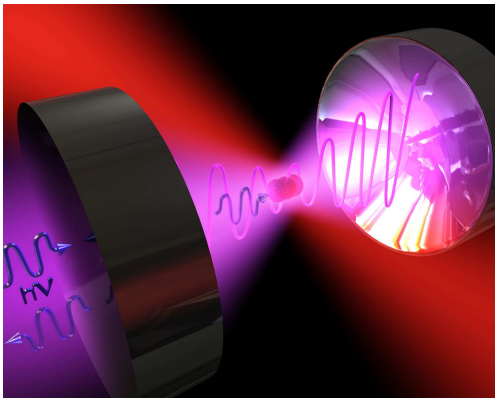


FIG. 1: Schematic illustration of a levitated nano-dielectric trapped in a high-finesse optical cavity.

Then we propose different protocols to couple the mechanical motion with the field out of the cavity, and to use non-Gaussian states of light to prepare non-Gaussian states of the mechanical object, such as superposition of Fock states [1, 3]. This is an effective way to obtain non-linearities in quantum optomechanical systems. We analyze how tomography of the mechanical state can be performed by time-of-flight experiments, exploiting the analogy of levitating objects with atomic physics. We also argue that time-of-flight experiments can be used to prepare macroscopic superposition states of nano-spheres (preparing the center-of-mass in a superposition of two positions separated by a distance of the order of the radius of the sphere), and discuss how this can be used to test objective reduction of the wave-function models [3]. Finally, we argue that the theory presented here for dielectric nano-spheres can be extended to arbitrarily shaped dielectric objects, and in particular even to microorganisms.

A part from the application of levitated nano-objects to experimentally address the above mentioned fundamental questions, we will also discuss how these systems might be used as quantum transducers for quantum information and computation purposes.

* Electronic address: oriol.romero-isart@mpq.mpg.de

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# Experimental verification of quantum process in one-way quantum computing

Y. Tokunaga,[1] S. Okamoto,[2] R. Ikuta,[2] R. Namiki,[3] T. Yamamoto,[2] M. Koashi,[2] and N. Imoto[2]

[1]*NTT Information Sharing Platform Laboratories, NTT Corporation, Tokyo, Japan* *

[2]*Division of Materials Physics, Graduate School of Engineering Science, Osaka University, Japan*

[3]*Department of Physics, Graduate School of Science, Kyoto University, Japan*

In experimental demonstration of quantum gates or channels, it is important to verify the process is beyond a classical limit, namely, the result cannot be obtained by classical processes. In this work, we demonstrate photonic one-way quantum computing and verify the process is in quantum-domain with several methods using the concept of entanglement breaking channels, entanglement capability, or remote state preparation. In this abstract, we mainly describe our proposed verification method that uses the idea of entanglement breaking channels and process fidelity.

First, we briefly introduce our experiment. We produce four-photon cluster states, e.g., $|C_4\rangle = \frac{1}{2}(|HHHH\rangle + |HHVV\rangle + |VVHH\rangle - |VVVV\rangle)$, as a resource for one-way quantum computing. Our schemes for preparing $|C_4\rangle$ and for experimental one-way quantum computing are based on Ref. [1, 2] (Fig. 1 (a)). The fidelity of the obtained four-photon cluster state is $F \geq 0.860 \pm 0.015$ [1, 3]. The quantum circuits in Fig. 1 (b) and (c) show implementation of a single-qubit rotation and the controlled-phase gate. The obtained process fidelities of the single-qubit rotation and the controlled-phase gate are $F_p = 0.90 \pm 0.02$ and $F_p = 0.86 \pm 0.02$, respectively.

We verify our experimental process using the concept of entanglement breaking channels [4]. A process is called *entanglement breaking* (EB) if and only if the process is written in measurement-and-prepare scheme $\mathcal{E}_{EB}(\rho) = \sum_i \langle\phi_i|\rho|\phi_i\rangle|\varphi_i\rangle\langle\varphi_i|$, which implies that the output of the process is produced merely by classical data processing from the measurement outcomes and that the process breaks quantum entanglement shared between the input of the process and other systems. Thus, it is a natural benchmark for a given experimental process to test whether it outperforms EB channels. Here we propose a simple verification method using process fidelity and the concept of EB channels. Process fidelity is a natural evaluation of a quantum process and bounds on a process fidelity can be obtained with significantly less experimental effort [5] than process tomography. Thus, this method will be widely useful for experiments. Suppose the experimentally obtained process fidelity $F_p = \mathrm{Tr}(\chi_{exp}\chi)$ exceeds the maximal process fidelity between any EB channel and an ideal process, $\max_{\mathcal{E}_{EB}} \mathrm{Tr}(\chi_{\mathcal{E}_{EB}}\chi)$. This demonstrates that the experimental process outperforms any EB channel. This bound is easily calculated using Jamiolkowski isomorphism, which relates a quantum operation $\mathcal{E}$ to a quantum state $\rho_{\mathcal{E}} = (I \otimes \mathcal{E})|\Phi\rangle\langle\Phi|$, where $|\Phi\rangle$ is a $d$-dimensional maximally entangled state with $d$ being the dimension of the input of the process $\mathcal{E}$. When a process is EB, the corresponding state $\rho_{\mathcal{E}_{EB}}$ is separable, i.e., the Schmidt number [6] is 1. On the other hand, when the process is an ideal unitary operation $\mathcal{E}_U$, $\rho_{\mathcal{E}_U}$ is a maximally entangled state and the Schmidt number is $d$. Consequently, the maximal process fidelity is calculated as $\max_{\mathcal{E}_{EB}} \mathrm{Tr}(\chi_{\mathcal{E}_{EB}}\chi) = 1/d$ and surpassing process fidelity $1/d$ assures that the process is not EB. Our experimentally obtained process fidelities of a single-qubit rotation and the controlled-phase gate both clearly exceed $1/d$ (1/2 and 1/4, respectively), which demonstrates that the processes outperform any EB channel. We can also extend this idea to Schmidt number greater than 1. For process $\mathcal{E}'$ such that the Schmidt number of $\rho_{\mathcal{E}'}$ is $k < d$, we similarly obtain $\max_{\mathcal{E}'} \mathrm{Tr}(\chi_{\mathcal{E}'}\chi) = k/d$. Thus, when an experimentally obtained process fidelity surpasses $k/d$, it is assured that the process at least preserves Schmidt number $k+1$ of entanglement between the input of the process and other systems. Our experimentally obtained process fidelity to the controlled-phase gate surpasses 3/4, which demonstrates that the process preserves full Schmidt number between the input and other systems.

We also present evaluations of the entanglement capability of the controlled-phase gate from the fully reconstructed process matrix, and another verification method [1] that analyzes the classical limits of remote state preparation [7], the general framework of which was recently reported [8].

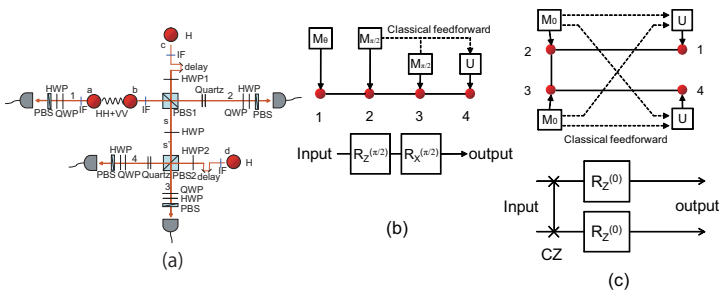


FIG. 1: (a) Experimental setup. (b) Physical implementation and the quantum circuit of a single-qubit rotation and (c) the controlled-phase gate.

* Electronic address: tokunaga.yuuki@lab.ntt.co.jp

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# Manipulation and control of non-classical field states in a cavity by quantum non-demolition measurements and quantum Zeno dynamics

Serge Haroche[1]
[1] *École Normale Superieure, Paris, France*

By performing quantum non-demolition (QND) counting of photons in a cavity, we prepare and reconstruct Fock and Schrdinger-cat states of the trapped field. Quantum feedback exploiting QND can be implemented to protect non-classical field states against decoherence. Dynamical decoupling methods related to the quantum Zeno effects can also be used to manipulate the field in the cavity and realize various kinds of coherent field-state superpositions. The talk will review the present status of these studies and describe experiments in preparation.

# Disentanglement for Finite Losses in Bipartite Systems

F. A. S. Barbosa[1], A. S. Coelho[1], A. J. de Faria[1], K. N. Cassemiro[2], A. S. Villar[2,3], P. Nussenzveig[1], and M. Martinelli[1]

[1] *Instituto de Física, Univ. de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, SP, Brazil.*
[2] *Max Planck Inst. for the Science of Light, Günther-Scharowsky-str. 1 / Bau 24, 91058 Erlangen, Germany.*
[3] *Univ. of Erlangen-Nuremberg, Staudtstr. 7/B2, 91058 Erlangen, Germany.*

The study of non-classical correlations is a fundamental issue in quantum theory [1, 2]. Among those non-classical correlated systems, entangled states are possibly the most prominent ones. From a practical point of view, quantum entanglement is a key feature in many applications in quantum information processing, storage and communication. In this context, the robustness of entanglement under the unavoidable action of the environment is necessary for these applications to be successful.

We experimentally analyzed the robustness of continuous-variable bipartite states possessing Gaussian statistics. We used an above threshold optical parametric oscillator (OPO) to generate entangled twin beams. Such states are fully characterized, except for a phase space translation, by the covariance matrix of the phase space operators, $q$ and $p$. Those are defined by the commutation relations $[q, p] = 2i$, that are satisfied by the quadrature operators of the electromagnetic field. In our case, the action of the environment is limited to dissipation only, which is implementated by losses in a beam-spliter. In this context, we call robust those states that maintain their entanglement until total loss.

The entanglement test is done using the criterion of positivity under partial transposition (PPT). It is a necessary and sufficient condition for two-mode Gaussian states. We observed regions of both robust entanglement and states that becomes separable for partial losses [5], a finite-loss parallel to the finite-time disentanglement, so-called entanglement sudden death (ESD) [3]. This phenomenon was recently observed in continuous-variable systems for a tripartite Gaussian state [4]. In the present experiment, we change from robust entanglement to ESD states adding phase noise to the system by increasing the pump power. Only one beam wss attenuated and even with this simple dissipation we are able to observe ESD. The results are presented in Fig. 1

We also theoretically studied the robustness of entanglement in continuous-variable two-mode bipartite systems. We derived, from the PPT criterion, one condition that attests the robustness of entangled states. For a suitable choice of quadrature basis it becomes

$$\delta_1\delta_2 - (c_p - c_q)^2 < 0, \qquad (1)$$

where $\delta_i = \langle(\Delta q_i)^2\rangle + \langle(\Delta p_i)^2\rangle - 2$, $c_q = \langle\Delta q_1 \Delta q_2\rangle$ and $c_p = \langle\Delta p_1 \Delta p_2\rangle$, with $\Delta\xi = \xi - \langle\xi\rangle$ for any operator $\xi$. Since (1) is derived from the PPT criterion, it is a sufficient condition for robust entanglement and becomes also necessary for Gaussian states. We also showed that (1) is an optimized form of the Duan criterion [6].

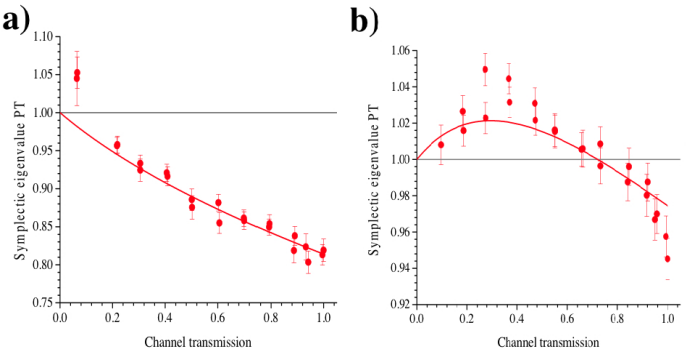


FIG. 1. Bipartite entanglement as a function of losses for different values of the pump power relative to threshold power ($\sigma = P_{pump}/P_{th}$). The state is entangled when the symplectic eigenvalue of the partially transposed covariance matrix is less than unity. a) Robust entangled state with $\sigma = 1.20$. b) ESD state with $\sigma = 1.38$.

In our experiment, we generated ESD states from robust ones by adding noise. However, this can also be done only by local unitary transformations. Actually, for Gaussian states, local squeezing can transform robust states in ESD or vice versa. In other words, one can change the entanglement behavior under losses wihtout change the amount of entanglement and the purity. This seems surprising and warrants further investigations for use in quantum information tasks.

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# Heralded Noiseless Linear Amplification and Distillation of Entanglement

G. Y. Xiang,[1] T. C. Ralph,[2] A. P. Lund,[1] N. Walk,[2] and G. J. Pryde[1,*]

[1] *Centre for Quantum Dynamics and Centre for Quantum Computer Technology, Griffith University, Brisbane, 4111, Australia*
[2] *Department of Physics, University of Queensland, Brisbane, 4072, Australia*

The idea of signal amplification is ubiquitous in the control of physical systems, and the ultimate performance limit of amplifiers is set by quantum physics. Increasing the amplitude of an unknown quantum optical field must introduce noise. This linear amplification noise prevents the perfect copying of the quantum state, enforces quantum limits on communications and metrology, and is the physical mechanism that prevents the increase of entanglement via local operations. It is known that non-deterministic versions of ideal cloning [1] and local entanglement increase (distillation) [2] are allowed, suggesting the possibility of non-deterministic *noiseless* linear amplification. Here we introduce, and experimentally demonstrate, such a noiseless linear amplifier for continuous-variables optical states, and use it to demonstrate distillation of field-mode entanglement [3].

We consider a device that performs the transformation

$$|\alpha\rangle\langle\alpha| \rightarrow \rho(\alpha) = P|g\alpha\rangle\langle g\alpha| + (1-P)|0\rangle\langle 0|. \qquad (1)$$

where $g$ (the amplitude gain) is a real number obeying $|g| > 1$ and $|\alpha\rangle$ is a coherent state of the field or oscillator with complex amplitude $\alpha$. Such a device can be realized for small $|\alpha|$ by the circuit in Fig. 1. For larger amplitudes, there exists a probabilistic (but heralded) method for dividing the state into smaller amplitude components that are each amplified separately and then recombined.

Phase-averaged coherent states provide a very general test state for a phase-independent amplifier. We generated states of the form $\rho_{\rm in} = (1-k)|0\rangle\langle 0| + k|1\rangle\langle 1|$, choosing $k = |\alpha|^2$, by attenuating one arm of a weak, polarization-unentangled spontaneous parametric down-conversion (SPDC) source. Such a state $\rho_{\rm in}$ has a theoretical fidelity of $> 0.9998$ with the desired mixed coherent state when the average photon number is 0.02 in both cases. We used the other photon from the SPDC pair as the ancilla photon that drives the amplifier stage.

In order to verify that amplification has occurred, we used photon counting to compare the measured average photon number at the input and output of the amplifier stage. Table I shows the measured intensity gain, $g^2$, as a function of the gain control reflectivity $\eta$.

In order to verify that the gain process is coherent and does not add noise, we embedded the amplifier stage in an interferometer. If the conditional amplification is perfect, then no noise is added and because the two arms are balanced in power—ideally the fringe visibility is unity. In contrast, ordinary linear amplification introduces phase noise, decreasing the visibility significantly. Measured interference visibilities are shown in Table I.

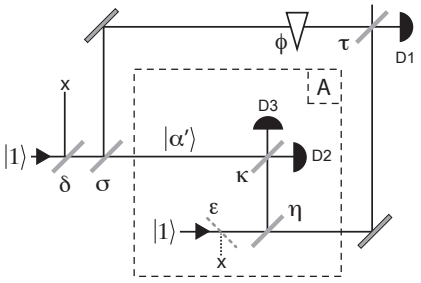


FIG. 1: Conceptual circuit for experimental demonstration of the basic amplifier stage "A", embedded in a test interferometer. The gain is controlled by beam splitter ratio $\eta$.

| $\eta$ | $g^2$ (exp) | $g^2$ (thy) | $V$ (exp) | $V$ (lin) |
|---|---|---|---|---|
| 1/3 | $2.05 \pm 0.06$ | 2 | $0.929 \pm 0.024$ | 0.675 |
| 1/4 | $2.97 \pm 0.08$ | 3 | $0.910 \pm 0.029$ | 0.514 |
| 1/5 | $3.85 \pm 0.10$ | 4 | $0.936 \pm 0.022$ | 0.419 |

TABLE I: Measured gains and interferometric visibilities, for several settings of $\eta$ and for $|\alpha|^2 = 0.02$, compared with the theoretical values for a standard linear amplifier (lin) with the same gain, input photon number, an interferometer configuration. The higher visibility of the experimental demonstration is evidence of the coherent operation of the amplifier.

Our results can also be seen as a demonstration of distillation of field entanglement. We detemine the concurrence of the field entanglement in the interferometer, with and without amplification, using the method of Ref. [4]. We compare the measured post-amplification concurrence ($c_{\rm out}$) with the theorectical value ($c_{\rm in}$) for the interferometer input configuration—this provides the most stringent test. For $g^2 \approx 4$, we determine $c_{\rm in} = 0.016$, and measure $c_{\rm out} = 0.028 \pm 0.002$, which is greater than $c_{\rm in}$.

* Electronic address: g.pryde@griffith.edu.au

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# Permutationally Invariant Quantum Tomography

Géza Tóth,[1,2,3] Witlef Wieczorek,[4,5,*] David Gross,[6]
Roland Krischek,[4,5] Christian Schwemmer,[4,5] and Harald Weinfurter[4,5]

[1]*Department of Theoretical Physics, The University of the Basque Country, P.O. Box 644, E-48080 Bilbao, Spain*
[2]*IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain*
[3]*Research Institute for Solid State Physics and Optics, Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary*
[4]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*
[5]*Fakultät für Physik, Ludwig-Maximilians-Universität, D-80797 München, Germany*
[6]*Institute for Theoretical Physics, Leibniz University Hannover, 30167 Hannover, Germany*

We propose permutationally invariant tomography in many-qubit quantum experiments. Concretely, instead of the density matrix $\varrho$, we propose to determine the permutationally invariant part of the density matrix defined as

$$\varrho_{\rm PI} = \frac{1}{N!}\sum_k \Pi_k \varrho \Pi_k, \qquad (1)$$

where $\Pi_k$ are all the permutations of the qubits.

We develop a provably optimal scheme, which is feasible for large multi-qubit systems: For our method, the number of local measurement settings needed increases *quadratically* with the size of the system. For a setting, the same operator $A_j$ has to be measured on all the qubits. Our approach is further motivated by the fact that almost all multipartite experiments are done with permutationally invariant quantum states. We demonstrate that these techniques are viable in practice by applying them to a photonic experiment observing a four-qubit symmetric Dicke state. The results are shown in Fig. 1 and Fig 2.

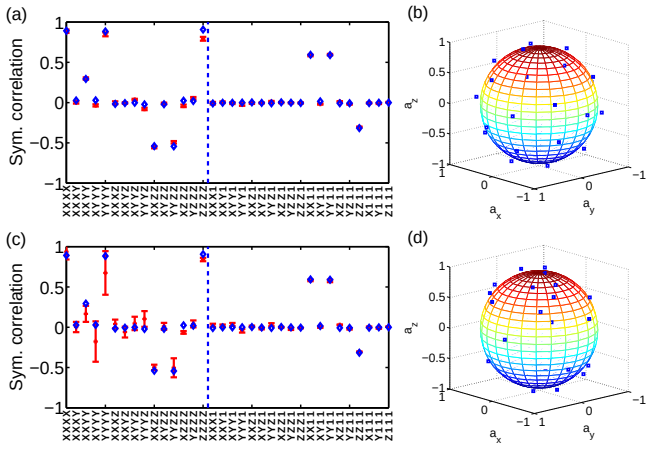


FIG. 1: (a) The 34 symmetrized correlations coming from (crosses with error bars) 15 permutationally invariant measurement settings with optimized $A_j$ matrices for $N = 4$ qubits and (diamonds) from full tomography requiring 81 local settings. The average uncertainty of all symmetrized correlations obtained from full tomography is $\pm 0.022$, and is not shown on the figure. The labels refer to symmetrized correlations. The results corresponding to the 15 full four-qubit correlations are left from the vertical dashed line. All correlations are between $-1$ and $+1$. (b) Measurement directions. A point at $(a_x, a_y, a_z)$ corresponds to measuring operator $a_x X + a_y Y + a_z Z$. (c) Results for randomly chosen $A_j$ matrices and (d) corresponding measurement directions.

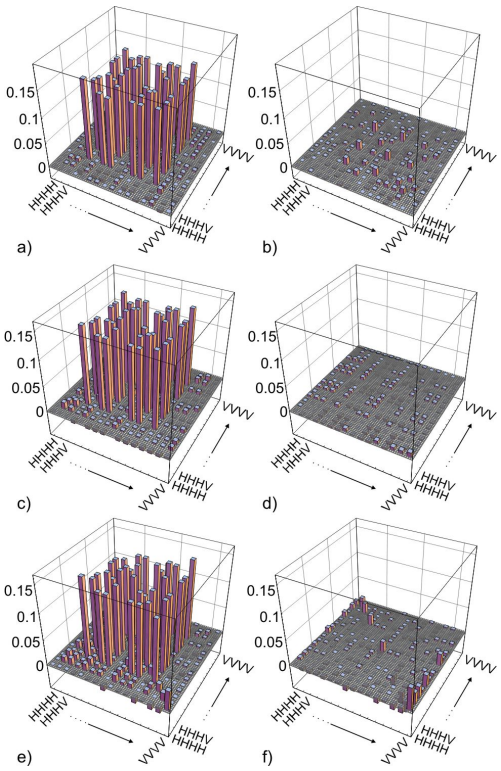


FIG. 2: (a) The real and (b) imaginary parts of the density matrix coming from full tomography. (c-d) The same for permutationally invariant tomography with optimized and (e-f) random measurement directions, respectively.

* Present address: Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Wien, Austria

[1] G. Tóth *et al.*, arxiv:1005.3313.

# The invariant-comb approach and its relation to the balancedness of multipartite entangled states

Andreas Osterloh[1] and Jens Siewert[2, 3, 4]

[1] *Fakultät für Physik, Campus Duisburg, Universität Duisburg-Essen, Lotharstr. 1, 47048 Duisburg, Germany.*
[2] *Departamento de Química Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Apdo. 644, 48080 Bilbao, Spain*
[3] *Ikerbasque, Basque Foundation for Science, Alameda Urquijo 36, 48011 Bilbao, Spain*
[4] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*

Entanglement is an important feature of coherent quantum systems. As an inherently non-local phenomenon, this entanglement must be defined modulo certain local operations. This fact has lead to the insight that entanglement measures can be expressed in termes of polynomial invariants of the corresponding local invariance group. For $q$ qubits we will consider the $SL(2,\mathbb{C})^{\otimes q}$. [1] Polynomial invariants of this group can be constructed in many ways, ranging from the Cayley-Hilbert Omega Process [2], singlet contractions [3], or more efficiently [4] using mutually orthogonal local invariant operators [5]. By asking which property of a state determines whether or not it is detected by a polynomial $SL(2,\mathbb{C})$ invariant we find that it is the presence of a minimal *balanced part* that persists under local unitary transformations. [6] We analyse to some extent the maximally entangled states [7, 8] detected by such polynomial invariants, leading to the concept of *irreducibly balanced* states. The latter indicates a tight connection with SLOCC classifications of qubit entanglement.

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# Deterministic Creation of NOON States in Two Superconducting Resonators

John Martinis[1]
[1] *University of California, Santa Barbara, USA*

The quantum entanglement of two or more degrees of freedom is a key requirement for quantum computation, and has been demonstrated in a variety of spin-like physical systems, ranging from atoms to electronic circuits. These systems share the common trait of very strong nonlinearity, and are used because the nonlinearity allows straightforward quantum control by classical means. Quantum control of linear systems, such as harmonic oscillators, is by contrast significantly more difficult, and has only been achieved using nonlinear intermediaries: Beam-splitters, nonlinear crystals and photon detectors to control traveling optical photons, and atoms, ions and more recently superconducting qubits to control microwave photons and phonons in cavities and resonators. Here we demonstrate a quantum circuit in which a pair of qubits is used to deterministically generate entangled photon states in two microwave resonators. We use as a benchmark the generation of NOON states, with $N$ photons in one resonator and 0 in the other, superposed with the state with the occupation numbers reversed. The resonator states are analyzed using bipartite Wigner tomography required to distinguish entanglement from an ensemble of mixed states.

# Quantum superpositions of nanocantilevers

C. Joshi,[1] A. Hutter,[1,2] F. Zimmer,[1,3] M. Jonson,[1,4,*] E. Andersson,[1] and P. Öhberg[1]

[1] *SUPA, Department of Physics, Heriot-Watt University, Edinburgh, EH14 4AS, UK*
[2] *Department of Physics and Astronomy, University of Würzburg, 97074 Würzburg, Germany*
[3] *Max Planck Institute for the Physics of Complex Systems, D-01187 Dresden, Germany*
[4] *Department of Physics, University of Gothenburg, SE-412 96 Göteborg, Sweden*

There have been long-standing arguments regarding the validity of quantum mechanics in the macroscopic world. The last two decades have seen an unprecedented rise of interest in studying the crossover of quantum and classical mechanics. These studies, both theoretical and experimental, have potential to improve our understanding of decoherence, which is assumed to degrade any quantum system to its classical counterpart. Over the past few years, there have been many fascinating experiments to prepare and detect quantum states of macroscopic objects. In this quest for studying the level of "quantumness" present in macroscopic objects, tremendous progress has been achieved in exploring the quantum regime of micro- and nano-mechanical systems. These mechanical systems offer a very promising playground for studying the quantum-classical crossover. This is mainly because miniaturized cantilevers contain a macroscopic number of atoms and can be fabricated to have very high resonant frequencies and exceedingly large quality factors, thereby guarding against the effects of decoherence.

Very recently, O'Connell et al. [1] were able to cryogenically cool a mechanical resonator to its quantum ground state, and were also successful in strongly coupling it to a superconducting qubit to read out the motion of the resonator. This success heralds a new era in investigating the quantum behavior of these nanomechanical systems. Enthralled by these experimental results, in the present work we theoretically investigate the possibility of entangling two nanocantilevers coupled via an ultracold Bose gas [2]. We model the nanocantilever as a quantum harmonic oscillator which has been cooled near to its ground state so that $\langle n_{thermal} \rangle \ll 1$. The ultracold Bose gas is assumed to comprise $N$ two-level atoms, and is modeled by the Dicke formalism. In our present work, due to the possibility of near ground state cooling of the nanocantilevers, we have restricted the excitation subspace of the two nanocantilevers — assumed to be identical — to the excitation of one, two and three quanta of their fundamental bending mode vibrations. We investigate two methods to generate entanglement between these "macroscopic" entities. One is entanglement by projection and the other dissipation assisted entanglement. We find that unitary evolution of the system leads to generation of time-varying entanglement between the two nanocantilevers, with the dynamics becoming complex in higher excitation manifolds. In the presence of dissipation in the ultracold gas, the system of two nanocantilevers relaxes to a statistical mixture of *dark* states corresponding to different excitation manifolds. We have shown that it is possible to generate long lived entangled states of nanocantilevers even if these were initially prepared in coherent or thermal states. This opens the possibility of achieving long lived "macroscopic" entangled states.

We have also studied quantum properties of nonlinear nanomechanical oscillators and found that for certain initial states of two nanocantilevers interacting with an ultracold gas of atoms, nonlinearity favors the generation of maximally entangled states of two nanocantilevers.

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* Also at the Department of Physics, Division of Quantum Phases & Devices, Konkuk University, Seoul 143-701, Korea

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# Entangled Quantum States of Photons and Superconducting Circuits

S. Filipp,[1,*] M. Baur,[1] P. Leek,[1] and A. Wallraff[1]

[1] *Department of Physics, ETH Zurich, CH-8093 Zurich, Switzerland*

The coupling of superconducting quantum circuits to a coplanar transmission line resonator, an architecture known as circuit-QED, allows for strong coupling between superconducting qubits and the electromagnetic field generated by single photons in the resonator. This provides a versatile platform for scalable quantum computation with fast operation times, and quantum optics experiments, where the interaction of photons and matter at the single quantum level can be probed.

In our experiments, we have used sideband pulses to deterministically transfer excitations of the qubit to resonator photons. These transitions involve simultaneous excitation of both qubit and resonator with two photons of equal energy. By increasing the number of photons one-by-one we could generate states with well-defined number of photons in a high-quality resonator mode [1].

Moreover, these resonator photons mediate interactions between distant qubits. By entangling first one qubit with the resonator and subsequently distributing the entanglement between two qubits (Figure 1(a)) we have prepared a complete set of maximally entangled quantum states. To determine the combined state of the two qubits, a time-resolved transmission measurement is used. Since the measurement response to a state change in one qubit is correlated to the state of the respective other qubit, information about qubit-qubit correlations can be obtained without individual qubit-readout; the resonator is used as a single entanglement-sensitive measurement apparatus. State tomography measurements [2] confirm the successful preparation of entangled quantum states between two superconducting circuits, see Figure 1(b).

State tomography techniques can also be used to analyze higher excited states of the superconducting transmon circuits. Transmons – modified version of cooper-pair box qubits with reduced 1/f noise sensitivity – feature an energy level structure with small anharmonicity and higher energy levels can be accessed conveniently with similar frequencies. Using optimal control techniques, we can generate arbitrary three-level states and analyze them with high fidelity using full quantum state tomography [3].

* Electronic address: filipp@phys.ethz.ch

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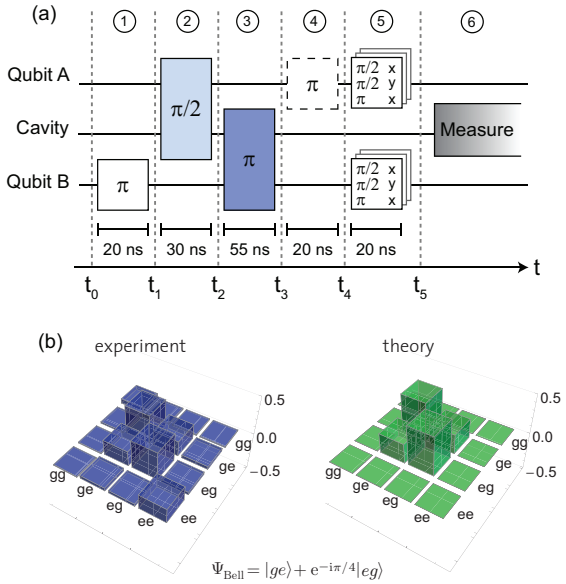


FIG. 1: (a) Pulse sequence for generation of a two-qubit entangled state using sideband. (b) Experimental density matrix of one of the generated states compared to theory.

# Towards electron-electron entanglement in Penning traps

L. Lamata,[1] D. Porras,[1] J. I. Cirac,[1] J. Goldman,[2] and G. Gabrielse[2]
[1]*Max-Planck-Institut für Quantenoptik, Garching, Germany*
[2]*Harvard University, Cambridge, MA, USA*

Entanglement of isolated elementary particles other than photons has not yet been achieved. We show how building blocks demonstrated with one trapped electron might be used to make a model system and method for entangling two electrons. Applications are then considered, including two-qubit gates and more precise quantum metrology protocols.

Entanglement is one of the most remarkable features of quantum mechanics. Two entangled systems share the holistic property of nonseparability: their joint state cannot be expressed as a tensor product of individual states. Entanglement is also at the center of the rapidly developing field of quantum information science. A variety of systems have been entangled, including photons, ions, atoms, and superconducting qubits. However, no isolated elementary particles other than photons have been entangled.

It is possible to perform quantum information protocols with electrons in Penning traps, as opposed to ions, even though the former cannot be laser cooled. This is possible because at low temperature in a large magnetic field (100 mK and 6 T in Harvard experiments [1]) the cyclotron motion radiatively cools to the ground state. We describe how one could use this mode for quantum information applications since there is suffciently small coupling to other modes.

In this work, we describe a possible method for entangling two electrons [2]. The model system and method we investigate are largely based on building blocks already demonstrated with one trapped electron. On the way to measuring the electron magnetic moment to 3 parts in $10^{13}$ [1], quantum nondemolition (QND) methods were used to reveal one quantum cyclotron and spin transitions between the lowest energy levels of a single electron suspended for months in a Penning trap. We demonstrate how the two-electron entanglement could make a universal two-qubit gate. We show how this gate could enable a metrology protocol that surpasses the shot-noise limit, and as an example we consider in detail the requirements for implementing this protocol in a measurement of the electron magnetic moment using two trapped entangled electrons. The payoffs and requirements for moving from two-electron to $N$-electron entanglement are listed. Possible applications include quantum simulators, analysis of decoherence, and more precise electron magnetic moment measurements using improved quantum metrology protocols.

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# Quantum information processing with trapped individual atoms

Philippe Grangier[1]
[1] *Institut d'Optique, Palaiseau, France*

We experimentally demonstrate a set of tools for quantum information processing, using qubits encoded on single atoms trapped in submicrometer tweezers. This includes single qubit operations with Raman transition, qubit motion in the tweezer [1], and entangling atoms in separate tweezers, using the Rydberg dipole blockade [2, 3]. Perspective towards multiqubits operations will be discussed [4].

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# Quantum interference of photons emitted by two remotely trapped atoms

Michael Krug,[1,*] Julian Hofmann,[1] Florian Henkel,[1] Christoph Kurz,[1] Mathias Müller,[1] Wenjamin Rosenfeld,[1] Markus Weber,[1] and Harald Weinfurter[1]

[1] *Fakultät für Physik, Ludwig-Maximilians-Universität München, D-80799 München, Germany*

Entanglement between atomic quantum memories at remote locations is a key resource for future applications in quantum communication. One possibility to achieve this is entanglement swapping of two entangled atom-photon pairs, from which two requirements follow. The first is the ability to entangle each memory device with a photonic qubit carrier of quantum information. The second requirement is to perform a Bell-state measurement on photonic carriers via quantum interference thereby swapping the entanglement onto the atomic system.

Here we present our recent progress on establishing entanglement between two single $^{87}$Rb atoms over a large distance. For this purpose we set up two independently operating atomic traps situated in two neighboring laboratories (separated by 20 meter). On each trap we capture a single neutral $^{87}$Rb atom in an optical dipole trap with life times exceeding 10 seconds. Next, both atoms are prepared in an excited state by a short optical pulse. In the following spontaneous decay process each atom emits a single photon whose polarization is entangled with the atomic spin [1]. The emitted photons are collected with high-NA objectives into single mode optical fibers and are guided to a non-polarizing 50-50 beam-splitter where they interfere (see Fig 1). To ensure good temporal overlap of the photonic wave functions, the excitation pulses in the experiment are synchronized with sub-nanosecond precision (see Fig 2).

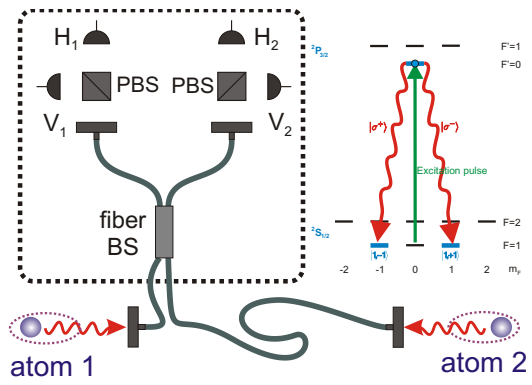


FIG. 1: Experimental setting: Two single atom traps (distance 20m) are connected via optical fibers. The emitted single photons arrive simultaneously on a 50-50 beam splitter and are detected by single photon counting avalanche photo detectors (APDs). The inset shows the scheme for the generation of single photons whose polarization is entangled with the atomic spin.

