Circuit
Quantum Electrodynamics (QED)

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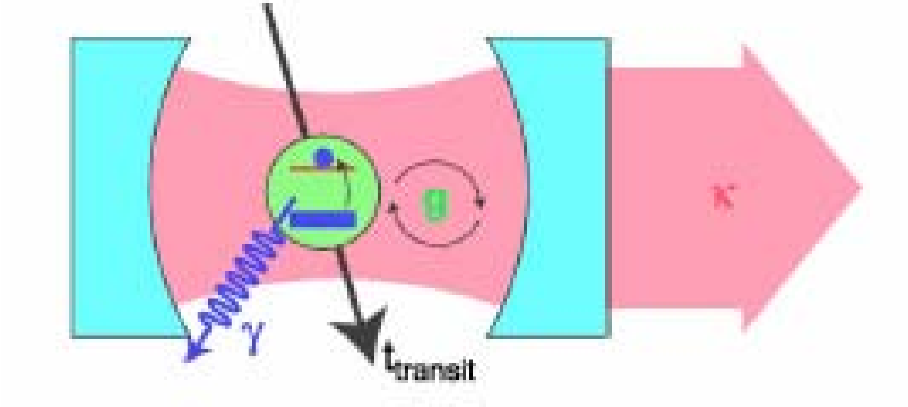
Atom in a Cavity

Consider only two levels of atom, with energy separation $\hbar\Omega$

Atom drifts through electromagnetic resonant cavity with very high Q



Jaynes-Cummings Hamiltonian:

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a) + H_\kappa + H_\gamma.$$

coupling strength $g = \mathcal{E}_{rms} d/\hbar$

cavity decay rate $\kappa = \omega_r/Q$.

decay rate to non-cavity modes - γ

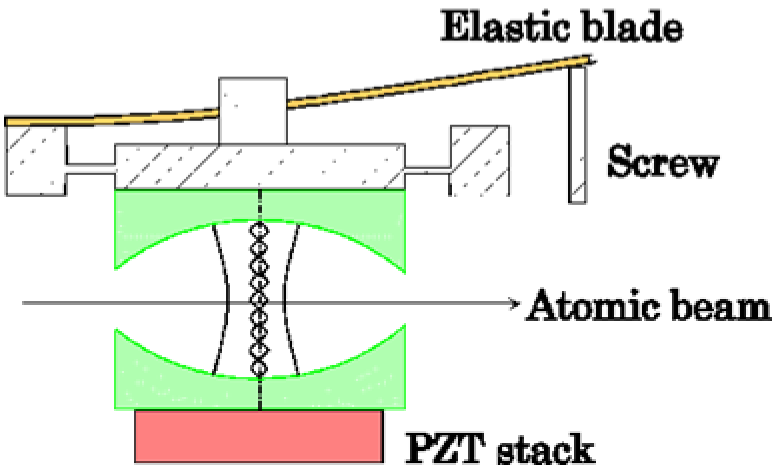
Strong Coupling limit

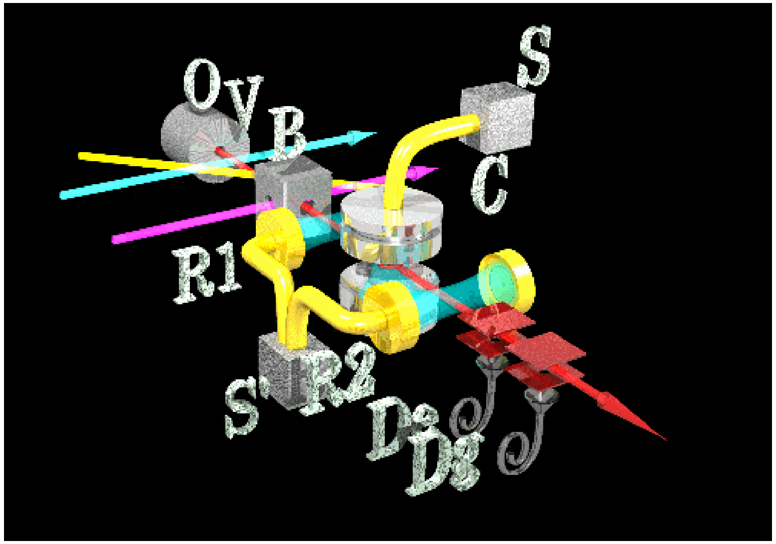
$$g >> \kappa, \gamma$$

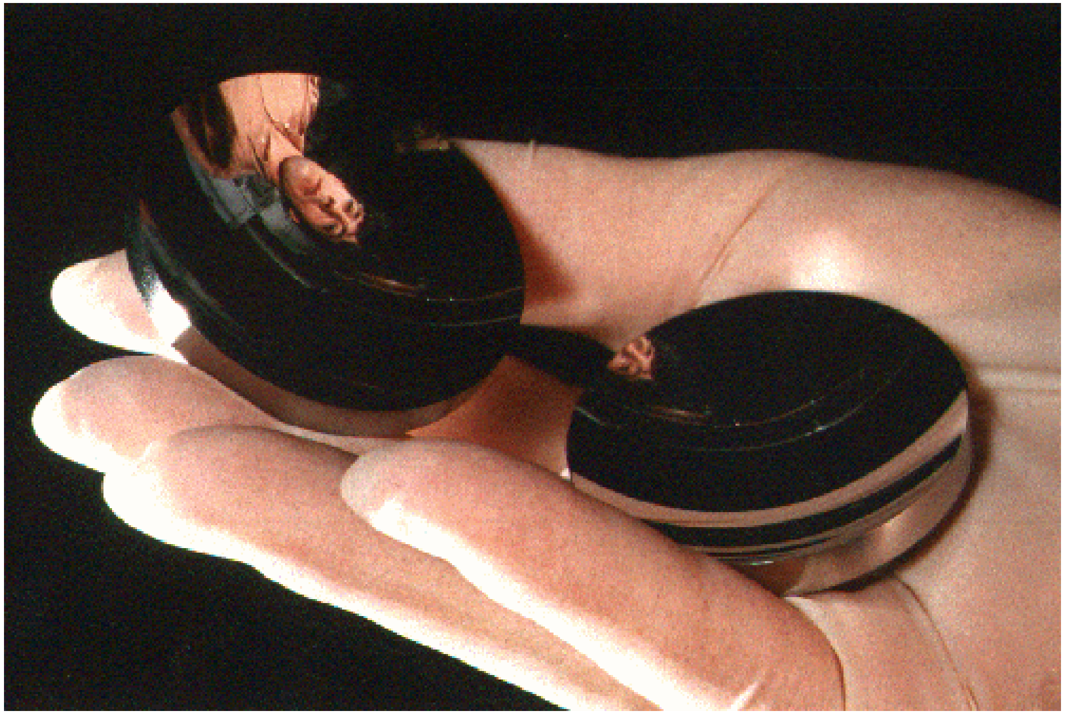
Needed:
High Q
Large d

Blais et. al, Phys. Rev. A, 2004

Electropolished superconducting Nb cavity







50 mm diameter and a 40 mm radius of curvature

Nice presentation at: http://www.lkb.ens.fr/recherche/qedcav/english/englishframes.html

Energy Eigenstates

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a) + \cancel{H_\kappa} + \cancel{H_\gamma}$$

Neglect damping for the moment, exact diagonalization gives energy eigenstates:

$$|\overline{+,n}\rangle = \cos\,\theta_n|\downarrow,n\rangle + \sin\,\theta_n|\uparrow,n+1\rangle,$$

$$|\overline{-,n}\rangle = -\sin\,\theta_n|\downarrow,n\rangle + \cos\,\theta_n|\uparrow,n+1\rangle,$$

$$\theta_n = \frac{1}{2}\tan^{-1}\left(\frac{2g\sqrt{n+1}}{\Delta}\right), \qquad E_{\pm,n} = (n+1)\hbar\omega_r \pm \frac{\hbar}{2}\sqrt{4g^2(n+1)+\Delta^2}, \qquad E_{\uparrow,0} = -\frac{\hbar\Delta}{2}.$$

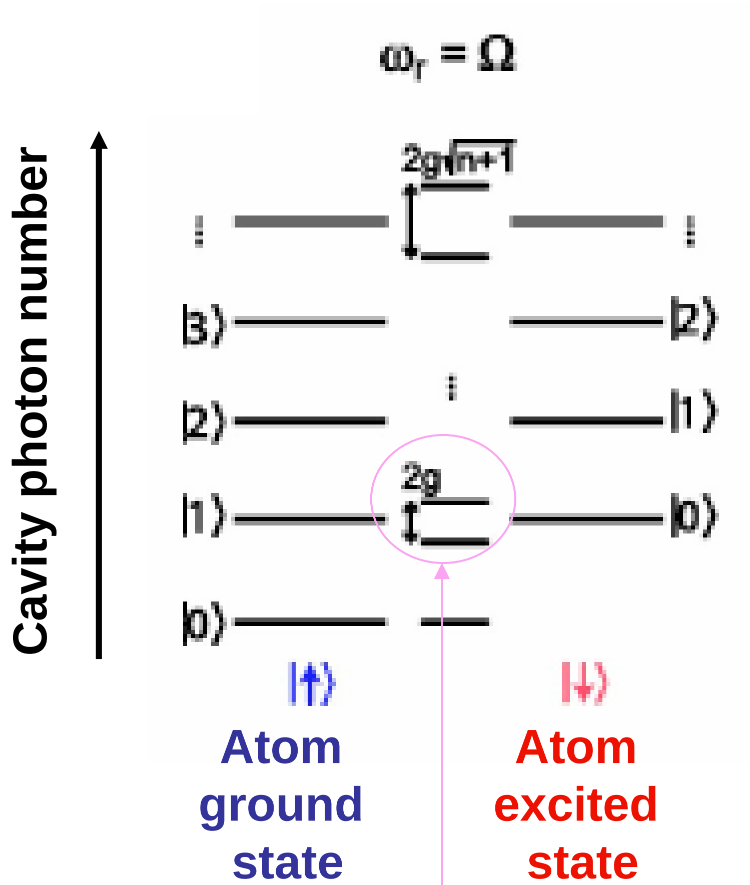
$$\boxed{\Delta \equiv \Omega - \omega_r \text{ the atom-cavity detuning.}}$$

For zero detuning the degeneracy of the photon states with the atom state is lifted by the coupling. These “dressed states” are “maximally entangled” atom – field states

$$|\overline{\pm,0}\rangle = (|\uparrow,1\rangle \pm |\downarrow,0\rangle)/\sqrt{2}$$

Blais et. al, Phys. Rev. A, 2004

Spectrum



$|\pm,0\rangle = (|\uparrow,1\rangle \pm |\downarrow,0\rangle)/\sqrt{2}$

Vacuum Rabi flopping:

$$\left|\downarrow,0\right\rangle \qquad \left|\uparrow,1\right\rangle$$

0 photons in cavity
atom in excited state

atom in GS
1 photon in cavity

Level splitting depends on number of photons in cavity

$$2g\sqrt{n+1}$$

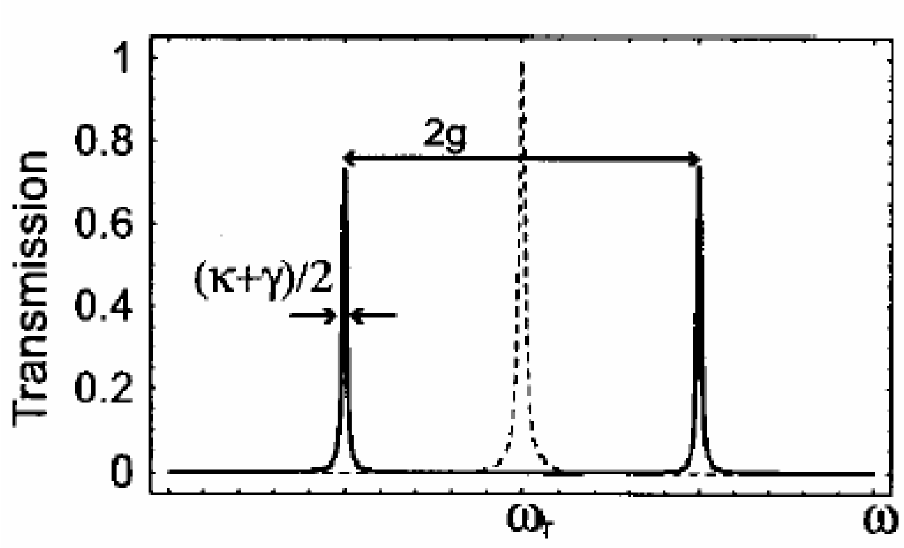
Blais et. al, Phys. Rev. A, 2004

Splitting of Cavity Resonance

Now consider damping: excitation is ½ photon, ½ atom $\Rightarrow$ decay rate: $\dfrac{\kappa+\gamma}{2}$

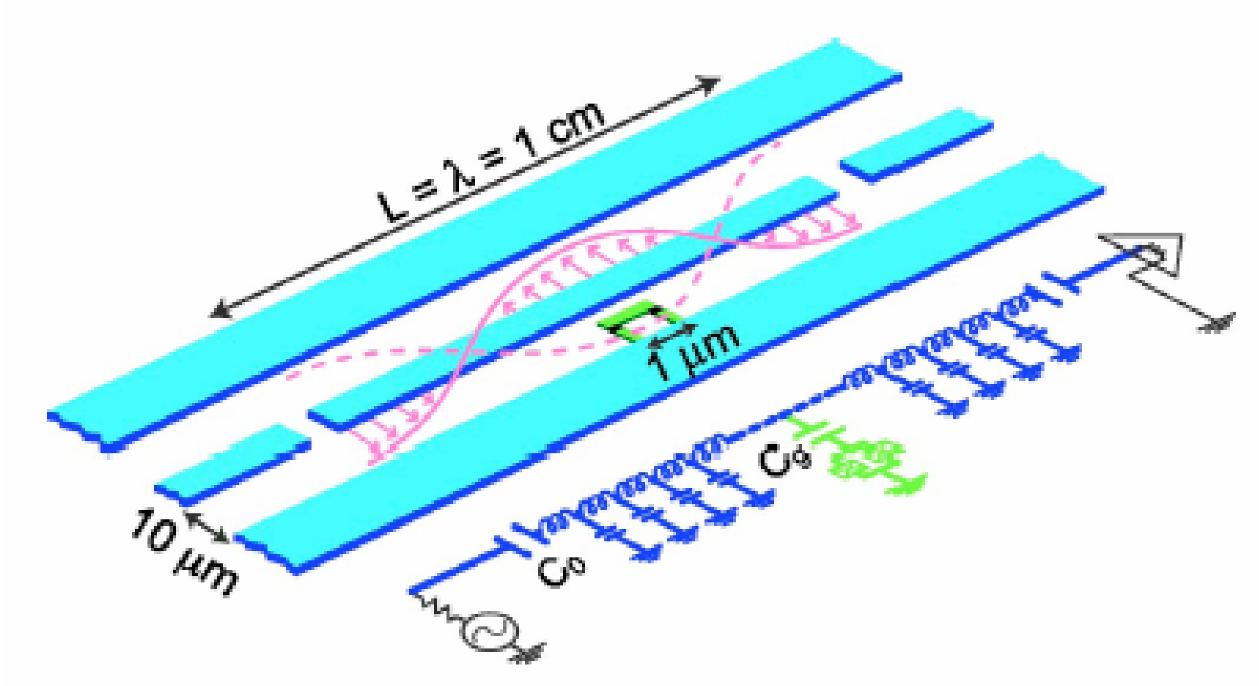
In *strong coupling limit* there is a splitting of cavity resonance which can be resolved because:

$$\boxed{g = \frac{\mathcal{E}_{\mathrm{rms}} d}{\hbar} >> \kappa, \gamma}$$



Blais et. al, Phys. Rev. A, 2004

CP box in Microstrip line cavity



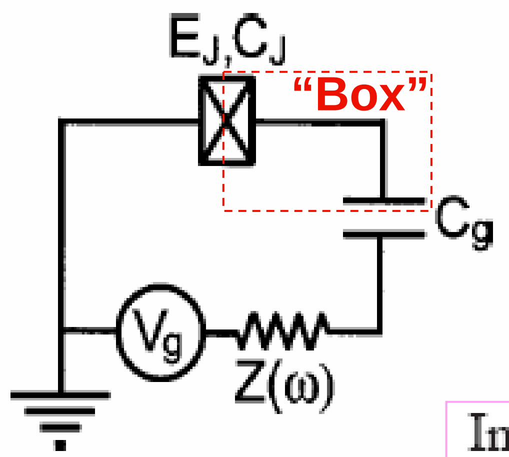
At a resonant frequency of 10 GHz ($h\nu/k_B \sim 0.5$ K)

