Exchange cotunneling
in
quantum dots
with
spin-orbit coupling

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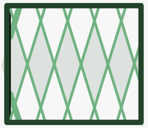
The Niels Bohr Institute
& Nano-Science Center

Stockholm, September, 2010

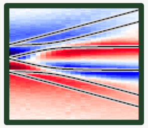
Collaborators (Phys. Rev. B **82**, 081309 (R) (2010)):

- Andreas Andersen
- Karsten Flensberg

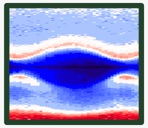
Outline



1. Coulomb blockade and cotunneling *(Brief reminder)*

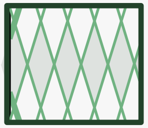


2. Spin-orbit coupling in quantum dots *(Exp)*



3. Probing spin-orbit coupling by inelastic cotunneling

Outline



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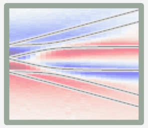

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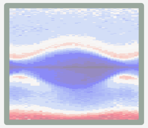
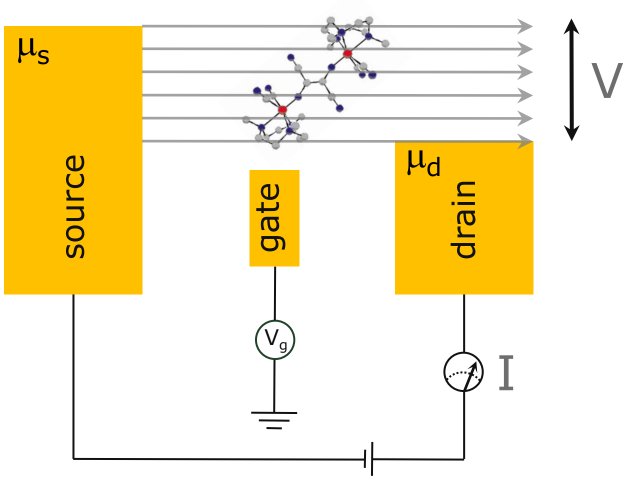
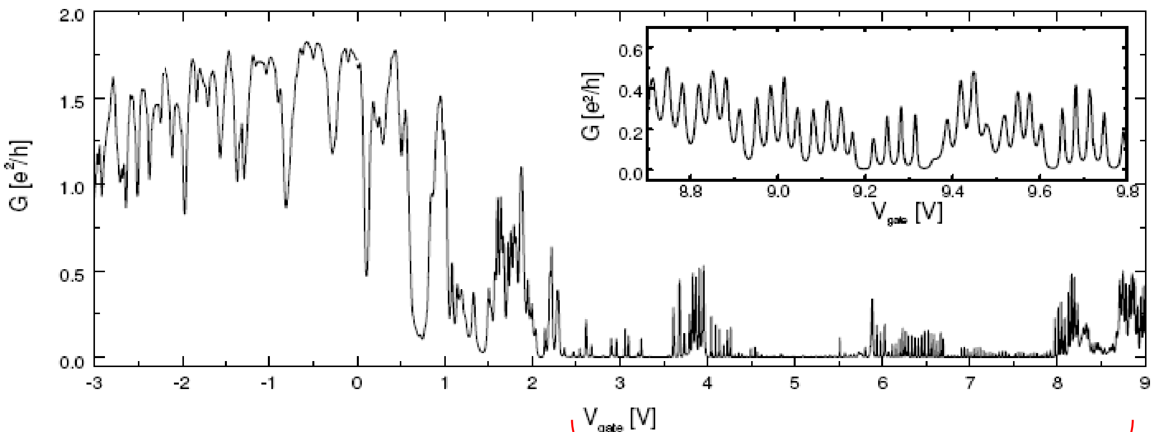


3. Probing spin-orbit coupling by inelastic cotunneling

'Canonical' nano-junction

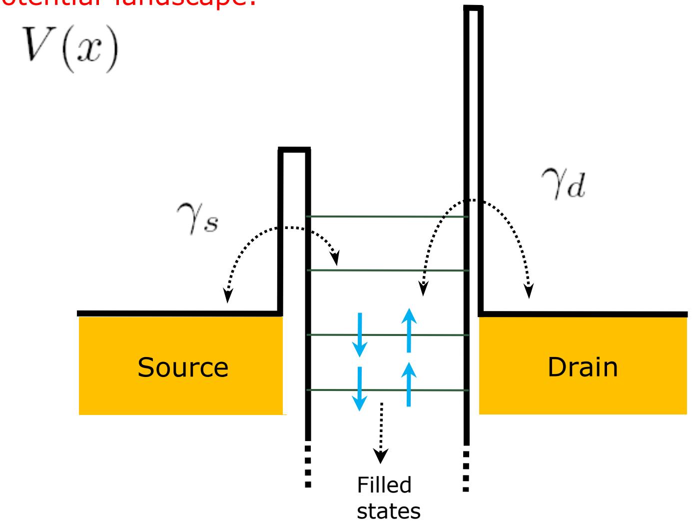




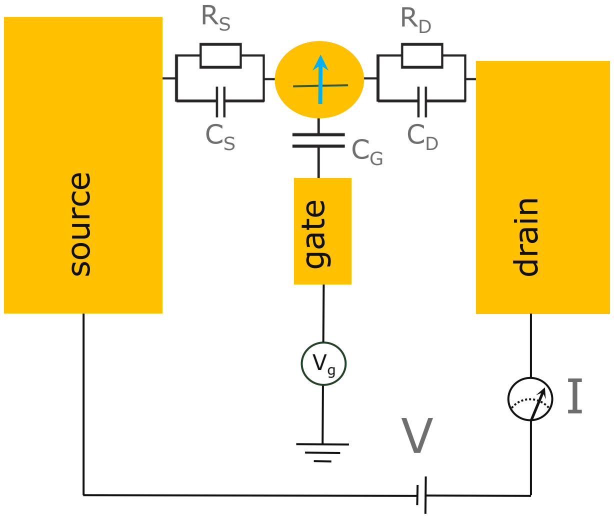
Coulomb blockade regime

Charge conduction: Quantum tunneling + Classical charging

Potential-landscape:



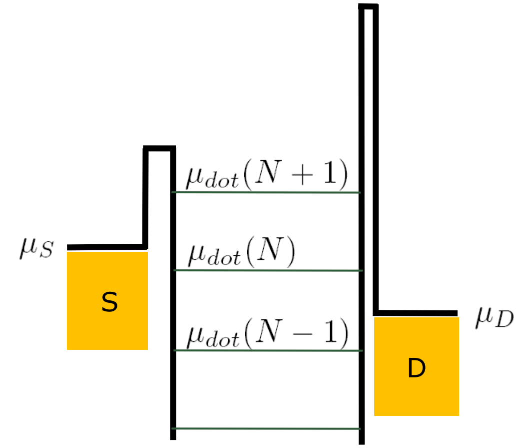
Elctrostatic-landscape:



$$U = \frac{Q_{dot}^2}{2C}$$

$$C = C_s + C_g + C_d$$

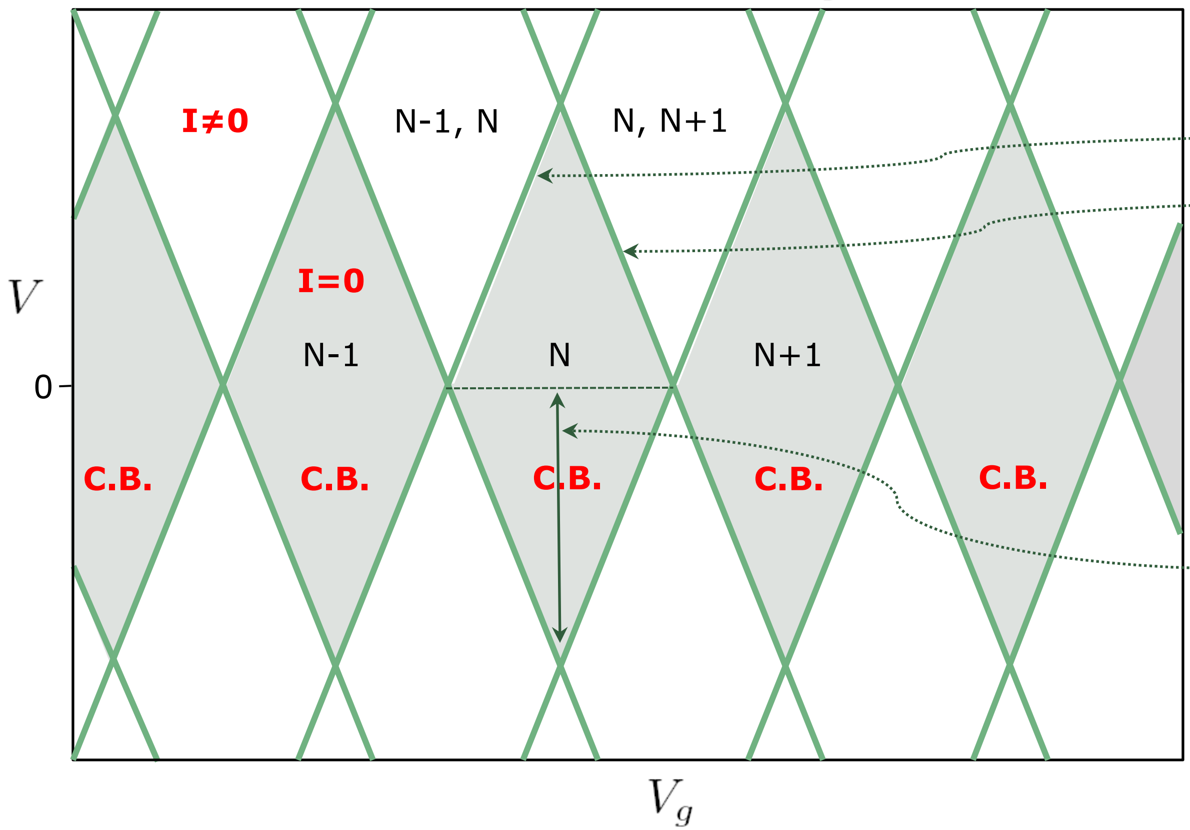
Diamond plot: Coulomb Blockade



Chemical potential of *dot* or molecule:

$$\mu_{dot} = E_N + U(N) - U(N-1)$$
$$= E_N + \frac{e^2}{C}(N - N_0 - 1/2) - (\mu_s C_s + |e|V_g C_g + \mu_d C_d)/C$$