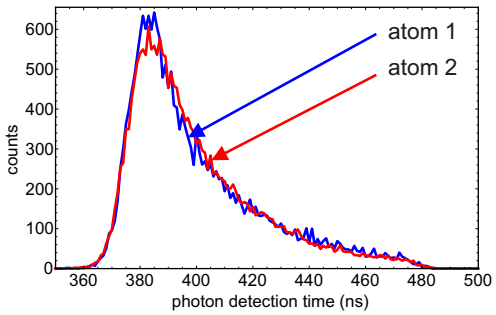


FIG. 2: Timedependent detection of photons emitted by both atoms after synchronized excitation: The photons emitted by the first atom (blue) and the second atom (red) arrive simultaneously with a sub nano second synchronization precision

After interference, the two photons leaving the output ports of the beam-splitter are detected by avalanche photo detectors. We observe bunching of indistinguishable photons, i.e. photons of the same polarization go preferentially into the same output port. First measurements show a bunching ratio of 1:3 with respect to orthogonally polarized photons which do not interfere. This bunching is the basis to perform a Bell-state measurement on the photons. By analyzing the polarization we are able to detect two out of four Bell-states (heralded by coincidence detection of two photons in particular combinations of the four detectors).

In the future, by combining this procedure with the analysis of the atomic state, we intend to detect the resulting long-distance entanglement between the two atoms. This will be a first step towards creating a basic node of a quantum communication network as well as a key prerequisite for a future loophole-free test of Bell's inequality with entangled pairs of neutral atoms [2].

* Electronic address: michael.krug@physik.uni-muenchen.de

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# Approximate reversal of quantum channels

Cédric Bény[1] and Ognyan Oreshkov[2]

[1] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543*
[2] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria*

Recently, we and other authors introduced several new analytical results concerning the approximate reversal of a quantum channel, i.e. given a channel $\mathcal{N}$, the problem of finding a channel $\mathcal{R}$ such that $\mathcal{RN} \approx \mathrm{id}$, where the quality of the approximation is measured in terms of the entanglement fidelity [1, 2], worst-case entanglement fidelity [3] or worst-case fidelity [4]. We will review these results and establish relations between them. The main new result is an explicit closed-form expression for the correction channel proposed in Ref [3]. This channel is closely related to the channel proposed by Tyson [2], and bears certain similarities to the "transpose channel" used in Refs. [1, 4], and is a generalisation of the channel used in Ref [5].

Exact correctability of a given quantum code is characterized in general terms by the Knill-Laflamme (KL) conditions [6] which specify the set of correctable errors for a particular code. However, practically useful codes need not be exactly correctable for any given noise model. In fact, a few exceptional examples show that allowing for a negligible error in the recovery can lead to surprisingly better codes [7, 8]. This indicates that assuming exact correctability is too strong a restriction. It is therefore of considerable interest to find appropriately weaker error-correction conditions.

In the context of error correction, the channel to be inverted is the encoding (given by an isometry $V$) followed by the noise (a channel with Kraus operators $E_i$), i.e.

$$\mathcal{N}(\rho) = \sum_i E_i V \rho V^\dagger E_i^\dagger. \tag{1}$$

The measure of success considered in Refs. [1, 2] is the entanglement fidelity $\langle\psi|(\mathcal{RN}\otimes\mathrm{id})(|\psi\rangle\langle\psi|)|\psi\rangle$. In Ref. [3] it is the minimum over $|\psi\rangle$ of this quantity that is considered, hence the *worst-case*.

For some fixed states $\sigma$ and $\rho$, we introduce the completely positive map $\mathcal{S}_{\rho,\sigma}$ defined by

$$\mathcal{S}_{\rho,\sigma}(\tau) = \sum_{ij} \mathrm{Tr}(\sigma V^\dagger E_i^\dagger E_j V)\, \rho V^\dagger E_j^\dagger \tau E_i V \rho \tag{2}$$

for any state $\tau$. The estimate for the optimal entanglement fidelity after correction in terms of the input $\rho = \mathrm{Tr}_2|\psi\rangle\langle\psi|$ in Ref. [2] is the quantity $\mathrm{Tr}\sqrt{\mathcal{S}^\dagger_{\rho,\rho}(I)}$ where the $\dagger$ denotes the conjugate map in terms of the Hilbert-Schmidt inner product. It was shown that this near-optimal estimate is achieved by the state-dependant correction channel

$$\mathcal{R}_\rho(\tau) = \mathcal{S}_{\rho,\rho}(\mathcal{S}^\dagger_{\rho,\rho}(I)^{-\frac{1}{2}}\,\tau\,\mathcal{S}^\dagger_{\rho,\rho}(I)^{-\frac{1}{2}}). \tag{3}$$

A direct application of the minimax theorem also yields an near-optimal estimate of the worst-case entanglement fidelity through the quantity [9]

$$\min_\rho \mathrm{Tr}\sqrt{\mathcal{S}^\dagger_{\rho,\rho}(I)}. \tag{4}$$

The near-optimal estimate for the worst-case entanglement fidelity proposed in Ref. [3] is

$$\min_\rho \mathrm{Tr}\sqrt{\mathcal{S}^\dagger_{\rho,\sigma}(I)} \tag{5}$$

where $\sigma$ is a state which can be chosen arbitrarily. As was shown in [3] under certain general conditions, a corresponding near-optimal correction channel can be computed. Here we show that it has the explicit form

$$\mathcal{R}(\tau) = \mathcal{S}_{\rho_\sigma,\sigma}(\mathcal{S}^\dagger_{\rho_\sigma,\sigma}(I)^{-\frac{1}{2}}\,\tau\,\mathcal{S}^\dagger_{\rho_\sigma,\sigma}(I)^{-\frac{1}{2}}) \tag{6}$$

where $\rho_\sigma$ is the state achieving the minimum in the definition of the corresponding estimate.

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# Improving zero-error classical communication capacity with entanglement

Toby S. Cubitt,[1] Debbie W. Leung,[2] William Matthews,[2] and Andreas Winter[1]
[1]*Department of Mathematics, University of Bristol, Bristol, United Kingdom*
[2]*Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, Canada*

*Abstract* Given one or more uses of a classical channel, only a certain number of messages can be transmitted with zero probability of error. The study of this number and its asymptotic behaviour constitutes the field of classical zero-error information theory [1, 2], the quantum generalisation of which has started to develop recently [3–5]. We show that, given a single use of certain classical channels, entangled states of a system shared by the sender and receiver can be used to increase the number of (classical) messages which can be sent with no chance of error. In particular, we show how to construct such a channel based on any proof of the Bell-Kochen-Specker theorem [6, 7]. This is a new example of the use of quantum effects to improve the performance of a classical task. We investigate the connection between this phenomenon and that of "pseudo-telepathy" games. The use of generalised non-signalling correlations to assist in this task is also considered. In this case, a particularly elegant theory results and, remarkably, it is sometimes possible to transmit information with zero-error using a channel with *no* unassisted zero-error capacity.

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## Cracking commercial quantum cryptography

L. Lydersen,[1,2] C. Wiechers,[3,4,5] C. Wittmann,[3,4] D. Elser,[3,4] J. Skaar,[1,2] and V. Makarov[1,*]

[1] *Department of Electronics and Telecommunications, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway*
[2] *University Graduate Center, NO-2027 Kjeller, Norway*
[3] *Max Planck Institute for the Science of Light, Günther-Scharowsky-Str. 1/Bau 24, 91058 Erlangen, Germany*
[4] *Institut für Optik, Information und Photonik, University of Erlangen-Nuremberg, Staudtstraße 7/B2, 91058, Erlangen, Germany*
[5] *Departamento de Física, Universidad de Guanajuato, Lomas del Bosque 103, Fraccionamiento Lomas del Campestre, 37150, León, Guanajuato, México*

Most of today's single-photon detectors based on avalanche photodiodes can be blinded by bright-light illumination, through several different mechanisms connected to detector electronic and thermal properties. Furthermore, in the blinded state the detector will respond deterministically to a short bright-light pulse, producing a click when the pulse peak power exceeds a certain threshold. We show how this can be used to launch a perfect attack against a quantum key distribution system, eavesdropping the complete secret key without alerting the legitimate users. We show experimentally [1] that this vulnerability is fully present in commercial quantum cryptosystems, Clavis2 from ID Quantique and QPN 5505 from MagiQ Technologies. We propose how to build a plug-and-play eavesdropper for both cryptosystems, using off-the-shelf components.

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* Electronic address: `makarov@vad1.com`; URL: `http://www.iet.ntnu.no/groups/optics/qcr/`

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# Hybrid Quantum Systems

Jörg Schmiedmeyer[1]
[1] *Atominstitut der Österreichischen Universitäten, TU-Wien, Wien, Austria*

Presently quantum physics is confined to its separate worlds, separated by deserts of classical physics. For quantum physics to emerge from fundamental research, one of the main challenges is how to pool the advantages of the different quantum systems by linking them to each other and preserving the quantum nature also over the link. One has to be able to quantum interconnect the different domains. A robust technological basis for this is currently not available. In the talk I will discuss two such links: (1) The connection of a flying photonic qubit which can be transmitted through quantum communication channels to an atomic quantum memory, and (2) a pathway to integrate solid state qubits with the help of CPW resonators to an ensemble quantum memory. The talk will analyse the physics and discuss the prospects of experimental realizations and how they can be implemented for quantum technology applications.

# Discrimination and classification of four-qubit entanglement by means of polynomial invariants

Oliver Viehmann,[1] Christopher Eltschka,[2] Andreas Osterloh,[3] and Jens Siewert[4,5,*]

[1] *Physik Department, Ludwig-Maximilians-Universität, Theresienstr. 37, D-80333 München, Germany*
[2] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*
[3] *Fakultät für Physik, Campus Duisburg, Universität Duisburg-Essen, Lotharstr. 1, 47048 Duisburg, Germany*
[4] *Departamento de Química Física, Universidad del País Vasco - Euskal Herriko Unibertsitatea, Apdo. 644, 48080 Bilbao, Spain*
[5] *IKERBASQUE, Basque Foundation for Science, Alameda Urquijo 36, 48011 Bilbao, Spain*

It is well known that both the number of SLOCC classes and the number of polynomial invariants increases dramatically on going beyond the relatively simple cases of two and three qubits [1–5]. Nonetheless, criteria for discriminating and classifying pure four-qubit states are highly desirable, bearing in mind the rapid evolution of experimental technology. We have developed a general criterion for the discrimination of pure multipartite entangled states based on polynomial $SL(2,C)^{\otimes N}$ invariants. By applying the generating polynomials for four-qubit invariants to a classification scheme of four-qubit entanglement [5] we show that various families of entangled states can be characterized/identified based on their "tangle patterns".

* Electronic address: jens_siewert@ehu.es

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# Geometric measure of nonclassicality

Paulina Marian[1,*] and Tudor A. Marian[1]

[1] *Centre for Advanced Quantum Physics, University of Bucharest, P.O.Box MG-11, R-077125 Bucharest-Măgurele, Romania*

In quantum optics a nonclassical state is defined as having a negative or a highly singular $P$ representation (more singular than Dirac's $\delta$). Otherwise we term it as a classical state. Signatures of nonclassicality could be also identified through the negativity of the Wigner function or by analyzing the characteristic function and the higher-order moments of the state. A first proposal to define a measure of nonclassicality of a state was formulated by Hillery [1] as the trace distance between the given non-classical state and the set of all classical ones. Following this idea, a good deal of work was done using distances with recognized ability to distinguish between density operators such as Bures distance [2, 3] and the relative entropy measure [3].

For a pure state $|\Psi\rangle$ *a geometric measure of nonclassicality* can be defined in terms of its distance to the set of classical pure states, namely the coherent ones,

$$D_{GM} = 1 - \max_{\{|\alpha\rangle\}} |\langle\Psi|\alpha\rangle|^2.$$

In the present work we use this definition to efficiently evaluate nonclassicality of some single-mode non-Gaussian pure states of recent experimental and theoretical interest. Then we investigate the possibility of extending this idea to mixed states by convex roof construction. For Gaussian states we find perfect agreement with the Bures nonclassicality [2], the entanglement potential [4], and the nonclassical depth.

* Department of Chemistry, University of Bucharest, Boulevard Regina Elisabeta 4-12, R-030018 Bucharest, Romania; Electronic address: paulina.marian@g.unibuc.ro

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# Optimal and scalable telecloning in a limited-control scenario

F. Ciccarello,[1] M. Paternostro,[2] S. Bose,[3] D. E. Browne,[3] G. M. Palma,[4] and M. Zarcone[1]

[1] *CNISM and Dipartimento di Fisica e Tecnologie Relative, Universita' degli Studi di Palermo, Viale delle Scienze, Edificio 18, I-90128 Palermo, Italy**

[2] *School of Mathematics and Physics, Queen's University, Belfast BT7 1NN, United Kingdom*

[3] *Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*

[4] *NEST Istituto di Nanoscienze-CNR and Dipartimento di Scienze Fisiche ed Astronomiche, Universita' degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy*

Quantum telecloning [1] is a Quantum Information Processing task, which allows to achieve optimal spreading among $N$ receivers of the quantum information initially possessed by one sender. So far, to the best of our knowledge, no scheme for the actual implementation of such a process in a scalable way has been proposed. In this work [2], we show the existence of a class of many-qubit singlets allowing for optimal and scalable telecloning. Next, we illustrate a protocol to prepare such states in a setting where scattering centers possessing a spin degree of freedom mutually interact by means of mobile particles. The scheme is understood simply by resorting to an appropriate coupling scheme for the addition of angular momenta and Hamiltonian symmetries. Major practical advantages of our protocol lie on the management of stationary and well separated spins along with the mild requirement to perform simple Geiger measurements over the mobile particles to establish the necessary multipartite entanglement [3, 4]. We also show strategies that allows to generate in the same setting other important many-qubit entangled states such as Aharonov, W and GHZ states.

* Electronic address: ciccarello@difter.unipa.it

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# The entropy gain and the Choi-Jamiolkowski correspondence for infinite-dimensional quantum evolutions

Alexander Holevo[1]

[1] *Steklov Mathematical Institute, Moscow, Russia*

We study the entropy gain $H(\Phi[\rho]) - H(\rho)$ for infinite-dimensional channels $\Phi$. We show that unlike finite-dimensional case where the minimal entropy gain is always non-positive, there are plenty of channels with positive minimal entropy gain. We obtain the new lower bound and compute the minimal entropy gain for a broad class of Bosonic Gaussian channels by proving that the infimum is attained on the Gaussian states.

The second part of the talk is devoted to the mathematical aspects of the Choi-Jamiolkowski (CJ) correspondence between channels and positive forms in the infinite-dimensional case. We show that there is no need to use a limiting procedure to define an "unnormalized maximally entangled state" and the corresponding analog of the CJ state since they can be defined rigorously as, in general, non-closable positive forms on an appropriate dense subspace. We obtain explicit expressions for the CJ operator defining a general non-degenerate Bosonic Gaussian channel and compute its norm.

# Invited talk: Quantum Computation and Cryptography in the presence of Closed Timelike Curves

Charles Bennett[1]

[1] *IBM Research, Yorktown Heights, USA*

We study discuss various models of closed timelike curves (CTCs), and their utility (assuming they exist) for distinguishing nonorthogonal quantum states or speeding up hard computations. CTCs are notorious for giving rise to the "grandfather paradox" — initial conditions admitting no consistent future. The most widely used model, Deutsch's 1991 mixed-state-fixed-point model, abolishes the grandfather paradox, and was formerly thought to have dramatic consequences for quantum computation and cryptanalysis. However, we show that when the tasks of computation and state discrimination are properly formulated, Deutsch type CTCs provide no help in state discrimination and have not been shown to help speed up hard computations.

An alternate CTC model, employing postselection, exploits the grandfather paradox instead of avoiding it and appears to have significant computational and state-discriminatory power, equivalent to the power of an unambiguous state discrimination protocol with the "failure" outcome guaranteed not to occur. We discuss whether the existence of such CTCs would have major consequences for computation, cryptography and ordinary notions of causality.

Joint work with Debbie Leung, Graeme Smith, John Smolin, and Aram Harrow.

## Applying the Variational Principle to quantum field theories

Frank Verstraete[1]

[1] *University of Vienna, Faculty of Physics, Boltzmanngasse 5, 1090 Wien, Austria*

Abstract not available at the time of printing.

# Explicit information locking from low distortion embeddings

Patrick Hayden[1]

[1]*School of Computer Science, McGill University, Montreal, Quebec, H3A 2A7, Canada*

As a result of "information locking" effects, classical messages can in principle be encrypted in quantum systems using logarithmic-sized keys, assuming a min-entropy bound on the plaintext message. The encryption is information-theoretically secure although the amount of information leakage cannot be made exponentially small. Previous work had only demonstrated the existence of such encryption schemes, using impractical Haar-random unitary encryption operators.

In this talk, I will explain how information locking is closely related to finding low distortion embeddings of Euclidean (l2) spaces in l1(l2) spaces. Using that connection, I will then explain how to construct explicit encryption operations than would run in polynomial time on a quantum computer. Other applications include explicit string commitment schemes and explicit encodings for quantum identification over noiseless bit channels.

## Quantum evolution from partial information

Michael M. Wolf[1]
[1] *Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*

We study the consistency of various constraints with the existence of a quantum quantum mechanical time evolution. More precisely, we will address the following questions: when can a map defined on a subspace be extended to a quantum channel? When does a time-series have a representation in terms of a Markovian process? Which sets of complex numbers can arise as spectra of quantum channels? Which quantum channels can be embedded into continuous one-parameter semigroups?

## Göran Lindblad and Stig Stenholm—a tribute

Ingemar Bengtsson[1]

[1] *Department of Physics, Stockholm University, SE-109 61 Stockholm, Sweden*

Professor Göran Lindblad and Professor Stig Stenholm, both professors emeriti at the Royal Institute of Technology, have contributed significantly to the development of quantum mechanics, quantum optics, quantum information, and quantum electronics. With this talk we wish to honor these two pioneers and to give a few glimpses from their respective professional lives.

# Unfrustrated ground states: an algebraic perspective

Reinhard Werner[1]

[1] *Technische Universität Braunschweig, Braunschweig, Germany*

We give a general method for analyzing states on a spin chain with $d$-dimensional one-site Hilbert space, which minimize every translate of a nearest neighbor interaction separately. We establish a one-to-one correspondence between the pure states on the half chain satisfying the ground state condition, and $d$-tuples of bounded operators satisfying a set of homogeneous quadratic equations, and an irreducibility condition. This sometimes suffices to determine all such states, many of which will typically not be translation invariant. We show how to extend the analysis to the full chain and how to impose the condition of translation invariance.

Examples treated are ferromagnetic, so the admissible ground states are symmetric on nearest neighbors, and antiferromagnetic, with the opposite symmetry, in each case allowing also a small number of essentially arbitrary states of the opposite symmetry.

Joint work with Bruno Nachtergaele done at the Institut Mittag-Leffler workshop "Quantum Information Theory".

# Characterizing the nonlocal correlations created in entanglement swapping experiments

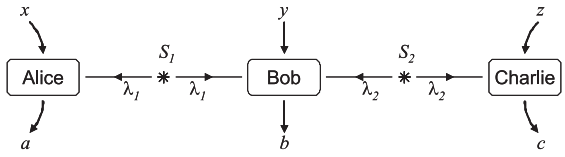
Cyril Branciard,[1,*] Nicolas Gisin,[2] and Stefano Pironio[3]

[1] *Centre for Quantum Computer Technology, University of Queensland, Australia*
[2] *Group of Applied Physics, University of Geneva, Switzerland*
[3] *Laboratoire d'Information Quantique, Université Libre de Bruxelles, Belgium*

Quantum theory predicts that nonlocal correlations can be established from measurements on entangled particles; this nonlocal nature of entangled states can be revealed by the violation of Bell inequalities [1].

However remarkable it is that quantum interactions can establish such nonlocal correlations, it is even more remarkable that particles that never directly interacted can also become nonlocally correlated. This is possible through a process called entanglement swapping [2]. To characterize nonlocality in this context, we introduce local models where quantum systems that are initially uncorrelated are described by *uncorrelated local variables* [3].

Consider the scenario depicted below:




A source $S_1$ sends particles to Alice and Bob, and a separate source $S_2$ sends particles to Charles and Bob. All parties can perform measurements on their system, labeled $x, y$ and $z$ for Alice, Bob and Charles respectively, and they obtain outcomes denoted $a, b$, and $c$, respectively. Bob's measurement $y$ might correspond to a joint measurement on the two systems that he receives from each source. The correlations between the measurement outcomes of the three parties are described by the joint probability distribution $P(a, b, c|x, y, z)$.

Under the usual assumption, the tripartite distribution $P(a, b, c|x, y, z)$ would be said to be local if it can be written in the factorized form

$$P(a,b,c|x,y,z) = \int \mathrm{d}\lambda\, \rho(\lambda)\; P(a|x,\lambda)P(b|y,\lambda)P(c|z,\lambda)\,. \quad (1)$$

Here we introduce a stronger assumption: since the two sources $S_1$ and $S_2$ are supposed to be independent and uncorrelated, it is natural to assume that this property carries over to the local model, which should introduce two independant local variables $\lambda_1$ and $\lambda_2$. The correlation would then decompose as

$$P(a,b,c|x,y,z) = \iint \mathrm{d}\lambda_1 \mathrm{d}\lambda_2\, \rho(\lambda_1)\rho(\lambda_2)\; P(a|x,\lambda_1)P(b|y,\lambda_1,\lambda_2)P(c|z,\lambda_2). \quad (2)$$

We refer to models satisfying this assumption as "bilocal" models, since they aim at explaining the correlations $P(a, b, c|x, y, z)$ with two independent sources of local variables.

This additional assumption leads to *stronger tests of nonlocality*. In fact, the set of bilocal correlations is not convex, contrary to the polytope of local correlations, and can thus not be simply delimited by Bell inequalities. We show however that one can derive *quadratic Bell ineqalities* for bilocal correlations, of the form

$$I \leq 1 + E^2 \quad (3)$$

(see [3] for a definition of the terms $I$ and $E$ above, which are essentially defined as linear combinations of the probabilities $P(a, b, c|x, y, z)$.)

The quantum correlations obtained in an entanglement swapping experiment can violate our inequalities for bilocality more easily than standard Bell inequalities; in particular, while a pair of maximally entangled qubits prepared in the usual way (i.e., emitted from a common source) requires a visibility close to 70% to violate a standard Bell inequality, an entangled pair generated through entanglement swapping will already violate the above inequality for visibilities as low as 50%. This simplifies the requirements for the demonstration of quantumness in entanglement swapping experiments.

This work raises many interesting questions: is this visibility threshold of 50% a fundamental limit, or could we find stronger tests, more robust to noise? What exactly is the role played by quantum entanglement, and could we still observe non-bilocal correlations if some of the quantum states in the game are separable? What happens if we consider scenarios with $n$ independant sources; how would the tolerance to noise scale with $n$...?

* Electronic address: c.branciard@physics.uq.edu.au

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# Measuring small longitudinal effects: weak measurements or interferometry?

Nicolas Brunner[1] and Christoph Simon[2]

[1]*H.H. Wills Physics Laboratory, University of Bristol, United Kingdom*

[2]*Institute for Quantum Information Science and Department of Physics and Astronomy, University of Calgary, Canada*

A cornerstone of quantum mechanics is that a measurement perturbs the system. Indeed during the process of a (standard) quantum measurement, the state of the system is projected onto one of the eigenstates of the measured observable, and consequently undergoes an irreversible transformation. In 1988, as part of an exotic research on the arrow-of-time in quantum theory, Aharonov, Albert and Vaidman [1] discovered that quantum mechanics offers in fact a much larger variety of measurements. As a matter of fact, the only restriction quantum mechanics imposes on measurements is a trade-off between information-gain and disturbance. Therefore, strong (or standard) quantum measurements are only part of the game. There exist also "weak" measurements, which disturb only very little the system, but as a counter part give only little information about its quantum state.

Weak measurements lead to striking results when post-selection comes into play. In particular, the result of a weak measurement on a pre and post-selected system can be arbitrarily large, the celebrated example being the measurement of a spin-half particle leading to a value of 100 [1]. Partly because of such intriguing predictions, weak measurements appeared first as controversial, and were for long considered as a weirdness for theorists. However, they turn out to be a key ingredient for exploring the foundations of quantum mechanics, and find applications in other fields of modern physics, such as superluminal light propagation in dispersive materials, polarization effects in optical networks, cavity QED experiments, and solid-state physics.

Already in 1990 Aharonov and Vaidman [2] pointed out the potential offered by weak measurements for performing very sensitive measurements. More precisely, when weak measurements are judiciously combined with pre- and post-selection, they lead to an amplification phenomenon, much like a small image is magnified by a microscope. This effect is of great interest from an experimental perspective, since it gives access to an experimental sensitivity beyond the detector's resolution, therefore enabling the observation of very small physical effects. Hosten and Kwiat [3] recently used this technique to perform the first observation of the spin Hall effect of light. More recently, Refs [4, 5] took advantage of the same method to amplify small transverse deflections of an optical beam, in order to measure the angular deflection of a mirror with an impressive resolution of 10 femtometer.

The effects measured in Refs. [3–5] are all transverse to the light propagation direction. Here we investigate the power offered by weak measurements for measuring small longitudinal phase shifts. We perform a comparison with standard interferometry, which is the natural reference in this context, taking into account the influence of experimental errors. We consider the situation where the most important errors are not due to statistics (i.e. the total number of photons detected), but to imperfections in the setup. For example, this is true if the phase that is to be measured is stationary (e.g. a weak-contrast, but stable, microscopic sample), making it possible to integrate over arbitrarily long times. While infinitely long integration times are of course an idealization, situations where the precision limit is not set by statistics, but by other factors such as unavoidable alignment errors, are very common in practice. We first consider a scenario involving a large real weak value, combined with an analysis in the time-domain, and show that it is greatly outperformed by interferometry. Then we present an interferometric setup involving a purely imaginary weak value, combined with an analysis in the frequency-domain, and show that it has potential to outperform standard interferometry by three orders of magnitude [6].

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# Memory cost of quantum contextuality

Matthias Kleinmann,[1] Otfried Gühne,[2] José R. Portillo,[3] Jan-Åke Larsson,[4] and Adán Cabello[5]

[1] *Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstr. 21A, A-6020 Innsbruck, Austria*
[2] *Institut für Theoretische Physik, Universität Innsbruck, Technikerstr. 25, A-6020 Innsbruck, Austria*
[3] *Departamento de Mathemática Aplicada I, Universidad de Sevilla, E-41012 Sevilla, Spain*
[4] *Institutionen för Systemteknik och Matematiska Institutionen, Linköpings Universitet, SE-581 83 Linköping, Sweden*
[5] *Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*

A fundamental property of quantum systems is that the result of a measurement may depend on which compatible observables were previously measured. This property is called contextuality. While the resources needed to simulate quantum nonlocality have been extensively investigated [1], there is no similar knowledge about the resources needed to simulate quantum contextuality. Here we show that any model which reproduces quantum contextuality requires individual systems to posses a minimum amount of memory, and obtain the minimum memory (or bounds on it) for some relevant cases.

We consider the following scenario: An individual two-qubit system initially prepared in an arbitrary state is submitted to a sequence of dichotomic measurements with possible results $-1$ or $+1$, chosen from a finite set of observables. The simplest example of a set of observables exhibiting state-independent contextuality is the Peres-Mermin set [2]:

$$\begin{array}{lll} A=\sigma_z\otimes\mathbb{1}, & B=\mathbb{1}\otimes\sigma_z, & C=\sigma_z\otimes\sigma_z, \\ a=\mathbb{1}\otimes\sigma_x, & b=\sigma_x\otimes\mathbb{1}, & c=\sigma_x\otimes\sigma_x, \\ \alpha=\sigma_z\otimes\sigma_x, & \beta=\sigma_x\otimes\sigma_z, & \gamma=\sigma_y\otimes\sigma_y, \end{array} \quad (1)$$

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ denote the Pauli operators. The observables within each row and column commute and the product of the operators in a row or column yields $\mathbb{1}$, except for the last column which yields $-\mathbb{1}$. Hence the product of the measurement results for the row/column measurement sequences will yield $+1$ ($-1$ in the last column), independent of the input state. This behavior cannot be reproduced by a noncontextual model, i.e., by a model for which the measurement results of compatible observables are independent of the choice of compatible observables. Similar to the Bell inequalities for local models, any noncontextual model satisfies

$$\chi \equiv \langle ABC\rangle+\langle abc\rangle+\langle\alpha\beta\gamma\rangle+\langle Aa\alpha\rangle+\langle Bb\beta\rangle-\langle Cc\gamma\rangle \leq 4, \quad (2)$$

while quantum mechanics predicts $\chi = 6$, independent of the initial state [3]. Recently, this inequality has been experimentally tested [4]. The results show a good agreement with quantum predictions.

A model that explains the contextual behavior in the Peres-Mermin set must eventually predict that the system attains different states during certain sequences of compatible measurements. Hence, there are at least two different states in such a model, and one can thus assign at least one bit of internal memory to the system. We ask how much memory is necessary and sufficient for the contextuality observed in the Peres-Mermin set and introduce the following results:

*Result 1.* For sequences of mutually compatible measurements, the necessary and sufficient memory (in bits) $\mu_1$ for the Peres-Mermin set, and also the memory needed to achieve the quantum violation of inequality (2), is $\mu_1 = \log_2 3 \approx 1.585$.

*Result 2.* For a larger class of sequences, at least $\mu_2 = 2$ bits of memory are needed.

*Result 3.* For the extended set of 15 observables proposed in [5] even 2 bits of memory do not suffice.

*Result 4.* For arbitrary sequences, the necessary and sufficient memory $\mu_3$ for reproducing all perfect correlations in the Peres-Mermin set is upper bounded by $\mu_3 \leq \log_2 10 \approx 3.322$.

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# Non-disturbing quantum measurements

Teiko Heinosaari[1,*] and Michael M. Wolf[1]
[1] *Niels Bohr Institute*

One of the main features of quantum mechanics is that measurements of different observables usually disturb each other. This property often comes along with non-commutativity or the impossibility of measuring observables simultaneously. Strictly speaking, however, *non-disturbance, joint measurability* and *commutativity* are different concepts.

We show that (i) non-disturbance is not a symmetric relation, (ii) it is equivalent to commutativity in the case of qubit measurements but (iii) inequivalent to joint measurability and commutativity in general.

* Electronic address: heinosaari@gmail.com

# Recent Quantum Tests with Photon Pairs

Anton Zeilinger[1]
[1] *Universität Wien, Wien, Austria*

In the talk, I will report on three recent tests on the foundations of quantum mechanics performed in Vienna. These include a non-local quantum eraser where, using entangled photons, one photon passes through an interferometer and the other one is sent to a distant station. It is the decision of the experimentalist that the distant station which decides whether the other photon passing through the interferometer shows an interference pattern or not, that is, wave-like or particle-like features.

In a second experiment, we performed a long-distance test of Bells inequality on the Canary Islands. The topic of that experiment was to exclude a subset of the freedom of choice loophole, that is, we excluded the possibility that the polarizer settings might be influenced by the emission event of the entangled pair at the source.

Finally, in a test on single photons in three-state systems, we experimentally implemented the proposal of Klyachko *et al.* to test the existence of probability distributions for individual quantum systems, a single-particle realisation of a version of the Kochen-Specker Paradox. I will also discuss the implications of these experiment for the ongoing discussion about the foundations of quantum physics.

# Driving universal quantum operations using a fixed interaction

Janet Anders,[1] Daniel K. L. Oi,[2] Elham Kashefi,[3] Dan E. Browne,[1] and Erika Andersson[4]

[1]*Department of Physics & Astronomy, University College London, London, WC1E 6BT, UK*
[2]*SUPA, Department of Physics, University of Strathclyde, Glasgow, G4 0NG, UK*
[3]*School of Informatics, University of Edinburgh, Edinburgh, EH8 9AB, UK*
[4]*SUPA, Department of Physics, Heriot-Watt University, Edinburgh, EH14 4AS, UK**

We show how a quantum register can be manipulated with the help of a single ancillary qubit that "drives" universal evolution of the register. The fully controlled ancillary qubit is coupled to the computational register via a *fixed* unitary two-qubit interaction, and then measured in suitable bases; see Figure 1. This drives universal single- and two-qubit operations on the register. Arbitrary single-qubit operations directly on register qubits are not needed. We characterize all interactions $E$ that induce universal, unitary, step-wise deterministic measurement back-action on the register. This scheme offers experimental advantages since no tunable control of the register is required.

Previous attempts to construct 'programmable', deterministic and universal quantum operations have concluded this to be impossible [1]. These schemes also use 'program qubits', i.e. ancillas, that are coupled to a register with a fixed interaction. The difficulty lies not in implementing the entangling operation, but in implementing an *arbitrary* (single-qubit) operation using a fixed interaction. Our results bypass existing no-go theorems as we allow *feedback* within the programmable part and either final fixed single-qubit correction operations or a final *local* redefinition of the computational basis.

In gate-based quantum computation, qubits are actively manipulated by a network of quantum gates. In measurement-based quantum computing (MBQC), computation is implemented "passively", as a sequence of adaptive single-qubit measurements on an entangled multi-partite resource state [2]. Ancilla-driven quantum computation (ADQC) [3] is a hybrid scheme. Implementing operations on a static register is similar to gate-based quantum computing, whereas the fixed ancilla-register interaction and measurement driven computation resembles MBQC. The computation neither requires direct control of the register nor the preparation of a large entangled state. The processing of information is driven by active manipulation of the ancilla alone.

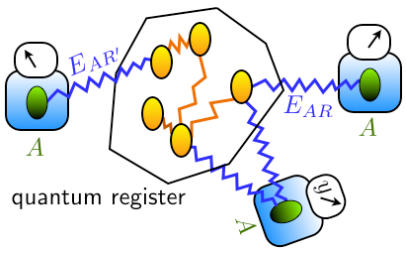


FIG. 1: Illustration of an ancilla-driven computation on a register consisting of several qubits. A single ancilla, $A$, is sequentially coupled to one, or at most two, register qubits, $R$ and $R'$ etc., and measured. The coupling, $E_{AR}$, is fixed throughout the computation while the measurements on the ancilla, indicated by the arrows, can differ.

Many existing schemes use 'flying' qubits for mediating interactions between register qubits, e.g. [4, 5]. The key difference of ADQC is that register qubits are only addressed with a fixed coupling operation. This is experimentally advantageous as not only can the computational register be separated from state preparation and measurement, but it also does not require bespoke control. Long-lived static qubits are addressed by mobile short-lived ancilla qubits using a fixed entangling interaction. Realizations of interest include neutral atoms in optical lattices, micro ion trap arrays, nuclear-electron spin systems, cavity QED and superconducting qubits.

For MBCQ, no characterization currently exists of exactly what classes of entangled states lead to universal computation. For ADQC, we are able to characterize the necessary and sufficient entangling interactions. An example of a universal interaction between register and ancilla is the Control-Z ($CZ$) interaction followed by local Hadamards, $E_{AR} = H_A H_R CZ_{AR}$, where $X, Y, Z$ are the Pauli matrices, $H = (X + Z)/\sqrt{2}$, and $CZ = \mathbf{1} - 2|11\rangle\langle 11|$. This is reminiscent of MBQC where resource states are constructed using the $CZ$ operation. For ADQC, however, local Hadamards are necessary, since otherwise one cannot implement arbitrary single-qubit operations on register qubits.

* Electronic address: E.Andersson@hw.ac.uk

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## Universal dynamical decoupling from slow noise with minimal control

G. Bensky,,[1] E. Brion,[2] F. Carlier,[2] V. M. Akulin,[2] and G. Kurizki[1]

[1] *Weizmann Institute of Science, Department of Chemical Physics - Rehovot 76100, Israel*
[2] *Laboratoire Aime Cotton, CNRS, Batiment 505, Campus d'Orsay, 91405 Orsay Cedex, France*

We propose a technique that allows to simultaneously perform universal control of the evolution operator of a system and compensate for the first-order contribution of any Hermitian constant noise and the action of the environment. We show that, at least, a threevalued Hamiltonian is needed in order to protect the system against any such noise and propose an explicit algorithm for finding an appropriate control sequence. This algorithm is applied to numerically design a safe gate in an atomic qutrit.

