Non-equilibrium transport through double quantum dot devices: A non-Fermi liquid critical point

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Motivation

- Interacting electron systems near quantum phase transitions, often exhibit non-Fermi liquid states
- Technically complicated
- Quantum impurities: interaction is local in space.
- Similar phenomena occur, but exact solutions are possible
- Non-equilibrium generalizations

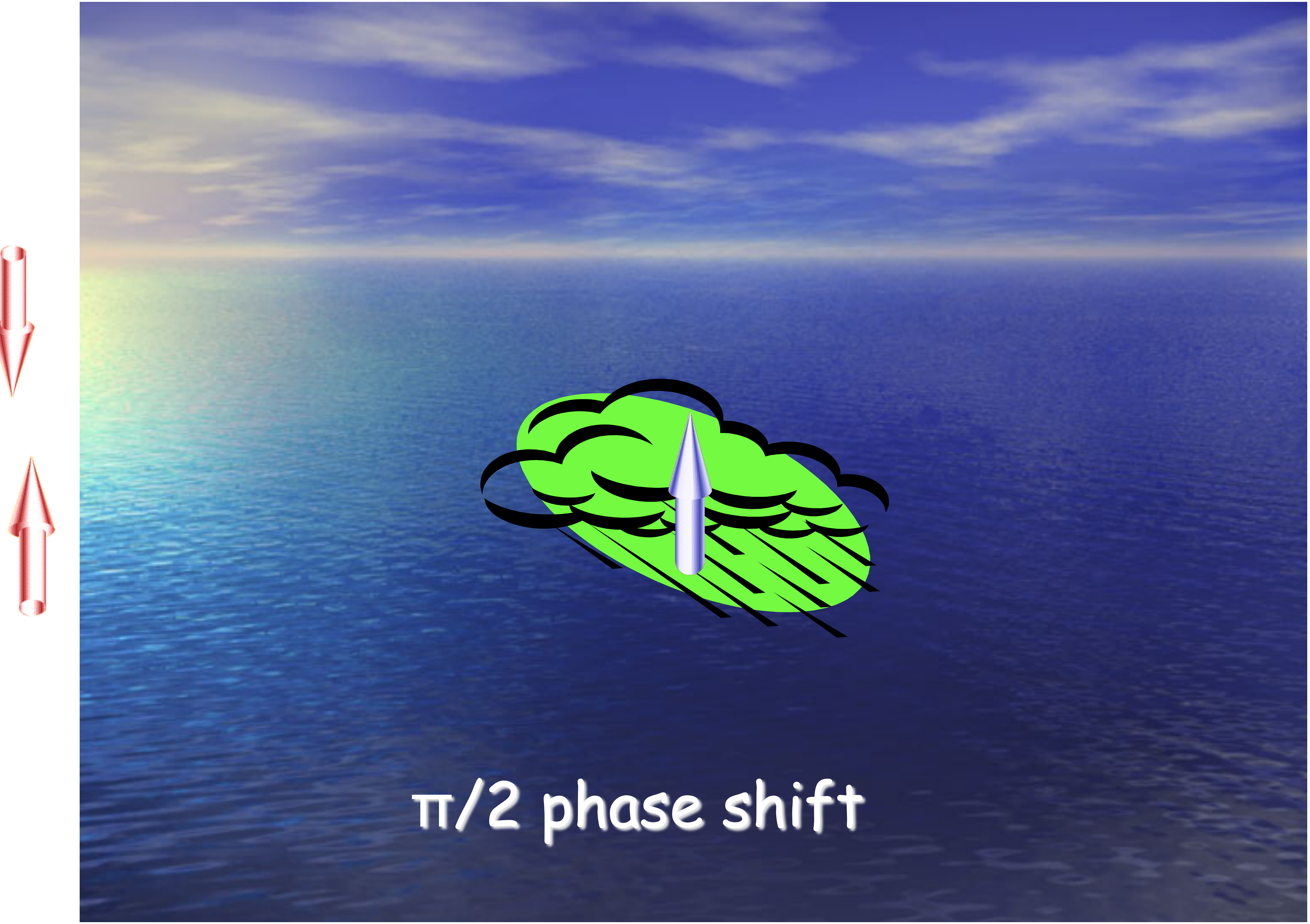
Outline

- Critical point in 2 impurity Kondo model
- Experimental realizations
- Exact critical behavior
- Non-equilibrium transport and noise

