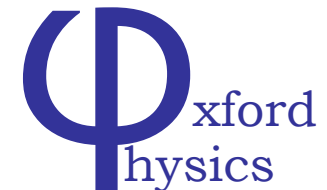
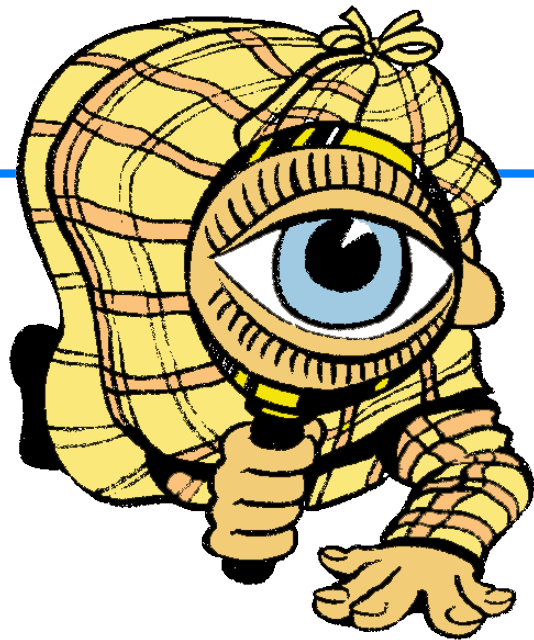


$$\nu = 5/2$$

*A condensed matter who-done-it
(or “what is it”)*



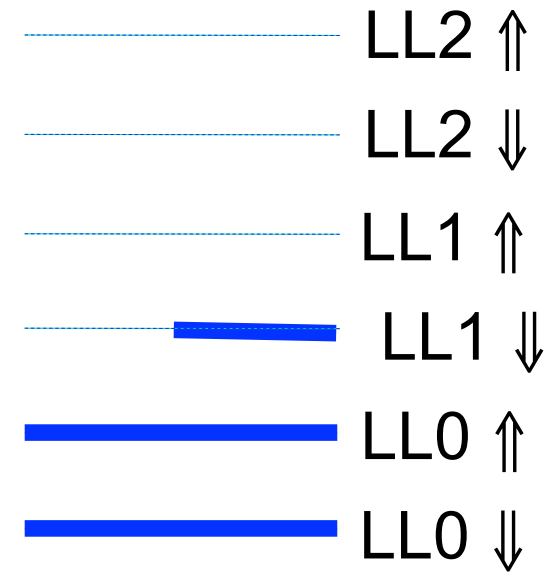
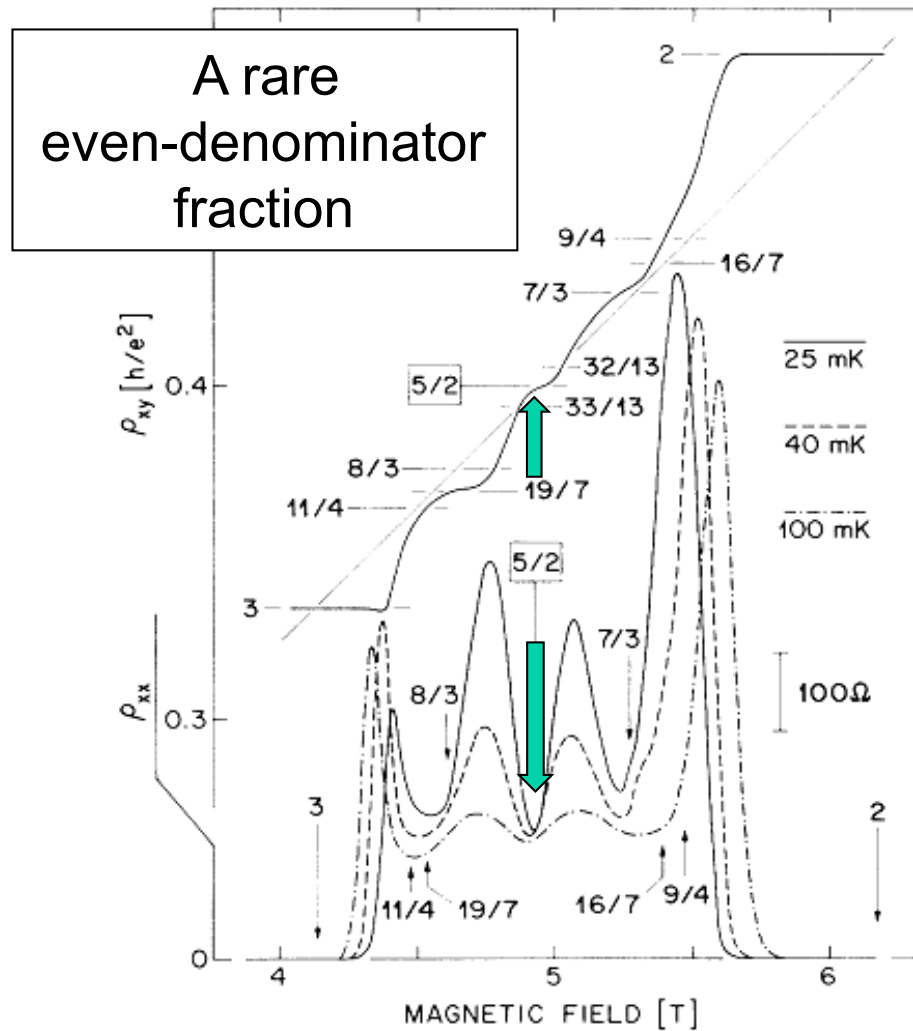
Steven H. Simon

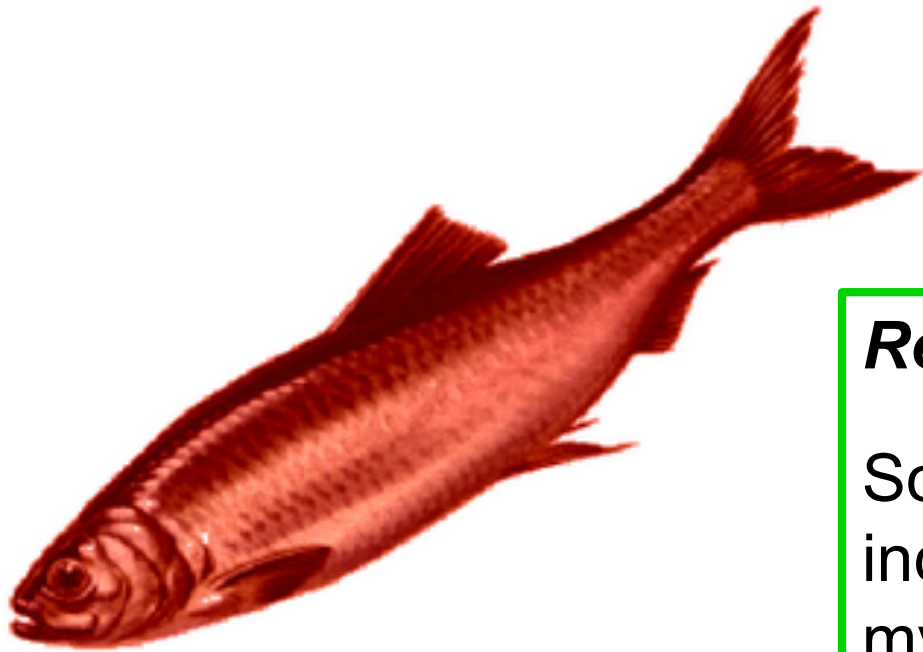
Work with Ed Rezayi (Calstate LA)



The mystery: a quantum Hall state at $\nu=5/2$

(Willett et al 1987)

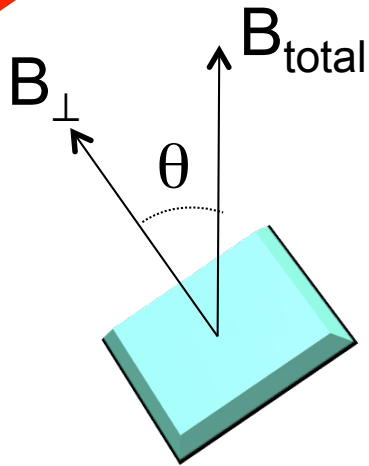
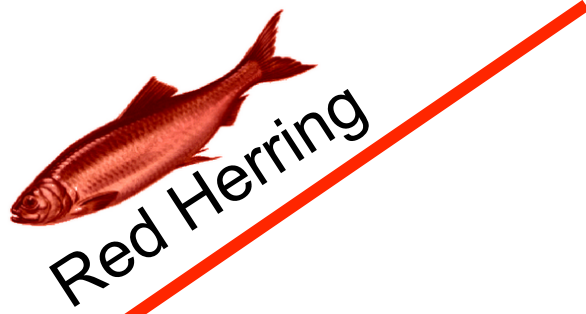




Red herring :

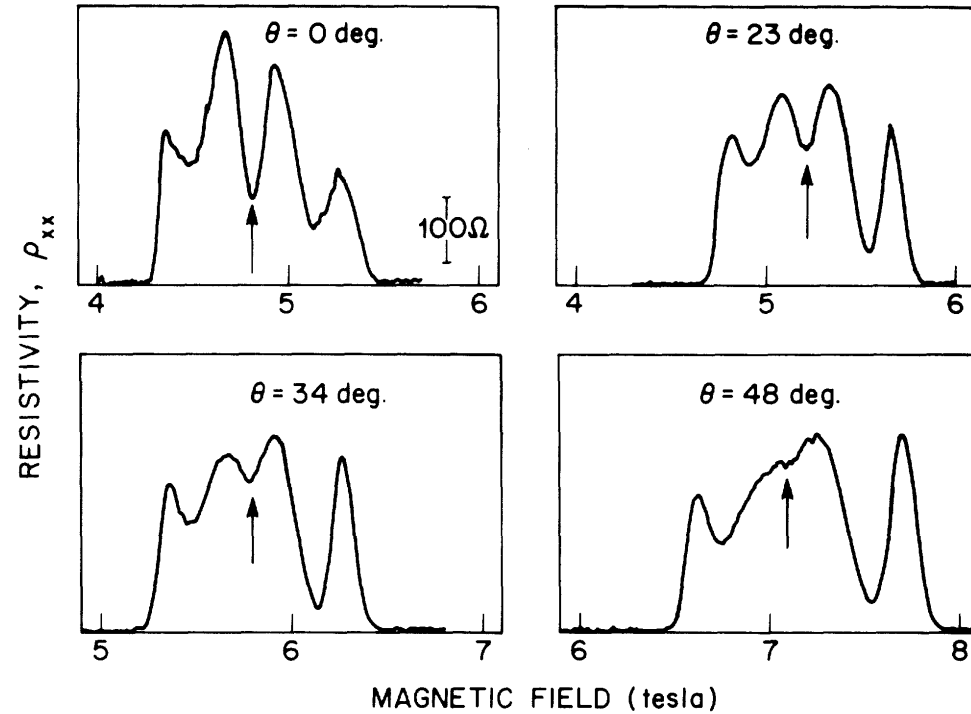
Something that diverts attention incorrectly. For example, in mystery fiction, an innocent party may be cast as highly suspicious so that attention is drawn away from the true guilty party.

The scent of the red herring confuses the pursuing hounds...



B_{\perp} Fixed
 B_{total} changes

(Arrows point to 5/2 – increasing θ increases Zeeman energy)



Eisenstein et al 1988

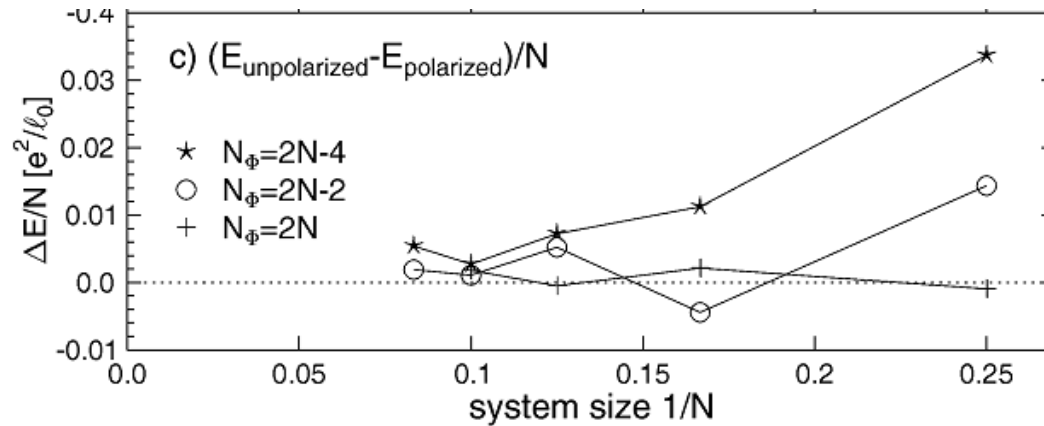
Tilting field kills 5/2 plateau

Conclusion at the time:

- State is *spin-unpolarized*.
- Increasing Zeeman forces spins to align and kills the state.