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# Indirect Estimation of Quadratic Hamiltonians

Daniel Burgarth,[1] Koji Maruyama,[2] and Franco Nori[2]
[1] *IMS and QOLS, Imperial College, London SW7 2PG, UK*
[2] *RIKEN, Wako-shi, Saitama 351-0198, Japan*

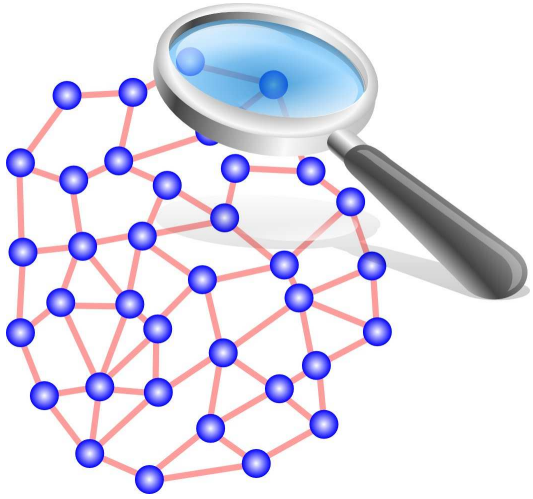

FIG. 1. The inner dynamics of fermions or bosons interacting via a quadratic Hamiltonian can be estimated by only looking at the surface of the system.

Estimating the nature and strength of interactions has been one of the main goals in quantum theory for almost a century. But it is only now that we are capable, in principle, of experimentally identifying the specific individual interactions of large systems of particles. However; in general the complexity of such Hamiltonian estimation is vast, as each particle needs to be initialised and its dynamics monitored. In this talk, we present efficient methods for estimating quadratic interactions, which are amongst the most frequently encountered in experiments.

We show that these can be identified indirectly- by only monitoring the quantum dynamics of the surface of the system, the inner dynamics is revealed. This is analogous to tomography as used in, e.g., seismology and ultrasonography. Our results provide further evidence for a holographic principle of many-body Hamiltonians, which was previously conjectured based on the scaling of entanglement in these systems [1].

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# Continuous variable entanglement in a time dependent two-particle system

Fernanda Raquel Pinheiro[1,2] and A. F. R. de Toledo Piza[2,*]
[1] *KoF - Physics Department, Stockholm University*[†]
[2] *DFMA - Physics Institute, University of Sao Paulo*

A known property of entanglement in compound systems is its dependence on the choice of the dynamical variables employed in its description [1,2]. This choice is directly related to the definition of each of its constituent parts and is reflected in the partition of the associated Hilbert space [3]. Explicitly, this means that in general, by redefining what the constituent parts of a system are, the amount of entanglement shared between its parts may vary for different choices. At the same time, much of the interest in quantum mechanics relies on studying systems' dynamics. This, in turn, depends on time evolution of the dynamical variables chosen to provide theoretical explanation for experimental phenomena. Given the above context, in this work we present some features of the dynamics of entanglement for a system of two particles interacting via a harmonic potential. The relative motion thereby evolves in accordance to a harmonic potential, and the center-of-mass motion evolves as a free particle. For an initial ground state in the relative coordinate and a Gaussian localized state in the center-of mass coordinate, time evolution manifests itself as broadening of the Gaussian. While the center-of-mass and relative motions remain disentangled, the entanglement between the two separate particles is not necessarily zero. In particular, we quantify entanglement and the way its amount changes with time is compared for different choices of dynamical variables. These results can be used to estimate the amount of entanglement shared between two atoms forming a diatomic molecule. The approximation of the interaction to the harmonic potential reduces the study of the entanglement in the diatomic molecule to the entanglement between Gaussian states, reassuring then the validity of its quantification by the measures applied.

* Electronic address: `piza@fma.if.usp.br`
† Currently at Stockholm University

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# Three-Level States in Superconducting Phase Circuits: Decoherence Characterization and Coupling to Two-Level Defects

Y. Shalibo,[1] Y. Rofe,[1] D. Shwa,[1] F. Zeides,[1] M. Neeley,[2] J. M. Martinis,[2] and N. Katz[1,*]

[1] *Racah Institute of Physics, Hebrew University of Jerusalem 91904, Israel*
[2] *Department of Physics, University of California, Santa Barbara, California 93106, USA*

Benchmarking the fidelity of quantum state preparation and evolution is vital for further advances in quantum engineering. State and process tomography are normally used for such benchmarking of individual qubit and coupled qubit systems. We extend this procedure to a superconducting phase circuit operating with three levels (qutrit), and measure the 3X3 density matrix for a set of arbitrary prepared states and their evolution. Representative prepared states are shown in figure (1). We quantify the diagonal and off diagonal decays due to relaxation and decoherence and compare to simulation.

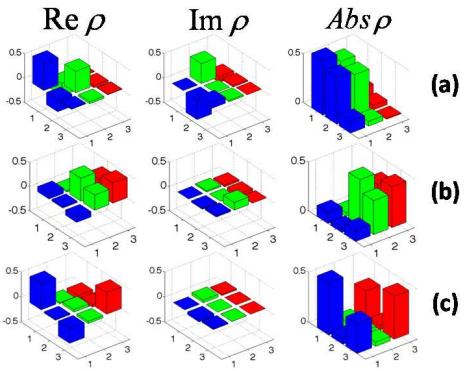


Figure 1: Qutrit Tomography. The measured density matrix of the target states (a) $|\psi\rangle = (|1\rangle + e^{i3\pi/4}|2\rangle)/\sqrt{2}$ (b) $|\psi\rangle = (|2\rangle + |3\rangle)/\sqrt{2}$ and (c) $|\psi\rangle = (|1\rangle - |3\rangle)/\sqrt{2}$, where $|1\rangle$, $|2\rangle$ and $|3\rangle$ are the ground state, 1st and 2nd excited states respectively.

Superconducting phase circuits suffer from various sources of decoherence, the most prominent of them being dielectric loss and coupling to microscopic two-level defect states (TLSs) inside the Josephson junction. The energy tunability of the phase circuit allows us to dynamically couple the qutrit to a single TLS, and generate entangled states of the combined system. We coherently transfer a photon into a long lived TLS and by shifting the circuit frequencies dynamically and loading another photon we extract from the TLS to the second excited state of the macroscopic phase qutrit. This procedure generalizes the qubit-TLS memory gate, demonstrated by Neeley et. al [1] and could be useful as a controlled memory readout operation.

We utilize similar gates on the qubit-TLS system (see sequence in the lower inset of figure (2)a) to probe the energy relaxation and decoherence processes of the TLS itself. The effect of coupling of the qubit with different TLSs is shown in figure (2). We measure the lifetime and coherence of an ensemble of TLSs and compare to their interaction strength with the phase qubit. We find a power law relation between the average decay times and the corresponding interaction strengths, and for the average coherence times we find an optimum at intermediate coupling strengths. We explain both the lifetime and the coherence results using the standard TLS model [2], which is consistent with dipole radiation by phonons and anti-correlated noise in the environmental parameters that set the defect's energy and the resulting dephasing.

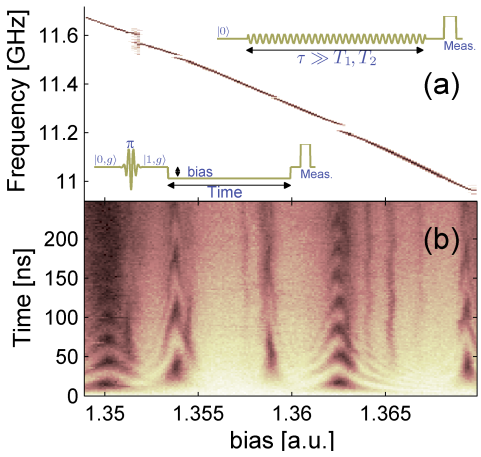


Figure 2: Frequency-domain and time-domain appearence of TLSs in qubit measurement. (a) qubit spectrum taken at different bias values. (b) probability of the qubit to remain in the excited state after a "free" evolution at different bias values. Upper and lower insets: control sequences used to produce (a) and (b) respectively

* Electronic address: katzn@phys.huji.ac.il

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# Quantify the robustness of optimal control signals in quantum control

Antonio Negretti,[1,*] Tommaso Calarco,[1] and Rosario Fazio[2]
[1]*Institut für Quanteninformationsverarbeitung, Universität Ulm, Albert-Einstein-Allee 11, D-89069 Ulm, Germany*
[2]*NEST, Scuola Normale Superiore and Istituto di Nanoscienze - CNR, Pisa, Italy*

The theory of optimal control (OC) has been instrumental for the achievement of highly reliable electronic devices used to control, for instance, mechanical systems such as airplanes, cars, etc., but also to control chemical reactions or to design ultra-fast laser pulses for manipulating molecules, and today even to optimize (stochastic) financial analyses.

The topic has recently attracted the attention of physicists working in quantum information and computation science, because of the need to engineer tailored quantum information processing and matter. To this aim different numerical techniques have been devised in order to minimize (or maximize) some performance criterion or, alternatively called, objective functional J. The most used ones in open-loop quantum control are: the Krotov iterative method [1] and the gradient ascent pulse engineering algorithm [2]. Although these are powerful tools, once the optimal solution of a particular control problem is found, there is no systematic way, to our knowledge, to give an assessment of the robustness of the obtained OC signal. A possible (empirical) approach relies on applying an arbitrary distortion to the OC solution and then looking at the error that it produces on the objective functional, or by selecting a region in the Hilbert space that is robust against noise (decoherence free-subspace) [3], whose existence follows from the symmetry properties of the noise.

Thus, our work aims to provide a general method to quantify how robust a given optimal control scheme is and to which extent it can tolerate errors [4]. More precisely: The usual problem is the engineering of a control Hamiltonian $\hat{H}(u_t)$, with $u_t \equiv u(t)$ being the control parameter, such that the initial state $|\psi_{\rm in}\rangle$ at time $t = t_0$ of the quantum system under consideration is brought at time $t = T$ to some desired goal state $|\psi_{\rm g}\rangle$ (see also Fig. 1). The search of the optimal control pulse $u_t^{\rm o}$ is performed, for example, by using one of the aforementioned numerical techniques. Hence, given the desired cost, $\mathcal{J}$, that the system (at least) has to attain, we aim to quantitatively assess the robustness of an OC signal by defining a suitable norm in the space of control parameters. This norm, which is an average of the cost functional J in the space of control parameters, allows to evaluate the degree of distiortion $\delta u_t = u_t - u_t^{\rm o}$ we may apply to the OC parameter $u_t^{\rm o}$ such that the distance between the final state $|\psi(T)\rangle$ and the target one $|\psi_{\rm g}\rangle$ is smaller, or at most equal, than $\mathcal{J}$.

Even though usually it is not possible to know analytically the OC parameter, we underscore that our philosophy is that the parameter obtained with some numerical algorithm is very close to the global optimum. In other words, the error on the cost functional obtained with the numerically found OC parameter has to be much smaller than the error allowed by the process we are interested to optimize. Besides the interest on its own, we believe that our approach is of importance for experiments, where, typically, optimal pulses are extremely difficult to achieve. Indeed, our method can help to find a control signal that is experimentally viabler than the OC one and still able to minimize passably the performance criterion we are interested in.

The formalism is applied to a toy model, which is analytically solvable, and to the Landau-Zener model, whose optimal control problem is solvable only numerically.

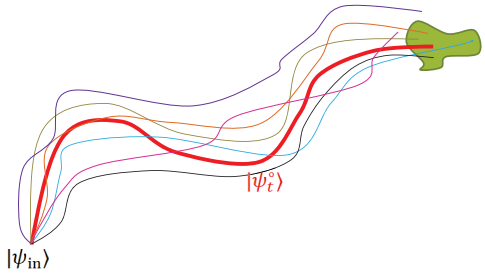


FIG. 1: Pictorial representation of the optimal trajectory $|\psi_t^{\rm o}\rangle$ (red-thick line) obtained with the OC parameter $u_t^{\rm o}$ and trajectories (thin lines) for non-optimized control pulses. The shaded (green) area represents the portion of Hilbert space within which the cost functional $\mathsf{J} \leq \mathcal{J}$ (see also text), that is, the subset of state vectors close to the goal state $|\psi_{\rm g}\rangle$.

* Electronic address: antonio.negretti@uni-ulm.de

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# Deterministic multi-mode photonic source and simulation of topological matter with collective encoding and Rydberg blockade in atomic ensembles

Anne E. B. Nielsen[1,*] and Klaus Mølmer[1]

[1] *Lundbeck Foundation Theoretical Center for Quantum System Research, Department of Physics and Astronomy, Aarhus University, Denmark*

The discovery that measurements provide a way to induce an effective interaction between photons has led to significant progress in optical quantum computing. Optical quantum computing does, however, require suitable resource states, and here we propose the implementation of a photonic source, which is able to produce a large variety of entangled multi-mode states of light deterministically [1].

The basic idea of our proposal is to manipulate the state of an ensemble of atoms and then convert the resulting state to a photonic state via cooperative spontaneous emission. The ensemble is only addressed collectively, and the desired multi-mode state is encoded in the populations of the internal atomic levels [2]. The state manipulation is achieved by application of the Rydberg blockade mechanism, which allows one to move precisely one atom at a time from one internal atomic level to another and to achieve controlled quantum gates between the populations in different atomic states. In the most simple case, the photon emission proceeds by transferring the populations in the internal levels one after the other to an excited state as illustrated in Fig. 1. Spontaneous emission occurs in a well-defined direction [3] whenever a level is occupied. State manipulation may also take place in between the emission events, which makes the proposal very flexible. We provide explicit schemes to generate trine states and one- and higher-dimensional cluster states with applications within quantum cryptography and quantum computing.

Topological matter shows a special kind of correlated order with no local order parameter. Fractional quantum Hall effect and high-temperature superconductivity

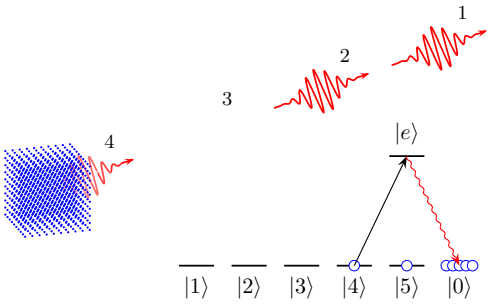


FIG. 1: Conversion of the collective state of an atomic ensemble to light. Here, the initial state of the ensemble is $|11011\rangle$, but it could also be a superposition of more terms. The state $|0\rangle$ is the reservoir state, which contains most of the atoms.

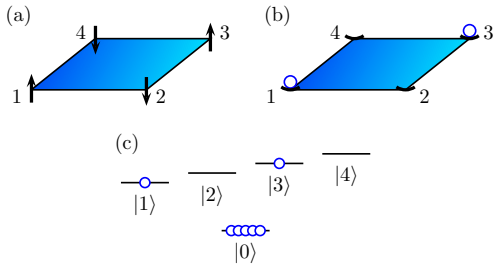


FIG. 2: (a) A plaquette of four spin 1/2 particles. (b) In the hard core boson model, the spin states are replaced by the presence or absence of an atom. (c) Plaquette encoded in the collective populations of four internal states of the atoms in an atomic ensemble. The illustrated state is $|\uparrow\downarrow\uparrow\downarrow\rangle$ or $|1010\rangle$.

have been proposed to originate from topological states. While unambiguous demonstration of topological matter in natural physical materials is difficult, a number of proposals have appeared, which prepare topological states artificially. In [4], it is proposed to prepare and manipulate minimum instances of topological matter in atoms trapped in an optical lattice. The states are prepared in four atoms sitting at the corners of a square as in Fig. 2(a), where $\uparrow$ and $\downarrow$ denote two internal atomic states. The same states can be encoded in the presence or absence of an atom at the four lattice sites (Fig. 2(b)). We propose [5] to prepare topological states in the populations of four internal levels of atoms in an ensemble as in Fig. 2(c), which is interesting because the properties of atoms in optical lattices and atomic ensembles are different. In an ensemble, it is, e.g., possible to have more than one atom in each level, which enlarges the available Hilbert space, and a number of transformations that may be difficult to achieve in an optical lattice can be carried out simply by use of Raman transitions.

* Also at Max-Planck-Institut für Quantenoptik, Garching, Germany; Electronic address: annebn@phys.au.dk

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# Molecular solution for the subset-sum problem on DNA -based quantum computing

M. Rezaei,[1,*] Z. Azimi,[2] and S. Ahadpour[2]

[1] *Department of Theoretical and Astrophysics, Faculty of Physics, University of Tabriz, Tabriz, Iran.*
[2] *Department of Physics, University of Mohaggeg Ardabili, Ardabil, Iran.*

Molecular computation was proposed by Feynman [1] in 1961 and it was showed that computing devices based on quantum theory are able to finish computations faster than the standard Turing machines. In 1994, Adleman [2] succeeded to solve an instance of the Hamiltonian path problem in a test tube, just by handling DNA strands. Lipton [3] investigated a special case of more general methods that could solve NP-complete problems using DNA experiments. Deutsch [4] presented a general model of quantum computation i.e., the quantum Turing machine. Molecular solution for the subset-sum problem on DNA-based supercomputing has been offered in [5]. It has been proved, the subset-sum problem is the NP-complete problem (Cormen et al., 2003; Garey and Johnson, 1979; Cook, 1971; Karp, 1972). Here, a finite set $S = \{s_1, ..., s_q\}$ is defined for solving subset-sum problem using DNA-based algorithm [5, 6] and it is supposed that every elements in $S$ are positive integer. Now the aim is finding possible subsets $S_i \subseteq S$ such that the sum of all elements in $S_i$ be exactly equal to b, where b is a positive integer and can implement by Hadamard gates, NOT gates, CNOT gates, CCNOT gates, Grover's operators, and quantum measurements on a quantum computer. In order to achieve this, first we use q number Hadamard gates to construct $2^q$ possible subsets of a q-element set S, then we apply NOT gates, CCNOT gates and Grover's operators to construct solution space. It is demonstrated, the DNA-based quantum algorithm[7, 8] of an n-qubit parallel adder and a DNA based quantum algorithm of an n-qubit parallel comparator can implement using quantum gates and Grover's operators to formally verify our designed molecular solutions for the subset-sum problem.

For carrying out the solution space from subsets of a finite set $S$, auxiliary quantum bits $\mid c_i >, \mid r_i >, \mid k_{ij} >$ , $\mid s_{ij} >$, are needed for $0 \leq i \leq q$ and $0 \leq j \leq n$. Each quantum bit in $\mid c_i >$ is initially prepared in state of $\mid 0 >$, in $\mid r_i >$ is initially prepared in state of $\mid 1 >$, in $\mid s_{ij} >$ is initially prepared in state of $|0 >$ and $\mid k_{ij} >$ is a $j-th$ bit of a binary number that is $i-th$ element of a finite set $S$. $\mid k_{ij} >$ is used to change the state of $\mid s_{ij} >$ after influence of $\mid c_i >$ , i.e., $\mid s_{ij} > \leftarrow \mid s_{ij} > \bigoplus \mid c_i > \wedge \mid k_{ij} >$. Changing $\mid s_{ij} >$ using this algorithm is quantum equivalent of Append operator in classical procedure value $(T_0, q, n)$ algorithm [7, 8]. Assume $((q+1) \times n)$ one- digit binary numbers, $y_w$ , are used to represent the sum of $q$ elements in $S$ for $1 \leq w \leq ((q+1) \times n)$. For carrying out quantum equivalent of Append operator in parallel adder $(T_0, q, n)$ Algorithm [8] suppose that $|y_{i \times n+j} >$ and $|z_{(i-1)\times(n+1)+(j+1)} >$ are in state $|0 >$, then change after influence $|y_{(i-1)\times n+j} >$ and $|z_{(i-1)\times(n+1)+j} >$ and $|s_{ij} >$. For this propose, we introduce some quantum registers again and compute $\mid y >, \mid z >$, in each step by using 19-CCNOT gates , and NOT gates. Last algorithm is parallel comparator for comparing the sum of elements for subsets of a finite set with any given positive integer by using quantum gates and Grover's operators.

* karamaty@tabrizu.ac.ir

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# Experimental Realization of the Deutsch-Jozsa Algorithm with a Six-Qubit Cluster State

Giuseppe Vallone,[1,2,*] Gaia Donati,[3,*] Natalia Bruno,[3,*] Andrea Chiuri,[3,*] and Paolo Mataloni[3,*]

[1] *Museo Storico della Fisica e Centro Studi e Ricerche Enrico Fermi, Via Panisperna 89/A, Compendio del Viminale, 00184 Roma, Italy*
[2] *Dipartimento di Fisica, Università Sapienza di Roma, 00185 Roma, Italy*
[3] *Istituto Nazionale di Ottica (INO-CNR), L.go E. Fermi 6, 50125 Florence, Italy*

It is possible to create multiqubit cluster states by encoding more qubits in different degrees of freedom of the particles. Here we report the experimental realization of the Deutsch-Jozsa Algorithm performed with a 6-qubit cluster state $|\mathsf{E}\rangle$ [1] based on the hyperentanglement of two photons in three independent degree of freedoms (DOFs) [2].

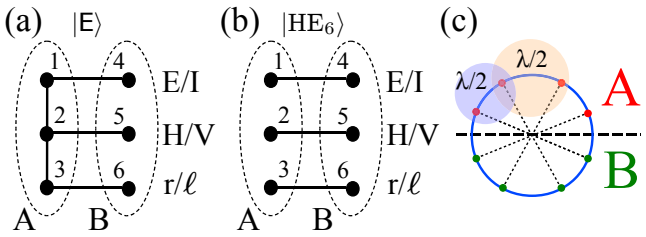


FIG. 1. Graphs associtated to (a) the $|\mathsf{E}\rangle$ cluster and (b) the $|\mathrm{HE}_6\rangle$ hyperentangled state. Qubits 1, 4 are encoded in the E/I momentum, qubits 2, 5 are encoded in polarization (H/V) and qubits 3, 6 in the r/$\ell$ momentum. (c) Annular section of the conical SPDC emission of a Type I phase matched crystal.

Referring to Fig. 1, the two-dimensional cluster state $|\mathsf{E}\rangle$ is obtained from a six-qubit hyperentangled state [2], $|\mathrm{HE}_6\rangle$, whose graph is shown in Fig. 1(b). Our experimental setup adopts a source of two-photon states based on a Spontaneous Parametric Down-Conversion (SPDC) process where the two particles are entangled at the same time in the polarization and in two linear momentum degrees of freedom (DOFs). By a proper interferometric setup [3, 4] it is possible to measure the two spatial DOFs; these variables, labeled as the "right/left" momentum ($r/\ell$) and the "external/internal" momentum ($E/I$), are both associated to each of the eight modes on which the two photons are emitted. The graph associated to the six-qubit cluster state $|\mathsf{E}\rangle$ exhibits two links between qubits 1 and 2 and between qubits 2 and 3, hence the corresponding $\mathsf{CZ}_{12}$ and $\mathsf{CZ}_{23}$ logic gates only involve qubits belonging to photon $A$. The optical implementation of the two controlled-Z gates is realized by means of two half-wave plates, as shown in Fig. 1(c).

We now go into the details of the experimental realization of the DJ algorithm for a function acting on $n = 2$ bits. In this case, the boolean function which we are interested in is such that $f : \{0,1\}^2 \to \{0,1\}$. We focus our attention on the balanced function $f_B$, such that $f_B(0) = f_B(3) = 0$, $f_B(1) = f_B(2) = 1$. The implementation of the DJ algorithm requires an additional ancillary qubit in the initial state $|y\rangle_A \equiv |y\rangle_{\overline{3}}$, where $\overline{3}$ is the logical qubit associated to the ancilla. The previously defined functions is implemnted with two controlled-NOT gate between logical qubits $\overline{i}$ and $\overline{j}$. The "$\mathsf{E}$ cluster" just described is our quantum computer. Let's consider the balanced function. By measuring qubits 1, 3 and 5 in the bases $B_1(0)$, $B_3(0)$ and $B_5(\pi)$ we implement, at the logical level, the two CNOT gates ($U_{f_B}$) needed to implement the oracle function $f_B$ (see Fig. 2). Then we proceed with the measurement of the output qubits 4, 6 and 2 in the computational basis.

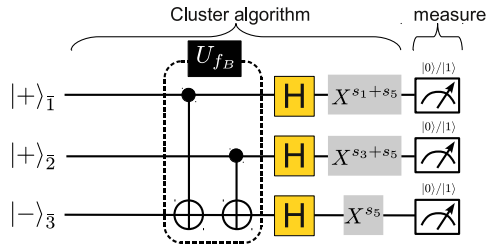


FIG. 2. Balanced function: circuit realized according to the pattern of single-qubit measurements implementing the DJ algorithm for $n = 2$. Gray gates represent Pauli errors.

The $|\mathsf{E}\rangle$ state was used to realize an all-optical implementation of the Deutsch-Jozsa algorithm for n = 2 qubits [1]. We were able to evaluate a two-bit balanced function as well as a constant one and to discriminate between them in one single run of the executed program in contrast to the three runs needed with a classical computer. The correct output is identified with fidelity $\simeq 75\%$ at a frequency of almost 1kHz without feed-forward (error-free case). This result overcomes by several orders of magnitude what can be currently achieved with six-photon cluster states.

* http://quantumoptics.phys.uniroma1.it/

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# Developing scalable quantum computer hardware from naturally trapped ions

Ying Yan, Jenny Karlsson, Lars Rippe and Stefan Kröll[1]
[1]*Dept. of Physics, Lund University, Box 118, S-221 00 Lund, Sweden*

Many physical systems are investigated as test-beds for quantum computer hardware, such as trapped ions [1], nuclei in molecules [2], single photons [3], Josephson Junctions in superconductors [4], nitrogen vacancy centers in diamond [5] etc. Here randomly doped rare earth ions in an inorganic crystal are studied (specifically praseodymium(Pr) doped $Y_2SiO_5$). One attractive point for this material is the $100\mu s-$ long optical coherence time at temperature below 4K which is due to the unfilled 4f shells are well shielded from the environment by the full filled outter-lying 5s and 5p shells. The qubit is represented by two ground state hyperfine levels of Pr ions and addressed by a dye laser with 1kHz linewidth via the optical transition $^3H_4 \rightarrow^1 D_2$ around 600nm. For the qubit levels the coherence time can be close to 1 second at appropriate magnetic fields [6].

High fidelity single qubit gates [7] has been demonstrated and a scalable approach has also been proposed [8] in this material for ensemble qubits which contain billions of ions. Still it is anticipated that a single instance approach where each qubit consists of only one single ion would be much easier to scale [9]. However to read out a single ion qubit state is a challenge since a Pr ion has an optical lifetime around $164\mu s$. We are investigating the possibility of having a nearby Cerium(Ce) ion as a read out ion using the dipole-dipole interaction between the Pr qubit and the Ce readout ion. To detect the qubit state, a laser is tuned to the Ce ion transition and the fluorescence from the Ce ion is monitored. Then a $\pi$ pulse resonant with a transition from the Pr qubit $|0\rangle$ state to an excited state is applied. If the qubit is in the $|0\rangle$ state, it will be promoted to the excited state. Because of the different permanent dipole moments in ground and excited states for both the Pr ion and Ce ion, the electric field generated by the Pr dipole shifts the nearby Ce readout ion energy levels out of resonance with the original driving laser. The fluorescence signal from the Ce ion is turned off. On the other hand if the Pr ion in the $|1\rangle$ state the Ce ion will continue to fluoresce.

One of the key requirements for the readout ion is that it has to have large dipole moment difference between the ground and excited state. This dipole moment difference can be measured by carrying out photon echo experiments on one ion (Pr ion in the experiments) while exciting the other ion (Ce ion) during the echo dephasing period and observe the reduction of the echo intensity as a result of the excitation induced frequency shift (EFS) [10]. Therefore the photon echo intensity from Pr ions was recorded as a function of separation time between the two photon echo pulses for two different situations: with and without Ce ion excitation. Up to 68% percent echo intensity reduction was observed.

The homogenous linewidth broadening of the Pr ion transition due to Ce (scrambler) excitation was observed from the experimental data, shown in Fig.1, which shows

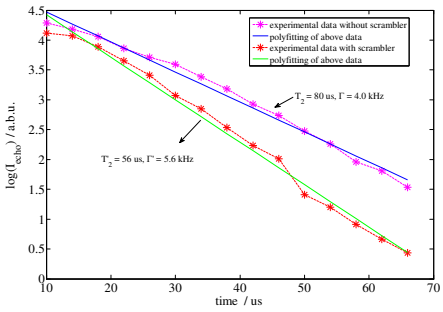


FIG. 1: 1.6kHz linewidth broadening was observed.

the dipole moment difference for the Ce ion to be 70 times as large as that of the Pr ions using the same model as in [11]. However the exact quantitative result need to be confirmed by future measurements.

An important issue for quantum computer approaches is to find ways to achieve large scale scalability. We are considering photon-based entanglement techniques for remote ions as one part of such a scheme. We have therefore theoretically simulated the situation where we enhance the evanescent field from a photon in a toroidal micro-cavity that interacts with an ion in a crystal just next to the micro-cavity. The simulation shows that the bow-tie antenna can increase the single photon Rabi frequency by three orders of magnitude [12]. These simulations will also be presented in the poster.

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# Heralded generation of entangled photon pairs

Stefanie Barz,[1,2,*] Gunther Cronenberg,[1,2] Anton Zeilinger,[1,2] and Philip Walther[1,2]

[1] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria*
[2] *Institute for Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria*

Entangled photons are a crucial resource for quantum communication and linear optical quantum computation. Therefore the controlled generation of these fundamental states attracts a worldwide effort. The majority of current experiments is based on the production of photon pairs in the process of spontaneous parametric down-conversion, where the entangled photon pair is concluded from post-selection of randomly occurring coincidences. Here we present the heralded generation of photon states that are maximally entangled in polarization with linear optics and standard photon detection from spontaneous parametric down-conversion. We utilize the down-conversion state corresponding to the generation of three pairs of photons, where the coincident detection of four auxiliary photons unambiguously heralds the successful preparation of the entangled state. This controlled generation of entangled photon states is a significant step towards the applicability of a linear optics quantum network, in particular for entanglement distribution, entanglement swapping, quantum teleportation, quantum cryptography and scalable approaches towards photonics-based quantum computing schemes.

* Electronic address: stefanie.barz@univie.ac.at

## Optimal and equilibrium solutions to quantum minority games

Puya Sharif[1,*] and Hoshang Heydari[1]
[1] *Physics Department, Stockholm University.*

Game theory is the mathematical framework for analyzing strategic interactions in conflict and competition situations. In recent years quantum game theory has earned the attention of physicists, and has emerged as a branch of quantum information theory [1]. With the aid of entanglement and linear superposition of strategies, quantum games are shown to yield significant advantage over their classical counterparts. In this paper we explore optimal and equilibrium solutions to quantum minority games. Initial states with different level of entanglement are investigated. Focus will be on 4 and 6 player games with some N-player generalizations.

* Electronic address: ps@puyasharif.net

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## Quantum filtering equation with a coherent state filter

Anita Dąbrowska* and Przemysław Staszewski†
*Department of Theoretical Foundations of Biomedical Sciences and Medical Informatics*
*Collegium Medicum, Nicolaus Copernicus University*
*ul. Jagiellońska 13, 85-067 Bydgoszcz, Poland*

We apply the theory of measurement continuous in time based on quantum stochastic calculus [1, 2] to derive the Belavkin filtering equation for a quantum system in- teracting with the Bose field prepared in a coherent state. To get the posterior dynamics of the system we use the generating functional approach described in [3, 4], but, in contrast to the derivation given by Belavkin when the Bose field is initially in the vacuum state, we consider the situation when the information about the system is extracted from a measurement of a coherent channel.

* Electronic address: adabro@cm.umk.pl
† Electronic address: przemek.staszewski@cm.umk.pl

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# Entanglement localization on a three-photon system

Miroslav Gavenda,[1] Radim Filip,[1] Eleonora Nagali,[2] Fabio Sciarrino,[2,3] and Francesco De Martini[2,4]

[1] *Department of Optics, Palacký University, 17. listopadu 12, 771 46 Olomouc, Czech Republic*
[2] *Dipartimento di Fisica, Sapienza Università di Roma, piazzale Aldo Moro 5, 00185 Roma, Italy*
[3] *Istituto Nazionale di Ottica Applicata, Largo E. Fermi 6, 50125 Firenze, Italy*
[4] *Academia Nazionale dei Lincei, Via della Lungara 10, 00165 Roma, Italy*

Quantum entanglement a key resource in quantum information tasks willingly interacts with surrounding systems. Due to this interaction the entangled system under interest couples to environment leading to the reduction of entanglement or even to its total loss. If the entanglement is completely destroyed distillation protocol does not work. Therefore other correcting protocols were suggested such as unlocking of hidden entanglement or entanglement localization [1]. Entanglement localization can concentrate back redistributed entanglement at least partially just by measurement on the environment and proper feed-forward quantum correction. In this presentation we deal with the situation when the input state is maximally entangled state of two qubits and another qubit serves as a surrounding system. The surrounding qubit is inaccessible before the coupling that's why is in an unknown state - totally mixed state. The qubits in our case are represented by polarization states of single photons. After a linear coupling between photons, modeled by a beam splitter with transmissivity $T$, the entanglement of the input state is reduced and for some $T$ entanglement completely vanishes. We study the influence of two-photon coherence between the surrounding photon and one photon from the entangled pair on the localization protocol. We introduce parameter $p$ which determines the level of indistinguishability between photons: with probability $p$ the photons are indistinguishable at the beam splitter. We theoretically prove that for any linear coupling it is possible to localize non-zero entanglement back to the pair just by proper polarization sensitive detection of photon in surrounding mode (after coupling the surrounding photon is accessible). After measurement on the surrounding photon we may use additional single-copy filtration on both photons from the pair to further increase the concurrence. Qualitatively this localization is independent on the level of coherence between coupling photons.

The theoretical curves of concurrences after mixing, measuring and filtering procedures are shown for distinguishable photons ($p = 0$) in FIG. 1 and for indistinguishable photons ($p = 1$) in FIG. 2. Further we show that single-copy filtration produces the state violating the Bell inequalities for any $p$ and $T$ (except point $T = p = 0$). For complete analysis of entanglement localization on a three-photon system see [2]. The theoretical results were experimentally tested using polarization entangled photonic qubits created in SPDC process [3]. An extension of the localization procedure was calculated for multiple consecutive couplings to the independent surrounding photons.

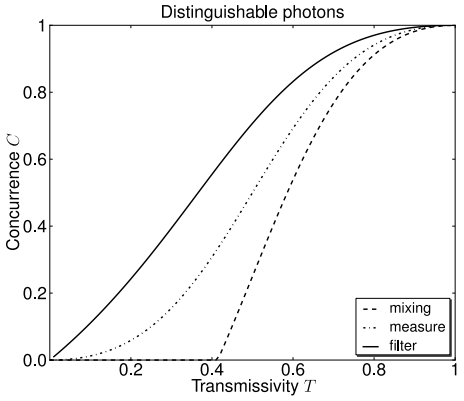


FIG. 1: Concurrences for distinguishable photons

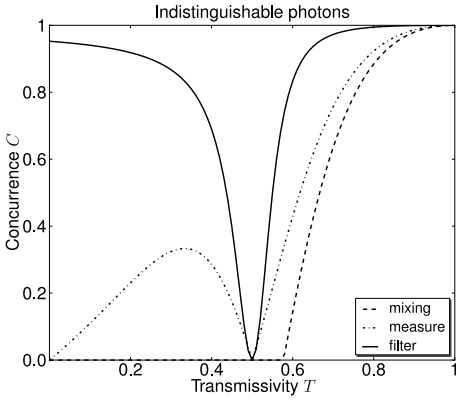


FIG. 2: Concurrences for indistinguishable photons

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# Non-Markovian dynamics of a driven qubit

Pinja Haikka[1,*] and Sabrina Maniscalco[1,†]
[1] *Turku Center for Quantum Physics*
*University of Turku, Finland* [‡]

All quantum systems are open, i.e., they interact with an environment. The interaction with the environment leads to dissipation and decoherence due to a flow of energy and/or information from the system to the environment. The coupling of the quantum system to the environment is described by the spectral density function. If the spectral density function strongly varies with the frequency of the environmental oscillators, the environment is said to be structured. In these systems the reservoir memory effects induce a feedback of information from the environment into the system. We call these systems non-Markovian.

We study a driven two-state system interacting with a structured environment. We introduce the non-Markovian master equation ruling the system dynamics, and we derive its analytic solution for general reservoir spectra [1, 2]. We compare the non-Markovian dynamics of the Bloch vector for two classes of reservoir spectra: the Ohmic and the Lorentzian reservoir.