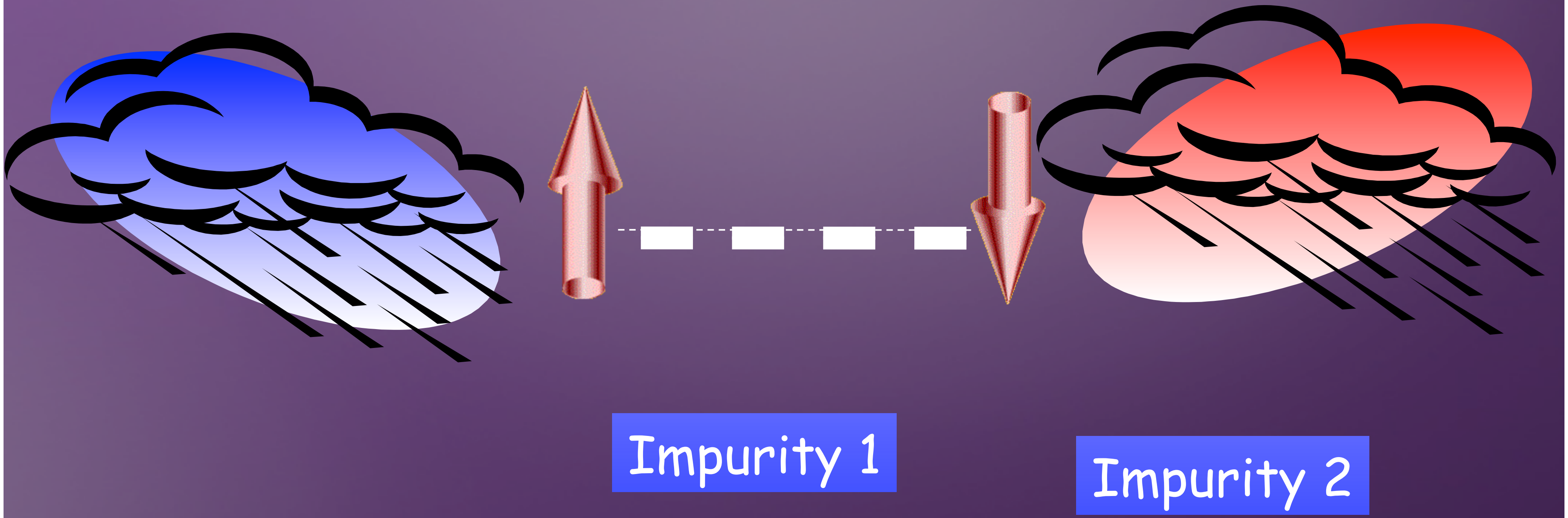
Critical point in 2-impurity Kondo model

Single impurity case:

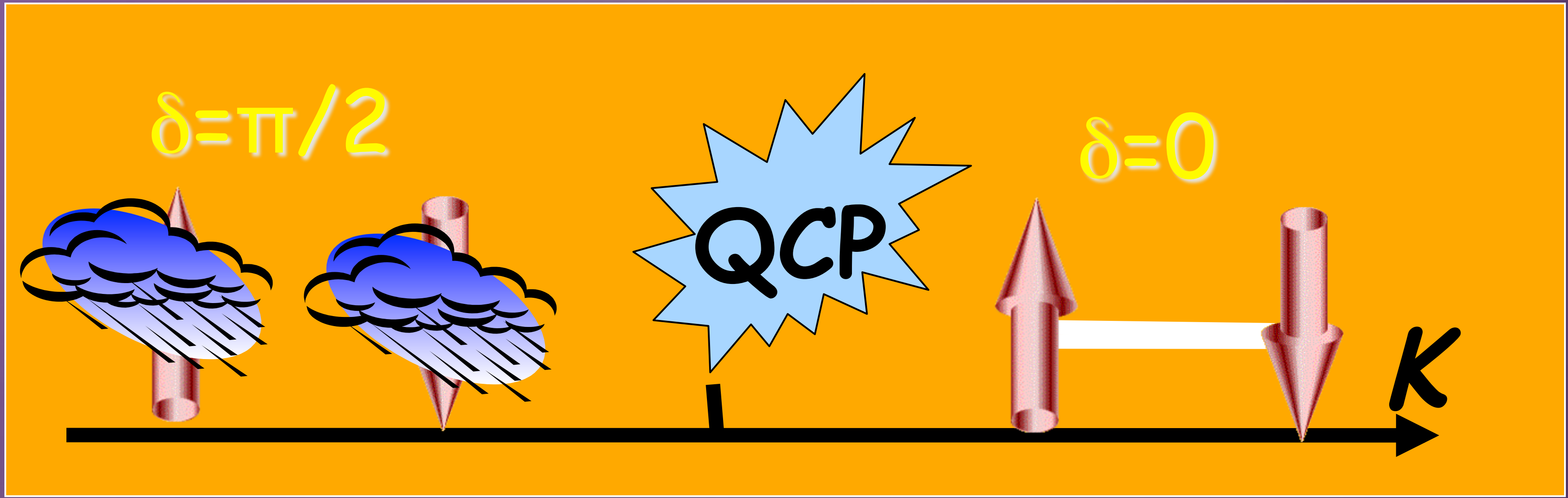
- A single S=1/2 impurity (at r=0) has an exchange interaction with a single channel of conduction electrons:
- J renormalizes to infinity at low energies
- Low energy (E<TK) fixed point corresponds to an electron from the Fermi sea forming an entangled singlet state with impurity