Transition from Quantum Hall to Compressible States in the Second Landau Level: New Light on the $\nu = 5/2$ Enigma

R. H. Morf



$$E_{\text{unpolarized}} > E_{\text{polarized}}$$

5/2 is spin-polarized

... but lies very close to a phase transition.

Tilting field can tweak the effective electron-electron interaction thereby crossing phase boundary and killing the FQHE.

The Moore-Read “Pfaffian” Wavefunction?

NONABELIONS IN THE FRACTIONAL QUANTUM HALL EFFECT

Gregory MOORE

Department of Physics, Yale University, New Haven, CT 06511, USA

Nicholas READ

Departments of Applied Physics and Physics, Yale University, New Haven, CT 06520, USA

Received 31 May 1990

(Revised 5 December 1990)

$$\Psi_{MR} = \text{Pf} \left(\frac{1}{z_i - z_j} \right) \prod_{i < j} (z_i - z_j)^2 = \text{Paired chiral p-wave composite fermions}$$

Since the combination $\psi^\dagger U^q$ is always a fermion at $\nu = 1/q$, q even, and so these must pair if they are to have any chance to condense, and since the pfaffian state is the simplest way for them to do so, we feel that it is likely that if an incompressible state is ever observed at these filling factors with full spin polarization, it should be this state. Such a state will inevitably have neutral fermion and charged nonabelion excitations.

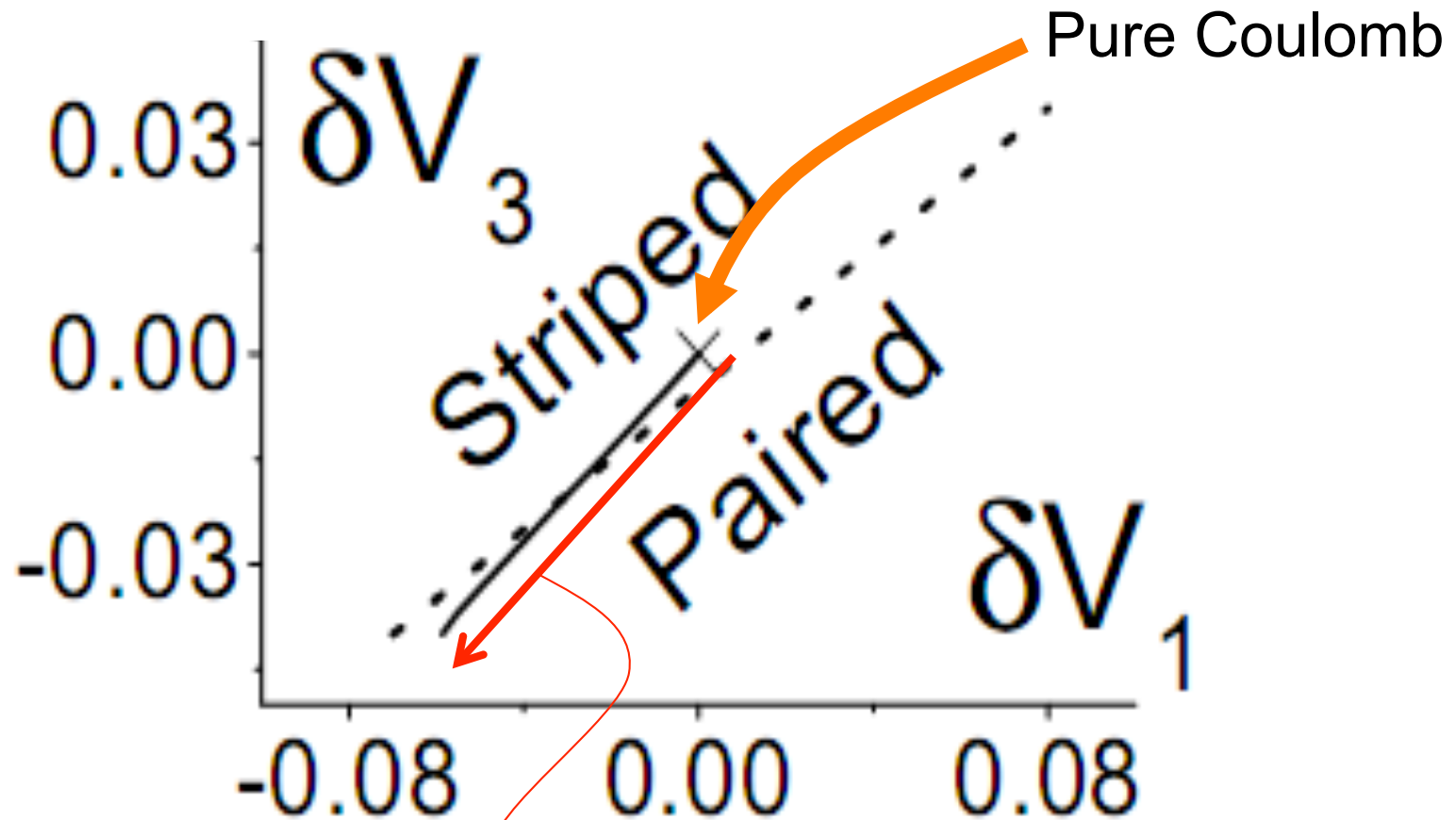
Incompressible paired Hall state, stripe order and the composite fermion liquid phase in half-filled Landau levels

E. H. Rezayi^a and F. D. M. Haldane^b

^aDepartment of Physics, California State University, Los Angeles, California 90032

^bDepartment of Physics, Princeton University, Princeton, New Jersey 08544

(June 1999; revised March 24, 2000)



As quantum well is widened goes into the Moore-Read Phase

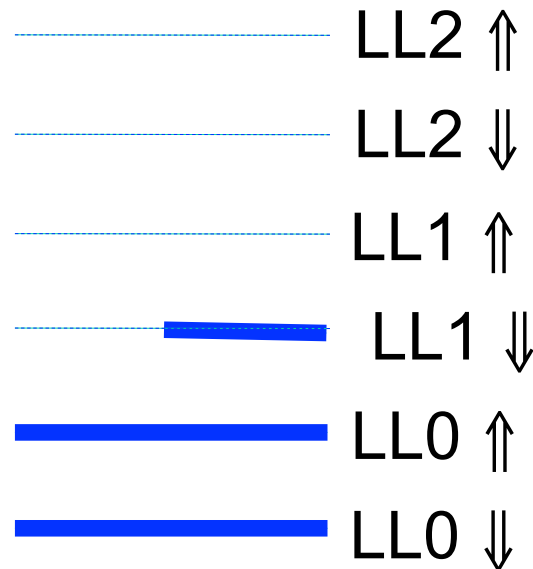
The Morf-Orthodoxy:

- *5/2 is polarized but close to a phase transition*
- *likely Moore-Read in nature*

Many numerical works have since supported this picture:

Haldane, Rezayi, Yang, Feiguin, Nayak, Das Sarma, Moller, Simon, Peterson, Wojs, Quinn, Schoutens, Regnault, Jolicoeur, Storni, Morf,

The reason you can do this numerical work is that you can project the problem to one LL, and diagonalize within a “smallish” Hilbert space.



$\nu = 5/2$ Fractional Quantum Hall Effect at 10 T: Implications for the Pfaffian StateChi Zhang,^{1,2} T. Knuuttila,¹ Yanhua Dai,¹ R. R. Du,^{1,2,*} L. N. Pfeiffer,^{3,4} and K. W. West^{3,4}

Solid State Communications 119 (2001) 641–645

Experimental evidence for a spin-polarized ground state
in the $\nu = 5/2$ fractional quantum Hall effectW. Pan^{a,b,*}, H.L. Stormer^{c,d}, D.C. Tsui^a, L.N. Pfeiffer^d, K.W. Baldwin^d, K.W. West^dVery high density samples ($n > 6 \times 10^{11} \text{ cm}^{-2}$) $\nu = n / B = 5/2$ plateau at $B_{\perp} > 10 \text{ T}$... and at tilts up to 25°

$$E_z \sim B \gg E_{\text{gap}} \sim B^{1/2}$$

**Confirms spin polarization!
(at least for these samples)**Also NMR Experiments by L. Tiemann, G. Gamez,
N. Kumada, and K. Muraki, unpublished HMF-19 2010 (Japan)

.... a possible wrench in the works?



PRL 105, 096801 (2010)

Selected for a *Viewpoint in Physics*
PHYSICAL REVIEW LETTERS

week ending
27 AUGUST 2010



Optical Probing of the Spin Polarization of the $\nu = 5/2$ Quantum Hall State

M. Stern,^{1,*} P. Plochocka,² V. Umansky,¹ D. K. Maude,² M. Potemski,² and I. Bar-Joseph¹

Also: March APS 2010 Abstract: Y2.00003. T.-D. Rhone (A. Pinczuk group)
March APS 2009 Abstract: W2.00005. A. Pinczuk

Optical experiments: claim consistency with $\nu = 5/2$ being *unpolarized*

My Claim: *Not an accurate probe of ground state polarization*

possibly related to: PRL 2010 Wojs, Moller, Simon, Cooper

(Ask me more at the end....)



Red Herring

(Plus some possible questions from transport as well... Dean et al PRL 2008)

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Many numerical works have since supported this picture:

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CAUTION

LL mixing is ALWAYS neglected even though $E_{coulomb}/E_{cyclotron} \approx 1$

LL mixing expected to be quantitatively but not qualitatively important

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5/2 = Moore-Read Pfaffian partially justified by:

“we see a plateau, what else can it be?”