$V_{\mathrm{rms}}^0 \sim \sqrt{\hbar\omega_r/cL} \sim 2\ \mu\mathrm{V}$ $\qquad \mathcal{E}_{\mathrm{rms}} \sim 0.2\ \mathrm{V/m}$

Very small effective volume, ~10^{-5} cubic wavelengths

Blais et. al, Phys. Rev. A, 2004

Cooper Pair Box as TLS



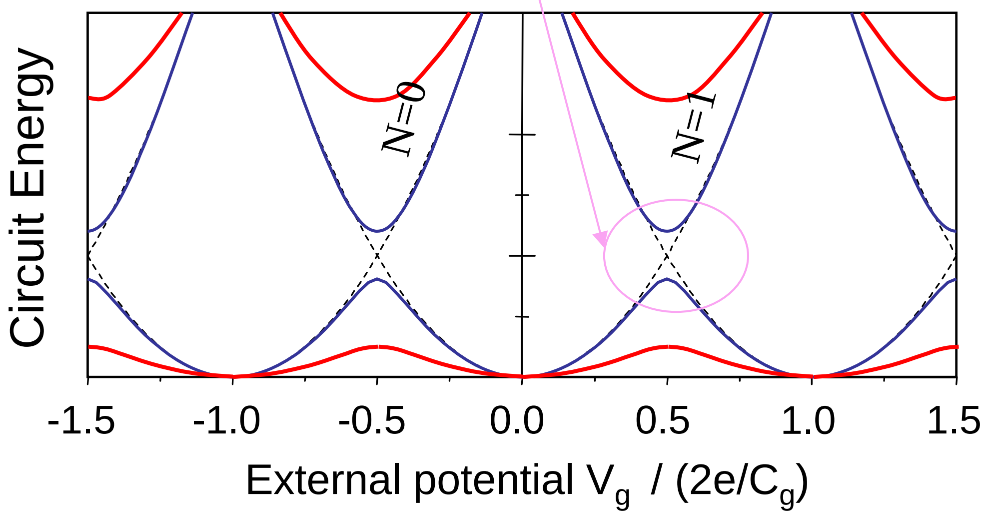
$$H_Q = 4E_c \sum_N (N - N_g)^2 |N\rangle\langle N| - \frac{E_J}{2} \sum_N \left(|N+1\rangle\langle N| + \mathrm{H.c.}\right),$$

$$E_c = e^2 / 2C_\Sigma \qquad E_J = \frac{\hbar}{2e} I_C = \frac{\hbar \pi \Delta(T)}{4e^2 R_N}$$

$$\boxed{H_Q = -\frac{E_{\mathrm{el}}}{2} \sigma^z - \frac{E_J}{2} \sigma^x,}$$

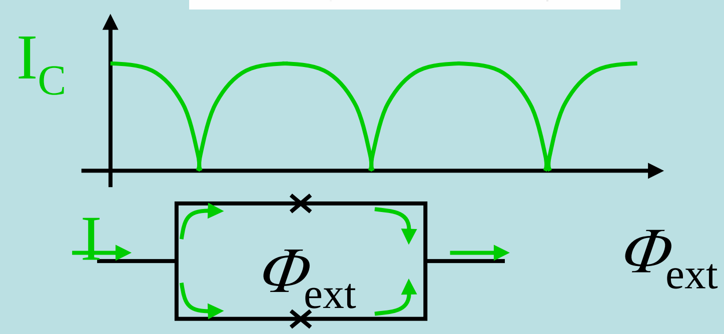
$$E_{\mathrm{el}} = 4E_C(1 - 2N_g)$$

In the charge regime $4E_c \gg E_J$



SQUID junction →tune E_J

$E_J \cos(\pi \Phi_{\mathrm{ext}} / \Phi_0) / 2$



Map to James-Cummings Hamiltonian

$$H = \hbar\omega_r\left(a^\dagger a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^z + \hbar g(a^\dagger\sigma^- + \sigma^+ a) + H_\kappa + H_\gamma.$$

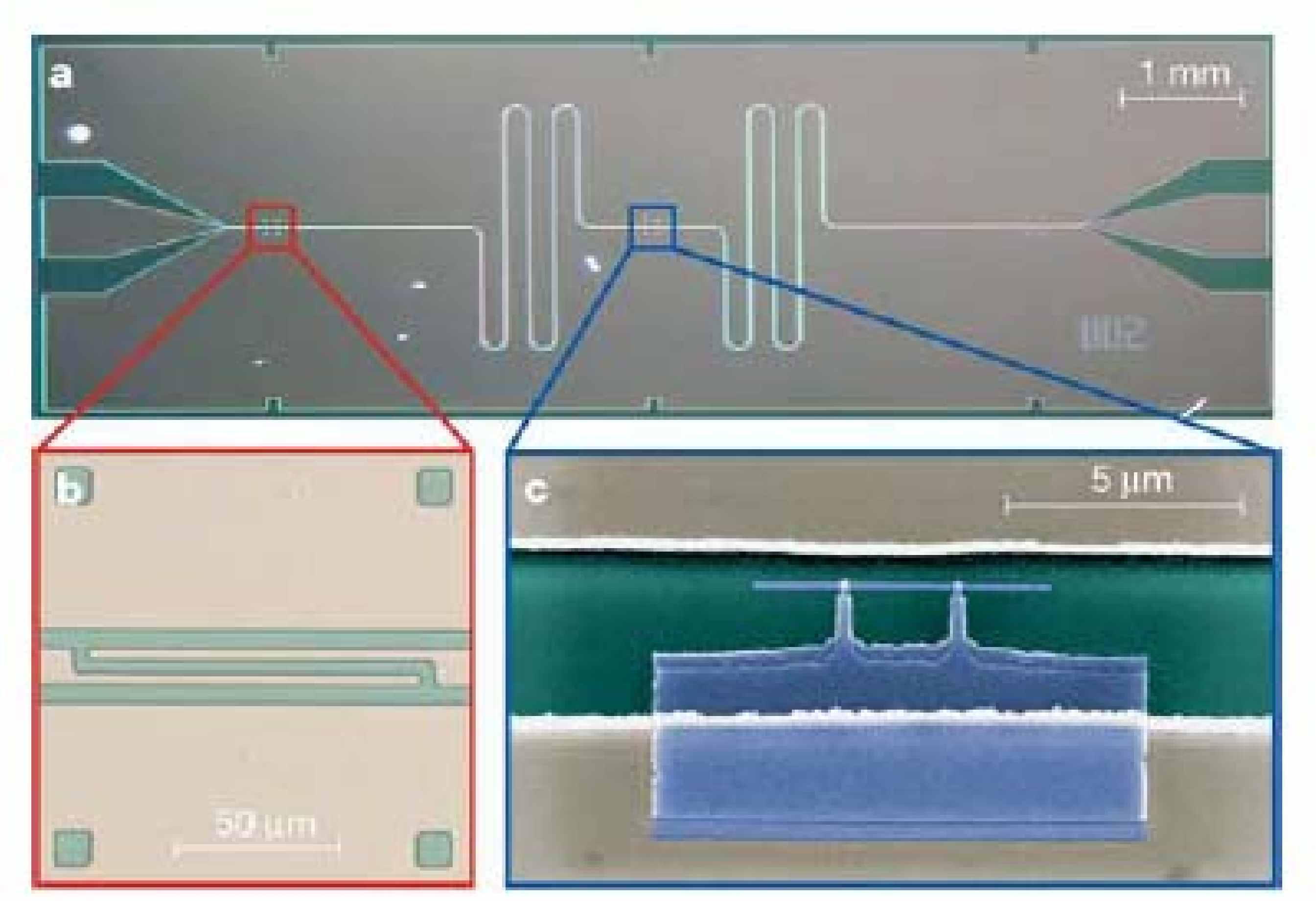
$$\Omega = E_J/\hbar \qquad g = \frac{\beta e}{\hbar}\sqrt{\frac{\hbar\omega_r}{cL}} \qquad \beta \equiv C_g/C_\Sigma$$

Very large effective dipole moment:

$$d \equiv \hbar g/\mathcal{E}_{\text{rms}} \sim 2 \times 10^4 \text{ atomic units } (ea_0)$$

Blais et. al, Phys. Rev. A, 2004

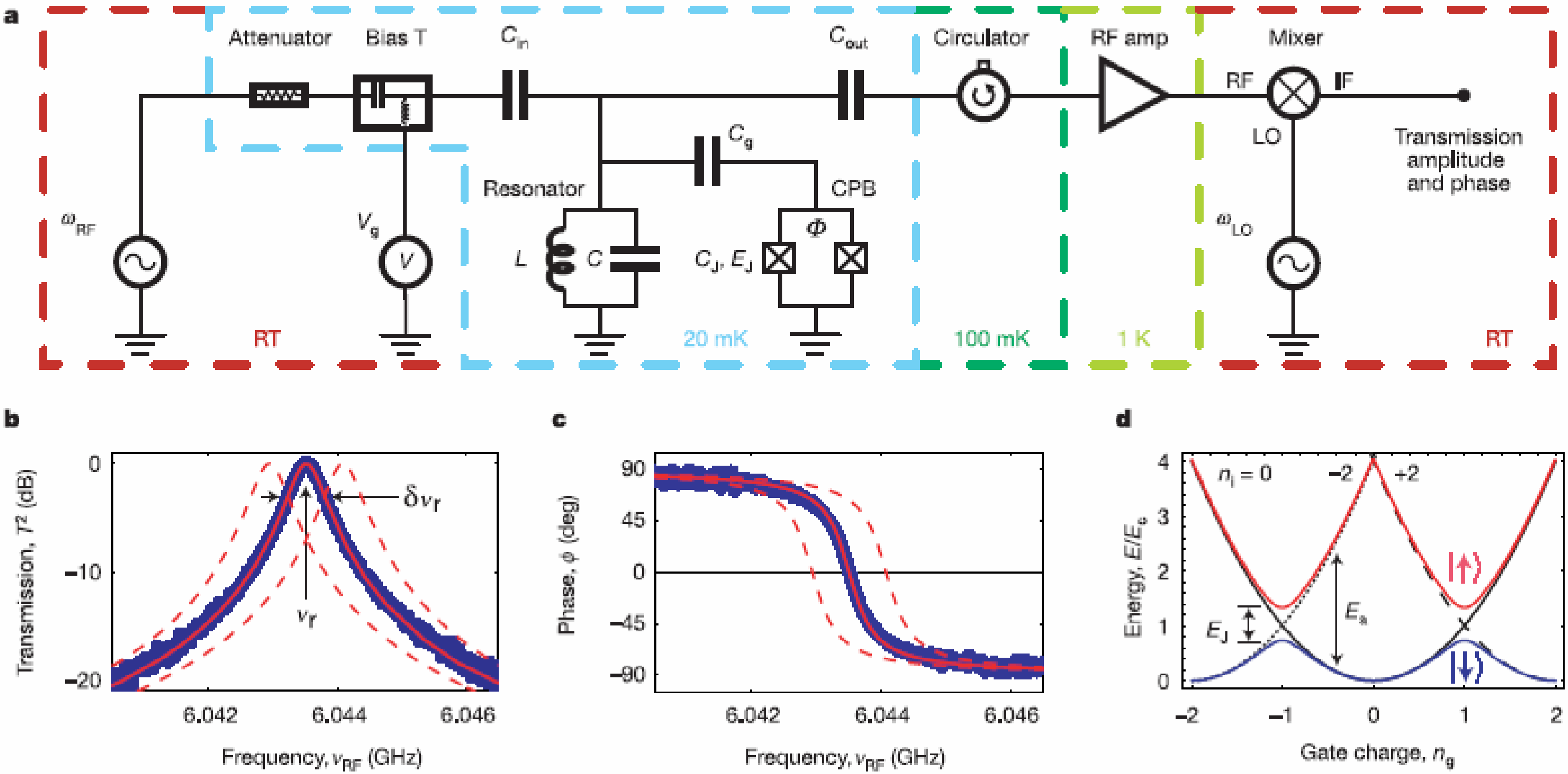
Experiment 1 by Yale Group



Nb cavity on Si/SiO2 substrate, length 24 cm,

Wallraff et. al, NATURE, 2004

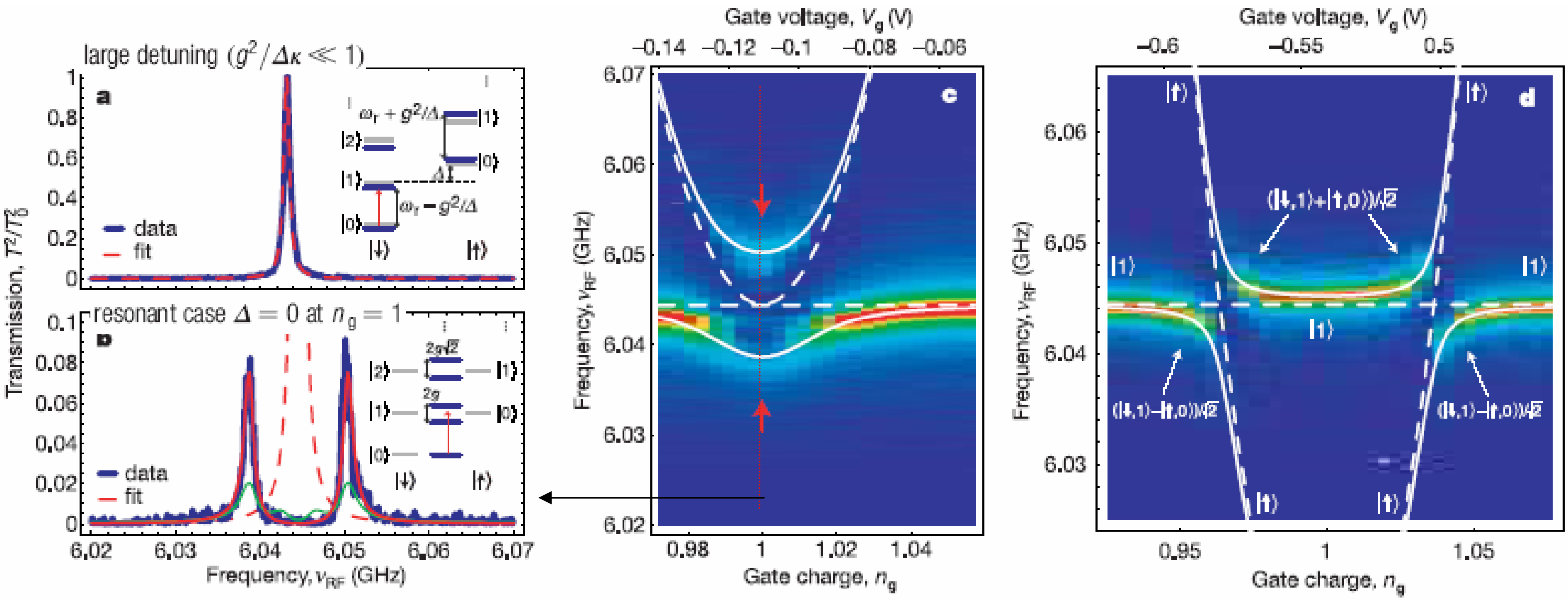
Schematic of measurement



$$Q = \nu_r / \delta\nu_r \approx 10^4.$$

Wallraff et. al, NATURE, 2004

Vacuum Rabi splitting



n=0.06

n=0.5

Blue – small T, Red – large T

By fitting the split cavity resonance, they can determine the mean number of thermal photons in the cavity

Wallraff et. al, NATURE, 2004

Atom vs. Circuit implementation

Atomic Physics:

Measure shift of atom level which drifts through cavity and infer the state of photons in the cavity

Circuit QED:

Directly measure transmission of cavity and observe splitting of cavity resonance.

