Challenges & Opportunity in Low Dimensional Systems
NORDITA, Stockholm 2010

# Novel Phases: From Light to Electrons


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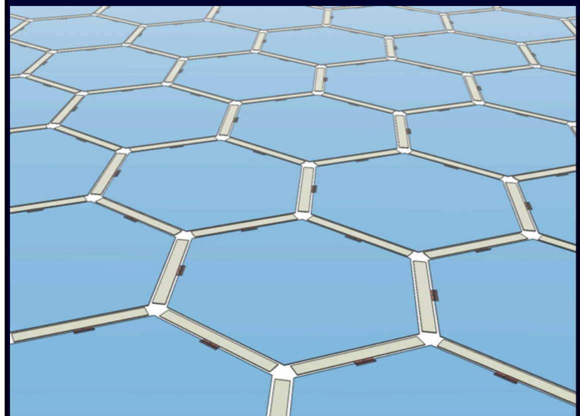
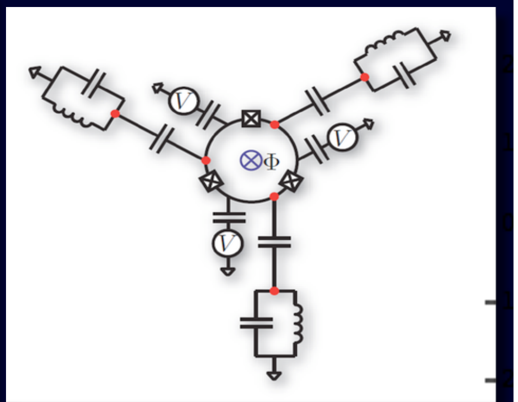
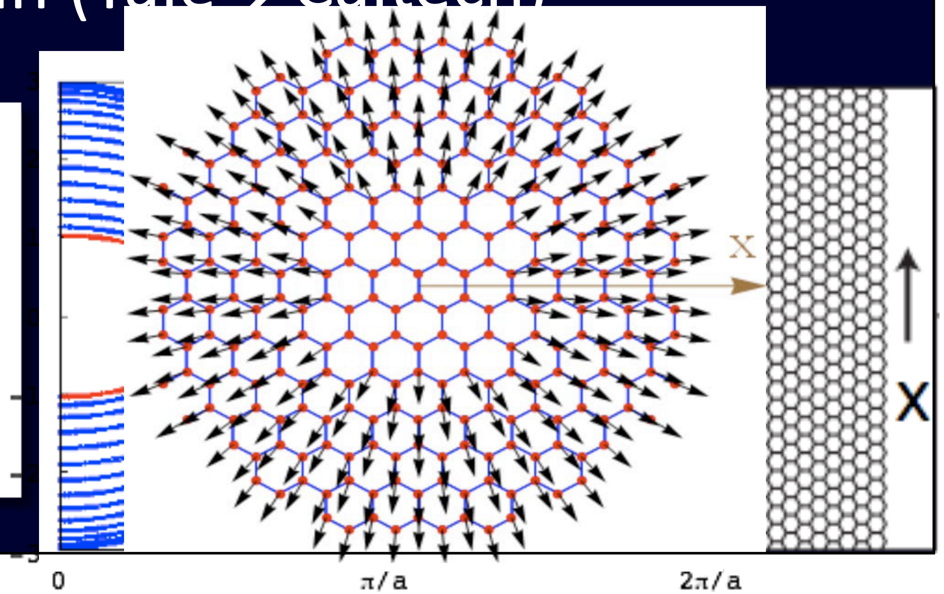
**Yale**

Jens Koch (Yale→NorthWestern), Steve Girvin
S. Rachel, D. Bergman (Yale→Caltech)

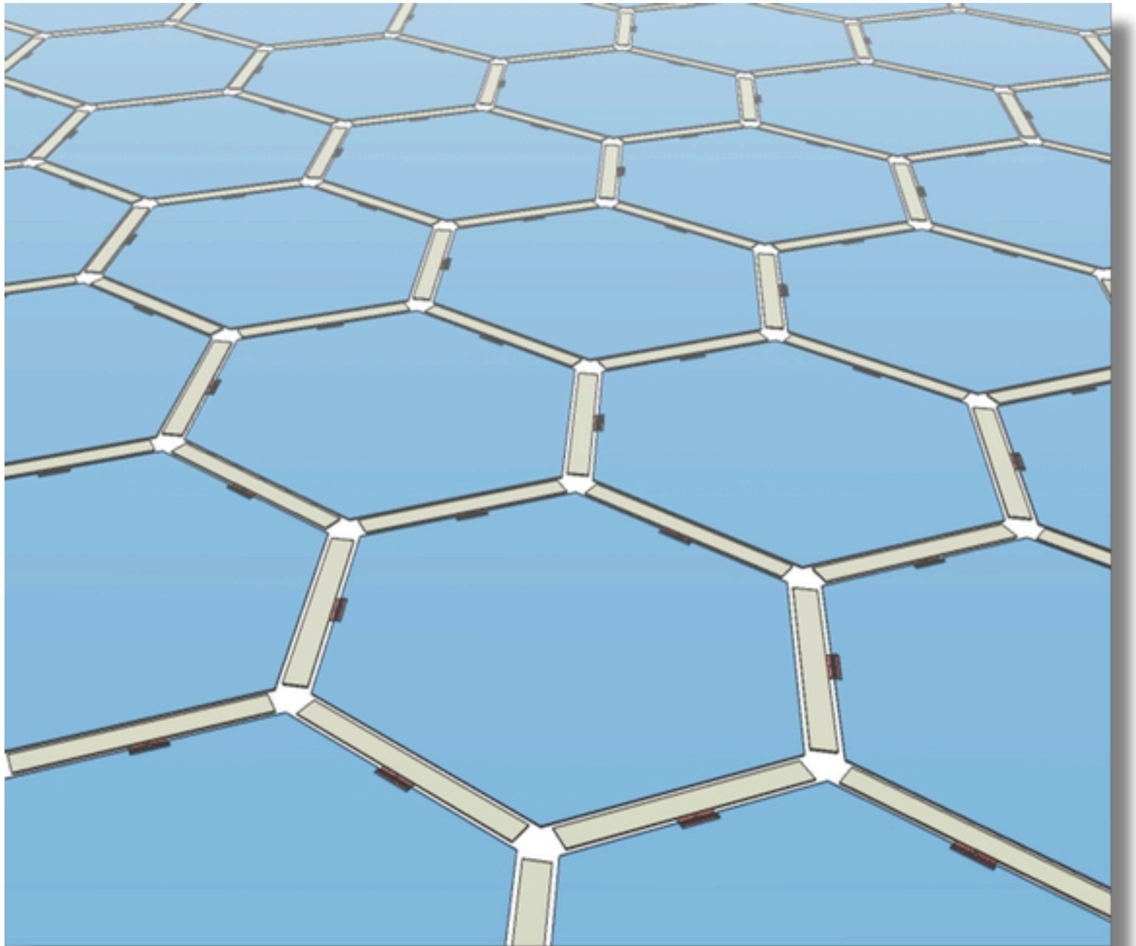









# Outline



*Theorists daydreaming:*
*systems of interacting photons*

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**Jaynes-Cummings lattice model**

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**Artificial Gauge Fields**
*for photons using on-chip circuitors*
*(topological phases)*

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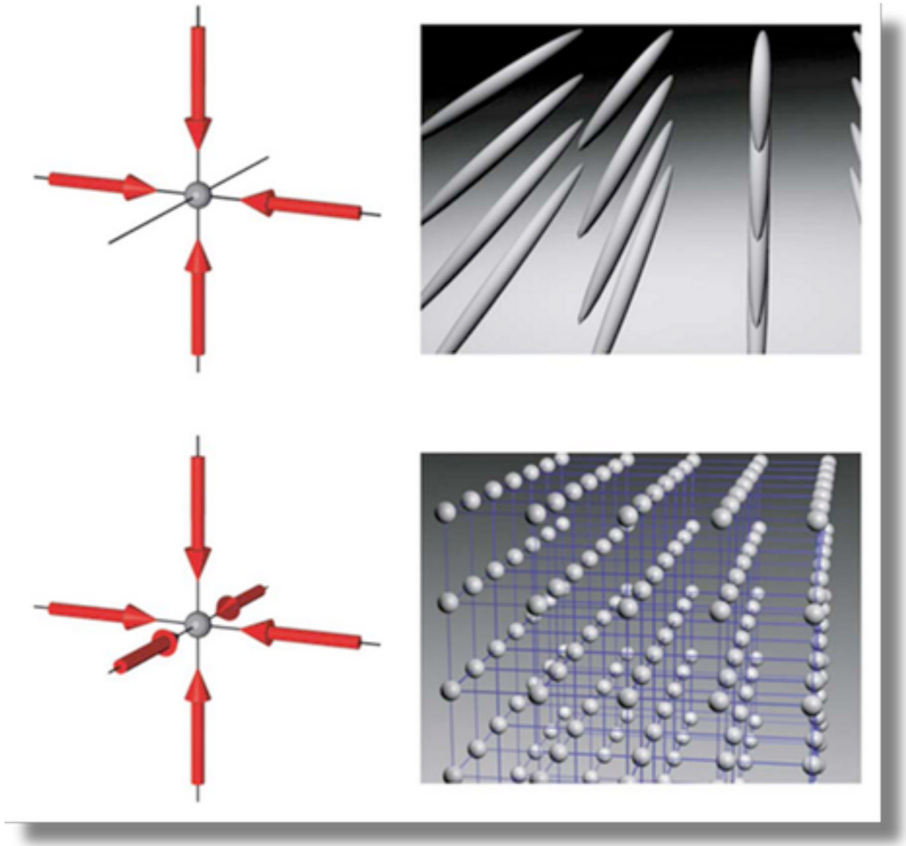
Electrons:
Topological insulators in 2D & Interactions (work with S. Rachel) PRB 2010 & SC of Dirac fermions (work with D. Bergman, 2009)

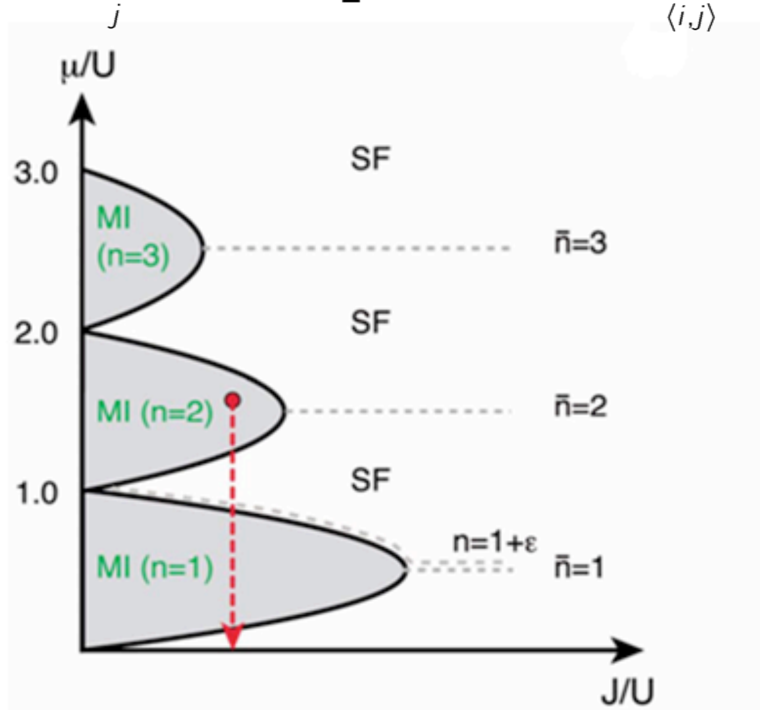
Jens Koch and KLH, PRA **80**, 023811 (2009)
Jens Koch, A. Houck, KLH & S. Girvin, arXiv:1006.0762, 2010
KLH, 0909.4822, chapter in the book on Understanding quantum phase transitions, ed. L.D. Carr

# Ultracold atomic gases



e.g., realization of the Bose-Hubbard model:

$$H = \sum_j [-\mu a_j^\dagger a_j + \frac{1}{2} U n_j (n_j - 1)] - J \sum_{\langle i,j \rangle} (a_j^\dagger a_i + \text{h.c.})$$




Josephson junction arrays!
B. Doucot, D. Haviland (Friday)

I. Bloch, J. Dalibard, W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)

# Systems of interacting photons: Theory surveys

► M. Hartmann et al., Laser & Photonics Review 2, 527 (2008)
A. Tomadin & R. Fazio, J. Opt. Soc. Am B 27, A130 (2010)