We study the system dynamics with and without the widely used secular approximation, singling out its limits of validity. The investigation of the effects of nonsecular terms on the dynamics of the Bloch vector brings to light the existence of nonsecular oscillations in the population of the two-state systems. Contrarily to oscillations due to non-Markovianity, such oscillations persist for times much longer than the reservoir correlation time. Moreover, our analysis shows that the nonsecular terms affect also the asymptotic long time values of the Bloch vector components.

Finally, a very important result we present is the analysis of conditions for complete positivity (CP) of the system with and without the secular approximation. Interestingly, the CP conditions have a transparent physical interpretation in terms of the characteristic timescales of phase diffusion and relaxation processes.

* pmehai@utu.fi
† smanis@utu.fi
‡ http://www.openq.fi

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# Entanglement in Two-Mode Gaussian Open Quantum Systems

Aurelian Isar[1, *]

[1] *Department of Theoretical Physics, National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania*

In the framework of the theory of open systems based on completely positive quantum dynamical semigroups, we investigate the continuous variable entanglement for a system composed of two modes embedded in a thermal environment. We are interested in discussing the correlation effect of the environment, therefore we assume that the two modes are uncoupled, i.e. they do not interact directly. We write the Markovian Lindblad master equation in the Heisenberg representation and the initial state of the considered system is taken of Gaussian form. The quantum dynamical semigroup assures the preservation in time of the Gaussian form of the state and we solve the time evolution equation for the covariance matrix. By using the Peres-Simon necessary and sufficient criterion for separability of two-mode Gaussian states [1, 2], we investigate the dynamics of entanglement of the system in terms of the covariance matrix. For certain values of the environment temperature, the state keeps for all times its initial type: separable or entangled. For other values of the temperature, entanglement generation, entanglement suppression (entanglement sudden death) or a repeated collapse and revival of entanglement take place. Using the upper and lower bounds on the symplectic invariant obtained in terms of global and marginal purities [3], we determine the asymptotic Gaussian maximally entangled mixed states (GMEMS) and the corresponding asymptotic maximal logarithmic negativity, which characterizes the degree of entanglement of these states [4–6].

* Electronic address: `isar@theory.nipne.ro`; URL: `http://www.theory.nipne.ro/~isar`

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# Investigation of magnetic field effect on thermal entanglement in a spin 1/2 chain

Safa Jami,[1] Maryam Divsalar,[1] and Mohsen Sarbishei[2]
[1]*Department of Physics, Islamic Azad University, Mashhad Branch, Mashhad, Iran*
[2]*Department of Physics, Ferdowsi University of Mashhad, Mashhad, Iran*

Entanglement is a particularly useful resource in quantum computation and quantum information theoretical studies of the last decade. And has many applications such as teleportation, quantum dense coding, cryptography and so on. However, when it comes to generate, the temperature should be considered since in most cases the thermal fluctuation can result in disentanglement. Therefore, the entanglement is very fragile because of the effect of the temperature, and how to create stable entanglement is becoming a significant task. Compared with other entanglement, thermal entanglement has its advantage of stability and requires neither measurement nor controlled switching of interaction in the preparing process. Entanglement of quantum systems changes under environment condition like, temperature, magnetic field, noise and so on.

The two-dimension spin-1/2 $j_1 - j_2$ model has been the subject of many studies in last two decades. Entanglement measure we use is the concurrence which was shown by Wootters to be a valid measure of entanglement for a pair of qubits.

In the present research, we have investigated the thermal entanglement properties of a system which consists of five spins 1/2 by calculating the pair wise concurrence in a uniform magnetic field. First we study the effect of changing temperature and exchanging coupling on thermal entanglement of a system. Then we apply magnetic field on the system and study the effect of external magnetic field on thermal entanglement. It is found that an increase in temperature and magnetic field lead to a decrease of pair wise entanglement. We also, have found that there is the entanglement-quantum phase transition, QPT, at the critical points.

Here, we report the result of investigations on the dependence of nearest-neighbor, next-nearest-neighbor concurrence on the frustration, applied magnetic field and temperature for a 5-qubit $j_1 - j_2$ chain. Our investigation considers pairwise entanglement between two qubits of the 5-qubit chain and is not expected to reveal all the entanglement of the system which might be multipartite in character. Furthermore, the change reported for the concurrence might be due to an opposite change in multipartite entanglement.

# Quantifying entanglement of two relativistic particles using optimal entanglement witnesses

M. A. Jafarizadeha[1,*] and M. Mahdiana[1,†]

[1] *Department of Theoretical Physics and Astrophysics, University of Tabriz, Tabriz 51664, Iran*

We consider two spin-1/2 massive fermions where entangled particles are generated both in the spin and momentum degrees of freedom. For simplicity, instead of using the superposition of momenta we use only two momentum eigenstates ($p_1$ and $p_2$), so we restrict ourselves to 2D momentum subspace. Then, for our problem, we present a new method of convex optimization to obtain optimal entanglement witness (OEW) that determines the amount of entanglement of two-particle system. Finally, we have shown that the entanglement is not relativistic invariant when the momentum and the Lorentz boost are parallel.

* Electronic address: jafarizadeh@tabrizu.ac.ir
† Electronic address: Mahdian@tabrizu.ac.ir

[1] M. A. Jafarizadeh and M. Mahdian , to be published in International Journal of Quantum Information, (IJQI).

# Measurement games against Nature in a world that minimizes changes in the number of black swan phenomena are quantum

Edward W. Piotrowski[1,*] and Jan Sładkowski[2,†]

[1] *Institute of Mathematics, The University of Białystok, ul. Lipowa 41, Pl-15424 Białystok, Poland*
[2] *Institute of Physics, The University of Silesia, ul. Uniwersytecka 4, Pl-40007 Katowice, Poland*

A wide class of activities is interpreted as a two-person game, in which one agent (observer) gathers knowledge about the second agent which is recognized as Nature (a collective opponent including other agent and environment). The goal is of the game is to obtain by measurement (subjective [1]) knowledge of the state of Nature. Knowledge can only be gained by analysis of the course of the game. We distinguish the class of calm measurement games with the following attributes:

- the game is against a collective opponent (Nature) [2]
- the goal is of the game is to obtain (subjective) knowledge of strategy of Nature
- the gathered information must be effective and the resulting "entities must not be multiplied beyond necessity" (Ockham's razor) [3]

  **a** observer can be a part of Nature

  **b** Nature avoids changes in the amount of extreme phenomena (called black swans)

Having this in mind, the following conclusions can be drawn:

- the risk of measurement error is quantized [4]
- in calm measurement games "negative probabilities" might happen [5]
- a natural analysis of calm measurement games can be performed with help of the quantum or quantum-like formalism [7]-[11]
- calm measurement games have features of bootstrap in the statistical sense.

* Electronic address: qmgames@gmail.com
† Electronic address: jan.sladkowski@us.edu.pl

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# Optimal entanglement witnesses

Justyna Pytel[1,*] and Dariusz Chruściński[1]

[1] *The Faculty of Physics, Astronomy and Informatics Nicolaus Copernicus University, POLAND*

Entanglement is one of the essential features of quantum physics and is fundamental to future quantum technologies. Therefore, there is a tremendous interest in developing efficient theoretical and experimental methods to detect entanglement, which has given rise to the notion of entanglement witnesses. There has been a considerable effort devoted to constructing the latter, but a general procedure is still not known. Recently, we provided two classes of indecomposable entanglement witnesses [1, 2]. The first class yields new entanglement witnesses for composite quantum systems of combined dimension $(2k)^2$ (or, equivalently, gives new indecomposable atomic maps in the algebra of $2k \times 2k$ complex matrices). These witnesses are optimal and atomic, i.e., they are able to detect the weakest quantum entanglement contained in states with positive partial transposition (PPT). The second class produce not only optimal but even nd-optimal indecomposable entanglement witnesses for composite quantum systems of combined dimension $(4k)^2$ (or, equivalently, indecomposable positive maps in the algebra of $4k \times 4k$ complex matrices). This construction provides a natural generalization of the Robertson map. Both classes reduce to the well-known Breuer-Hall witness in dimension 4. We will discuss these new classes of entanglement witnesses as well their relation to entanglement breaking channels.

* Electronic address: `letypka@fizyka.umk.pl`

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# Geometric phases and invariants of four-qubit states

H. Cavit Sezer[1,*] and Hoshang Heydari[1]
[1] *Physics Department, Stockholm University.*

Geometrical and topological structure of multi-qubit systems are quit interesting research topics in the field of quantum information and quantum computation with potential to provide less error prone schemes for quantum computation. In this paper, we study the entanglement properties of four-qubit states under action of topological quantum gates. In particular, we investigate evolution of four-qubit states based on geometric phases and polynomial and geometric invariants [1, 2]. We also compare and discuss entanglement dynamics of these multi-qubit states using geometric phases and invariants.

* Electronic address: cavitsezer@hotmail.com

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# Entanglement in an Infinite Ising Spin Chain with a Time Dependent Coupling in an External Time-Varying Magnetic Field

Gehad Sadiek[1,*]

[1] *Department of Physics, King Saud University, Riyadh 11451, Saudi Arabia*
*Department of Physics, Ain Shams University, Cairo 11566, Egypt*

Interacting Heisenberg spin chains represent very reliable model for constructing quantum computing schemes in different solid state systems. This spin chain can be experimentally realized, for instance, as a chain of coupled nano quantum dots where the coupling between the dots can be controlled by gate voltages. The dynamics of entanglement in a one dimensional XY spin chain system has been studied before for a constant Heisenberg coupling in presence of a time-dependent magnetic field [1]. In this work, we consider an infinite one dimensional Ising spin chain with a next nearest neighbor time-dependent coupling $J(t)$ in presence of a time-dependent magnetic field $h(t)$. We discuss a general solution for the system and present an exact solution for step function forms $J(t) = J_0 + (J_1 - J_0)\theta(t)$ and $h(t) = h_0 + (h_1 - h_0)\theta(t)$ which is of great practical interest. The Hamiltonian of the system takes the form

$$H = -J(t)\sum_{i=1}^{N}\sigma_i^x\sigma_{i+1}^x - \sum_{i=1}^{N}h(t)\sigma_i^z \qquad (1)$$

Applying the standard Lieb and Schultz [2] approach, the Hamiltonian is transformed into the form $H = \sum_{p=1}^{N/2}\tilde{H}_p$ with $\tilde{H}_p$ given by

$$\tilde{H}_p = \alpha_p(t)[c_p^\dagger c_p + c_{-p}^\dagger c_{-p}] + iJ(t)\delta_p[c_p^\dagger c_{-p}^\dagger + c_p c_{-p}] + 2h(t)\,, \qquad (2)$$

where $\alpha_p(t) = -2J(t)\cos\phi_p - 2h(t)$, $\delta_p = 2\gamma\sin\phi_p$, $c_i^\dagger$ and $c_i$ are fermionic creation and annihilation operators, $\phi_p = \frac{2\pi p}{N}$ and N are the number of sites. Initially the system is assumed to be in a thermal equilibrium state represented by an initial density matrix $\rho_p(t) = e^{-\beta\tilde{H}_p(0)}$ where $\beta = 1/kT$ is the temperature parameter. Solving Liouville equation $i\dot{\rho}_p(t) = [H_p(t), \rho_p(t)]$ and utilizing the magnetization and spin-spin correlation functions of the system, we were able to evaluate the density function $\rho(t)$ and use it to evaluate the concurrence $C$ at any time at both zero and nonzero temperature [3]. The asymptotic behavior of entanglement at the infinite time limit at zero temperature and constant $J$ and $h$ depends only on the parameter $\lambda = J/h$ rather than the individual values of $J$ and $h$ but becomes dependent for nonzero temperature. Furthermore, the asymptotic behavior is very sensitive to the initial values of $J$ and $h$ and for particular choices we may create and maintain finite asymptotic entanglement regardless of the final values of $J$ and $h$. The persistence of quantum effects in the system as the temperature is raised and as it evolves in time is studied by monitoring the entanglement in the system. We find that the quantum effects dominates within certain region of the $kT$-$\lambda$ space in the vicinity of both the critical phase transition point and zero temperature as the system evolves in time as shown in Figs. 1 and 2.

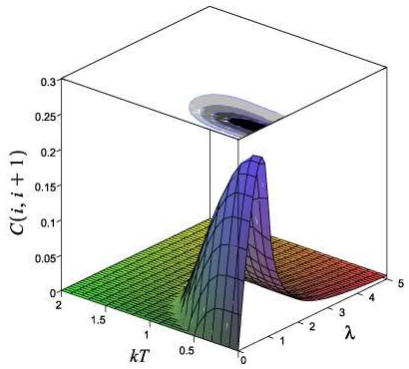


FIG. 1: The asymptotic behavior of $C(i, i+1)$ as a function of $\lambda = J/h$ and $kT$ with $h = h_0 = h_1 = 1$ and $J = J_0 = J_1$.

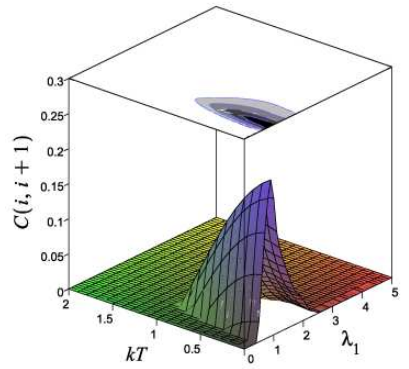


FIG. 2: The asymptotic behavior of $C(i, i+1)$ as a function of $\lambda_1 = J_1/h_1$ and $kT$ with $h_0 = h_1 = 1$ and $J_0 = 1$.

* Electronic address: gehad@ksu.edu.sa

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# Optimal replication of Von Neuman measurements

Michal Sedlák,[1,*] Alessandro Bisio,[1] Paolo Perinotti,[1] and Giacomo Mauro D'Ariano[1]
[1] *QUIT Group, Dipartimento di Fisica A. Volta and INFN Sezione di Pavia, via Bassi 6, 27100 Pavia, Italy*

Suitable mathematical representation of objects emerging in quantum mechanics is crucial for solving most of the optimization problems. Introduction of Process Positive Operator Valued Measures (PPOVM) [1] and Quantum Combs [2] allowed to solve several problems in which the most general (thought) experiments involving $N$ uses of a tested quantum channel (completely positive trace preserving map) have to be optimized. These are the problems like quantum channel discrimination [2], comparison [3], learning [4] and cloning [5] of unitary transformations and some two-party quantum games that were not solvable before. In this contribution, we apply quantum combs to optimize quantum circuits achieving transformations of measurements. More precisely, such a circuit has to work as one big POVM after $N$ measurements are inserted into the open slots of the circuit. The aim of the circuit is to create $M$ replicas of the inserted measurements, which are assumed to be unknown POVMs of the Von Neuman type (i.e. non-degenerate projective measurements). We show that for arbitrary figure of merit the presence of measurements in the circuit allow us to restrict the optimization to a subclass of quantum combs, which are called diagonal. We investigate two tasks, called Learning and Cloning of a qudit POVM. The goal of $N \to M$ Learning of a

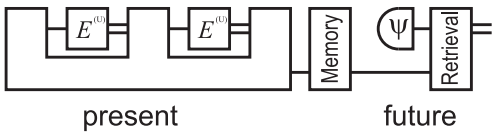


FIG. 1: Quantum circuit/comb diagram for $2 \to 1$ learning of POVM

POVM is to use the unknown measurement $N$ times, store what was learned about it in a quantum memory and later retrieve $M$ uses of the original measurement on a state that is not available in the learning phase (see Figure 1). In contrast, in $N \to M$ Cloning of a POVM the state to be measured is available from the very beginning, but we have to mimic $M > N$ uses of the unknown measurement by using it just $N$ times (see Figure 3).

In order to assess the performance of the circuits we chose particular figure of merit that expresses the closeness of the replicated measurement to the original one. The function we chose achieves its maximum value 1 if and only if the two measurements coincide. Moreover, it can be interpreted as an average probability of getting the correct outcome of the replicated measurement for an eigenstate of the original Von Neuman measurement.

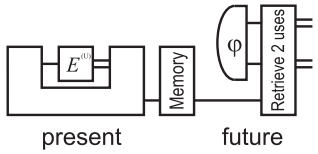


FIG. 2: Quantum circuit/comb diagram for $1 \to 2$ learning of a POVM

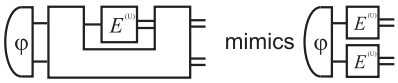


FIG. 3: Quantum circuit/comb diagram for $1 \to 2$ cloning of a POVM

Using diagonal quantum combs we solve $N \to 1$ Learning ($N = 1, 2, 3$), $1 \to 2$ Learning and $1 \to 2$ Cloning of a qudit POVM. We compare the performance of the optimal $1 \to 2$ Learning with $1 \to 2$ Cloning of a POVM (see Figures 2, 3). Similarly, to the analogous tasks for unitary channels the performance of cloning is much better than that of learning. We discovered that the uses of the unknown measurements in the optimal circuit can not be parallelized for $3 \to 1$ Learning of a qudit POVM. Thus, $N \to 1$ Learning of a qudit POVM represents a task, where the optimal strategy is necessarily sequential. This feature of non-parallelizability is present also in Grover algorithm, where the calls to the oracle can not be parallelized as was shown by Zalka [6]. Indirectly, our findings can help to understand how to search for optimal quantum circuits i.e. optimal quantum algorithms with oracle callings, which can not be parallelized.

* Electronic address: michal.sedlak@unipv.it

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# Quantum games that make sense

Edward W. Piotrowski[1,*] and Jan Sładkowski[2,†]

[1] *Institute of Mathematics, The University of Białystok, ul. Lipowa 41, Pl-15424 Białystok, Poland*
[2] *Institute of Physics, The University of Silesia, ul. Uniwersytecka 4, Pl-40007 Katowice, Poland*

What do quantum game theory have to do with real world? To answer this question we briefly describe the idea behind quantum game theory and show that they really form a distinct class of games. The study of quantum games is a quite new field of research, arising from seminal papers of D. Meyer [1] who for the first time used the term quantum game and J. Eisert et al [2]. Ever since there have been a continuously growing number of contributions and discussions over what exactly a quantum game really is and if there is anything new in quantum game theory [3, 4]. Many of the issues still lack a satisfactory mathematical formalism in which these important issues can be precisely expressed and analyzed [5]-[7]. Actually, there are genuine quantum games that involve the mysteries of quantum world and quantum information processing and quantum-like games. The former class is reserved for quantum physics[**?** ] and the later comprises game-theoretical models that go beyond classical randomization (mixed strategies).

Quantum-like games can be successfully applied to biology, finances and economy, psychology, decision and cognitive science [8]-[10]. They have already some interesting applications.

On the other hand, genuine quantum games have the potential for developing new technologies that might be of use. The research already performed suggests that there are several possible niches for quantum game products to be launched e.g. quantum auctions [11]-[13]. We will also point out some key problems that have to be solved before commercial use would be possible. But even with present technology, optical networks, single photon sources and detectors we envisage experimental realization in the near future.

Even if commercial use would turn out to be problematic, quantum game theory will remain a powerful analytic tool [14]-[17].

* Electronic address: qmgames@gmail.com
† Electronic address: jan.sladkowski@us.edu.pl

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[18] But they might be of practical use.

# Experimental demonstration of local expansion for W states

Toshiyuki Tashima,[1,*] Tsuyoshi Kitano,[1] Şahin Kaya Özdemir,[2]
Takashi Yamamoto,[1] Masato Koashi,[1] and Nobuyuki Imoto[1]

[1] *Graduate School of Engineering Science, Osaka University, Toyonaka, Osaka 560-8531, Japan*
[2] *Department of Electrical and Systems Engineering, Washington University in St. Louis, St. Louis, MO 63130 USA*

Multipartite entanglement is the key to our understanding of quantum theory and realization of intriguing quantum tasks ranging from quantum communication to quantum computation. Although entanglement of two quantum systems is well-understood, there still remains many challenging problems in the study of entanglement among three or more quantum systems. Even when we limit the constituent systems to qubits, we still encounter many nonequivalent classes of entanglement which differ in the structure of how the qubits are correlated, e.g., GHZ, W and cluster states. The preparation of such entangled states in large scales is one of necessary studies towards realization of quantum information processing. One of the promising approaches here is to increase the number of qubits by a few at a time using expansion gates[1–6], while retaining the structure of the desired entanglement class.

GHZ states and cluster states are, in principle, deterministically expanded with a local gate — a two-qubit unitary gate between one of the entangled qubits and an ancillary qubit to be added [1, 2]. Experimentally, probabilistic versions of these gates have been implemented [3, 4]. On the other hand, the expansion of W states is not straightforward for several reasons. First of all, the local expansion process cannot be achieved unitarily, because the marginal states of the remaining untouched $N-1$ qubits are different for $|\,\mathrm{W}_N\rangle$ and $|\,\mathrm{W}_{N+1}\rangle$. Secondly, W states have a unique entanglement structure which is different from the GHZ and cluster states. In a GHZ state, the entanglement is sustained by all of the qubits, i.e., a removal of one qubit completely disentangles the rest. In the case of the $N$-qubit linear cluster state, entanglement is sustained by at least half of the qubits, i.e., accessing $N/2$ qubits is enough to destroy the entanglement completely. In W states, on the other hand, the remaining bipartite state is still entangled even after N-2 qubits are discarded [7, 8]. In order to expand an $N$-qubit W state while retaining such a structure, the newly added qubits should not only get entangled with the accessed qubit in the initial W state, but also form independent pairwise entanglement with each of the untouched $N-1$ qubits. Several probabilistic methods for expanding W states have been proposed [5, 6], but there has been no experimental demonstration.

In this study, we report an experimental demonstration of local state expansion by expanding a single photon state and a bipartite entangled state into a tri-partite and a four-partite W state, respectively. The expansion gate is composed of two 50:50 beamsplitters, a half wave plate (HWP) for phase shifting, and two photons in the Fock state as an ancilla [6]. In Figure 1, we show our experimental setup. We first prepare a photon pair with the same polarization (or an EPR pair of photons) using parametric down-conversion at a type-I phase-matched $\beta$-barium borate (BBO) crystal, and then mix two photons from weak coherent pulses (WPC). Successful events are selected by a four-fold coincidence detection at the detectors ($\mathrm{D}_{\mathrm{A,B,C,D}}$). We have fully characterized the final states using quantum state tomography and probed the entanglement structure of all sub-components of the final states. The fidelities of the expanded three-photon and four-photon W states are $0.83\pm0.03$ and $0.79\pm0.03$.

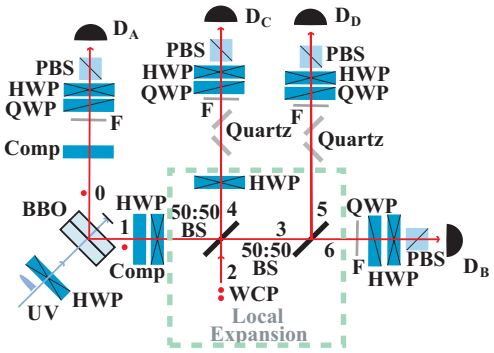


FIG. 1: Experimental setup.

* Electronic address: tashima@qi.mp.es.osaka-u.ac.jp

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# Non-Markovian dynamics of quantum discord in continuous variable systems

R. Vasile,[1,*] P. Giorda,[2] S. Olivares,[3] M. G. A. Paris,[3] and S. Maniscalco[1]

[1] *Turku Center for Quantum Physics, Department of Physics and Astronomy University of Turku, FIN-20014 Turku, Finland*

[2] *ISI Foundation, I-10133 Torino, Italy*

[3] *Dipartimento di Fisica dellUniversit degli Studi di Milano, I-20133 Milano, Italy*

In the last decade there has been a growing interest towards the investigation of genuinely quantum properties of states in different physical situations. Great attention has been given to the quantum nature of correlations between different parts of a multipartite system. The concept of quantum discord, in its various definitions [1], appears to be of great help in the identification of those states that possess correlations of a purely quantum nature.

Until now the attention has been focused on the discrete variable domain, where the evaluation of quantum discord, especially in the case of a two-qubit system, is mathematically feasible. However very recently attention have been devoted to the analysis of quantum discord in continuous variable systems, in particular in the context of two-mode Gaussian states [2].

Here we present a detailed study of quantum discord in the context of two-mode continuous variable quantum systems weakly coupled to external thermal environments [3]. We focus on the short-time non-Markovian behavior in the case of a common or independent reservoirs characterized by a Lorentz-Drude spectrum. We analyze the time evolution of the quantum discord comparing its behavior with the evolution of the entanglement.

* Electronic address: rugvas@utu.fi

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# Efficient estimation of entanglement measures for large experimentally created graph states via simple measurements

Harald Wunderlich,[1] Shashank Virmani,[2] and M. B. Plenio[1]

[1] *Institute for Theoretical Physics, Ulm University, Germany*
[2] *SUPA, Physics Department, University of Strathclyde, United Kingdom*

Detecting and quantifying entanglement is one of the standard tasks in quantum information science. Experimentally created entanglement can in principle be quantified by determining the quantum state via full tomography, and calculating an entanglement measure of choice for this state. Apart from exceptions like the negativity, entanglement measures usually involve optimization problems, hence they are hard to calculate. Another issue is the tensor-structure of the Hilbert space, which implies that the number of measurement settings grows exponentially with the number of constituents involved in the system. Thus, full-state tomography becomes infeasible for even medium-sized systems. For this reason, sophisticated methods are required to quantify entanglement of many-body systems. Here, we present methods to quantify entanglement of arbitrarily large two-colorable graph states, which represent a significant resource for applications in quantum information science. They encompass GHZ states, CSS error correction codeword states, and cluster states. We will show that the entanglement of two-colorable graph states can be estimated efficiently via measurements of the stabilizer operators only, thus reducing the experimental effort in measuring the state exponentially. Furthermore our method of entanglement estimation is purely analytical, thus avoiding computationally costly post-processing of measurement data.

A graph state is completely described by a set of stabilizer operators $K_i := X_i \bigotimes_{Ngb(i)} Z_j$, $i = 1, \ldots, n$, which generate the abelian group $\mathcal{S}$, called the *stabilizer*. An experimentally created graph state could in principle be verified by measuring the $2^n$ elements of the stabilizer. As mentioned earlier, full-state tomography is not an option for determining the properties of a quantum many-body system due to an exponentially fast growing measurement effort. We will see that merely the measurement results of the generators of the stabilizer suffice to attain highly useful bounds on entanglement measures. Let us suppose the goal of an experiment is the creation of a two-colorable graph state, and the generators of the stabilizer are measured with outcomes $a_i = tr(\rho K_i)$, $i = 1, \ldots, n$. Given this tomographically incomplete data, one is now interested in finding the minimal entanglement (according to a certain entanglement measure) compatible with the measurement data. Mathematically, this is a formulated as the semidefinite program [1–3, 6]:

$$E_{min} = min_\rho \{E(\rho) : tr(\rho K_i) = a_i, \rho \geq 0\}, \qquad (1)$$

where $E(\rho)$ is the entanglement quantifier of choice [4]. The entanglement measures considered here are the robustness of entanglement and the relative entropy of entanglement. We show that the minimization of the robustness of entanglement can be related to the minimal fidelity with the target state consistent with the measurement data [3], and we will relate the minimization of the relative entropy of entanglement to the max-entropy problem [5]. We provide optimal solutions to the minimal fidelity and max-entropy problem for arbitrarily large graph states. Thus, one attains analytical upper and lower bounds to the minimal entanglement consistent with the measurement data. In order to illustrate the power of our method we apply it to large linear cluster states subjected to local dephasing (cf. Fig. 1).

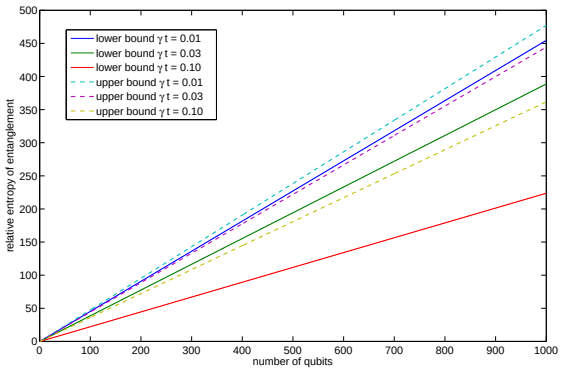


FIG. 1: Lower and upper bounds on the relative entropy of entanglement for stabilizer measurements on a linear graph state up to 1000 qubits subjected to local dephasing with dephasing rate $\gamma$.

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# Redundant imprinting of information in non-ideal environments: Quantum Darwinism via a noisy channel

Michael Zwolak,[1,*] H. T. Quan,[1] and Wojciech H. Zurek
[1] *Theoretical Division*
*Los Alamos National Laboratory*
*Los Alamos, NM 87545, USA*

Quantum Darwinism provides an information-theoretic framework for the emergence of the classical world from the quantum substrate. It recognizes that we - the observers - acquire our information about the "systems of interest" indirectly from their imprints on the environment. Objectivity, a key property of the classical world, arises via the proliferation of redundant information into the environment where many observers can then intercept it and independently determine the state of the system. We give a general introduction to this framework and demonstrate how non-ideal initial states of the environment (e.g., mixed states) affect its ability to act as a communication channel for information about the system [1, 2]. The environments capacity for transmitting information is directly related to its ability to increase its entropy. Therefore, environments that remain nearly invariant under the Hamiltonian dynamics, such as very mixed states, have a diminished ability to transmit information. Further, we show that - when decoherence of the system is not complete - the deviation of the mutual information from the environments entropy change is quantified by the quantum discord, i.e., the excess mutual information regards the initial coherence between pointer states of the system. Our results demonstrate that Quantum Darwinism is robust with respect to non-ideal initial states of the environment: the environment almost always acquires redundant information about the system but its rate of acquisition can be reduced.

* Electronic address: mpzwolak@gmail.com; URL: http://mike.zwolak.org

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# Single-shot information theory and statistical mechanics

Johan Åberg[1]

[1] *Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland**

The single-shot information theoretic measures of entropy, the smooth min- and max-entropy, can in a certain sense be regarded as more 'fundamental' than the Shannon/von Neumann entropy. Here we raise the question whether this is true also in statistical mechanics. The recent finding [1] that the smooth min- and max-entropy are the relevant measures in certain settings of work extraction, suggests that this indeed could be the case. We discuss and generalize these results via an alternative approach.

Already at the very birth of information theory the link to statistical mechanics was noted, the most obvious correspondence being the use of the Shannon/von Neumann entropy [2]. Another connection is Landauer's erasure principle [3], which tells us that we have to dissipate $kT\ln 2$ of energy for every bit we wish to erase in a memory. This is the basis for the most popular argument to exorcise Maxwell's demon [4]; the idea being that the resetting of the demon's memory requires a dissipation of energy that balances any potential energy gain it could make. A somewhat less famous cousin to Landauer's erasure principle is the work extraction problem [5], where we make use of our knowledge of the state of the system to extract work. These links between statistical mechanics and information theory have so far to a large extent been based on standard information theory.

In 'traditional' information theory one typically determines the optimal rates for various information theoretic tasks. A simple example is the source coding theorem [2], which says that in the limit of infinitely many independent repetitions of a source over an alphabet $\mathcal{X}$, we in average only need to store $H(P) = -\sum_{x\in\mathcal{X}} P(x)\log_2 P(x)$ bits per output, where $P$ is the distribution of the source. (The assumption of an independent and identically distributed (iid) source is often relaxed and replaced with a Markov process [2].)

However, when we consider more complex types of correlations, or do not take an asymptotic limit, the standard machinery is no longer applicable. Lately there has been a development that allows us to handle the general case [6–8]. In this approach we do not consider an infinite iid limit, but rather one single instance of a task, e.g., the problem of storing one single output from a source. Since an iid repetition can be regarded as a single instance on a larger alphabet, this contains the iid setting as a special case. In this 'single-shot' setting it turns out that the Shannon/von Neumann entropy is not the correct operational measure of entropy. This role is taken over by the smooth min- and max-entropies [6–8]. In the asymptotic iid limit the Shannon/von Neumann entropy can be recovered from these measures. This generalization has turned out to be fruitful, especially in quantum cryptography, where there is no reason to assume that an adversary attacks the protocol in an iid manner [6].

Since the smooth min- and max-entropies in the above sense can be regarded as the more 'fundamental' entropy measures in information theory, an obvious question is whether this is the case also in statistical mechanics. The recent results in [1] can be seen as a first step towards an answer to this general question. In a setting where we extract work once from a single system, it was found that the smooth max-entropy is related to how much work we can extract if we wish to be essentially certain of succeeding, while the smooth min-entropy is related to the work that can be extracted if we accept almost any risk of failure. These results were obtained mainly by information theoretic methods. Most of these methods implicitly assume the Hamiltonian of the system to be degenerate. Although a very reasonable assumption for quantum information theory, such a restriction is less satisfying from the point of view of statistical mechanics. We discuss a somewhat more 'physics oriented' approach that does not suffer from this restriction, thus bringing these results closer to the proper realm of statistical mechanics. To this end we use a model where we let the system evolve under sequences of thermalizations and adiabatic changes of energy levels. (For similar models see, e.g., [9, 10].) As an outlook we consider possible generalizations.

* Electronic address: jaaberg@phys.ethz.ch

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# Simulation of static and random errors on Grover's search algorithm implemented in a Ising nuclear spin chain quantum computer with few qubits

Gustavo V. López,[1,*] T. Gorin,[1,†] and L. Lara[2,‡]

[1] *University of Guadalajara*
*Blvd. Marcelino Gaarcía B., 1421, eqs. Calzada Olímpica*
*44420 Guadalajara, Jalisco, México*
[2] *University of Guadalajara*

We consider Grover's search algorithm on a model quantum computer implemented on a chain of four or five nuclear spins with first and second neighbor Ising interactions. Noise is introduced into the system in terms of random fluctuations of the external fields. By averaging over many repetitions of the algorithm, the output state becomes effectively a mixed state. We study its overlap with the nominal output state of the algorithm, which is called fidelity. We analyse the behavior of the fidelity as a function of the noise intensity for static and random noise on either the Larmor frequencies or the Rabi frequency, and we compare our results to theoretical predictions and numerical simulations which are based on more abstract quantum computer models.

* Also at Physics Department, University of Guadalajara.
† Electronic address: thomas.gorin@red.cucei.udg.mx
‡ URL:

# Correlation induced non-Abelian quantum holonomies

Markus Johansson,[1,*] Marie Ericsson,[1] Kuldip Singh,[2] Erik Sjöqvist[1,2] and Mark Williamson[2]

[1] *Department of Quantum Chemistry, Uppsala University, Box 518, Se-751 20 Uppsala, Sweden*

[2] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore, Singapore*

In the spirit of Pancharatnam [1] we develop the idea of in-phase conditions and geometric phases, in the context of two-particle interferometry. We construct a non-Abelian holonomy that does not depend on degeneracy, but on the ability to divide the system into spatially separated subsystems. The setting considered here is a Franson interferometer [2, 3] and a source of two-particle states as depicted in fig.1. The two subsystems can have different dimensionality $N$ and $M$ respectively and the state can be mixed as well as pure.

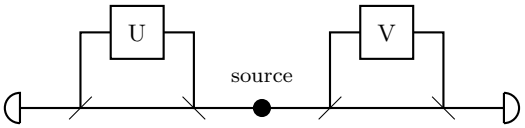


FIG. 1: The Franson interferometry setup. Unitary operations are performed in the long arms of each Franson loop.

In the first of the two Franson loops a $SU(N)$ operation is introduced in the long arm of the loop, and in the second Franson loop a $SU(M)$ operation is introduced in the long arm, and the controllers of these unitaries are called Alice and Bob respectively. Given a choice of $SU(N)$ operation $U$ applied by Alice, Bob is instructed to choose a $SU(M)$ operation $V$ such as to maximize the measured coincidence intensity of detectors measuring the output of the two Franson loops. It is found that $V$ is a function of both $U$ and the correlations between the subsystems, and a closed form for $V$ was derived for the case where $V$ is a $SU(2)$ operation. This maximization procedure defines the "in-phase" condition for the system. The in-phase condition is then used as a starting point for the construction of a iterative procedure where the maximization is repeated for a sequence of $SU(N)$ operations chosen by Alice. In each step Bob finds the $V$ that maximizes coincidence intensity given $U$, and then the input to the interferometer is transformed by $U$ and $V$.

