

Superconformal invariance of scattering amplitudes in $\mathcal{N} = 4$ super-Yang-Mills theory

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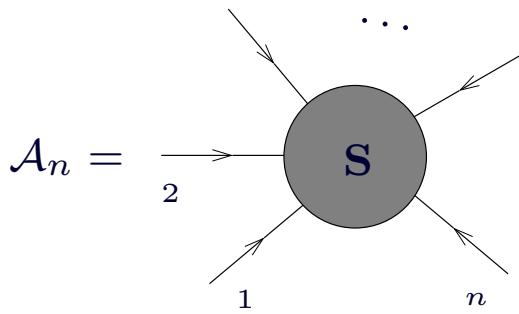
LAPTH, Annecy, France

Based on work in collaboration with

James Drummond, Johannes Henn and Gregory Korchemsky

On-shell scattering amplitudes in $\mathcal{N} = 4$ SYM

- ✓ Scattering amplitudes in $\mathcal{N} = 4$ SYM



- ✗ Quantum numbers of on-shell states $|i\rangle = |p_i, h_i, a_i\rangle$: momentum ($p_i^2 = 0$), helicity (h_i), color (a_i)
- ✗ IR divergences \mapsto dimensional regularization
 $\mathcal{A}_n = \text{Div}(p_i, 1/\epsilon, \mu) \times \text{Fin}(p_i)$ \rightarrow subject of this talk
- ✗ Perturbative expansion in 't Hooft coupling $a = \frac{g^2 N}{8\pi^2}$:

$$\mathcal{A}_n(p_i, h_i) = \mathcal{A}_{n;0} + a \sum_H \mathcal{A}_{n;1}^H M_{n;1}^H(p_i) + O(a^2)$$

- ✓ Simplest example: Maximally Helicity Violating (MHV) amplitudes, e.g. for gluons:
 $(- - + \dots +)$, $(- + - + \dots +)$, etc.
Unique helicity structure (tree):

$$\mathcal{A}_n^{\text{MHV}}(p_1^-, p_2^-, p_3^+, \dots, p_n^+) = \mathcal{A}_{n;0}^{\text{MHV}} M_n^{\text{MHV}}(p_i), \quad M_n^{\text{MHV}} = 1 + a M_n^{(1)} + O(a^2)$$

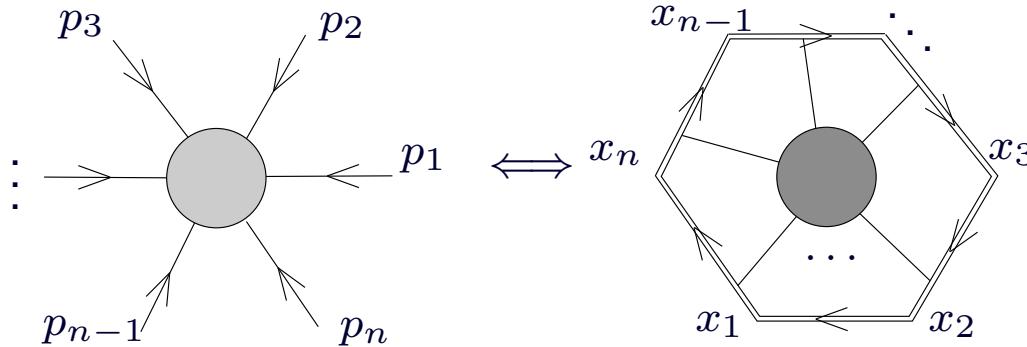
- ✓ $\mathcal{N} = 4$ SYM is a (super)conformal theory \Rightarrow conformal symmetry of $\mathcal{A}_n(p_i)$?

Two problems:

- Conformal boosts act non-locally (2nd-order differential operators)
- IR divergences break conformal symmetry - how?