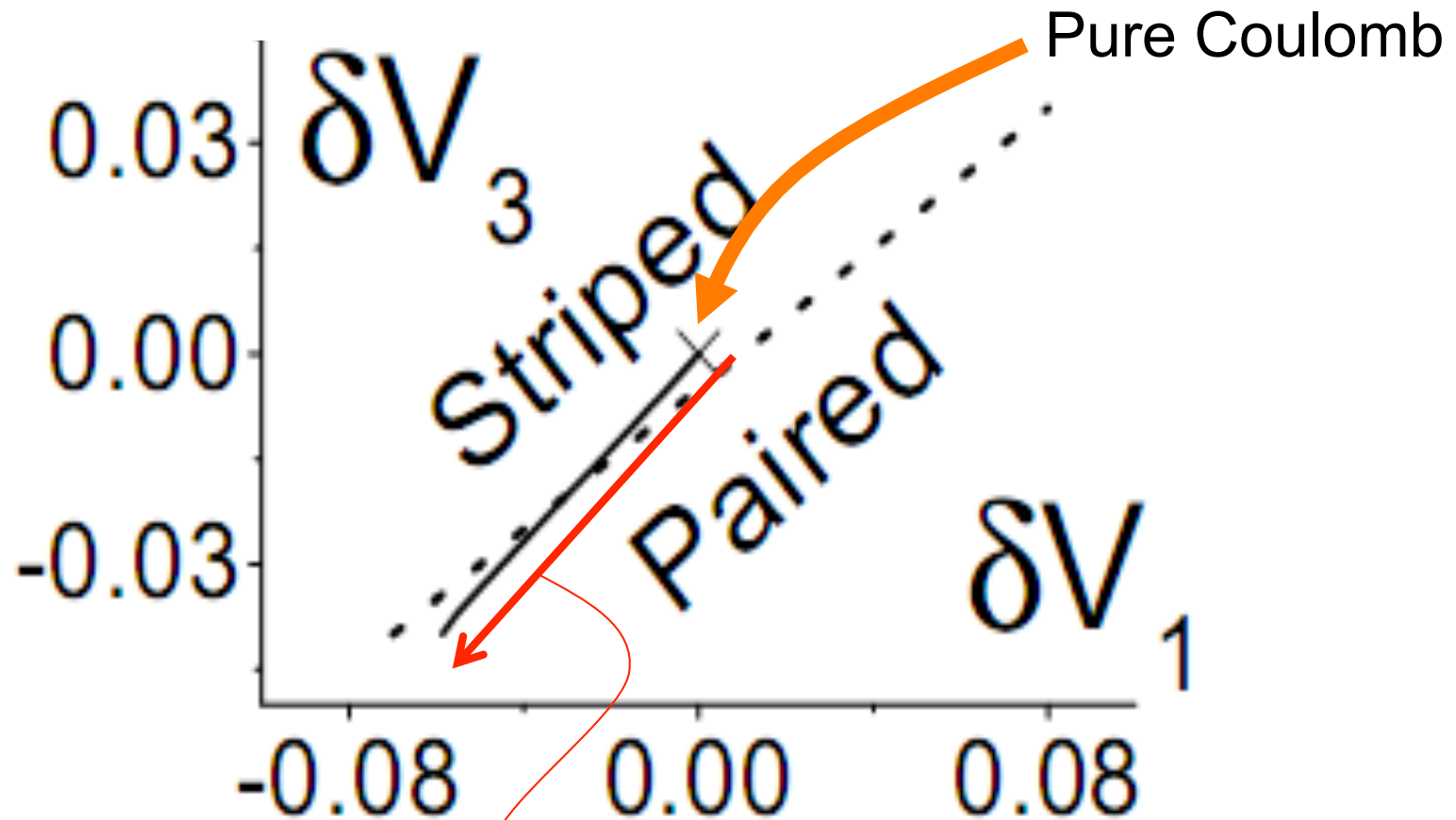
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(June 1999; revised March 24, 2000)



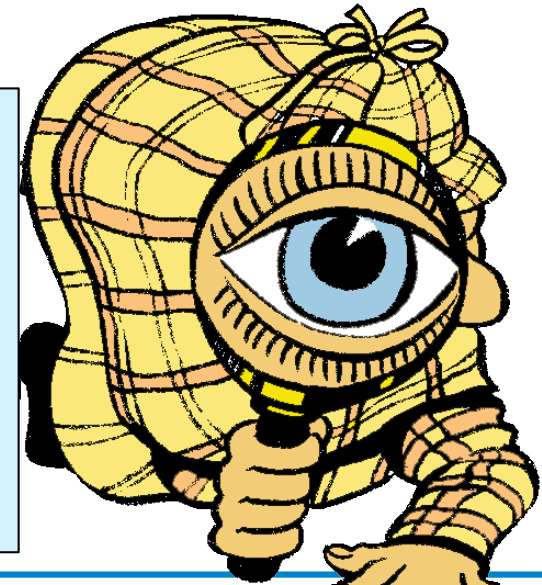
As quantum well is widened goes into the Moore-Read Phase

The Morf-Orthodoxy:

- 5/2
- like

Many numerical
Haldane, Rezayi
Schoutens, Reg

Case Closed?



The reason you can do this numerical work is because you can project the problem to one LL, and diagonalize within a “smallish” Hilbert space.

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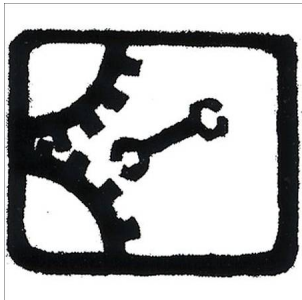
we see a plateau, what else can it be?

What else can it be? : *The AntiPfaffian*

PRL 99, 236806 (2007)

PHYSICAL REVIEW LETTERS

week ending
7 DECEMBER 2007



Particle-Hole Symmetry and the Pfaffian State

Michael Levin, Bertrand I. Halperin, and Bernd Rosenow

Particle-Hole Symmetry and the $\nu = \frac{5}{2}$ Quantum Hall State

Sung-Sik Lee,¹ Shinsei Ryu,¹ Chetan Nayak,^{2,3} and Matthew P. A. Fisher²

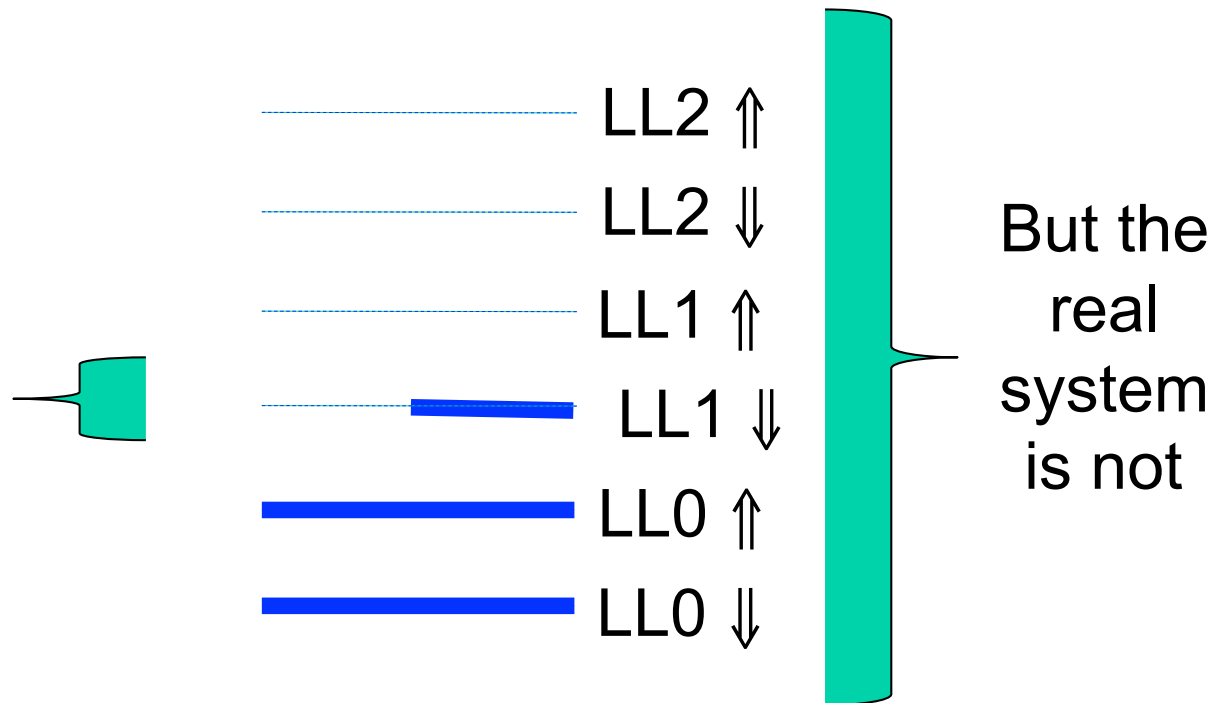
The particle hole conjugate of half-filled LL Moore-Read State is *a different phase of matter with e.g. different edge physics (must have backwards propagating edge modes...)*

No Prior Numerics could have distinguished the two!

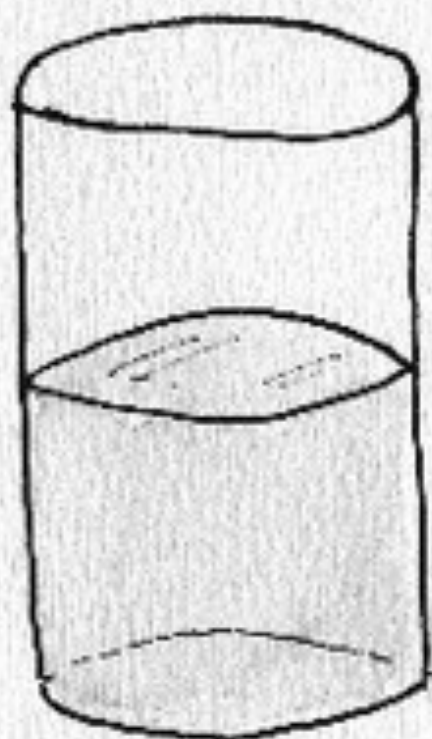
Only LL mixing breaks p-h symmetry.

Is the glass half-full or half-empty?

one LL alone
Is p-h symmetric



Without LL mixing, the Coulomb interaction between electrons is the same as the Coulomb interaction between holes.



Optimist:
Glass half-full

Pessimist:
Glass half-empty

Physicist:

$$\frac{1}{\sqrt{2}} (\Psi_{\text{full}} - \Psi_{\text{empty}})$$



So far:

(1) Polarization:

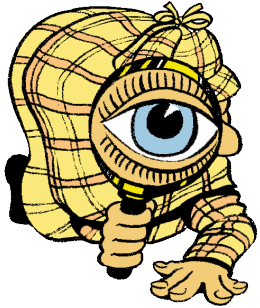
- Morf-odoxy supports polarized state

(3) Pfaffian vs. AntiPfaffian

- Morf-odoxy does not distinguish the two
- Only LL mixing breaks p-h symmetry
- Without LL projection Hilbert space is too big to handle
- But perturbation theory in LL mixing cannot be trusted for

$$E_{coulomb}/E_{cyclotron} \approx 1$$

Breaking of Particle-Hole Symmetry by Landau Level Mixing in the $\nu = \frac{5}{2}$ Quantized Hall State



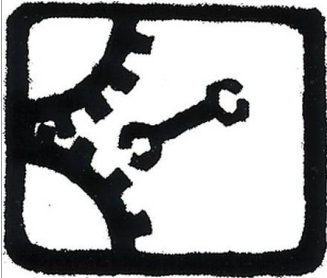
Edward H. Rezayi¹ and Steven H. Simon²

(arXiv:0912.0109v2)

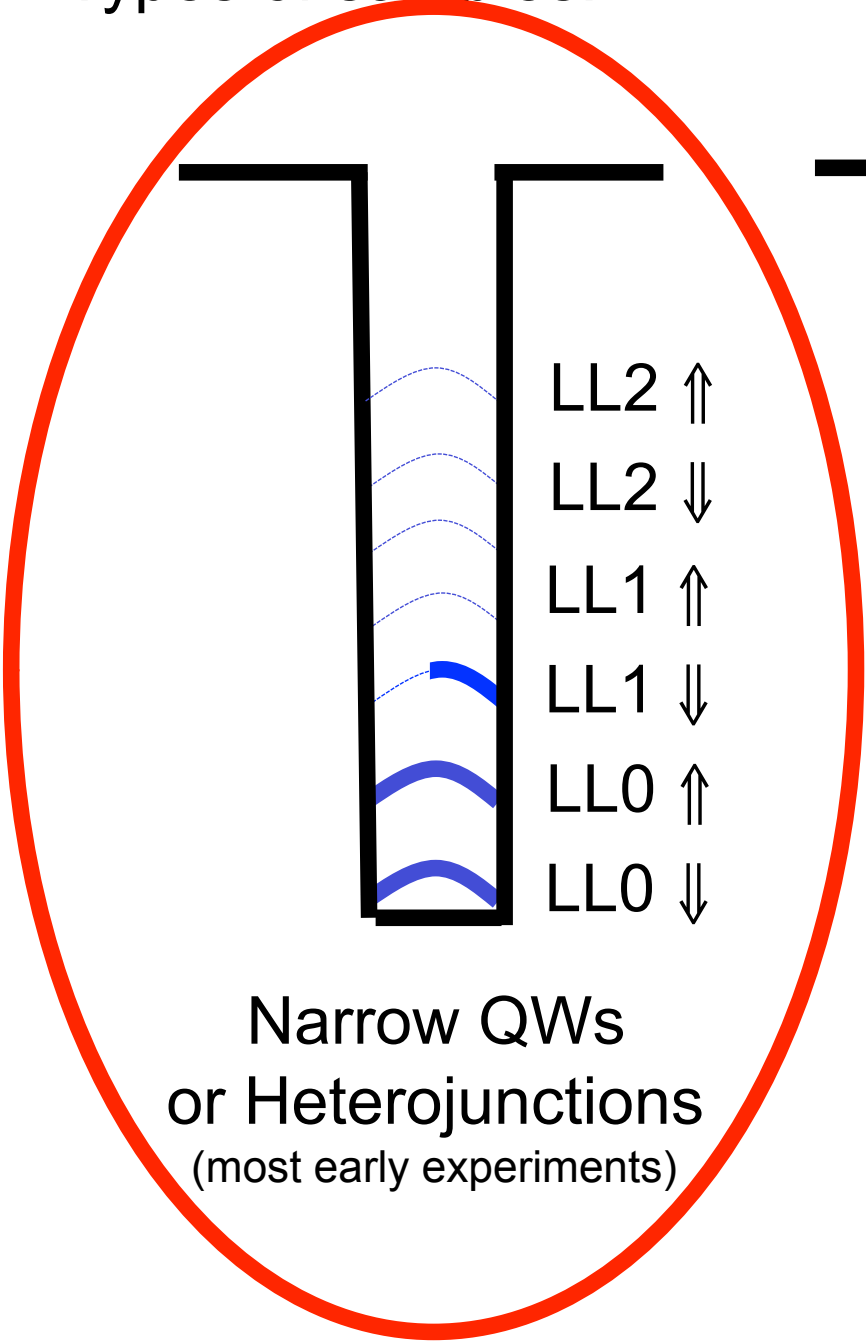
Hilbert Space Truncation Technique (Torus)

- Keep all states within the valence LL
- Allow only a “few” excitations outside of the valence LL
- Variational: as “few” \rightarrow “many”, becomes exact.