Plotting conductance as a function of V_g and $|e|V = \mu_S - \mu_D$



Current thresholds:

$$\mu_{dot} = \mu_D$$
$$\mu_{dot} = \mu_S$$

gives the slopes:

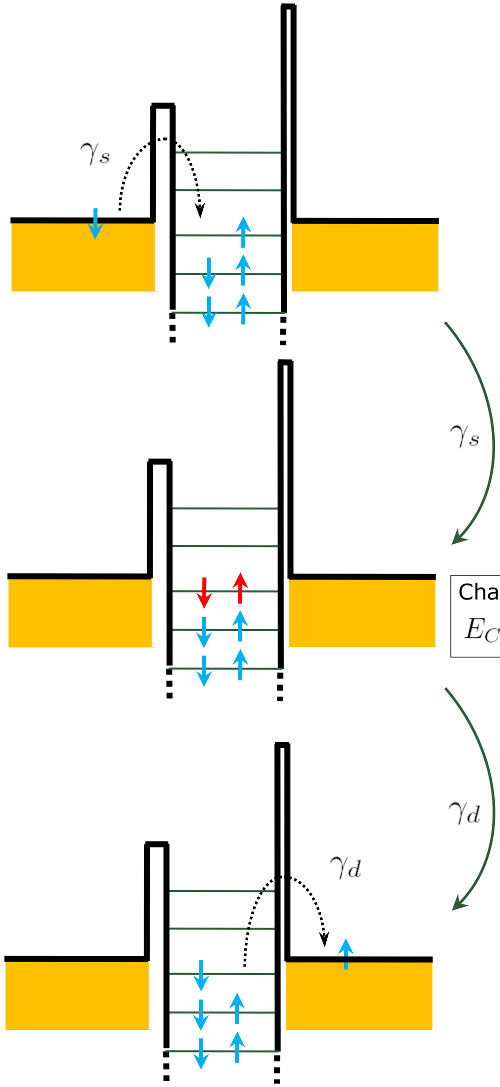
$$\pm 2C_g/C$$

for $C_S = C_D$.

Addition energy:

$$\mu_{dot}(N+1) - \mu_{dot}(N) = e^2/C$$

Cotunneling: *Lifting Coulomb blockade by quantum fluctuations*



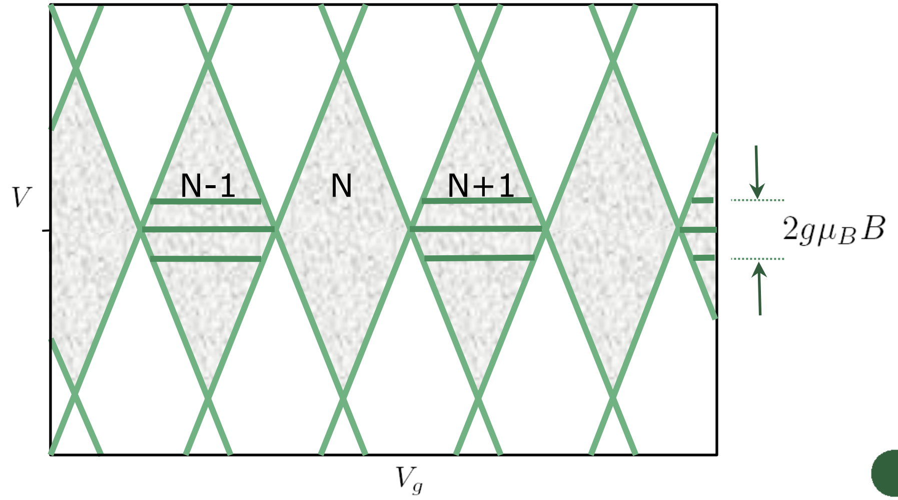
Charging cost: $E_C = e^2/C$

Cotunneling rate (2.-order PT): $J \sim \frac{\gamma_s^* \gamma_d}{E_C}$

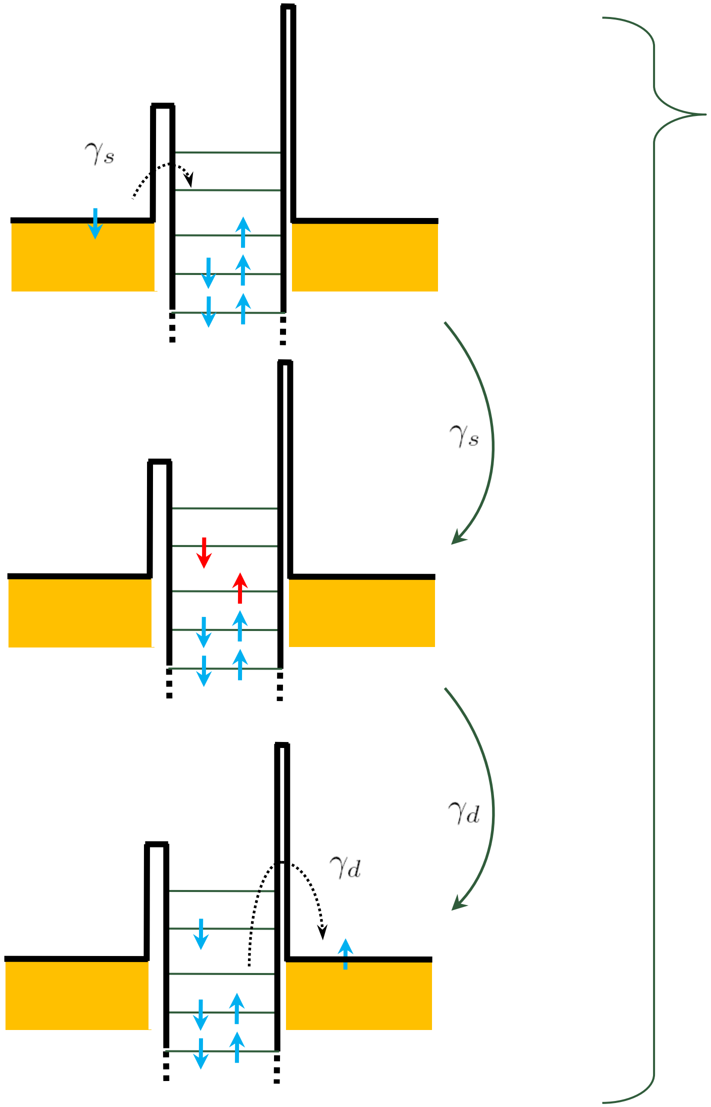
⟹ Finite current: $I \sim \frac{e^2}{h} |g_0 J|^2 V$

Spinful dot (odd occ.) (∞-order PT):

⟹ "Kondo-effect": $I \sim \frac{2e^2}{h} V, \quad V \ll T_K$

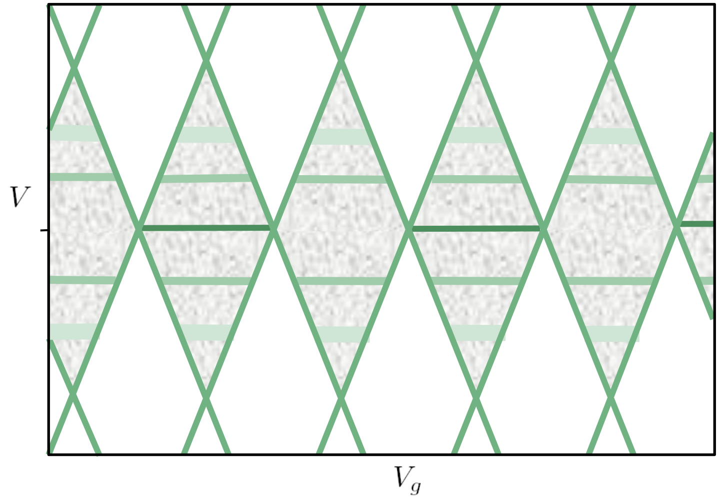


Inelastic Cotunneling: *Bias spectroscopy*



Extra contribution to the current:

$$I \sim \frac{e^2}{h}|g_0 J|^2 V\theta(V-\Delta)$$



Excited state spectroscopy !!

Specific signatures:

- spin-flip transitions (*Kondo-sharpened!*)
- vibrationally assisted transitions (*sidebands!*)

Kondo effect and spin-orbit coupling (not complete)

Ce-compounds etc.

- B. Coqblin, J. R. Schrieffer, Phys. Rev. (1969)
- P. Nozières, A. Blandin, J. Phys (1980)
- K. Yamada, K. Yosida, K. Hansawa, Prog. Theor. Phys. (1984)

so-coupling in conduction electrons

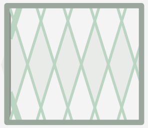
- Y. Meir, N. S. Wingreen, Phys. Rev. B (1994)
- O. Újsághy, A. Zawadowski, Phys. Rev. B (1998)
- L. Zsunyogh, G. Zaránd, S. Gallego, M. C. Muñoz, B. L. Györffy, Phys. Rev. Lett. (2006)

so-coupling + QDots

- J. Danon, Y. V. Nazarov, Phys. Rev. B (2009)
- D. F. Mross & H. Johannesson, Phys. Rev. B (2009)

- SO-coupling (isotropic LS-coupling) changes atomic spectrum, new g-factor etc.
- Degenerate Kramers doublet gives Kondo-effect.
- Spin-anisotropy from crystal fields.