[Witten]

MHV scattering amplitudes/Wilson loop duality



MHV amplitudes are dual to light-like Wilson loops

[Alday,Maldacena], [Drummond,Korchemsky,ES],
[Brandhuber,Heslop,Travaglini]

$$W(C_n) = \langle 0 | \text{Tr} \, \text{P} \exp \left(ig \oint_{C_n} dx^\mu A_\mu(x) \right) | 0 \rangle, \quad C_n = \text{light-like polygon}$$

$$\ln \mathcal{A}_n^{(\text{MHV})} \sim \ln W(C_n) + \text{const} + O(\epsilon) + O(1/N_c^2),$$

✓ At **strong** coupling, agrees with the BDS ansatz for $n = 4$, but **not** for $n \rightarrow \infty$

[Alday,Maldacena]

✓ At **weak** coupling, the duality was verified for:

✗ $n = 4$ (rectangle) to two loops

[Drummond,Henn,Korchemsky,ES]

✗ $n \geq 5$ (polygon) to one loop

[Brandhuber,Heslop,Travaglini]

✗ $n = 5, 6$ to two loops

[Drummond,Henn,Korchemsky,ES][Bern et al]

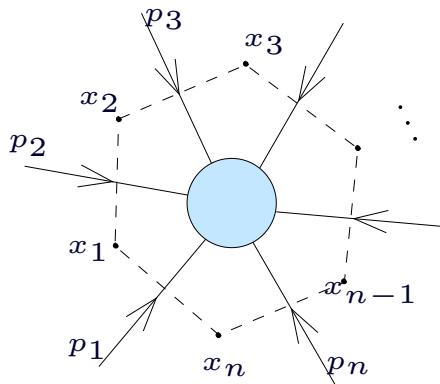
Wilson loops match the BDS ansatz for $n = 4, 5$ but not for $n \geq 6$. Why?

Answer: **Dual conformal symmetry**.

Dual conformal symmetry I

- ✓ Hidden symmetry of \mathcal{A}_n of dynamical origin
- ✓ Linear action on the particle momenta in **dual space**

[Broadhurst],[Drummond,Henn,Smirnov,ES]



- \cancel{x} $p_i = x_i - x_{i+1} \equiv x_i \cdot x_{i+1} \Leftrightarrow \sum_i p_i = 0$ if $x_{n+1} \equiv x_1$
- \cancel{x} $p_i^2 = 0 \Leftrightarrow x_i^2 \cdot x_{i+1}^2 = 0$
- \cancel{x} Conformal group $SO(4, 2)$ acting on the dual coordinates $x_i \implies$ **dual conformal symmetry.**

- ✓ Example: MHV amplitudes

$$\ln M_n^{\text{MHV}} = \ln Z_n(x_i, \epsilon, \mu) + \ln F_n(x_i) + O(\epsilon), \quad \ln Z_n = \sum_{l \geq 1} a^l \sum_{i=1}^n (-x_{i,i+2}^2 \mu^2)^{l\epsilon} \left(\frac{\Gamma_{\text{cusp}}^{(l)}}{(l\epsilon)^2} + \frac{\Gamma^{(l)}}{l\epsilon} \right)$$

- ✓ Wilson loop has conformal invariance in dual space \Rightarrow Anomalous CWI : [Drummond,Henn,Korchemsky, ES]

$$K^\mu \ln F_n = \frac{1}{2} \Gamma_{\text{cusp}}(a) \sum_{i=1}^n \ln \frac{x_{i,i+2}^2}{x_{i-1,i+1}^2} x_{i,i+1}^\nu \Rightarrow \text{Fixes } \ln F_n \text{ for } n = 4, 5 \text{ but not for } n \geq 6$$

Dual conformal symmetry II

- ✓ What about non-MHV amplitudes? Need to study the helicity structures and the loop corrections
- ✓ Spinor helicity formalism: commuting spinors λ^α (helicity -1/2), $\tilde{\lambda}^{\dot{\alpha}}$ (helicity 1/2) [Xu,Zhang,Chang]

$$p_i^2 = 0 \Leftrightarrow p_i^{\alpha\dot{\alpha}} \equiv p_i^\mu (\sigma_\mu)^{\alpha\dot{\alpha}} = \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}}$$