“Atom” replaced by a superconducting circuit with quantized energy.

Circuit does not drift, no transit time.

Circuit two-level-system can be tuned with external voltage and current.

Comparison

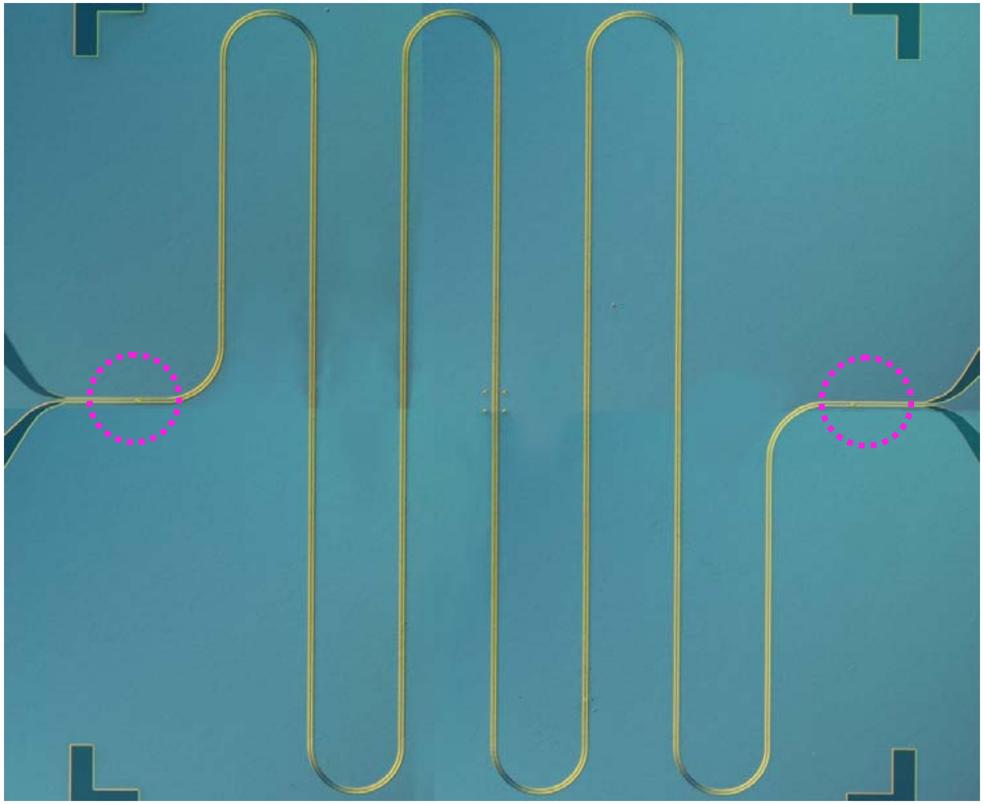
TABLE I. Key rates and CQED parameters for optical [2] and microwave [3] atomic systems using 3D cavities, compared against the proposed approach using superconducting circuits, showing the possibility for attaining the strong cavity QED limit ($n_{\mathrm{Rabi}} \gg 1$). For the 1D superconducting system, a full-wave ($L=\lambda$) resonator, $\omega_r/2\pi = 10$ GHz, a relatively low Q of 10^4, and coupling $\beta = C_g/C_\Sigma = 0.1$ are assumed. For the 3D microwave case, the number of Rabi flops is limited by the transit time. For the 1D circuit case, the intrinsic Cooper-pair box decay rate is unknown; a conservative value equal to the current experimental upper bound $\gamma \lesssim 1/(2\ \mu\mathrm{s})$ is assumed.

Parameter	Symbol	3D optical	3D microwave	1D circuit
Resonance or transition frequency	$\omega_r/2\pi$, $\Omega/2\pi$	350 THz	51 GHz	10 GHz
Vacuum Rabi frequency	g/π, g/ω_r	220 MHz, 3×10^{-7}	47 kHz, 1×10^{-7}	100 MHz, 5×10^{-3}
Transition dipole	d/ea_0	~1	1×10^3	2×10^4
Cavity lifetime	$1/\kappa, Q$	10 ns, 3×10^7	1 ms, 3×10^8	160 ns, 10^4
Atom lifetime	$1/\gamma$	61 ns	30 ms	2 μs
Atom transit time	t_{transit}	$\geqslant 50\ \mu$s	100 μs	∞
Critical atom number	$N_0 = 2\gamma\kappa/g^2$	6×10^{-3}	3×10^{-6}	$\leqslant 6\times10^{-5}$
Critical photon number	$m_0 = \gamma^2/2g^2$	3×10^{-4}	3×10^{-8}	$\leqslant 1\times10^{-6}$
Number of vacuum Rabi flops	$n_{\mathrm{Rabi}} = 2g/(\kappa+\gamma)$	~10	~5	$\sim10^2$

- Cooper Pair Box does not drift, stays in place $t_{\mathrm{transit}} = \infty$
- Examine one (and the same) quantum system
- Tune parameters of CPB Hamiltonian with external gate voltage and magnetic flux