Given a sequence of unitaries chosen by Alice, such that the subsequent product of the unitaries is equal to unity, the total transformation that has been applied to Bobs subsystem at the end of the sequence of maximizations defines the holonomy associated to the input state and Alices sequence. The holonomy group is the set of holonomies corresponding to all possible choices of sequences by Alice, and a given state, and is typically a non-Abelian subgroup of the full $SU(M)$. For $V \in SU(2)$, where a closed form expression was derived, it could be shown that the holonomy group is sensitive to correlation in the sense that it is an Abelian subgroup whenever the input state from the source is a product state.

For the special case of pure two-qubit input states, the parallel transport condition and associated holonomies are identical to the construction by Lévay [4] restricted to local unitary transports.

* Electronic address: markus.johansson@kvac.uu.se

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# Ambiguity in the quantum trajectory model of open quantum systems

Erik Sjöqvist[1,*] and Patrik Pawlus[1]

[1] *Department of Quantum Chemistry, Uppsala University, Box 518, Se-751 20 Uppsala, Sweden*

Quantum systems may undergo open system effects induced by entanglement with environmental degrees of freedom; effects that may be detrimental in various quantum information protocols. This feature has led to a revived interest in the theory of open quantum systems. One important class of open system evolution is governed by the Lindblad master equation

$$\dot{\rho} = -i[H,\rho] + \sum_m \left( L_m \rho L_m^\dagger - \frac{1}{2} L_m^\dagger L_m \rho - \frac{1}{2} \rho L_m^\dagger L_m \right) \quad (1)$$

which describes Markovian evolution of the system's density operator $\rho$ with $L_m$ the Lindblad operators [1].

Instead of solving the full master equation one sometimes strive for physical insights by solving simpler approximate equations involving for instance only pure states. The quantum trajectory or quantum jump model is one such approach that has become particularly fashionable in quantum optics [2]. These trajectories are obtained by dividing the evolution given by Eq. (1) into small time steps, leading to smooth curves in state space interrupted by random jumps, generated by jump operators proportional to $L_m$. Each of these trajectories can be realized experimentally by continuously monitoring the environment.

We find a class of open system models where individual quantum trajectories may depend on parameters that are undetermined by the full open system evolution. These parameters $f_m$ are defined as shifts of Lindblad operators, i.e., $L_m \to L_m - f_m \hat{1}$. We illustrate the general result by demonstrating that the no-jump curves of qubit dephasing can be transformed into no-jump curves of qubit decay (amplitude damping). Our finding [3] indicates a potential source of ambiguity in the quantum trajectory approach to open quantum systems.

* Electronic address: erik.sjoqvist@kvac.uu.se

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# Bipartite entanglement of permutation symmetric states

Ludovic Arnaud[1,*] and Nicolas J. Cerf[1]

[1] *QuIC, Université Libre de Bruxelles, 50 avenue. F.D. Roosevelt, 1050 Brussels, Belgium.* [†]

Recently, different works [1–3] concerning multipartite entanglement used a special familly of states : $n$-qubits states invariant under an arbitrary permutation of qubits. This kind of states are interesting because they form a subspace of the total Hilbert space, the so-called *symmetric subspace* $\mathcal{H}_S$ whose size increases only linearly with the number of qubits. However, at the same time, those states can be highly entangled as the $|W\rangle$ and $|GHZ\rangle$ states for 3 qubits. Thanks to different possible mathematical representations (e.g. the Majorana's representation or the expansion in the Dicke basis [1–3]) those states can be manipulated easily and results based on the *geometric measure of entanglement* were published for up to 12 qubits [2, 3].

We investigate bipartite entanglement for such states. Bipartite entanglement remains an interesting property for those states because it does not depend on the way the system is partitioned. Finally, the only relevant parameter is the ratio of the sizes of the parties. Our results concern statistical properties of the von Neumann and linear entropies of such states.

* Electronic address: larnaud@ulb.ac.be

† URL: http://quic.ulb.ac.be/

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# Evolution of polynomial invariants of four-qubit systems controlled by local unitary operation

Duy Ngoc Hoang[1,*] and Hoshang Heydari[1,†]

[1] *Department of Physics, Stockholm University 10691 Stockholm Sweden*

In the field of quantum computation it is very important to be able to control the flow of quantum information encoded in qubits by unitary transformations. Such controlled quantum network are necessary for construction of a possible quantum computer. In this paper we study evolution of some classes of four-qubit states under local bipartite unitary operation using polynomial invariants [1]. The evolution is controlled by single-qubit local unitary operation. In particular, we study the evolution of four-qubit state $|\Psi(t)\rangle = \mathcal{U}_{KL}(t)|\Psi(0)\rangle = (I_A \otimes U_{KL}(t) \otimes I_D)(I_A \otimes U_K \otimes I_C \otimes I_D)\Psi(0)\rangle$, where $|\Psi(0)\rangle$ is some classes of four qubit states and $U_{KL}(t) = \exp\{-i(e\sigma_x^K \otimes \sigma_x^L + f\sigma_y^K \otimes \sigma_y^L + g\sigma_z^K \otimes \sigma_z^L)t\}$ is a global bipartite unitary operator and

$$U_K = e^{i\vartheta} \begin{pmatrix} e^{-i(\beta+\gamma)/2}\cos\frac{\theta}{2} & e^{-i(\beta-\gamma)/2}\sin\frac{\theta}{2} \\ e^{i(\beta-\gamma)/2}\sin\frac{\theta}{2} & e^{i(\beta+\gamma)/2}\cos\frac{\theta}{2} \end{pmatrix},$$

is a local unitary operator and $K, L = A, B, C, D$ are subsystems of four qubit states. Then, we study the entanglement properties of different classes of four-qubit states based on geometric invariant rate $R_{GI}(U_K)$ controlled by $U_K$

$$R_{PI}(U_K) = \frac{|PI(|\Psi(t)\rangle)_{t=t^*} - PI|\Psi(0)\rangle|}{t^*}, \qquad (1)$$

where $t^* = \min_{U_K}(t_1)$ and $t_1$ is the minimal positive number satisfying $\frac{dPI(t,U_K)}{dt}|_{t=t_1} = 0$ and $PI = |H|, |M|, |L|, |\Delta_4|$ are polynomial invariants [1].

* Electronic address: ngocduy138@gmail.com

† URL: http://www.physto.se/~hoshang/; Electronic address: hoshang@physto.se

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# Sub shot-noise interferometry and multiparticle entanglement

Philipp Hyllus[1], Luca Pezzé[2], Augusto Smerzi[1], Roland Krischek[3,4], Christian Schwemmer[3,4], Wiesław Laskowski[3,4,5], Witlef Wieczorek[3,4,6], and Harald Weinfurter[3,41]

[1] [1] *INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, I-38123 Povo, Italy*
[2] *Laboratoire Charles Fabry, Institut d'Optique, 91127 Palaiseau, France*
[3] *Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*
[4] *Department für Physik, Ludwig-Maximilians-Universität, D-80797 München, Germany*
[5] *Institute of Theoretical Physics and Astrophysics, University of Gdansk, ul. Wita Stwosza 57, 80-952 Gdanks, Poland*
[6] *Faculty of Physics, University of Vienna, Boltzmanngasse 3, 1090 Wien, Austria*

Entangled quantum states allow for sub shot-noise sensitivity with linear interferometers [1–3], with applications in various fields such as quantum frequency standards, quantum lithography, quantum positioning and clock synchronization, and quantum imaging [4]. The possibility of reaching sub shot-noise sensitivity has has been demonstrated in experiments with entangled states of up to 8 photons [5–7], and up to 5 ions [8]. These works applied a linear error model to estimate the possible sensitivity. Here, we perform a full estimation protocol with a Bayesean strategy using a 4-photon twin-fock state [9], which can assign a meaningful phase error even to a single measurement. We demonstrate sub shot-noise sensitivity for a large range of phase shifts. We investigate the connection to multiparticle entanglement by estimating the so-called Fisher information in several ways. This quantity is related to the optimal phase sensitivity achievable *via* the Cramer-Rao bound [10, 11]. By using a recent classification of multipartite entanglement based on the Fisher information [12, 13], we show that the high sensitivity we observe in the experiment is connected to the presence of genuine 4-partite entanglement in the input state.

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# Bi-Partite Separability of Werner State in $2^{2p}$ Dimensions

Wei-Chi Su,[1] Kuan-Peng Chen,[1] Ming-Chung Tsai,[2] and Zheng-Yao Su[1, 2, *]

[1] *National Center for High-Performance Computing, Hsinchu, Taiwan, R.O.C.*
[2] *Department of Physics, National Tsing-Hua University, Hsinchu, Taiwan, R.O.C.*

Quantum entanglement is a characteristic of two or more quantum systems which reveals the correlations that cannot be explained by classical physics. It plays a key resource in the processes of quantum computation and quantum information theory, for instance, teleportation, dense coding and many quantum protocols. Yet, it is difficult to determine whether or not a quantum system, a density operator, is separable. In this report, a criteria (a necessary condition) for the bi-partite separability of a Werner state

$$\rho_w = \frac{1}{2^{2p}}((1-x)I + x|\psi\rangle\langle\psi|) \tag{1}$$

in the $2^p \times 2^p$ system is proposed, where $0 \leq x \leq 1$ and $|\psi\rangle = \frac{1}{\sqrt{2^p}} \sum_{\alpha \in Z_2^p} |\alpha, \alpha\rangle$. It is known on the occasion $p = 1$ that $\rho_w$ is separable iff $x \leq \frac{1}{3}$. We extend this result to the occasion $p > 1$ and acquire the inequality

$$x \leq \frac{1}{2^p + 1} \tag{2}$$

as a necessary condition by virtue of the criterion for positiveness of a Hermitian matrix [1].

As described in [1], if an $N \times N$ hermitian matrix $R = \sum_{i,j=1}^{N} R_{ij}$, the inequalities hold

$$|R_{ii}||R_{jj}| \geq |R_{ij}|^2 \tag{3}$$

for all $1 \leq i, j \leq N$, here $R_{ij} \in \mathbb{C}$ and $R_{ji} = R_{ij}^*$. Suppose a Werner state in $2^p \times 2^p$ system is bi-partite separable and expressed as, for $p_k \geq 0$ and $\sum_{k=1}^{M} p_k = 1$,

$$\rho_w = \sum_{m,n,s,t=1}^{2^p} \rho_{ms,nt} |m\rangle\langle n| \otimes |s\rangle\langle t| = \sum_{k=1}^{M} p_k \rho_k^A \otimes \rho_k^B,$$

where $\rho_k^A = \sum_{m,n=1}^{2^p} \rho_{k,mn}^A |m\rangle\langle n|$ and $\rho_k^B = \sum_{s,t=1}^{2^p} \rho_{k,st}^B |s\rangle\langle t|$ are a density operator, and the relation is valid

$$\rho_{ms,nt} = \sum_{k=1}^{N} p_k \rho_{mn}^A \rho_{st}^B \tag{4}$$

for all $1 \leq m, n, s, t \leq 2^p$. Remind that a matrix is a density operator if it is trace-unit, Hermitian and positive. Since both $\rho_k^A$ and $\rho_k^B$ are a density operator, the positiveness and hermiticity of these two matrices leads to the following inequalities by Eq. 3

$$|\rho_{k,mm}^A||\rho_{k,nn}^A| \geq |\rho_{mn}^A| \text{ and } |\rho_{k,ss}^B||\rho_{k,tt}^B| \geq |\rho_{st}^B|. \tag{5}$$

Through the straightforward calculation, the inequality of Eq. 2 is a consequence of Eqs. 1, 4 and 5. A such criteria is not only useful to test the separability of a Werner state, but is a sufficient condition rendered by representing the state in an appropriate coordinate, for instance, in terms of a spinor representation.

* Electronic address: zsu@nchc.org.tw

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# A Tripartite Entanglement Witness Independent of the Hilbert Space

Jean-Daniel Bancal,[1] Nicolas Gisin,[1] and Stefano Pironio[2]
[1] *Group of Applied Physics, University of Geneva*
*20 rue de l'Ecole-de-Médecine, CH-1211 Genève 4, Switzerland*
[2] *Laboratoire d'Information Quantique, Université Libre de Bruxelles, Belgium*

It is well-known that entanglement of composite quantum systems can be manifested by measuring the expectation value of physical observables known as *entanglement witnesses*[1]. These physical observables are usually constructed with a specific quantum state in mind, and verification of entanglement with them requires in practice knowledge of the actual measurements being performed, as well as the dimension of the underlying Hilbert spaces.

In contrast, Bell inequalities – which detect nonlocal correlations – can witness entanglement without requiring such knowledge about the physical implementation. In other words, they can be used as *Device Independent Entanglement Witnesses (DIEW)*. In fact for bipartite quantum systems, it was shown that any DIEW is a Bell inequality.

In analogy to the bipartite case, one might expect that violation of a Svetlichny inequality[2] – which certifies the presence of genuine multipartite nonlocality – is necessary in order to demonstrate genuine multipartite entanglement in a device independent manner. But this is not the case as was already noticed by Cereceda [3]. There must thus be witnesses of multipartite entanglement which are more easily violated than Svetlichny inequalities.

Here we focus on the tripartite scenario. Using a hierarchy of Semi-Definite Programs [4], we are able to approximate the set of correlations that can be produced when a biseparable quantum state is measured. This allows us to find a DIEW for genuine tripartite-entanglement which is neither a Bell nor a Svetlichny inequality. This witness can be violated by measurement statistics obtained from genuine tripartite-entangled states.

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## Quantum non-locality is a non-additive resource

Antonio Acín,[1] Mafalda Almeida,[2] Daniel Cavalcanti,[3,*] and Valerio Scarani[4]

[1] *ICFO-Institut de Ciencies Fotoniques, 08860 Castelldefels (Barcelona), Spain. ICREA-Institucio Catalana de Recerca i Estudis Avançats, 08010 Barcelona, Spain*
[2] *ICFO-Institut de Ciencies Fotoniques, 08860 Castelldefels (Barcelona), Spain.*
[3] *Centre for quantum technologies, National University of Singapore, Singapore*
[4] *Centre for quantum technologies, University of Singapore, Singapore Department of Physics, National University of Singapore, Singapore*

Bell's theorem proves the impossibility of reproducing the results of local measurements performed in distant quantum systems through local models [1]. This milestone in the foundations of quantum theory has found several applications such as quantum cryptography [3], quantum communication [4], and randomness extraction [6]. However deciding if a quantum state can demonstrate nonlocality is extremely hard, and the nonlocal properties of even the simplest examples of mixed states are not well understood. We demonstrate that more stringent nonlocality criteria can be obtained in the multipartite scenario. One-way entanglement distillability is shown to be a sufficient condition for the appearance of multipartite nonlocality. This is used to prove that many copies of a local state can give rise to nonlocal correlations and to improve the noise threshold for which isotropic states can display nonlocality. These results imply that nonlocality is not an additive resource, and can indeed be activated.

* Electronic address: dcavalcanti@nus.edu.sg

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# A short and efficient error correcting code for polarization coded photonic qubits in a dissipative channel

J. Almlöf[1,*] and G. Björk[1]

[1] *School of Information and Communication Technology, Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden*

To combat qubit errors originating from unwanted but unavoidable interaction with the quantum channel, quantum-error-correcting codes can be used. A code that will protect a logical qubit from any single bit flip, phase flip, or combination of both is called a Pauli code. It has been shown that the shortest such code codes a logical qubit onto five physical qubits. However, errors caused by dissipation (involving a reservoir ancilla) cannot be exactly corrected by such codes, and therefore codes that can only approximately correct for such errors have been developed [1]. Codes for the dissipative channel need to handle an additional requirement, namely that any dissipation quantum can originate from $|0_L\rangle$ or $|1_L\rangle$ with equal probabilities. In spite of typically only correcting to first order in the dissipation, they work very well.

Codes adapted to dissipative channels typically assume that the qubits are coded onto a ground state that we will label $|0\rangle$, unaffected by the dissipation, and an excited state $|1\rangle$. The latter state is assumed to undergo a quantum jump $|1\rangle \rightarrow |0\rangle$ with probability $\gamma$ when passing through the channel. This is a good model for many systems, but not for all. E.g., in systems where quantum information needs to be transmitted long distances, single photons are typically used as the qubit carrier. Such "flying qubits" are typically encoded on single photons in a superposition of two orthogonal states. Examples of such systems include quantum repeaters, long-distance entanglement-distribution, entanglement swapping, long-distance quantum key-distribution and quantum teleportation.

We shall call the two orthogonal qubit states $|H\rangle$ and $|V\rangle$, alluding to a single photon in a well defined spatio-temporal mode, linearly polarized either in the horizontal or in the vertical direction. **For these states dissipation will imply that both states undergo a quantum jump to the ground state** $|0\rangle$ due to absorption or scattering. That is, with probability $\gamma$ the jumps $|H\rangle \rightarrow |0\rangle$ or $|V\rangle \rightarrow |0\rangle$ will occur. We note that the jump process is now symmetric between the two basis states in contrast to what is usually assumed.

Exploiting this difference from the assumption Pauli-codes are based on, one finds that it is possible to correct up to one error with a three-qutrit code, making use of the two photon modes and also the vacuum mode. The explicit coding of the logical qubits $|0_L\rangle$ and $|1_L\rangle$ is

$$|0_L\rangle \rightarrow \frac{1}{\sqrt{3}}(|0VH\rangle + |H0V\rangle + |VH0\rangle),$$
$$|1_L\rangle \rightarrow \frac{1}{\sqrt{3}}(|000\rangle + |HHH\rangle + |VVV\rangle).$$

This coding is non-degenerate, that is, the codewords and the single qubit-error syndromes are all mutually orthogonal. To the best of our knowledge, this is the shortest, and hence simplest, proposed code for this particular channel.

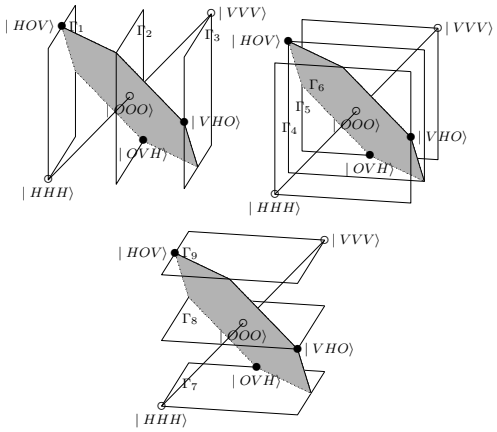


FIG. 1: $|0_L\rangle$ and $|1_L\rangle$ are marked with dots and circles respectively. Note that each of the 9 planes representing the photon state of a given mode contains exactly two kets – one circle from $|1_L\rangle$ and one dot from $|0_L\rangle$. The 6 planes $\Gamma_1, \Gamma_3, \Gamma_4, \Gamma_6, \Gamma_7, \Gamma_9$ represent the modes $|H\rangle$ and $|V\rangle$ which can dissipate. Therefore any one dissipated photon will not reveal if it came from the $|0_L\rangle$ or $|1_L\rangle$ codeword.

* Electronic address: jalml@kth.se

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# Reduced density matrix after long time walking

Mostafa Annabestani,[1] Seyed Javad Akhtarshenas,[2,*] Seyed Masoud Amini,[3,†] and Mohamad Reza Abolhassani[1]

[1] *Department of Physics, Basic Sciences Faculty, Tarbiat Modares University, Tehran, Iran*[‡]
[2] *Department of Physics, University of Isfahan, Isfahan, Iran*[§]
[3] *Department of Mathematics, Basic Sciences Faculty, Tarbiat Modares University, Tehran, Iran*

In this article we show that for any quantum walker with $m$-dimensional coin subspace, There is an scalar $m^2 \times m^2$ matrix $\mathcal{C}$ where the reduced density matrix of the long time behavior of the local state of the coin is given by $Tr_1\left(P_0 \otimes I\,\mathcal{C}\right)$, where $P_0$ is the projector of initial state and $Tr_1$ is the partial trace over the first part of the system. Also the characteristic matrix $\mathcal{C}$ is independent of the initial state and just depends on the coin operator, so by finding this matrix for a given QW, the long time behavior of QW, such as asymptotic local state of the coin and asymptotic entanglement between coin and position (**CE**) will be completely known for any initial states.

We have found the characteristic matrix $\mathcal{C}$ for general coin operator $SU(2)$, which is the most general form of coin operator in 1DQW. The general form of $SU(2)$ can be parameterized by three parameters $\alpha, \beta$ and $\theta$

$$U_{\theta,\alpha,\beta} = \begin{pmatrix} \mathrm{e}^{i\alpha}\cos\theta & \mathrm{e}^{i\beta}\sin\theta \\ \mathrm{e}^{-i\beta}\sin\theta & -\mathrm{e}^{-i\alpha}\cos\theta \end{pmatrix}, \tag{1}$$

this type of coin operator leads to the characteristic matrix

$$\mathcal{C}_{\theta,\alpha,\beta} = \frac{1}{2}\begin{pmatrix} 2-\sin\theta & f^* & f^* & g^* \\ f & \sin\theta & \sin\theta & -f^* \\ f & \sin\theta & \sin\theta & -f^* \\ g & -f & -f & 2-\sin\theta \end{pmatrix} \tag{2}$$

where

$$\begin{aligned} f &= \tan\theta\,(1-\sin\theta)\,\mathrm{e}^{i(\alpha-\beta)}, \\ g &= \frac{\sin\theta\,(\sin\theta-1)}{\sin\theta+1}\,\mathrm{e}^{2\,i(\alpha-\beta)}. \end{aligned} \tag{3}$$

This matrix completely determines the reduce density matrix of the walker with initial state $|\psi_0\rangle$,

$$\hat{\rho}_c = Tr_1\left(P_0 \otimes I\,\mathcal{C}\right) = Tr_2\left(I \otimes P_0\,\mathcal{C}\right) \tag{4}$$

where $P_0 = |\psi_0\rangle\langle\psi_0|$.

We show that the eigenvalues of the reduce density matrix $\hat{\rho}_c$, needed to calculate entanglement between coin and position, can be calculated from the characteristic polynomial

$$\lambda^2 - \lambda + det\left(\hat{\rho}_c\right) = 0 \tag{5}$$

where

$$det\left(\hat{\rho}_c\right) = Tr\left(P_0\mathbf{A}P_0\mathbf{A}' - P_0\mathbf{B}^\dagger P_0\mathbf{B}\right), \tag{6}$$

for

$$\begin{aligned} \mathbf{B} &= \frac{1}{2}\begin{pmatrix} f & \sin\theta \\ g & -f \end{pmatrix} \\ \mathbf{A} &= \frac{1}{2}\begin{pmatrix} 2-\sin\theta & f^* \\ f & \sin\theta \end{pmatrix} \\ \mathbf{A}' &= \frac{1}{2}\begin{pmatrix} \sin\theta & -f^* \\ -f & 2-\sin\theta \end{pmatrix} = I - \mathbf{A}. \end{aligned} \tag{7}$$

The CPE calculated by our analytic method matches with the result of direct numerical simulation. Fig.1 plots the coin-position entanglement for two initial states.

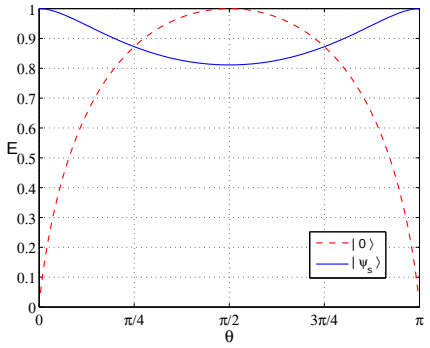


FIG. 1: Entanglement between coin and position for 1DQW with $SU(2)$ coin operator for initial states $|0\rangle$ (dash line) and the symmetric initial state, $|\psi_s\rangle = 1/\sqrt{2}\,(|0\rangle + i|1\rangle)$ (solid line)

The speed of spreading versus CPE, as well as some sufficient conditions for maximum CPE are discussed in this article, and a family of initial states with maximum CPE is obtained.

* Electronic address: akhtarshenas@phys.ui.ac.ir
† Electronic address: mamini@modares.ac.ir
‡ Electronic address: annabestani@modares.ac.ir
§ Also at Quantum Optics Group, University of Isfahan, Isfahan, Iran

# Superadditive quantum coding in atomic systems

Adetunmise Dada,[1,*] Erika Andersson,[1,†] Mark Everitt,[2,‡] Martin Jones,[3,§] and Vivien Kendon[3,¶]

[1] *Department of Physics, Heriot-Watt University, Edinburgh EH14 4AS, UK*
[2] *National Institute of Informatics, Chiyoda-ku, Tokyo, Japan.*
[3] *School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, UK*

Generalized quantum measurements are a key feature in quantum communication schemes. One example of such measurements is the unambiguous discrimination of non-orthogonal quantum states, also known as the Ivanovic-Dieks-Peres (IDP) measurement [1–3]. All the realizations of the IDP measurement have been optical [4, 5]. We describe a realization of this measurement in a cavity quantum electrodynamics (CQED) setting. Another interesting example is the measurement required to demonstrate that quantum channel capacities can be superadditive. This demonstration requires at least two uses of a quantum channel. To date, the only realization of this measurement is optical and the two uses of the quantum channel were encoded using the path and polarization degrees of freedom of a single photon [6]. While this demonstrates the principle of the measurement, extension of the coding to a higher number of uses of the channel would be impractical. This is because the number of linear optical elements scales exponentially with the number of channels. We hereby describe a scheme to realize superadditive quantum coding and the associated generalized quantum measurement in CQED. In our scheme, we use two atoms, and encode each use of the channel in the internal states of a separate atom. As shown in [6, 7] superadditive quantum coding can significantly boost the communication performance of conventional coding techniques.

* acd8@hw.ac.uk
† erika.andersson@hw.ac.uk
‡ mark.s.everitt@gmail.com; Also at School of Physics and Astronomy, University of Leeds, Leeds LS2 9JT, UK
§ martin.jones@quantuminfo.org
¶ v.kendon@leeds.ac.uk

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# Isotropic index for unitary quantum errors

André L. Fonseca de Oliveira,[1,*] Efrain Buksman,[1,†] and Jesús García-López[2,‡]

[1] *Facultad de Ingeniería, Universidad ORT Uruguay*

[2] *Escuela Universitária de Informática, Universidad Politécnica de Madrid*

Quantum Computing has traditionally studied the effects, and correction, of noise [1] [2]. Beside decoherence errors [3], due to the interaction with the environment, exists unitary errors [4] that appear in various ways: errors in initial state, gates errors (hardware errors) and read-out errors. In this case it's worth knowing what is the shape, and symmetry, of the of these errors as a result of states propagation in quantum circuits.

This article describes a proposal for a double index to estimate the isotropic components of errors in any processes of quantum computation. In the context of this work the error is considered isotropic if it has spherical symmetry about the state of interest [5]. As an example, given the quantum state $|\psi\rangle = |0\rangle$ (qubit 1) we consider an isotropic unitary error model as in equation 1.

$$|\psi\rangle \rightarrow |\psi'\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\varphi}|1\rangle \qquad (1)$$

where $\theta \in [0, \theta_{MAX}]$ and $\varphi \in [0, 2\pi)$ are random variables, and $\theta_{MAX}$ is the maximum dispersion of error.

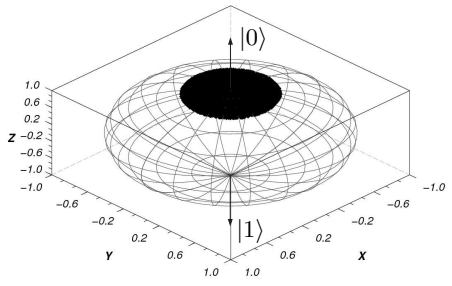


FIG. 1. Isotropic error model.

Consider a one qubit reference state $|\psi\rangle$ and a set of random states characterized by a probability density function $f(\varphi, \theta)$. If the coordinate system is properly rotated (aligning $Z$ axis with the $|\psi\rangle$ state) we have the following definition:

*Definition.* Double isotropic index: is the pair $i_{\text{iso}} = (i_\varphi, i_\theta)$ where, $\theta$ and $\varphi$ are the spherical coordinates in the Bloch sphere($\theta \in [0, \pi]$, $\varphi \in [0, 2, \pi]$) and

$$k_\varphi(\varphi) = \lim_{\Delta\varphi\to 0} \frac{2\pi}{\Delta\varphi} \int_{\varphi-\frac{\Delta\varphi}{2}}^{\varphi+\frac{\Delta\varphi}{2}} \int_0^\pi f(\varphi,\theta)\mathrm{d}\theta\mathrm{d}\varphi$$

$$k_{\varphi\_\max} = \max_\varphi k_\varphi(\varphi), \quad k_{\varphi\_\text{av}} = \frac{1}{2\pi}\int_0^{2\pi} k_\varphi(\varphi)\,\mathrm{d}\varphi$$

$$k_\theta(\varphi) = \lim_{\Delta\varphi\to 0} \frac{2\pi}{\Delta\varphi} \int_{\varphi-\frac{\Delta\varphi}{2}}^{\varphi+\frac{\Delta\varphi}{2}} \int_0^\pi f(\varphi,\theta)\ \theta\mathrm{d}\theta\mathrm{d}\varphi$$

$$k_{\theta\_\max} = \max_\varphi k_\theta(\varphi), \quad k_{\theta\_\text{av}} = \frac{1}{2\pi}\int_0^{2\pi} k_\theta(\varphi)\,\mathrm{d}\varphi$$

$$i_\varphi = \frac{k_{\varphi\_\text{av}}}{k_{\varphi\_\max}}, \quad i_\theta = \frac{k_{\theta\_\text{av}}}{k_{\theta\_\max}} \qquad (2)$$

As examples we study by numerical simulation [6] the propagation of this double index in error correcting codes as the 9-qubit Shor code [7], considering quantum gates with unitary errors [8] [9].

* fonseca@ort.edu.uy
† buksman@ort.edu.uy
‡ jglopez@eui.upm.es

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# Limiting distributions and statistical measures of quantum walks under weak measurements and weak values regimes

Debabrata Ghoshal,[1] Marco Lanzagorta,[2,1] and Salvador E. Venegas-Andraca[3]

[1] *College of Science, George Mason University**

[2] *ITT Technical Division*

[3] *Quantum Information Processing Group, Tecnológico de Monterrey Campus Estado de México*

The field of quantum walks has become an established discipline in quantum computation due to several celebrated results, including quantum algorithms for graph traversing [1] and, more recently, for being a universal model of quantum computation [2, 3]. Furthermore, quantum walks are an active research area due to their potential application on new quantum algorithms, as well as for being powerful tools to describe biological phenomena such as photosynthetic energy transfer [4].

Quantum walks have been studied under several regimes. Motivated by experimental results on quantum weak measurements and weak values [5, 6] as well as by the need to develop new insights for quantum algorithm development, we are extending our knowledge by studying the behavior of quantum walks under the regime of quantum weak measurements and weak values of pre- and postselected measurements (QWWM hereinafter) [7].

In particular, we investigate the limiting position probability distribution and several statistical measures (such as standard deviation) of a QWWM on an infinite line, and compare such results with corresponding classical and quantum walks position probability distributions and statistical measures, stressing the differences provided by weak measurements and weak values with respect to results computed by using canonical observables. We first introduce results for a QWWM under Hadamard evolution and extend our analysis to quantum evolution ruled by general unitary operators. Moreover, we extend our analysis by presenting limiting position distribution and statistical measures of a QWWM on a circle and compare those results with corresponding previous work on both classical and quantum walks.

We then propose a definition and focus on the properties of mixing time of QWWM on an infinite line, followed by a comparison of known corresponding results for classical and quantum walks mixing times.

Finally, we study the effects of decoherence on QWWM on an infinite line and compare those results with other definitions of quantum walks under the effects of decoherence.

* Electronic address: dghoshal@gmu.edu

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# Adaptive versus non-adaptive strategies for quantum channel discrimination

Aram W. Harrow,[1] Avinatan Hassidim,[2] Debbie W. Leung,[3] and John Watrous[3]

[1] *Department of Mathematics, University of Bristol, Bristol, United Kingdom*
[2] *Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA*
[3] *Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, Canada*

*Abstract* We provide a simple example that illustrates the advantage of adaptive over non-adaptive strategies for quantum channel discrimination. In particular, we give a pair of entanglement-breaking channels that can be perfectly discriminated by means of an adaptive strategy that requires just two channel evaluations, but for which no non-adaptive strategy can give a perfect discrimination using any finite number of channel evaluations.

*Introduction* This talk concerns the problem of *quantum channel discrimination*. In this problem, two quantum channels $\Phi_0$ and $\Phi_1$ are fixed, and access to one of the two channels is made available. It is not known which of the two channels has been made available, however, and the goal is to correctly identify which of $\Phi_0$ and $\Phi_1$ it is.

An *optimal* discrimination strategy maximizes the probability that the unknown channel is correctly identified, assuming it is selected according to a fixed distribution that is known ahead of time.

*Adaptive Strategies* An interesting aspect of channel discrimination is the distinction between *adaptive* and *non-adaptive* strategies when multiple uses of the unknown channel are made available. In an adaptive strategy, one may use the outputs of previous uses of the channel when preparing the input to subsequent uses, whereas a non-adaptive strategy requires that the inputs to all uses of the given channel are chosen before any of them is evaluated. It was found in [1] that unitary channels are insensitive to this distinction; adaptive strategies do not give any advantage over non-adaptive strategies for unitary channel discrimination. In the same paper, a pair of *memory channels* was shown to require an adaptive scheme for optimal discrimination. However, whether memoryless channels can have similar property was left open.

Here, we present two quantum channels that can be perfectly discriminated given two adaptive channel evaluations, but for which *no finite number* of non-adaptive channel evaluations allows for a perfect discrimination. This proves the necessity of adaptive schemes for optimal quantum channel discrimination. Furthermore, this result shows that the diamond norm ceases to characterize optimal quantum channel when multiple channel evaluations are available.

Our example also demonstrates another distinction between quantum and classical channel discrimination problems. If a pair of classical channels cannot be perfectly distinguished with one evaluation, then they cannot be perfectly distinguished with any finite number of evaluations. In contrast, it is possible for a pair of quantum channels to be discriminated perfectly when multiple evaluations are available, but not in the single evaluation case. Prior examples involve unitary channels [2]. The channels in our example are *entanglement-breaking* channels, which suggests that entanglement-breaking channels share similar properties to general quantum channels with respect to channel discrimination tasks.

*Related Work* Three channels that require adaptive strategies for an optimal identification was presented in [3]. A recent paper [4] has provided a criterion for the perfect discrimination of pairs of quantum channels, as well as a general method to find adaptive strategies that allow for perfect discrimination.

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# Photonic Communications in Biological Systems

S.Mayburov[1, *]

[1] *Lebedev Institute of Physics*
*Moscow, Russia, RU - 117924*

The existence of low intensity radiation in optical and soft UV range from various bio-systems is now established [1, 2]. The multiple experiments evidence that such photons can perform the effective communications between distant bio-systems [2, 3]. In particular, being radiated by one bio-system, they can rise the rate of cell division (mitosis) in another bio-system of the same or similar specie up to 40%, if this system is irradiated by them during the time interval larger than $10^{1\div 2}$ min, this phenomenon called mitogenetic effect (ME) [1, 3]. The biophotonic communications of some other types were reported, in particular, suppressing the cell divisions or resulting in the cell destruction [4]. Recent experiments evidence that the biophotons are emitted in form of short ( about $10^{-4}$ sec or less) periodic or quasi-periodic bursts with the period between $10^{-1}$ and 10 sec [1]. So its temporary structure is similar to the binary computer communications where each such burst transfers one bit. From that we proposed the model in which such bursts encode the signals, which contain the commands, changing the functioning of other bio-system; this encoding is supposedly similar to the standard methods used for noisy communication channels [5]. In its framework the registration of photons by the receiver bio-system is supposedly performed in photocounting regime, i.e. as the independent clicks.

