Spin Chains for $\mathcal{N} = 2$ Superconformal Theories

Integrability in the Veneziano Limit?

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Abhijit Gadde, Elli Pomoni, L.R. arXiv:0912.4918, 1006.0015

and in progress with Gadde, Pedro Liendo, Pomoni and Wenbin Yan

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Paradigm: $\mathcal{N} = 4$ SYM $\leftrightarrow$ IIB on $AdS_5 \times S^5$ Maldacena

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All well-understood string duals of 4d gauge theories are rather close cousins of original paradigm. Motivated from $D3$ branes at local singularities in critical string theory.

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Q1: How general is the gauge/string duality?

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Some common features:

- Adjoint or bifundamental matter (quivers).
  Fundamental flavors can be added in probe approximation $N_f \ll N_c$
- Susy can be broken but there are always remnants of the "extra" matter
- Anomaly coefficients $a = c$ at large N_c. "No-go theorem" (?)
- Dual geometries are 10d
- Radius of curvature R related to coupling λ (a modulus),
  $R \sim \lambda^{1/4}$, can be taken arbitrarily large (but $\lambda \to 0$ not always an option)

’t Hooft gave a very general heuristic argument for

“Large N field theory = closed string theory with $g_s \sim 1/N$”

So far we understand “well” only a limited class of dualities,
for the theories “in the universality class” of $\mathcal{N} = 4$ SYM

$\exists$ many string constructions of field theories with genuinely fewer d.o.f. in the IR (say pure $SU(N)$, or $\mathcal{N} = 1$ SYM).

However if one takes a limit that decouples the unwanted UV d.o.f, the dual string is described (at best) by a strongly curved sigma model.

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Independent question,
Q2: How general are integrability/solvability of $\mathcal{N} = 4$ SYM?

Attack "next simplest case"

Ideal case study: $\mathcal{N} = 2$ superconformal QCD,
$\mathcal{N} = 2$ SYM with $N_f = 2N_c$ fundamental hypermultiplets

Large N limit à la Veneziano: $N_f \sim N_c$
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$\lambda = g_{YM}^2 N_c$ is an exactly marginal coupling, just as in $\mathcal{N} = 4$ SYM.

Simplification for large λ? a weakly-curved gravity description?

String theory on... $AdS_5 \times \ldots$? Long-standing open problem

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Q2: Integrable structures?

The Veneziano limit and dual strings

Veneziano limit:
$N_c \to \infty$, $N_f \to \infty$ with N_f/N_c and $\lambda = g_{YM}^2 N_c$ fixed.

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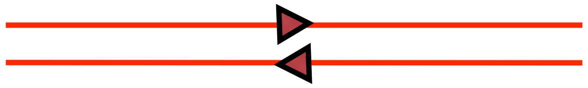
Schematically: adjoint fields ${\phi^a}_b$ $a = 1, \ldots, N_c$ color index

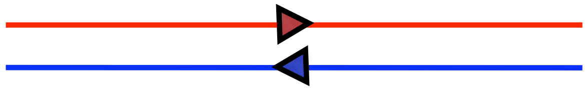
fundamental fields ${q^a}_i$ $i = 1, \ldots, N_f$ flavor index

Two kinds of double lines:

Adjoint lines $\langle \phi\ \phi \rangle$



Quark lines $\langle q\ q \rangle$



Quark lines *not* suppressed.

Vacuum Feynman diagrams $\to$ bi-colored Riemann surfaces $\sim N^{2-2g}$
suggesting a dual closed string theory describing the flavor singlet sector, with $g_s = 1/N$.

Main novelty: *glueball* operators $\mathrm{Tr}(\phi \ldots \phi)$ (color-trace)

mix at leading order with

flavor-singlet *mesons* $\bar{q}^{i}\phi \ldots \phi q_{i}$

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In flavor-singlet sector, basic building blocks are the single-trace operators

$$\mathrm{Tr}(\phi^{k_1}\mathcal{M}^{l_1}\phi^{k_2}\mathcal{M}^{l_2}\ldots)$$

Usual large N factorization arguments apply.

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• In the (conjectural) dual string theory, large meson/glueball mixing interpreted as large backreaction of the "flavor" branes (need to resum open string perturbation theory).

Back to $\mathcal{N} = 2$ superconformal QCD

From the "top-down":

- Engineer it with branes in string theory.

We found some evidence for a non-critical string dual,
with seven "geometric" dimensions, containing both an AdS_5 and an S^1 factor.

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From the "bottom-up" :

- Study the perturbative dilation operator:
  integrable spin-chain? asymptotic Bethe ansatz? clues of a dual sigma-model?

In both approaches, very useful to consider more general family of SCFTs,
interpolating between a $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ and $\mathcal{N} = 2$ SCQCD.

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In the rest of this talk, I'll describe the very first step of the bottom-up approach:
one-loop dilation operator in the scalar sector.

$\mathcal{N}=2$ **SCQCD**

$\mathcal{N}=2$ hypermultiplet

$$\begin{matrix} & \psi_\alpha & \\ q & & \tilde{q}^* \\ & \bar{\tilde{\psi}}^{\dot\alpha} & \end{matrix}$$

$\mathcal{N}=2$ vector multiplet

$$\begin{matrix} & A_\mu & \\ \lambda^1_\alpha & & \lambda^2_\alpha \\ & \phi & \end{matrix}$$

	$SU(N_c)$	$U(N_f)$	$SU(2)_R$	$U(1)_r$
$\mathcal{Q}^{\mathcal{I}}_{\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$
$\mathcal{S}_{\mathcal{I}\,\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
A_μ	Adj	$\mathbf{1}$	$\mathbf{1}$	0
ϕ	Adj	$\mathbf{1}$	$\mathbf{1}$	-1
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$Q_{\mathcal{I}}$	$\square$	$\square$	$\mathbf{2}$	0
ψ_α	$\square$	$\square$	$\mathbf{1}$	$+1/2$
$\tilde{\psi}_\alpha$	$\overline{\square}$	$\overline{\square}$	$\mathbf{1}$	$+1/2$
$\mathcal{M}_{\mathbf{1}}$	Adj $+\ \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0
$\mathcal{M}_{\mathbf{3}}$	Adj $+\ \mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	0