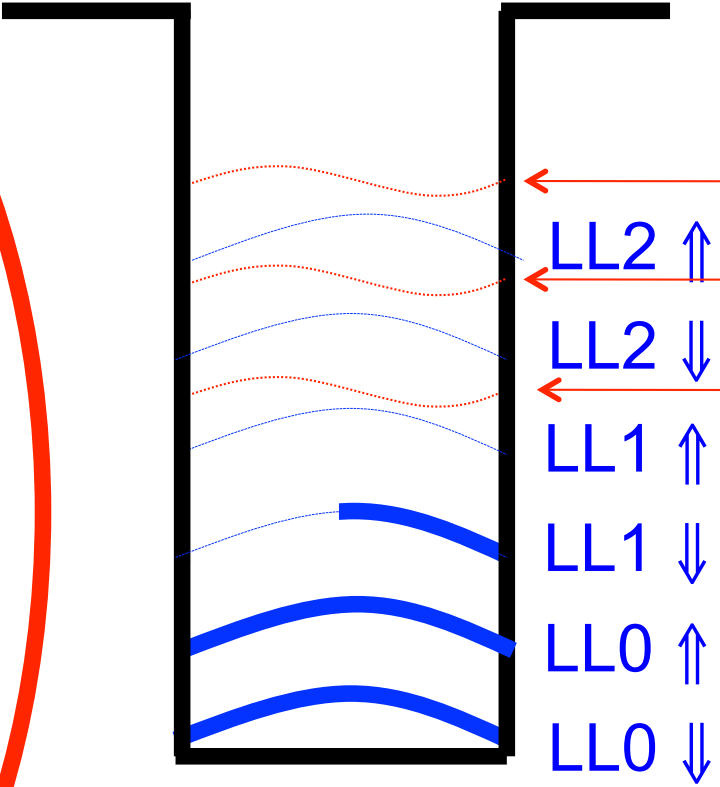
Makes sense because matrix elements to high LL's rapidly become less important.



Types of samples:



Narrow QWs
or Heterojunctions
(most early experiments)



SB2-LL1 \downarrow
SB2-LL0 \uparrow
SB2-LL0 \downarrow

Wider QWs
(many recent experiments)

Step 1: Is it polarized?

Example: 6 orbitals per LL (x2 for spin)

Allow complete freedom within LL1

Also allow up to 2 holes in LL0, 2 electrons in LL2

Hilbert Space =
 10^7 dimension



Result: Ground state is still polarized.

Can vary these parameters, ex, 6 becomes 8
2 becomes 1
transitions to LL3 etc...

Result: Ground state is still polarized.

Step 2: Does it matter if we allow transitions of minority spins?



Again, explore allow increasing number of inter-LL transitions

Projected overlap > 0.98 even for very large systems

Conclude: Can ignore minority spins

Assumption of spin polarized ground state, plus neglect of minority spin species transitions allows us to examine reasonably large systems.

Step 3: With these assumptions look at large systems (with various Hilbert space truncation schemes) and see if ground state is Pfaffian or AntiPfaffian.

Note: For small systems $|\langle \text{Pf} | A \text{Pf} \rangle|$ can be very large, so if the ground state overlaps well with one, it also overlaps well with the other.

... now for the results...

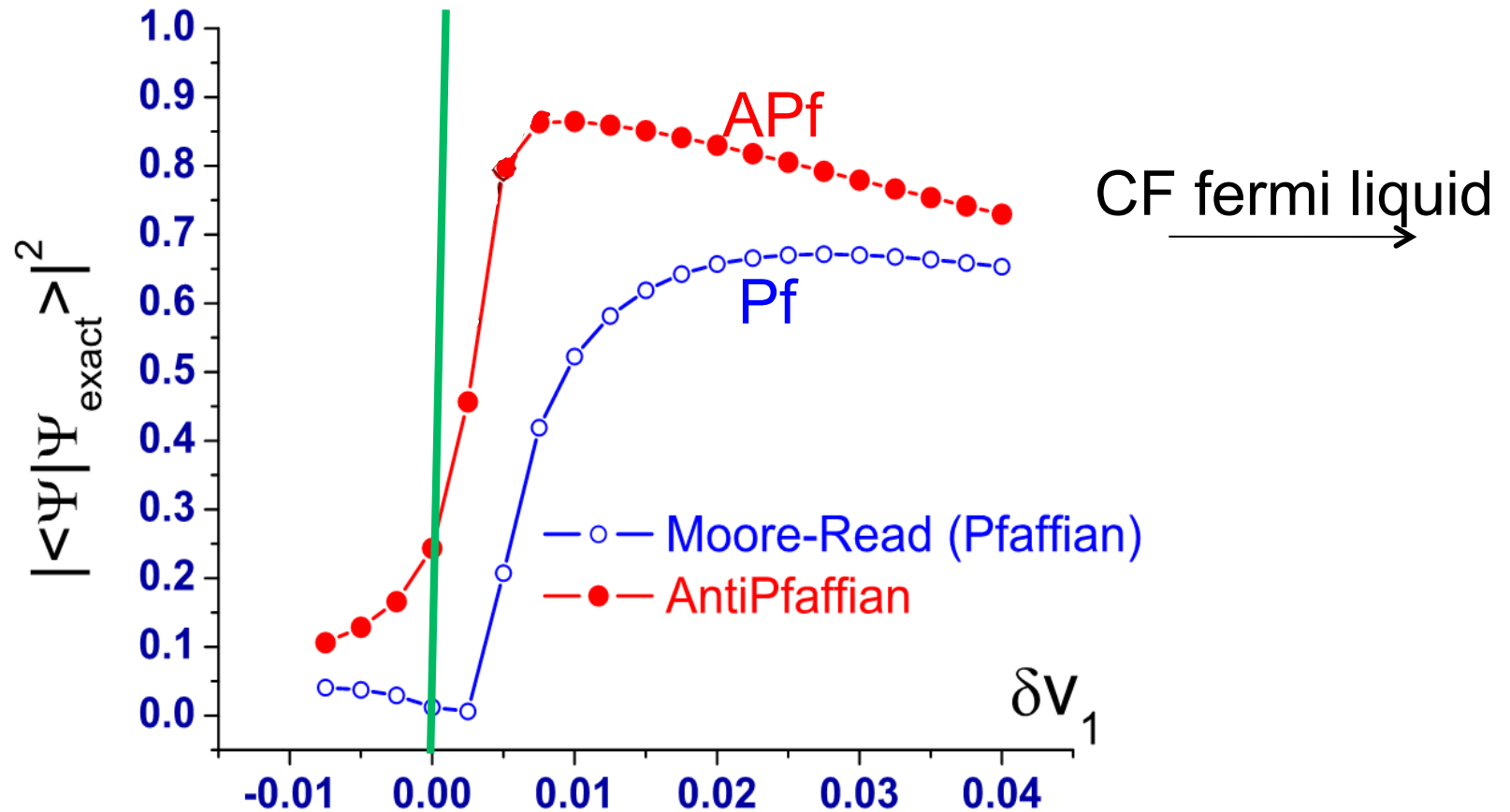
$$E_{coulomb}/E_{cyclotron} = 1.34 \quad (n = 2.3 \times 10^{11})$$

$$|\langle \text{Pf} | \text{APf} \rangle|^2 = 0.29$$

$N_e = 50$

20 states per LL

Hexagonal Unit Cell (ground state unique – 3 fold degen)



2 holes in LL0 allowed

1 electron in LL2 allowed

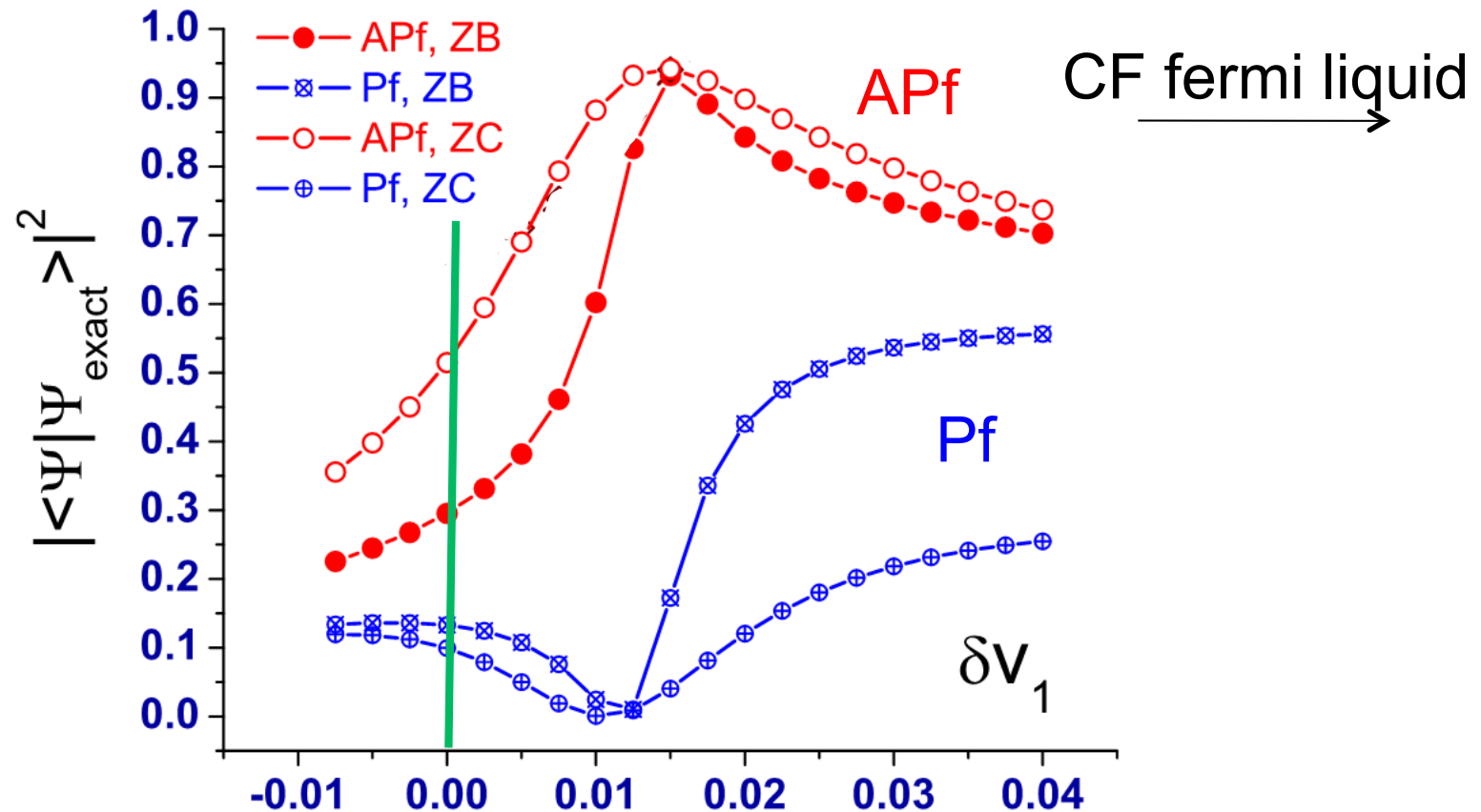
Anti-Pfaffian Wins !

$$E_{coulomb}/E_{cyclotron} = 1.34 \quad (n = 2.3 \times 10^{11})$$

$N_e = 50$
20 states per LL

$$|\langle Pf | APf \rangle|^2 = \begin{cases} 0.12 & \text{Zone Boundary} \\ .008 & \text{Zone Corner} \end{cases}$$

Square Unit Cell : Zone Corner = 1 fold, Zone Boundary = 2 fold



2 holes in LL0 allowed
1 electron in LL2 allowed

Anti-Pfaffian Wins Big!