π/2 phase shift

2-Impurity Kondo

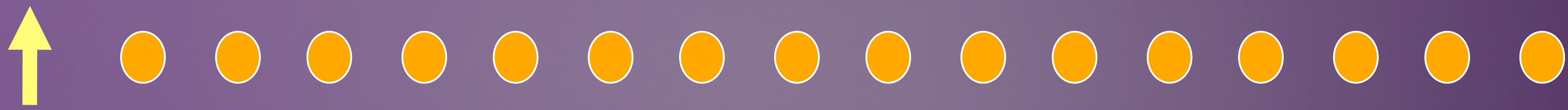


$$+K\, S_1 \cdot S_2$$



$K_c \sim 2.2 T_K$

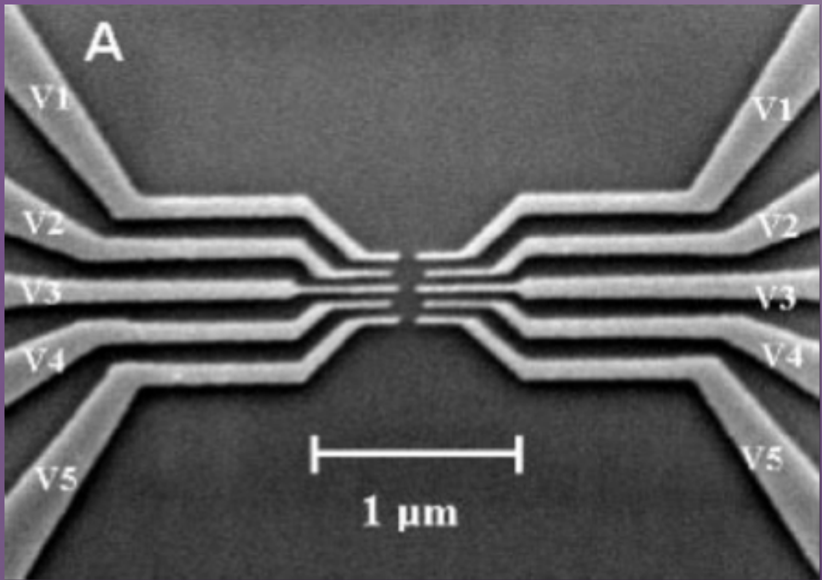
- Phase shifts are a sort of trivial conformally invariant boundary condition (BC)
- At quantum critical point (QCP) a non-trivial conformally invariant BC occurs
- Inter-impurity singlet and many-body Kondo singlet are degenerate at this point
- Behavior near QCP at K=Kc is equivalent to that of critical Ising chain with boundary field (Affleck and Ludwig)

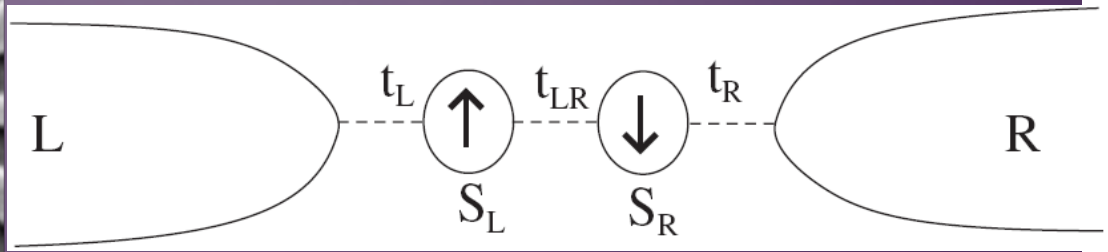


$$H = -hS_0^z - \sum_{i=0}^{\infty}\left[S_i^z S_{i+1}^z + S_i^x\right]$$

- Bulk chain is tuned to critical transverse field
- At h=0, 2 states of "impurity" at i=0 are degenerate
- h is a relevant boundary perturbation that leads to renormalization group (RG) flow to a fixed spin boundary condition at i=0.
- Spin up or down state of boundary spin correspond to inter-impurity singlet or Kondo singlet in 2 impurity Kondo model
- An exact correspondence