- ✓ Simplest case: MHV tree level [Parke, Taylor]

$$\mathcal{A}_{n;0}^{\text{MHV}}(\dots i^- \dots j^- \dots) = \delta^{(4)}(\sum_{k=1}^n p_k) \frac{\langle i j \rangle^4}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle}, \quad \langle i j \rangle = -\langle j i \rangle = \epsilon^{\alpha\beta} \lambda_{i\alpha} \lambda_{j\beta}$$

- ✓ Dual conformal symmetry = Poincaré + conformal inversion: [Drummond,Henn,Korchemsky, ES]

$$I[x_i^\mu] = \frac{x_i^\mu}{x_i^2}, \quad I[\lambda_i^\alpha] = \frac{\lambda_i^\alpha (x_i)_{\alpha\dot{\alpha}}}{x_i^2} \Rightarrow I[\langle i i+1 \rangle] = \frac{\langle i i+1 \rangle}{x_i^2}$$

- ✓ Split-helicity tree amplitudes $\mathcal{A}_{n;0}(+ + + - - - + + + +)$ are manifestly dual conformal
- ✓ Non-split-helicity tree amplitudes are dual conformal as parts of superamplitudes
- ✓ Superamplitudes in dual superspace exhibit dual superconformal symmetry.

Superamplitudes in on-shell superspace I

- ✓ $\mathcal{N} = 4$ gluon supermultiplet \rightarrow PCT self-conjugate \rightarrow holomorphic (chiral) description

$$\Phi(p, \eta) = G^+(p) + \eta^A \Gamma_A(p) + \eta^A \eta^B S_{AB}(p) + \eta^A \eta^B \eta^C \epsilon_{ABCD} \bar{\Gamma}^D(p) + \eta^A \eta^B \eta^C \eta^D \epsilon_{ABCD} G^-(p)$$

η^A ($SU(4)$ index $A = 1 \dots 4$, helicity $1/2$) are Grassmann variables of **on-shell superspace**

- ✓ Superamplitudes $\mathcal{A}_n(\Phi(1) \dots \Phi(n))$ = expansion in powers of η_i^A

- ✓ Example: **Nair's** description of tree MHV amplitudes

[Nair]

$$\mathcal{A}_n^{\text{MHV}} = \frac{\delta^{(4)}(\sum_{i=1}^n p_i) \delta^{(8)}(\sum_{j=1}^n \lambda_j \alpha \eta_j^A)}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle} = \frac{\delta^{(4)}(\sum_{i=1}^n p_i)}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle} \left(\langle 1 2 \rangle^4 \eta_1^4 \eta_2^4 \eta_3^0 \dots \eta_n^0 + \dots \right)$$

- ✓ On-shell $\mathcal{N} = 4$ supersymmetry

✗ super-Poincaré

$$q_\alpha^A = \lambda_\alpha \eta^A, \quad \bar{q}_A \dot{\alpha} = \tilde{\lambda}_{\dot{\alpha}} \frac{\partial}{\partial \eta^A}, \quad p_{\alpha \dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}} : \quad \{q_\alpha^A, \bar{q}_B \dot{\alpha}\} = \delta_B^A p_{\alpha \dot{\alpha}}$$

✗ super-conformal \rightarrow **non-local** (2nd-order)

[Witten]

$$s_A^\alpha = \frac{\partial^2}{\partial \lambda_\alpha \partial \eta^A}, \quad \bar{s}_{\dot{\alpha}}^A = \eta^A \frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}}, \quad k_{\alpha \dot{\alpha}} = \frac{\partial^2}{\partial \lambda^\alpha \partial \tilde{\lambda}^{\dot{\alpha}}} : \quad \{s_A^\alpha, \bar{s}_{\dot{\alpha}}^B\} = \delta_A^B k_{\alpha \dot{\alpha}}$$

Superamplitudes in on-shell superspace II

- ✓ Invariance of the superamplitude: $p_{\alpha\dot{\alpha}} \mathcal{A}_n = q_\alpha^A \mathcal{A}_n = 0 \Rightarrow$

$$\mathcal{A}_n(\lambda, \tilde{\lambda}, \eta) = \delta^{(4)}(p) \delta^{(8)}(q) \left[\mathcal{A}_n^{(0)} + \mathcal{A}_n^{(4)} + \dots + \mathcal{A}_n^{(4n-16)} \right]$$