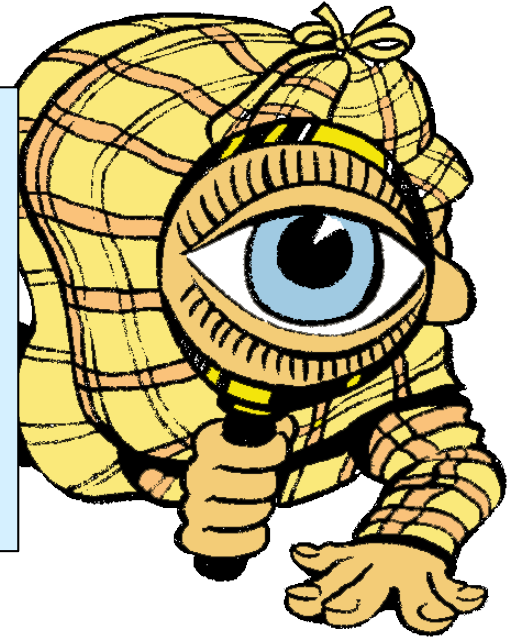
The Pfaffian never wins if even a single virtual electron is allowed in LL2.

(b)	N_ϕ	N	$0\downarrow$	$1\downarrow$	$2\downarrow$	dim	Pf	APf	$\langle P A \rangle^2$
$\mathcal{H}_{p,6}$	12	30	12	6	0	14	.90	.90	.69
$\mathcal{H}_{p,8}$	16	40	12	8	0	1.0×10^2	.53	.53	.016
$\mathcal{H}_{p,10}$	20	50	12	10	0	9.2×10^2	.71	.71	.29
$\mathcal{H}_{p,12}$	24	60	12	12	0	9.4×10^3	.56	.56	.059
$\mathcal{H}_{r,1}$	12	30	10-12	6-8	0	6.0×10^2	.94	.83	.69
$\mathcal{H}_{r,2}$	12	30	10-12	5-8	0-1	1.1×10^4	.80	.89	.69
$\mathcal{H}_{r,3}$	12	30	10-12	4-8	0-2	7.6×10^4	.83	.89	.69
$\mathcal{H}_{r,4}$	12	30	9-12	3-9	0-3	1.1×10^6	.82	.89	.69
$\mathcal{H}_{r,5}$	16	40	14-16	8-10	0	9.1×10^3	.63	.33	.016
$\mathcal{H}_{r,6}$	16	40	14-16	7-10	0-1	2.1×10^5	.34	.51	.016
$\mathcal{H}_{r,7}$	16	40	14-16	6-10	0-2	1.8×10^6	.37	.56	.016
$\mathcal{H}_{r,8}$	20	50	18-20	10-12	0	1.4×10^5	.40	.00	.29
$\mathcal{H}_{r,9}$	20	50	18-20	9-12	0-1	3.8×10^6	.01	.24	.29

Plus dozens of other exact diags...

... the trends are very clear....

Case Closed?



Perturbative approach: integrating out virtual transitions for weak interaction...

PHYSICAL REVIEW B **80**, 121302(R) (2009)

Effect of Landau level mixing on the effective interaction between electrons in the fractional quantum Hall regime

Waheb Bishara¹ and Chetan Nayak^{2,3}

Result:

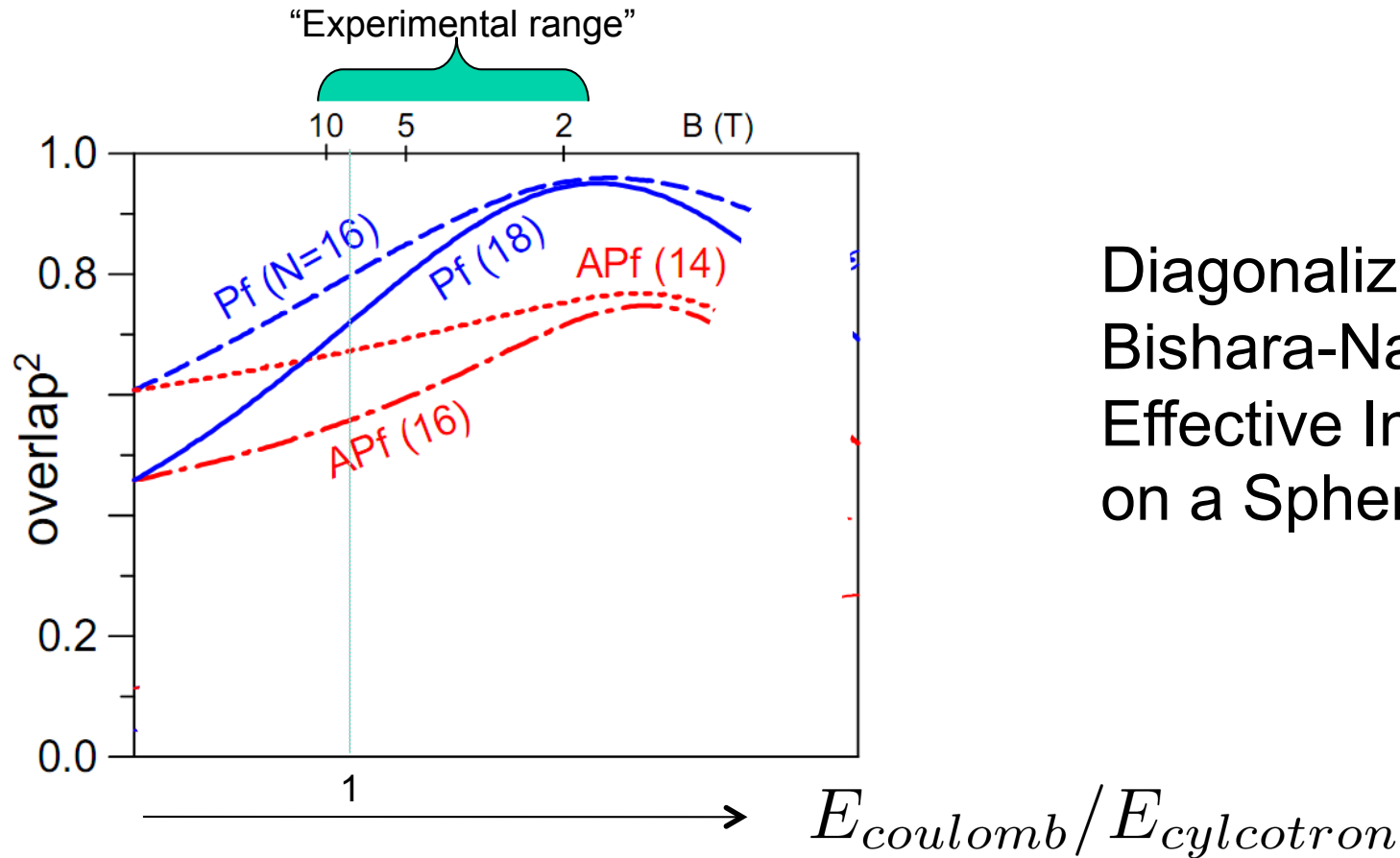
3 electrons at closest approach (L=3)	E= - 0.0147
at next closest approach (L=5)	E= - 0.0054
at next closest approach (L=6)	E= - 0.0099
at next closest approach (L=7)	E= + 0.0005
⋮	⋮

E in units of $E_{coulomb}$ ($E_{coulomb}/E_{cyclotron}$)

(also small modification of 2-body interaction at same order)

Landau-Level Mixing and the Emergence of Pfaffian Excitations for the 5/2 Fractional Quantum Hall Effect

Arkadiusz Wójs,^{1,2} Csaba Tóke,³ and Jainendra K. Jain⁴



Diagonalization of
Bishara-Nayak
Effective Interaction
on a Sphere.

1. Repeated the diagonalizations of Wojs, Toke, Jain :
we agree with their result
3. Hilbert Truncation approach for small $E_{coulomb}/E_{cyclotron}$
we still get APf ! $\Rightarrow\Leftarrow$
3. Repeated Bishara-Nayak calculation we do **not** agree with their results!

(At least one of us must be wrong....)

If we use our corrected effective interaction in the
Wojs Toke Jain calculation ... now we get APf !



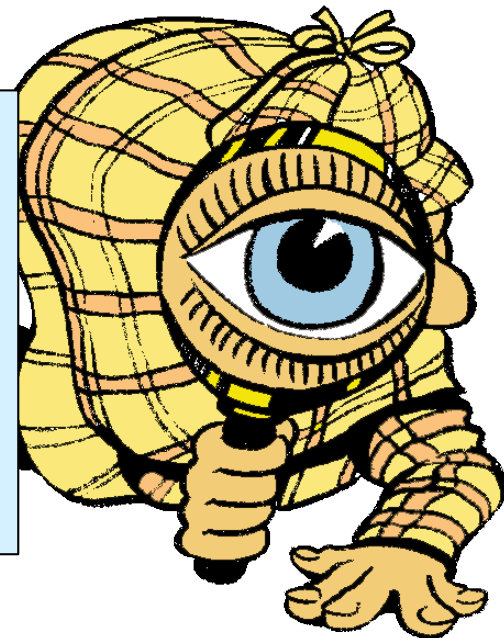
Summary:

We say $5/2$ is the spin-polarized Anti-Pfaffian!

Hopefully soon there will be agreement...

What do the experiments say...

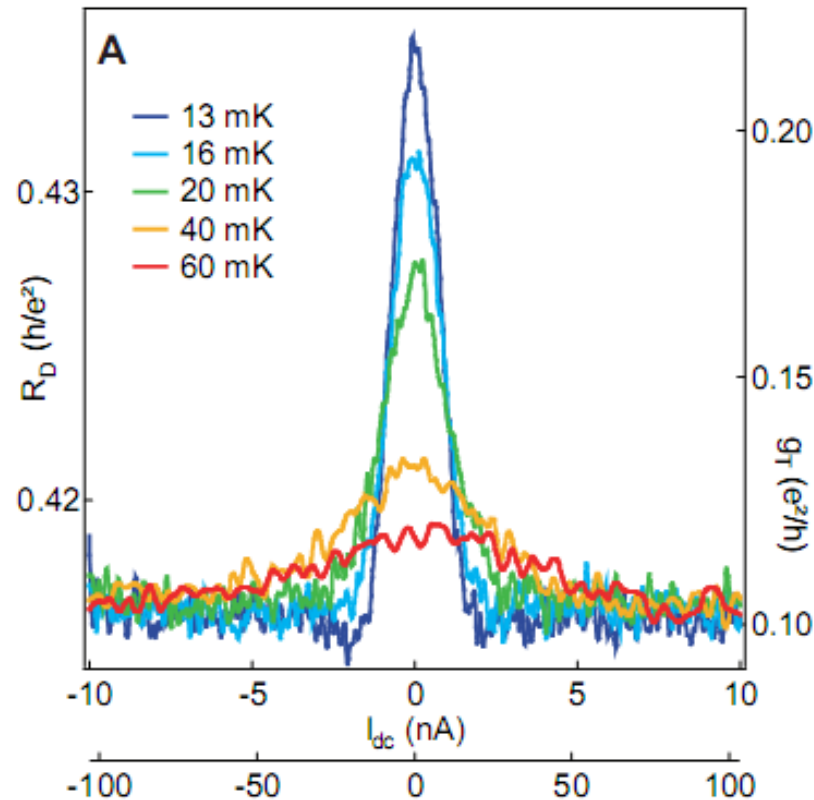
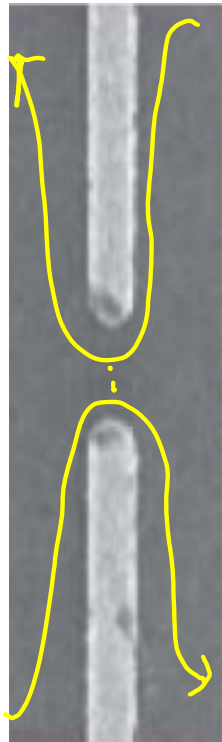
Case Closed?