Outline



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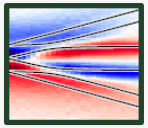

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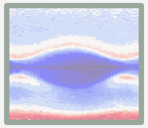


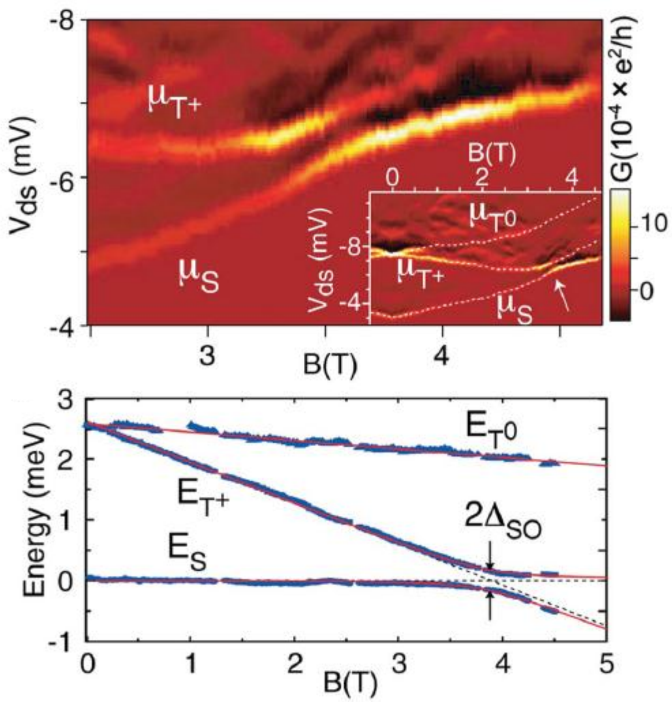
3. Probing spin-orbit coupling by inelastic cotunneling

InAs quantum wire dots

PRL **98,** 266801 (2007) PHYSICAL REVIEW LETTERS week ending 29 JUNE 2007

Direct Measurement of the Spin-Orbit Interaction in a Two-Electron InAs Nanowire Quantum Dot

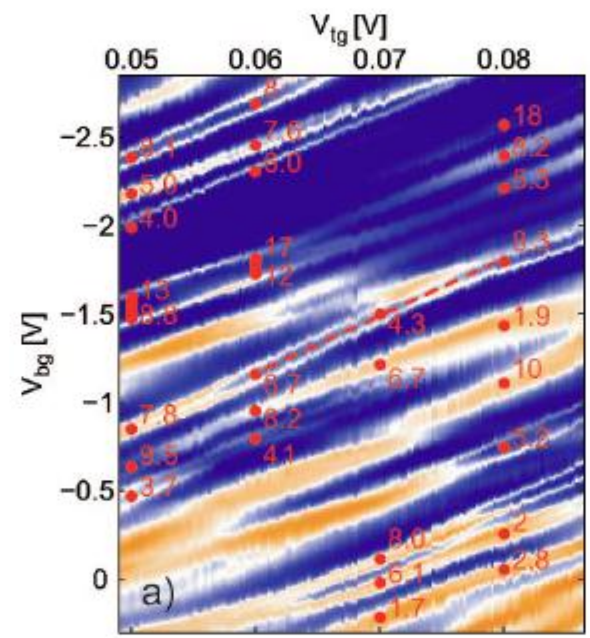
C. Fasth,[1] A. Fuhrer,[1,*] L. Samuelson,[1] Vitaly N. Golovach,[2] and Daniel Loss[2]
[1]*Solid State Physics/Nanometer Consortium, Lund University, P.O. Box 118 Lund, Sweden*
[2]*Department of Physics and Astronomy, University of Basel, Klingenbergstrasse 82, CH-4056 Basel, Switzerland*
(Received 8 January 2007; published 26 June 2007)



NANO LETTERS
2008
Vol. 8, No. 11
3932-3935

Giant Fluctuations and Gate Control of the *g*-Factor in InAs Nanowire Quantum Dots

S. Csonka,* L. Hofstetter, F. Freitag, S. Oberholzer, and C. Schönenberger
Department of Physics, University of Basel, Klingelbergstr. 82, CH-4056 Basel, Switzerland
T. S. Jespersen, M. Aagesen, and J. Nygård
Nano-Science Center, Niels Bohr Institute, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark



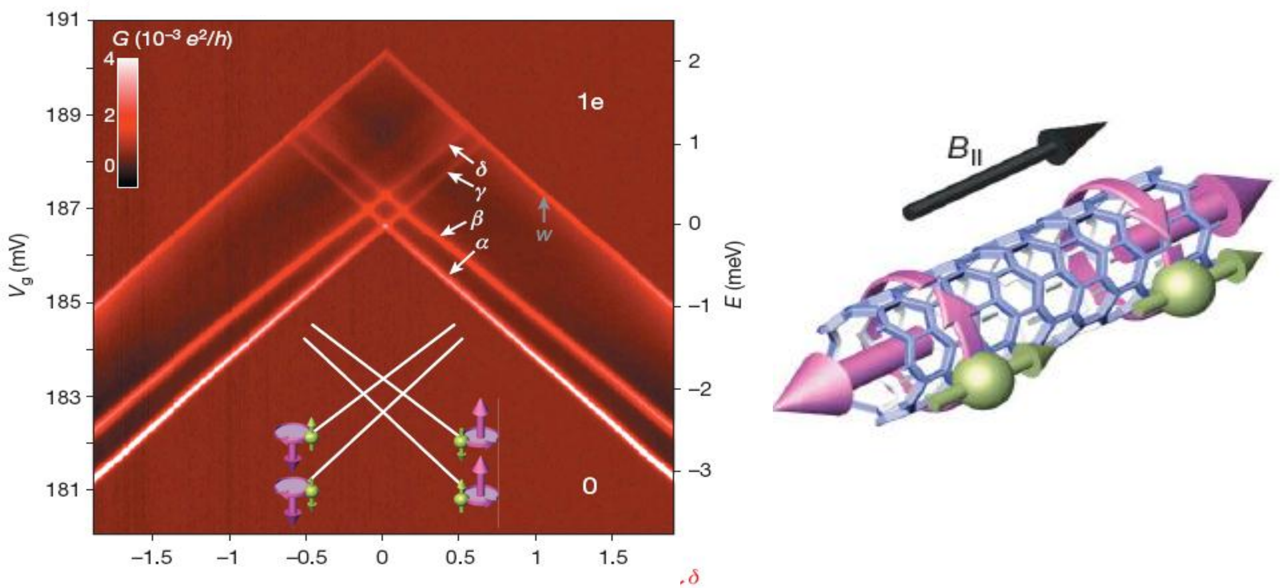
Spin-Orbit Coupling in carbon nanotube quantum dots

nature Vol 452|27 March 2008|**doi:10.1038/nature06822**

LETTERS

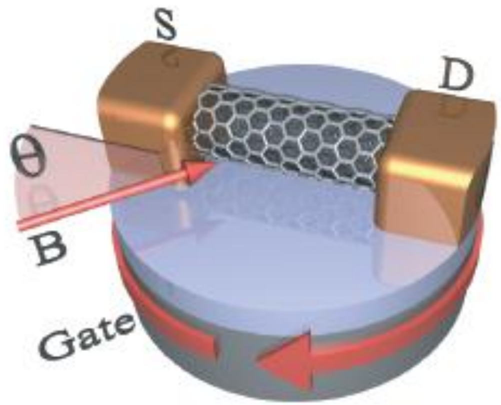
Coupling of spin and orbital motion of electrons in carbon nanotubes

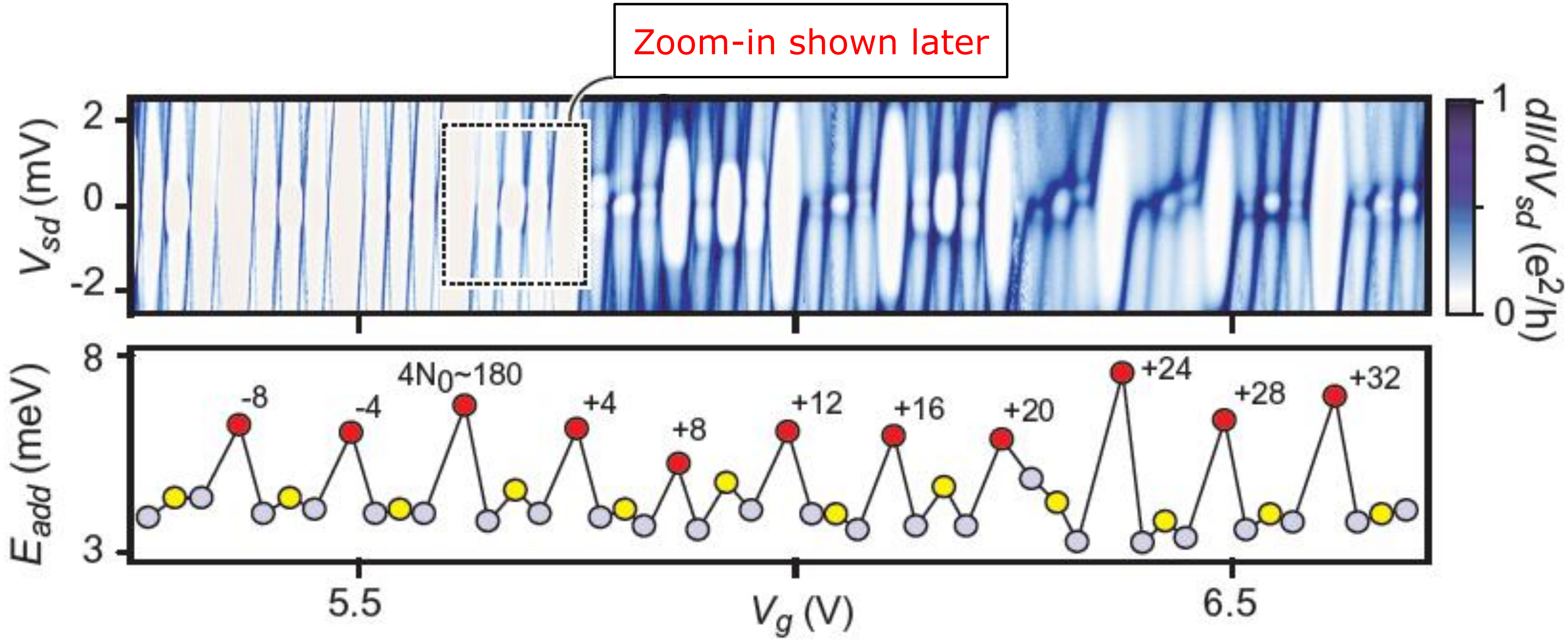
F. Kuemmeth[1]*, S. Ilani[1]*, D. C. Ralph[1] & P. L. McEuen[1]