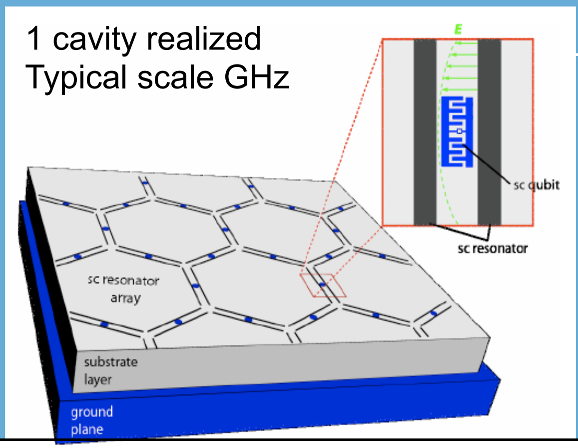
**Interacting photons:**

R. Y. Chiao, T.H. Hansson, J.M. Leinaas & S. Viefers, 2003

M. Lukin, E. Demler et al: Fermionizing light

*proposed realizations*

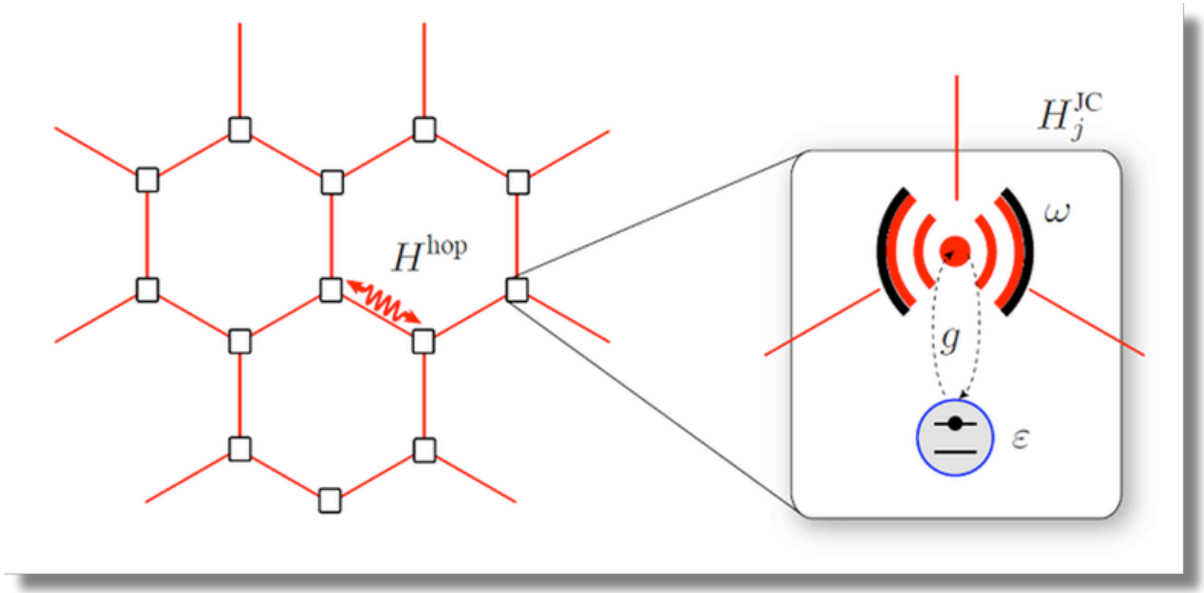
* photonic band gap cavities
* arrays of silicon micro-cavities
* fibre based cavities
* circuit QED: J. Koch & KLH




*some pros and cons*

+ tunability
+ access to single lattice site
- must be treated as open system
+ interesting: transitions between different steady states

# The Jaynes-Cummings "Lattice" Model




Jaynes-Cummings model: 1963
(famous model in quantum optics)

Greentree et al., Nat. Phys. **2**, 856 (2006)
Angelakis et al., PRA **76**, 031805 (2007)
Jens Koch and KLH, PRA **80**, 023811 (2009)

Other groups: R. Fazio, G. Blatter, S. Bose, Y. Yamamoto, P. Littlewood, M. Plenio, B. Simons, A. Sandvik,...

Jaynes-Cummings lattice model $H = \sum_j H_j^{\mathrm{JC}} + H^{\mathrm{hop}} - \mu N$ "chemical potential"

- *Jaynes-Cummings:* $H_j^{\mathrm{JC}} = \omega a_j^\dagger a_j + \varepsilon\sigma_j^+\sigma_j^- + g(a_j^\dagger\sigma_j^- + \sigma_j^+ a_j)$
- *nearest-neighbor photon hopping:* $H^{\mathrm{hop}} = -\kappa \sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i)$
- *polariton number:* $N = \sum_j (a_j^\dagger a_j + \sigma_j^+\sigma_j^-)$

# Simple analysis: Jaynes-Cummings Model

"Atomic" limit $\kappa \to 0$

$H = \sum_j (H_j^{JC} + \mu N_j)$

$\boxed{\Delta = \varepsilon - \omega}$

*eigenenergies:* $E_{n\pm}^{\mu} = E_{n\pm} - \mu n$

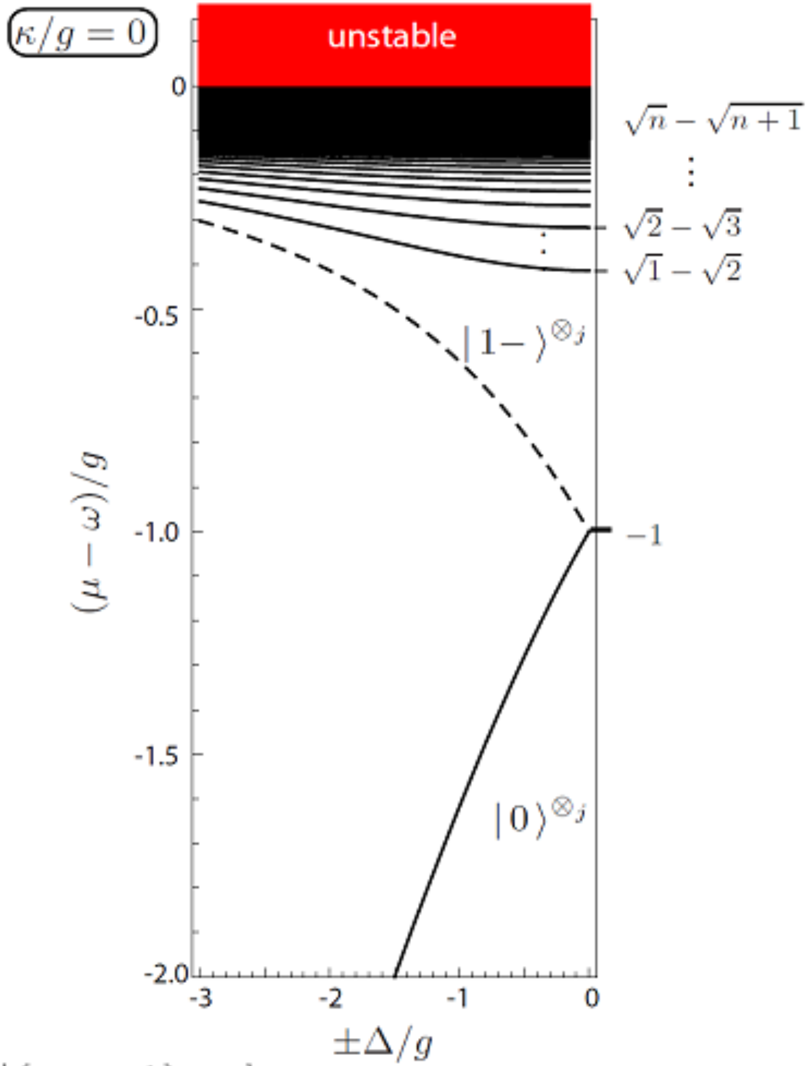
$$\begin{cases} E_0 = 0 \\ E_{n\pm} = n\omega + \Delta/2 \pm [(\Delta/2)^2 + ng^2]^{1/2} \quad (n \geq 1) \end{cases}$$

*ground state:* $E_{n\alpha}^{\mu} = \min\{E_0^{\mu}, E_{1\pm}^{\mu}, E_{2\pm}^{\mu}, \ldots\}$

► *fixed polariton number on each site*

‣ *extra polariton on site j does not propagate to other sites*

‣ *MOTT-INSULATING STATE (gapped, incompressible)*

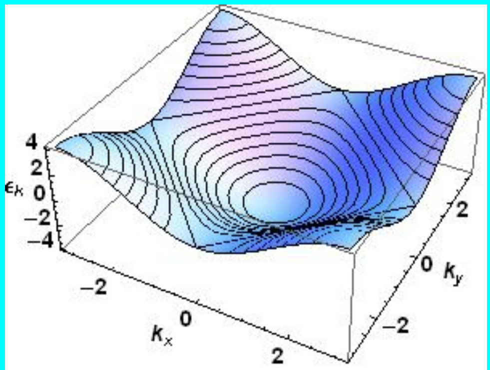



$$\begin{aligned} |n+\rangle &= \sin\theta_n |n\,,g\rangle + \cos\theta_n |(n-1)\,,e\rangle \\ |n-\rangle &= \cos\theta_n |n\,,g\rangle - \sin\theta_n |(n-1)\,,e\rangle \end{aligned}$$

# Other simple limit

Hopping dominated limit $\kappa/g \gg 1$

$$H^{\mathrm{tb}} = (\omega - \mu)\sum_i a_i^\dagger a_i - \kappa \sum_{\langle i,j\rangle}\left(a_i^\dagger a_j + a_j^\dagger a_i\right)$$

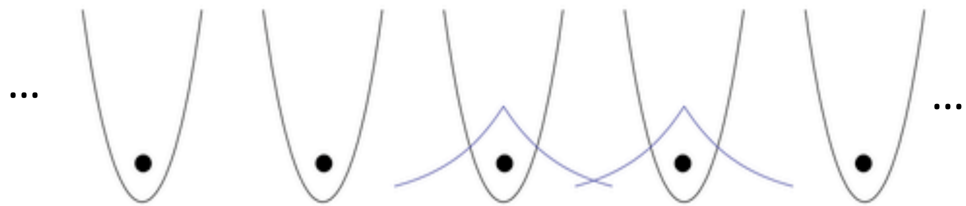



*dispersion of 2d cubic lattice:*
$\epsilon_k = -2\kappa\sum_i \cos(k_i a)$

▶ *polaritons condense into **k**=0 state SUPERFLUID STATE (not gapped)*

▸ *polaritons delocalize over lattice*

▸ *something bad happens for (instability)* $\omega - \mu < \kappa z_c$




Tight-binding model

▶ *eigenstates:* $|\,\mathbf{k}\,\rangle = \frac{1}{\sqrt{N}}\sum_{\mathrm{r}_j} e^{i\mathrm{k}\cdot\mathrm{r}_j}\,|\,\mathbf{r}_j\,\rangle$

▸ geometry dependent band structure, e.g., cubic lattice: $\epsilon_k = -2\kappa\sum_i \cos(k_i a)$

⚠ *fermions ≠ bosons !*

*electromagnetic mode function ≠ wavefunction !*

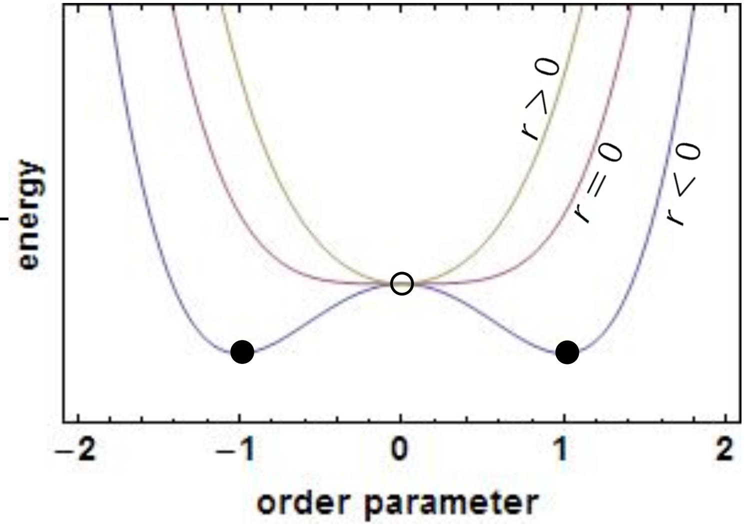
# MFT: How, to go beyond: $\psi = z_c\, \kappa \langle a_i \rangle$

$$h_j^{\mathrm{mf}} = \frac{1}{2}(\varepsilon - \mu)\sigma_j^z + (\omega - \mu)a_j^\dagger a_j + g(a_j^\dagger \sigma_j^- + \sigma_j^+ a_j) - \underbrace{(a_j \psi^* + a_j^\dagger \psi) + \frac{1}{z_c \kappa}|\psi|^2}_{\text{treat perturbatively!}}$$

*Sufficiently close to the phase boundary:* $\psi \sim \langle a_j \rangle \ll 1$

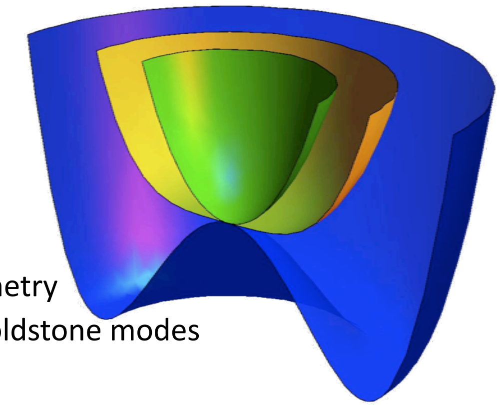
Expansion in orders of

$$E_0(\psi) = E_0^{\mathrm{mf}} + r|\psi|^2 + \tfrac{1}{2}u|\psi|^4 + \mathcal{O}(|\psi|^6)$$




- ▶ standard situation for a MFT phase transition: for *u>0* transition occurs at *r=0*

- ▶ perturbation theory gives analytical expressions for phase boundary!