$SU(2)_R$ doublet $Q_{\mathcal{I}} = (q\,,\ \tilde{q}^*)$

Flavor-contracted "mesonic" operator: $\mathcal{M}^{\mathcal{I}}{}_{\mathcal{J}}{}^{a}{}_{b} = Q_{\mathcal{I}}{}^{a}{}_{i}\,\bar{Q}^{\mathcal{J}}{}_{b}{}^{i}$

$$\mathcal{M}_{\mathbf{1}} \equiv \mathcal{M}^{\mathcal{I}}{}_{\mathcal{I}} \quad \text{and} \quad \mathcal{M}_{\mathbf{3}} \equiv \mathcal{M}^{\mathcal{J}}_{\mathcal{K}} - \frac{1}{2}\mathcal{M}^{\mathcal{I}}{}_{\mathcal{I}}\delta^{\mathcal{J}}_{\mathcal{K}}$$

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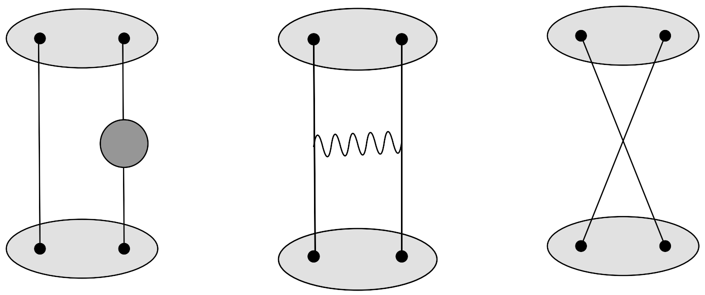
The One-Loop Hamiltonian in the Scalar Sector

We have evaluated the complete one-loop hamiltonian acting on single-trace operators made of scalars,

$$\mathrm{Tr}\left[\,\phi^k \bar{\phi}^\ell \mathcal{M}_{\mathbf{1}}^m \mathcal{M}_{\mathbf{3}}^n\,\right] \qquad \text{(arbitrary permutations thereof)}$$

As usual, large N ensures locality of the hamiltonian.
Nearest neighbor at one-loop, next-to nearest at two loops, ...
(Still true in the Veneziano limit).



Each site of the chain occupied by 6d vector space spanned by $\phi, \bar{\phi}, Q_{\mathcal{I}}, \bar{Q}^{\mathcal{J}}$.

Nearest neighbour Hamiltonian $H_{l,l+1}$ acting on $V_l \otimes V_{l+1}$

$$\phi_{\mathfrak{m}} = (\phi, \bar{\phi})$$

$$
\begin{array}{c} \\ \phi_{\mathfrak{p}'}\phi_{\mathfrak{q}'} \\ \bar{Q}^{\mathcal{I}'}Q_{\mathcal{J}'} \\ Q_{\mathcal{K}'}\bar{Q}^{\mathcal{L}'} \\ \bar{Q}^{\mathcal{I}'}\phi_{\mathfrak{p}'} \end{array}
\begin{array}{c} \begin{array}{cccc} \phi^{\mathfrak{p}}\phi^{\mathfrak{q}} & Q_{\mathcal{I}}\bar{Q}^{\mathcal{J}} & \bar{Q}^{\mathcal{K}}Q_{\mathcal{L}} & Q_{\mathcal{I}}\phi^{\mathfrak{p}} \end{array} \\
\begin{pmatrix}
2\delta^{\mathfrak{p}}_{\mathfrak{p}'}\delta^{\mathfrak{q}}_{\mathfrak{q}'} + g^{\mathfrak{p}\mathfrak{q}}g_{\mathfrak{p}'\mathfrak{q}'} - 2\delta^{\mathfrak{p}}_{\mathfrak{q}'}\delta^{\mathfrak{q}}_{\mathfrak{p}'} & \sqrt{\frac{N_f}{N_c}}g_{\mathfrak{p}'\mathfrak{q}'}\delta^{\mathcal{J}}_{\mathcal{I}} & 0 & 0 \\
\sqrt{\frac{N_f}{N_c}}g^{\mathfrak{p}\mathfrak{q}}\delta^{\mathcal{I}'}_{\mathcal{J}'} & (2\delta^{\mathcal{I}'}_{\mathcal{I}}\delta^{\mathcal{J}}_{\mathcal{J}'} - \delta^{\mathcal{J}}_{\mathcal{I}}\delta^{\mathcal{I}'}_{\mathcal{J}'})\frac{N_f}{N_c} & 0 & 0 \\
0 & 0 & 2\delta^{\mathcal{K}}_{\mathcal{L}}\delta^{\mathcal{L}'}_{\mathcal{K}'} & 0 \\
0 & 0 & 0 & 2\delta^{\mathcal{I}'}_{\mathcal{I}}\delta^{\mathfrak{p}}_{\mathfrak{p}'}
\end{pmatrix}\end{array}
$$

$SU(2)_R$ indices $\mathcal{I}, \mathcal{J}, \mathcal{K}, \mathcal{L} \cdots = 1, 2$ $\quad$ $U(1)_r$ indices $\mathfrak{m}, \mathfrak{n} \cdots = 1, 2$ $\quad$ $g_{\mathfrak{m}\mathfrak{n}} = g^{\mathfrak{m}\mathfrak{n}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\Gamma^{(1)} \equiv g^2 H \,, \qquad g^2 \equiv \frac{\lambda}{8\pi^2} \,, \quad \lambda \equiv g_{YM}^2 N_c$$

Elementary operators acting on each site of the chain, transforming "incoming" $\mathcal{O}^{\mathcal{I}}{}_{\mathcal{J}}$ to "outgoing" $\mathcal{O}^{\mathcal{L}}{}_{\mathcal{K}}$:

Trace operator $\mathbb{K}^{\mathcal{J}\mathcal{L}}_{\mathcal{I}\mathcal{K}} = \delta^{\mathcal{J}}_{\mathcal{I}}\delta^{\mathcal{L}}_{\mathcal{K}}$

Permutation operator $\mathbb{P}^{\mathcal{J}\mathcal{L}}_{\mathcal{I}\mathcal{K}} = \delta_{\mathcal{I}\mathcal{K}}\delta^{\mathcal{J}\mathcal{L}}$

Identity operator $\mathbb{I}^{\mathcal{J}\mathcal{L}}_{\mathcal{I}\mathcal{K}} = \delta^{\mathcal{L}}_{\mathcal{I}}\delta^{\mathcal{J}}_{\mathcal{K}}$

$$H_{k,k+1} = \begin{array}{c} \\ \phi\phi \\ \bar{Q}Q \\ Q\bar{Q} \\ \bar{Q}\phi \end{array} \begin{array}{c} \begin{array}{cccc} \phi\phi & Q\bar{Q} & \bar{Q}Q & Q\phi \end{array} \\ \begin{pmatrix} 2\mathbb{I} + \mathbb{K} - 2\mathbb{P} & \sqrt{\frac{N_f}{N}}\mathbb{K} & 0 & 0 \\ \sqrt{\frac{N_f}{N}}\mathbb{K} & (2\mathbb{I} - \mathbb{K})\frac{N_f}{N_c} & 0 & 0 \\ 0 & 0 & 2\mathbb{K} & 0 \\ 0 & 0 & 0 & 2\mathbb{I} \end{pmatrix} \end{array}$$

The $N_f = 0$ case has been considered by Di Vecchia and Tanzini

Vacuum $\mathrm{Tr}(\phi^\ell)$.