- ✓ $\mathcal{A}_n^{(4k)}(\eta)$ – homogeneous polynomials in η of degree $4k$:
 $k = 0 \rightarrow \text{MHV}, k = 1 \rightarrow \text{Next-to-MHV}, \dots, k = n - 4 \rightarrow \overline{\text{MHV}}$

- ✓ Define ‘ratio’ $R = \text{general/MHV superamplitude}$:

$$\mathcal{A}_n = \mathcal{A}_n^{\text{MHV}} \times \left[R_n(\lambda, \tilde{\lambda}, \eta) + O(\epsilon) \right] = \mathcal{A}_n^{\text{MHV}} \left[1 + R_n^{(4)} + \dots + R_n^{(4n-16)} + O(\epsilon) \right]$$

$R_n^{(4k)}$: finite homogeneous polynomials in $\eta \rightarrow$ helicity structures and loop corrections

- ✓ Conjecture: $R_n^{(4k)}$ are exactly dual conformal. Conformal anomaly in IR divergent MHV factor.
 Checked at:

- ✗ one loop for $k = 1$ (NMHV), any n
- ✗ two loops for $k = 1, n = 6$

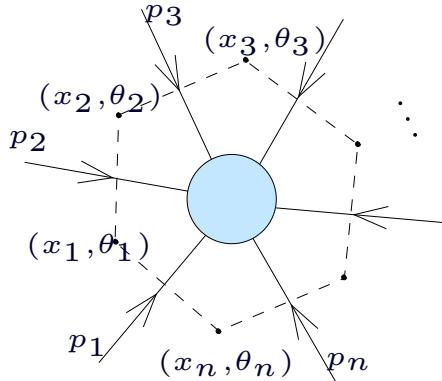
[Drummond,Henn,Korchemsky, ES]

[Kosower, Roiban, Vergu]

- ✓ Ordinary superconformal at tree level, but broken by IR divergences at loop level - how?

Dual $\mathcal{N} = 4$ superconformal symmetry

- ✓ Chiral dual superspace $(x_{\alpha\dot{\alpha}}, \theta_\alpha^A, \lambda_\alpha)$:



$$\begin{aligned} \times \quad p &= \sum_{i=1}^n p_i = 0 \rightarrow \quad p_i = x_i - x_{i+1}, \quad x_{n+1} = x_1 \\ \times \quad q &= \sum_{i=1}^n \lambda_i \eta_i = 0 \rightarrow \quad \lambda_{i\alpha} \eta_i^A = (\theta_i - \theta_{i+1})_\alpha^A, \quad \theta_{n+1} = \theta_1 \end{aligned}$$

- ✓ Dual $\mathcal{N} = 4$ superconformal symmetry in dual superspace

\times $\mathcal{N} = 4$ super-Poincaré algebra

$$Q_{A\alpha} = \sum_{i=1}^n \frac{\partial}{\partial \theta_i^{A\alpha}}, \quad \bar{Q}_{\dot{\alpha}}^A = \sum_{i=1}^n \theta_i^{A\alpha} \frac{\partial}{\partial x_i^{\dot{\alpha}\alpha}}, \quad P_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial x_i^{\dot{\alpha}\alpha}}; \quad \{Q_{A\alpha}, \bar{Q}_{\dot{\alpha}}^B\} = \delta_A^B P_{\alpha\dot{\alpha}}$$

\times Conformal inversion: $I[x_i] = x_i^{-1}, \quad I[\theta_i] = \theta_i x_i^{-1}, \quad I[\lambda_i] = \lambda_i x_i^{-1}$

Dual superconformal symmetry: MHV superamplitudes

- ✓ MHV superamplitude in dual superspace

$$\mathcal{A}_n^{\text{MHV}}(x, \theta, \lambda) = \frac{\delta^{(4)}(x_1 - x_{n+1}) \delta^{(8)}(\theta_1 - \theta_{n+1})}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle} [1 + a M_n^{(1)}(x_{ij}) + O(a^2)]$$