Quasiparticle Tunneling in the Fractional Quantum Hall State at $\nu = 5/2$

Iuliana P. Radu,¹ J. B. Miller,² C. M. Marcus,² M. A. Kastner,¹ L. N. Pfeiffer,³ and K. W. West³

(Dated: 7 March 2008)



Theory: X.-G. Wen 1991

Differential conductance g_T

$$g_T = AT^{(2g-2)} F\left(g, \frac{e^* I_{dc} R_{xy}}{kT}\right)$$

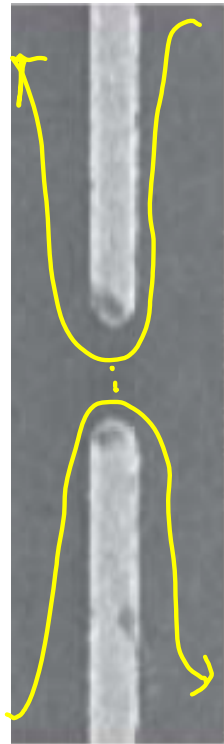
F = a known functional form

g = Interaction parameter

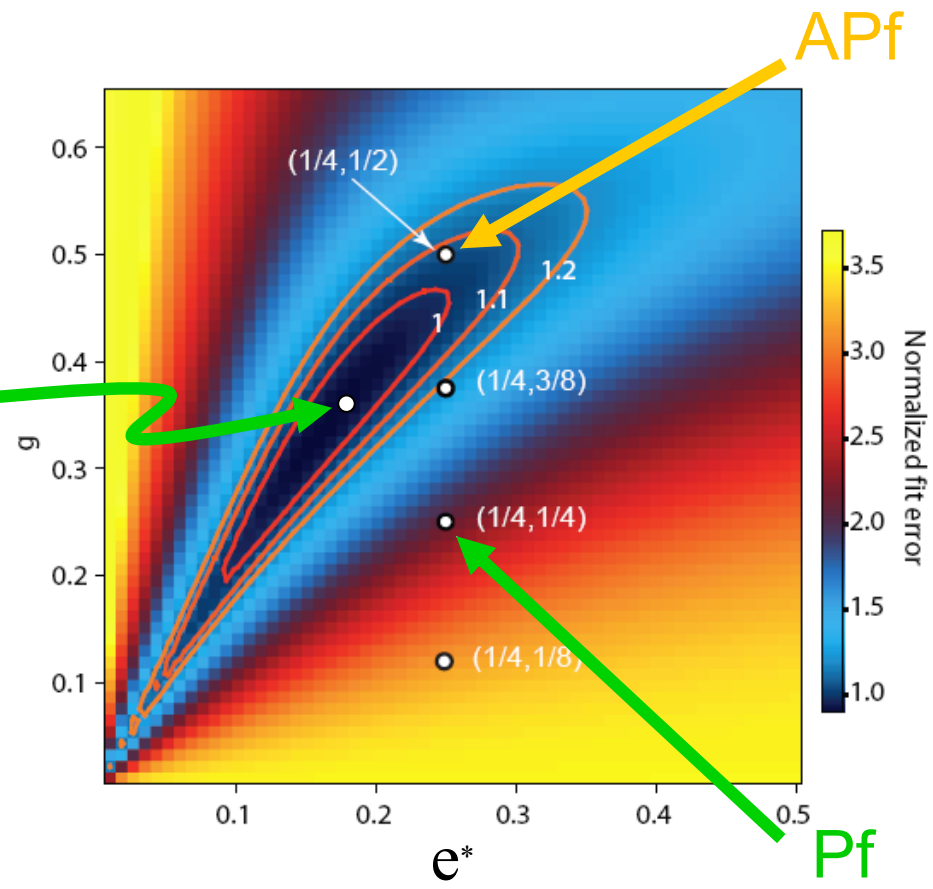
e^* = tunneling qp charge

Quasiparticle Tunneling in the Fractional Quantum Hall State at $\nu = 5/2$

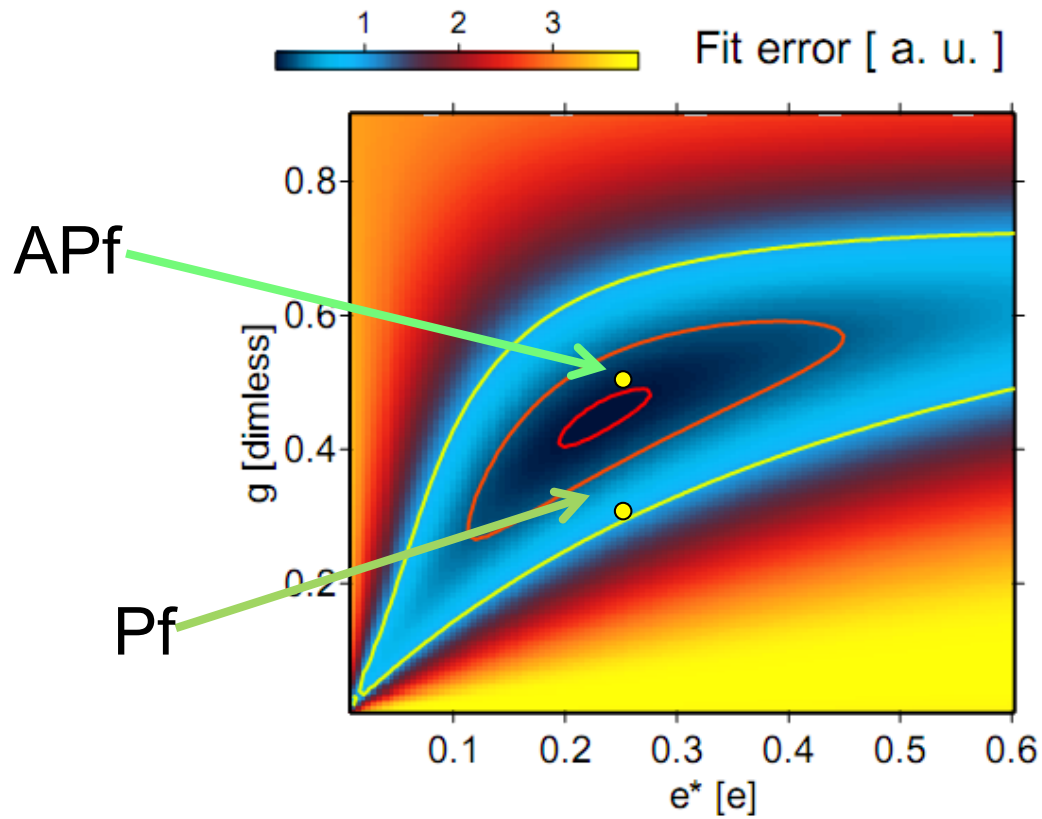
Iuliana P. Radu,¹ J. B. Miller,² C. M. Marcus,² M. A. Kastner,¹ L. N. Pfeiffer,³ and K. W. West³



Best fit
 $e^* = .17 e$
 $g = .35$



AntiPfaffian fits better!



Better Data:

From Thesis
of Yiming Zhang
C. Marcus Group

[Available on Web]

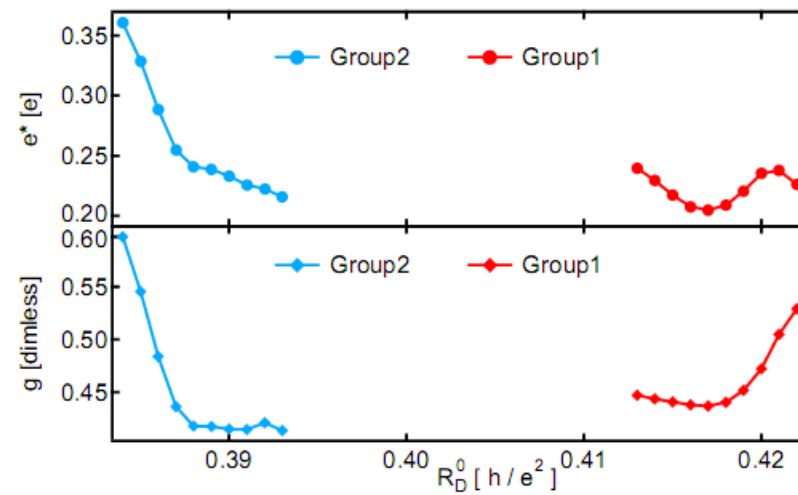
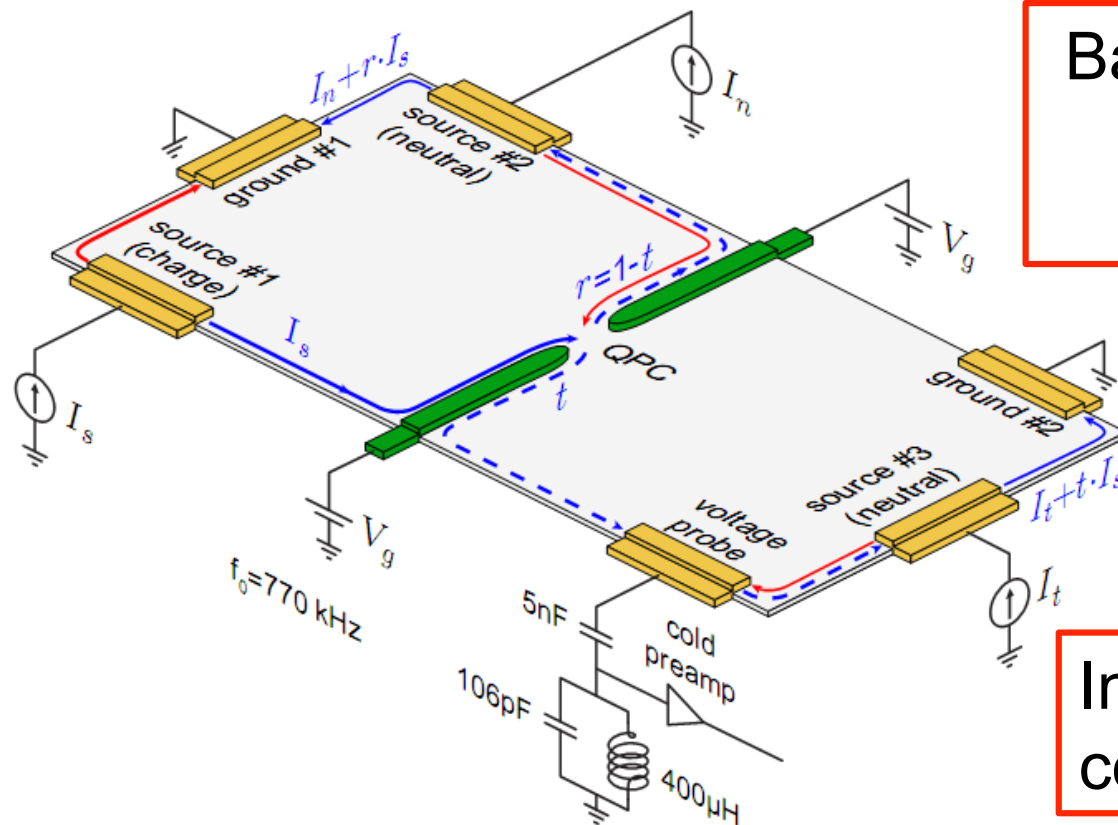


Figure 9.8: Best-fit e^* and g as a function of R_D^0 for both groups of tunneling peaks.