Spin-Orbit Coupling in multi-electron carbon nanotubes

T. Sand Jespersen, K. Grove-Rasmussen,
J. Paaske, K. Muraki, T. Fujisawa, J. Nygård, K. Flensberg
(arXiv:1008.1600)





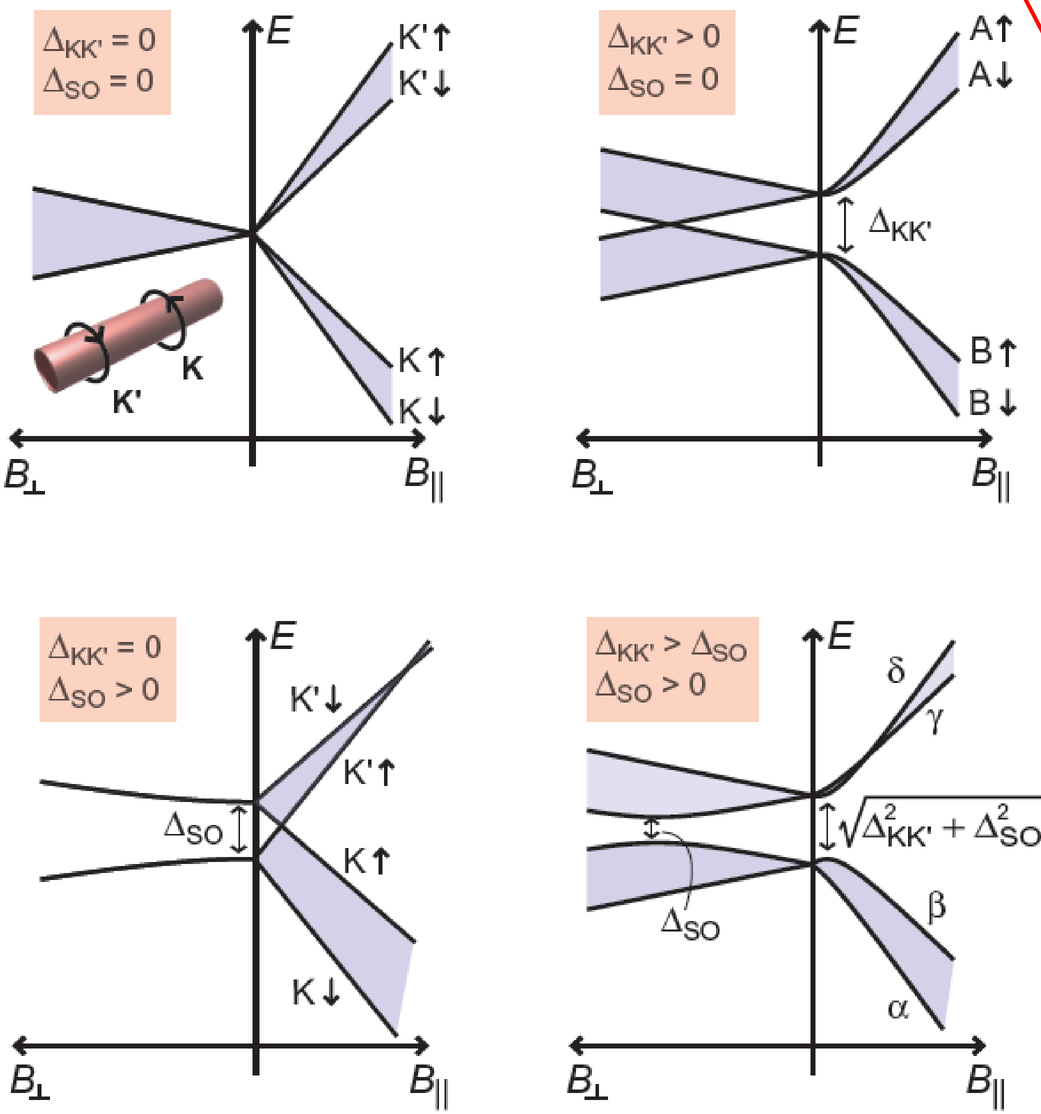
Single-particle states: $E_{s,\tau} = s\tau\Delta_0 \pm \sqrt{(\Delta_g + \tau\Delta_\Phi + s\tau\Delta_1)^2 + {\varepsilon_n}^2}$

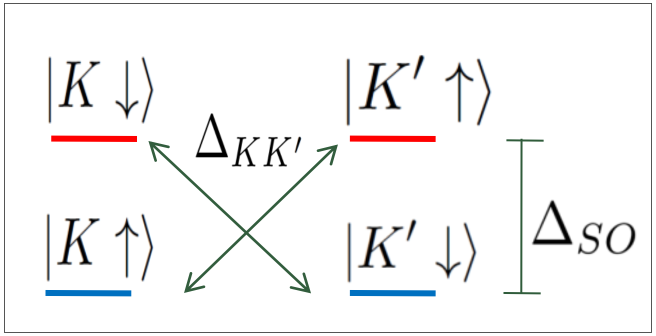
AB flux

Curvature gap

Ando, 2000
Huertas-Hernando, Guinea, Brataas, 2006

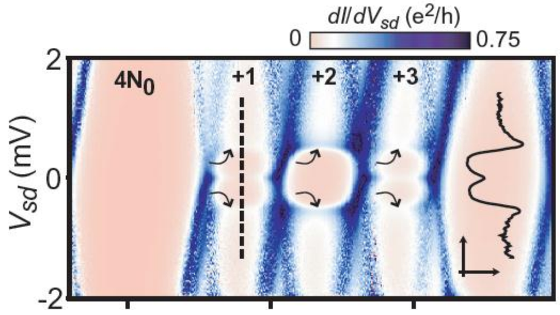
Izumida, Sato, Saito, 2009
Jeong and Lee, 2009



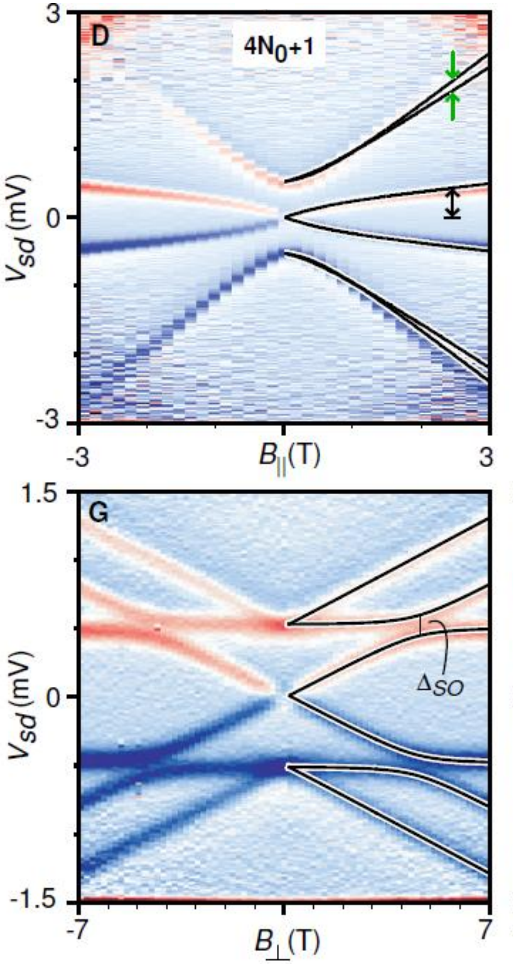


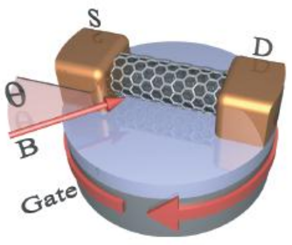
Cotunneling spectroscopy

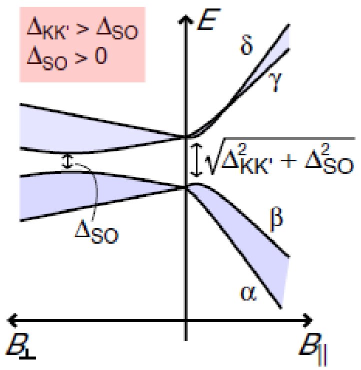
Fit parameters: $\Delta_{\mathrm{S0}}, \Delta_{KK'}$ and g_{orb}



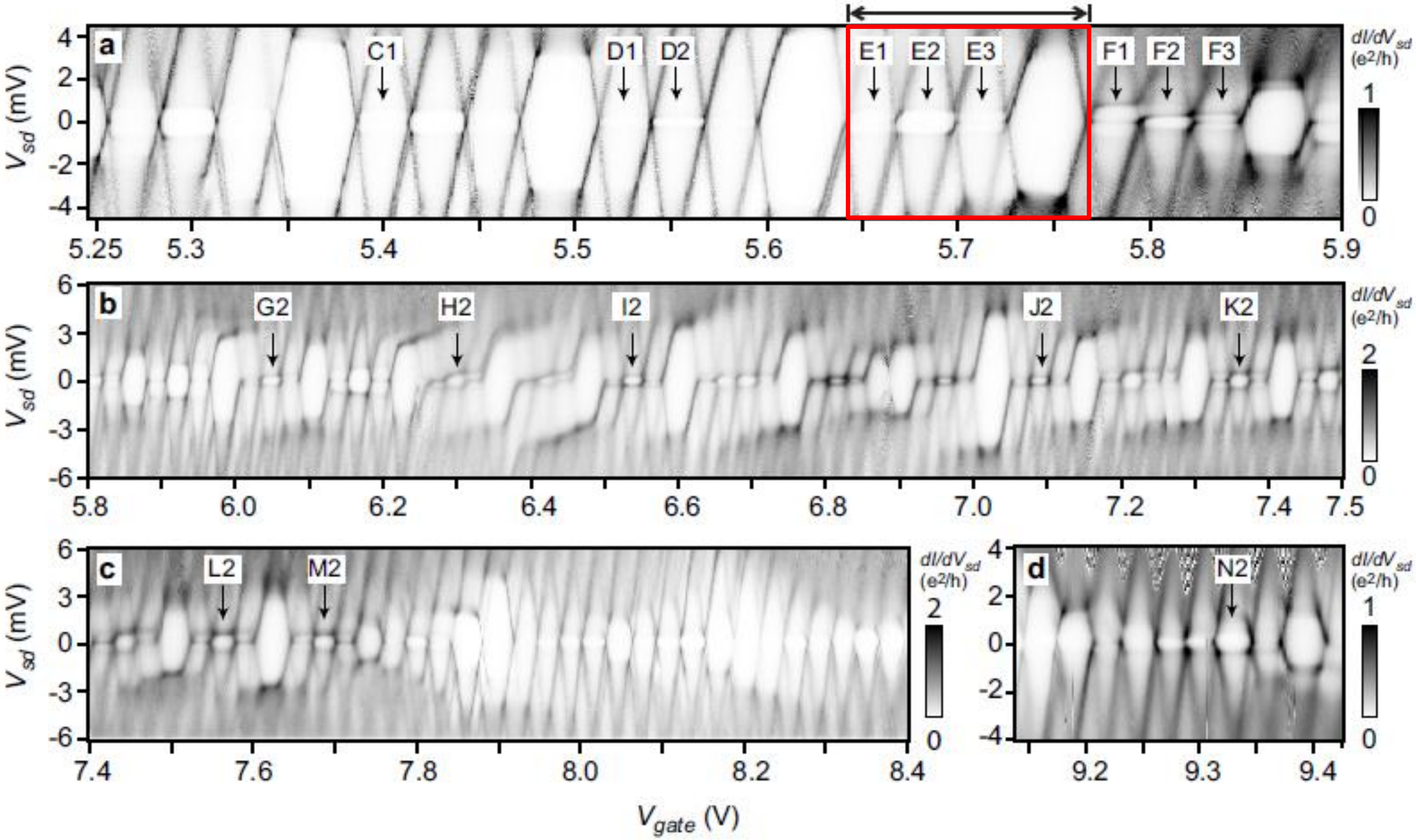
1st electron **2nd electron** **3rd electron**