- ✗ Tree – manifestly dual superconformal covariant.
- ✗ Loops – IR divergent factor $M_n^{(1)}(x_{ij})$ satisfies anomalous dual conformal Ward identity
- ✗ Loops – dual supersymmetry \bar{Q} broken by $M_n(x_{ij})$

- ✓ Multi-particle discontinuity (branch cut)

$$\text{Disc}_{x_{1,j+1}^2} M_n^{(1)} = \ln \left[\frac{(1 - u_{1,n,j,j+1})(1 - u_{1,2,j+2,j+1})}{(1 - u_{1,2,j,j+1})(1 - u_{1,n,j+2,j+1})} \right], \quad u_{ijkl} = \frac{x_{il}^2 x_{jk}^2}{x_{ik}^2 x_{jl}^2}$$

IR finite and dual conformal, but not dual supersymmetric, why?

Dual superconformal symmetry: Holomorphic anomaly

- ✓ $\mathcal{A}_n^{\text{MHV}}$ is naively holomorphic in λ , but in reality

[Cachazo,Svrcek,Witten]

$$\frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}} \frac{1}{\langle \lambda \chi \rangle} = 2\pi \tilde{\chi}_{\dot{\alpha}} \delta(\langle \lambda, \chi \rangle) \delta([\tilde{\lambda}, \tilde{\chi}]) \rightarrow \text{holomorphic anomaly}$$

- ✓ Important when loops are made from trees via unitarity

[Cachazo],[Bena,Bern,Kosower,Roiban]

$$\text{Disc}_{s_1 \dots j} \mathcal{A}_n^{\text{MHV};1} = \mathcal{A}^{\text{MHV};0}(-\ell_1, 1, \dots, j, -\ell_2) \star \mathcal{A}^{\text{MHV};0}(\ell_2, j+1, \dots, n, \ell_1),$$

- ✓ Difference between K and \bar{Q}

$$K_{\alpha \dot{\alpha}} \frac{1}{\langle i i + 1 \rangle} = \sum_k (x_{k+1})_{\alpha}^{\dot{\beta}} \tilde{\lambda}_{k \dot{\alpha}} \frac{\partial}{\partial \tilde{\lambda}_k^{\dot{\beta}}} \frac{1}{\langle i i + 1 \rangle} = 0$$

$$\bar{Q}_{\dot{\alpha}}^A \frac{1}{\langle i i + 1 \rangle} = \sum_k \eta_k^A \frac{\partial}{\partial \tilde{\lambda}_k^{\dot{\alpha}}} \frac{1}{\langle i i + 1 \rangle} \neq 0$$

- ✓ Can we control the \bar{Q} -anomaly?

- ✗ The Wilson loop is only dual conformal, not **super**conformal
- ✗ The Wilson loop knows nothing about helicity
- ✗ Can we imagine a dual supersymmetric model for amplitudes with broken Poincaré SUSY???

Dual superconformal symmetry: NMHV superamplitudes

- ✓ General superamplitude: $\mathcal{A}_n = \mathcal{A}_n^{\text{MHV}}(a, 1/\epsilon) \left[1 + R_n^{(4)} + \dots + R_n^{(4n-16)} + O(\epsilon) \right]$
- ✓ Conjecture: $R_n^{(4)}$ are finite dual (super)conformal invariants [Drummond,Henn,Korchemsky,ES]

$$R_n^{(4)} = \sum_{p,q,r=1}^n R_{pqr} [1 + a M_{pqr}^{(1)}(x_{ij}) + O(a^2)]$$

✗ dual superconformal invariant

$$R_{pqr} = \frac{\langle q-1|q\rangle\langle r-1|r\rangle \delta^{(4)}(\langle p|x_{pq}x_{qr}|\theta_{rp}\rangle + \langle p|x_{pr}x_{rq}|\theta_{qp}\rangle)}{x_{qr}^2 \langle p|x_{pr}x_{r-1}|q-1\rangle \langle p|x_{pr}x_{r-1}|q\rangle \langle p|x_{pq}x_{q-1}|r-1\rangle \langle p|x_{pq}x_{q-1}|r\rangle}$$