ARTICLES

Observation of neutral modes in the fractional quantum Hall regime

Aveek Bid^{1*}, N. Ofek^{1*}, H. Inoue¹, M. Heiblum¹, C. L. Kane², V. Umansky¹ & D. Mahalu¹

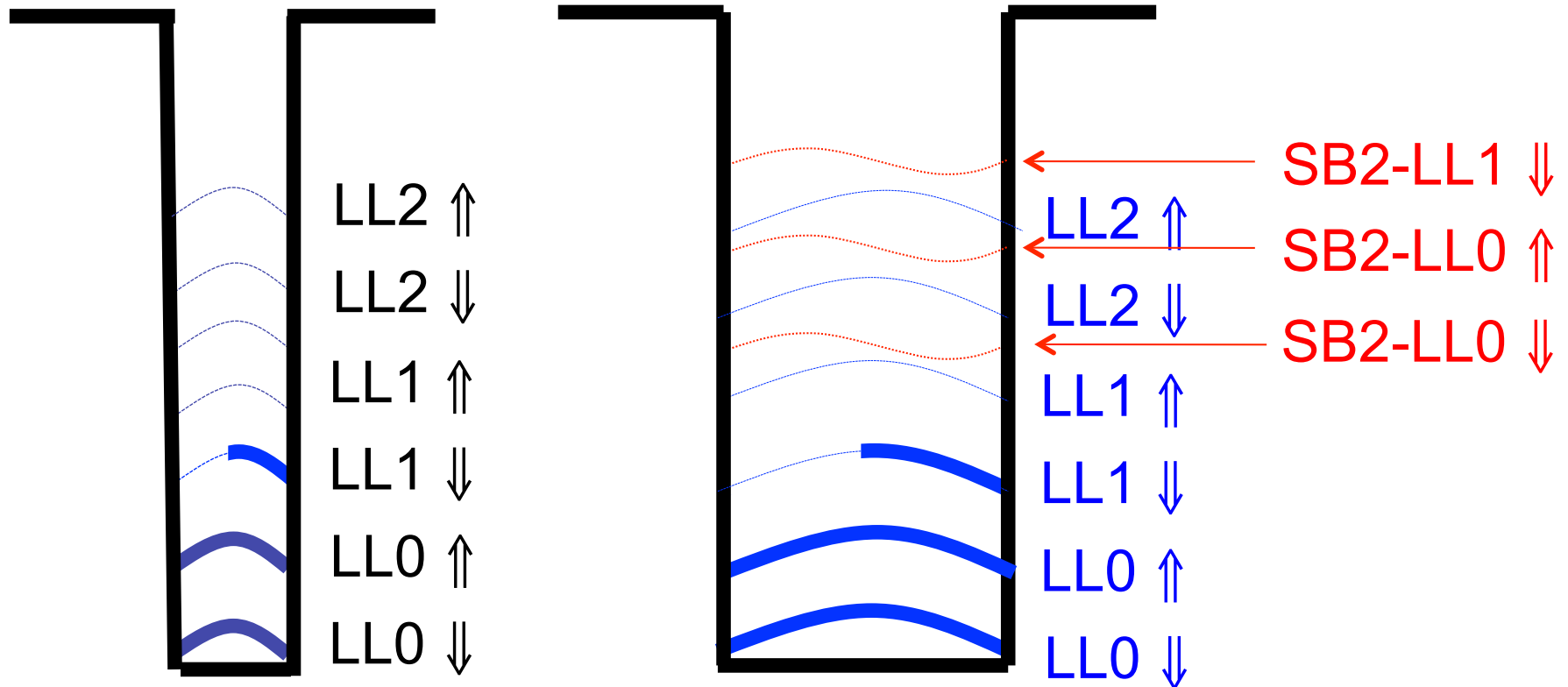


Back-propagating neutral mode heats QPC. Detected by noise.

In simplest picture consistent with APf not Pf

Truth in Advertising:

Calculations for wide quantum well samples are *much* harder.
So far it looks like the AntiPfaffian still wins... but still in progress

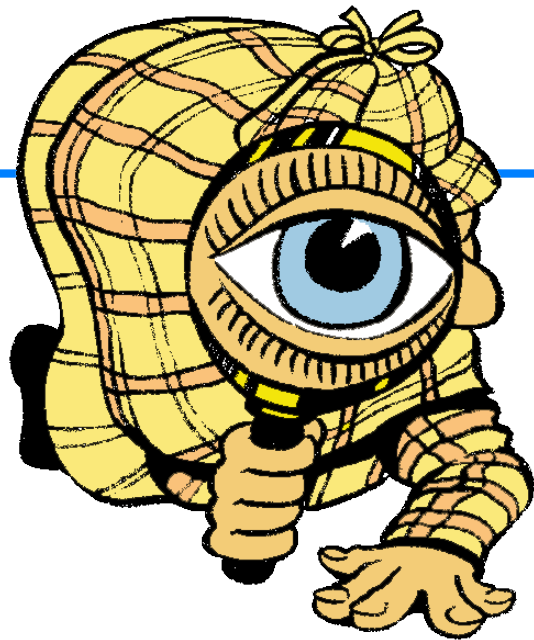


Narrow QWs
or Heterojunctions
(most early experiments)

Wider QWs
(many recent experiments)

$$\nu = 5/2$$

*A condensed matter who-done-it
(or “what is it”)*

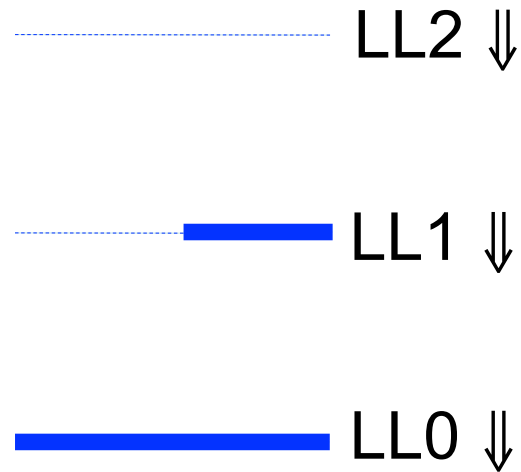


Steven H. Simon

Work with Ed Rezayi (Calstate LA)



A bit more understanding?



When no transitions are allowed to LL2
(but holes allowed in LL0), then Pfaffian is favored

But when we also allow transitions to LL2 and
higher, the Anti-Pfaffian wins

Handwaving argument (“valid” for weak interaction) :

- LL2 ↓ At 2nd order perturbation theory energy is lowered.
- LL1 ↓ Can generate interactions of 3-electrons by virtual transitions up to higher LL's.
- LL0 ↓ Stabilizes three electrons getting close to each other. Favors AntiPfaffian.

When no transitions are allowed to LL2 (but holes allowed in LL0), then Pfaffian is favored

But when we also allow transitions to LL2 and higher, the Anti-Pfaffian wins

Handwaving argument (“valid” for weak interaction) :

- LL2 ↓ At 2nd order perturbation theory energy is lowered.
- LL1 ↓ Can generate interactions of **3-holes** by virtual transitions down to **lower LLs**.
- LL0 ↓ Stabilizes three **holes** getting close to each other. Favors **Pfaffian**.

(why transitions up are more important than transitions down, I don't know!)

When no transitions are allowed to LL2
(but holes allowed in LL0), then Pfaffian is favored

But when we also allow transitions to LL2 and
higher, the Anti-Pfaffian wins

.... a possible wrench in the works?



PRL 105, 096801 (2010)

Selected for a *Viewpoint in Physics*
PHYSICAL REVIEW LETTERS

week ending
27 AUGUST 2010



Optical Probing of the Spin Polarization of the $\nu = 5/2$ Quantum Hall State

M. Stern,^{1,*} P. Plochocka,² V. Umansky,¹ D. K. Maude,² M. Potemski,² and I. Bar-Joseph¹

Also: March APS 2010 Abstract: Y2.00003. T.-D. Rhone (A. Pinczuk group)
March APS 2009 Abstract: W2.00005. A. Pinczuk

Optical experiments: claim consistency with $\nu = 5/2$ being *not* polarized

(Ask me at the end.....)

related to: PRL 2010 Wojs, Moller, Simon, Cooper

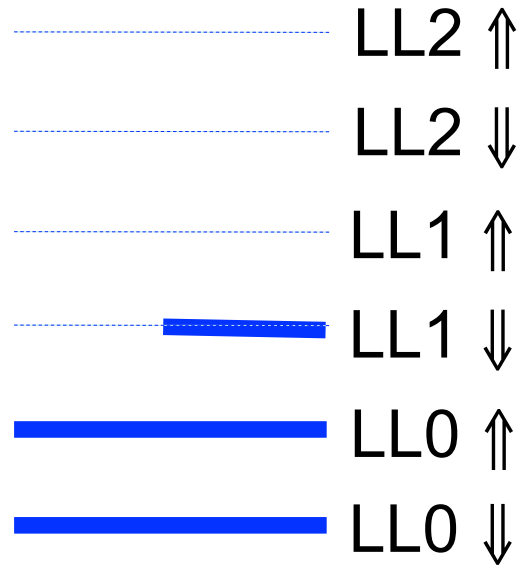
(Plus some possible questions from transport as well... Dean et al PRL 2008)



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(Also similar from Rhone, Pinczuk...)



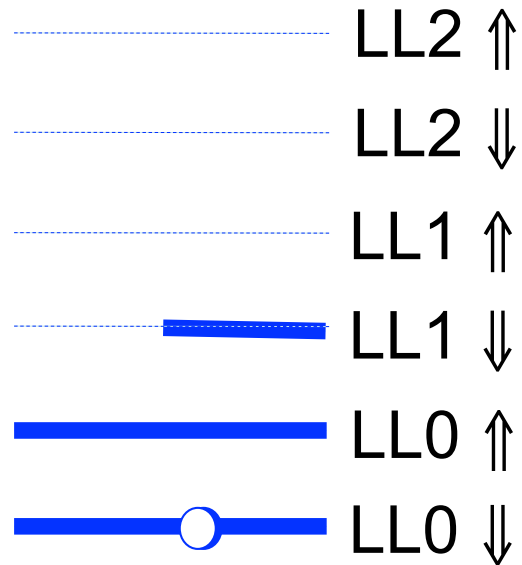
Photoluminescence Experiments:



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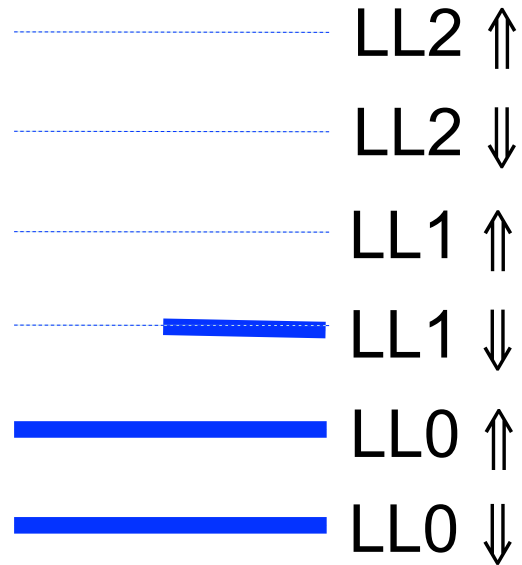
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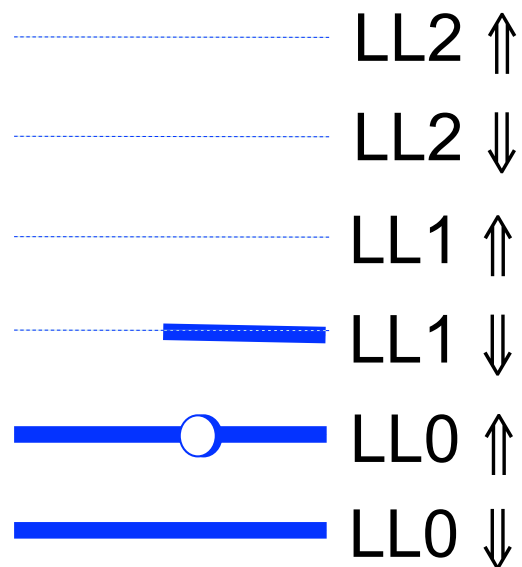
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Photoluminescence Experiments:



Valence band

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Photoluminescence Experiments:

Measures energy difference between σ_+ (\uparrow) and σ_- (\downarrow) recombination.