Excitations are either the elementary $\bar{\phi}$ or the composite $\mathcal{M}_{\mathbf{1}}$, $\mathcal{M}_{\mathbf{3}}$.

In the infinite chain, diagonalize H in the one-impurity sector.

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For the $SU(2)_R$ triplet, $H[\mathcal{M}_{\mathbf{3}}(x)] = 8\mathcal{M}_{\mathbf{3}}(x)$, or $H[\mathcal{M}_{\mathbf{3}}(p)] = 8\mathcal{M}_{\mathbf{3}}(p)$

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The singlets $\bar{\phi}$ and $\mathcal{M}_{\mathbf{1}}$ mix:

$$\bar{\phi}(p) \equiv \sum_x \bar{\phi}(x) e^{ipx}\,, \quad \mathcal{M}_{\mathbf{1}}(p) \equiv \sum_x \mathcal{M}_{\mathbf{1}}(x) e^{ipx}$$

$$H \begin{pmatrix} \bar{\phi}(p) \\ \mathcal{M}_{\mathbf{1}} \end{pmatrix} = \begin{pmatrix} 6 - e^{ip} - e^{-ip} & (1+e^{-ip})\sqrt{\frac{2N_f}{N_c}} \\ (1+e^{ip})\sqrt{\frac{2N_f}{N_c}} & 4 \end{pmatrix} \begin{pmatrix} \bar{\phi}(p) \\ \mathcal{M}_{\mathbf{1}} \end{pmatrix}$$

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For $N_f = 2N_c$ one of the two singlet eigenstates is gapless.

Henceforth $N_f \equiv 2N_c$.

The eigenstates are

$$T(p) \equiv -\frac{1}{2}(1+e^{-ip})\bar{\phi}(p) + \mathcal{M}_{\mathbf{1}}(p)\,, \qquad \widetilde{T}(p) \equiv \bar{\phi}(p) + \frac{1}{2}(1+e^{ip})\mathcal{M}_{\mathbf{1}}(p)$$

with eigenvalues

$$HT(p) = 4\sin^2(\frac{p}{2})\,T(p)\,, \qquad H\widetilde{T}(p) = 8\,\widetilde{T}(p)\,.$$

Protected Operators

From explicit one-loop calculation in the scalar sector, the single-trace operators with $\gamma = 0$ are

- $\mathrm{Tr}\,\mathcal{M}_{\mathbf{3}}$
- $\mathrm{Tr}\,\phi^{\ell}$, with $\ell \geq 2$.
- $\mathrm{Tr}\,T\,\phi^{\ell}$, with $\ell \geq 0$, where $T \equiv \bar{\phi}\phi - \mathcal{M}_{\mathbf{1}}$.

Scalar Multiplets	SCQCD operators	Protected
$\mathcal{B}_{R,r(0,0)}$	$\mathrm{Tr}[\bar{\phi}^r \mathcal{M}_3^R]$	
$\mathcal{E}_{r(0,0)}$	$\mathrm{Tr}[\bar{\phi}^r]$	✓
$\hat{\mathcal{B}}_R$	$\mathrm{Tr}[\mathcal{M}_3^R]$	✓ for $R = 1$
$\mathcal{C}_{R,r(0,0)}$	$\mathrm{Tr}[T\mathcal{M}_3^R\bar{\phi}^r]$	
$\mathcal{C}_{0,r(0,0)}$	$\mathrm{Tr}[T\bar{\phi}^r]$	✓
$\hat{\mathcal{C}}_{R(0,0)}$	$\mathrm{Tr}[T\mathcal{M}_3^R]$	
$\hat{\mathcal{C}}_{0(0,0)}$	$\mathrm{Tr}[T]$	✓
$\mathcal{D}_{R(0,0)}$	$\mathrm{Tr}[\mathcal{M}_3^R\bar{\phi}]$	

Note that $\mathrm{Tr}\,T$ $(\Delta = 2)$ is the lowest weight state of the $\mathcal{N} = 2$ stress-tensor multiplet.

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These operators are superconformal primaries.
In the free theory they are the lowest weight states of (semi-)short multiplets.
In the interacting theory (semi-)short multiplets can a priori combine into long multiplets with $\gamma \neq 0$.

Protection of $\mathrm{Tr}\phi^{\ell}$ easily proved to all orders from superconformal representation theory: such multiplets never appear in decomposition of long multiplets. Dolan-Osborn

Protection of $\mathrm{Tr}\,\boldsymbol{\mathcal{M}_3}$ and of $\mathrm{Tr}\,T\,\phi^{\ell}$ more subtle,
we prove it by computing (essentially) a superconformal index.
Most easily done in interpolating family of SCFTs (coming up soon).

(Situations more intricate than in $\mathcal{N} = 4$ SYM where the only single-trace protected multiplets are the 1/2 BPS multiplets.)

An interpolating family of super CFTs

$\mathcal{N} = 2$ SCQCD can be viewed as a limit of a family of $\mathcal{N} = 2$ SCFTs.