For the start we studied how the bio-system signals in the darkness can be discriminated from the stochastic background, induced by natural radioactivity, etc. The experimental data for optical radiation from the sample of 200 fish eggs ($Misgurnusfossilis$) were detected by photomultiplier; the integral photocurrent over each .1 sec was recorded in data files. The average background intensity is about .6 of one measured from fish egg sample. For the discrimination the recognition template (algorithm) RT demands that N or more bursts with amplitude $I$ larger than some threshold $I_b$ are registrated; in addition this bursts should be periodic up to $T\pm\sigma_t$ where $\sigma_t \sim .2T$. For $T \approx 8$ sec defined from the best fit to the data, it was found that for N=5 RT gives noise/signal ratio 2.1*10-3 , this is enough for effective signaling between two egg samples at the distance of several cm. This results demonstrate that the radiation from fish eggs is essentially structurized in comparison with the stochastic patterns. We find also that for fish eggs the periodic patterns of $5 \div 10$ bursts are interspersed by the long intervals of practically stochastic radiation, their duration ratio is about $1 : 5$. Concerning the encoding algorithm, the average $T$ value for early development stages ($10 \div 15$ hours) is 9.6 sec, whereas for late stages ($30 \div 40$ hours) it is 6.5 sec. Thus, $T$ value can encode the information about this stage; such signals supposedly sinchronize the development inside the egg colonies, which was confirmed experimentally [3].

The average intensity of biophoton radiation is about 10 photons per sec from $cm^2$ of bio-system surface [1, 2], so this communications are supposedly described as one-photon processes. Our model assumes that the radiation field generated by bio-system is noncoherent, yet such field, in principle, can also possess short-time coherence within this photon bursts, similarly to the laser pulses; some experiments evidence that such coherence really takes place [6]. In our framework it is reasonable to suppose that such coherent field would produce the collective excitations of cells more effectively than noncoherent one. If this hypothesis is correct, then such influence of coherence will not change the principal scheme of communications proposed here, rather, it would gain its efficiency.

* Electronic address: mayburov@sci.lebedev.ru

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# Information Transfer Constraints and Their Role in Quantum Measurements

S.Mayburov[1, *]

[1] *Lebedev Institute of Physics Moscow, Russia, RU - 117924*

The principal feature of any measurement process is the transfer of information from the measured object $S$ to the information system $O$, which stores and processes it; thus, any measuring system (MS) constitutes the information channel. It was found earlier that due to the severe constraints, induced by Heisenberg commutation relations, and the decoherence effects, the capacity of quantum information channels is relatively small and the information losses are significant [1]. Here the influence of such constraints and the decoherence on measurement outcomes will be considered both separately of each other and together [2].

As the model example we consider the measurement of dichotomic $S$ observable $\Lambda$ performed by MS, which includes the detector $D$ and $O$, both of them are regarded as the quantum objects. The measurement of two $S$ ensembles $E_{1,2}$ is considered; $E_1$ includes the identical pure states which are the superposition of $\Lambda$ eigenstates $|\lambda_{1,2}\rangle$, another ensemble $E_2$ is the probabilistic mixture of such eigenstates. For the start we analyzed the information transfer during $S, D$ and $D, O$ interactions, neglecting $D, O$ decoherence effects. It was found that in this case this Heisenberg constraints obstacle the transfer of information about the purity of S state, for the studied ensembles it is described by the expectation value of $S$ observable $\Lambda'$, conjugated to $\Lambda$. As the result, $O$ can't discriminate the pure and mixed $S$ ensembles with the same $\bar{\Lambda}$ [3]. Applying the formalism of restriction maps in Hilbert space, it is shown that this information losses induce the appearance of stochasticity in the measurement of $S$ pure state, so that in the individual events $O$ would detect the random 'pointer' values $O_{1,2}$, which correspond to the outcomes for $O$ measurement of incoming $|\lambda_{1,2}\rangle$ [4]. For the start, the analysis of $D, O$ decoherence effect by the environment was performed , neglecting the influence of Heisenberg constraints. It's argued that by itself, due to the unitarity of decoherence interactions, it doesn't result in the appearance of stochasticity in the measurement outcomes. Yet the account of its dynamics together with Heisenberg constraints conserves the stochasticity of outcomes and stabilizes the final MS states additionally.

* Electronic address: mayburov@sci.lebedev.ru

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# Limit Distributions of Discrete Time Quantum Walks with Environment

Yutaka Shikano[1,*]

[1]*Department of Physics, Tokyo Institute of Technology, Meguro, Tokyo, 152-8551, Japan*
*Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02140, USA*

The discrete time quantum walk (DTQW) initiated in Ref. [1] has various applications like the classical random walk, for instance, the efficient algorithm on quantum computation and the quantum simulator for quantum critical phenomena. Recently, there are physical realizations of a few steps DTQW in some quantum systems.

Let us define the one-dimensional DTQW as follows. First, we assume that the quantum walker is in the one-dimensional discrete lattice denoted as $\mathbb{Z}$ and the coin state is a qubit system with the orthonormal basis, $|L\rangle = (1,0)^T$ and $|R\rangle = (0,1)^T$, where $T$ is transposition. To simplify the discussion, we assume that the initial state is localized at the origin ($x = 0$). Second, the time evolution of the DTQW is described by a unitary operator $U$. A quantum coin flip corresponding to the coin flip in the random walk is described by a unitary operator $C$ acting on the coin state given by $C = a|L\rangle\langle L| + b|L\rangle\langle R| + c|R\rangle\langle L| + d|R\rangle\langle R| \in U(2)$ noting that $abcd \neq 0$ except for the trivial case. Thereafter, the position shift $S$ is described as the move depending on the coin state; $S|x\rangle|L\rangle := |x-1\rangle|L\rangle$ and $S|x\rangle|R\rangle := |x+1\rangle|R\rangle$. Therefore, the unitary operator describing the one-step time evolution for the DTQW is defined as $U = S(I \otimes C)$. We repeat this procedure. Finally, we obtain the probability distribution on the position $x$ at $t$ step as $\Pr(X_t = x) = \mathrm{Tr}\left[\left(\mathrm{Tr}_c U^t \left(|0\rangle\langle 0| \otimes \rho_0\right) U^{t\dagger}\right)|x\rangle\langle x|\right]$, where $X_t$ means a random variable at $t$ step since the measurement outcome of the position measurement is probabilistically determined according to the Born formula.

In the following, we consider the two typical models of the DTQW with environment. One is the DTQW with the periodic position measurement. The other model is the inhomogeneous quantum walk, which is to prepare the spatially different coin.

**Theorem 1** ([3]). *Let $\{Y_i^{(d)}\}$ be an independently identical distributed sequence of the DTQW on $\mathbb{Z}$ with the initial position $x = 0$ and the initial coin state $\rho_0 = (|L\rangle\langle L| + |R\rangle\langle R|)/2$, and the quantum coin flip $C = a|L\rangle\langle L| + b|L\rangle\langle R| + c|R\rangle\langle L| + d|R\rangle\langle R| \in U(2)$ noting that $abcd \neq 0$. Let $X_t = \sum_{i=1}^{M} Y_i^{(d)}$ be a random variable on a position with $d$ step between measurements and the number of the measurements $M$ with the final time $t = dM$. If $d \sim t^\beta$, then, as $t \to \infty$, we have the limit distribution as follows:*

$$\frac{X_t}{t^{(1+\beta)/2}} \Rightarrow \begin{cases} N(0,1) & \text{for } \beta = 0 \\ N(0, 1-\sqrt{1-|a|^2}) & \text{for } 0 < \beta < 1 \\ K(|a|) & \text{for } \beta = 1, \end{cases} \quad (1)$$

*where "$\Rightarrow$" means the weak convergence and $N(m,\sigma^2)$ is the normal distribution with the mean $m$, the variance $\sigma^2$. Note that, the random variable $K(r)$ has the probability density function $f(x;r)$ with a parameter $r \in (0,1)$: $f(x;r) = \sqrt{1-r^2}\, I_{(-r,r)}(x) / \left(\pi(1-x^2)\sqrt{r^2-x^2}\right)$. Here, $I_{(-r,r)}(x)$ is the indicator function, that is, $I_A(x) = 1$ $(x \in A), = 0$ $(x \notin A)$.*

The generalization to the general initial coin state and the extension to the continuous time quantum walk will be shown in Ref. [4].

In the second model, the quantum coin at the position $m$ is described as

$$C_m = \begin{pmatrix} \cos(2\pi\alpha m) & \sin(2\pi\alpha m) \\ -\sin(2\pi\alpha m) & \cos(2\pi\alpha m) \end{pmatrix} \quad (2)$$

with the constant number $\alpha$. This model is called an inhomogeneous DTQW [5].

**Theorem 2** ([6]). *For any irrational $\alpha$, the limit distribution of the inhomogeneous quantum walk under any time scale is localized at the origin;*

$$\frac{X_t}{t^\theta} \Rightarrow \delta(x), \quad (3)$$

*where $\theta > 0$ is arbitrary time scale parameter and $\delta(\cdot)$ is the Dirac delta function.*

According to Theorem 1 and 2, the DTQW with environment does not generally correspond to the classical random walk. In Theorem 1, we show that the DTQW with the periodic position measurement is not time scale invariant. In Theorem 2, we show that the inhomogeneous DTQW has the localization property like the Anderson localization.

Furthermore, we propose the conjecture on the localization or delocalization of the DTQW distinguished by the eigenvalues of the time evolution operator [6].

* Electronic address: shikano@mit.edu; URL: http://www.th.phys.titech.ac.jp/~shikano/

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# Entanglement-annihilating channels

Mário Ziman[1,2,*] and Lenka Moravčíková

[1] *Research Center for Quantum Information, Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 84511 Bratislava, Slovakia*

[2] *Faculty of Informatics, Masaryk University, Botanická 68a, 60200 Brno, Czech Republic*

Consider a pair of spatially separated qubits, each of them interacting with its own environment. Let us denote by $\mathcal{E}_1, \mathcal{E}_2$ the noisy channels acting on these qubits, respectively. Let us denote by $\omega_{12}$ the initial state of these qubits. Which channels $\mathcal{E}_1, \mathcal{E}_2$ destroy any entanglement? Clearly, if at least one of the channels is entanglement-annihilating, then $\mathcal{E}_1 \otimes \mathcal{E}_2[\omega_{12}]$ is a separable state. Is this the only option? The goal of this talk will be to answer these type of questions, i.e. which channels completely destroy any entanglement.

Let $\mathcal{H}_Q = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n$ be a system composed of $n$ particles. Consider a subset of particles $A$ and set $B = Q \setminus A$. By $\mathcal{S}_{\rm sep}(\mathcal{H}_A)$ we will denote the set of separable states on the composite system $A$ and by $\mathcal{S}(\mathcal{H}_{AB})$ the set of separable states on $\mathcal{H}_Q$ with respect to splitting into $A$ and $B$. We say the channel $\mathcal{E}_A$ acting on the subsystem $\mathcal{H}_A$ is *entanglement-annihilating* (EA) if

$$\mathcal{E}_A[\mathcal{S}(\mathcal{H}_A] \subset \mathcal{S}_{\rm sep}(\mathcal{H}_A)\,,$$

and *entanglement-breaking* (EB) if

$$\mathcal{E}_A \otimes \mathcal{I}_B[\mathcal{S}(\mathcal{H}_{AB})] \subset \mathcal{S}_{\rm sep}(\mathcal{H}_{AB})$$

for arbitrary ancillary system $B$.

In words, the entanglement-annihilating (EA) channels are defined as the ones that completely destroy/annihilate any entanglement within the subset $A$ of the composite system (see Figure). On contrary, the entanglement-breaking (EB) channels are those that completely disentangle the subsystem they are acting on from the rest of the system.

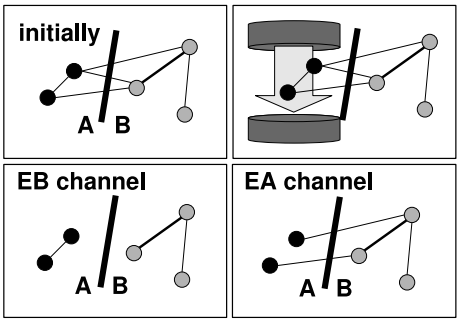


FIG. 1: The action of entanglement-breaking and entanglement-annihilating channel is illustrated. Lines between the systems exhibits the existence of entanglement.

In order to give answer to our original problem we need to consider special subclass of EA and EB channels. Let us assume that $A$ consists of the particles of the same type and consider channels of the form $\mathcal{E}_A = \mathcal{E} \otimes \cdots \mathcal{E} = \mathcal{E}^{\otimes |A|}$. We say a single-particle channel $\mathcal{E}$ is a *$k$-locally entanglement annihilating* channel ($k$-LEA), if $\mathcal{E}^{\otimes k}$ is entanglement annihilating. Similarly, $\mathcal{E}$ is a *$k$-locally entanglement breaking* channel ($k$-LEB) if $\mathcal{E}^{\otimes k}$ is entanglement breaking. By $\mathsf{T}_{k-\rm LEA}$, $\mathsf{T}_{k-\rm LEB}$ we shall denote the subsets of $k$-LEA and $k$-LEB channels, respectively. Since elements of these sets are uniquely associated with single-particle channels $\mathcal{E} \in \mathsf{T}_{\rm chan}(\mathcal{H})$, we can understand these sets as subsets of $\mathsf{T}_{\rm chan}(\mathcal{H})$, i.e. $\mathsf{T}_{k-\rm LEA}, \mathsf{T}_{k-\rm LEB} \subset \mathsf{T}_{\rm chan}(\mathcal{H})$. Moreover, let us denote by $\mathsf{T}^1_{\rm EB} \subset \mathsf{T}_{\rm chann}(\mathcal{H})$ the subset of entanglement breaking channels acting on the single system $\mathcal{H}$, i.e. $\mathcal{E} \in \mathsf{T}^1_{\rm EB}$ means that $(\mathcal{E} \otimes \mathcal{I}_{\rm anc})[\omega]$ is separable for all $\omega \in \mathcal{S}(\mathcal{H} \otimes \mathcal{H}_{\rm anc})$.

In the talk we will analyze the structure and properties of entanglement-annihilating channels. We will show that there are channels which are simultaneously entanglement-breaking and entanglement-annihilating, but also channels possessing only one of these features, i.e. $\mathsf{T}_{\rm EA} \cap \mathsf{T}_{\rm EB} \neq \emptyset$ and $\mathsf{T}_{\rm EB} \not\subset \mathsf{T}_{\rm EA} \not\subset \mathsf{T}_{\rm EB}$. The set of entanglement-annihilating channels $\mathsf{T}_{\rm EA}$ is convex. Moreover, a composition of an entanglement-annihilating channel and an arbitrary channel results in an entanglement-annihilating channel, i.e. the property of being entanglement-annihilating is preserved under channel composition. Further, we will prove the following set relations

$$\mathsf{T}^1_{\rm EB} = \mathsf{T}_{2-\rm LEB} = \mathsf{T}_{3-\rm LEB} = \cdots = \mathsf{T}_{\infty-\rm LEB}\,,$$
$$\mathsf{T}^1_{\rm EB} \subset \mathsf{T}_{\infty-\rm LEA} \subset \cdots \subset \mathsf{T}_{3-\rm LEA} \subsetneq \mathsf{T}_{2-\rm LEA}\,,$$

where $\mathsf{T}^1_{\rm EB}$ is the set of entanglement-breaking channels of a single particle. Finally, we will discuss the problem of identification whether a given channel is entanglement annihilating and interactions leading to entanglement-annihilating channels. The presented results are mostly based on Ref.[1].

* Electronic address: ziman@savba.sk

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# Extended phase space and higher order correlation

Hilal Al Hadhrami[1]

[1]*Nizwa College of Applied Sciences*
*Nizwa 611 P.Box 699, Sultanate of Oman*

Extended phase-space $q-p-Q-P$ (position, momentum, position increment, momentum increment) is studied. Extended Wigner and extended Weyl functions are introduced. Higher order correlation can be achieved using extended Wigner and Weyl functions. Examples of fourth order interference are given for harmonic oscillator and finite systems in extended phase-space.

For any two quantities $f(q), f(p)$ related to each other through Fourier transform, we can define a Wigner function $W(q,p)$ and a Weyl function $\tilde{W}(Q,P)$ such that [1]:

$$\begin{aligned} W(q,p) &\equiv \frac{1}{2\pi}\int f^*(q+\frac{Q}{2})f(q-\frac{Q}{2})\exp(-ipQ)\,dQ \\ \tilde{W}(Q,P) &\equiv \int f^*(q+\frac{Q}{2})f(q-\frac{Q}{2})\exp(-iqP)\,dq \end{aligned} \quad (1)$$

Since these two functions are related through Fourier transform, then an extended Wigner function $W_e(q,p,Q,P)$ can be defined as[2]

$$\begin{aligned} W_e(q,p,Q,P) \equiv\ & (2\pi)^2\int\int W(q+\frac{1}{2}q',p+\frac{1}{2}p')\times \\ & W(q-\frac{1}{2}q',p-\frac{1}{2}p')\times \\ & \exp[i(p'Q-q'P)]dq'dp' \end{aligned} \quad (2)$$

Extended Weyl function $\tilde{W}_e(q',p',Q',P')$ can be defined as well. These two functions are also related to each other through Fourier transform. So by this way we can get higher order Wigner and Weyl functions.
Analogy of such functions in finite Hilbert space can also be obtained [3]. The extended wigner and Weyl functions reveal more properties of the state. Higher order correlation and interference can be observed. For example, a mixed state described by the density matrix

$$\rho = \frac{1}{n}\sum_{i=1}^{n}\rho_i \quad (3)$$

The Wigner function of this density matrix is:

$$W = \frac{1}{2n\pi}\sum_{i=1}^{n}W(\rho_i) \quad (4)$$

Thence, using Eqn(2), the extended Wigner function is

$$\begin{aligned} W_e(\rho) = & \frac{1}{n^2}\sum_{j=1}^{n}W_e(\rho_j)+ \\ & \int\sum_{jk} W_j(q+\frac{1}{2}q',p+\frac{1}{2}p')W_k(q-\frac{1}{2}q',p+\frac{1}{2}p')\times \\ & \exp(i[p'Q-q'P])dq'dp' \end{aligned} \quad (5)$$

The second part gives the fourth order interference. As an example we choose

$$\rho = \frac{1}{3}(|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta| + |\gamma\rangle\langle\gamma|) \quad (6)$$

where $|\alpha\rangle, |\beta\rangle, |\gamma\rangle$ are coherent states. Fig(1) shows fourth order interference (the middle Gaussians).

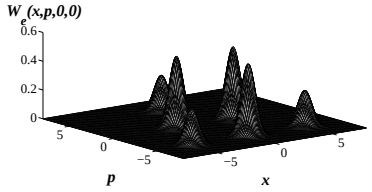


FIG. 1: $W_e(x,p,0,0)$ for the mixed state of Eqn.(6) as a function of $x,p$. $\alpha=3,\beta=-3,\gamma=3i$

In a second example we study a finite system with dimension $d=11$. We calculate the mixed state

$$\rho = \frac{1}{2}(|p_1\rangle\langle p_1| + |p_2\rangle\langle p_2|) \quad (7)$$

where $p_i$ are momentum states. The fourth order interference is shown in Fig(2)(the middle wall).

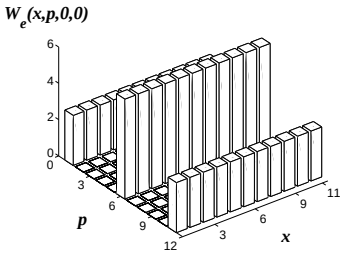


FIG. 2: $W_e(x,p,0,0)$ for the mixed state of Eqn.(7) as a function of $x,p$. $p_1=|1\rangle, p_2=|11\rangle$

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## Quantum key distribution robust against photon number splitting attacks

Steve Brierley[1]

[1] *Center for Quantum Information and Communication, Universit Libre de Bruxelles, Belgium*

Many implementations of quantum key distribution protocols use photons produced by attenuated lasers as a source of quantum states. Such schemes are vulnerable to photon number splitting (PNS) attacks since signals sometimes contain more than one photon. For protocols such as BB84, such attacks greatly reduce the distance over which the protocol remains secure.

In this poster, we introduce a new family of quantum key distribution protocols designed to be robust against photon number splitting (PNS) attacks. Each protocol uses $d$-level quantum states that are elements of $c$ mutually unbiased bases. The case of $d = c = 2$ corresponds to the SARG protocol and we see that by making use of all three Pauli operators ($c = 3$) the protocol is more robust against a PNS attack. When implementing a storage PNS attack the information gained by any eavesdropper is lower than that of the SARG protocol allowing the secure implementation of the protocol over longer distances. Analysis of the protocol against other attacks is given to motivate the claim that the protocol is worthy of further study.

# Assessing the Hilbert space dimension of arbitrary quantum systems

Rodrigo Gallego,[1] Nicolas Brunner,[2] Christopher Hadley,[1] and Antonio Acín[1]
[1] *ICFO–Institut de Ciències Fotòniques, E-08860 Castelldefels, Barcelona, Spain*
[2] *H.H. Wills Physics Laboratory, University of Bristol, United Kingdom*

In quantum mechanics, experimental observations are usually described using theoretical models which make specific assumptions on the physical system under consideration, including the size of the associated Hilbert space. The Hilbert space dimension is thus intrinsic to the model. In this work, the converse approach is considered: is it possible to assess the Hilbert space dimension from experimental data without an *a priori* model?

This is also relevant in the context of quantum information science, in which dimensionality enjoys the status of a resource for information processing. Higher dimensional systems may potentially enable the implementation of more efficient and powerful protocols. It is therefore desirable to design methods for testing the Hilbert space dimension of quantum systems, which are 'device-independent'; that is, where no assumption is made on the devices used to perform the tests.

Though no systematic study of this problem has yet been performed, several recent studies have obtained interesting results applicable in specific scenarios. Firstly, the concept of dimension witnesses was introduced in the context of non-local correlations [1]. Such witnesses are essentially Bell-type inequalities, the violation of which imposes a lower bound on the Hilbert space dimension of the entangled state on which local measurements have been performed. [2–8]. Wehner *et al.* subsequently [9] showed how the problem relates to random-access codes, and could thus exploit previously known bounds. Finally, Wolf *et al.* addressed the question from a dynamical viewpoint, showing how bounds on the dimensionality may be obtained from the evolution of an expectation value.

Though these works represent significant progress, they all have significant drawbacks. The approach of Brunner *et al* [1] may not be applied to single-party system; the bounds of Wehner *et al* are based on Shannon channel capacities, which are, in general, difficult to compute; whilst the approach of Wolf *et al.* cannot be applied to the static case.

In the present work we formalise the problem of testing the Hilbert space dimension of arbitrary quantum systems in the simplest scenarios in which the problem is meaningful. We introduce natural tools for addressing the problem, starting by developing methods for determining the minimal dimensionality of classical systems, given certain data. Using geometrical ideas, we introduce the idea of 'tight classical dimension witnesses', leading to a generalisation of quantum dimension witnesses to arbitrary systems. We also provide simple examples of such classical and quantum dimension witnesses. We believe that the simplicity of our techniques makes them appealing for experimental perspectives.

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# Bell inequality violations with bipartite-entangled photonic qudits

Adetunmise Dada,[1,*] Erika Andersson,[1,†] Gerald Buller,[1,‡] Jonathan Leach,[2,§] and Miles Padgett[2,¶]

[1] *Department of Physics, Heriot-Watt University, Edinburgh EH14 4AS, United Kingdom*
[2] *Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom*

Entanglement, a key ingredient in many quantum communication schemes, can be demonstrated using Bell inequalities. Experiments have been proposed and performed with photons entangled in polarization as well as orbital angular momentum (OAM) to demonstrate the violation of Bell inequalities for systems with dimensionality $d$=2. However, OAM entangled photons generated by spontaneous parametric down-conversion (SPDC) are in higher dimensional entangled states. This has been demonstrated by the violation of a generalized Clauser-Horne-Shimony-Holt (CHSH)-type Bell inequality with $d$=3 [1]. We now describe experimental violations of corresponding Bell inequalities for $d = 4, 5, 6, 7, 8, 9, 10$ as well as the extension of this experiment to $d = 24$. Our experiments use OAM entangled photons and the family of Bell inequalities derived in [2]. A violation of a $d$-dimensional Bell inequality requires a reduction of the OAM state space to $d$ dimensions. The reduction is never perfect in practice and this leads to contamination from other modes. We demonstrate the effect of modal contamination, coincidence photon counting accuracy, and misalignment on Bell inequality violation experiments with higher $d$. Higher dimensional entangled states allow the realization of new types of quantum communication protocols that can offer high information-density coding and resilience to noise. They also enable quantum key distribution protocols with higher margins of security [3]. It has also been shown recently that closing the detection loophole in Bell experiments is more experimentally feasible in qu*d*it based Bell experiments with current detection technologies[4].

* acd8@hw.ac.uk
† erika.andersson@hw.ac.uk
‡ g.s.buller@hw.ac.uk
§ j.leach@physics.gla.ac.uk
¶ m.padgett@physics.gla.ac.uk

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# SIC-POVM: Star Product and Relation to Other Probability Representations

Sergey N. Filippov[1,*] and Vladimir I. Man'ko[2,†]

[1] *Moscow Institute of Physics and Technology (State University), 9 Institutskii Pereulok, Dolgoprudny, Moscow Region 141700, Russia*

[2] *P. N. Lebedev Physical Institute, Russian Academy of Sciences, 53 Leninskii Prospect, Moscow 119991, Russia*

The report deals with a symmetric informationally complete positive operator valued measure (SIC-POVM) in $d$-dimensional Hilbert space $\mathscr{H}_d$. The core of any SIC-POVM is a set of $d^2$ rank-1 projectors $\hat{\Pi}_i = |\psi_i\rangle\langle\psi_i|$ acting on $\mathscr{H}_d$ and satisfying the condition

$$\mathrm{Tr}\big[\hat{\Pi}_i\hat{\Pi}_j\big] = |\langle\psi_i|\psi_j\rangle|^2 = \frac{d\delta_{ij}+1}{d+1}, \qquad (1)$$

where $\delta_{ij}$ is the Kronecker symbol. The existence of SIC-POVMs in any finite dimension still remains an unsolved problem, though the existence is demonstrated (partly analytically and numerically) in dimensions $d \leq 67$.

The SIC representation of quantum states (see, e.g. the review [1]) is based on the idea that a quantum state, usually described by the density operator $\hat{\rho}$, is also fully determined by $d^2$ probabilities $p_i$. The set of probabilities $\{p_i\}_{i=1}^{d^2}$ and the density-operator reconstruction read

$$p_i = \frac{1}{d}\mathrm{Tr}\big[\hat{\rho}\hat{\Pi}_i\big], \qquad \hat{\rho} = (d+1)\sum_{i=1}^{d^2} p_i\hat{\Pi}_i - \hat{I}. \qquad (2)$$

In the SIC representation, every quantum state can be represented as a set of probabilities $\{p_i\}_{i=1}^{d^2}$ in the simplex of all probability vectors with $d^2$ components.

The aim of the report is to show that the SIC-POVM is a partial case of the probability representation of quantum states and can be related to other known kinds of the probability representations like the spin tomography and the unitary spin tomography (reviewed in [2]) as well as the spin tomography with a finite number of rotations [3]. Also, we cannot help mentioning a conceptual drawback of the SIC representation, namely, the absence of a measurable physical quantity which can give rise to the SIC probability distribution.

The essence of the star product scheme is as follows.

Any operator $\hat{A}$ acting on $\mathscr{H}_d$ can be alternatively described by the function $f_A(\mathbf{x})$ (symbol of the operator):

$$f_A(\mathbf{x}) = \mathrm{Tr}\big[\hat{A}\hat{U}(\mathbf{x})\big], \qquad \hat{A} = \int f_A(\mathbf{x})\hat{D}(\mathbf{x})d\mathbf{x}, \qquad (3)$$

where $\hat{U}(\mathbf{x})$ and $\hat{D}(\mathbf{x})$ are a dequantizer operator and quantizer operator, respectively [4]; the set of variables $\mathbf{x}$ as well as the integration $\int d\mathbf{x}$ depends on a system under study and a concrete quantization scheme.

A symbol $f_{AB}(\mathbf{x})$ of the product of two operators $\hat{A}$ and $\hat{B}$ acting on $\mathscr{H}_d$ is referred as the star product of symbols $f_A(\mathbf{x}_1)$ and $f_B(\mathbf{x}_2)$ and reads

$$f_{AB}(\mathbf{x}) = \int f_A(\mathbf{x}_1)f_B(\mathbf{x}_2)K(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x})d\mathbf{x}_1d\mathbf{x}_2, \qquad (4)$$

where the star-product kernel $K(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x})$ reads

$$K(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}) = \mathrm{Tr}\big[\hat{D}(\mathbf{x}_1)\hat{D}(\mathbf{x}_2)\hat{U}(\mathbf{x})\big]. \qquad (5)$$

The star product is associative but not necessarily commutative. The associativity property results in the star-product kernel $K^{(N)}(\mathbf{x}_1,\ldots,\mathbf{x}_N,\mathbf{x})$ of an arbitrary number $N$ of symbols being expressed through the kernel (5). For example, in the case of three operators we have

$$\begin{aligned} K^{(3)}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3,\mathbf{x}) &= \int K(\mathbf{x}_1,\mathbf{x}_2,\mathbf{y})K(\mathbf{y},\mathbf{x}_3,\mathbf{x})d\mathbf{y} \\ &= \int K(\mathbf{x}_1,\mathbf{y},\mathbf{x})K(\mathbf{x}_2,\mathbf{x}_3,\mathbf{y})d\mathbf{y}. \end{aligned} \qquad (6)$$

Comparing (2) and (3), we see that the SIC projectors $\hat{\Pi}_i$ can be considered (up to a normalization factor and the identity operator) as dequantizers $\hat{U}_i = \frac{1}{d}\hat{\Pi}_i$ and quantizers $\hat{D}_i = (d+1)\hat{\Pi}_i - \hat{I}$ of the SIC star-product quantization scheme with $\mathbf{x} = i$ and $\int d\mathbf{x} = \sum_{i=1}^{d^2}$. It follows immediately that the existence of SIC-POVM means the existence of the associative product $K_{ijk} = \mathrm{Tr}\big[\hat{D}_i\hat{D}_j\hat{U}_k\big]$ which is a solution of Eq. (6). Moreover, utilizing the standard equations for a generic star-product kernel, we derive some properties of the triple products $T_{ijk} = \mathrm{Tr}\big[\hat{\Pi}_i\hat{\Pi}_j\hat{\Pi}_k\big]$ of SIC projectors found in [1] and establish new relations on them. Thus, we interpret such properties of $T_{ijk}$ as standard properties of the star-product kernel (including the dual star-product scheme). From the same point of view, the Lie algebraic structure found in [1] is an immediate and known consequence of the antisymmetrized kernel of associative product. Further, the problem of SIC-POVM existence is formulated in terms of symbols of the SIC projectors and the corresponding kernel of associative product. The approach to solve the modified problem is also developed in [5].

* Electronic address: sergey.filippov@phystech.edu
† Electronic address: manko@sci.lebedev.ru

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# Quantum polarization of coherent and thermal states

Iulia Ghiu,[1,*] Gunnar Björk,[2] Paulina Marian,[1,3] and Tudor A. Marian[1]

[1] *Centre for Advanced Quantum Physics, Department of Physics, University of Bucharest, P. O. Box MG-11, R-077125 Bucharest-Măgurele, Romania*
[2] *School of Information and Communication Technology, Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden*
[3] *Department of Chemistry, University of Bucharest, Boulevard Regina Elisabeta 4-12, R-030018 Bucharest, Romania*

The most convenient quantum measures of polarization for two-mode coherent and thermal states of the radiation field are re-examined. In parallel, we consider the phase-averaged coherent states, which are non-Gaussian and at the classicality threshold. The basic idea is to exploit the recently introduced definition of the degree of polarization in terms of the block-diagonal density matrix $\hat{\rho}_b := \mathcal{B}[\hat{\rho}]$ [1, 2]. The state $\hat{\rho}_b$ is hence obtained from the given quantum state $\hat{\rho}$ by a non-selective measurement $\mathcal{B}$ of the total number of photons:

$$\hat{\rho} \xrightarrow{\mathcal{B}} \hat{\rho}_b = \sum_{N=0}^{\infty} \hat{P}_N \hat{\rho} \hat{P}_N,$$

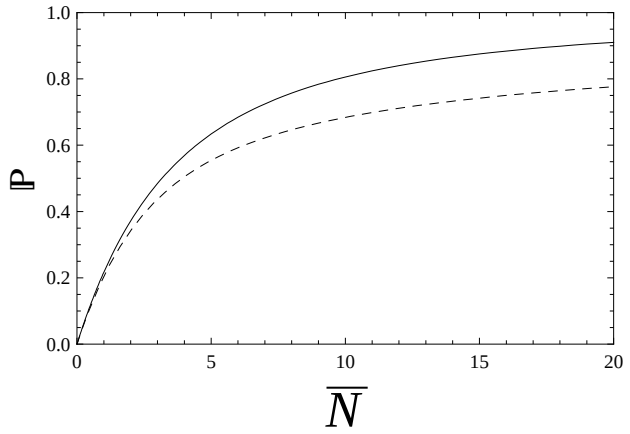


FIG. 1: Degree of polarization of the coherent states: the Chernoff measure $\mathbb{P}_C(\hat{\rho})$ (full line) and the Bures measure $\mathbb{P}_B(\hat{\rho})$ (dashed line).

where $\hat{P}_N = \sum_{k=0}^{N} |\, k, N-k \,\rangle\langle\, k, N-k \,|$ is the projection operator onto the vector subspace of the $N$-photon states.

We first review the traditional Stokes and $SU(2)$ $Q-$function degrees of polarization. Then we evaluate and discuss some computable distance-type measures such as the Hilbert-Schmidt, Bures, Chernoff degrees of polarization, and the relative entropy of polarization. We are particularly interested in their dependence on the mean occupancy in the states involved. A *bona fide* distance-type measure of polarization is expected to have a monotone variation of the degree of polarization with respect to this parameter. As a significant example we show in Fig. 1 the Chernoff and Bures degrees of polarization as functions of the average photon number $\overline{N} = |\alpha_1|^2 + |\alpha_2|^2$ of a two-mode coherent state. Our plot exhibits the general property [2]:

$$\mathbb{P}_C(\hat{\rho}) \geq \mathbb{P}_B(\hat{\rho}).$$

* Electronic address: iulia.ghiu@g.unibuc.ro

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# Graph states, algebraic curves and Mutually Unbiased bases

A.B. Klimov[1,*]

[1] *Departamento de Física, Universidad de Guadalajara, México.*

The graph states were introduced [1] as eigenstates of a commuting set of $n$ operators acting in the $2^n$ dimensional Hilbert space, where each element in this set is a product of single qubit Pauli operators and which particular form is related to mathematical graphs. The the result of appicatons of local (except the local complements), non-local operations to graphs is not transparent, especially for a large number of qubits. Also, the classification of graph states presents difficulties, since the number of graphs grows very fast with $n$ [2].

Here we analize an algebraic way of characterization of graph states using recently intriduced in [3] concept of Abelian curves over $\mathbb{F}_{2^n}$ and compare these two approaches. The principal operations applied to graph states are formulated on the algebraic languge. Connecton to the construction of Mutually Unbiased Bases is discussed.

A complete set of commuting monomials $\{K_a, a = 1,..,n\}$ for $n$ qubit system is associated with $n$-vertex graph $G = (V, E)$: to each vertex $(V)$ a single qubit is assigned and each edge $(E)$ represents the interaction between two correposnding qubits: $K_a = \sigma_z^a \Pi_{b\in N_a} \sigma_x^b$, where $N_a$ the neighborhood of the vertex $a$, $N_a = \{b \in V, [a,b] \in E\}$. The whole set of $2^n$ commuting monomials generated by $\{K_a\}$ form an Abelian group $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus ...,$, where each term has the form $Z_{\vec{a}} X_{\vec{b}} \in \mathcal{P}^n$, $\mathcal{P}^n = \mathcal{P}^1 \otimes \mathcal{P}^1 \otimes \cdots$, $\mathcal{P}^1 = \{I, \sigma_z, \sigma_x, \sigma_y\}$, $a_j, b_j \in \mathbb{Z}_2$,

$$Z_{\vec{a}} = \sigma_z^{a_1} \otimes \ldots \otimes \sigma_z^{a_n},\ X_{\vec{b}} = \sigma_x^{b_1} \otimes \ldots \otimes \sigma_x^{b_n}. \quad (1)$$

To label both the states of $n$-qubit systems and the elements of $\mathcal{P}^n$ we use elements of the finite field $\mathbb{F}_{2^n}$, which is a linear space spanned on an "abstract" basis $\{\theta_1, ..., \theta_n\}$, so that $(a_1, a_2, ..., a_n) \Leftrightarrow \alpha = \sum_{i=1}^n a_i \theta_i \in \mathbb{F}_{2^n}$. The basis is chosen to be "orthonormal" (self-dual) with respect to the trace operation, $tr(\theta_i \theta_j) = \delta_{ij}$, $tr(\beta) = \beta + \beta^2 + ... + \beta^{2^{n-1}} \in \mathbb{Z}_2$. The basis of eigenstates of the operators $Z_{\vec{a}}$ is labelled by a single element of $\mathbb{F}_{2^n}$: $|\alpha\rangle \Leftrightarrow |a_1\rangle_1 ... |a_n\rangle_n$, $a_i = tr(\alpha\theta_i)$, i.e. each qubit is associated with an element of the basis: $\text{qubit}_i \Leftrightarrow \theta_i$.