Experimental Realizations

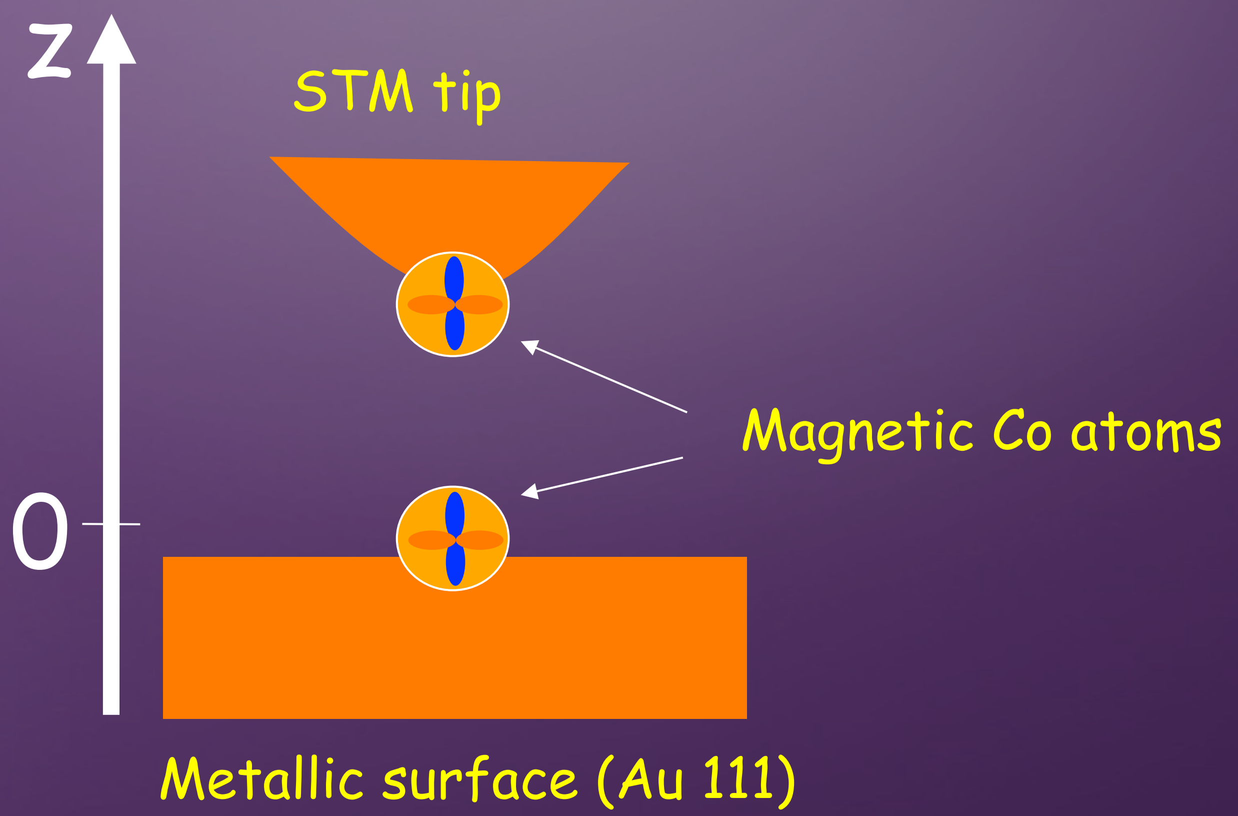




Jeong et al Science '01

- Two gated semi-conductor quantum dots in series between two leads
- Tunable exchange $K \sim t_{LR}^2/U$ where U is dot charging energy
- System has been studied experimentally as a 2-qubit realization

System was recently realized in STM experiment (K. Kern, private communication) with one magnetic atom picked up by the STM tip and one on surface



Exact Critical Behavior

- Connection with Ising model can be used by mapping to 8 Majorana fermions
- First bosonize, then refermionize introducing charge spin, flavour and “spin-flavour” fermions
- Each of these 4 Dirac fermions can then be split into 2 Majorana fermions

- Various fixed points correspond to various simple BC's on Majorana fermions, not original electrons.

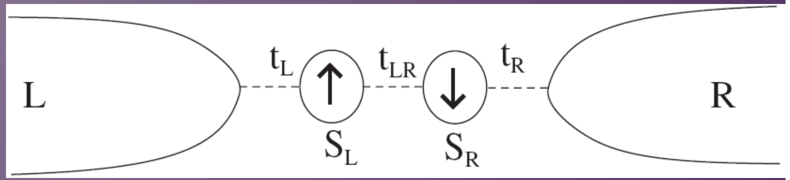
- Zero Kondo coupling: χ_i (out)= χ_i(in)
- QCP: sign change in BC for 1 Majorana component of spin-flavor fermion, χ_1 (out)= -χ_1(in), other BC's unchanged.
- Relevant (K-Kc) interaction at QCP can be written in terms of χ_1 at origin after transforming to scattering basis

- Here $\lambda_1 \propto$ K-Kc
- a is an extra Majorana fermion that lives at x=0 only and flips impurities between interimpurity- and Kondo singlets

- In series double dot set-up there is generally another relevant operator which transmits charge between 2 leads ("channels")