In opposite limit the family reduces to a well-known $\mathbb{Z}_2$ orbifold of $\mathcal{N} = 4$ SYM

Start with $\mathcal{N} = 4$ SYM: X_{AB}, λ_α^A, A_μ

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$$X_{AB}=\frac{1}{\sqrt{2}}\left(\begin{array}{cc|cc} 0 & X_4+iX_5 & X_7+iX_6 & X_8+iX_9 \\ -X_4-iX_5 & 0 & X_8-iX_9 & -X_7+iX_6 \\ \hline -X_7-iX_6 & -X_8+iX_9 & 0 & X_4-iX_5 \\ -X_8-iX_9 & X_7-iX_6 & -X_4+iX_5 & 0 \end{array}\right)$$

Pick $SU(2)_L\times SU(2)_R\times U(1)_r$ subgroup of $SU(4)_R$

$$\begin{array}{c} 1\ + \\ 2\ - \\ 3\ \hat{+} \\ 4\ \hat{-} \end{array}\left(\begin{array}{c|c} SU(2)_R\times U(1)_r & \\ \hline & SU(2)_L\times U(1)_r^* \end{array}\right)$$

$$\mathcal{Z}\equiv\frac{X_4+iX_5}{\sqrt{2}}\,,\qquad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}}\equiv\frac{1}{\sqrt{2}}\begin{pmatrix} X_7+iX_6 & X_8+iX_9 \\ X_8-iX_9 & -X_7+iX_6 \end{pmatrix}$$

$\mathcal{I},\mathcal{J}=\pm$ $SU(2)_R$ indices, $\hat{\mathcal{I}},\hat{\mathcal{J}}=\hat{\pm}$ $SU(2)_L$ indices

$SU(2)_L\times SU(2)_R\cong SO(4)$ are 6789 rotations,

$U(1)_R\cong SO(2)$ 45 rotations.

In R-space, orbifold by $\mathbb{Z}_2 \subset SU(2)_L$, $\mathbb{Z}_2 = \{\pm \mathbb{I}_{2\times 2}\}$

$$(X_6, X_7, X_8, X_9) \to \pm (X_6, X_7, X_8, X_9)$$

In color space, start with $SU(2N_c)$ and declare non-trivial element of orbifold

$$\gamma \equiv \begin{pmatrix} \mathbb{I}_{N_c\times N_c} & 0 \\ 0 & -\mathbb{I}_{N_c\times N_c} \end{pmatrix} \qquad A_\mu \to \gamma A_\mu \gamma\,, \quad Z_{\mathcal{I}\mathcal{J}} \to \gamma Z_{\mathcal{I}\mathcal{J}} \gamma\,, \quad \lambda_{\mathcal{I}} \to \gamma \lambda_{\mathcal{I}} \gamma\,, \quad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} \to -\gamma \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} \gamma\,, \quad \lambda_{\hat{\mathcal{I}}} \to -\gamma \lambda_{\hat{\mathcal{I}}} \gamma\,.$$

Fields surviving the projections are:

$$A_\mu = \begin{pmatrix} A^a_{\mu b} & 0 \\ 0 & \check{A}^{\check{a}}_{\mu \check{b}} \end{pmatrix} \qquad Z = \begin{pmatrix} \phi^a{}_b & 0 \\ 0 & \check{\phi}^{\check{a}}{}_{\check{b}} \end{pmatrix} \qquad \lambda_{\mathcal{I}} = \begin{pmatrix} \lambda^a_{\mathcal{I}b} & 0 \\ 0 & \check{\lambda}^{\check{a}}_{\mathcal{I}\check{b}} \end{pmatrix}$$

$$\lambda_{\hat{\mathcal{I}}} = \begin{pmatrix} 0 & \psi^a_{\hat{\mathcal{I}}\check{a}} \\ \tilde{\psi}^{\check{b}}_{\hat{\mathcal{I}}b} & 0 \end{pmatrix} \qquad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} = \begin{pmatrix} 0 & Q_{\mathcal{I}\hat{\mathcal{I}}\check{a}}{}^{a} \\ -\epsilon_{\mathcal{I}\mathcal{J}}\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}\bar{Q}^{\check{b}\hat{\mathcal{J}}\mathcal{J}}{}_b & 0 \end{pmatrix}$$

	$SU(N_c)$	$SU(N_{\tilde{c}})$	$SU(2)_R$	$SU(2)_L$	$U(1)_R$
$Q^{\mathcal{I}}_{\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$+1/2$
$\mathcal{S}_{\mathcal{I}\,\alpha}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$-1/2$
A_{μ}	Adj	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0
$\check{A}_{\mu}$	$\mathbf{1}$	Adj	$\mathbf{1}$	$\mathbf{1}$	0
ϕ	Adj	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	-1
$\check{\phi}$	$\mathbf{1}$	Adj	$\mathbf{1}$	$\mathbf{1}$	-1
$\lambda^{\mathcal{I}}$	Adj	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$-1/2$
$\check{\lambda}^{\mathcal{I}}$	$\mathbf{1}$	Adj	$\mathbf{2}$	$\mathbf{1}$	$-1/2$
$Q_{\mathcal{I}\hat{\mathcal{I}}}$	$\square$	$\overline{\square}$	$\mathbf{2}$	$\mathbf{2}$	0
$\psi_{\hat{\mathcal{I}}}$	$\square$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$
$\check{\psi}_{\hat{\mathcal{I}}}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{2}$	$+1/2$

Two gauge-couplings g_{YM} and $\check{g}_{YM}$ can be independently varied

while preserving $\mathcal{N} = 2$ superconformal invariance

For $\check{g}_{YM} \to 0$, recover $\mathcal{N} = 2$ SCQCD $\oplus$ decoupled $SU(N_{\check{c}})$ vector multiplet

For $\check{g}_{YM} = 0$, global symmetry enhancement $SU(N_{\check{c}}) \times SU(2)_L \to U(N_f = 2N_c)$:
$(\check{a}, \hat{\mathcal{I}}) \equiv i = 1, \dots N_f = 2N_c$

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$$c = a = \frac{N_c^2}{2}$$ along the whole marginal deformation

For $\check{g}_{YM} = 0$, interpret as

$$a = \left(\frac{7}{24} + \frac{5}{24}\right) N_c^2 \qquad c = \left(\frac{8}{24} + \frac{4}{24}\right) N_c^2$$

Interpolating theory has vastly more protected "closed" states than $\mathcal{N} = 2$ SCQD:

towers of states with arbitrary high (and equal) $SU(2)_L$ and $SU(2)_R$ spins

For $\check{g} \to 0$, they are re-interpreted as multiparticle states of short open strings