- ✗ dual conformal invariant $M_{pqr}^{(1)}$, made of finite combinations of one-loop scalar box integrals
- ✗ All coefficients = 1, why? Fixed by analytic properties:
 - Absence of spurious singularities at $\langle p|x_{pr}x_{r-1}|q\rangle = 0$, etc. [Korchemsky,ES]
 - “Deform” generators to absorb the anomaly [Bargheer, Beisert, Galleas, Henn, Loebbert, McLoughlin, Plefka]

- ✓ Complete n -particle tree found by BCFW recursion and shown to be dual superconformal [Brandhuber,Heslop,Travaglini], [Drummond,Henn]

Conventional conformal symmetry and twistor transform

- ✓ Are trees both **dual** and **conventional** superconformal? Apply 2-nd-order conventional superconformal generators (not easy):

✗ MHV tree is invariant

[Witten]

✗ NMHV: each R_{pqr} is invariant by itself; need analytic properties to fix coeffs

[Korchemsky,ES]

- ✓ Better way: do twistor ("half-Fourier") transform: $\tilde{\lambda}_{\dot{\alpha}} \rightarrow \mu^{\dot{\alpha}}$, $\eta^A \rightarrow \psi_A$, but not λ^{α}

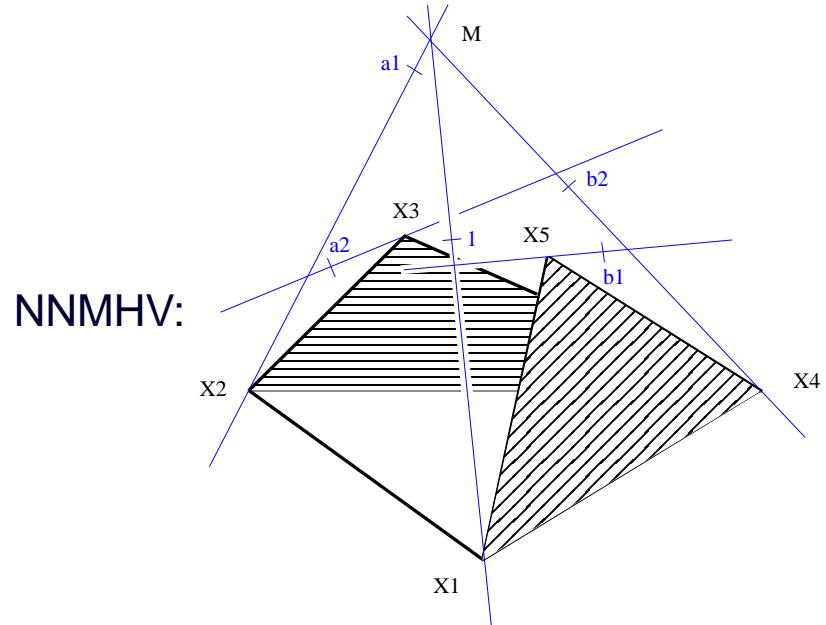
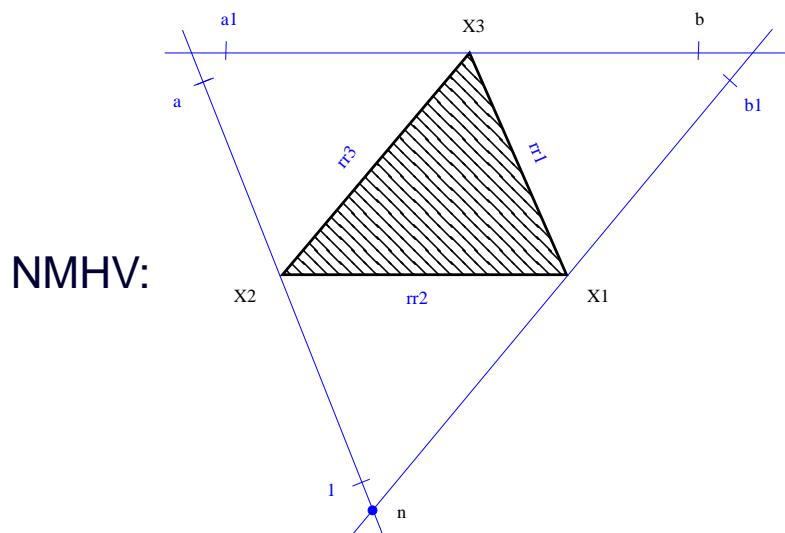
[Witten]