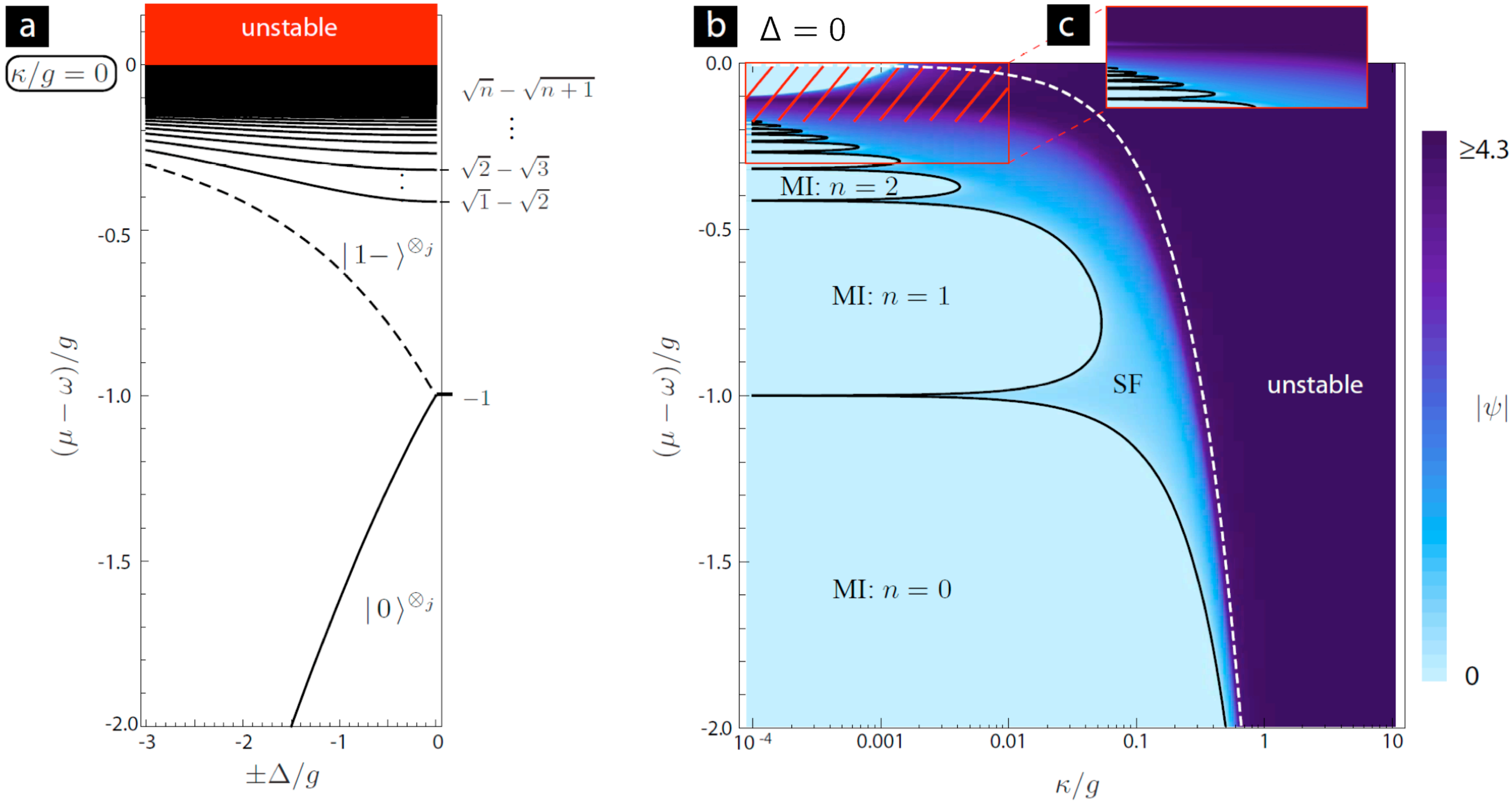


breaking of **U(1)** symmetry
gapless excitations: Goldstone modes

# MFT results for the JC lattice

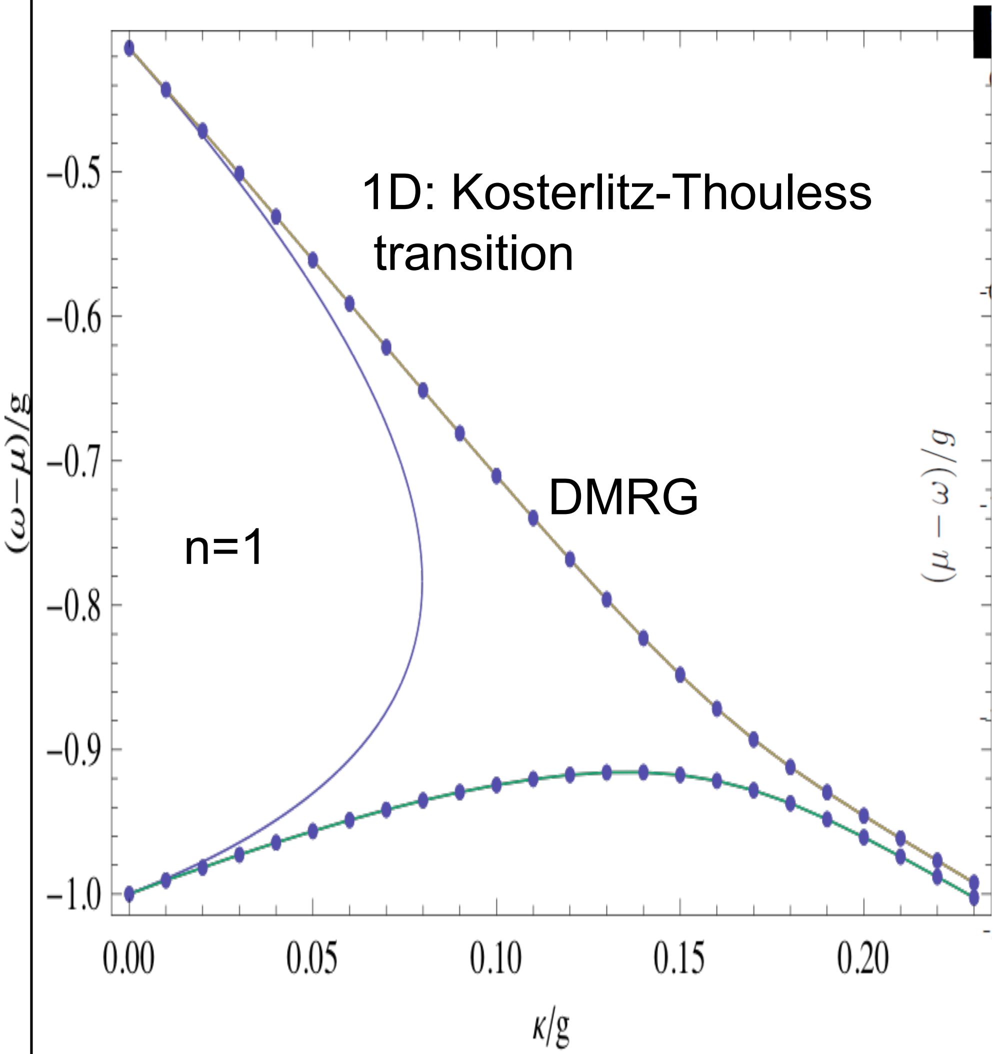
Greentree et al., Nat. Phys. **2**, 856 (2006)

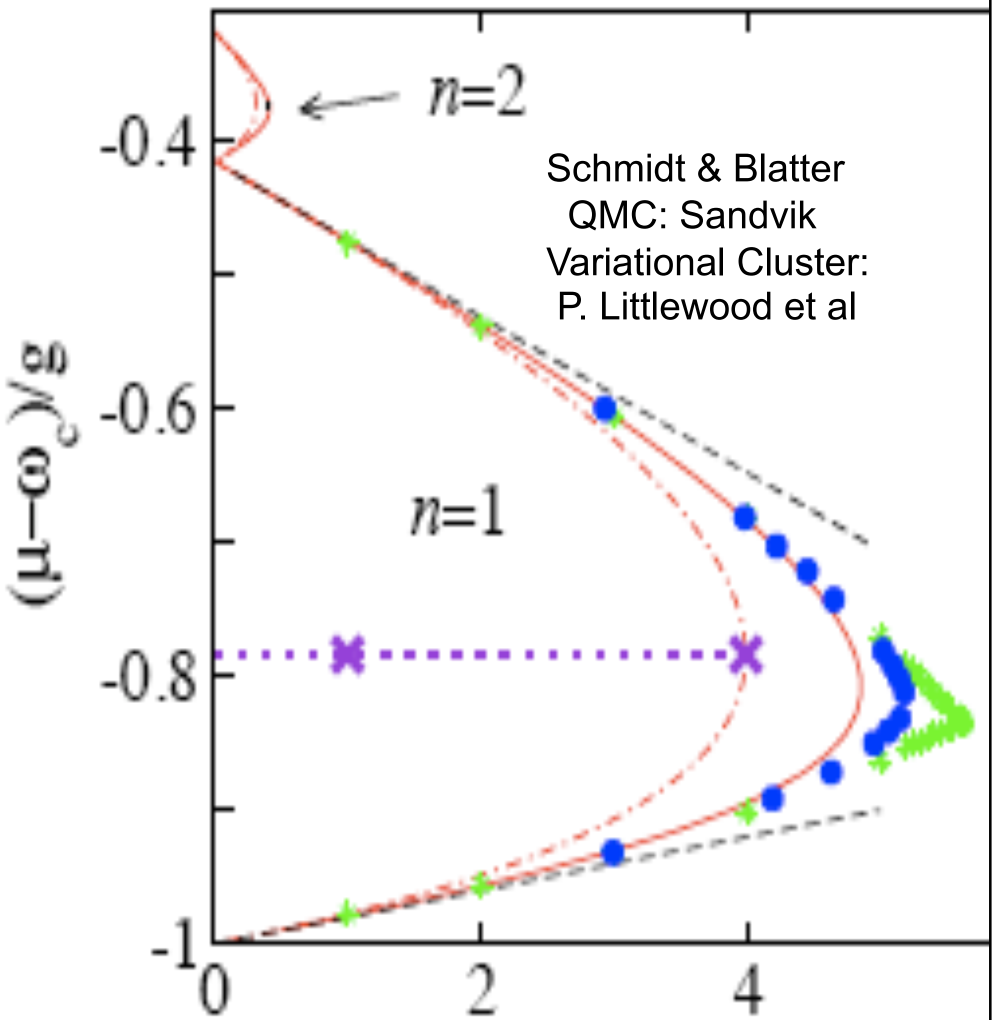
Angelakis et al., PRA **76**, 031805 (2007)




Jens Koch and KLH, PRA **80**, 023811 (2009)