Sees only bare Zeeman energy:

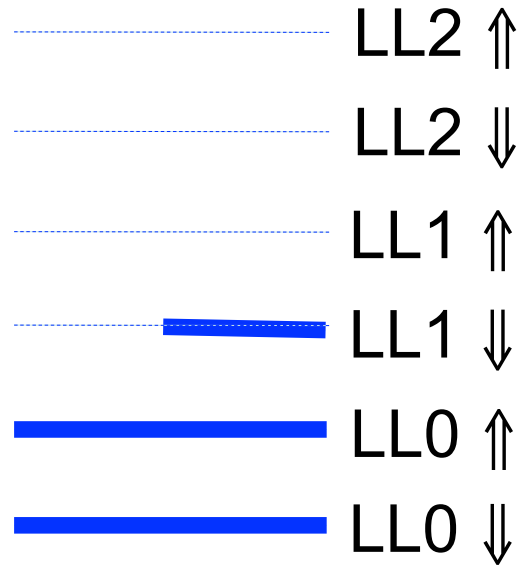
no “enhanced” Zeeman expected due to interaction with a polarized LL1 !

Valence band

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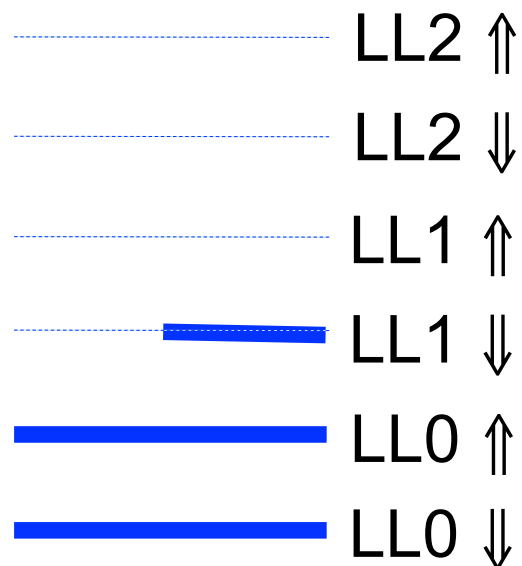
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(Also similar from Rhone, Pinczuk...)



A hole sits in the valence band and thermalizes.
The electrons see a strong potential from this hole!

Could the strong potential effect the outcome of the experiment?
1. Local shift of filling fraction?



Valence band

Could Skyrmions be Involved?

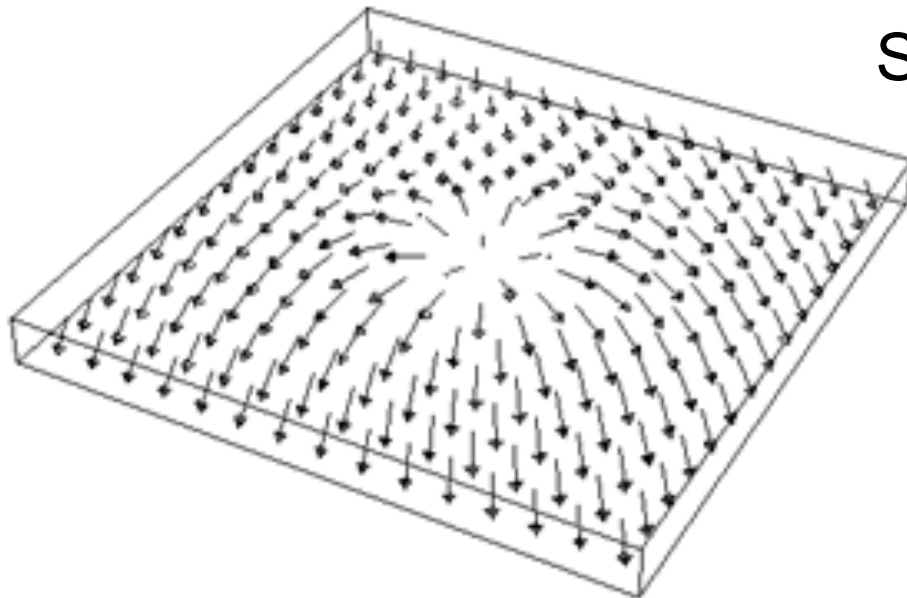
PRL **104**, 086801 (2010)

PHYSICAL REVIEW LETTERS

week ending
26 FEBRUARY 2010

Skyrmions in the Moore-Read State at $\nu = \frac{5}{2}$

Arkadiusz Wójs,^{1,2} Gunnar Möller,¹ Steven H. Simon,³ and Nigel R. Cooper¹



Skyrmion:

Spin structure creates 1 flux worth of effective flux due to Berry's phase

Therefore charge is $e\nu = e/2$.

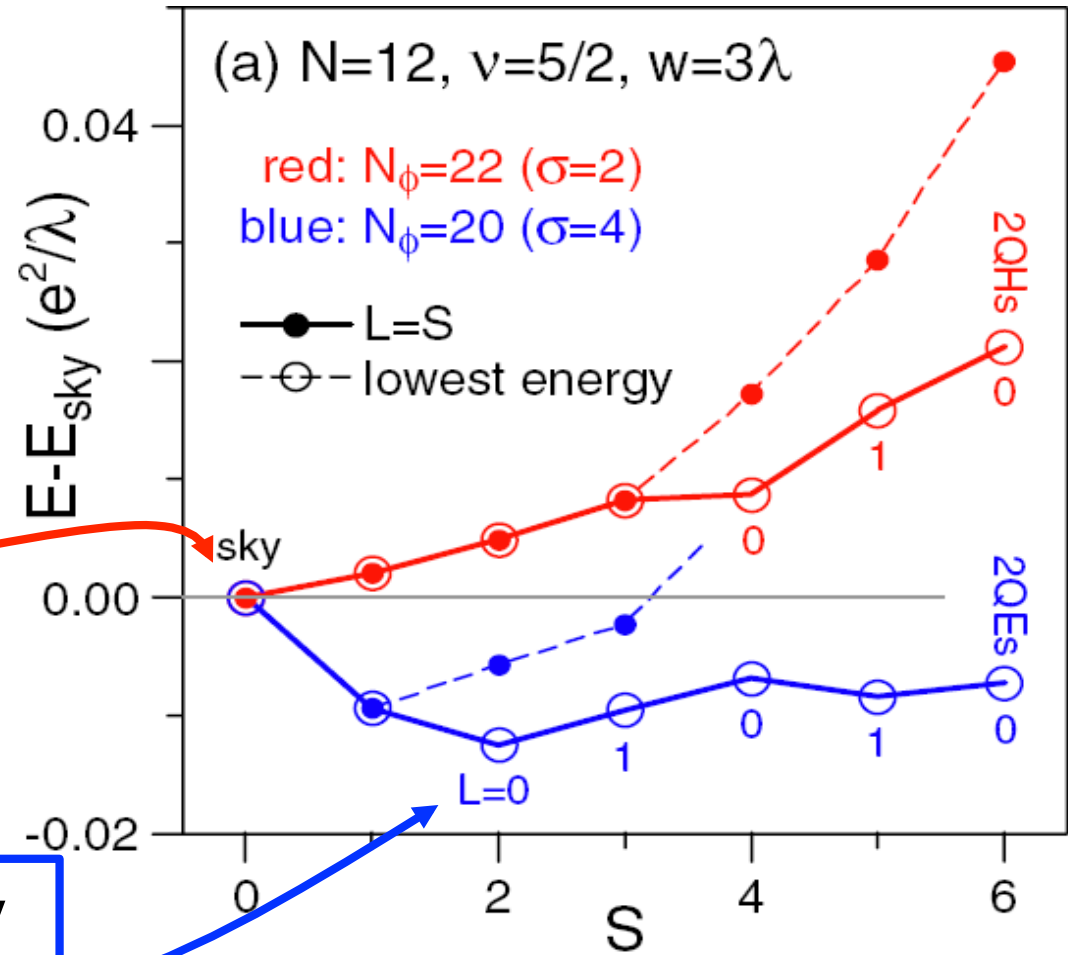
Moore-Read is different from Laughlin case, because the elementary charged qp is smaller charge than skyrmion

Disorder that “holds together” charge could favor skyrmion

Without Zeeman

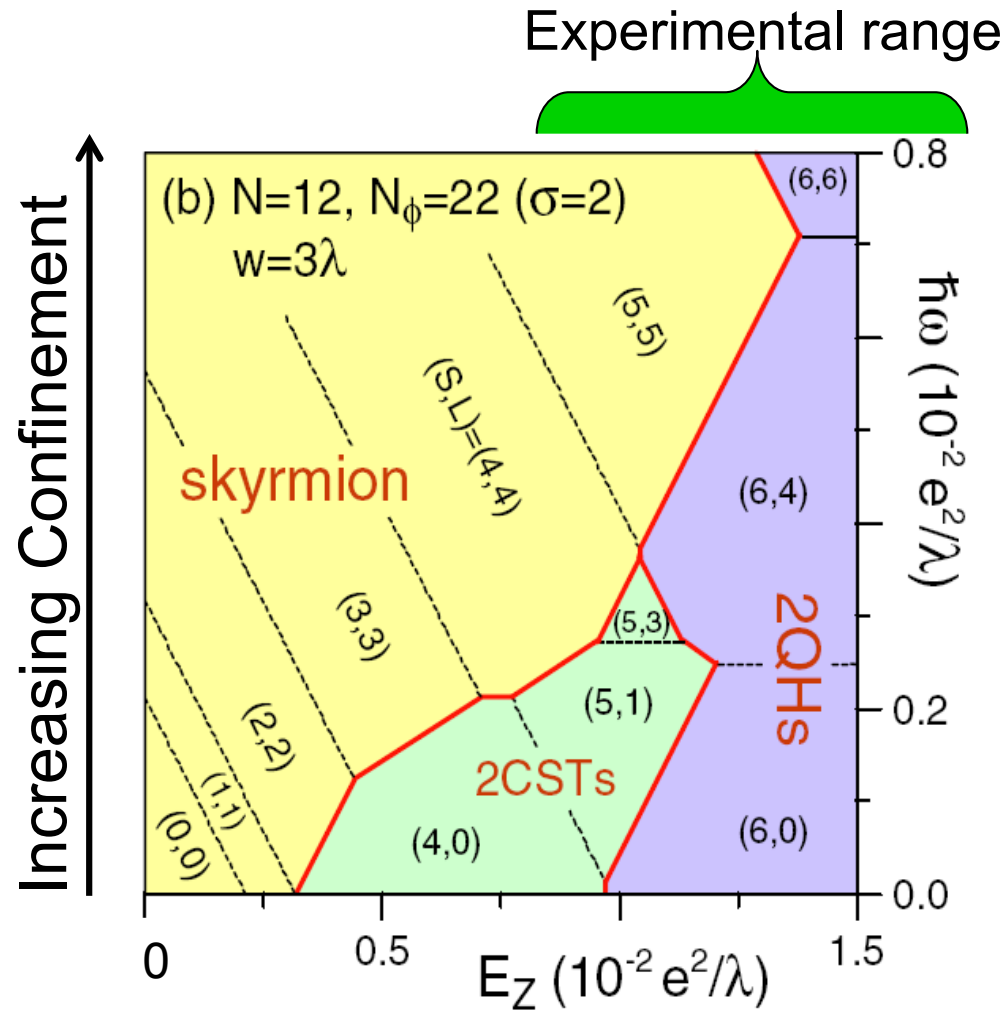
For Pf QH lowest energy is spinless once one flux is added

For Pf QE lowest energy is mixed spin once one flux is removed



Pf quasiholes (APf quasielectrons) bind to form skyrmions

Pfaffian quasielectrons (APf quasiholes) maybe ...



- Disorder (potential well) holds 2 Pf qh's (APf qe's) together and favors Skyrmions.
- Zeeman disfavors Skyrmions.

(Assuming AntiPfaffian)

HOLE TRAPS SKYRMIONS

RECOMBINATION FROM REGION
OF SCRAMBLED SPIN LOOKS
UNPOLARIZED