Spin chain for interpolating SCFT

$$H_{k,k+1} =$$

$$\begin{array}{c} \\ \phi\phi \\ Q\bar{Q} \\ \check{\phi}\check{\phi} \\ \bar{Q}Q \\ \phi Q \\ Q\check{\phi} \\ \check{\phi}\bar{Q} \\ \bar{Q}\phi \end{array}
\begin{array}{c} \begin{array}{cccccccc} \phi\phi & Q\bar{Q} & \check{\phi}\check{\phi} & \bar{Q}Q & \phi Q & Q\check{\phi} & \check{\phi}\bar{Q} & \bar{Q}\phi \end{array} \\
\begin{pmatrix}
(2+\mathbb{K}-2\mathbb{P}) & \mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbb{K} & (2-\mathbb{K})\hat{\mathbb{K}}+2\kappa^2\mathbb{K} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \kappa^2(2+\mathbb{K}-2\mathbb{P}) & \kappa^2\mathbb{K} & 0 & 0 & 0 & 0 \\
0 & 0 & \kappa^2\mathbb{K} & \kappa^2(2-\mathbb{K})\hat{\mathbb{K}}+2\mathbb{K} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & -2\kappa & 0 & 0 \\
0 & 0 & 0 & 0 & -2\kappa & 2\kappa^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2\kappa^2 & -2\kappa \\
0 & 0 & 0 & 0 & 0 & 0 & -2\kappa & 2
\end{pmatrix} \end{array}$$

$$\kappa \equiv \frac{\check{g}}{g}, \quad g^2 \equiv \frac{g_{YM}^2 N}{8\pi^2}, \quad \check{g}^2 \equiv \frac{\check{g}_{YM}^2 N}{8\pi^2}$$

More compactly, using the $\mathbb{Z}_2$-projected $SU(2N_c)$ adjoint fields

$$Z = \begin{pmatrix} \phi & 0 \\ 0 & \check{\phi} \end{pmatrix}, \qquad \mathcal{X}_{\mathcal{I}\hat{\mathcal{I}}} = \begin{pmatrix} 0 & Q_{\mathcal{I}\hat{\mathcal{I}}} \\ -\epsilon_{\mathcal{I}\mathcal{J}}\epsilon_{\hat{\mathcal{I}}\hat{\mathcal{J}}}\bar{Q}^{\hat{\mathcal{J}}\mathcal{J}} & 0 \end{pmatrix}$$

$$g^2 H = \begin{pmatrix} ZZ & \mathcal{X}\mathcal{X} & Z\mathcal{X} & \mathcal{X}Z \\ (g_+ + \gamma g_-)^2(2 + \mathbb{K} - 2\mathbb{P}) & (g_+ + \gamma g_-)^2\mathbb{K}\hat{\mathbb{K}} & 0 & 0 \\ (g_+ + \gamma g_-)^2\mathbb{K}\hat{\mathbb{K}} & \begin{array}{c}(g_+ + \gamma g_-)^2(2\hat{\mathbb{K}} - \mathbb{K}\hat{\mathbb{K}}) \\ +2(g_+ - \gamma g_-)^2\mathbb{K}\end{array} & 0 & 0 \\ 0 & 0 & 2(g_+ + \gamma g_-)^2 & -2({g_+}^2 - {g_-}^2) \\ 0 & 0 & -2({g_+}^2 - {g_-}^2) & 2(g_+ - \gamma g_-)^2 \end{pmatrix}$$

where $g_\pm \equiv (g \pm \check{g})/2$ and γ the twist operator.

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For $g \neq \check{g}$, bulk Hamiltonian truly different.

Parity is broken for generic $\check{g}$, but is recovered for $\check{g} \to 0$, as seen more clearly in terms of composite mesonic operators.

Dynamics of the interpolating spin chain

For $\check{g} \neq 0$, magnons are conventional, "elementary" Q or $\bar{Q}$, with dispersion relation

$$E(p;\kappa) = 2(1-\kappa)^2 + 8\kappa\left(\sin^2\frac{p}{2}\right) \qquad \text{where } \kappa \equiv \check{g}/g.$$

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The 2-body S-matrix is defined as usual, for $Q\bar{Q}$

$$\sum_{x_1 \ll x_2} \left(e^{ip_1x_1+ip_2x_2} + S(p_2,p_1)e^{ip_2x_1+ip_1x_2}\right) |\ldots\phi Q(x_1)\check{\phi}\ldots\check{\phi}\bar{Q}(x_2)\phi\ldots\rangle$$

(pure reflection)

and similarly for $\bar{Q}Q$,

$$\sum_{x_1 \ll x_2} \left(e^{ip_1x_1+ip_2x_2} + \check{S}(p_2,p_1)e^{ip_2x_1+ip_1x_2}\right) |\ldots\check{\phi}\bar{Q}(x_1)\phi\ldots\phi Q(x_2)\check{\phi}\ldots\rangle$$

Clearly we have $\quad S(p_1,p_2;g,\check{g}) = \check{S}(p_1,p_2;\check{g},g)\,.$

Each magnon is in the spin 1/2 representation of both $SU(2)_L$ and $SU(2)_R$.

Solving the 2-body problem in the four sectors with different $SU(2)$ quantum numbers,

$L \otimes R$	$S(p_1, p_2, \kappa)$
$1 \otimes 1$	$-\mathcal{S}(p_1, p_2, \kappa - \frac{1}{\kappa})\mathcal{S}^{-1}(p_1, p_2, \kappa)$
$1 \otimes 3$	$\mathcal{S}(p_1, p_2, \kappa - \frac{1}{\kappa})$
$3 \otimes 1$	-1
$3 \otimes 3$	$\mathcal{S}(p_1, p_2, \kappa)$

$$\mathcal{S}(p_1, p_2, \kappa) \equiv -\frac{1 - 2\kappa e^{ip_1} + e^{i(p_1+p_2)}}{1 - 2\kappa e^{ip_2} + e^{i(p_1+p_2)}}$$

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The magnon S-matrix factorizes into "left" and "right"