$$\delta H = V_{LR}\psi_1^+(0)\psi_2(0) + h.c.,$$

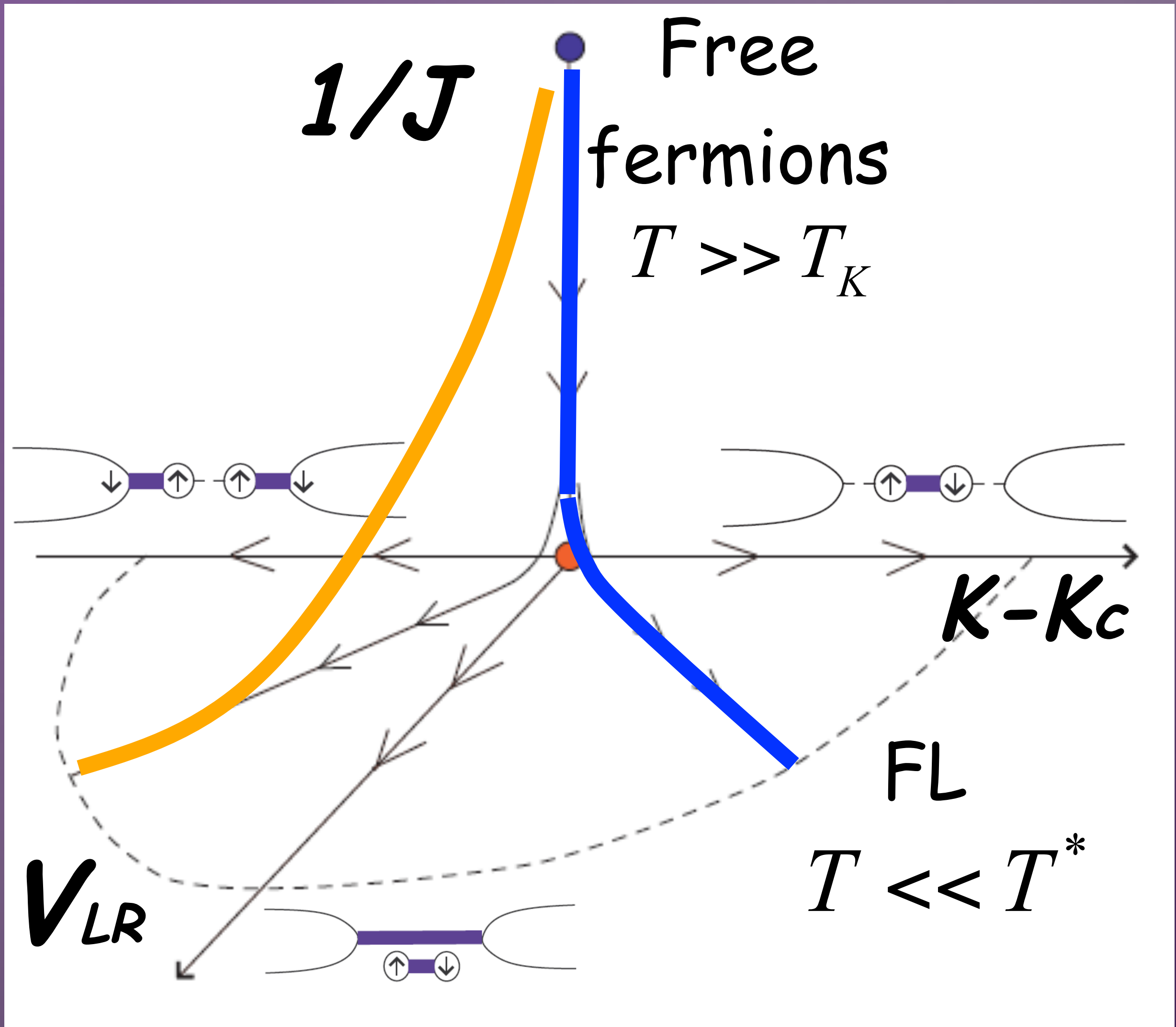
$$V_{LR} \propto t_L t_{LR} t_R / U^2$$



- At QCP this becomes linear in the flavor Majorana fermion,

- Here $\lambda_2 \propto V_{LR}$
- Both relevant interactions have RG scaling dimension ½
- Related to each other by SO(8) rotation

Flow diagram $T_K >> T^*$



Non FL

$T^* << T << T_K$

T^* is a low E crossover scale:

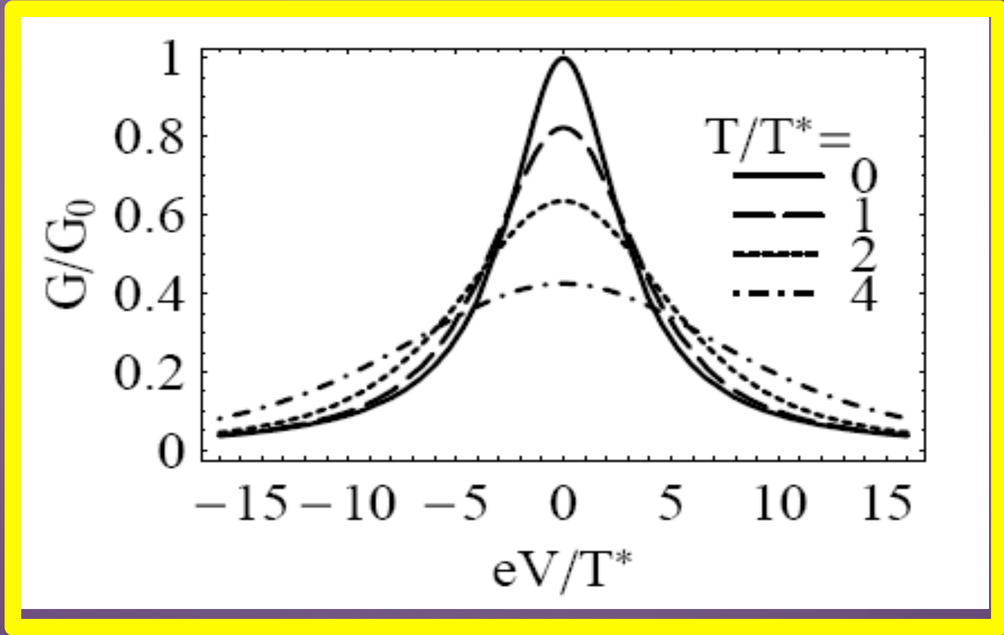
Here ν is density of states; c_i are constants of order 1