The abstact form of the operators (1) is

$$Z_\alpha = \sum_{\gamma\in\mathbb{F}_{2^n}} (-1)^{tr(\gamma\alpha)} |\gamma\rangle\langle\gamma|,\ X_\beta = \sum_{\gamma\in\mathbb{F}_{2^n}} |\gamma+\beta\rangle\langle\gamma|. \quad (2)$$

The monomials of the form $Z_\alpha X_{\beta=f(\alpha)}$, where

$$f(\alpha) = \sum_{m=0}^{n-1} \phi_m\, \alpha^{2^m}, \quad \phi_m \in \mathbb{F}_{2^n}, \quad (3)$$

are additive and commute among them if the expansion coefficients satisfy the conditions $\phi_j = \phi_{n-j}^{2^j}$, $j = 1, \ldots, [(n-1)/2]$ (for even $n$, additionaly $\phi_{n/2} = \phi_{n/2}^{2^{n/2}}$): $Z_\alpha X_{f(\alpha)} Z_{\alpha'} X_{f(\alpha')} = Z_{\alpha+\alpha'} X_{f(\alpha+\alpha')}$. The graph-state generators $\{K_a, a = 1,..,n\}$ can be written in an algebraic form $\{Z_\alpha X_{\beta=f(\alpha)}, \alpha = \theta_1, ..., \theta_n\}$, where $\theta_k$ are elements of the self-dual basis and the function $\beta = f(\alpha)$ additionaly to (3) satisfies the condition,

$$tr(\theta_k f(\theta_k)) = 0, \quad k = 1,..,n. \quad (4)$$

We call the structure (3) "curve" since in the discretre phase space which coordinates are labbeled by the idices of the operators $Z_\alpha$ and $X_\beta$ the pairs $(\beta, \alpha)$ form a discrete curve passing through the origing. The generic form of *graph curves* which satisfy (4): for odd $n$ it is sufficient that: $\phi_0 = 0$; for even $n$ the zero and the middle coefficients should be related: $\phi_0 = \sum_{k=1}^n tr\left(\phi_{n/2}\theta_k^{2^{n/2}}\theta_k\right)\theta_k^2$.

*Reconstruction relation*: given the adjacency matrix $\Gamma^G$ of a graph $G$, the expansion coefficients (3) for the corresponding curve are: $\phi_r = \sum_{p,q=1}^n \Gamma_{pq}^G \theta_q^{2^r} \theta_p$.The adjacency matrix for graphs corresponding to the curves satisfying (4) is $\Gamma_{kl}^{(f)} = tr(\theta_k f(\theta_l))$.

*Local operations*: Multy-qubit $x$, $z$- rotations are labelled by $\xi \in \mathbb{F}_{2^n}$ which indicates on what qubits the rotation is perfomed by its expansion in the self-dual basis, $\xi = \sum_i \xi_i \theta_i$: $\xi_i = 1$ means that the $i$-th qubit is rotated: $U_{x,z}^\xi = \otimes \prod_{i=1}^n U_{x,z}^{tr(\xi\theta_i)}$, $U_{x,z} = \exp(i\pi\sigma_{x,z}/4)$, e.g. local $x$- rotation: $f(\alpha) \to f(\alpha) + \sum_i \xi_i tr(\alpha\theta_i)\theta_i$. Local complement operator $V_a = U_r^{(a)} \Pi_{b\in N_a} U_r^{(b)}$ action: $\phi_r \to \phi_r + f^{2^r+1}(\theta_a) + \sum_i tr(f(\theta_a)\theta_i)\theta_i^{2^r+1}$.

*Non-local operations*: $XOR_{pq}$: $\beta \to \beta' = f(\alpha) + tr(\alpha\theta_q) f(\theta_p) + tr(f(\alpha)\theta_p)\theta_q$, so that a graph curve transforms into a graph curve; $SWAP_{pq}$: $\beta \to \beta' = f(\alpha) + tr(\alpha\varepsilon) f(\varepsilon) + tr(f(\alpha)\varepsilon)\varepsilon$, $\varepsilon = \sigma_p + \sigma_q$.

*Diagonalization of curves*: $Z_\alpha X_{f(\alpha)} \sim P_f Z_\alpha P_f$, where $P_f = \sum_\kappa c_\kappa^{(f)} |\widetilde{\kappa}\rangle\langle\widetilde{\kappa}|$, $|\widetilde{\kappa}\rangle$ are the eigenstates of $X_\beta$ and $c_\alpha^{(f)} = (-1)^{\frac{1}{2}\overrightarrow{\alpha}^\top \Gamma^{(f)} \overrightarrow{\alpha}}$, where $\Gamma^{(f)}$ is the adjacency matrix corresponding to the graph curve $\beta = f(\alpha)$.

*Factorization*: the $k$-th vertex is disconnected if $f(\theta_k) = 0$; for each disconnected subgraph $A_k$, $f(A_k) = A_k$.

*Locally non equivalent curves* for 3-6 qubits are explicitly given.

* Electronic address: klimov@cencar.udg.mx

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# A Lot of Fringes: Super-Resolving Phase Measurements Without Entanglement or Joint Detection

Christian Kothe,[1,*] Gunnar Björk,[1] and Mohamed Bourennane[2]

[1] *School of Information and Communication Technology, Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden*

[2] *Department of Physics, Stockholm University, SE-109 61 Stockholm, Sweden*

Interference plays a crucial role in many physical aplications, such as detection of gravitational waves, metrology, imaging, lithography or atomic spectroscopy. In interferometry *phase super-resolution* denotes the ability of seeing $n$ fringes in the interference pattern where one usually would expect only one fringe. Until some years ago it was believed that one would need entangled states [1] to be able to produce phase super-resolution, mainly by using so called NOON-states

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle), \tag{1}$$

which consist of a superposition of $N$ photons in one mode and zero photons in another mode, and vice versa. Such states are difficult to produce for $N \geq 2$, and so far only experiments with $n = N = 4$ have been reported [2, 3]. Recently, however, it was shown that the same effect also could be achieved by using "classical states" consisting of unentangled, coherent light, since quantum mechanics assures so-called *time-reversal symmetry* [4]. Using this approach, phase super-resolution has been shown for $n = 6$.

We show that even the constraint of time-reversal symmetry is unnecessary for achieving phase super-resolution, and that the whole effect goes back to a simple mathematical expression which neither requires entanglement, nor joint detection of photons [5]. By removing these limitations, we show how one can design different experiments including only standard optical components, allowing, in principle, for arbitrarily high numbers of $n$. We show experimental results for $n$ up to 30 (experimental results for $n = 10$ can be seen in Fig. 1), and explain how one can achieve even higher $n$. We also explain that *phase super-sensitivity*, i.e., the ability of getting a phase uncertainty in experiments which is lower than the standard quantum limit, cannot be achieved in these experiments, since this presumably requires entanglement.

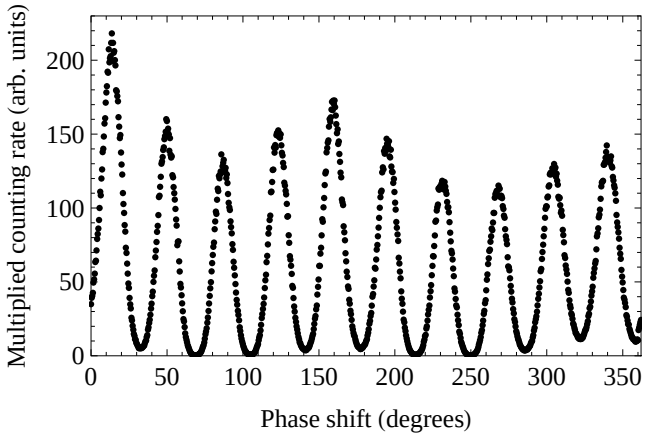


FIG. 1: Phase super-resolution with $n = 10$.

* Electronic address: kothe@kth.se; Also at Department of Physics, Stockholm University.

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# The wave function collapse as an effect of field quantization

K. Lewin[1]

[1] *Formerly: Humboldt-University Berlin**

**Background:** Since the sixtieth, quantum mechanics (QM) is seen as a complete theory because of the results of EPR-Bell experiments. Indeterminacy relations are fundamental and cannot be traced back to a lack of knowledge. However, conceptual problems around the wave function collapse and the nonlocality problem remained and the long-standing discussion continues.

**Purpose:** Indication that these problems are not due to the quantum mechanical formalism and its entangled states, but are connected exclusively to the Copenhagen interpretation and its conception of truth rejecting the discussion of particle presence during wave packet evolution.

**Method:** Assuming that the successful mathematical formalism of QM describes an objective reality independent of observers and their knowledge, QM represents a classical field theory within the frame work of field quantization. That is important to understand the particle structure of matter during wave function evolution and to account for the collapse as symmetry breakdown by detection. Symmetries and probabilities in context of superposition states appear as to sides of the same issue. The collapse is not an instantaneous event but retroactively reveals additional facts on the dynamics of the particle before approaching a measured detection point. A simple example is the decay of a two-particle s state. If the position of the detection point is characterized by a definite particle quantum number, (e.g., a spin component in a Stern-Gerlach experiment), the particle must have left the emitter with that quantum number so that in the case of decays of two-particle bound states the other constituent must have obtained a corresponding quantum number because of conservation laws at the emitter. The apparatus as a whole only provides correlations between macroscopically distinguishable detection points and quantum numbers of the quantum system under consideration.

**Result and conclusion:** Absence of the nonlocality problem in Bohms version of the EPR arrangement (and in corresponding EPR-Bell coincidence experiments) favours the approach described. The problematic assumption of superluminal correlations between the two detections of such experiments as necessary in Copenhagen-like interpretations - is no point of discussion.

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* Electronic address: klauslewin@hotmail.de

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# All-versus-nothing proofs with $n$ qubits distributed between $m$ parties

Adán Cabello[1,*] and Pilar Moreno[1,†]

[1] *Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*

All-versus-nothing (AVN) proofs [1–6] show the conflict between Einstein, Podolsky, and Rosen's (EPR) elements of reality [7] and the perfect correlations of some quantum states. The name of "all-versus-nothing" [8] reflects a particular feature of these proofs: If one consider a set of perfect correlations and asumes EPR elements of reality, then a subset of these correlations leads to a conclusion that is opposite of the one obtained from the complementary subset of correlations.

The perfect correlations among single qubit measurements required for AVN proofs are given by the $2^n$ stabilizer operators of an $n$-qubit graph state. The possibility of experimentally preparing new classes of graph states [9–11] naturally leads to the following problem: Does a distribution of an $n$-qubit graph state between $m$ parties allow an AVN proof? This problem has been solved for $m = 2$ [12]. Here we describe a method to decide whether a given $n$-qubit $m$-particle graph state allows an $m$-partite AVN proof specific for this state (i.e., which cannot be obtained using a graph state with fewer qubits) [13]. This method requires that two observables of each qubit are EPR elements of reality. This forces a series of constraints that are only satisfied by a reduced group of the graph state's stabilizer operators. We detail these requirements and apply them to decide whether some $n$-qubit $m$-particle graph states recently prepared in the laboratory [9–11] allow $m$-partite AVN proofs.

We also address the following problem: Given an $n$-qubit graph state, what is the minimum number $m$ of parties that allows a specific $m$-partite AVN proof? The solution of this problem enables us to obtain all inequivalent distributions allowing AVN proofs with $n < 9$ qubits and an arbitrary number $m$ of parties [13].

These results provide the tools to help experimentalists to design tests of new AVN proofs and new Bell inequalities based on these proofs [14].

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* adan@us.es
† mpmoreno@us.es

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# Semiclassical analysis for quantum systems involving two bosonic fields via a generalized Wigner function

José Luis Romero[1] and Andrei B. Klimov[1]
[1]*Departamento de Física, Universidad de Guadalajara, México.*

The phase space methods is a powerful tool for studying quantum systems. The language of quasidistribution functions was proved to be useful not only for representation of quantum states in the corresponding classical phase space but also provides a way to construct in the semiclassical limit a specific perturbation theory, allowing to establish a correspondence quantum and classical worlds on the dynamical level. The expansion parameter depends on the system's symmetry and is usually inversely related to the number of excitations stored in the system. The semiclassical expansion in the evolution equations is especially advantageous for the Wigner function, related to the symmetric ordering of operators from the dynamic algebra. The main idea consists in keeping only the first-order derivatives and neglecting of higher-order terms in the exact evolution equation for the Wigner function. This can usually be done when the semiclassical parameter is multiplied the higher derivatives. Such truncated evolution equation describes quite well sufficiently long-time dynamics for essentially nonlinear quantum systems.

Typical quantum optical systems are usually described in terms of either the flat $q$-$p$ space Wigner function or the $SU(2)$ Wigner function. In the first case the semiclassical parameter is the inverse number of excitations in each interacting mode, so that the naive semiclassical dynamics, in general, does not describe well evolution of a system when one of the modes contains a small amount of energy. For instance, the two-photon parametric down-conversion [1]:

$$H = \chi \left( a^2 b^\dagger + b a^{\dagger 2} \right),$$

being the initial state $|0\rangle_a |\beta\rangle_b$ and $|\beta|^2 \gg 1$. Another example is given in [2]:

$$H = \chi \left( a^\dagger a^\dagger a a + b^\dagger b^\dagger b b \right) + g a^\dagger a b^\dagger b,$$

when the initial state is again given by $|0\rangle_a |\beta\rangle_b$ and $|\beta|^2 \gg 1$.

In the case of the Wigner function defined on the two-dimensional sphere $\mathcal{S}_2(\theta, \phi)$ (spin-like systems) the semiclassical parameter is the inverse dimension of representation. This, in principle, allows to describe evolution of coupled modes even when one of the mode is not sufficiently excited. Thus, in some sense the perturbation theory is more effective in this case. The problem arises if the initial state does not belong to a single representation space, but distributed among several subspaces. Since the Wigner function is a "semiclassical" symbol of the density matrix, which contains all non-diagonal (on the $SU(2)$ irreducible subspaces) matrix elements, the Stratonovich-Weyl correspondence [3] is not faithfull for description of evolution of observables which cannot be represented as elements from enveloping $su(2)$ algebra (for instance, one of the interacting fields). The situation becomes even more complicated when the system's Hamiltonian contains terms which produces transitions between $SU(2)$ invariant subspaces (for instance external field).

Recently, a modified $SU(2)$ Wigner function has been introduced [4], which takes into account simultaneously all the irreducible representations of the $SU(2)$ group over which the initial state of a quantum system is expanded. The Wigner function [4] is defined in a 3 dimensional meta phase-space and depends on a single discrete index, which is related to the dimension of involved representations.

We apply this Wigner function to describe evolution of quantum systems wich cannot be treated semiclassically in the standard approaches.

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# Non-orthogonal mutually unbiased bases and quantum tomography

Isabel Sainz,[1] Andrei B. Klimov,[1,*] and Luis Roa[2]

[1]*Departamento de Física, Universidad de Guadalajara, México.*
[2]*Center of Quantum Optics and Quantum Information, Center for Optics and Photonics, Departamento de Física, Universidad de Concepción, Chile.*

The complementarity between two observables, $A$ and $B$, means that if a state of a quantum system is measured in the basis of eigenstates corresponding to observable $A$, a subsequent measurement, in the basis of eigenstates of observable $B$, produces no new information about the initial quantum state. In this sense, in a $d$- dimensional Hilbert space, two different bases $\{|a;A\rangle\}$ and $\{|b;B\rangle\}$ are unbiased, and the relation $|\langle a;A|\, b;B\rangle| = 1/\sqrt{d}$ is held for every $|a;A\rangle$ and $|b;B\rangle$.

If the dimension of the Hilbert space is a power of a prime number ($p$), it is always possible to construct $p+1$ orthogonal mutually unbiased bases (MUB) [1]. In this case, the $p(p+1)$ mutually unbiased operators $P_n^s = |\psi_n^s\rangle\langle\psi_n^s|$ define a complete set of projection measurements that decomposes the identity, and the measured probabilities $p_{sn} = Tr(P_n^s\hat{\rho})$ completely determine the density operator ($\hat{\rho}$) of the system, which allows to develop an optimal quantum tomographic procedure [2].

However, when only a limited access to the full state space is granted one should apply quantum tomography with non-orthogonal bases [3].

In this work we consider a linearly independent and non-orthogonal set of normalized states in a $p$-dimensional Hilbert space, $\{|\psi_n^0\rangle, n=0,...,p-1\}$, such that the scalar product between any two different states of the set is a real constant $\lambda$.

The corresponding (normalized) bi-orthogonal basis $\{|\phi_n^0\rangle, n=0,\ldots,p-1\}$, where

$$\langle\phi_m^0|\psi_n^0\rangle = \frac{\delta_{nm}}{\sqrt{\mu}}, \quad \mu = \frac{1+(p-2)}{(1-\lambda)[1+(p-1)\lambda]},$$

can be expressed in terms of the original basis, and the scalar product between different elements of this basis is a constant $\eta \neq \lambda$.

We find a non-unitary cyclic operator $Z$, such that the elements of basis $\{|\psi_n^0\rangle\}$ are eigenstates of this operator, while the bi-orthogonal basis is formed by eigenstates of its Hermitian conjugate [4]. In close analogy with the orthogonal case, we can introduce a unitary operator $X$ that forms a dual pair with operator $Z$, i.e. $ZX = \omega XZ$. We then find another $p-1$ bases whose elements correspond to the eigenstates of the monomials $Z^sX$, $s=1,\ldots,p$. And their corresponding sets of bi-orthogonal bases are also constructed.

The set of bases $\{|\psi_n^s\rangle\}$ are mutually unbiased with their corresponding bi-orthogonal counterparts $\{|\phi_m^t\rangle\}$,

$$\left|\langle\phi_m^t|\psi_n^s\rangle\right|^2 = \frac{\delta_{st}\delta_{nm}}{\mu} + \frac{(1-\delta_{st})}{\mu p}, \qquad (1)$$

for all $s,t=0,\ldots,p-1$ and $n,m=0,\ldots p-1$, [5].

The unbiasedness relation (1) can be used for an optimal reconstruction of a density matrix $\hat{\rho}$. The main idea consists in expanding the density matrix on the projectors $|\phi_n^s\rangle\langle\phi_n^s|$, while the measurements are accomplished in the bases $\{|\psi_n^s\rangle\}$. However, the set of projectors $\{|\phi_n^s\rangle\langle\phi_n^s|\}$ and $\{|\psi_n^s\rangle\langle\psi_n^s|\}$ for $n=0,\ldots p-1, s=0,\ldots p-1$ do not decompose the identity, and thus do not satisfy the condition to form a positive operator valued measure. To correct this problem we introduce the (orthonormal) basis of eigenstates of the unitary operator $X$ , $\{|\psi_n^x\rangle\}$, where $n=0,\ldots,p-1$.

And the density matrix can be expanded as a sum of projectors on $p$ non-orthogonal bases and the orthogonal basis as,

$$\begin{aligned}\hat{\rho} \;=\; & \mu\sum_{s=0}^{p-1}\sum_{n=0}^{p-1} p_{sn}\,|\phi_n^s\rangle\langle\phi_n^s| \\ & + \frac{1-\lambda}{1+(p-1)\lambda}\,(p_{x0}-1)\,|\psi_0^x\rangle\langle\psi_0^x| \\ & + \sum_{n=1}^{p-1}\left(p_{xn} - p_{x0}p\frac{\lambda}{1-\lambda} - 1\right)|\psi_n^x\rangle\langle\psi_n^x|\,,\end{aligned} \qquad (2)$$

where the expansion coefficients are linear combinations of the probabilities measured in the corresponding bases $\{|\psi_n^s\rangle\}$, $s=x,0,\ldots,p-1$. We find that, as in the orthonormal case, each measurement determines a single element of the density matrix, providing in this sense an optimal reconstruction scheme [5]. Finally, we analyze the limits $\lambda\to 0$ and $\lambda\to 1$.

* Electronic address: klimov@cencar.udg.mx

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## Bell Inequalities from Multilinear Contractions

Alejo Salles,[1] Daniel Cavalcanti,[2] Antonio Acín,[2,3] David Pérez-García,[4] and Michael M. Wolf[1]

[1]*Niels Bohr Institute, Blegdamsvej 17, 2100 Copenhagen, Denmark*
[2]*ICFO—Institut de Ciències Fotòniques, E-08860 Castelldefels, Barcelona, Spain*
[3]*ICREA—Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, 08010, Barcelona, Spain*
[4]*Departamento de Análisis Matemtico & IMI, Universidad Complutense de Madrid, 28040 Madrid, Spain*

We provide a framework for Bell inequalities which is based on multilinear contractions. The derivation of the inequalities allows for an intuitive geometric depiction and their violation within quantum mechanics can be seen as a direct consequence of non-vanishing commutators. The approach is motivated by generalizing recent work on non-linear inequalities which was based on the moduli of complex numbers, quaternions and octonions. We extend results on Peres conjecture about the validity of Bell inequalities for quantum states with positive partial transposes. Moreover, we show the possibility of obtaining unbounded quantum violations albeit we also prove that quantum mechanics can only violate the derived inequalities if three or more parties are involved.

# Quantum Degrees of Polarization and Stokes Measurements

Jonas Söderholm,[1,*] Gunnar Björk,[2] Luis L. Sánchez-Soto,[3] and Andrei B. Klimov[4]

[1]*Max Planck Institute for the Science of Light, Günther-Scharowsky-Str. 1, 91058 Erlangen, Germany*
[2]*School of Communication and Information Technology, Royal Institute of Technology (KTH), Electrum 229, SE-164 40 Kista, Sweden*
[3]*Departamento de Óptica, Facultad de Física, Universidad Complutense, 28040 Madrid, Spain*
[4]*Departamento de Física, Universidad de Guadalajara, 44420 Guadalajara, Jalisco, Mexico*

Despite that the operators corresponding to the Stokes parameters were identified long ago, it is still not clear how to quantify polarization in quantum optics. Denoting the annihilation operators of the modes with horizontal and vertical polarization as $\hat{a}$ and $\hat{b}$, respectively, the Stokes operators can be expressed as

$$\hat{S}_0 = \hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}\,, \qquad \hat{S}_x = \hat{a}\hat{b}^\dagger + \hat{a}^\dagger\hat{b}\,, \tag{1}$$
$$\hat{S}_y = i(\hat{a}\hat{b}^\dagger - \hat{a}^\dagger\hat{b})\,, \qquad \hat{S}_z = \hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}\,,$$

Since the Stokes parameters are given by the average values of these operators, the classical degree of polarization becomes

$$\mathbb{P}_{\rm S} = \frac{\sqrt{\langle\hat{S}_x\rangle^2 + \langle\hat{S}_y\rangle^2 + \langle\hat{S}_z\rangle^2}}{\langle\hat{S}_0\rangle}\,. \tag{2}$$

However, there exist quantum states that obviously carry some information about polarization, but have a vanishing degree of polarization according to this definition [1].

We note that apart from a factor two, the operators coincide with the Schwinger realization of the angular momentum operators. Hence, the Stokes operators are the generators of the SU(2) transformations induced by linear optics.

Now, assume that the polarization properties can be fully characterized by Stokes' original setup employing linear optical devices and measurements corresponding to the Stokes operators. Since the total photon number is invariant under such transformations, and the corresponding operator commutes with all the other Stokes operators $[\hat{S}_0, \hat{S}_\alpha] = 0$, $\alpha \in \{x, y, z\}$, quantum mechanics allows us to determine the total number of photons in any such Stokes measurement. The polarization properties can therefore be decomposed into excitation manifolds corresponding to different number of photons.

It is now natural to consider any state that under SU(2) transformations leaves all measurement statistics of all the Stokes operators invariant to be unpolarized. As a quantum non-demolition measurement of the total photon number before the measurement setup would not affect the outcomes, it is then clear that any state that becomes SU(2) invariant by a projection onto the excitation manifolds must be unpolarized.

The SU(2)-invariant states are known to be those of the form [2, 3]

$$\hat{\sigma} = \bigoplus_{N=0}^{\infty} p_N\,\hat{\sigma}_N, \tag{3}$$

where $p_N$ is the probability that the state is found to contain $N$ photons, and the only SU(2)-invariant $N$-photon state is

$$\hat{\sigma}_N = \frac{\hat{\mathbb{1}}_N}{N+1}\,. \tag{4}$$

Here, $\hat{\mathbb{1}}_N$ denotes the projector onto the $N$-photon excitation manifold.

The general decomposition into different excitation manifolds and the simple form of the SU(2)-invariant states make it now fairly easy to calculate different measures of distinguishability between any given state and the set of unpolarized states. For this purpose, one can think of several generalizations of the overlap of two pure states. Two attractive choices are the trace and Chernoff distances [4, 5], which are directly related to error probabilities in quantum hypothesis testing [6, 7]. Due to the fact that all unpolarized states have the same projection $\hat{\sigma}_N$ in any given manifold $N$, it is straightforward to obtain analytical expressions for the polarization degrees based on different distance measures [5] by averaging over the manifolds. This allows us to compare them with other proposed degrees of polarization [8].

* Electronic address: jonas.soederholm@mpl.mpg.de

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# Controllable decoherence of single photon polarization qubits

A. Shaham[1] and H.S. Eisenberg[1,*]

[1] *Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel*

Light depolarization is a fundamental phenomenon. Methods for total depolarization such as the Lyot depolarizer[1] were invented more than eighty years ago. From the quantum information point of view, where the information is encoded in the polarization of a single photon, depolarization has the role of decoherence, i.e. noise on the quantum information. The possibility to control and characterize the properties of this noise will enable the experimental study of its effects on various quantum protocols. In particular, realization of noisy quantum channels enables the test of Decoherence Free Subspaces[2], controlling the mixture level of entangled states[3] and the experimental testing of quantum key distribution protocols.

In this Work, we studied theoretically and experimentally the effects of a depolarizer composed of two birefringent crystals and wave-plates, on photons of short coherence times. Every crystal entangles the polarization degree of freedom with two temporal modes. The coupling depends on the relative angles between the symmetry axes of the crystals, the wave-plates and on the initial polarization state. Depolarization occurs because the photon detection is insensitive to the temporal degrees of freedom. The depolarization effect of the crystals and wave-plates can be calculated for any configuration.

We investigated a depolarizer composed of two equal crystals. The relative angle $\theta$ between their symmetry axes can be changed. When $\theta = 0°$, the symmetry axes of the two crystals are perpendicular and no depolarization occurs. When $\theta = 90°$ the channel is completely dephasing - the same as for a single crystal. Single photons that were generated by spontaneous parametric down-conversion of 390nm pulses, were transmitted through the depolarizer. The final polarization state was characterized by a quantum state tomography procedure, as a function of the angle $\theta$. Measurements were performed for horizontal, diagonal and circular initial polarizations, which are mutually unbiased states. The coherence time was determined by 5nm bandpass filters before the detectors. To eliminate a birefringent phase from the crystal rotation, the polarization was rotated by wave-plates before and after the first crystal instead.

Experimental results and their theoretical predictions are shown in Figure 1(a). The depolarization is mostly anisotropic for the three initial polarizations. One interesting angle is at $\theta = 54.7°$, where a completely symmetric depolarizer was obtained. Theoretical calculations show that for this angle, every initially polarized light, will lose 2/3 of its polarization amount after passing through the depolarizer. Our experimental results are in very good agreement with these calculations, which are also valid for classical light.

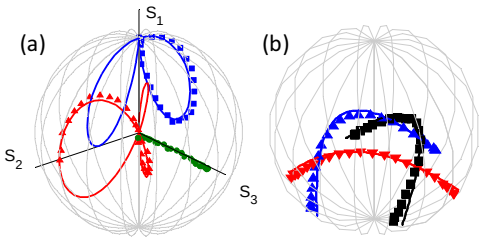


FIG. 1: Depolarization results in the Poincaré sphere representation. (a) Experimentally measured final states for horizontal (blue squares), diagonal (red triangles), and circular(green circles) inputs initial states, after depolarization with two equal crystals. The equivalent angle between the crystals is scanned between $0° > \theta > 90°$. Theoretical curves are also included. (b) Experimentally measured final states for three mutually unbiased states with initial $S_1 = -\sqrt{1/3}$ value . A quarter-wave plate that is placed between the two perpendicular fixed crystal, and rotated between 0° and 90°.

We have also studied the depolarizing configuration of two perpendicular fixed crystals and a rotatable quarter wave-plate in between them. If the two crystals are equal in length, there exist three mutually unbiased bases that always experience an equal depolarization. For these three bases, the depolarization ranges continuously between zero and 60% of polarization loss. Experimental and theoretical results for such bases, are shown in Figure 1(b). Another configuration that was studied included one crystal that is twice as long as the other, and a rotatable quarter wave-plate between them. This configuration results also with equal depolarization for the same mutually unbiased bases, this time with heavy polarization loss that ranges between 40% and 100%.

It is possible to combine the three studied schemes to achieve any arbitrary depolarizing channel. Such channels open new possibilities for the study of decoherence of quantum information systems.

* Electronic address: hagai.eisenberg@huji.ac.il

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# MEASURE FOR THE NON-MARKOVIANITY OF QUANTUM PROCESSES

Elsi-Mari Laine,[1,*] Jyrki Piilo,[1] and Heinz-Peter Breuer[2]

[1] *Turku Center for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 University of Turku, Finland*

[2] *Physikalisches Institut, Universität Freiburg, Hermann-Herder-Strasse 3, D-79104 Freiburg, Germany*

A Markov process in the evolution of an open quantum system gives rise to a quantum dynamical semigroup, for which the most general representation of the dynamics can be written in the Lindblad form. There exists complex systems for which the relatively simple description of the open system dynamics given by a Markovian master equation fails to give a comprehensive picture of the dynamics. Thus in many realistic physical systems the Markovian approximation of the dynamics gives an overly simplified picture of the open system evolution and a more rigorous description of the dynamics is required.

To give insight into the nature of non-Markovian effects many analytical methods and numerical simulation techniques have been developed in recent years. Non-Markovianity manifests itself in the different approaches in a variety of ways and there exists no recipe for comparing the degree of non-Markovianity between the different approaches. In order to give a general quantity determining the degree of non-Markovian behavior in the open system dynamics, one has to rigorously define what makes a dynamical map non-Markovian.

We introduce a general measure for the degree of non-Markovian behavior in an open quantum system by quantifying the information flow from the environment to the open system [1, 2]. The change in the distinguishability of states of the open system can be interpreted as information flow between the system and the environment. Thus the measure for non-Markovianity can be constructed from the concept of trace distance, which quantifies the distinguishability of quantum states. If the distinguishability is always decreasing, then the system is Markovian. Increase in the distinguishability at certain times indicates information flow from the environment to the system and therefore non-Markovian behavior.

This criterion for non-Markovianity does not require knowledge about the details of the environment or the system-environment interaction. Instead, tomographic measurements of a system can quantify the extent to which a system exhibits non-Markovian behavior.

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* Electronic address: emelai@utu.fi

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# Effects of Decoherence in the One-Way Model of Quantum Computation: The role of classical and quantum correlations

Rafael Chaves[1,*] and Fernando de Melo[2]

[1] *ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
[2] *Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Leuven, België*

Since the advent of quantum computation, entanglement, a clear-cut quantum mechanical feature, is commonly believed the *key resource* behind it. In a pure state quantum computation certainly some entanglement is necessary if the quantum protocol is not to be simulated efficiently in a classical computer [1]. However, entanglement is only necessary and not a sufficient condition for a exponential quantum speed-up. The so called Clifford-group operations may produce highly entangled states, however, can be efficiently simulated classically [1, 2].

In parallel to that, striking results in mixed state quantum computation continue to point, however, in another direction. A model for mixed state quantum computation introduced in [3], in which the input state consists of a single qubit in a pure state and all the others in an uniform incoherent sum of classical alternatives – and therefore not entangled–, offers an exponential speed-up to series of problems that are believed intractable by classical computers. Also room temperature NMR implementations of quantum information tasks, which employ a rather noisy state where entanglement is known not to be present, seem to still present gain over classical computations. Nevertheless, in these cases, generation of entanglement during the computation itself cannot be ruled out, and a definitive statement remain hitherto missing.

In this work [4], we explore still another facet of the entanglement role on quantum computations: Does higher entanglement always empower better computations? Or in more practical terms: should one always try to minimize the influence of the environment over the entanglement such as to maximize the "quality"(fidelity) of a computation?

To answer in the *negative* to these questions, we derive an expression for the fidelity of *any* computation within the one-way model [5] when the resource state undergoes various types of decoherence. In this model, local (projective) measurements on highly entangled graph states are responsible for input preparation, the required computation and final read-out. No entanglement is created during the computation. We have thus a clear distinction between the entanglement creation and its use as a resource. As a example of the developed formalism we can see that even for the simplest one-way protocol, a remote state preparation involving two entangled qubits as the resource, entanglement is neither *sufficient* nor *necessary* signature of higher quality quantum computation (see Figure 1). The computation fidelity can be higher when the entanglement is more fragile against disturbances. This reasoning can be extended to quantum discord, a recently proposed measure of quantum correlations which goes beyond entanglement.

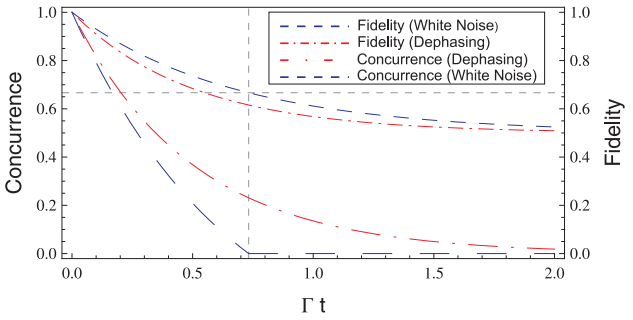


FIG. 1: **Entanglement decay vs. computation fidelity**. A less entangled state may lead to a higher computation fidelity.

Within our formalism we also show that the Deutsch-Jozsa [6] algorithm, which in a circuit model of quantum computation usually generate entangled states, within the one-way framework requires *no entanglement* for its execution.

Our results imply that entanglement cannot be the sole signature of efficient mixed state quantum computations, and it is not even necessary for protocols which present gain over their classical counterparts – as the one-way version of the DJ algorithm. The mere use of a quantum logic in a mixed state scenario, seems to entail considerable gain. This can be of great importance for biological systems, which, despite of being strongly influenced by the surroundings, are mesoscopic systems that may profit from their quantum nature.

* Electronic address: `rafael.chaves@icfo.es`

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# Programmable discrimination for mixed states

Ramon Muñoz-Tapia,[1,*] Gael Sentís,[1] John Calsamiglia,[1] and Emilio Bagan[1]

[1] *Física Teòrica:Informació i Fenòmens Quàntics, Facultat de Ciències, Edifici Cn, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona) Spain*

Discrimination between given hypothesis is one the most basic tasks in our every day lives. In the quantum realm it amounts to discriminate between quantum states. This task encompasses a plethora of non-trivial theoretical and experimental implications. In the usual setting the a priori states are known, i.e. the classical information characterising the possible states is known and the discrimination protocol is tailored for this specific information. In spit of its simplicity, only very recently closed expressions for the asymptotic error probability has been obtained (see [1] and references therein), the celebrated Chernoff bound from which metric distances and states densities [2] can be derived.