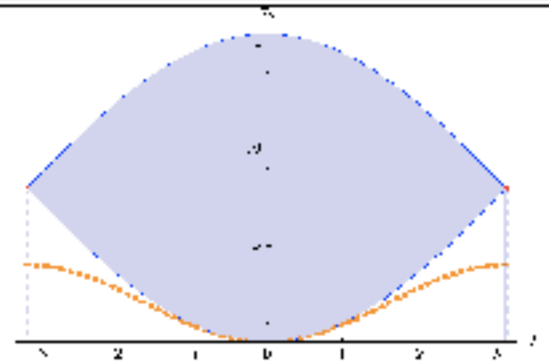
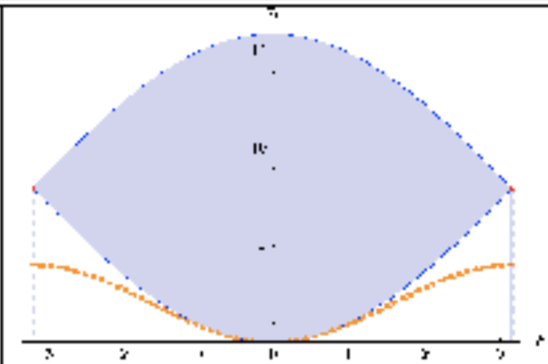
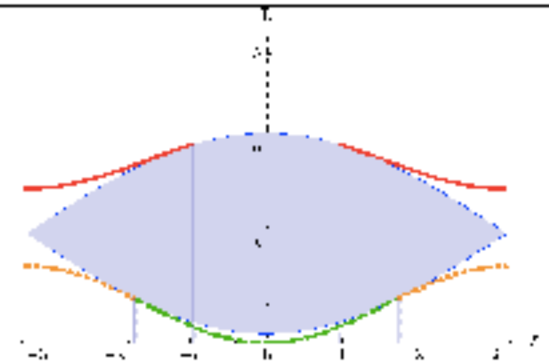
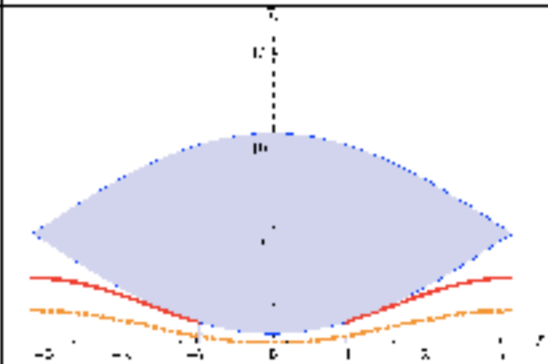
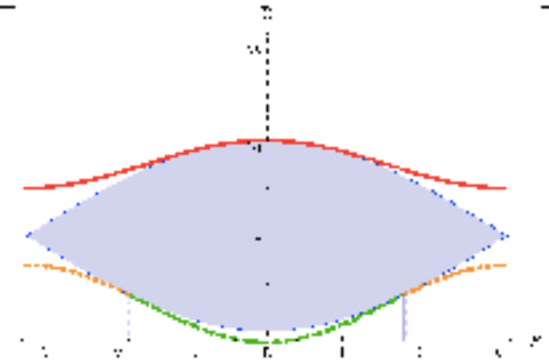
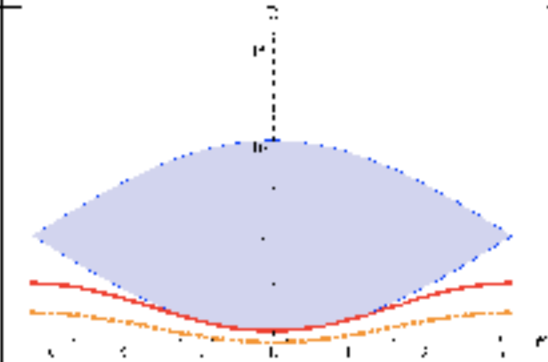
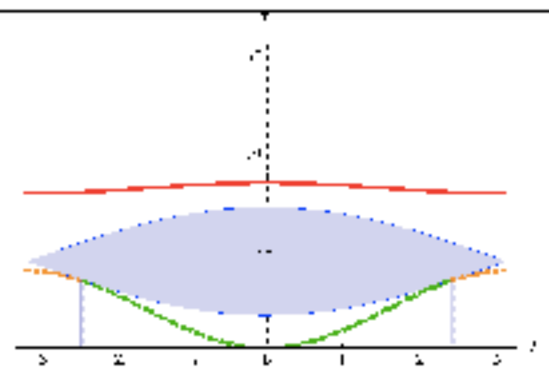
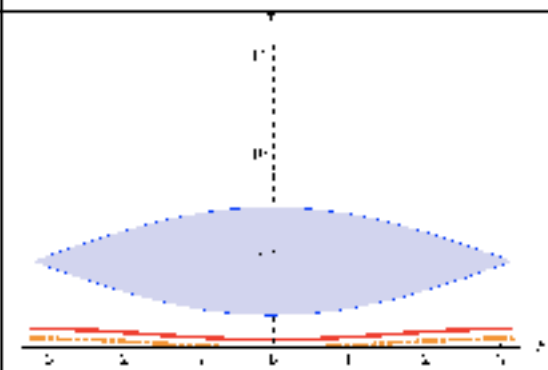
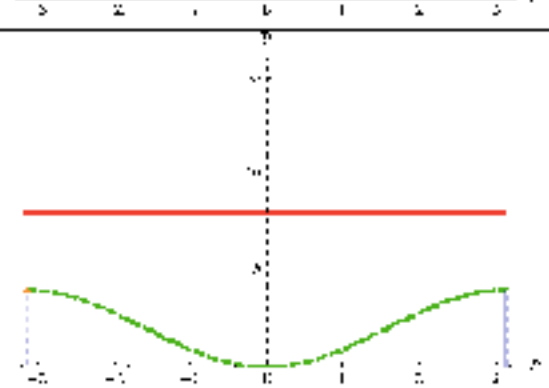
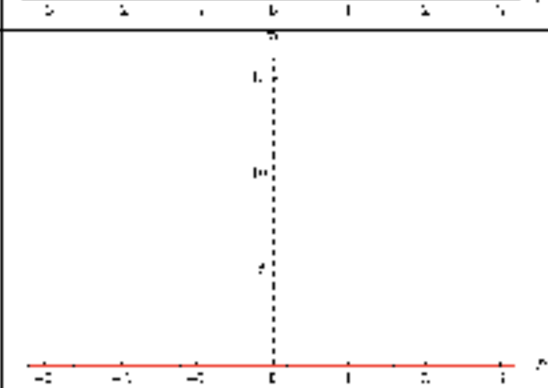
$$S(p_1, p_2; \kappa) = \frac{S_L(p_1, p_2; \kappa) S_R(p_1, p_2; \kappa)}{S_{3\otimes 3}(p_1, p_2; \kappa)}$$