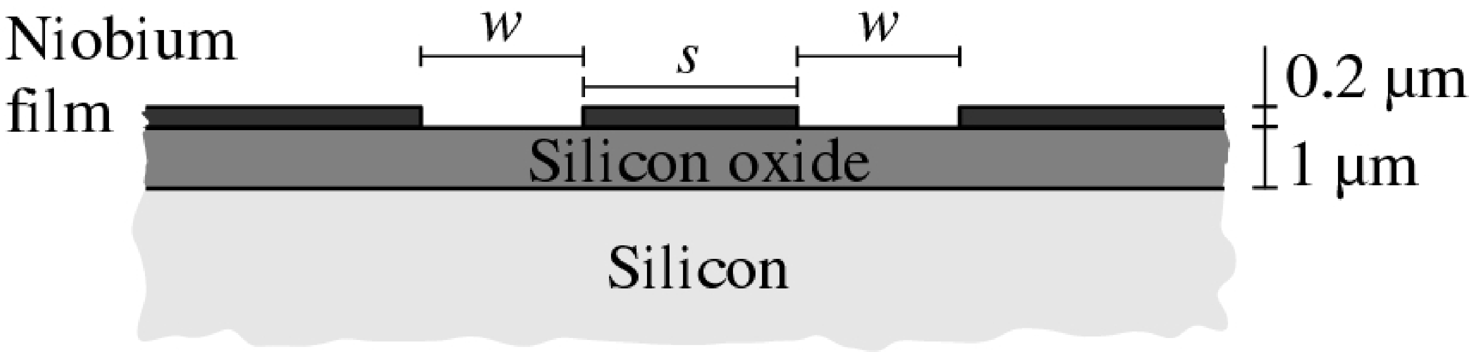
Blais et. al, Phys. Rev. A, 2004

Superconducting cavities at Albanova

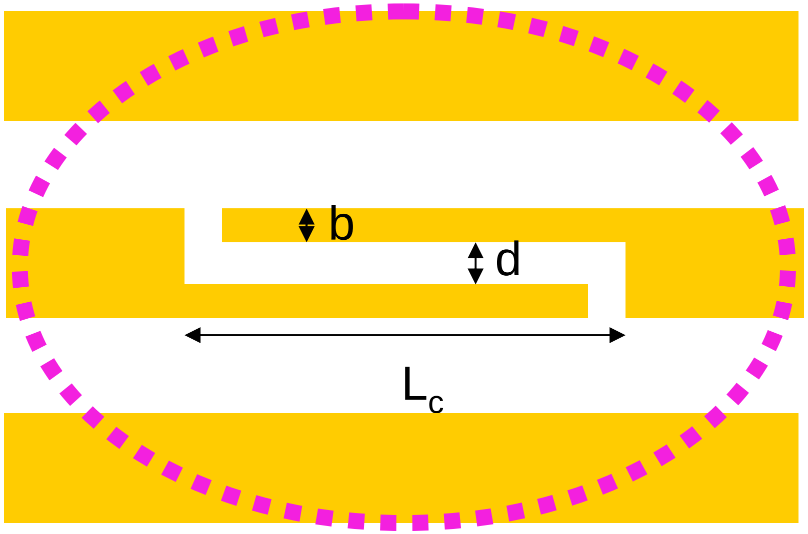


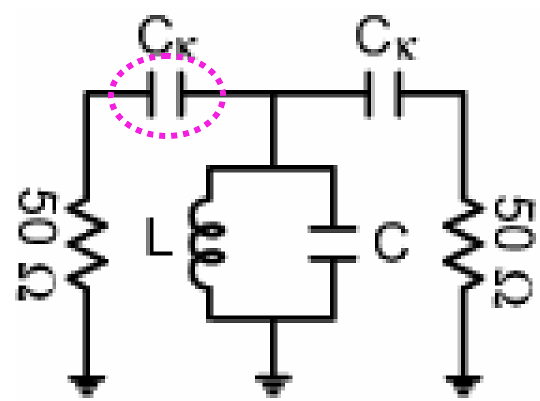
<u>Niobium process:</u>

- Nb thin film
- E-Beam Lithography
- Etch gapsBCl_3,



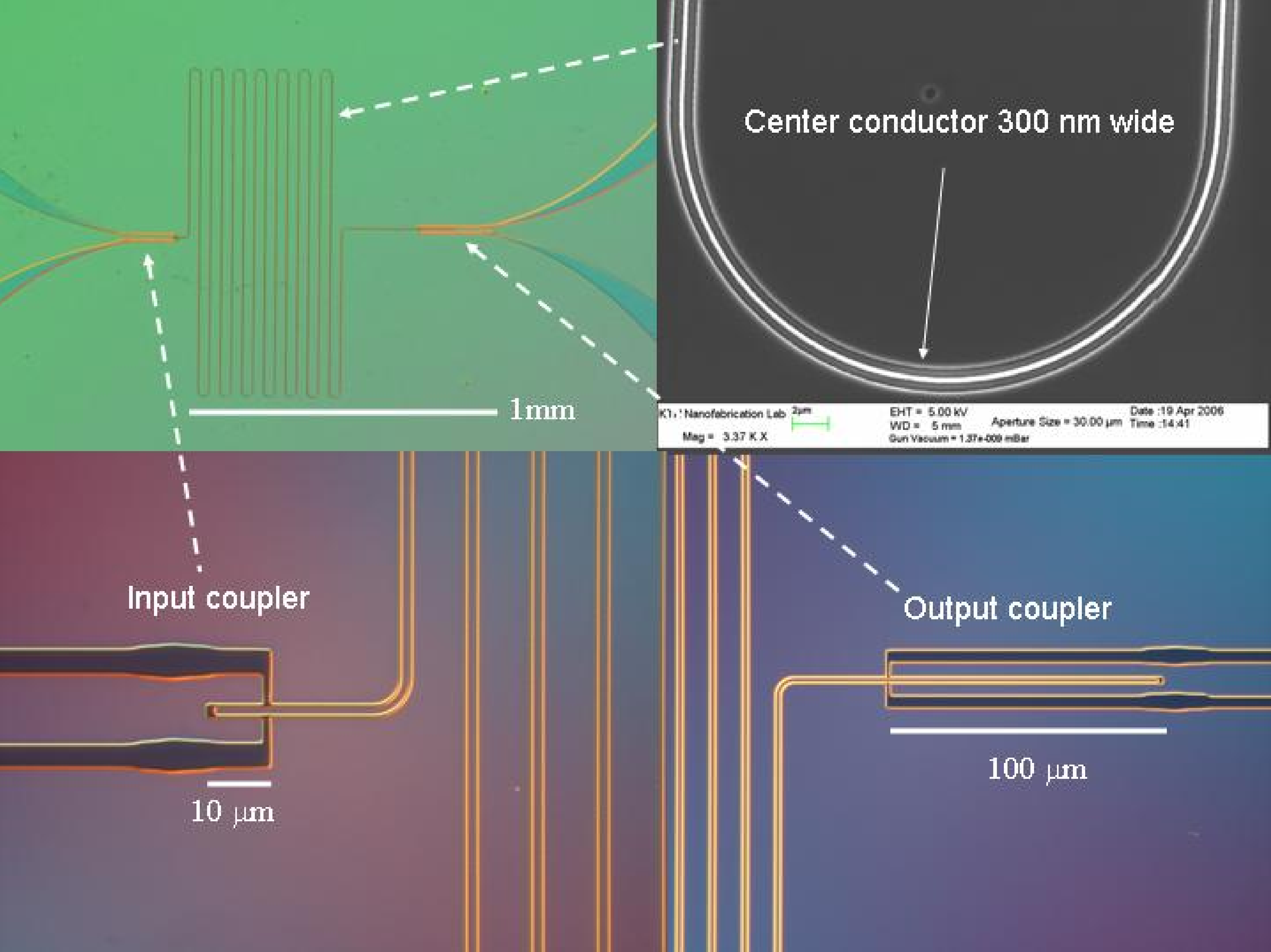
Coupling capacitors



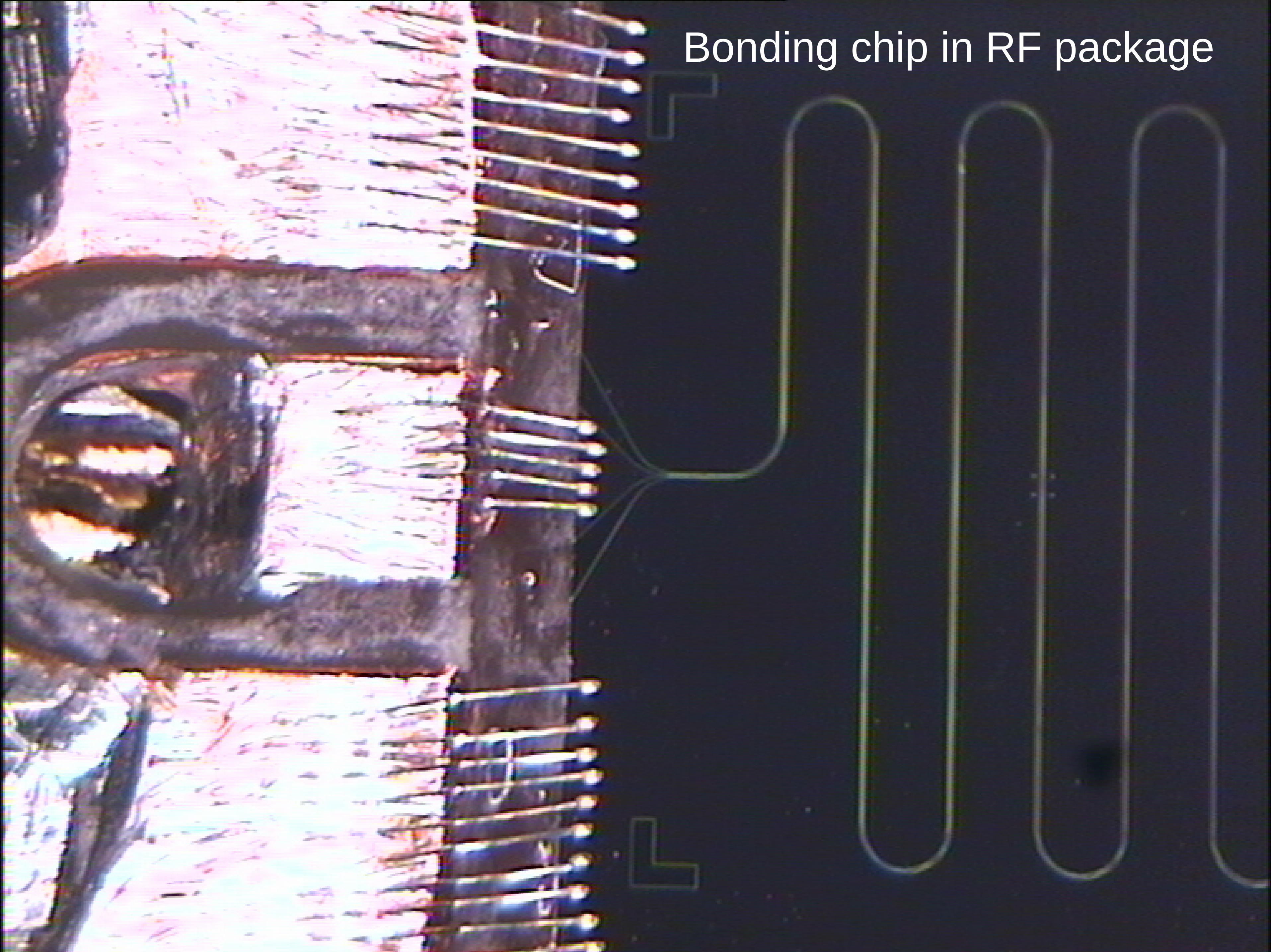


- **L_c =0, 25, 50, 100 μm, S=10μm, w=4μm, fabaricated and tested**
- **Coupling capacitances C_κ, between 0.1 and 5 fF**
- **C_κ determines loaded Q factor, Q_L**

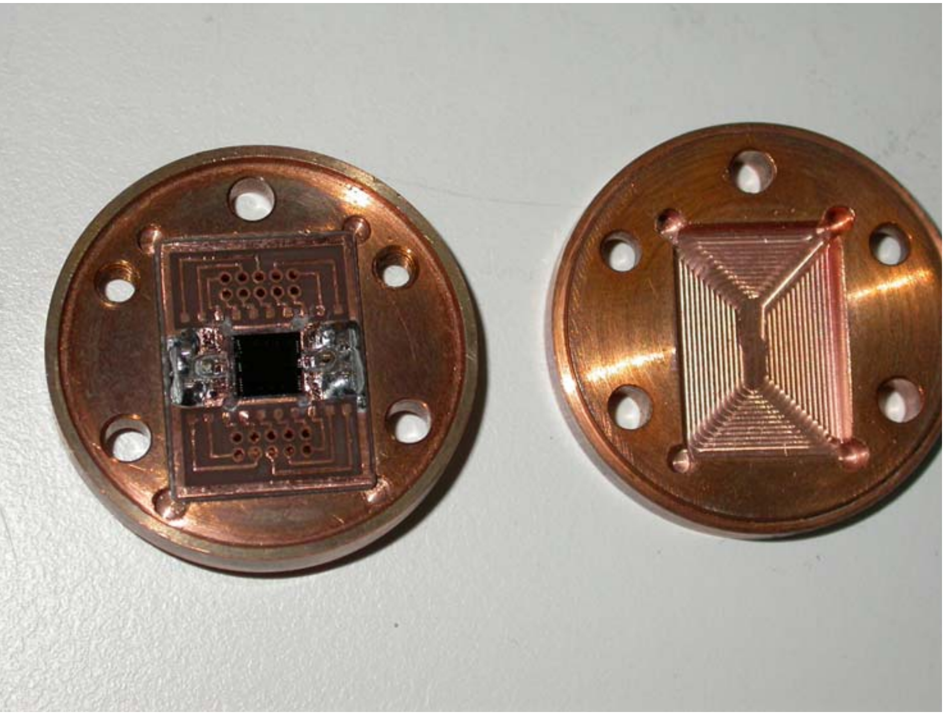
$$\frac{1}{Q_L} = \frac{1}{Q_{\mathrm{int}}} + \frac{1}{Q_{\mathrm{ext}}} \qquad Q_{\mathrm{ext}} = \frac{\omega C}{G_{\mathrm{ext}}} \qquad G_{\mathrm{ext}} = \frac{2R_L C_\kappa^2 \omega^2}{1 + R_L^2 C_\kappa^2 \omega^2}$$

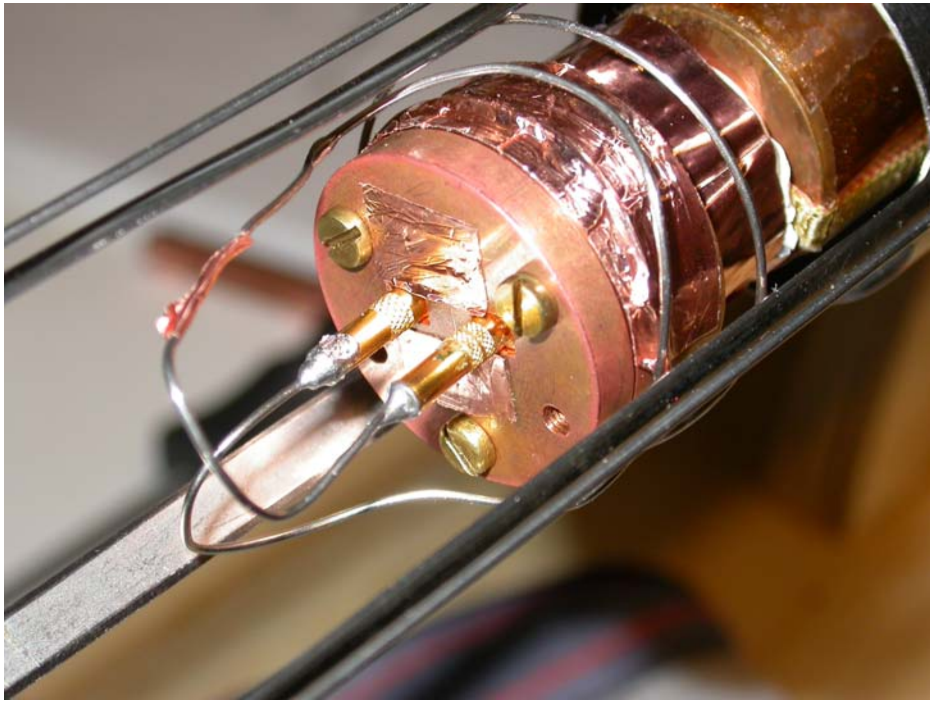


Center conductor 300 nm wide
1mm
Nanofabrication Lab
2µm
Mag = 3.37 K X
EHT = 5.00 kV
WD = 5 mm
Aperture Size = 30.00 µm
Date :19 Apr 2006
Time :14:41
Input coupler
10 µm
Output coupler
100 µm


Bonding chip in RF package

Sample holder and RF connection





Chip in PC board

Bonded with Al wires

Shielded by copper box

Mounted in cryostat

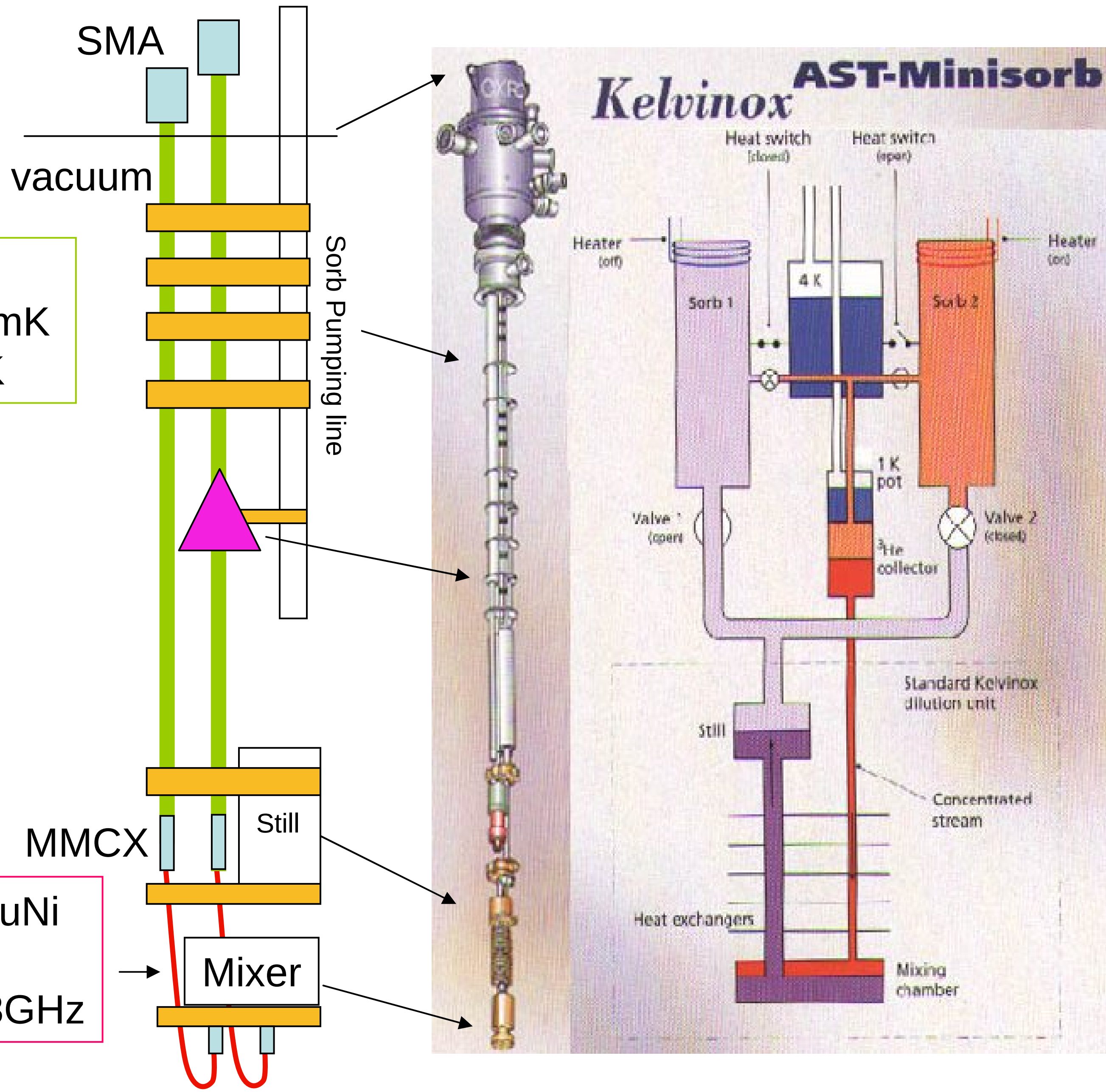
Connected via MMCX connectors

Broad band cabling to sample cryogenic low noise amplifier

2 coax lines to sample
Base T before coax, 15-20mK
Base T after coax 25-30mK

Cryogenic amplifier
Miteq APS series
Gain 36 dB, 2-4 GHz
T=30K
Tnoise <16K ??
0.260mW DC power