# MFT results for the JC lattice and Beyond…



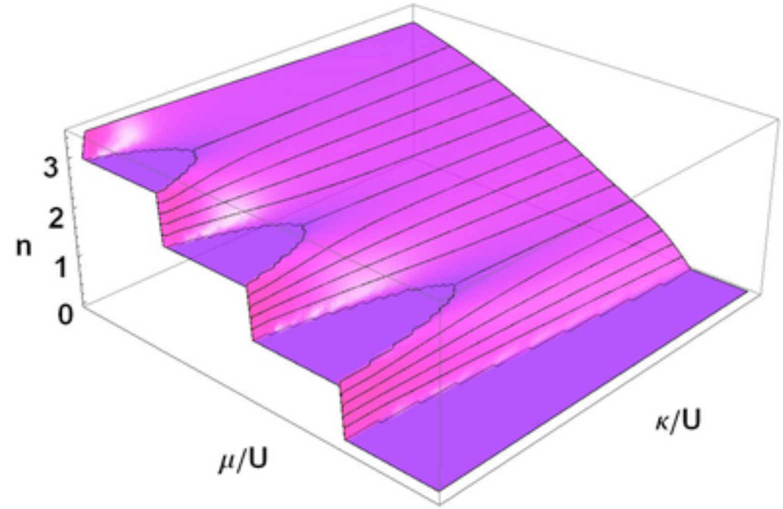
# Multicritical points (BHM)

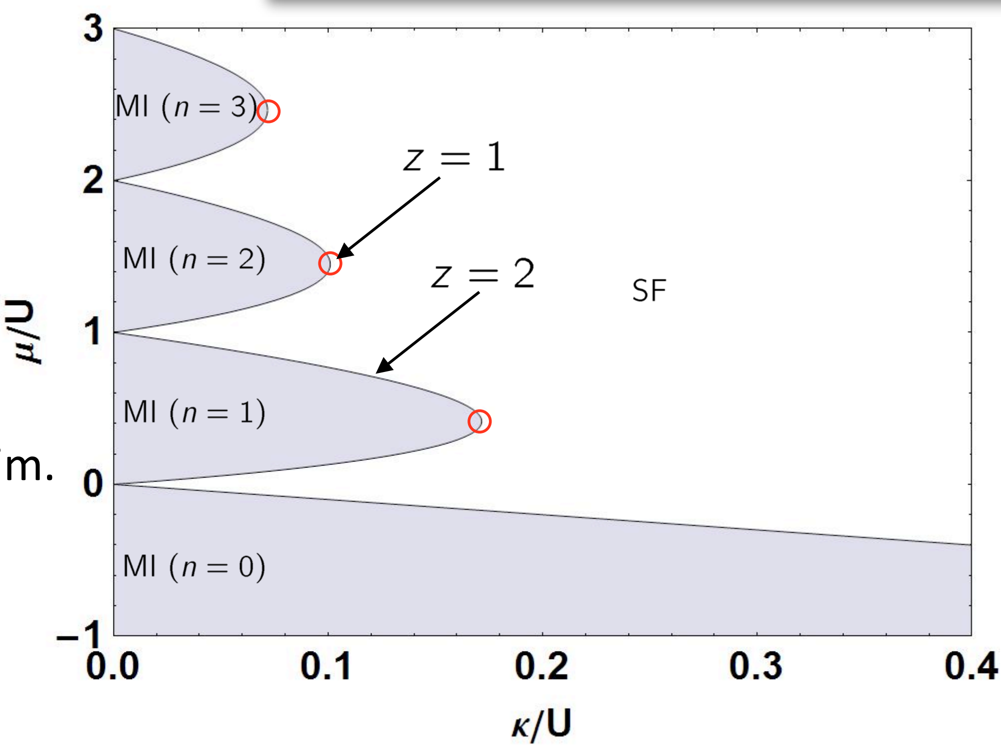
The tips of lobes are special!

► generically, crossing the phase boundary is associated with a *change in boson density*

► at lobe tips, density remains **constant**

► lobe tips are **multicritical points, universality class differs**




dynamical critical exponent

QPT in d dim. $\leftrightarrow$ classical PT in (d+1) dim.

$\xi \sim |\kappa - \kappa_c|^{\nu}$ diverging length scale (correlation length)

$\xi_\tau \sim \xi^{z}$ diverging time scale

dynamical critical exponent

## Field theory in terms of coherent state path integrals

partition function: $$Z = \int \prod_j \mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)\delta(\mathbf{N}_j^2 - 1)e^{-S[a_j^*, a_j, \mathbf{N}_j]}$$

action: $$S[a_j^*, a_j, \mathbf{N}_j] = \int_0^\beta d\tau \left\{ \sum_j \langle \mathbf{N}_j(\tau)| \frac{d}{d\tau} |\mathbf{N}_j(\tau)\rangle + \sum_j a_j^* \frac{\partial a_j}{\partial \tau} + \sum_j H_j^{\mathrm{JC}}(a_j^*, a_j, \mathbf{N}_j) + H^{\mathrm{hop}}(a_j^*, a_j) \right\}.$$

w/ $a_j \to a_j(\tau), \quad a_j^\dagger \to a_j^*(\tau), \quad \sigma_j^\alpha \to N_{j,\alpha}$

use *Hubbard-Stratonovich transformation* to decouple hopping term:

$$\exp\left[\int_0^\beta d\tau \sum_{j,j'} a_j^* \kappa_{jj'} a_{j'}\right] = \int \prod_j \mathcal{D}\psi_j^*(\tau)\psi_j(\tau) \exp\left[-\int_0^\beta d\tau \sum_{j,j'} \psi_j^* \kappa_{jj'}^{-1} \psi_{j'}\right] \exp\left[\int_0^\beta d\tau \sum_j \{\psi_j^* a_j + \psi_j a_j^*\}\right]$$

aux. field $\psi_j$ plays role similar to order parameter

$$Z = \int \prod_j \mathcal{D}\psi_j^*(\tau)\mathcal{D}\psi_j(\tau)\overbrace{\mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)}^{\text{integrate out}}\delta(\mathbf{N}_j^2 - 1)\exp\left(-S'[\psi_j^*, \psi_j, a_j^*, a_j, \mathbf{N}_j]\right)$$

## Effective field theory in the critical region

$$Z = \int \prod_j \mathcal{D}\psi_j^*(\tau)\mathcal{D}\psi_j(\tau)\mathcal{D}a_j^*(\tau)\mathcal{D}a_j(\tau)\mathcal{D}\mathbf{N}_j(\tau)\delta(\mathbf{N}_j^2 - 1)\exp\left(-S'[\psi_j^*, \psi_j, a_j^*, a_j, \mathbf{N}_j]\right)$$

$$= \int \prod_j \mathcal{D}\psi^*(x,\tau)\mathcal{D}\psi(x,\tau)\exp\left(-S_{\mathrm{eff}}[\psi^*, \psi]\right)$$

Gradient expansion for effective action:

proportional to
MFT params. *r, u*

$$S_{\mathrm{eff}}[\psi^*, \psi] = \int_0^\beta d\tau \int d^d x \left[K_0 + K_1\psi^*\frac{\partial\psi}{\partial\tau} + K_2\left|\frac{\partial\psi}{\partial\tau}\right|^2 + K_3|\nabla\psi|^2 + \tilde{r}\,|\psi|^2 + \frac{\tilde{u}}{2}|\psi|^4 + \cdots\right]$$

### $S'$ invariant under:

$$a_j \to a_j e^{i\varphi(\tau)}, \quad \psi_j \to \psi_j e^{i\varphi(\tau)},$$

$$\mathbf{N}_j \to \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}\mathbf{N}_j,$$

$$(\omega - \mu) \to (\omega - \mu) - i\frac{d\varphi}{d\tau}$$

$\Rightarrow$

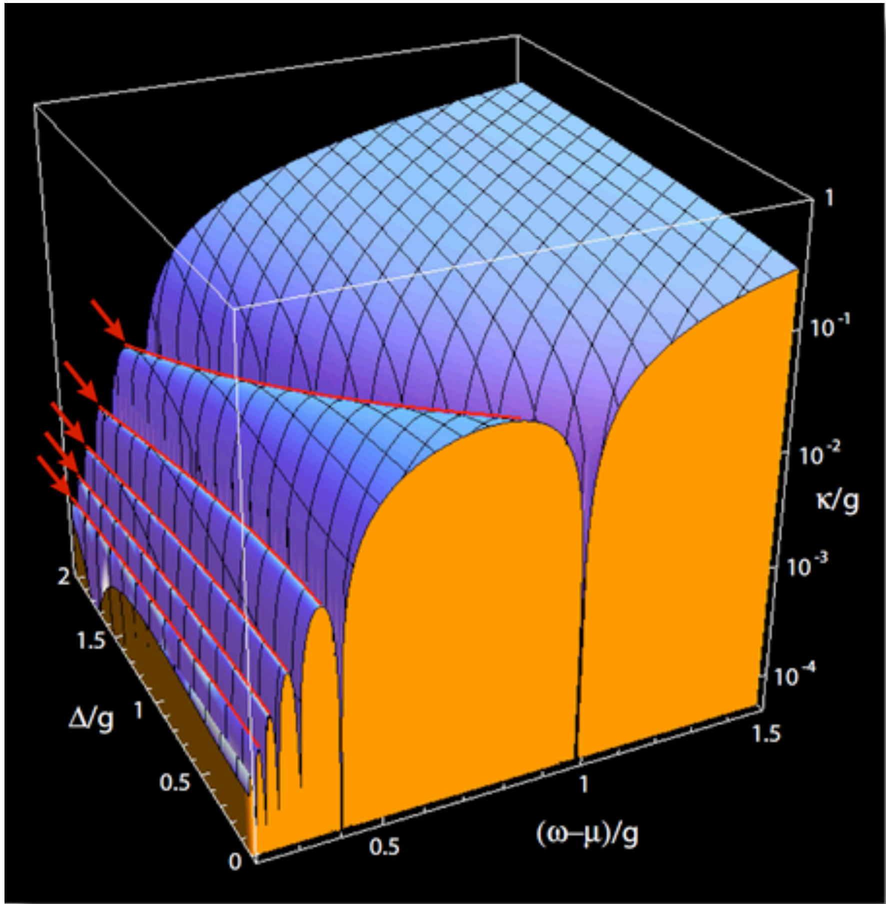
### *Coefficients* $K_1$, $K_2$

$$\underbrace{K_1 = \frac{\partial\tilde{r}}{\partial(\omega-\mu)}}, \qquad K_2 = -\frac{1}{2}\frac{\partial^2\tilde{r}}{\partial(\omega-\mu)^2}$$

$K_1 \neq 0$ generic case, *z=2*

$K_1 = 0$ multicrit. curves, *z=1*

# Multicritical curves for the JC lattice model



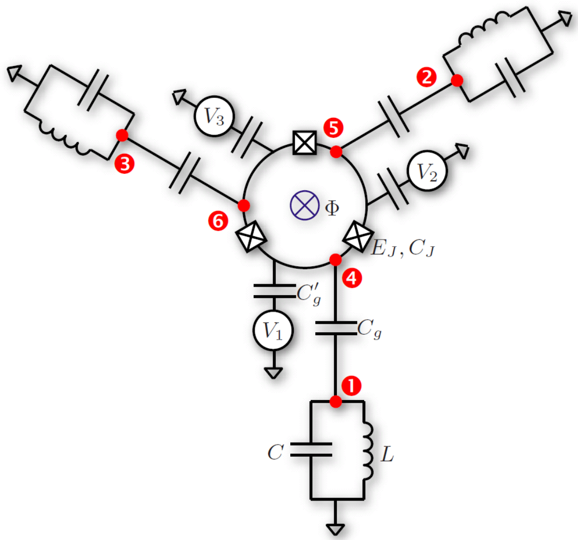

$K_1 \neq 0$ generic case, *z=2*
$K_1 = 0$ multicrit. curves, *z=1*

1D: Mapping onto XX spin model
between 2 Mott lobes
(J. Koch & KLH, 2009)

# Experimental Progress

Andrew Houck
*(Princeton)*

- *model system, potentially as versatile as ultracold atoms in optical lattices*
  - *quantum phase transitions*
  - *lattice dynamics for photons*
  - *realizable with circuit QED*

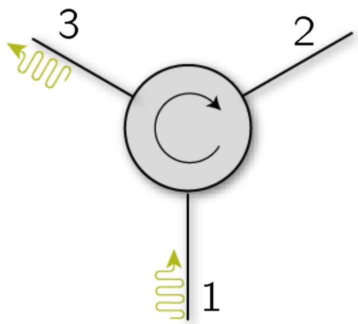


Circulator for photons

# photon circulators (commercial circulators=ferrites)

## Motivation

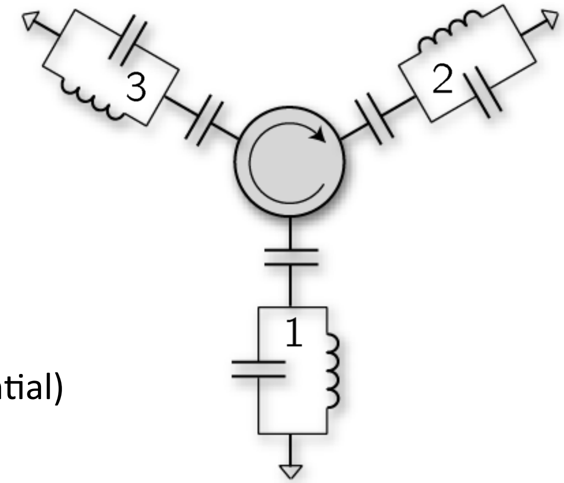
- on-chip circulators would be useful for a number of applications
- could incorporate into JC lattice, explore QPTs with broken t-reversal symmetry




$$S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

## Model

- in JC lattice, circulator would act as coupling element between resonators (not semi-infinite transmission lines)
- assume: only one resonator mode is relevant