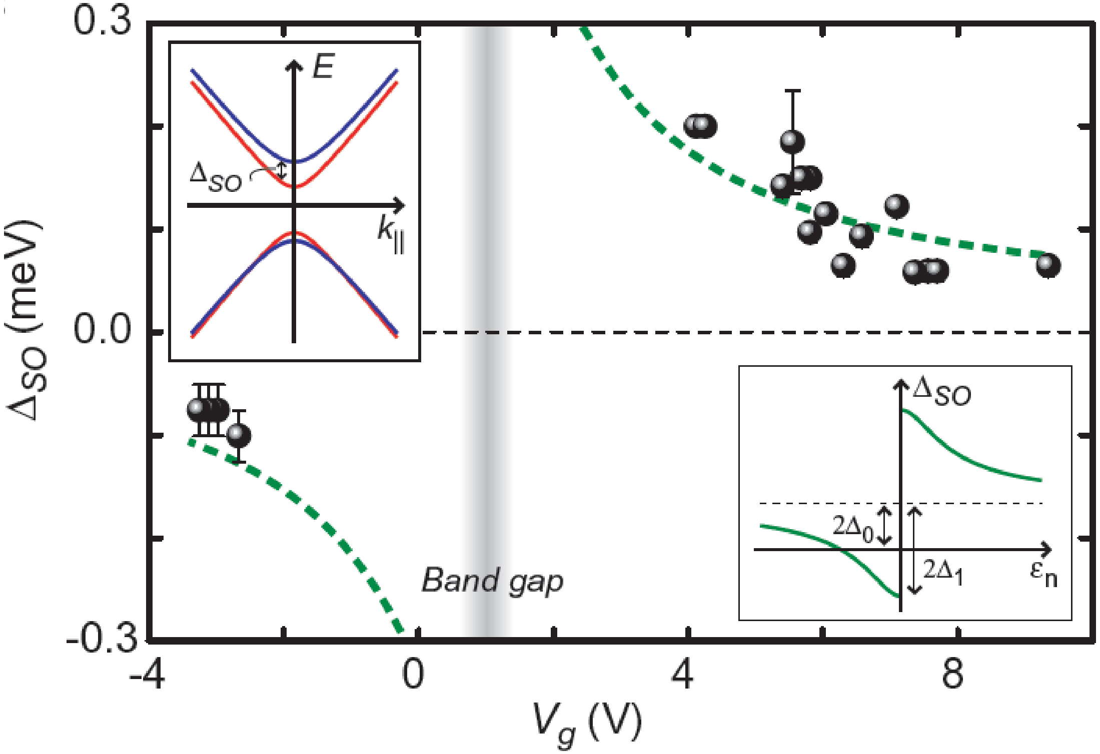
Bias-spectroscopy for large filling



Gate-dependence of spin-orbit coupling

$$E_{s,\tau} = s\tau\Delta_0 \pm \sqrt{(\Delta_g + \tau\Delta_\Phi + s\tau\Delta_1)^2 + {\varepsilon_n}^2}$$

$$\Delta_{\mathrm{SO},\pm} = E_{K\uparrow} - E_{K\downarrow} = 2\Delta_0 \pm \sqrt{(\Delta_g + \Delta_1)^2 + \varepsilon_n^2} \mp \sqrt{(\Delta_g - \Delta_1)^2 + \varepsilon_n^2}$$



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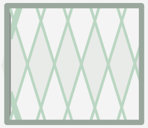

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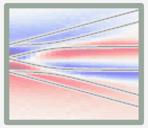

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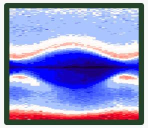


3. Probing spin-orbit coupling by inelastic cotunneling

Cotunneling and (S=1/2) Kondo effect with spin-orbit coupling

$$\mathcal{H}_d = \frac{(\mathbf{p}+e\mathbf{A})^2}{2m} + V(\mathbf{r}) + \frac{e\hbar}{4m^2c^2}[\mathbf{E}(\mathbf{r}) \times (\mathbf{p}-\mathbf{eA})] \cdot \boldsymbol{\tau} + \mathbf{g}\mu_{\mathbf{B}}\mathbf{B} \cdot \boldsymbol{\tau}$$

For $B = 0$, time-reversal symmetry $\Longrightarrow$ **Kramers doublets** (of 2-spinors)

$$\psi_{n\Uparrow}(\mathbf{r}) = \begin{pmatrix} u_n(\mathbf{r}) \\ v_n(\mathbf{r}) \end{pmatrix}, \quad \psi_{\mathbf{n}\Downarrow}(\mathbf{r}) = \begin{pmatrix} -v_n^*(\mathbf{r}) \\ u_n^*(\mathbf{r}) \end{pmatrix}$$

(Dot electrons)

$$\psi_{\alpha\mathbf{k}\uparrow}(\mathbf{r}) = \begin{pmatrix} a_{\alpha\mathbf{k}}(\mathbf{r}) \\ b_{\alpha\mathbf{k}}(\mathbf{r}) \end{pmatrix}, \quad \psi_{\alpha\mathbf{k}\downarrow}(\mathbf{r}) = \begin{pmatrix} -b_{\alpha\mathbf{k}}^*(\mathbf{r}) \\ a_{\alpha\mathbf{k}}^*(\mathbf{r}) \end{pmatrix}$$

(Conduction electrons)

Time-reversal symmetry-partners:

$$\boxed{\psi_{n\Uparrow,\sigma} \;=\; i\tau^y_{\sigma\sigma'}\psi^*_{n\Downarrow,\sigma'}}$$

Resulting Anderson-model (Constant interaction model)

$$H = \sum_{\substack{\alpha=L/R \\ \mathbf{k},\nu}} (\varepsilon_k - \mu_\alpha) c^\dagger_{\alpha\mathbf{k}\nu} c_{\alpha\mathbf{k}\nu} + \sum_{n,\eta} \varepsilon_n d^\dagger_{n\eta} d_{n\eta}$$

$$+ \sum_{\substack{\alpha=L/R \\ \mathbf{k},\nu,\eta,n}} \left(t^{\alpha k n}_{\nu\eta} c^\dagger_{\alpha\mathbf{k}\nu} d_{n\eta} + t^{n\alpha k}_{\eta\nu} d^\dagger_{n\eta} c_{\alpha\mathbf{k}\nu} \right) + H_{int}$$

$$t^{\alpha\mathbf{k}n}_{\nu\eta} = \int d\mathbf{r}\ \psi^*_{\alpha\mathbf{k}\nu}(\mathbf{r}) \mathcal{H}_{tot}(\mathbf{r}) \psi_{n\eta}(\mathbf{r}) = t_{\alpha\mathbf{k}n} \mathbb{U}^{\alpha\mathbf{k}n}_{\nu\eta}$$

(Unitary matrix !!)

$$\mathcal{H}_{tot}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r})\tau^0 + i\lambda_{so}\varepsilon_{ijk}\tau^i E_j(\mathbf{r})\partial_{x_k}$$

$$\psi_{n\Uparrow,\sigma} = i\tau^y_{\sigma\sigma'}\psi^*_{n\Downarrow,\sigma'}$$

$\Downarrow$ (m'th orbital)

$$\tilde{c}^\dagger_{\alpha\mathbf{k}\eta} = c^\dagger_{\alpha\mathbf{k}\nu} \mathbb{U}^{\alpha\mathbf{k}m}_{\nu\eta}$$

$$H = \sum_{\alpha\mathbf{k}\eta} (\varepsilon_k - \mu_\alpha) \tilde{c}^\dagger_{\alpha\mathbf{k}\eta} \tilde{c}_{\alpha\mathbf{k}\eta} + \sum_{\eta} \varepsilon_m d^\dagger_{m\eta} d_{m\eta}$$

$$+ \sum_{\alpha\mathbf{k}\eta} t_{\alpha\mathbf{k}m} \left(\tilde{c}^\dagger_{\alpha\mathbf{k}\eta} d_{m\eta} + d^\dagger_{m\eta} \tilde{c}_{\alpha\mathbf{k}\eta} \right) + H_{int}$$

No trace of *so*-interaction!

Kondo effect via degenerate Kramers doublets

What happens in applied magnetic field?

$$\mathcal{H}_B = \mu_B \left(\left[g\mathbf{B} + \frac{4mc^2}{e}\mathbf{B} \times \mathbf{r} \times \mathbf{E}(\mathbf{r}) \right] \cdot \boldsymbol{\tau} - \mathbf{B} \cdot \mathbf{L} \right)$$

Time-reversal symmetry is broken:
- **Non-degenerate Kramers doublets**
- **Splitting of Kondo peaks**
- **i.e. log-enhanced inelastic cotunneling**

m'th level is mixed in with other levels: $\langle n\eta' | \mathcal{H}_B | m\eta \rangle$

$\Rightarrow \quad t_{\nu\eta}^{\alpha k m}$ No longer Unitary matrix !!

Implications for the Kondo model

Do Schrieffer-Wolff transformation with general $t_{\nu\eta}^{\alpha k m}$:

$$H_K = \sum_{\alpha\mathbf{k}\eta}(\varepsilon_{k\eta} - \mu_\alpha)\tilde{c}^\dagger_{\alpha\mathbf{k}\eta}\tilde{c}_{\alpha\mathbf{k}\eta} + \frac{1}{2}\sum_{\substack{\alpha'\alpha,\mathbf{k}'\mathbf{k},\eta'\eta \\ i,j=0,x,y,z}} J^{ij}_{\alpha'\alpha} S^i \, \tilde{c}^\dagger_{\alpha'\mathbf{k}'\eta'}\tau^j_{\eta'\eta}\tilde{c}_{\alpha\mathbf{k}\eta}$$

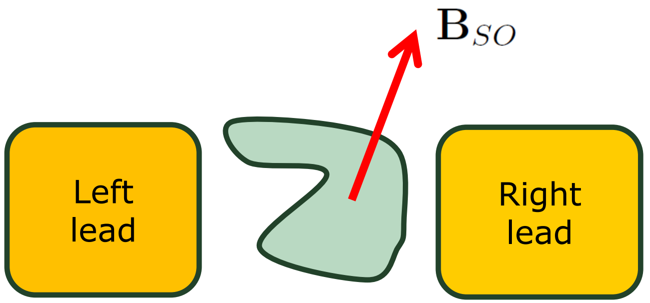
$$J^{ij}_{\alpha'\alpha} = \mathrm{Tr}[t_{\alpha' m}\tau^i t^\dagger_{\alpha m}\tau^j]\frac{\varepsilon_+ + (-1)^{\delta_{i0}}\varepsilon_-}{2^{(\delta_{i0}+\delta_{j0})}\varepsilon_+\varepsilon_-}$$

Effective cotunneling (Kondo) model with **anisotropic** exchange coupling.