$$TT \left[\mathcal{A}_{n;0}^{\text{MHV}} \right] = \int d^4 X d^8 \Theta \prod_{i=1}^n \frac{\delta^{(2)}(\mu_i + \langle \lambda_i | X) \delta^{(4)}(\psi_i + \langle \lambda_i | \Theta \rangle)}{\langle i | i+1 \rangle}$$

- ✗ Conventional conformal generators become 1-st-order – easy to check invariance.
 ✗ All particles lie on a line in twistor space with moduli $X_{\alpha\dot{\alpha}}$, Θ_A^{α}

- ✓ Twistor transform of N^k MHV tree: each term supported on $2k + 1$ intersecting lines

[Korchemsky,ES]



Grassmannian approach

- ✓ Conventional superconformal symmetry made manifest as an integral of supertwistors over a Grassmannian manifold $G(n, k+2)$ [Arkani-Hamed,Cachazo,Cheung,Kaplan]
- ✓ Dual superconformal symmetry made manifest as an integral of momentum supertwistors over a Grassmannian manifold $G(n, k)$
 - ✗ Momentum supertwistors [Mason,Skinner] [Hodges]

$$W = \begin{pmatrix} \lambda^\alpha \\ \nu_{\dot{\alpha}} \equiv x_{\dot{\alpha}\alpha} \lambda^\alpha \\ \chi^A \equiv \theta_\alpha^A \lambda^a \end{pmatrix} \Rightarrow \text{linear realization of dual } SU(2, 2|4)$$

- ✗ Most general dual superconformal invariant (John or "X-ray" transform)

$$R_n^k(W) = \int \frac{D^{k(n-k)} t}{\omega(t)} \prod_{a=1}^k \delta^{(4|4)} \left(\sum_{i=1}^n t_a^i W_i \right)$$

- ✗ Very special measure made from consecutive minors (Plücker coordinates on $G(n, k)$)

$$\omega(t) = \prod_{i=1}^n T_k^{(i)}, \quad T_k^{(i)} = \frac{1}{k!} \epsilon^{a_1 a_2 \dots a_k} t_{a_1}^{i} t_{a_2}^{i+1} \dots t_{a_k}^{i+k-1}$$

- ✗ **Uniqueness theorem:** only this measure assures both dual (manifest) and conventional (non-manifest) superconformal invariance [Korchemsky,ES] [Drummond,Ferro]
- ✓ Choice of contour? Leading singularity of loop integrals?

Classification of the superinvariants I

- ✓ General dual superconformal invariant in momentum twistor space

✗ Simple example: $SL(1|1)$:

$$\mathcal{Q} R_n(w_i, \chi_i) = \tilde{\mathcal{Q}} R_n(w_i, \chi_i) = 0, \quad \{\mathcal{Q}, \tilde{\mathcal{Q}}\} = \mathcal{C} = \text{helicity}$$

Solution:

$$\mathcal{Q}: \chi_1 = 0 \Rightarrow R_n = f(w) + \chi_{\hat{i}} f^{\hat{i}}(w) + \chi_{\hat{i}_1} \chi_{\hat{i}_2} f^{\hat{i}_1 \hat{i}_2}(w) + \dots + \chi_{\hat{i}_1} \dots \chi_{\hat{i}_{n-1}} f^{\hat{i}_1 \dots \hat{i}_{n-1}}(w)$$

$$\tilde{\mathcal{Q}}: \partial^{\hat{i}} f(w) = \partial^{[\hat{i}} f^{\hat{i}_1]}(w) = \dots = \partial^{[\hat{i}} f^{\hat{i}_1 \dots \hat{i}_{n-2}]}(w) = 0, \quad \text{no constraints on } f^{\hat{i}_1 \dots \hat{i}_{n-1}}(w)$$