*generic Hamiltonian*

$$H = \sum_{j=1}^{3} \omega a_j^\dagger a_j + \lambda \sum_{j=1}^{3} (a_j^\dagger + a_j) B_j + H_B$$

(assumption of capacitive coupling not essential)

J. Koch, A. Houck, KLH & SM Girvin, arXiv:1006.0762

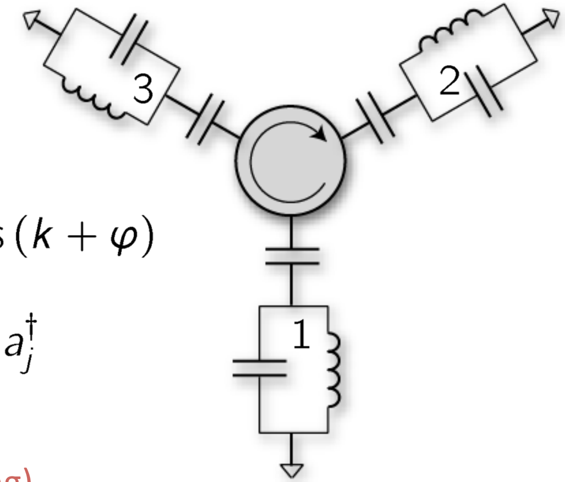
# "Quantum" circulators

$$H_{\text{eff}} = \sum_{j=1}^{3} \omega' a_j^\dagger a_j - \kappa \left[ e^{i\varphi} (a_1 a_3^\dagger + a_3 a_2^\dagger + a_2 a_1^\dagger) + \text{h.c.} \right]$$

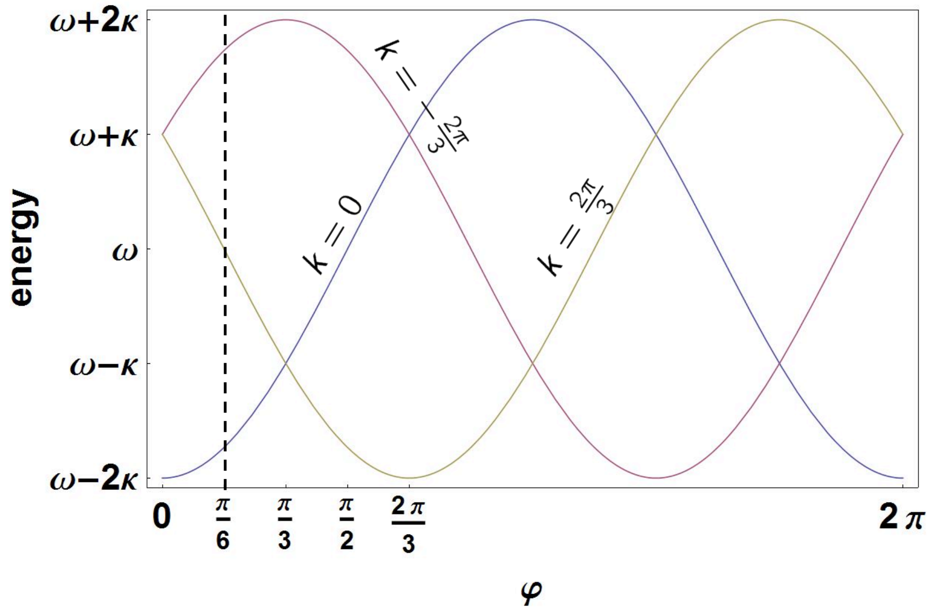
$$= \sum_k \Omega_k A_k^\dagger A_k \quad (k = -\tfrac{2\pi}{3},\, 0,\, \tfrac{2\pi}{3})$$

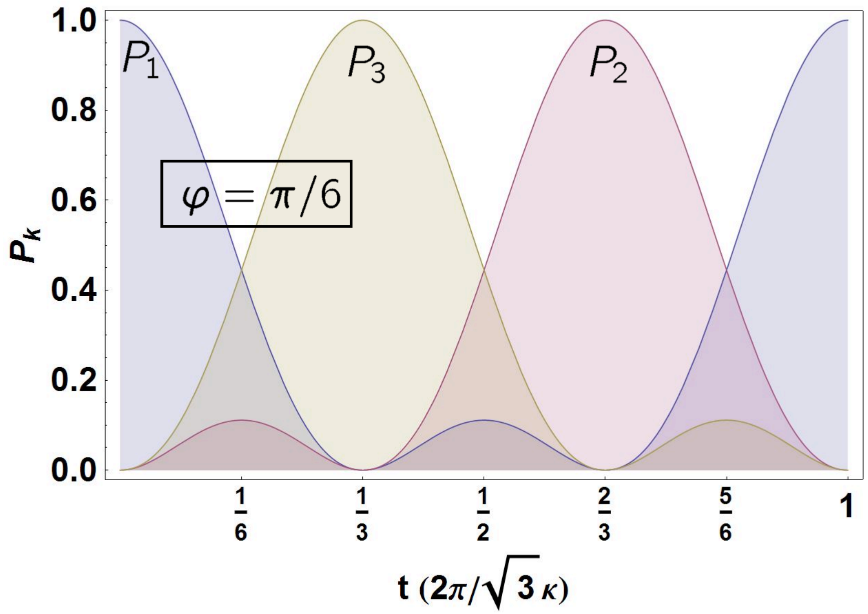
$$\Omega_k = \omega' - 2\kappa \cos(k + \varphi)$$

$$A_k^\dagger = \frac{1}{\sqrt{3}} \sum_{j=1}^{3} e^{ikj} a_j^\dagger$$



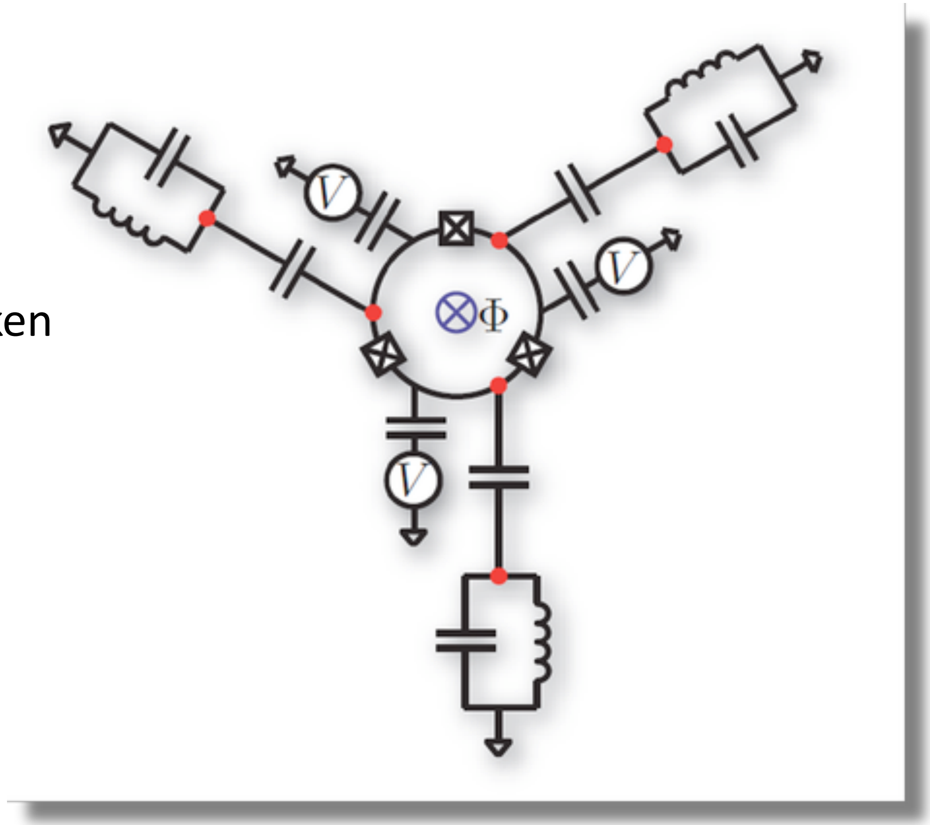

(again: tight binding)



# Outlook: J. Koch, A. Houck, KLH & SM Girvin, arXiv:1006.0762

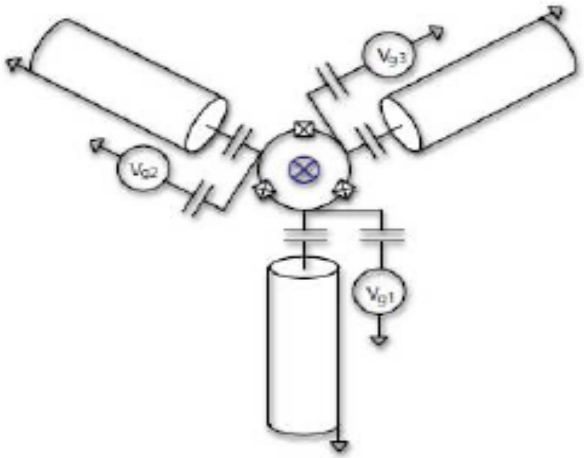
(numerical check at intermediate couplings)




- ► Josephson ring provides one way to generate complex phase factors
- ► need magnetic flux to break t-reversal sym. additionally, particle-hole sym. must be broken
- ► large $E_J/E_C$: no complex phases, but *tunable coupling strength*!

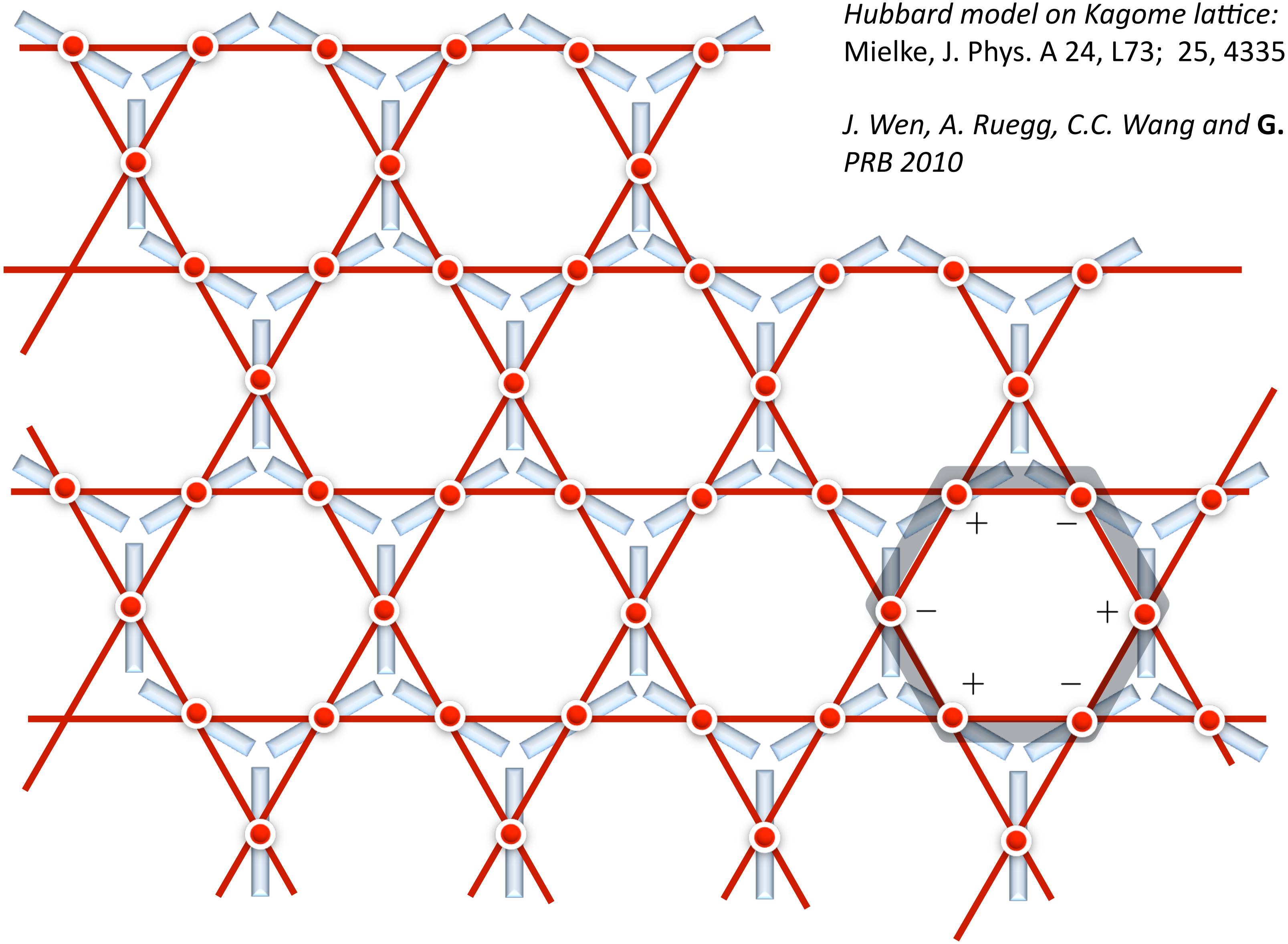
complex phases for intermediate $E_J/E_C$