$SU(2)_L$	$S_L(p_1, p_2; \kappa)$	$SU(2)_R$	$S_R(p_1, p_2; \kappa)$
1	$\mathcal{S}(p_1, p_2; \kappa - \frac{1}{\kappa})$	**1**	-1
3	$\mathcal{S}(p_1, p_2; \kappa)$	**3**	$\mathcal{S}(p_1, p_2; \kappa)$

The 2-body S-matrix of the interpolating theory reveals a rich spectrum of bound and anti-bound states:

	Pole of the S-matrix	Range of existence	Dispersion relation $E(P)$
$\mathcal{M}_{33}$	$e^{-q}=\cos(\frac{P}{2})/\kappa$	$2\arccos\kappa<\lvert P\rvert<\pi$	$4\sin^2(\frac{P}{2})$
T	$e^{q}=\cos(\frac{P}{2})/\kappa$	$0<\lvert P\rvert<2\arccos\kappa$	$4\sin^2(\frac{P}{2})$
$\widetilde{T}$ and $\mathcal{M}_3$	$e^{-q}=\cos(\frac{P}{2})/(\kappa-\frac{1}{\kappa})$	See equ.(5.26)	$\frac{4\kappa^2}{(1-\kappa^2)}(\frac{2}{\kappa^2}-1-\sin^2\frac{P}{2})$
$\check{\mathcal{M}}_{33}$	$e^{-q}=\kappa\cos(\frac{P}{2})$	$0<\lvert P\rvert<\pi$	$4\kappa^2\sin^2(\frac{P}{2})$
$\check{T}$	$e^{q}=\kappa\cos(\frac{P}{2})$	No solution	
$\check{\widetilde{T}}$ and $\check{\mathcal{M}}_3$	$e^{-q}=\cos(\frac{P}{2})/(\frac{1}{\kappa}-\kappa)$	See equ.(5.26)	$\frac{4\kappa^2}{(1-\kappa^2)}(1-2\kappa^2+\sin^2\frac{P}{2})$

The asymptotic wave-functions behave as
$e^{iP\frac{x_1+x_2}{2}-q(x_2-x_1)}$ for bound states,
$(-1)^{x_2-x_1}e^{iP\frac{x_1+x_2}{2}-q(x_2-x_1)}$ for anti-bound states.

	$Q\bar{Q}$ scattering channel	$\bar{Q}Q$ scattering channel
$\kappa = 0.999$		
$\kappa = 0.65$		
$\kappa = \frac{-1+\sqrt{5}}{2}$		
$\kappa = 0.35$		
$\kappa = 0.001$		

Remarkably, for $\check{g} \to 0$ the "dimeric" excitations $T(p)$, $\widetilde{T}(p)$ and $\mathcal{M}_{\mathbf{3}}$ of the SCQCD chain are recovered smoothly for $\kappa \to 0$ as "infinitely tight" bound states $(p \to \pm i\infty)$

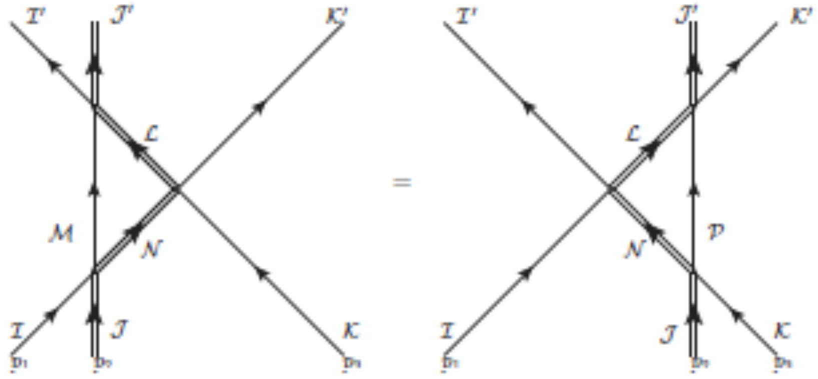
Recall the two-magnon energy,

$$E = 4(1-\kappa)^2 + 16\kappa \left(\sin^2 \frac{p}{2}\right) .$$

For some bound states of the interpolating theory, as $\kappa \to 0$ the "zero" in front of the kinetic energy multiplies the "infinity" arising from the large imaginary momenta.

In the limit $\kappa \to 0$ the elementary Q and $\bar{Q}$ excitations freeze, but their bound states retain a non-trivial dynamics.

Yang-Baxter



The YB equation for the 2-body S-matrix obviously holds for $g = \check{g}$.

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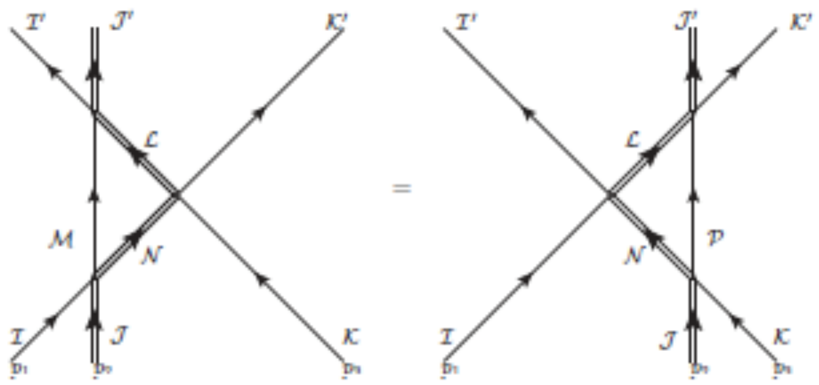

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We find that is is generically violated for $\check{g} \neq g$, showing conclusively that the interpolating spin-chain is not integrable.

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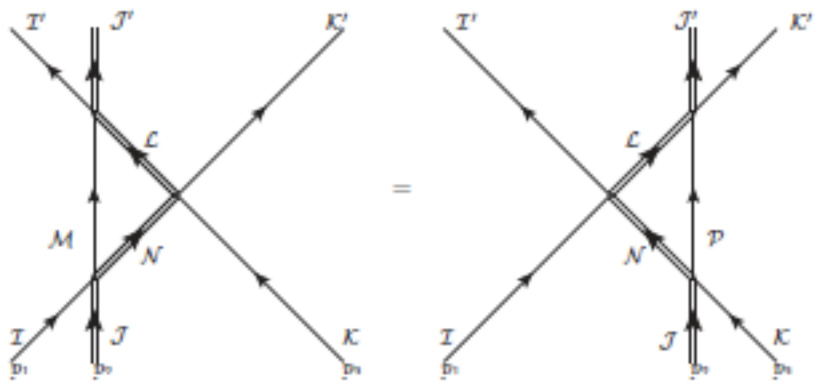


The YB equation for the 2-body S-matrix obviously holds for $g = \check{g}$.