Notice that $\alpha', \alpha = L/R$ is mixed with $\uparrow / \downarrow$: Potentially 2-channel Kondo

A simple model: 2 levels with SO-coupling and B-field

Diagonalize two levels with spin (4x4):



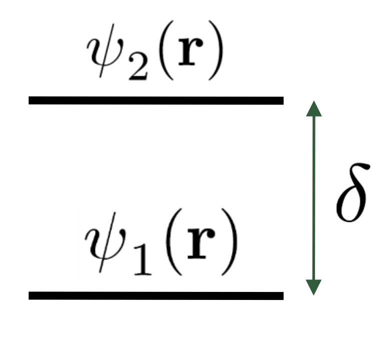
$$\mathcal{H}_d = \mathcal{H}_d^{(0)} + \mathcal{H}_{SO}$$

$$\mathcal{H}_{SO} = (e\hbar/2mc^2)(\mathbf{E}\times\mathbf{p})\cdot\boldsymbol{\sigma}$$

$$\langle\psi_1, s|H_{S0}|\psi_1, s'\rangle = 0$$

$$\langle\psi_1, s|H_{S0}|\psi_2, s'\rangle = \frac{e\hbar}{2mc^2}\langle\psi_1|(\mathbf{E}\times\mathbf{p})|\psi_2\rangle\cdot(\boldsymbol{\sigma})_{ss'}$$

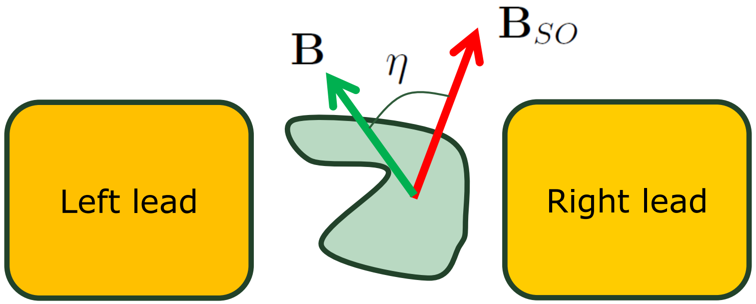
$$= \underbrace{-\hat{\mathbf{z}}\; i\Delta_{SO}/2}_{\mu_B\mathbf{B}_{SO}}$$

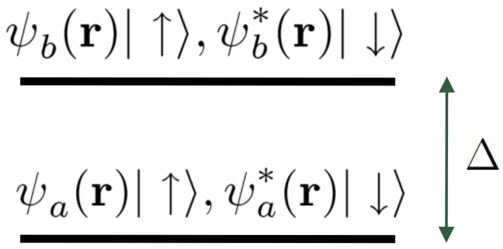


Diagonalize in zero B-field: (spin-quantization axis along $\mathbf{B}_{SO}$)

$$\mathcal{H}_0 = -\delta\tau_z\sigma_0/2 + \Delta_{SO}/2\ \tau_y\sigma_z$$

$$\Rightarrow E_c = \mp\frac{1}{2}\Delta \quad \begin{cases} c = a, b \\ \Delta = \sqrt{\delta^2 + \Delta_{SO}^2} \end{cases}$$





Now, apply magnetic field (linear terms):

$$\mathcal{H}_Z = \frac{1}{2}g_0\mu_B\mathbf{B}\cdot\boldsymbol{\sigma}, \quad \mathcal{H}_L = \mu_B\ \mathbf{L}\cdot\mathbf{B} \quad ?$$

Breaking time-reversal symmetry by external field:

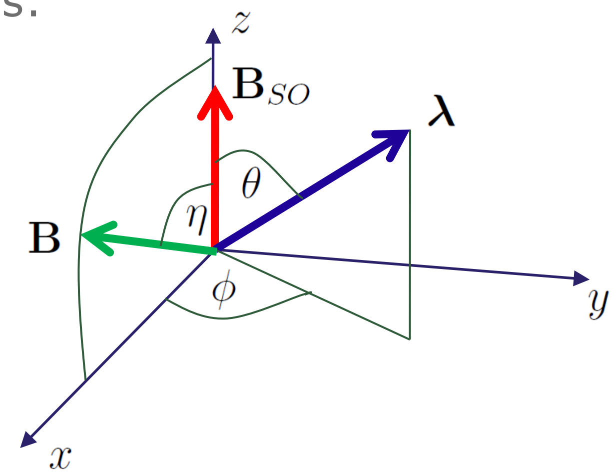
- In absence of B-field, there was no sign of entangled spin and orbit.
- In presence of B-field, there is ....

$$\mathcal{H}_B = \frac{1}{2} g_0 \mu_B \tau_0 \mathbf{B} \cdot \boldsymbol{\sigma} + i \mu_B \sigma_0 \tau_y \boldsymbol{\lambda} \cdot \mathbf{B}$$

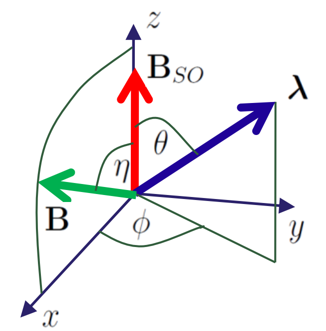
$$\boldsymbol{\lambda} = -i \langle \psi_1 | \mathbf{L} | \psi_2 \rangle$$

The magnetic field couples the doublets!

Three relevant vectors:

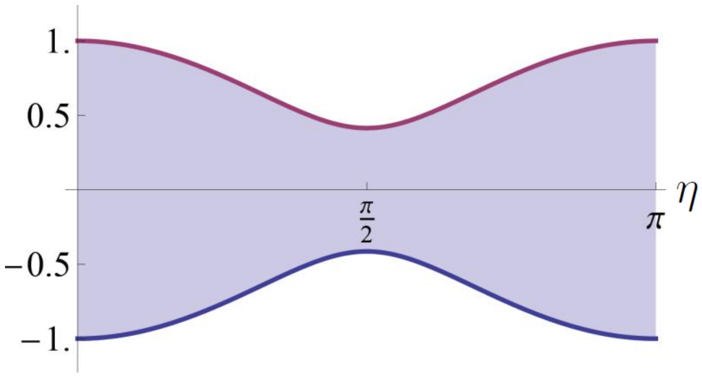


Within one Kramers doublet (a, b-basis)

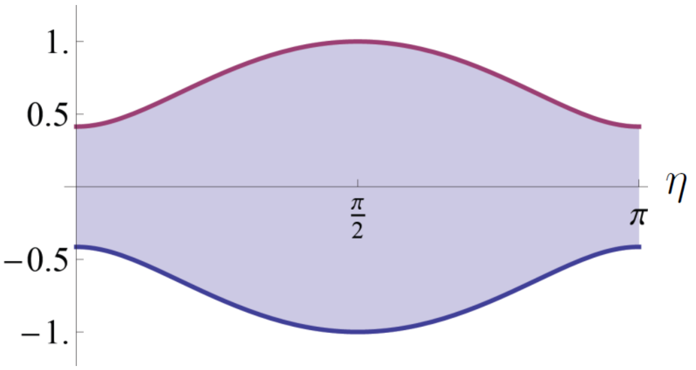


$\delta = \Delta_{SO} = 1,\ \boldsymbol{\lambda} \| \mathbf{B}_{SO}$

Splitting of Kramers doublet *b*



Splitting of Kramers doublet *a*



$\psi_b(\mathbf{r})|\uparrow\rangle, \psi_b^*(\mathbf{r})|\downarrow\rangle$

Δ

$\psi_a(\mathbf{r})|\uparrow\rangle, \psi_a^*(\mathbf{r})|\downarrow\rangle$

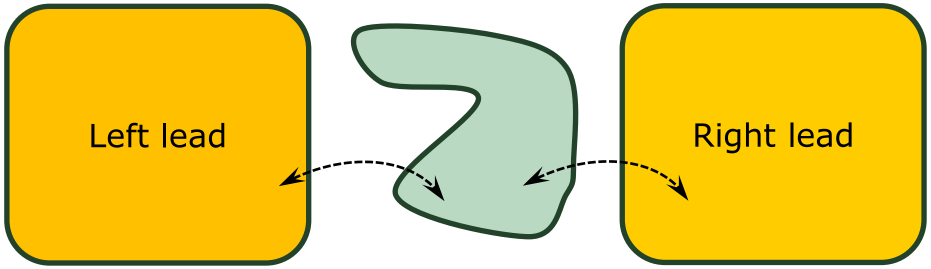
$$\mathcal{H}_c^{(1)} = \frac{1}{2} g_0 \mu_B B Z_c \begin{pmatrix} \cos\xi_c & \sin\xi_c \\ \sin\xi_c & -\cos\xi_c \end{pmatrix}$$

$$\sin\xi_c = \frac{\pm\delta \sin\eta}{Z_c \Delta}$$

$$Z_c = \sqrt{\frac{\delta^2 \sin^2\eta}{\Delta^2} + \left(\cos\eta \pm \frac{2\tilde{\lambda}\Delta_{SO}}{g_0 \Delta}\right)^2}$$

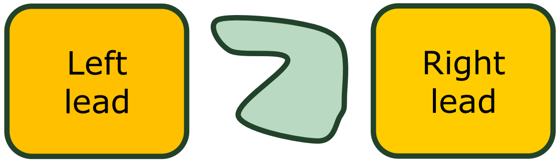
$$\tilde{\lambda} = |\boldsymbol{\lambda}| \left(\cos\eta\cos\theta + \sin\eta\cos\varphi\sin\theta\right)$$