In this talk [3], we will consider a more general scenario. Very much in the spirit of universal computers, it is interesting to consider programmable devices able to discriminate between any unknown states [4]. In these, the information is included in a quantum way. That is, the device include program ports loaded with the states to be discriminated, The observer has no other information rather than the physical systems. These ports act as analogic programs of the device. The state one wishes to identify enters the data port. In this spirit, we can also view these devices as learning machines [5]. The device instructed about the nature of the states from the program ports and uses this knowledge to identify the state entering the data port. Increasing the number of copies of states at the program and data ports, of course, increases the chances of correct identification. The error rates as a function of the number of states is one of the most relevant parameters assessing the performance of the device. We compute the unambiguous and minimum error probabilities. In the former the device is programmed to give an error free answer. This is only possible stochastically, i.e, unless the states are orthogonal, the device will give some of the times an inconclusive answer. In the latter, the device is forced to give a definite answer, which may be wrong some of the times. An optimal device is one which minimises the inconclusive or the error probability. We will consider two states discrimination and focus on qubit states, although most of the results can be generalised to higher dimensional systems.

We first study the performance of such devices for pure states. We compute the unambiguous and minimum error probabilities for any number of pure qubit states at the input ports. When the program ports are loaded with an infinitely large number of copies of the states we recover the usual state discrimination problem, since it is clear that one can have the classical information of the states entering the program ports . On the other hand, when the number of copies at the data port is infinitely large, while the number of copies at the program ports are kept finite we recover the state comparison problem.

We extend the applicability of the device when the ports are fed with mixed states. In this case no unambiguous answers can be given, but the minimum error probability can be readily computed. The performance of the device for a given purity of the input states allows to quantify the degradation of the discrimination power in the presence of noise. The asymptotic cases are also studied and we obtain for the first time the non-trivial $O(1/n)$ term of the error probability.

We compare the performance of these optimal devices with a measure-and-discriminate machine (MAD), i.e., one that first estimates the states and then tries to identify the input states. We see analytically that when the number of copies at the program ports is asymptotically large, but not infinite, the error probabilities coincide with the average of the same probabilities for known states, but at subleading order, $O(1/n)$, the minimum error probability can be up to a factor of two worse for these MAD machines. This fact reveals that indeed the optimal device is able to correlate the states quantumly inside the machine and obtain a significant advantage to MAD machines.

We also analyse the case of completely unknown input states, i.e. when their purity is randomly distributed according to some reasonable priors. We consider hard-sphere, Bures and Chernoff priors. Such a device can be regarded as the (trully) universal programmable device.

* rmt@ifae.es

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# Limitations to the phase-sensitivity in practically realizable quantum systems*

Oleg Żero[1] and Gunnar Björk[1]

[1]*School of Information and Communication Technology, Royal Institute of Technology, KTH Electrum 229, 16440 Kista, Sweden C*

Quantum interferometry offers the ultimate precision for phase estimation, provided appropriate quantum states are used [1]. In practice, interferometer losses degrade the resolution and requires states that are more "robust" than the the states that gives the highest resolution in absence of loss. Losses therefore lead to a somewhat lower phase sensitivity [2]. Moreover, existing technological imperfections and barriers impose even more strict limitations on what kind of states can be generated, and how they can be measured. E.g., the highest photon number for which the super-sensitivity has been demonstrated so far is $N = 4$ [3].

In our study, we search for optimal quantum states for interferometry in the presence of loss. In addition we have had a strong focus on setups implementable with existing technology. Therefore, we have refrained from considering adaptive measurements and post-selective methods. The sensitivity limits are theoretically investigated, strictly assuming the use of only commonplace optical elements such as beam splitters and photon counters. Hence, we have excluded from our study both Positive Operator Valued Measurements (POVMs) and Quantum Non-Demolition schemes (QNDs) in contrast to the study in Ref. [2]. Consequently, both the input state optimization and the estimated limits in precision are computed using the *classical* Fisher information, and the phase inference is performed using *classical* Estimation Theory.

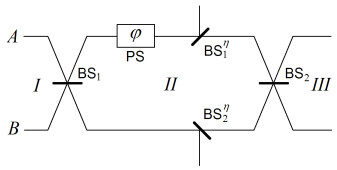


FIG. 1: The two-mode Mach-Zehnder interferometer, "equipped" with two fictitious beam splitters $BS_1^\eta$ and $BS_2^\eta$ with adjustable coupling strengths: $\eta_1$ and $\eta_2$ working in transmission to model the interferometer loss. Furthermore, we have assumed an *ideally-resolving* network of photon counters with unity efficiency, placed at $III$.

The considered setup is a Mach-Zehnder interferometer (Fig.: 1). Two primary cases of considering interferometer loss are investigated: an *overall* (symmetric) loss that is present in both arms: $(\eta_1 = \eta_2 \equiv \eta)$, and a loss-less interferometer but a *lossy phase-shift*: $(\eta_1 \equiv \eta; \eta_2 = 1)$.

Furthermore, the estimation of phase is based on the indistinguishability between the loss-originating outcomes is compared to the cases, in which we are granted the information about the specific occurrence of loss prior to the detection. In the latter case it is shown that, for specific states, we arrive at *over-estimation* of the phase, having up to 200% difference in the Fisher information.

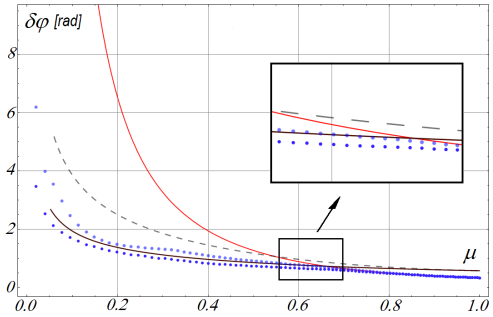


FIG. 2: The phase estimation uncertainty $\delta\varphi$ vs. *the efficiency* $\mu := \frac{2\eta}{1+\eta}$ shown for the loss-asymmetric, 3-photon case. Bright-blue dots: optimal state assumed real amplitudes; dark-blue dots: optimal state assumed complex amplitudes, red line: the N00N state, grey-dashed line: the coherent state with $BS_1$ set to 50:50 and black line: the coherent state with $BS_1$ optimally adjusted – SQL.

Finally, we show the importance of allowing the input amplitudes to be complex rather than real, before optimization. As shown on the Fig. 2, dismissing complex input amplitudes may lead to significant *under-estimation* of the phase-shift, even above the standard quantum limit (SQL).

In the next step, we are going to extend the analysis with the additional sources of error. We will, e.g., look at the consequences of having the detectors' network unable to resolve the events perfectly and also at the possible implications of a non-perfect preparation of the input state. The ultimate aim of this research will be to investigate the influence of various factors to the ultimate sensitivity limit of interferometers.

* Electronic address: gbjork@kth.se

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## How asymmetric and correlated errors affect the performance of quantum codes

Carlo Cafaro[1] and Stefano Mancini[1]
[1]*School of Science and Technology, Physics Division, University of Camerino, I-62032 Camerino, Italy*

It is known that in actual physical situations, qubits may interact with a common environment which unavoidably introduces correlations in the noise [1]. Furthermore, most of the realistic quantum computing devices may be characterized by asymmetric noise errors. Thus, it is a relevant issue describing and understanding the effect that asymmetric and correlated noise errors may have on the performances of error correction schemes.

Here we study the performance of common quantum stabilizer codes in the presence of asymmetric and correlated errors. Specifically, we consider the depolarizing noisy quantum memory channel and perform quantum error correction via the five and the CSS seven-qubit stabilizer codes. We characterize these codes by means of the entanglement fidelity as function of the error probability and the degree of memory. We show that their performances are lowered by the presence of correlations and we compute the error probability threshold values for codes eectiveness. Furthermore, we uncover that the asymmetry in the error probabilities does not affect the performance of the five- qubit code while it does affect the performance of the seven-qubit code which results less effective when considering correlated and symmetric depolarizing errors but more effective for correlated and asymmetric errors.

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## Transitions in the communication capacity of dissipative qubit channels,

D. Daems[1,*]

[1] *QuIC, Ecole Polytechnique, Université Libre de Bruxelles, 1050 Brussels, Belgium*

The information transmission is studied for quantum channels in which the noise includes dissipative effects, more specifically, non-unitality [1]. Noise is usually a nuisance but can sometimes be helpful. For these channels the communication capacity is shown to increase with the dissipative component of the noise and may exhibit transitions beyond which it increases faster. The optimal states are constructed analytically as well as the pertaining "phase" diagram.

* Electronic address: ddaems@ulb.ac.be

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## Necessary and sufficient condition for non-zero quantum discord

Borivoje Dakić,[1,*] Vlatko Vedral,[2,3,4] and Časlav Brukner[1,5]

[1] *Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria*
[2] *Centre for Quantum Technologies, National University of Singapore, Singapore*
[3] *Department of Physics, National University of Singapore, Singapore*
[4] *Clarendon Laboratory, University of Oxford, Oxford UK*
[5] *Institute of Quantum Optics and Quantum Information, Austrian Academy of Sciences, Boltzmanngasse 3, A-1090 Vienna, Austria*

Quantum discord characterizes "non-classicality" of correlations in quantum mechanics. It has been proposed as the key resource present in certain quantum communication tasks and quantum computational models without containing much entanglement. We obtain a necessary and sufficient condition for the existence of non-zero quantum discord for any dimensional bipartite states. This condition is easily experimentally implementable. Based on this, we propose a geometrical way of quantifying quantum discord. For two qubits this results in a closed form of expression for discord. We apply our results to the model of deterministic quantum computation with one qubit, showing that quantum discord is unlikely to be the reason behind its speedup.

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* Electronic address: borivoje.dakic@univie.ac.at

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# Searching the Optimal Quantum Circuits

Ming-Chung Tsai,[1] Wei-Chi Su,[2] Kuan-Peng Chen,[2] and Zheng-Yao Su[2,*]

[1] *Department of Physics, National Tsing-Hua University, Hsinchu, Taiwan, R.O.C.*
[2] *National Center for High-Performance Computing, Hsinchu, Taiwan, R.O.C.*

Mathematically, most of theoretical inquiries in quantum information can be cast into problems of path searching on the manifolds of unitary groups. In the quantum realm, the evolution of a dynamics is governed by unitary transformations. A classical problem in quantum computing is the design of quantum gates. For instance, what kinds of basic quantum gates are required such that the computation is universal, or what is the optimal bound of complexity to achieve the computational universality, such as the least number of nonlocal gates that are required? According to the theorem of computation universality, any unitary transformation can be decomposed to a set of CNOT gates and rotations of $SU(2)$. In this report, a method is proposed to acquire the optimal design of quantum gates using the lowest number of CNOT gates.

In order to portray the evolution *path* on the manifold of $SU(N)$, $2^{p-1} < N \leq 2^p$, the notion of the *Cartan decomposition* in its algebra $su(N)$ is essential. As described in [1], a Cartan decomposition $su(N) = \mathfrak{t} \oplus \mathfrak{p}$ is a composition of the subalgebra $\mathfrak{t}$ and the vector subspace $\mathfrak{p}$ satisfying the decomposition condition

$$[\mathfrak{t},\mathfrak{t}] \subset \mathfrak{t}, [\mathfrak{t},\mathfrak{p}] \subset \mathfrak{p}, [\mathfrak{p},\mathfrak{p}] \subset \mathfrak{t}, \text{ and } \mathrm{Tr}(\mathfrak{t}\mathfrak{p}) = 0. \quad (1)$$

As long as a Cartan decomposition $su(N) = \mathfrak{t} \oplus \mathfrak{p}$ is given, every group action $U \in SU(N)$ admits the factorization $U = Ke^{A}K'$ according to $KAK$-theorem [2], where $K$ and $K'$ are two group actions generated from the subalgebra $\mathfrak{t}$ and $A$ is a maximal abelian subalgebra in $\mathfrak{p}$. The two group actions $K$ and $K'$ can further undergo respective factorizations likewise and admits a Cartan decomposition. It thus allows a recursive process of decompositions until the subalgebra reduces to being simple, and a such recursive decomposition implies a revolutionary path on the manifold of the group. For example, a unitary action $U \in SU(8)$ can be factorized into the following form,

$$U = e^{ia_{[3]1}} e^{ia_{[2]1}} e^{ia_{[3]2}} e^{ia_{[1]1}} e^{ia_{[3]3}} e^{ia_{[2]2}} e^{ia_{[3]4}} e^{ia_{[0]1}} \times e^{ia_{[3]5}} e^{ia_{[2]3}} e^{ia_{[3]6}} e^{ia_{[1]2}} e^{ia_{[3]7}} e^{ia_{[2]4}} e^{ia_{[3]8}},$$

where $a_{[i]j}$ is a vector in the subspace $\mathcal{A}_{[i]}$ derived in the recursive process, $i = 0, \cdots, 3$ and $j = 1, 2, \cdots, 2^i$. In a sense, this decomposition provides a group-theoretic or even a geometric aspect for many factorizations well known in matrix analysis [1]. Mathematically, there have three types of the decompositions for unitary Lie algebras: the type AI yields the SVD (Singular Value Decomposition), the type AII leads to the symplectic decomposition in group actions and the type AIII to the cosine-sine (CS) decomposition.

A novel scheme which enables the systematic, exhaustive, and recursive generating of Cartan decompositions for $su(N)$, $2^{p-1} < N \leq 2^p$, of arbitrary dimensions is introduced in [3]. The success of the scheme is mainly attributed to the discovery of an algebraic structure, the *quotient algebra* that generally exists in Lie algebras. As explained in [3], every *Cartan subalgebra* $\mathcal{A}$, the maximal abelian subalgebra, in $su(N)$ can generate a quotient algebra, denoted as $\{\mathcal{Q}(\mathcal{A};q)\} = \{W_i, \hat{W}_i;\ i = 1, 2\cdots, q\}$, consisting of $q$ pairs of *conjugate pairs* satisfying the *conjugate partition*

$$[W_i, \mathcal{A}] \subset \hat{W}_i,\ [\hat{W}_i, \mathcal{A}] \subset W_i \text{ and } [W_i, \hat{W}_i] \subset \mathcal{A} \quad (2)$$

and *condition of closure*

$$[W_i, W_j] \subset \hat{W}_k,\ [W_i, \hat{W}_j] \subset W_k \text{ and } [\hat{W}_i, \hat{W}_j] \subset \hat{W}_k, \quad (3)$$

here $0 < i, j, k \leq q$. Thanks to the partition, the complete set of Cartan decompositions of $su(N)$. This yields all admissible factorizations of an arbitrary unitary transformation $U \in SU(N)$ according to different recursive Cartan decompositions.

To perform any group action in $SU(N)$, supposing $N = 2^p$, the raw complexity of quantum gates, counting the number of CNOT gates and ignoring all other local gates, is $O(p^2 4^p)$. With a straightforward application of the above scheme of Cartan decompositions, the complexity immediately reduces to $O(p4^p)$. Based on the same scheme and with more care on the gate reduction, the lowest bound $O(4^p)$ of the complexity of quantum gates is achieved. Furthermore, each factorization shares the same number of CNOT gates, the minimum number required to implement a unitary action. Through the scheme, we can construct the optimal quantum circuits precisely. The work will play a key role to design scalable quantum circuits and even help the realization of quantum computing hardwares.

* Electronic address: zsu@nchc.org.tw

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# Classifying Quantum Error Correction Codes

Kuan-Peng Chen,[1] Wei-Chi Su,[1] Ming-Chung Tsai,[2] and Zheng-Yao Su[1, 2, *]

[1] *National Center for High-Performance Computing, Hsinchu, Taiwan, R.O.C.*
[2] *Department of Physics, National Tsing-Hua University, Hsinchu, Taiwan, R.O.C.*

In realistic situations, quantum information are transmitted or manipulated in noisy environments. The information gets lost gradually due to the baneful interaction with the environment. To protect the fragile quantum states, error-correction codes are essential to safeguard the quantum data during the processes of quantum computation and communication. In this report, a systematic method based on the *group* structure of a unitary Lie algebra is proposed to exhaustively generate quantum codes, including the *additive* and *nonadditive* codes. Moreover, the generated quantum codes are classified into four kinds by the duality between states and operators, and the classical correspondences for some of these codes are rendered as well.

A single qubit state can suffer three types of errors respectively represented by the Pauli matrices: the bit error $\sigma_1 = |0\rangle\langle 1| + |1\rangle\langle 0|$, phase error $\sigma_3 = |0\rangle\langle 0| - |1\rangle\langle 1|$ and bit-phase error $\sigma_2 = -i|0\rangle\langle 1| + i|1\rangle\langle 0|$. For a $p$-qubit states, $p \geq 1$, a set of $N$ encountered errors $\mathcal{E} = \{E_0, E_1, \cdots, E_{N-1}\}$ is chosen from the set $G = \{I, \sigma_1, \sigma_2, \sigma_3\}^{\otimes p}$ comprising all tensor products of $p$ Pauli matrices, namely $E_{0\leq r<N} = \sigma_{i_1} \otimes \sigma_{i_2} \otimes \cdots \otimes \sigma_{i_p} \in G$ for the identity $I = \sigma_0$ and $0 \leq i_1, i_2, \cdots, i_p \leq p$.

By representing all the $2^{2p}$ generators (including the identity) of the Lie algebra $su(2^p)$ in terms of the spinors in $G$, the algebra $su(2^p)$ forms a group under the multiplication, namely $S_1 \cdot S_2 = S_3 \in su(2^p)$ for all $S_1, S_2 \in su(2^p)$. Let $\mathfrak{C} = \{S_i : \forall\, S_i, S_j \in G, [S_i, S_j] = S_i \cdot S_j - S_j \cdot S_i = 0 \text{ for } 0 \leq i, j < 2^p\}$ be a maximal abelian subalgebra of $su(2^p)$, which is called the *Cartan subalgebra* and spanned by the commuting spinors of $G$. Up to the sign factor, it is easy to check that the subalgebra $\mathfrak{C}$ containing in total $2^p$ generators is a subgroup of $su(2^p)$ under the same group operation. Thus as described in [1], the subalgebra $\mathfrak{C}$ can generate a partition, denoted as $\{\mathcal{P}(\mathfrak{C})\}$, in $su(2^p)$ consisting of $2^p$ subspaces $\{\mathcal{W}_i; i = 0, 1, \cdots, 2^p - 1 \text{ and } \mathcal{W}_0 = \mathfrak{C}\}$ satisfying the rule:

$$\forall\, S_1 \in \mathcal{W}_\zeta, S_2 \in \mathcal{W}_\eta,\ S_1 \cdot S_2 = S_3 \in \mathcal{W}_{\zeta+\eta}, \qquad (1)$$

here $\zeta, \eta$ being a $p$-digit binary string of the additive group $Z_2^p$. That is, the partition $\{\mathcal{P}(\mathfrak{C})\}$ composed of these $2^p$ subspaces is a structure isomorphic to $Z_2^p$.

To generate quantum codes against a given error set $\mathcal{E} = \{E_0, E_1, \cdots, E_{N-1}\} \subset G$, an algorithm is designed according to the group structure in the partition $\{\mathcal{P}(\mathfrak{C})\} = \{\mathcal{W}_\lambda : \forall\, \lambda \in Z_2^p\}$. Here the $N$ errors are distributed to $N$ different subspaces, namely $E_i \in \mathcal{W}_{\lambda_i}$ for $0 \leq i < N$, and $E_0 = I^{\otimes p}$. First, an initial state $|\ _0\rangle = \sum_{S\in\mathfrak{C}} S|00\cdots0\rangle$ is produced by applying all the spinors of the Cartan subalgebra $\mathfrak{C}$ to the $p$-qubit zero state. Then a set of $M$ *codeword spinors* $\mathcal{B} = \{S_0 = I^{\otimes p}, S_1, \cdots, S_{M-1}\} \subset G$, respectively chosen from the $M$ subspaces other than those including the errors, are applied to the initial state $|\ _0\rangle$ to generate the set of $M$ states $\{|\ _r\rangle : 0 \leq r < M\} = \{|\ _0\rangle = S_0|\ _0\rangle, |\ _1\rangle = S_1|\ _0\rangle, \cdots, |\ _{M-1}\rangle = S_{M-1}|\ _0\rangle\}$. This set forms a generating set of a code subspace $H_{code}$ with dimension $M$ against the set of errors $\mathcal{E}$. The subspace $H_{code}$ is as well denoted as $[[p, K]]$, which is a quantum code having the length $p$ and dimension $M$. Note that this code obey the so-called quantum Hamming bound $MN \leq 2^p$.

In brief, by creating an initial state $|\ _0\rangle$ and choosing an appropriate set of codeword spinors $\mathcal{B}$, a quantum code capable of correcting a given error set can be generated. Let $C = \{\alpha_r; r = 0, 1, 2, \cdots, k\}$ denote the set of strings of the initial state $|\ _0\rangle = \sum_{r=1}^{k} (-1)^{\epsilon_r} |\alpha_r\rangle$, here $\alpha_r \in Z_2^p$, $\epsilon_\alpha = 0$ or 1, and $0 \leq r \leq k < 2^p$. It is instructive to classify the generated quantum codes by the different options of $|\ _0\rangle$ and $\mathcal{B}$, as shown in the following table. In Table I, the quantum code of *type-I*, which is an additive code, corresponds to both $C$ and $\mathcal{B}$ being a subgroup of $Z_2^p$ and $\mathcal{P}(\mathfrak{C})$ respectively. The remaining codes are nonadditive. For the code of *type-II*, the $\mathcal{B}$ is not a subgroup but $C$ is. The set $\mathcal{B}$ is a subgroup yet $C$ is not for the code of *type-III*. Neither of $\mathcal{B}$ and $C$ is a subgroup in the last type of code. Apparently, the codes of types $I$ and $II$ have classical correspondences. The former refers to the linear code and the latter to the nonlinear one.

| Type | $\mathcal{B}$ | $C$ | |
|---|---|---|---|
| $I$ | g. | g. | additive |
| $II$ | n.g. | g. | nonadditive |
| $III$ | g. | n.g. | nonadditive |
| $IV$ | n.g. | n.g. | nonadditive |

TABLE I: The classification of quantum codes generated by the scheme in this report; here g. (n.g.) indicates that $\mathcal{B}$ and $C$ are a subgroup of Lie algebra $su(2^p)$ and the additive group $Z_2^p$ respectively.

* Electronic address: zsu@nchc.org.tw

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# Single NV center quantum phase gate

J.-H. Schönfeldt,[1,*] J. Twamley,[1] and S Rebić[1]

[1] *Centre for Quantum Computer Technology, Macquarie University, Sydney Australia*

Single photons are obvious candidates for the implementation of quantum information processing systems. They are robust against decoherence and single-qubit operations are easy to implement. Due to the very small interaction between photons the required two-qubit quantum phase gate (QPG) operation is very difficult to implement. It is possible to enhance the photon-photon interaction by making use of optical nonlinearities, eg. those generated during electromagnetically induced transparency (EIT) in an ensemble of atoms [1]. Current proposals make use of so called double-EIT in Rb atoms [2] which results in the two fields propagating with the same group velocity, leading to enhanced interaction. Single photon implementations of this scheme, however, typically have rather low maximum phase shifts and has a low QPG fidelity ($\sim 0.65$) [3]. The conditional phase shift (CPS) of the one field due to the other would allow, besides a deterministic QPG, the establishment of entanglement through measurement [4].

Another way to enhance photon-photon coupling is by exploiting the large optical nonlinearities achievable in cavity quantum electrodynamics (QED) where photons trapped in a high quality resonator interact strongly through their mutual coupling to a single intracavity atom [5, 6]. Rather than considering a gas cell of Rb atoms to attempt strong coupling we consider a perhaps more convenient (at least as far as scalability is concerned) condensed matter system. In the work we present we consider a cavity QED scheme where a single nitrogen-vacancy (NV) center coupled to two counter circulating high-Q whispering gallery modes (WGM) of a microresonator cavity provide the nonlinear interaction required to achieve a CPS. The NV center is confined in a diamond nanocrystal and is in the tripod energy level configuration. We will show that by utilizing quantum coherent processes we can achieve extremely high levels of nonlinear interaction and a significant CPS. Recent advances in toroidal and spherical microresonators have seen cavity QED systems with ultra-high quality factors [7, 8] with for example a stable $Q \approx 10^9$ obtained in silica microspheres. Experiments coupling nanocrystal NV centers and solid state cavity QED systems have recently been performed with silica microspheres [9] and microdisks [10]. These ultra-high quality factors allow for strong coupling between light and atomic systems, thus making a single atom approach feasible. Our scheme follows that of Rebić et al. (2005) [11] but instead of an ensemble of interacting centers our scheme only involves a single NV center.

Initial quantum trajectory calculations making use of weak coherent fields and including dephasing and decay in the NV center with values taken from literature yield promising results 1 with a CPS of $\sim \pi/2$ achieved in about a hundred nano seconds, well within the life time of the cavity.

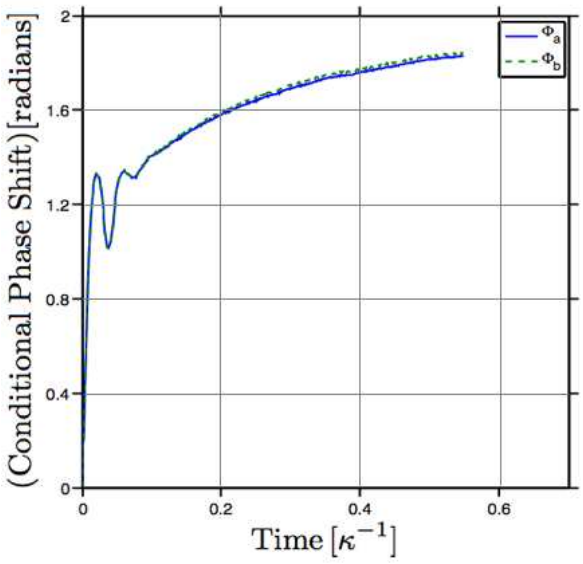


FIG. 1: The conditional phase shift of the probe $\Phi_P$ and trigger $\Phi_T$ fields. A CPS of $\pi/2$ was achieved well within the cavity lifetime $1/\kappa$.

* Electronic address: johann-heinrich.schonfeldt@mq.edu.au

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## Sequential measurement-based quantum computing with memories

Augusto J. Roncaglia,[1] Leandro Aolita,[1] Alessandro Ferraro,[1] and Antonio Acín[1,2]

[1] *ICFO-Institut de Ciencies Fotoniques, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain*
[2] *ICREA-Institució Catalana de Recerca i Estudis Avançats, Lluis Companys 23, 08010 Barcelona, Spain*

We introduce a scheme for sequential one-way quantum computation, where static systems with long quantum coherence interact with "flying" systems that may possess very short coherence times. Both the generation of the cluster state needed for the computation and its consumption by measurements are carried out simultaneously. As a consequence, effective clusters of one spatial dimension fewer than the standard approach are sufficient for computation. In particular, universal computation requires only a one-dimensional array of static systems. The scheme applies to discrete-variable systems of any dimensions as well as to continuous-variable ones, and both are treated equivalently within the unified framework of local complementation of graphs. The procedure is intrinsically well-suited for implementations with atom-photon interfaces.

# Observation of tunable Popescu-Rohrlich correlations through post-selection of a Gaussian state

D. S. Tasca,[1,*] S. P. Walborn,[1,†] F. Toscano,[1,‡] and P. H. Souto Ribeiro[1,§]

[1] *Instituto de Física, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, RJ 21941-972, Brazil*

In 1965, John Bell showed that quantum mechanics exhibits non-local correlations which can be stronger than any classical correlation [1]. Classical correlations obey the CHSH inequality $S \leq 2$, where $S$ is known as the Bell parameter [2]. For certain quantum states, $S$ can exceed 2, while the maximum value obtainable is $2\sqrt{2}$, and is known as Tsirelson's bound [3]. Popescu and Rohrlich (PR) have proposed and investigated correlations that could lead to the maximum possible value of $S = 4$ and therefore violate Tsirelson's bound. In their work, they have demonstrated that these PR correlations, produced by what are now known as PR boxes, are compatible with the causality imposed by special relativity, even though they are stronger than any known classical or quantum correlation [4].

The PR box relates Alice and Bob's respective input bits $A$ and $B$ with their respective output bits $a$ and $b$ according to the relation $a \oplus b = A \cdot B$, where $\oplus$ stands for addition modulo 2. This definition guarantees that $a = b$ whenever either $A$ or $B$ is zero, and that $a \neq b$ whenever $A = B = 1$. Also required for the definition of the PR box are that $(i)$ Alice and Bob's marginal distributions are completely random: $P_A(a) = P_B(b) = 1/2$ for all $A, B, a, b$; and $(ii)$ satisfy relativistic causality, i.e $P_{A,B}(a) = P_{A,B'}(a)$ for all $A, B, B', a$, etc [4]. These basic restrictions guarantee that it is impossible to communicate directly via the PR box.

We show that non-local Popescu-Rohrlich correlations can be observed in the post-selected results of binned position measurements on a two-party gaussian state [5]. Our experiment is based on the spatial correlations of entangled photons and lens systems. We obtain a maximum violation of the CHSH inequality of 3.42, which corresponds to the implementation of a non-local AND gate with success probability of 0.93 [6]. These results do not conflict with quantum mechanics due to the post-selection required, and open up the possibility of experimental investigation of fundamental aspects of Popescu-Rohrlich non-locality with a reliable and simple experimental setup. Our experiment demonstrates high fidelity PR correlations in variable post-selected dichotomic measurements, that are performed on a bipartite continuous-variable gaussian state [5]. We use twin pairs of photons produced in parametric down-conversion and perform measurements on the transverse spatial variables of the photons. This system is particularly interesting for the study of non-local correlations. On the one hand, it presents genuine quantum correlations between the transverse variables of the photon pairs [7–9], demostrated through the violation of continuous variable separability and ERP-like correlation criteria [10–12]. On the other hand, because the quantum state of the photon pair can be approximately described by a positive Wigner function, the non-local correlations can only be observed through the measurement of observables with a particular Weyl-Wigner representation [13]. In the experiment reported here, we measure dichotomic spatial observables that are not of this type. However, we show that the PR correlations appear through a spatial filtering process. Moreover, we show that the strength of the PR correlations can be readily tuned. This manipulation of the correlations is a clear demonstration of how a detection loophole can be exploited to obtain a false violation in a Bell's inequality experiment.

* tasca@if.ufrj.br
† swalborn@if.ufrj.br
‡ toscano@if.ufrj.br
§ phsr@if.ufrj.br

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# Multipartite entanglement and phase transitions in circuit QED

Jonas Larson[1,*]

[1] *Department of Physics, Stockholm University, AlbaNova University Center, 106 91 Stockholm, Sweden*

For more than a dacade, cavity QED have provided us with experiments studying fundamental aspects of quantum mechanics, such as entanglement and the appearance of a classical world [1]. These typically consider single atoms traversing a high-Q cavity, and operate in a parameter regime where the rotating wave approximation is applicable. In recent years, and alternative to cavity QED circuit QED, has emerged. Josephson junction qubits are coherently coupled to single modes of optical resonators. Contrary to cavity QED, these qubits are not flying and it is possible to couple and control several of them simultaneously. Moreover, the effective coupling is in the regime where the validity of the rotating wave approximation becomes questionable.

I show how multipartite entangled ground states in circuit QED setups can be achieved by letting the cavity field act as a quantum bus rendering a long range coupling between the qubits [2]. The effective model is a particular example of the Lipkin-Meshkov-Glick model, which posessess a second-order quantum phase transition between a ferromagnetic and a broken-symmetry phase.

* Electronic address: jolarson@fysik.su.se

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# Rare earth doped crystals for quantum memories and slow light

Atia Amari, Felix Beaudoin, Stefan Kröll, Huang Maomao, Lin Nan, Lars Rippe, Mahmood Sabooni and Andreas Walther[1] and Mikael Afzelius, Imam Usmani, Björn Lauritzen, Christoph Simon, Nicolas Sangouard, Jirí Minár, Hugues de Riedmatten, Nicolas Gisin[2]

[1]*Dept of Physics, Lund University, Box 118, S-221 00 Lund, Sweden*
[2]*Group of Applied Physics, University of Geneva, CH-1211 Geneva 4, Switzerland*

To efficiently use the additional resources like superposition and entanglement that the quantum world offers it is necessary to develop new devices. At low temperatures quantum superposition states in rare-earth-ion-doped inorganic crystals last for seconds [1], which offer unique opportunities for the construction of useful quantum devices based on these materials. Our quantum memory work is in particular focussed on the development of quantum memories for quantum repeaters for quantum key distribution across long distances.

Our recent developments in the construction of rare earth based quantum hardware will be described including development of quantum memories with 35% efficiency [2], spin state storage [3] with storage times >100 $\mu$s [4] and storage and recall of weak coherent pulses (∼0.1 photons/pulse) with 25% efficiency [5]. Arbitrary spectral structures for processing input pulses can also be created in rare-earth-ion doped crystals [6], e.g. polarization sensitive semi-permanent (>10 seconds) slow light structures [7] with high delay bandwidth products [8].

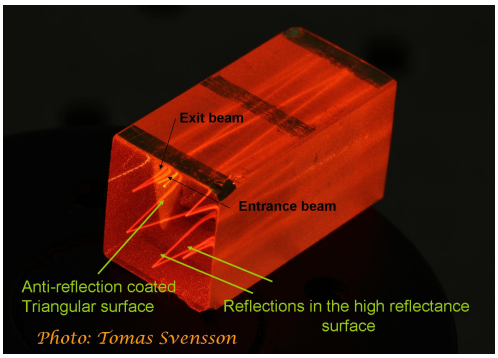


FIG. 1: A Pr doped yttrium silicate crystal with reflection coated and curved end faces forming a cavity where the incoming light beam can be reflected and undergo multiple passes. The beam is coupled into and out of the crystal through a triangularly shaped anti-reflection coated portion of the end facet. The beams in the crystal are clearly seen due to the strong fluorescence light from the Pr doped crystal. The dark bands on the top part of the crystal are electrodes. By applying a voltage on these electrodes the energy level spacing in the crystal can be controlled and manipulated.

Much of the work is strongly reliant on having highly absorbing materials. Figure 1 shows a Pr doped yttrium silicate crystal where part of the end faces have been reflection coated such that a beam can pass back and forth several times in the crystal before exiting. In this way optical densities well above 100 are possible to obtain.

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# Relative-states approach to correlations in quantum systems

Pierre Rudolfsson[1,*]

[1] *Department of Physical and Analytical Chemistry, Uppsala University*

Correlations in quantum systems are a notorious enigma of quantum mechanics (QM); technically, it poses problems of characterization and quantification, such as the determination of wether a given $N$-partite state is separable or not, and the construction of operationally meaningful measures of the different types of correlations. Entanglement, the correlation characteristic of composite systems in pure states, is also conceptually pivotal, i.e., at the heart of the so called measurement problem and an apparent choice between certain aspects of realism and locality via Bell's theorem.

One influential work that allegedly resolves the conceptual problems of QM is Hugh Everett's "relative-states formulation of quantum mechanics"[1], which inspired e.g. the many-worlds interpretation. Everett's rationale is that it is meaningless to speak about definite states of subsystems of a composite system in a correlated state, and that quantum mechanics only tells us about subsystems' *relative states*. For a bipartite system in an entangled state, the relative states are the the one to one correspondences between pure post-measurement states of one subsystem and the updated or "collapsed" states of the other.

Everett's formalism has been employed to study entanglement in e.g. [2], and the work to be presented follows this approach to quantify correlations in bipartite quantum states.

A bipartite state $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ induces an anti-linear map $L_\Psi : \mathcal{H}_A \to \mathcal{H}_B$, where an input state $|\varphi\rangle \in \mathcal{H}_A$ can be viewed as the state after a local measurement on subsystem A, and $|\phi\rangle \in \mathcal{H}_B$ is given by

$$|\phi\rangle = L_\Psi|\varphi\rangle = \langle\varphi|\Psi\rangle. \quad (1)$$

We then use the the exterior product

$$\omega^k : \mathcal{H}_B^{\otimes k} \to \Omega^k(\mathcal{H}_B),\ \omega^k(\phi_{i_1}, ...\phi_{i_k}) = \phi_{i_1} \wedge \cdots \wedge \phi_{i_k} \quad (2)$$

to quantify the difference within subsets of relative states $\{|\varphi_i\rangle, |\phi_i\rangle\}$, which we interpret as a measure of the correlation between $k$-dimensional subspaces. For pure states this approach reproduces known measures of entanglement, which are however given a somewhat different operational meaning, and the measure is generalized to quantify total correlations in mixed bipartite states and its features explored.

* Electronic address: pierre.rudolfsson@kvac.uu.se

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