20 cm UT20-CuNi-AuCuNi
0.5 mm OD 50 Ω coax
Insertion loss -6dB @ 3GHz



Cooling down

Dewar filled with liquid helium

Cryostat mounted on dewar

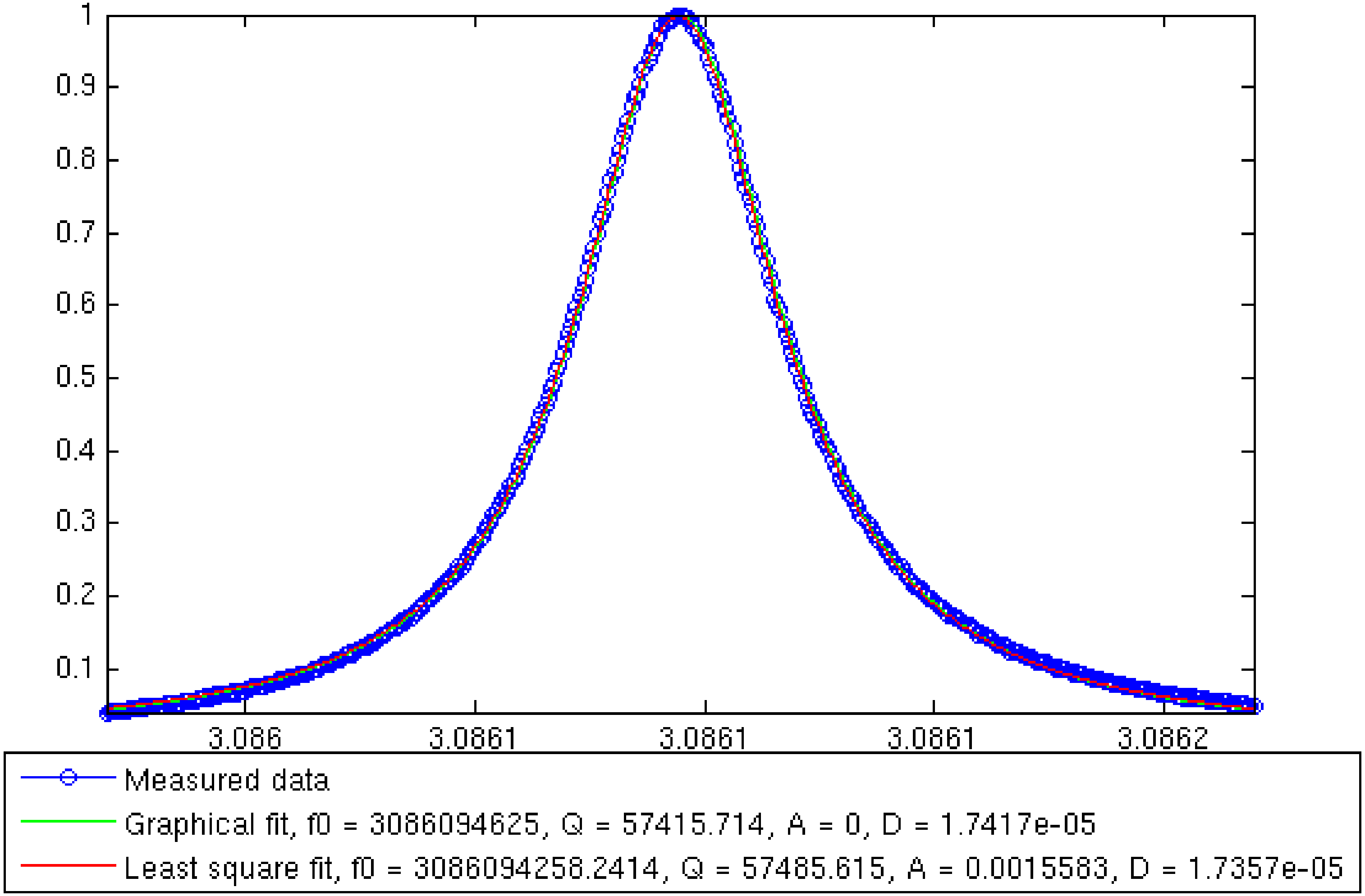
Slowly lowered into the helium

Dilution refrigeration T_{min}=20mK

Network analyzer (in rack) connected to crystat

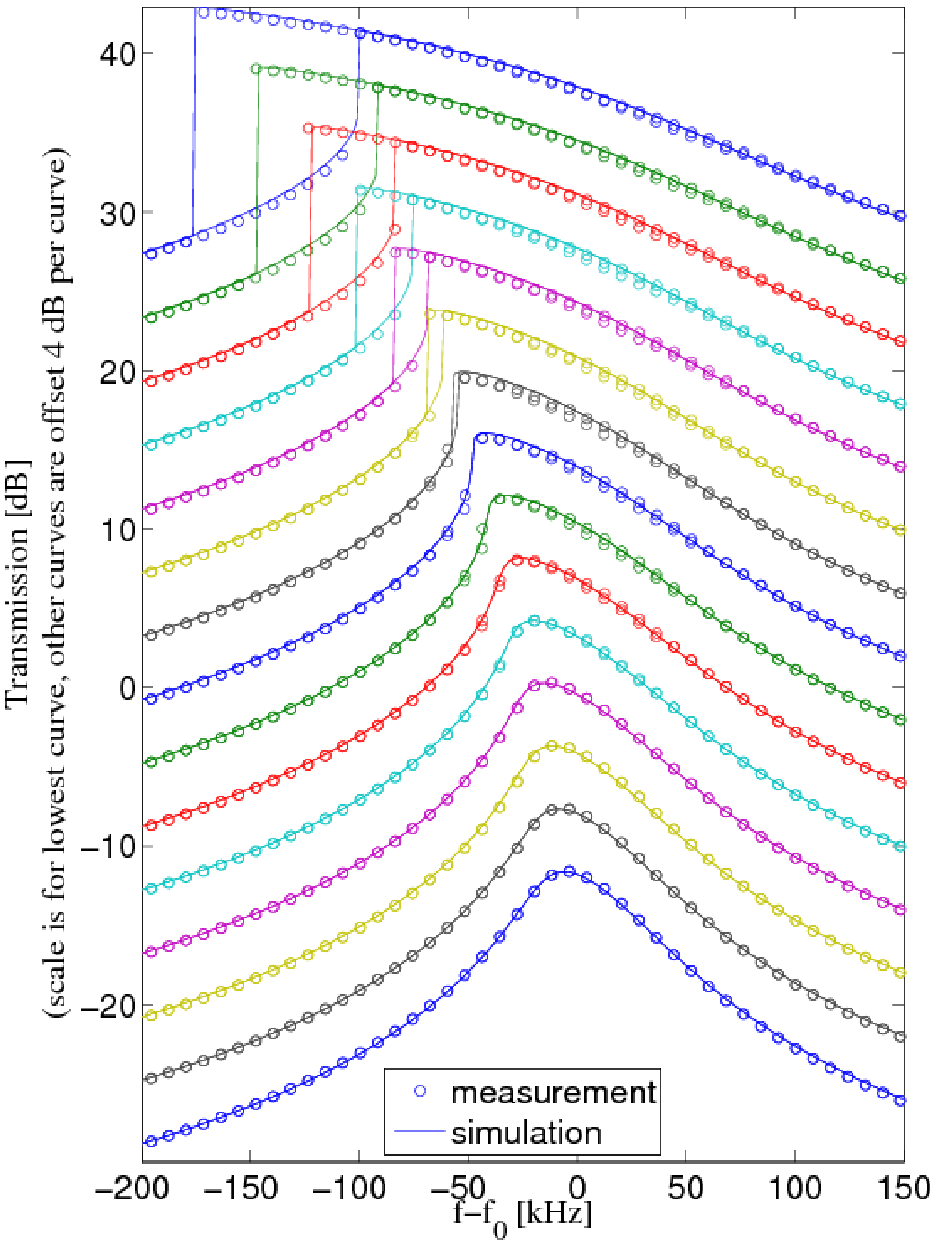
Microwave transmission up to 50 GHz

Low drive power, High Q oscillator



Increasing drive Power

- Bending of resonance curve to lower frequency
- Bifrication at critical power
- Classic “Duffing oscillator” behavior



Sources of Nonlinearity

Transmission line equations for strip line:

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t}$$

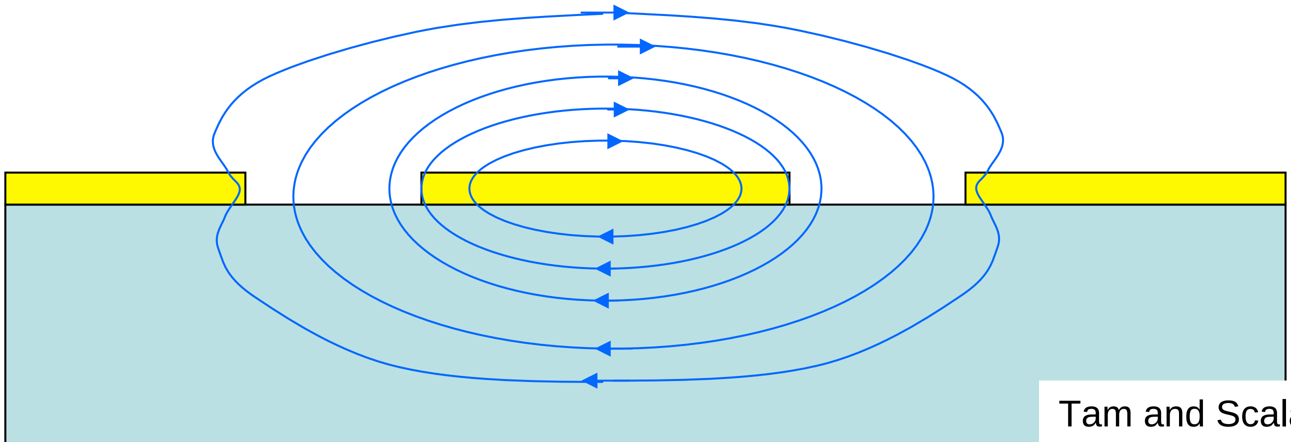
$$L = L_{EM} + L_{Kinetic}$$

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} - RI$$

Kinetic Inductance (kinetic energy in charge flow due to mass of charge carriers)

$$L(I) = L_0 + \Delta L\left(\frac{I}{I_c}\right)^2$$

Due to penetration of $\vec{B}$ field in to superconductor, Kerr-like nonlinearity.



Nonlinear damping:

$$R(I) = R_0 + \Delta R\left(\frac{I}{I_c}\right)^2$$

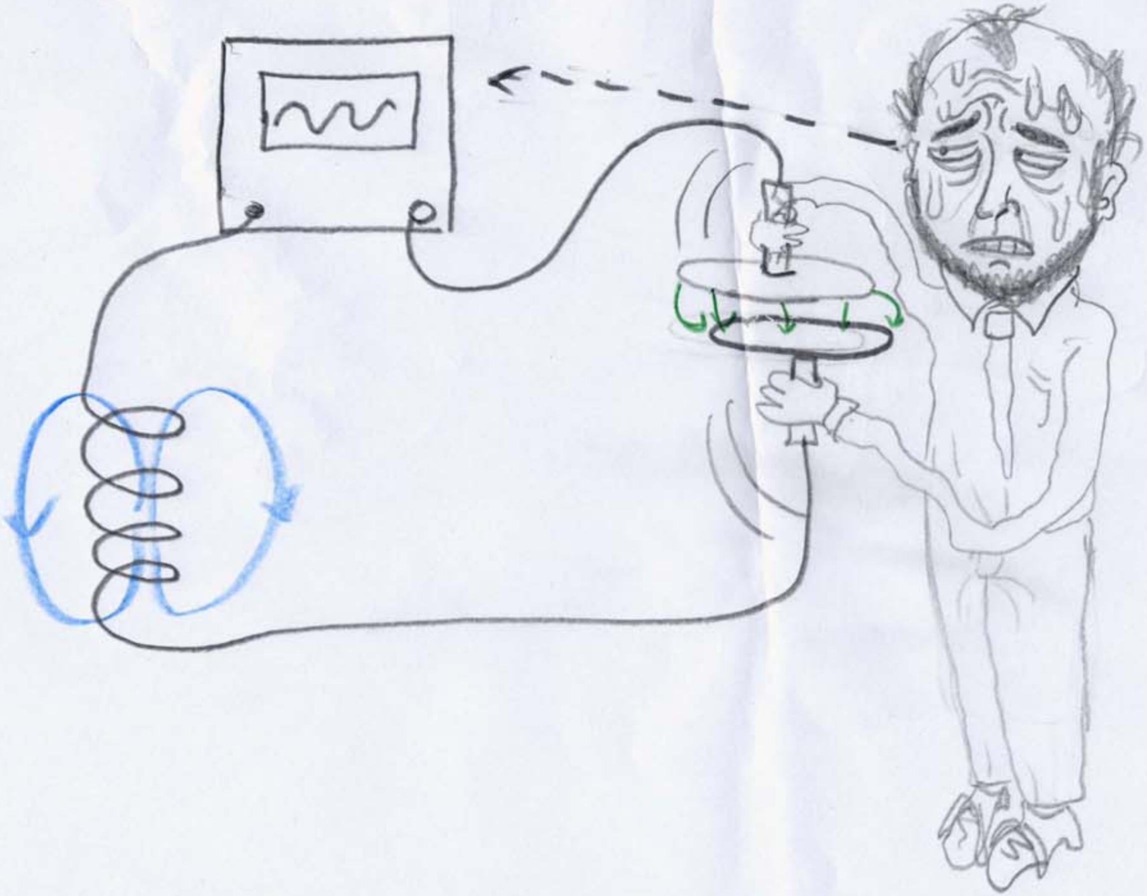
Tam and Scalapino, J. Appl. Phys. 81 2002 (1997)

The Parametric amplifier

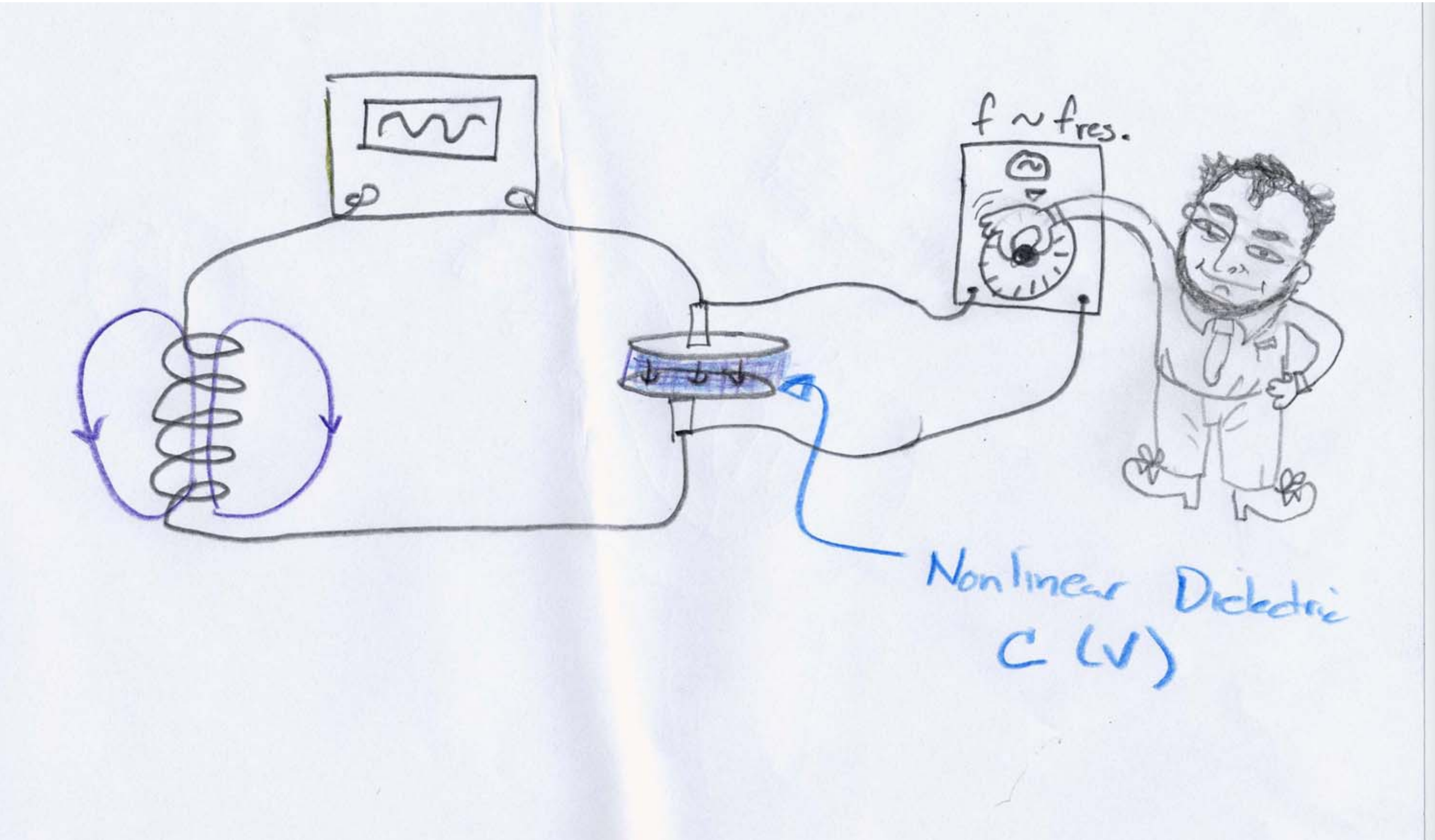
- Change parameter (moment of inertia) during oscillation.
- Small amplitude motion is amplified to large amplitude of swing
- Possible because swing stores energy (under damped oscillator, Q>1)
- Amplification is *phase sensitive*.
    - Pumping at correct phase gives maximum amplification
    - Pumping $\pi/2$ out of phase gives maximum deamplification.

Mechanical pumping , E&M oscillator

Hard Work!