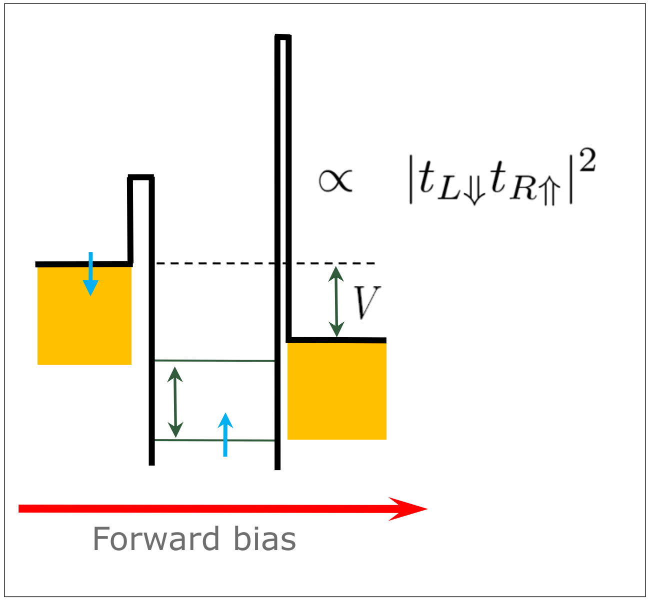
Tunneling amplitudes revisited in finite B-field



With only spin-orbit *or* magnetic field:	$t_{\uparrow} = t_{\downarrow}$
With both spin-orbit *and* magnetic field:	$t_{\Uparrow} \neq t_{\Downarrow}$

Inelastic cotunneling current



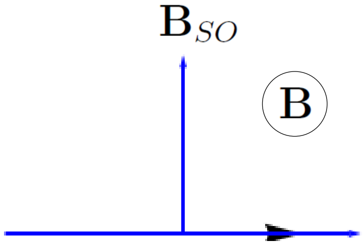


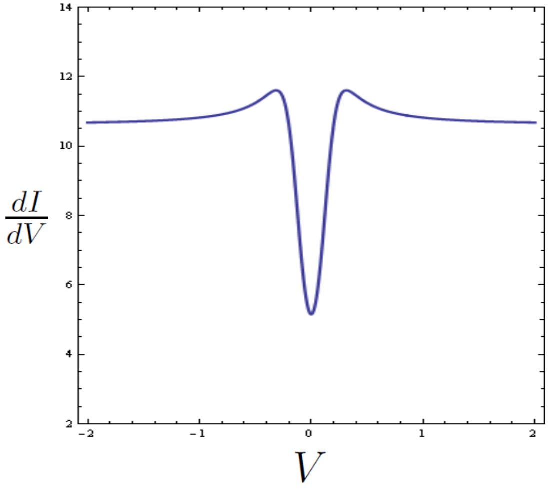
Calculate nonlinear dI/dV:
(Requires noneq. occupations).

$$\dot{P}(\Downarrow) = P(\Uparrow)\Gamma_{\Downarrow\Uparrow} - P(\Downarrow)\Gamma_{\Uparrow\Downarrow} = 0$$

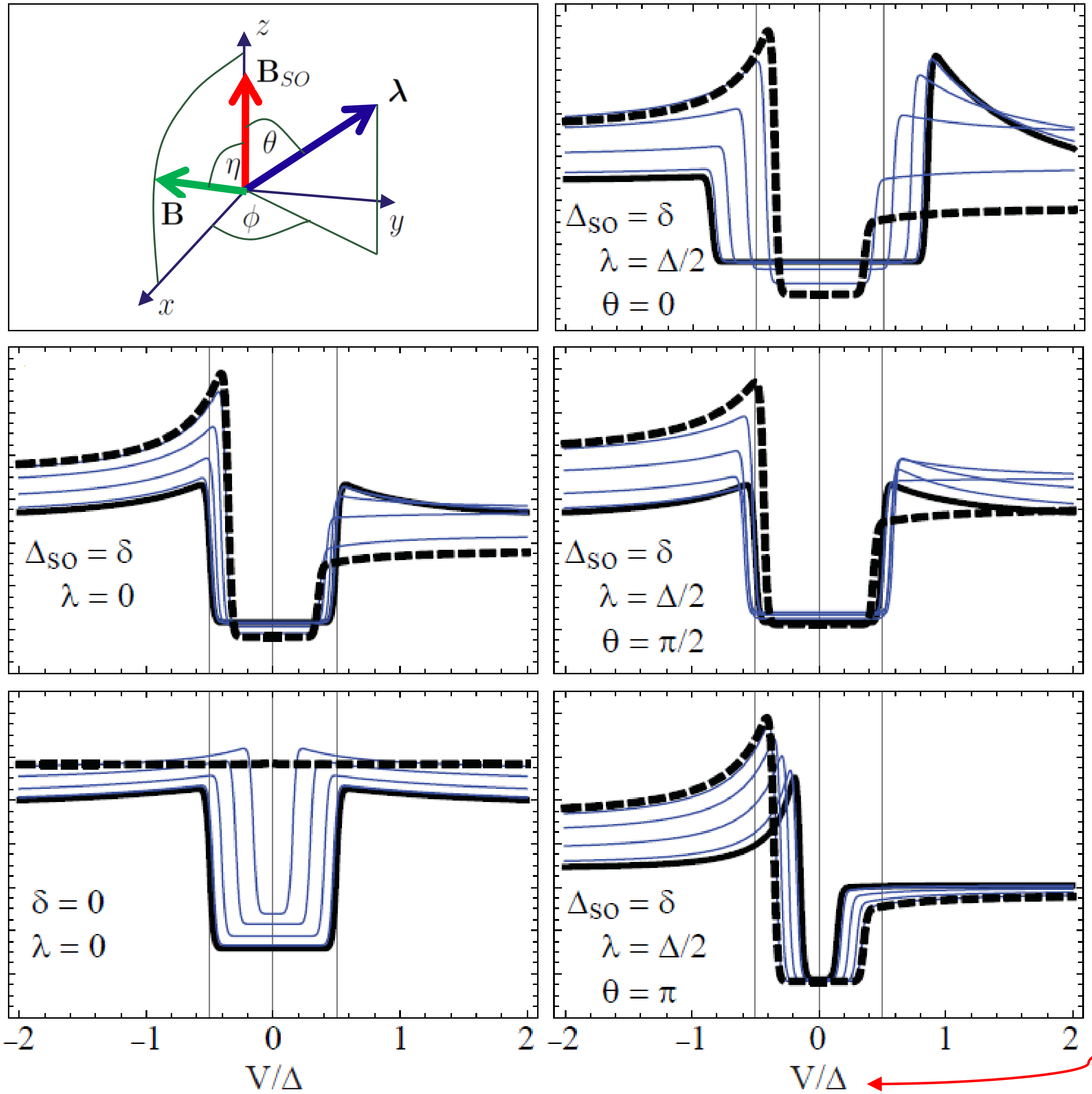
Difference between forward and reverse bias:

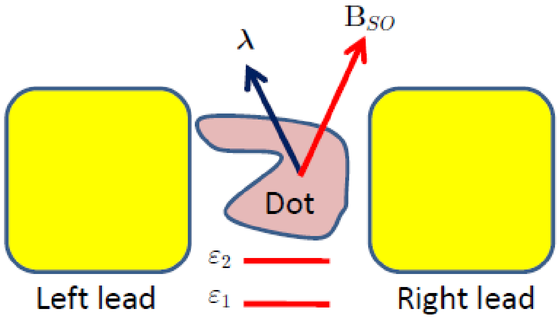
$$|t_{L\Downarrow} t_{R\Uparrow}|^2 - |t_{L\Uparrow} t_{R\Downarrow}|^2 \neq 0$$

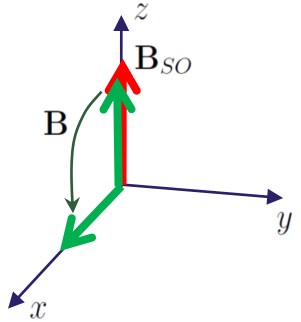




Lots of different behaviors now possible:

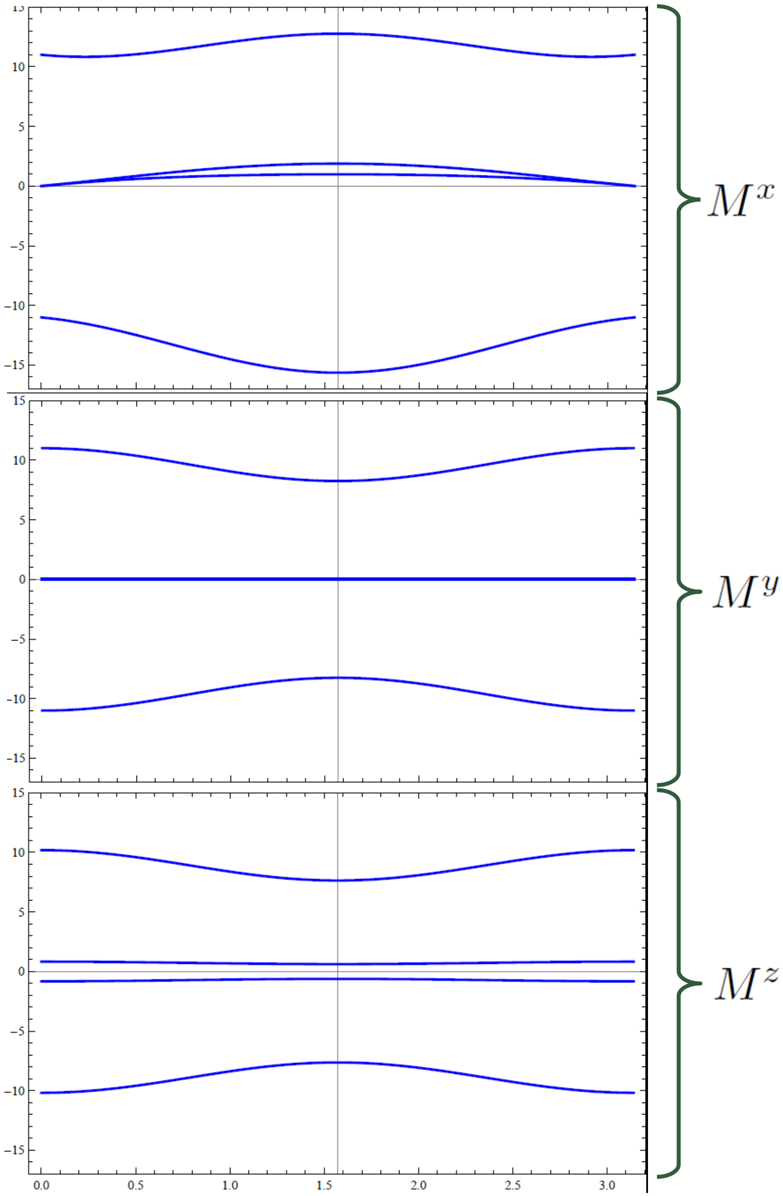






$$\Delta = \sqrt{\delta^2 + \Delta_{SO}^2}$$

Two-channel Kondo model at finite B-field



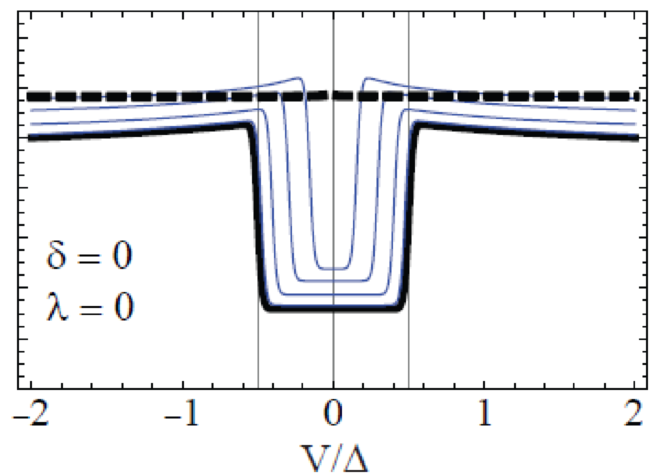
Calculate the 4 eigenvalues of:

$$M^i = \sum_{j=x,y,z} J^{ij}_{\alpha'\alpha}\, \tilde{c}^\dagger_{\alpha'\eta'} \tau^j_{\eta'\eta} \tilde{c}_{\alpha\eta}$$

considered as a matrix in the basis:

$$\{|L\uparrow\rangle, |L\downarrow\rangle, |R\uparrow\rangle, |R\downarrow\rangle\}$$

Not only two non-zero eigenvalues!