Non-equilibrium transport and noise

- The series double dot system is amenable to exact calculations near the critical point since:
- Perturbations are harmonic in χ's and a-fermions
- current operator is quadratic in χ's
- H_{eff} maps onto one studied earlier as a special (and highly unrealistic) limit of a single dot model by Schiller-Hershfield and Komnik-Gogolin, as does current operator
- We may simply borrow their results
- Results become exact when $T^* \ll T_K$

- Conductance at source-drain voltage V and temperature T

where ψ_1 is the trigamma function



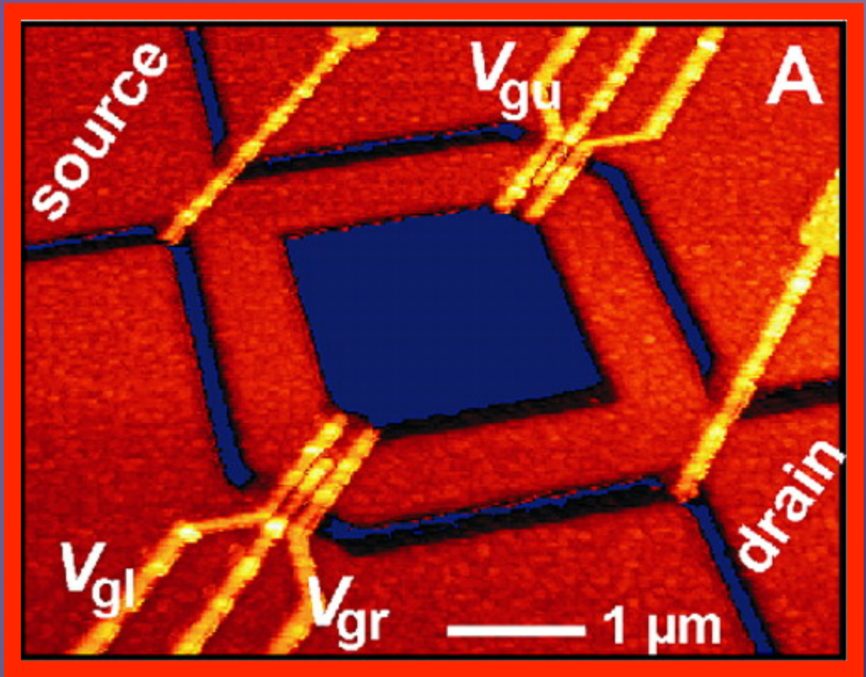
- At T=0 very narrow resonance at the Fermi energy of width T*
- Peak height is $2e^2/h$ at K=Kc
- Note: at larger V~K when T_K~T*, presumably get split peaks as predicted earlier and seen in experiments