We find that is is generically violated for $\check{g} \neq g$, showing conclusively that the interpolating spin-chain is not integrable.

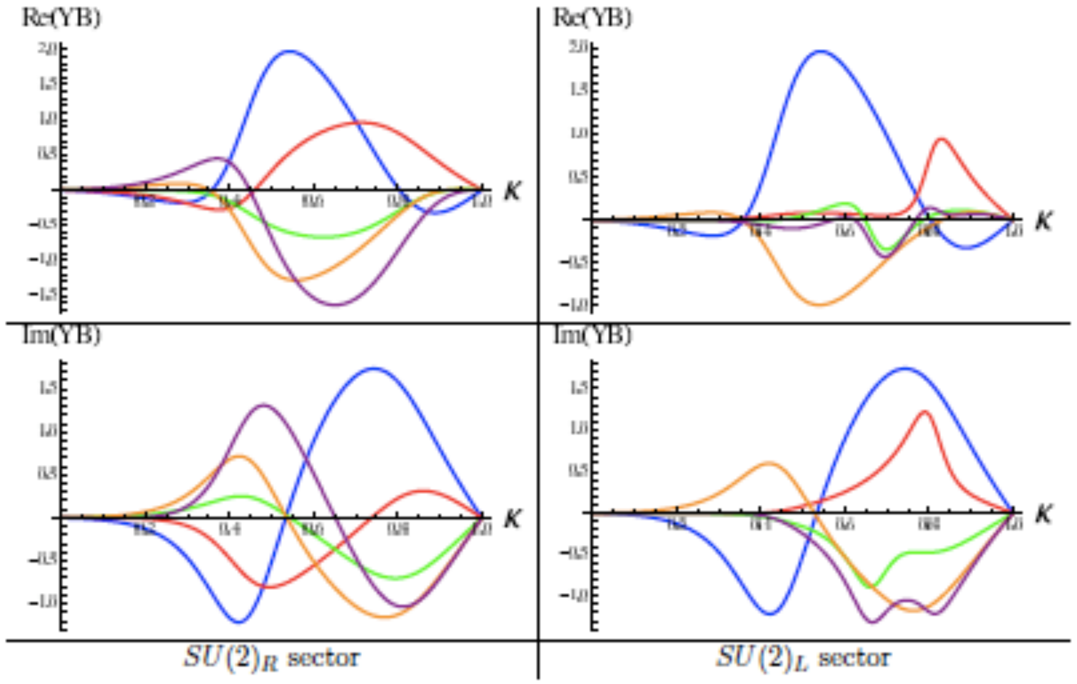
Remarkably, YB holds again for $\check{g} \longrightarrow 0$!

Hint that $\mathcal{N} = 2$ SCQCD may be integrable, at least at one loop.

Algebraic structure (to be discussed shortly) gives hope for higher-loops.

Ordinarily, YB is strong evidence, but in our case things may be more subtle.

Smooth limit is reason for optimism.



Outlook: Symmetry

We have focused on the scalar sector, where the S-matrix has $SU(2)_L \times SU(2)_R$ symmetry.

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In the right sector, same as $\mathcal{N} = 4$ SYM, but the representation is now reducible (two copies of the fundamental).

Repeating Beisert's argument, it appears one can constrain the right S-matrix to all orders (up to overall phase of course): we find a certain function of κ, which is consistent with the one-loop S-matrix in the scalar sector.

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Repeating Beisert's argument, it appears one can constrain the right S-matrix to all orders (up to overall phase of course): we find a certain function of κ, which is consistent with the one-loop S-matrix in the scalar sector.

• Calculation of the complete one-loop Hamiltonian is also in progress.

Outlook: Integrability?

- Elegant way to conclusely show integrability for the $\mathcal{N} = 2$ SCQCD chain would be to find an algebraic Bethe ansatz.
  Simplest guess for the R-matrix does not appear to work.

- Numerical checks of integrability are in progress.
  Difficult to work at $\kappa \equiv 0$
  (for example it seems hard to find the S-matrix of the dimeric magnons).

  Instead, use small κ as a regulator.

  Write Bethe equations for small κ, taking $\kappa \longrightarrow 0$ in the final result for the energies.
  Then compare with brute force diagonalization of H_{SCCD}.

  Similarly, determine the S-matrix of the bound states by fusion for small κ.

  Consistency of the procedure (?) would hinge on smoothness of the $\kappa \longrightarrow 0$ limit.

Outlook: String Theory

- Dual sigma-model is in principle known for large g, finite $\kappa \equiv \check{g}/g$.
  Start from orbifold sigma-model. Taking $\check{g} \neq g$ amounts to changing the period of B_{NSNS} through the collapsed cycle of the orbifold.

  2-body S-matrix at strong coupling?

  Natural to expect integrability only at the two parity-invariant values of B, corresponding to $\check{g} = g$ and $\check{g} = 0$.
  (Somewhat similar story in the ABJ model).

Outlook: String Theory

- Dual sigma-model is in principle known for large g, finite $\kappa \equiv \check{g}/g$.
  Start from orbifold sigma-model. Taking $\check{g} \neq g$ amounts to changing the period of B_{NSNS} through the collapsed cycle of the orbifold.

  2-body S-matrix at strong coupling?

  Natural to expect integrability only at the two parity-invariant values of B, corresponding to $\check{g} = g$ and $\check{g} = 0$.
  (Somewhat similar story in the ABJ model).

- On the string theory side, $\kappa \to 0$ limit is singular, need to change duality frame.
  In our first paper, argued that in the limit one finds a non-critical string background.

  Field theory results in qualitative agreement with this picture:
  protected states consistent with KK spectrum,
  counting of gapless magnons consistent with counting of transverse directions.

Conclusion

$\mathcal{N} = 2$ SCQCD is perhaps the simplest theory outside the $\mathcal{N} = 4$ universality class.

Continuously connected to the $\mathcal{N} = 4$ class by an interpolating $\mathcal{N} = 2$ SCFT.

Already the simplest calculations reveal a rich dynamics.