Zero splitting attained!

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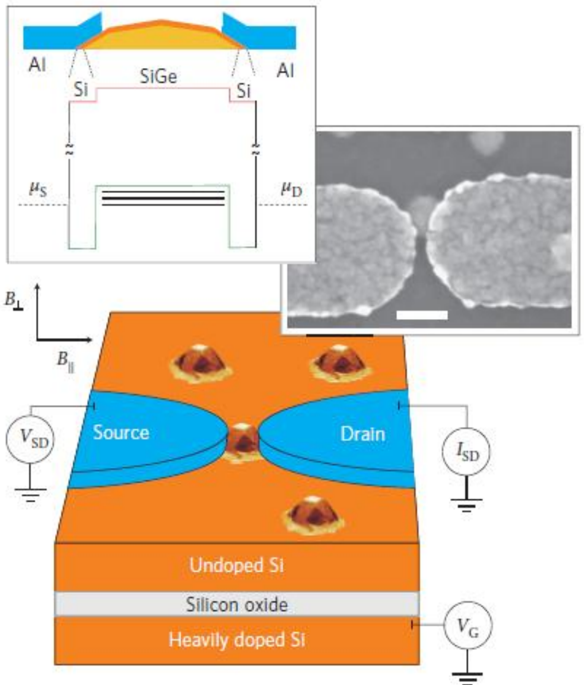
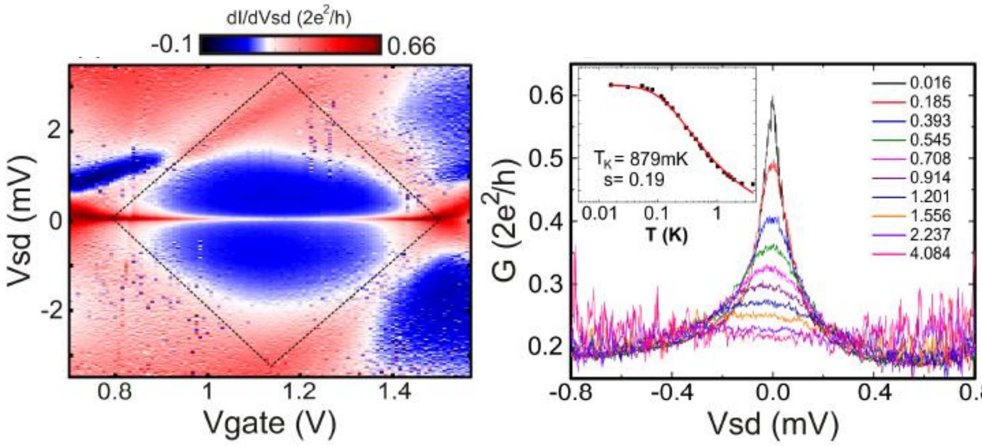
Hybrid superconductor–semiconductor devices made from self-assembled SiGe nanocrystals on silicon *('Stranski-Krastanow growth')*

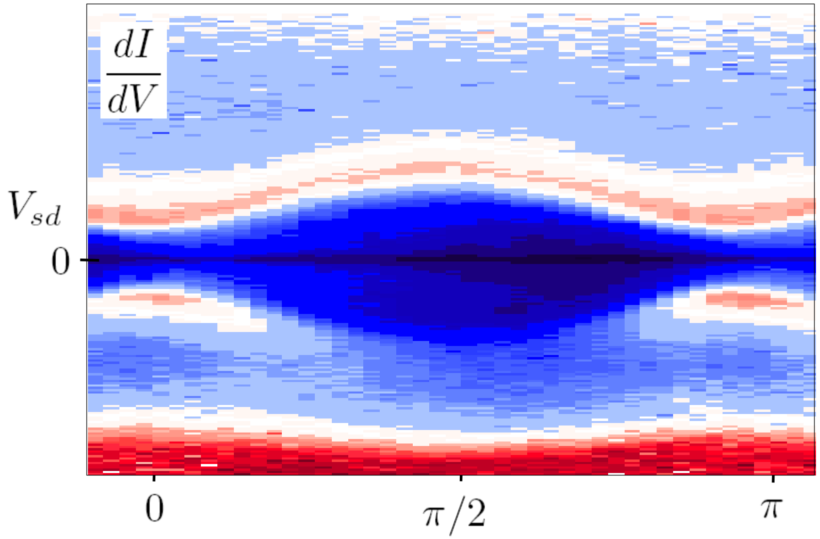
G. Katsaros[1]*, P. Spathis[1], M. Stoffel[2], F. Fournel[3], M. Mongillo[1], V. Bouchiat[4], F. Lefloch[1], A. Rastelli[2], O. G. Schmidt[2] and S. De Franceschi[1]*

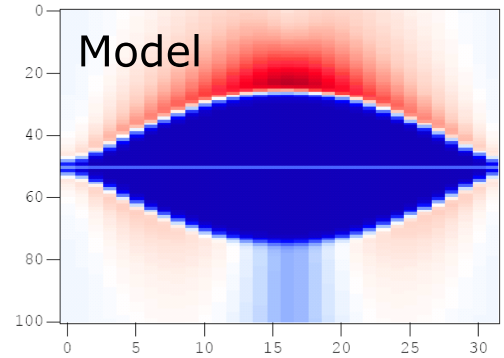


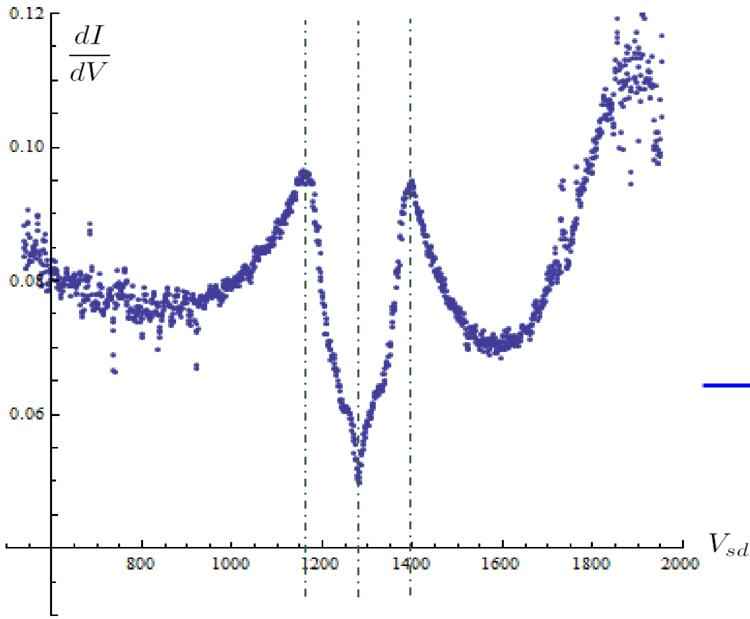


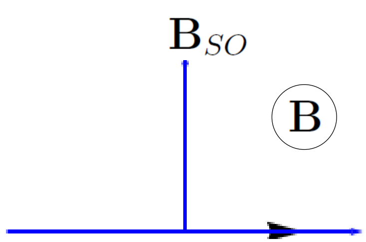
Split Kondo peak: rotating the magnetic field

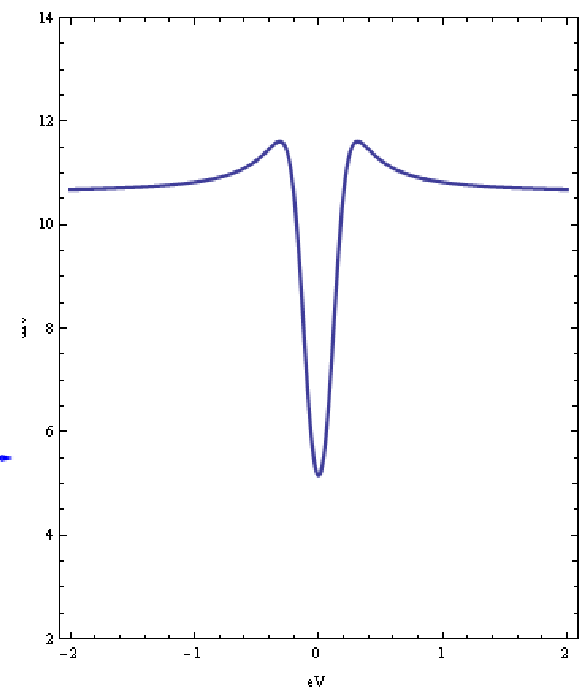
[**Giorgos Katsaros**, S. De Franceschi *et al.*, *unpublished*]



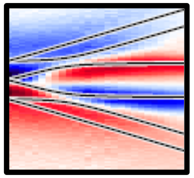




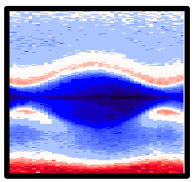




Summary



Spin-orbit coupling in CNT at large filling
Cotunneling spectroscopy with external magnetic field at different angles.



Spin-orbit coupling in Kondo regime
Efficiently probed by inelastic cotunneling with external magnetic field:

- Bias-asymmetry and strong angular dependence!
- Potential for 2-channel Kondo effect (?)