- ► *random off-set charges can be controlled*




- ► topological phases, Kagome lattice, steady states under continuous driving

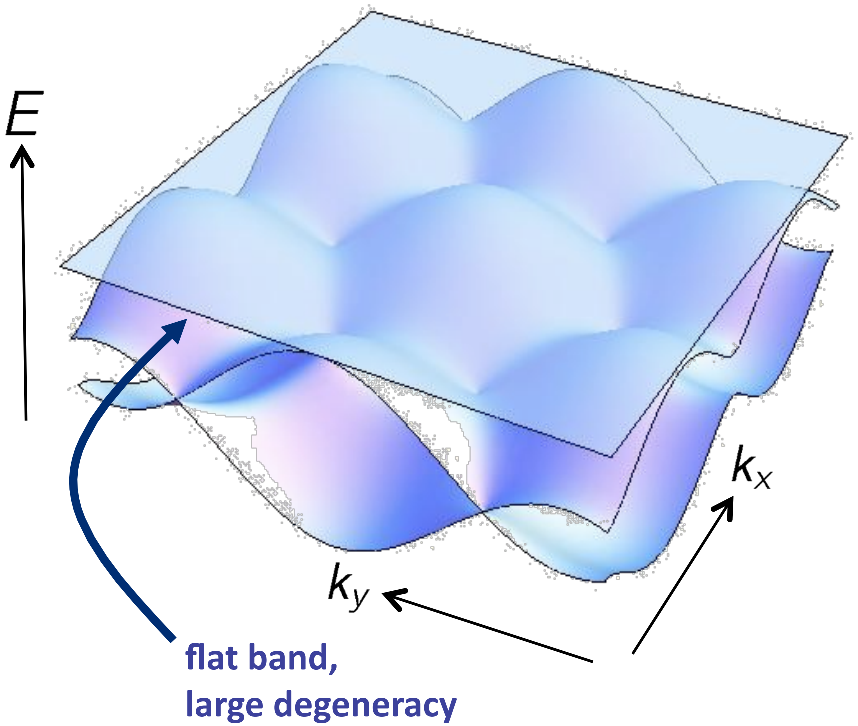
# From honeycomb to Kagome




*Hubbard model on Kagome lattice:*
Mielke, J. Phys. A 24, L73; 25, 4335 (1991/92)

*J. Wen, A. Ruegg, C.C. Wang and* **G. Fiete**, *PRB 2010*

# Kagome: flat band and photon localization

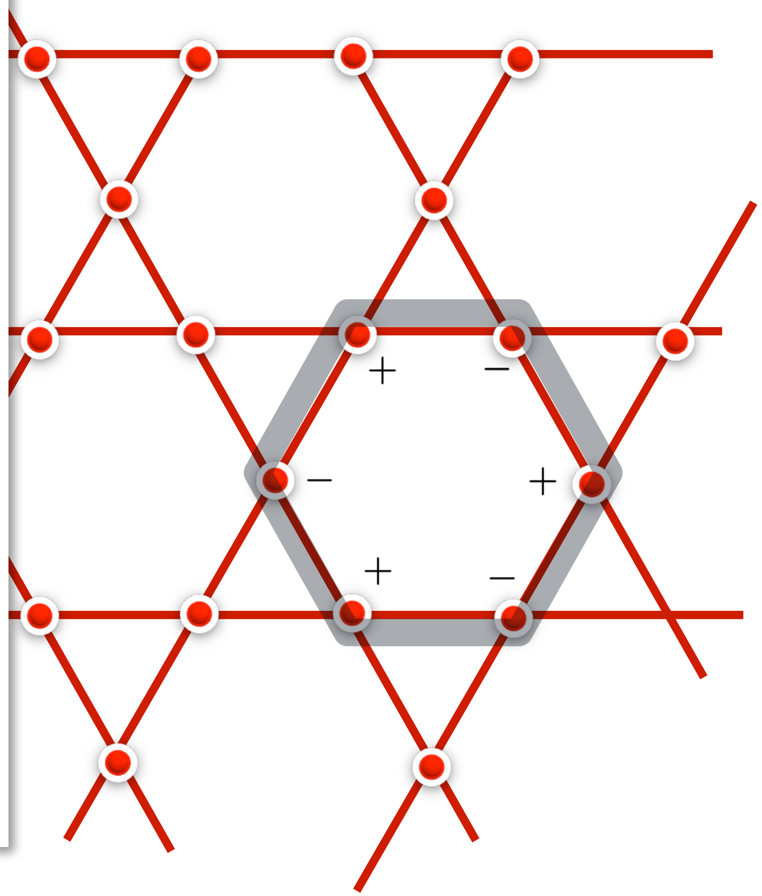



localized photon states
even *without* interaction

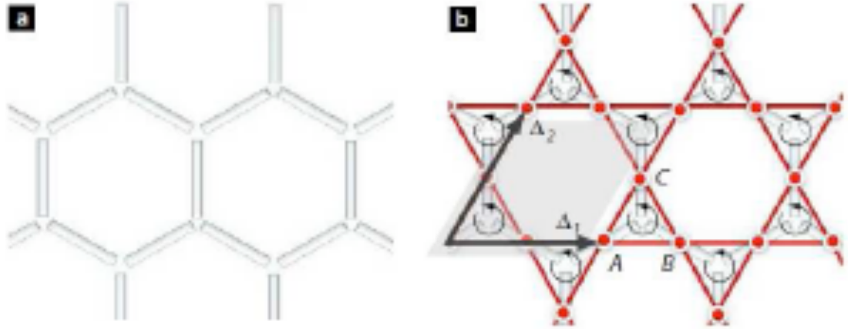
Future: add interaction and study
strong correlations

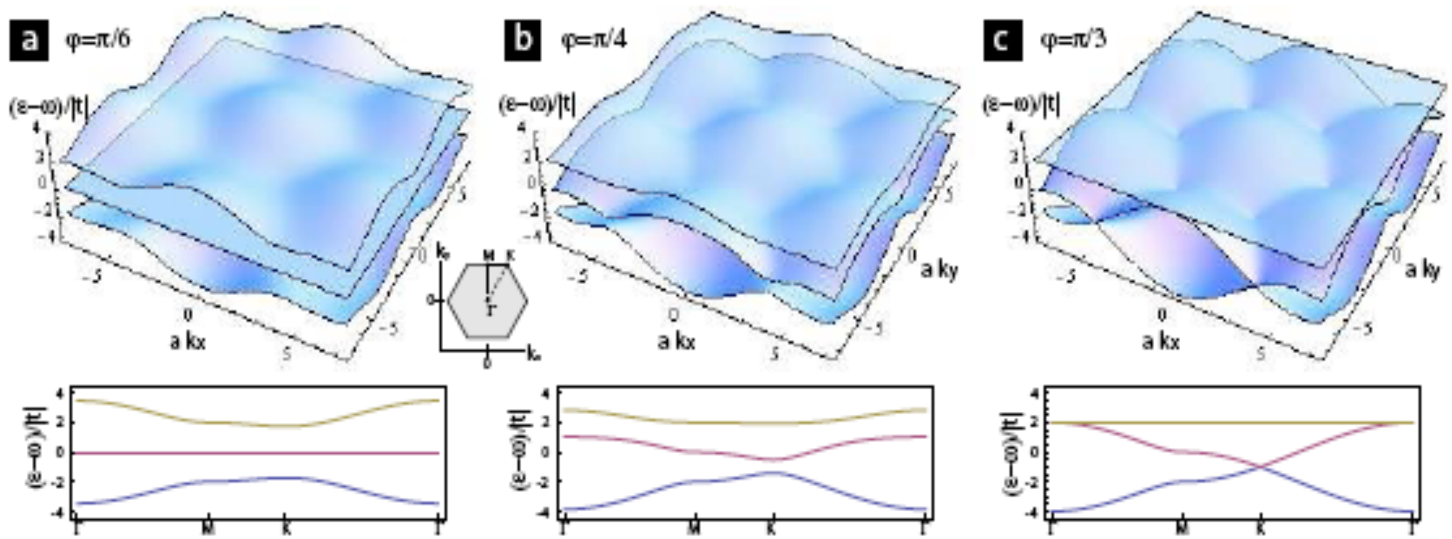
*Hubbard model on Kagome lattice:*
Mielke, J. Phys. A 24, L73;  25, 4335 (1991/92)

*Frustrated antiferromagnets:*
Chalker & Eastmond, PRB 46, 14201 (1992)
Balents, Fisher & Girvin, PRB 65, 224412 (2002)



# Breaking T-reversal symmetry



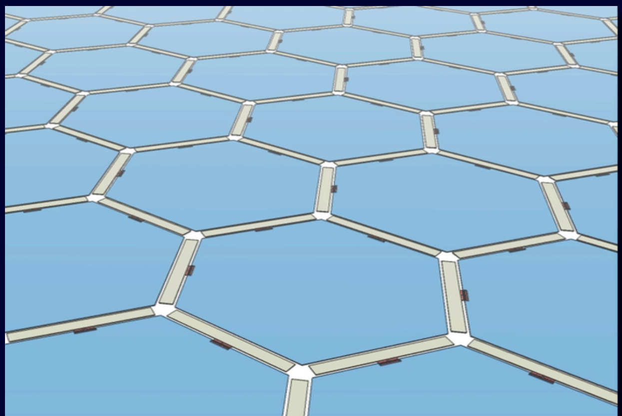



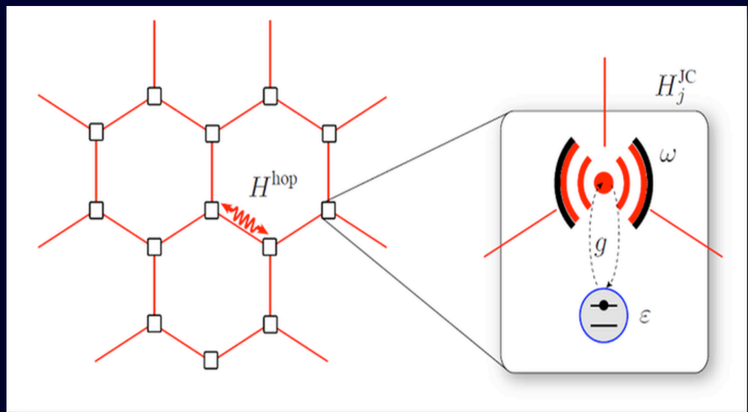

Other Realizations in Photonic crystals: Haldane & Raghu
Experiment: M. Soljacic Nature 461, 772 (2009)

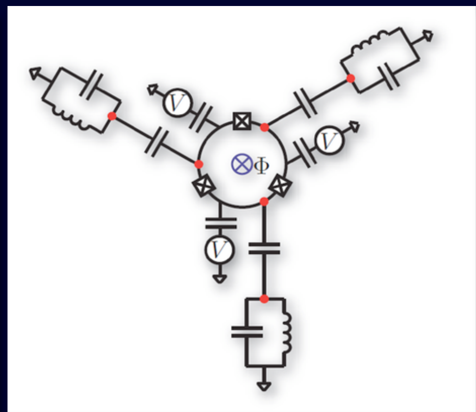
# Superfluid-Mott Insulator Transition of Light: Many Similarities with Bose Hubbard model

Multicritical curves (z=1)
1D: XX spin mapping between two Mott lobes
Disorder? Glassy phases?
Dissipation & open system
Other quantum phase transitions: Dicke model?