✗ Integral representation of the solution

$$R_n^k = \chi_{\hat{i}_1} \dots \chi_{\hat{i}_k} f^{\hat{i}_1 \dots \hat{i}_k}(w) = \int Dt \tilde{r}_k(t) \prod_{a=1}^k \delta(t_a^i w_i) \delta(t_a^i \chi_i)$$

John transform for the bosonic coefficients, with redundant image $\tilde{r}_k(t)$, e.g.,

$$f^{\hat{i}}(w) = \int Dt t_1^{\hat{i}} \tilde{r}_1(t) \delta(t_1^1 w_1 + \sum_{\hat{i}=2}^n t_1^{\hat{i}} w_{\hat{i}})$$

✗ Back to $SL(4|4)$: $R_n^k(W)$ as the top component of an invariant in a k -dimensional subspace

$$R_n^k(W) = \int [\mathcal{D}t]_{n,k} \prod_{a=1}^k \delta^{(4|4)} \left(\sum_{i=1}^n t_a^i W_i \right) \quad \text{with some measure } [\mathcal{D}t]_{n,k}$$

Classification of the superinvariants II

✓ Properties of the Grassmannian measure $[\mathcal{D}t]_{n,k}$

✗ Global $GL(n)$ and local $GL(k)$ invariance \leftrightarrow Grassmannian $G(k, n)$:

$$[\mathcal{D}t]_{n,k} = \tilde{r}(t) D^{k(n-k)} t$$

with the natural measure $D^{k(n-k)} t$ on $G(k, n)$ and some weight $\tilde{r}(t)$

✗ Helicity (local rescaling of each t^i) invariance achieved if we take

$$[\mathcal{D}t]_{n,k} = \tilde{r}(t) D^{k(n-k)} t, \quad \text{with} \quad \tilde{r}(t) = \frac{\omega(t)}{T_k^{(1)} T_k^{(2)} \dots T_k^{(n)}}$$

with an arbitrary helicity-less and $GL(k)$ invariant function $\omega(t)$.

✓ What can fix the freedom in $\omega(t)$? Answer: conventional conformal symmetry imposed

✗ either by second-order (non-local) conformal generators

[Drummond, Ferro]

✗ or by first doing a twistor transform which renders the symmetry local \rightarrow Twistor space support on intersecting twistor lines

[Korchemsky, ES]

✗ Conventional conformal symmetry and helicity \rightarrow Ward identities with

unique solution $\omega(t) = \text{const}$

✓ What can fix the constant?

Conclusions and outlook

- ✓ Dual superconformal symmetry is a universal feature of $\mathcal{N} = 4$ scattering amplitudes
- ✓ Field-theory origin unknown (dynamical)
- ✓ AdS/CFT: fermionic T-duality is a symmetry of the string sigma model, but what is the object dual to amplitudes with helicity? Minimal surfaces with fermions? [Berkovits, Maldacena], [Beisert,Ricci,Tseytlin]
- ✓ MHV/Wilson loop duality does not see the helicity structure. Need to supersymmetrize (?) the Wilson loop and test if it is dual to non-MHV superamplitudes
- ✓ The closure of ordinary & dual superconformal symmetries is an infinite-dimensional Yangian → integrability? [Drummond,Henn,Plefka]
- ✓ Caution: the symmetries alone do not completely fix even the tree, more input needed (BCFW, analytic properties, ...)
- ✓ Symmetries are anomalous at loop level → useless if we cannot control the breaking. Known how to do it for **dual** conformal symmetry (UV breaking), but not for **conventional** (IR breaking?)
- ✓ Grassmannian approach: is it more than just elegant representation theory? We hope so!
- ✓ What fixes the remainder function in the MHV amplitude? Integrability? New symmetries?