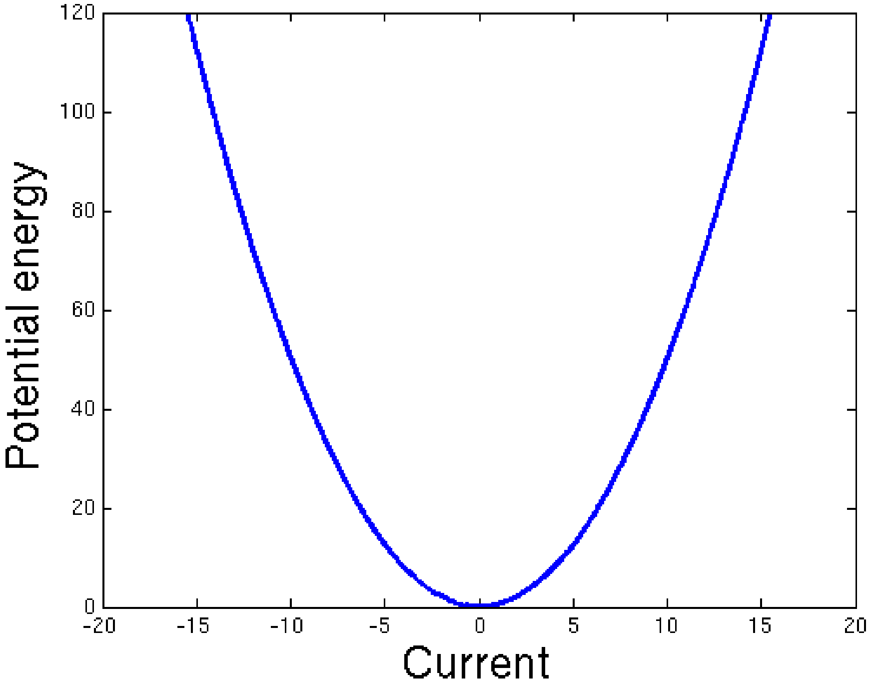


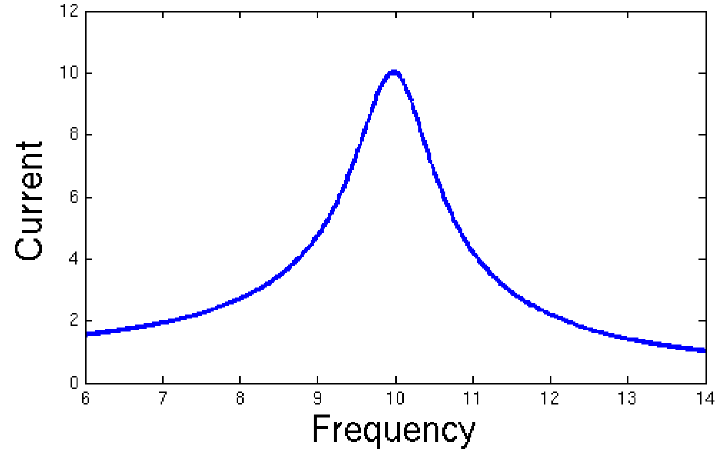
Non-linear element and E&M pump

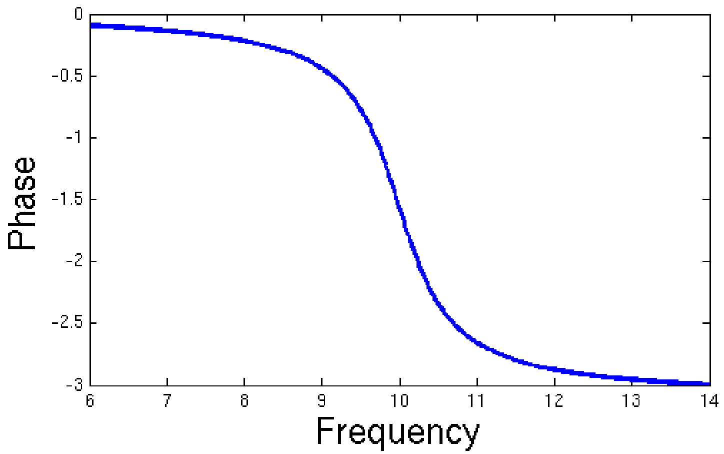


Resonance of a driven, damped, simple harmonic oscillator

$$U(i) = \tfrac{1}{2} L i^2$$

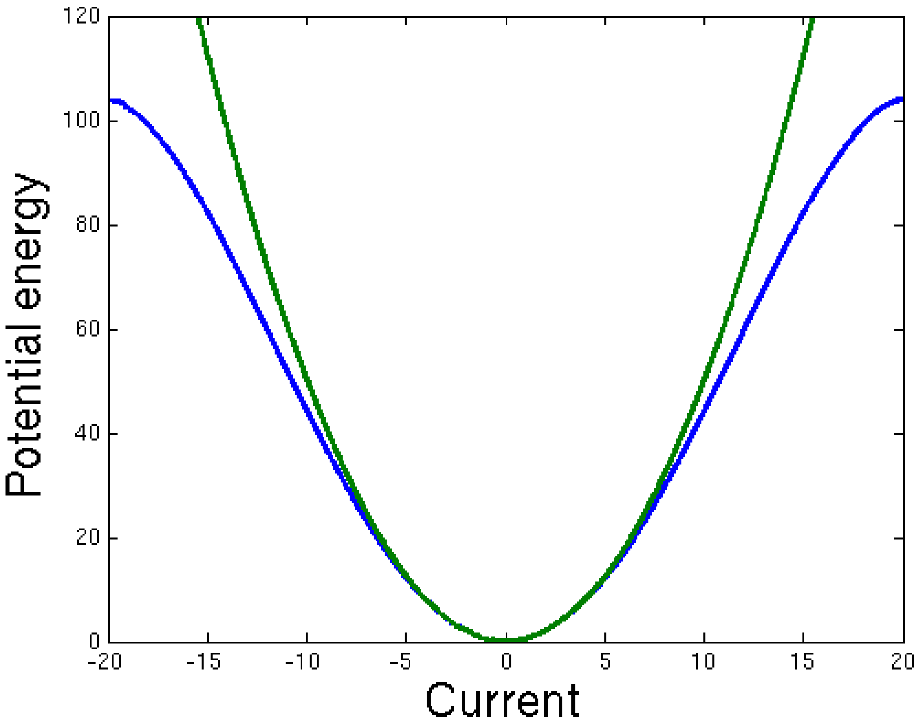


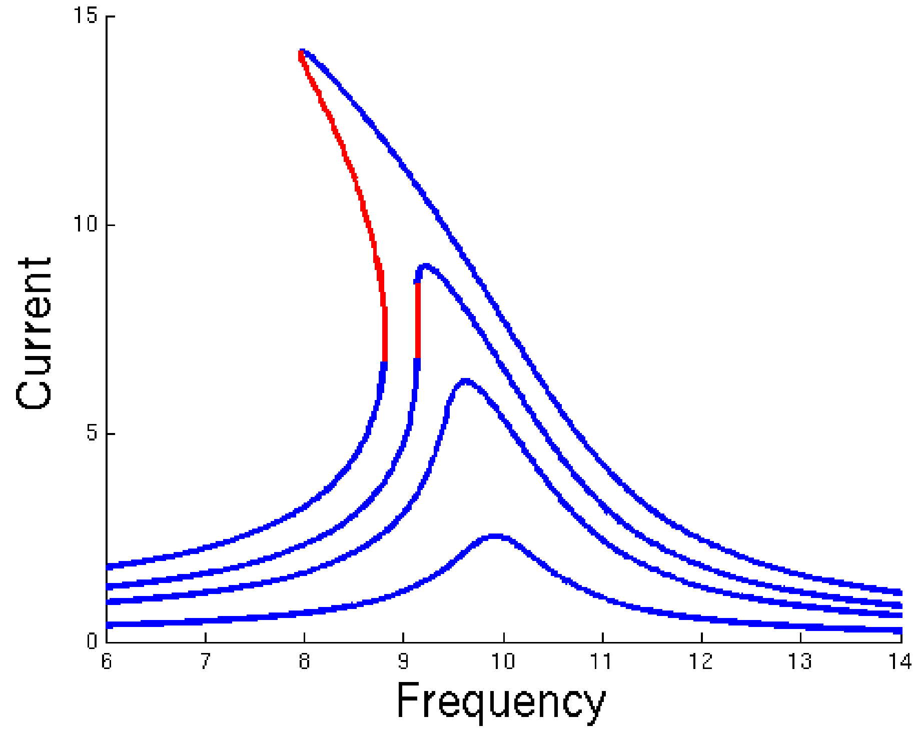




Driven, damped nonlinear oscillator

$$U(i) = \frac{1}{2} L i^2 - L_{NL} i^4$$





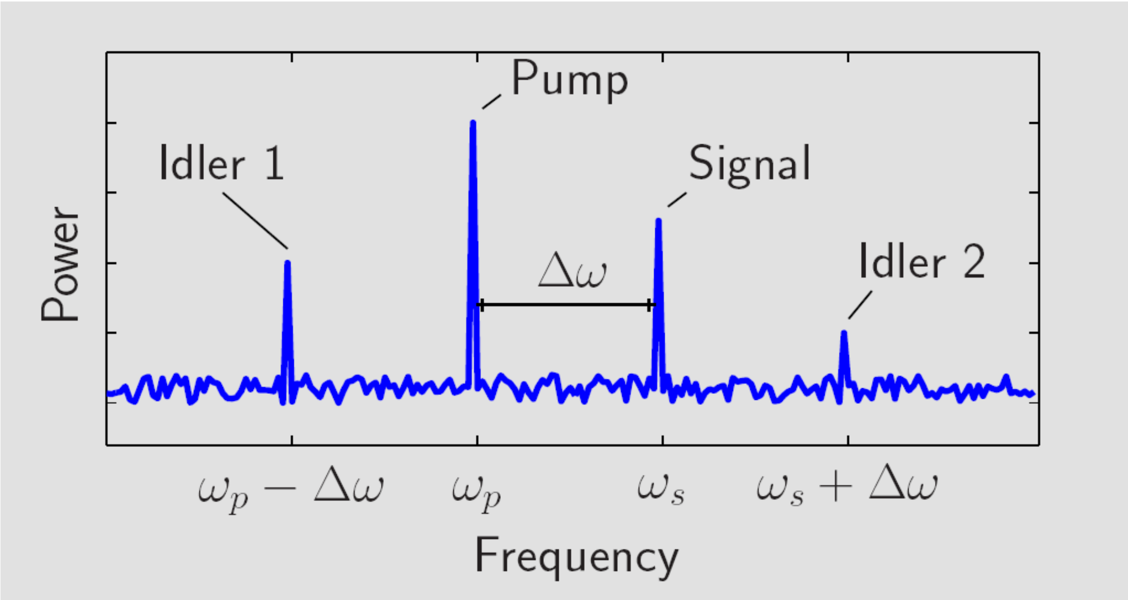
Pump, Signal and Idlers

$$i^3 = [i_p \cos(\omega_p t) + i_s \cos(\omega_s t)]^3 =$$

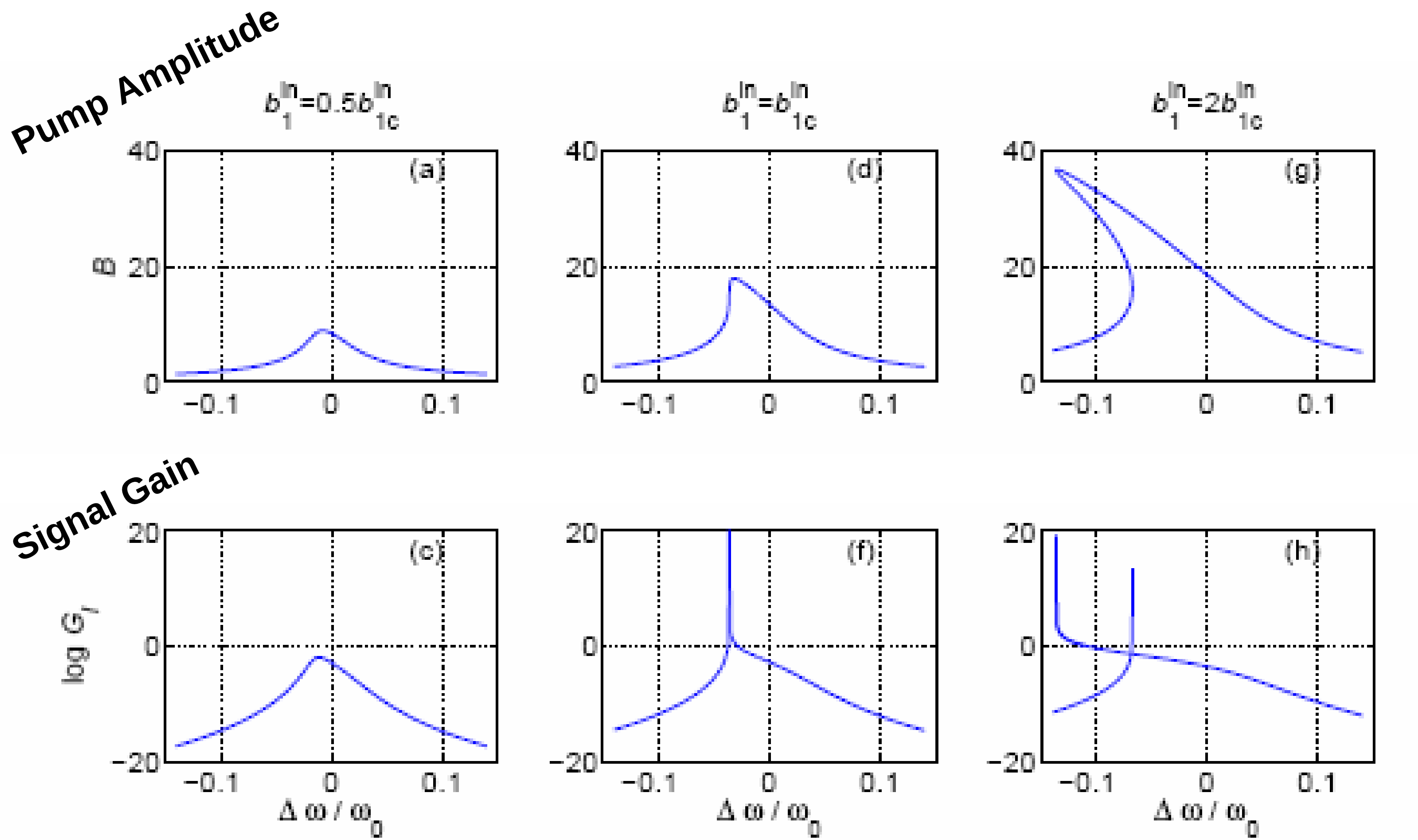
$$6 i_p^2 i_s \cos(\omega_s t)$$

$$+ 3 i_p^2 i_s \cos(\omega_p - \Delta\omega)$$

$$+ 3 i_p i_s^2 \cos(\omega_s + \Delta\omega) + \dots$$

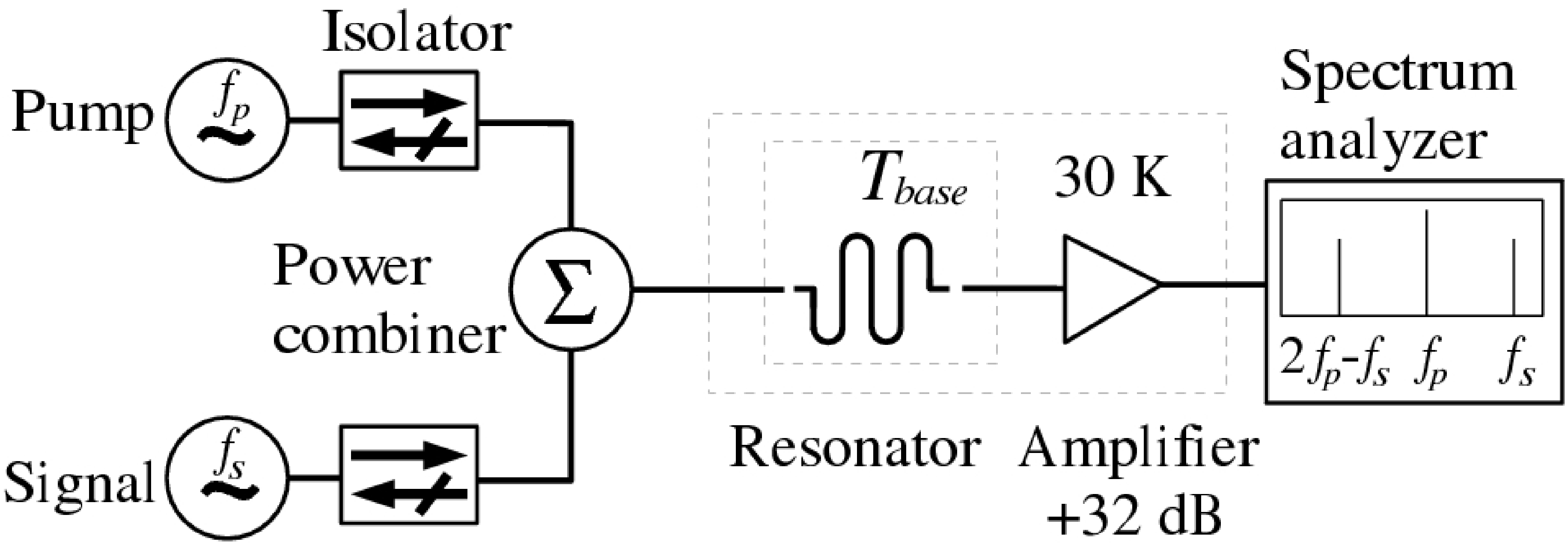


Kerr Non-Linearity for parametric Amplification

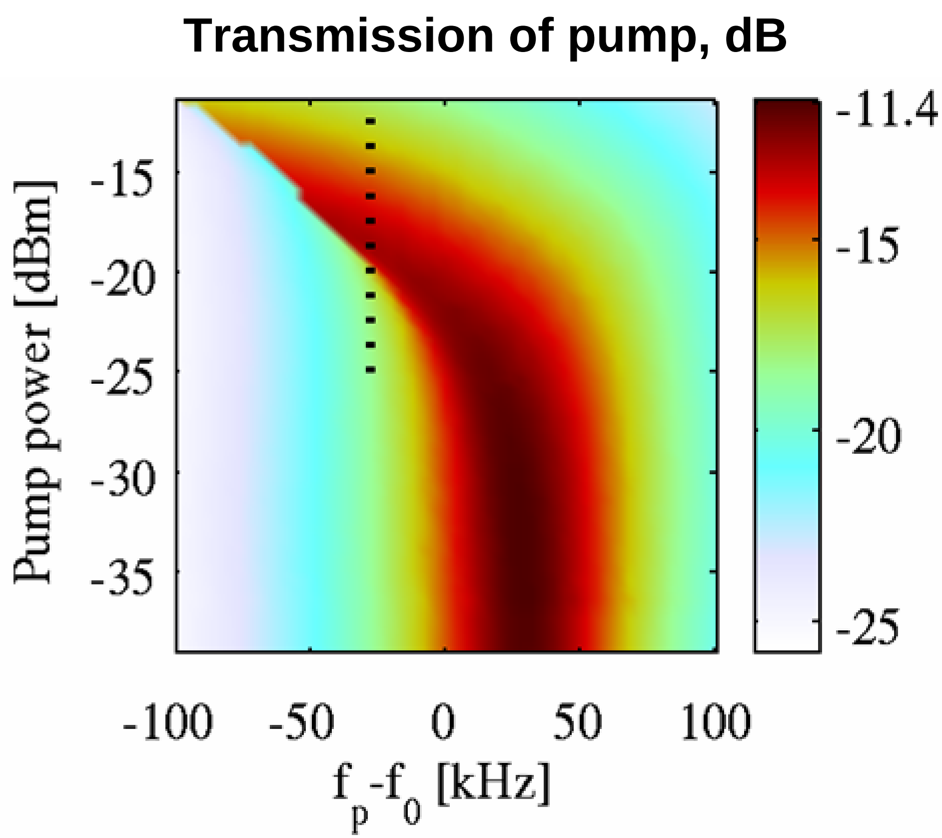


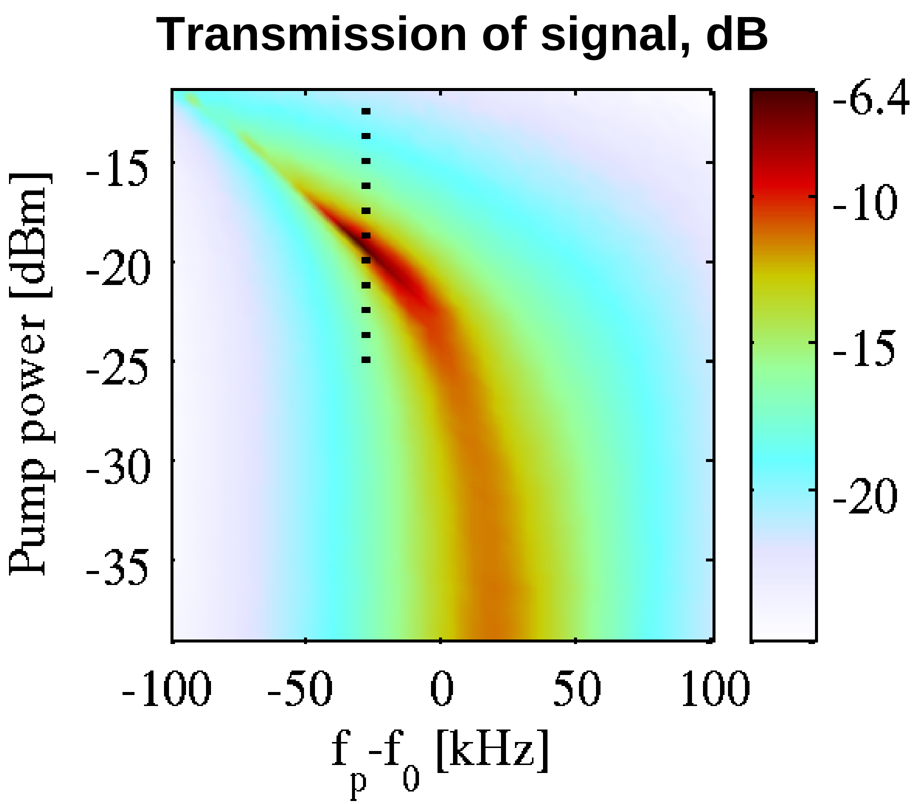
Yurke and Buks quant-ph/0505018

Measuring intermodulation

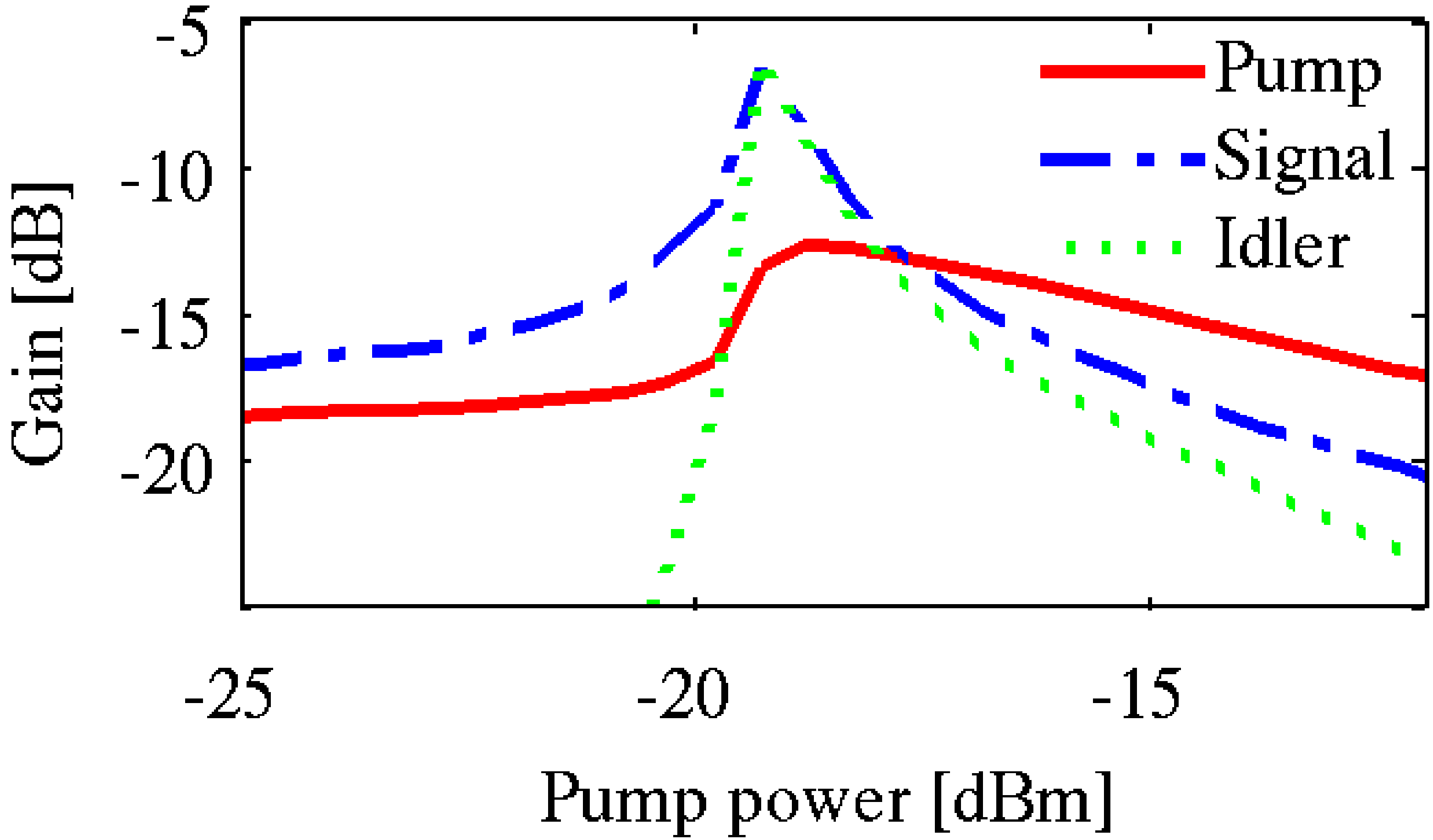


Pump power dependence : Sample 1

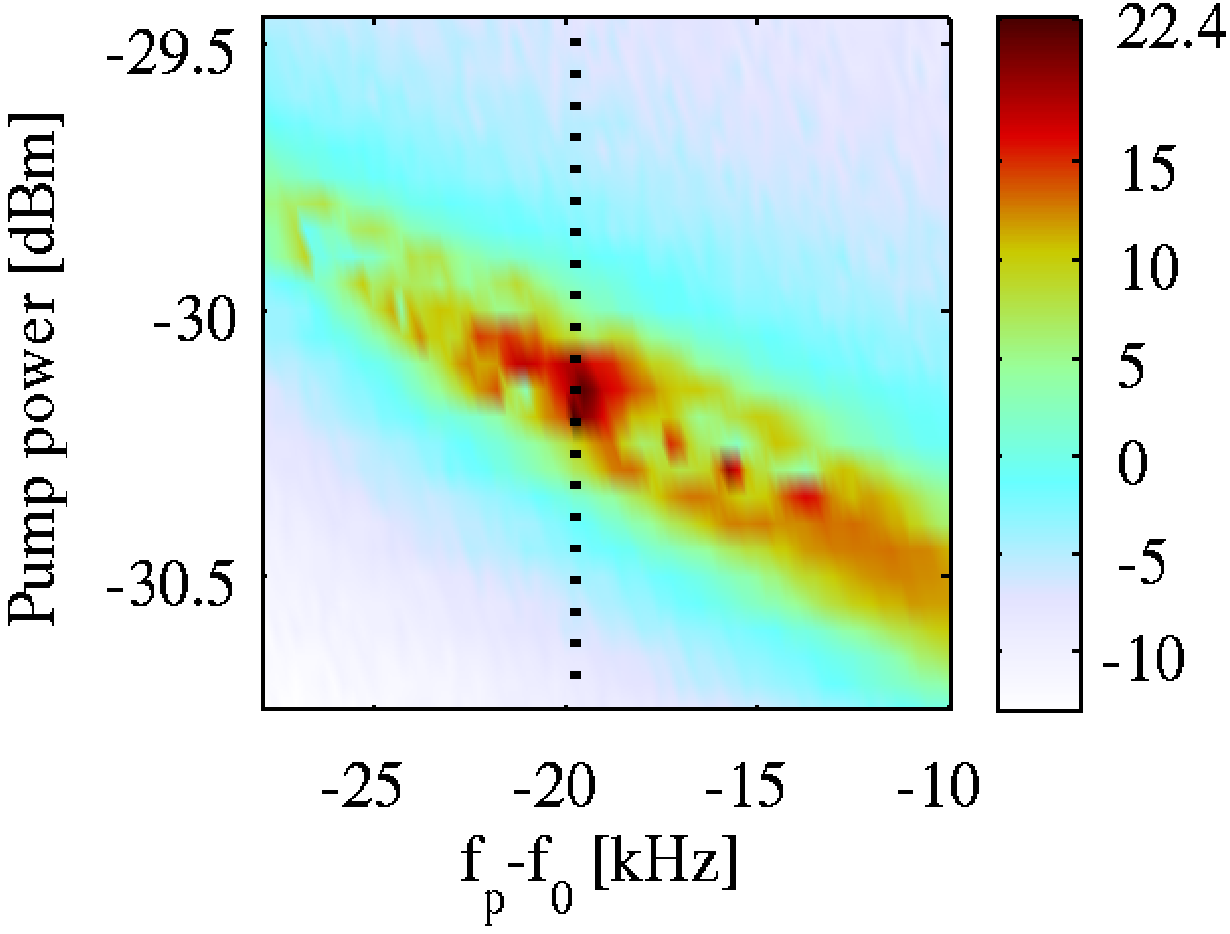