# Circulators: Artificial Gauge Fields





# Time reversal invariant version of Haldane model: Kane-Mele model

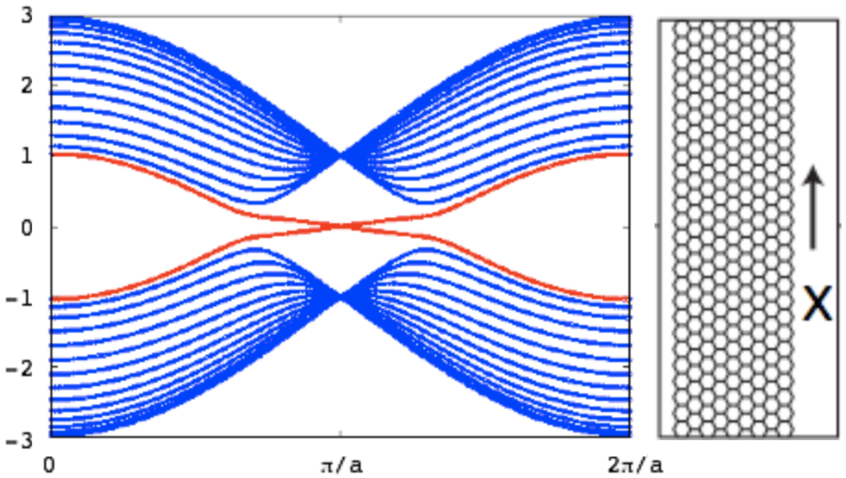
**Kane & Mele, PRL 95, 226801 (2005)**

***see also:*** **Bernevig, Hughes, and Zhang, Science 314, 1757 (2006) + Molenkamp-experiments**

$$\mathcal{H} = -t \sum_{<ij>\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma_{\sigma\sigma'}^{z} c_{i\sigma}^{\dagger} c_{j\sigma'}$$

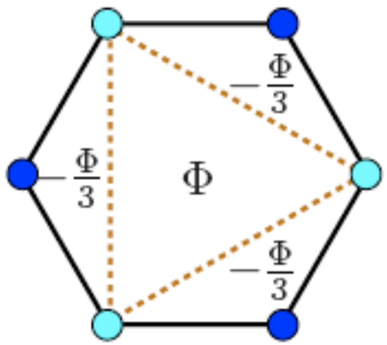
$\nu_{ij} = \pm 1$

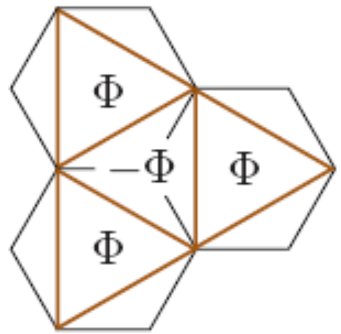
**strip geometry:**




**edge states: Kramer's pair**

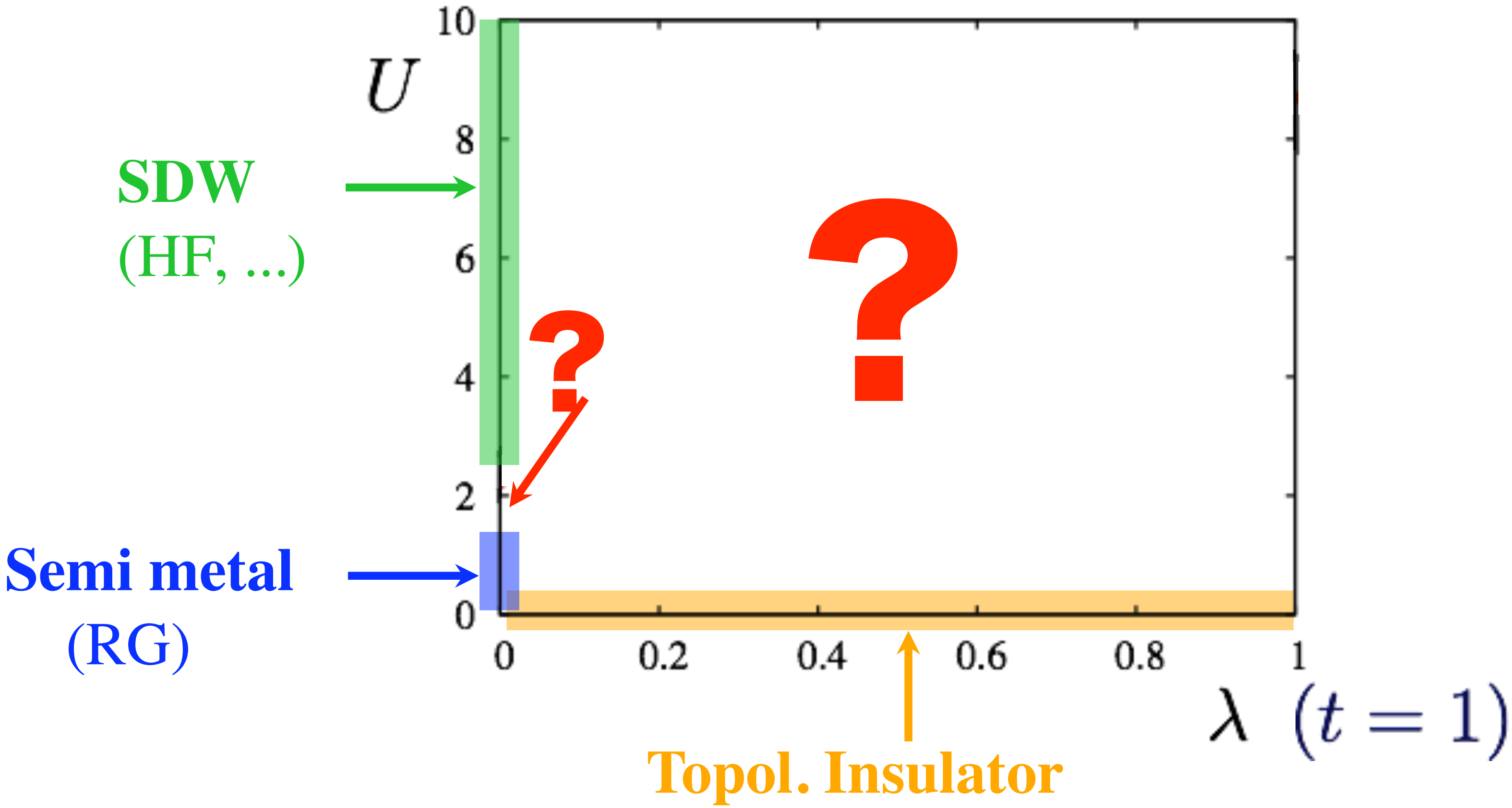
$$\mathcal{H} \propto \Psi_k^{\dagger} \sigma^z \tau^z \Psi_k$$



# Effect of electron-electron interaction?

$$\mathcal{H} = -t \sum_{<ij>\sigma} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{\ll ij \gg} \sum_{\sigma\sigma'} \nu_{ij} \sigma^z_{\sigma\sigma'} c^\dagger_{i\sigma} c_{j\sigma'} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$



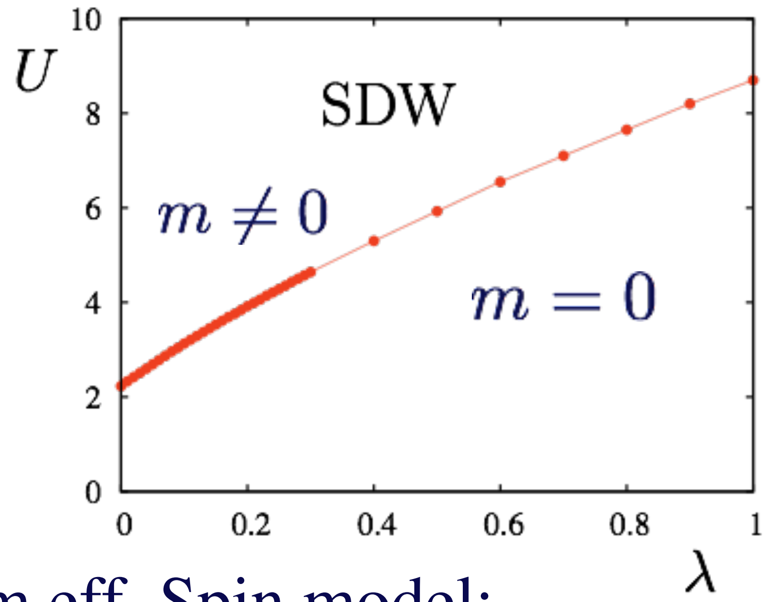

*Meng et.al., Nature 464, 848 (2010): (RVB) spin liquid in interacting graphene model (QMC)*

# Presence of spin orbit coupling

- **Simple Hartree Fock yields transition to magnetically ordered phase (Mott phase)**
- **critical *U* increases with increasing spin orbit coupling**

$$m = (-1)^{\zeta} \langle c^{\dagger}_{i\uparrow} c_{i\uparrow} - c^{\dagger}_{i\downarrow} c_{i\downarrow} \rangle$$

**(cf. Sorella & Tosatti EPL 19, 699 (1992)**




increase of $U_c$ can be understood from eff. Spin model:

$$\mathcal{H}^{\mathrm{eff}} = \sum_{\langle ij \rangle} \frac{4t^2}{U} \boldsymbol{S}_i \boldsymbol{S}_j + \sum_{\langle\langle ij \rangle\rangle} \frac{4\lambda^2}{U} \left( - S^x_i S^x_j - S^y_i S^y_j + S^z_i S^z_j \right)$$

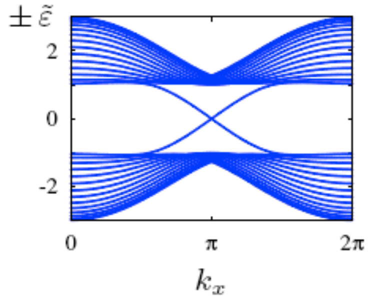
**favors Neel**

**competes with nearest neighbor term**

$\lambda \neq 0$ : **SDW order is turned into XY plane**

Different from $(J_1,J_2)$ model: C. Lhuillier et al (2001)

# Presence of spin orbit coupling




- **Apply Slave-Rotor theory of Florens & Georges, PRB 70, 035114 (2004)**
- **Rewrite fermions as rotors (charge degrees of freedom) and spinons**

$$c_{i\sigma} = e^{i\theta_i} f_{i\sigma}$$

- **Introduce constraint**

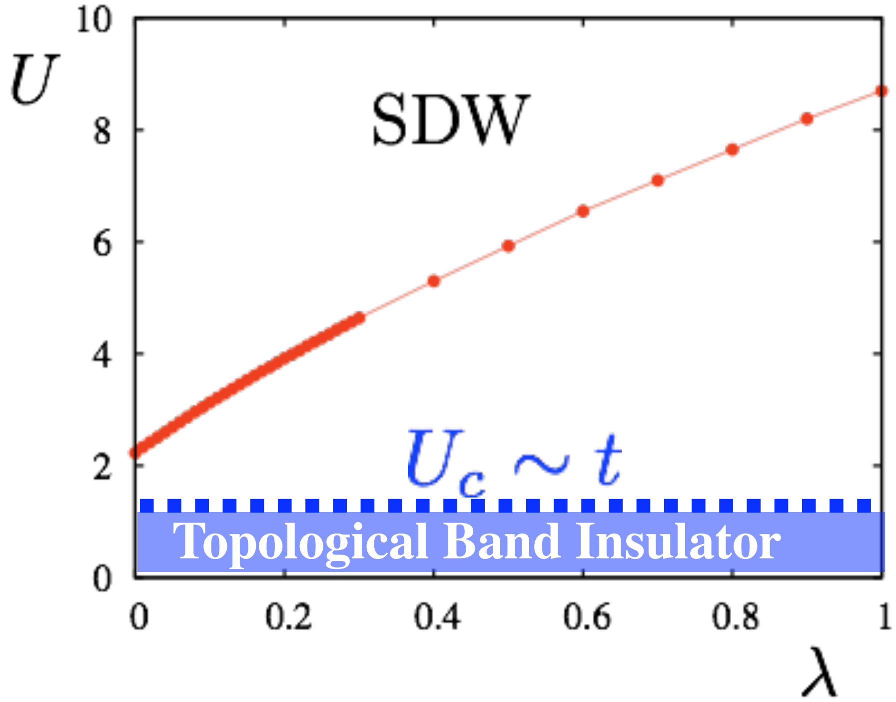
$$\sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} + L_i = 1$$

$$\frac{U}{2}\sum_i \left(\sum_\sigma n_{i\sigma} - 1\right)^2 \longrightarrow \frac{U}{2}\sum_i L_i^2$$

- **Hubbard interaction simplifies**

$$L = (i/U)\partial_\tau \theta.$$

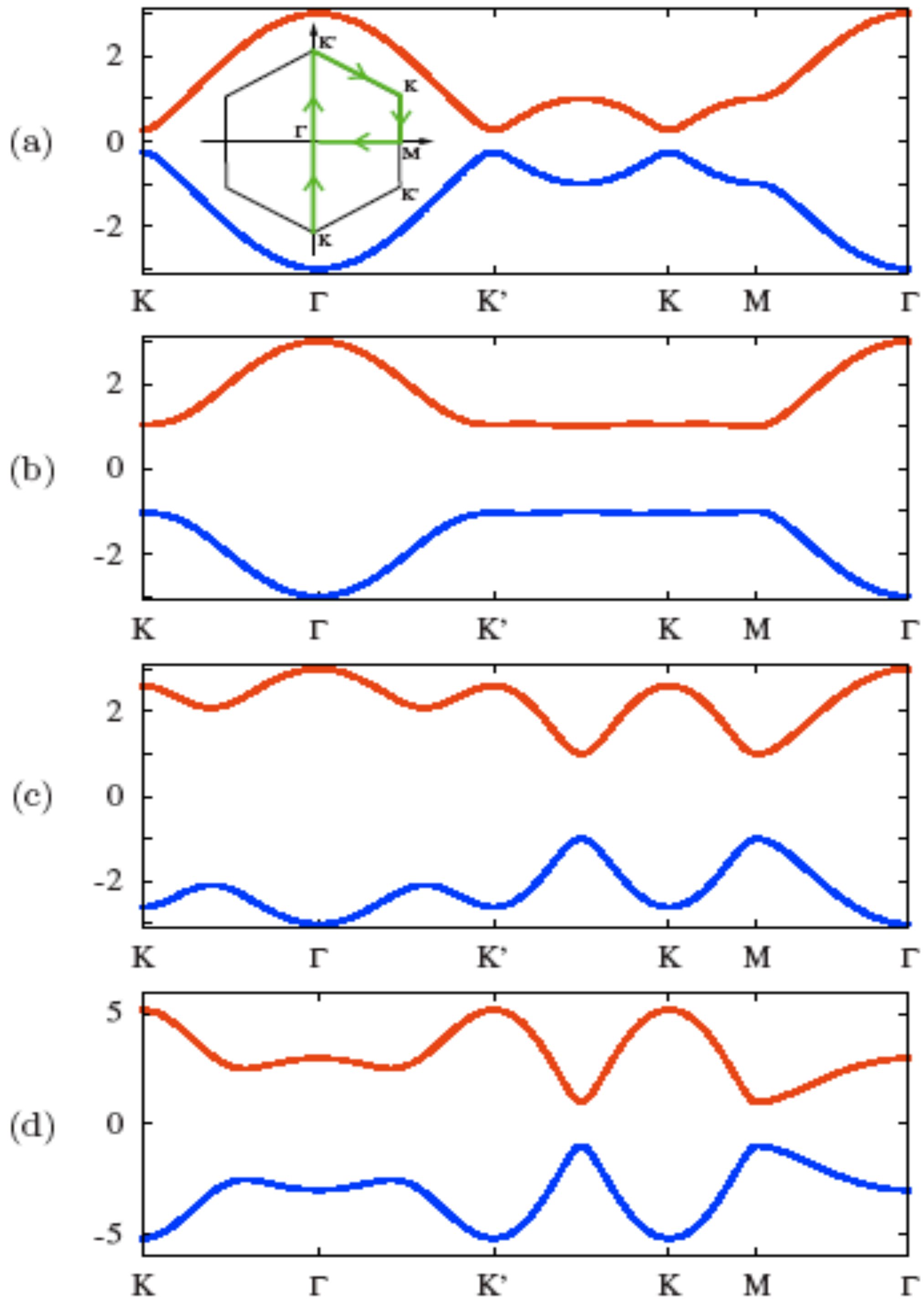
- **Interaction affects rotor only**
- **weak *U*: rotor condense,** $f_\sigma \propto c_\sigma$




S. Rachel & KLH, 2010

[+ other theory arguments]

Gap independent
of λ for λ~0.2t



# Presence of spin orbit coupling

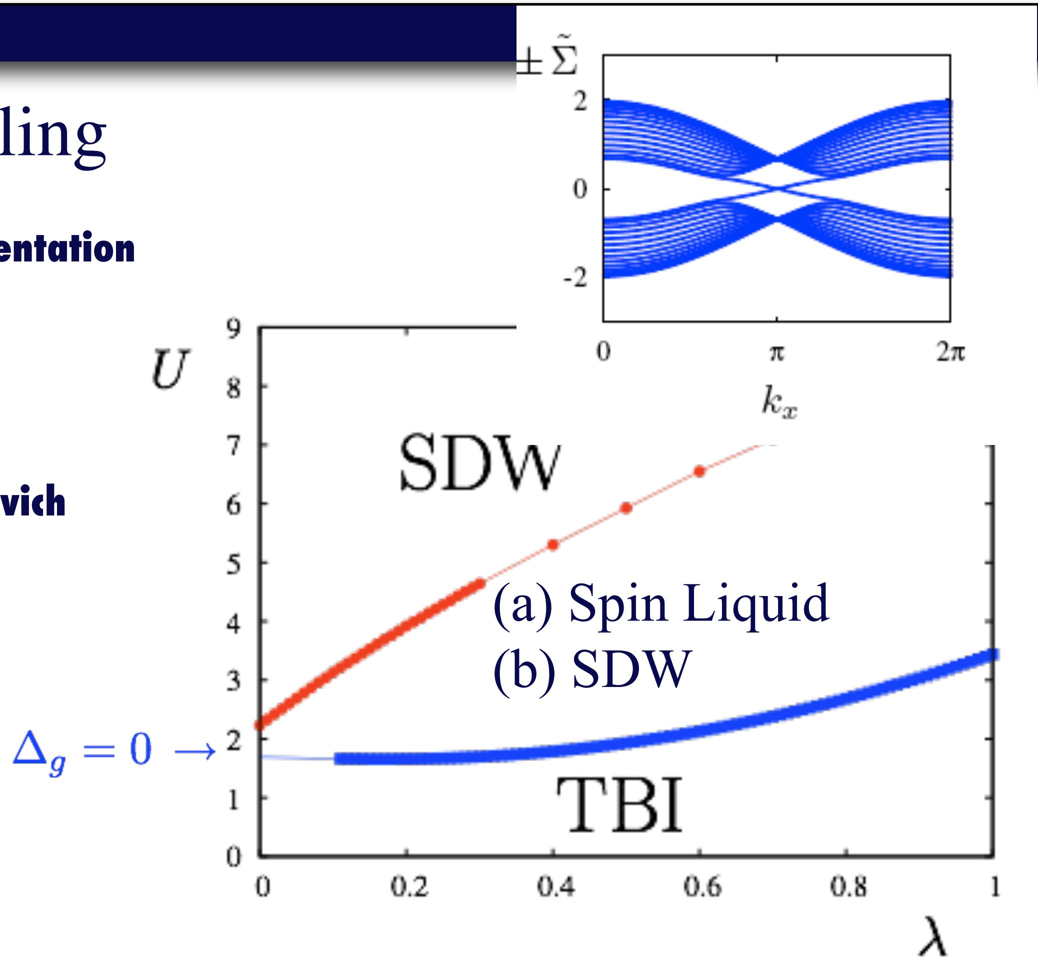
- **More Slave-Rotor: use sigma-model representation**
- **introduce bosonic *X*-fields**

$$X = e^{i\theta} \qquad |X|^2 = 1$$

- **mean-field decoupling or Hubbard Stratonovich**
- **insulating gap (zero at the transition)**

$$\Delta_g = 2\sqrt{U(\rho + \min \xi_k)}$$

*See also Lee & Lee PRL 2005*
*Young, Lee, Kallin, PRB 2008*
*Pesin & Balents, Nat. Phys. 2010*




- **(a) Quasi-2D System (additional Layers):**
- **charge frozen in Mott phase + spin liquid**

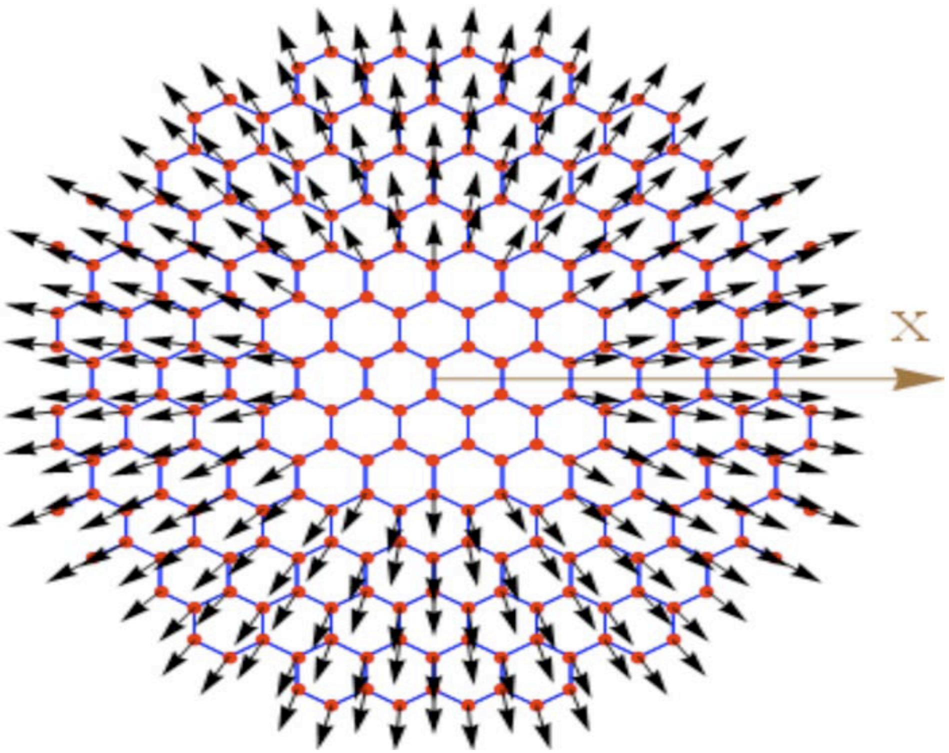
**S. Rachel & KLH, PRB 82, 075106 (2010); arXiv:1003.2238, 20 pages**

- **(b) Gauge Field Effects, pure 2D layer:**
- **compact gauge field, magnetic monopole condensation (Polyakov '75) => XY-instability**

*Y. Ran, A. Vishwanath & D.-H. Lee, arXiv:0806.2321*
*M. Hermele, Y. Ran, P. Lee & X-G. Wen, arXiv:0803.1150*

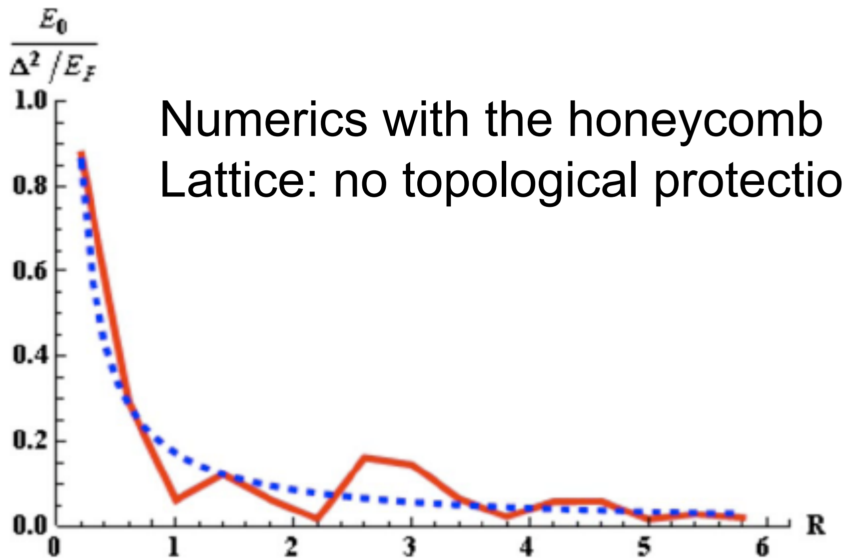
# “Near-Zero” Modes in Superconducting Graphene

D. Bergman & KLH, PRB 2009, 25 pages and also Ghaemi-Wilczek




Dirac theory for superconducting Graphene predicts the presence of 4 Majorana fermions at zero energy
(4 as a result of symmetries)

Already for a s-wave superconductor




Numerics with the honeycomb Lattice: no topological protection

R is the radius of the vortex

¼ of SC graphene is better!
(topological insulators coupled to s-wave SCs: Fu-Kane or 1D wires)