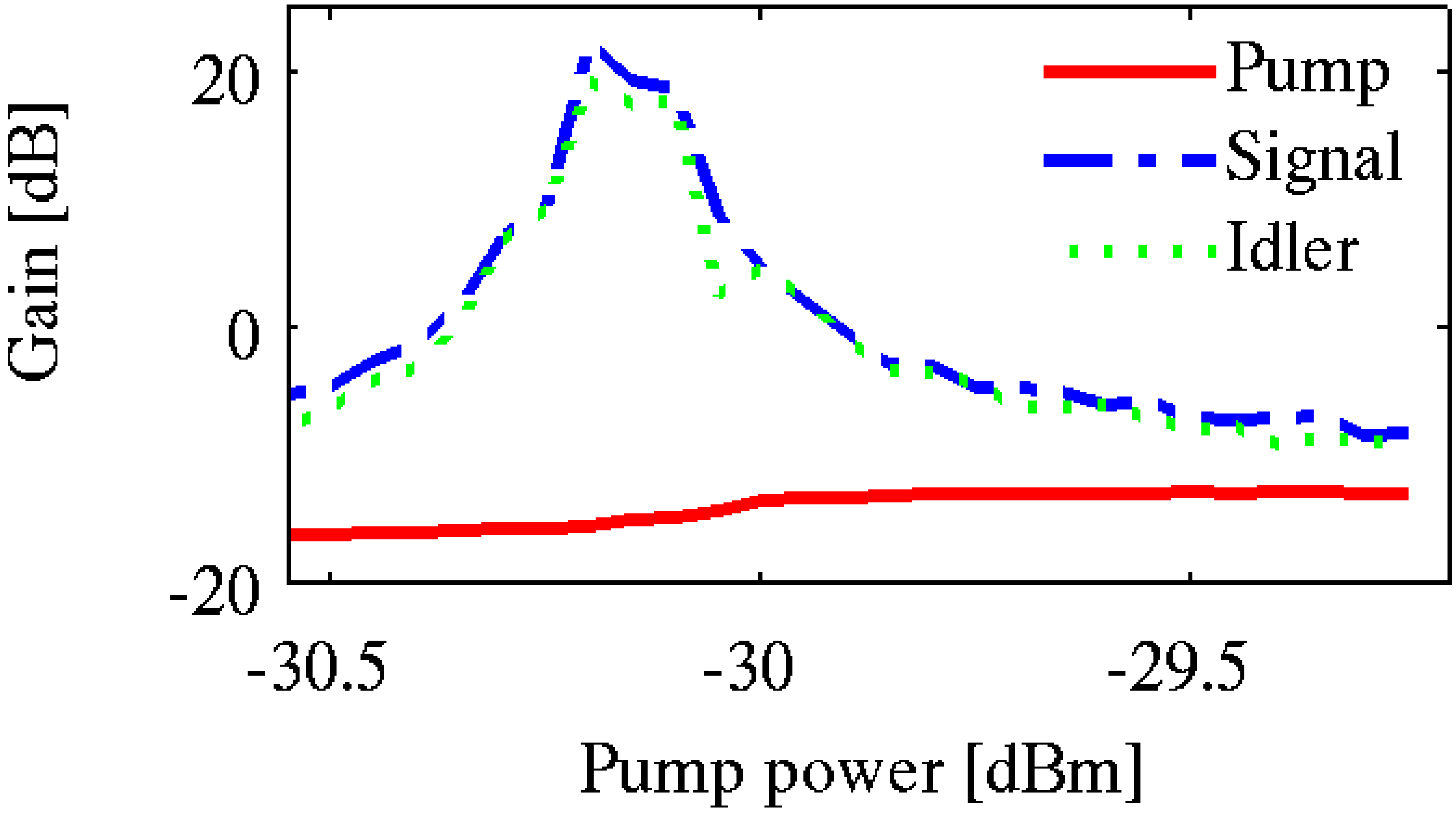
Maximum gain at instability



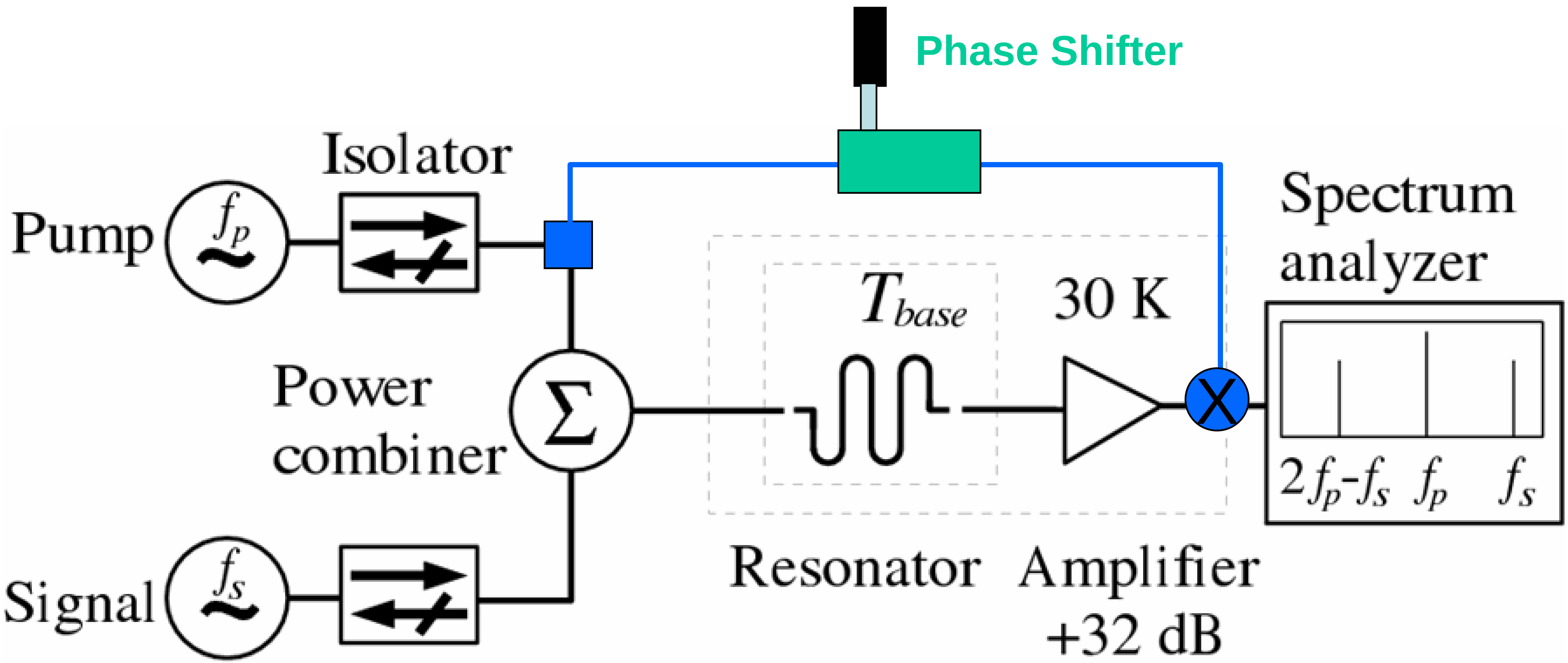
Zoom around maximum gain: sample 2



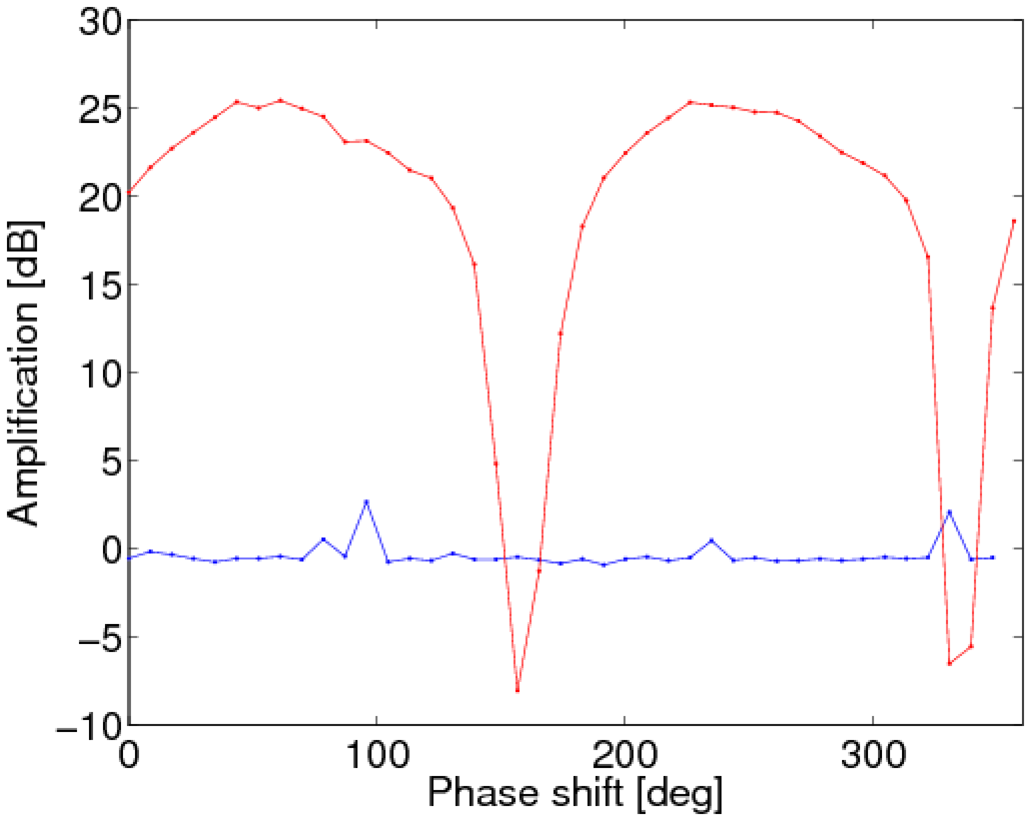
Gain is sharply peaked



Phase sensitive amplification



Deamplification of Signal



Applies to all signals in the cavity – even Noise

Deamplify quantum noise (zero point fluctuations)

$\Rightarrow$Squeezed vacuum states!

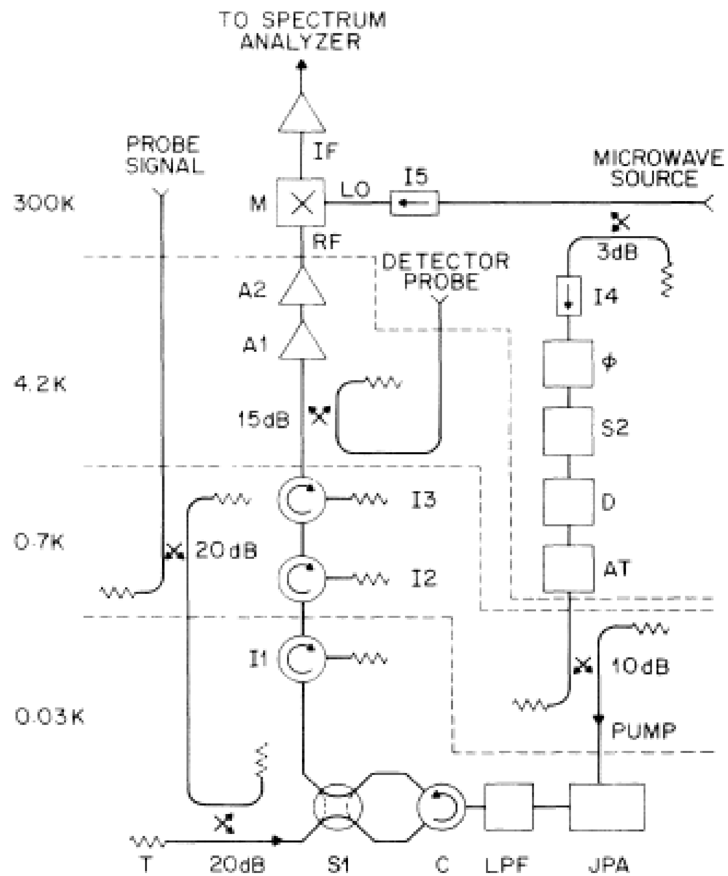
Observation of Zero-Point Noise Squeezing via a Josephson-Parametric Amplifier

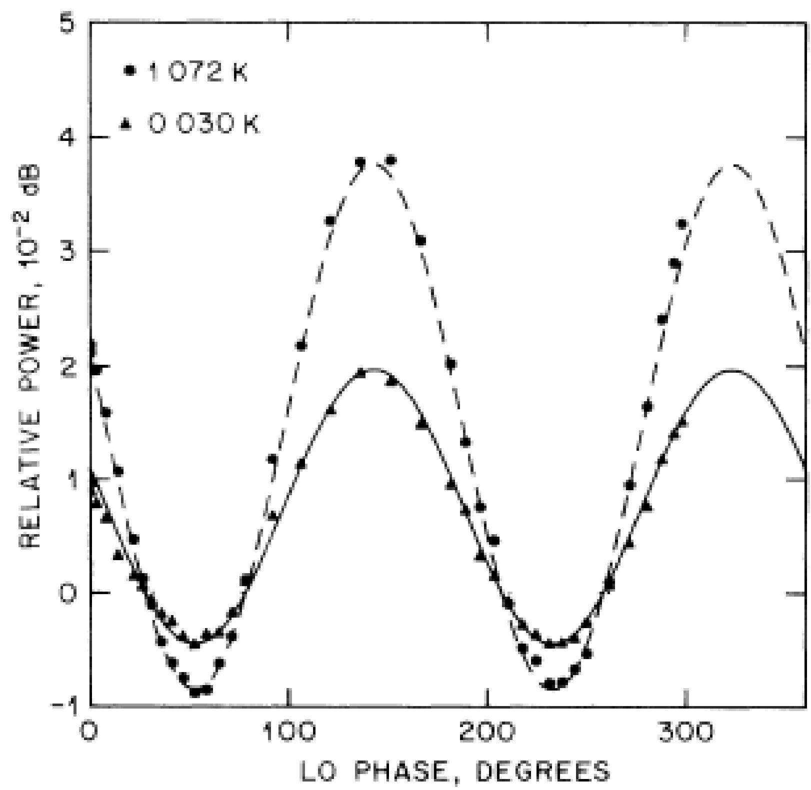
R. Movshovich, B. Yurke, and P. G. Kaminsky
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

A. D. Smith, A. H. Silver, and R. W. Simon
TRW Space and Technology Group, Redondo Beach, California 90275

M. V. Schneider
AT&T Bell Laboratories, Holmdel, New Jersey 07733

(Received 7 June 1990)





Need to measure low power

$$Q = \frac{\text{Energy Stored in cycle}}{\text{Energy lost per cycle}}$$

$$Q \cong \frac{\left(N+\tfrac{1}{2}\right)hf_0}{P_{\text{out}}/f_0} = \frac{\left(N+\tfrac{1}{2}\right)hf_0{}^2}{P_{\text{out}}}$$

$$P_{\text{out}}{}^{ZPF} = \left.\frac{\tfrac{1}{2}hf_0{}^2}{Q}\right|_{\substack{f_0=5\text{GHz} \\ Q=10^4}} = -151\,\text{dBm} \qquad (0\,\text{dBm} = 1\text{mW})$$

Network Analyzer

Cryogenic Amplifier

Presently we have (with 1 Hz bandwidth) P_{noise}= -125 - 32 dBm

Need to keep cavity cold

- Must have $k_B T > h f_0$ (5GHz $\Rightarrow$ 250mK)
- Self heating due to hard pumping and internal losses
  - Sample 1 $\rightarrow$ 900 mK
  - Sample 2 $\rightarrow$ 250 mK
- Optimize coupling and Q, should be possible to reach
- noise from pump, back action of cryo amplifier
  - Need to isolate input and output, attenuators, circulators…..

Conclusions

- QED with superconducting nano-circuits leads to interesting new possibilities for Quantum Electronics at Microwave frequencies
- Strong Coupling QED with Solid State systems
- Non-Linearity of Cavity can be used for parametric amplification, noise squeezing
- Beat “standard quantum limit” for signal amplifier

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