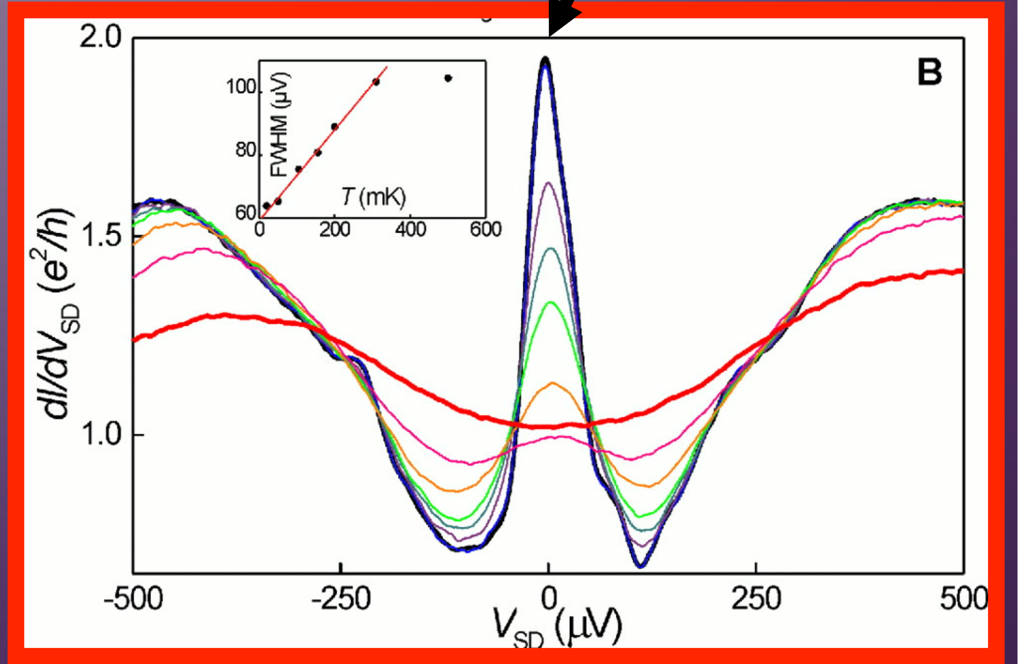
2000

The Kondo Effect in the Unitary Limit

W. G. van der Wiel,[1*] S. De Franceschi,[1] T. Fujisawa,[2] J. M. Elzerman,[1] S. Tarucha,[2,3] L. P. Kouwenhoven[1]

Perfect transmissiton





- We determined exactly the "Full counting statistics", as defined by Levitov and Lesovic: Letting P(Q) be the probability of transmitting charge Q in waiting time τ, the generating function defined by

$$\sum_{Q=-\infty}^{\infty} P(Q)e^{i\mu Q} = \chi(\mu)$$

is given by

with

$$A_1(\varepsilon) = \frac{2T^*_{\delta K}T^*_{LR}}{4\varepsilon^2 + T^{*2}}\left[n_F(\varepsilon)\left(1-n_L(\varepsilon)\right) + n_R(\varepsilon)\left(1-n_F(\varepsilon)\right)\right]$$

$$A_2(\varepsilon) = \frac{{T^*_{LR}}^2}{4\varepsilon^2 + T^{*2}}n_L(\varepsilon)\left(1-n_R(\varepsilon)\right)$$

$$n_F(\varepsilon) = 1/(e^{\varepsilon/T}+1), n_{L/R}(\varepsilon) = n_F(\varepsilon \pm eV)$$

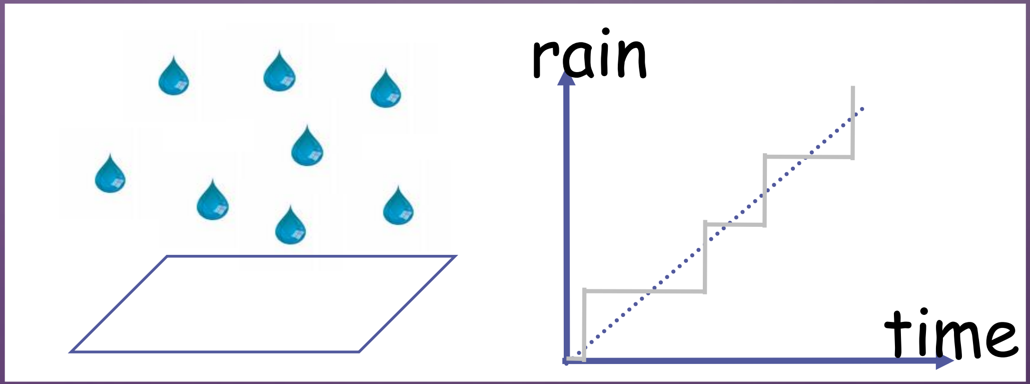
with A_{-n} obtained from A_n by switching L and R.

n=±2 terms signal two particle processes

From $\chi(\mu)$ the current I, current fluctuations S and higher cumulants can be extracted:

$$I = -ie \frac{\partial \ln \chi}{\partial \mu}\bigg|_{\mu=0} \bigg/ \tau$$

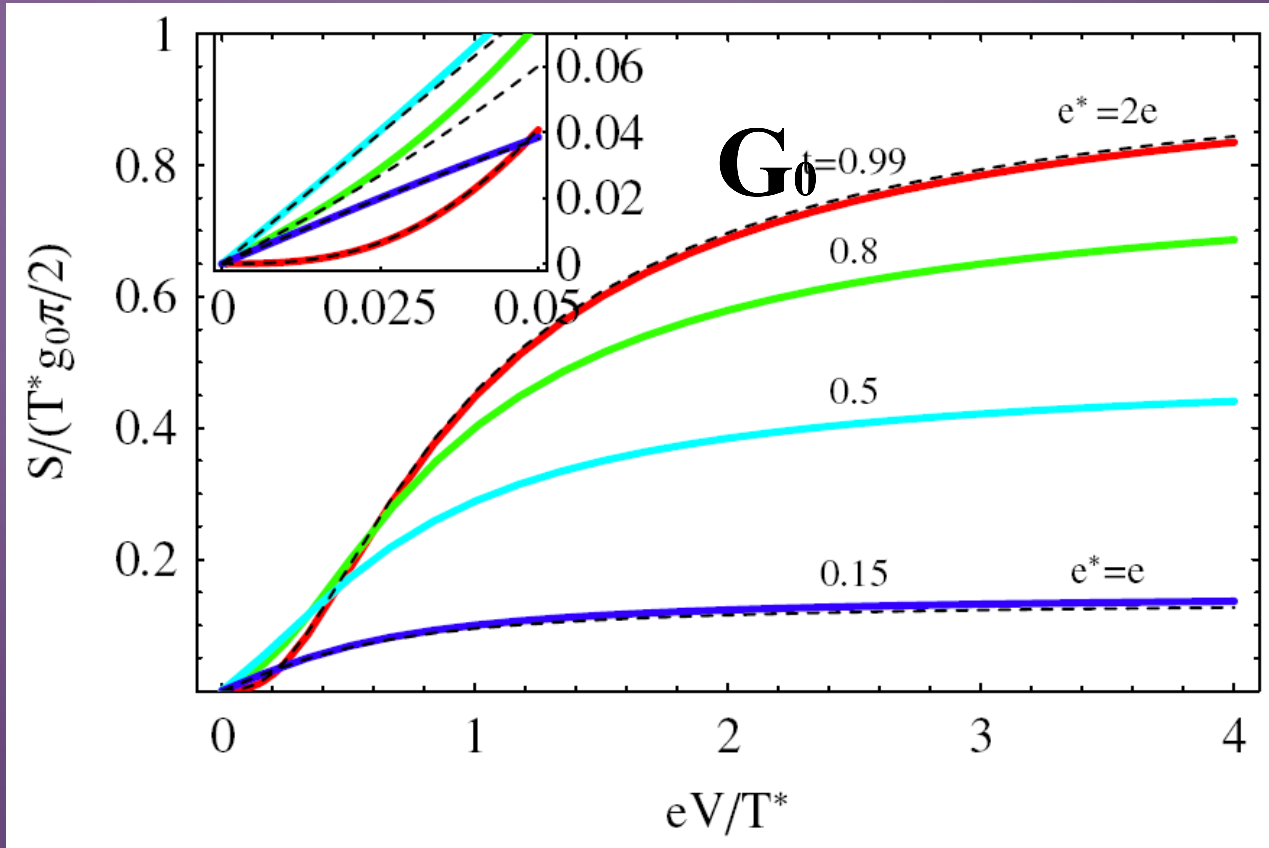
Current fluctuations at eV>>T → shot noise



Effective charge, e*, is often defined by:

Where t(V)=(h/2e^2)dI/dV

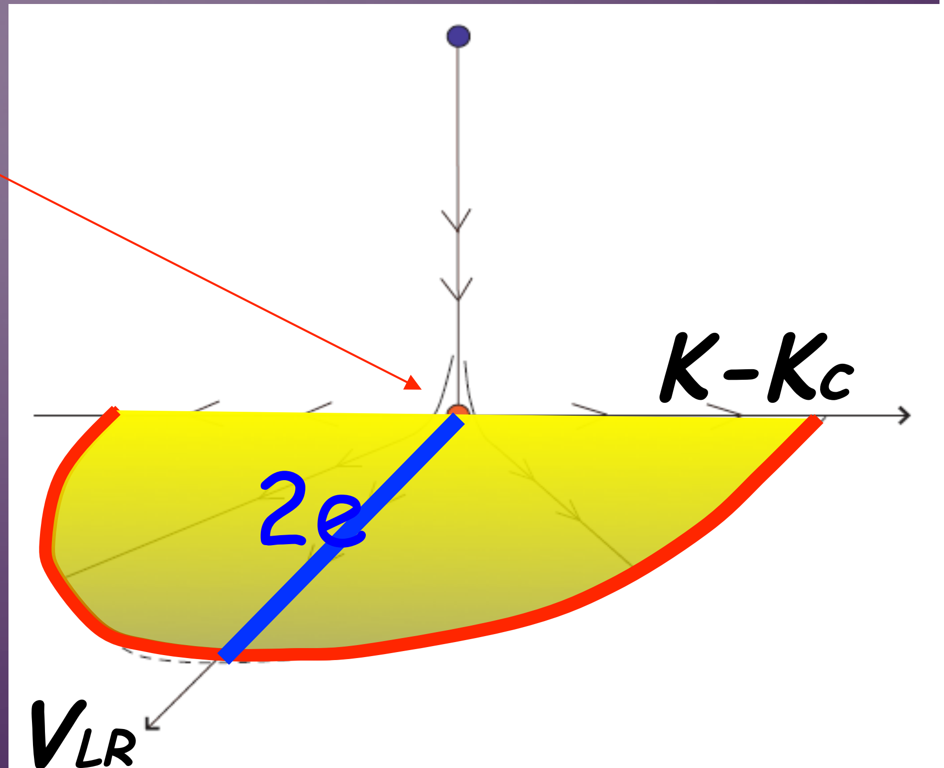
This definition is motivated by noise for non-interacting electrons noise



- Close to critical point (K=Kc, G_0=1) fit with e*=2e coincides with exact formula
- "Pairing" follows from interaction mediated by fliping between interimpurity- and many body Kondo singlets

Fermi Liquid theory

SO(8)→SO(7)



Where $J_j = \Psi_j^* \Psi_j, \vec{J}_j = \Psi_j^* \frac{\sigma}{2} \Psi_j$

$e^* = 2e$

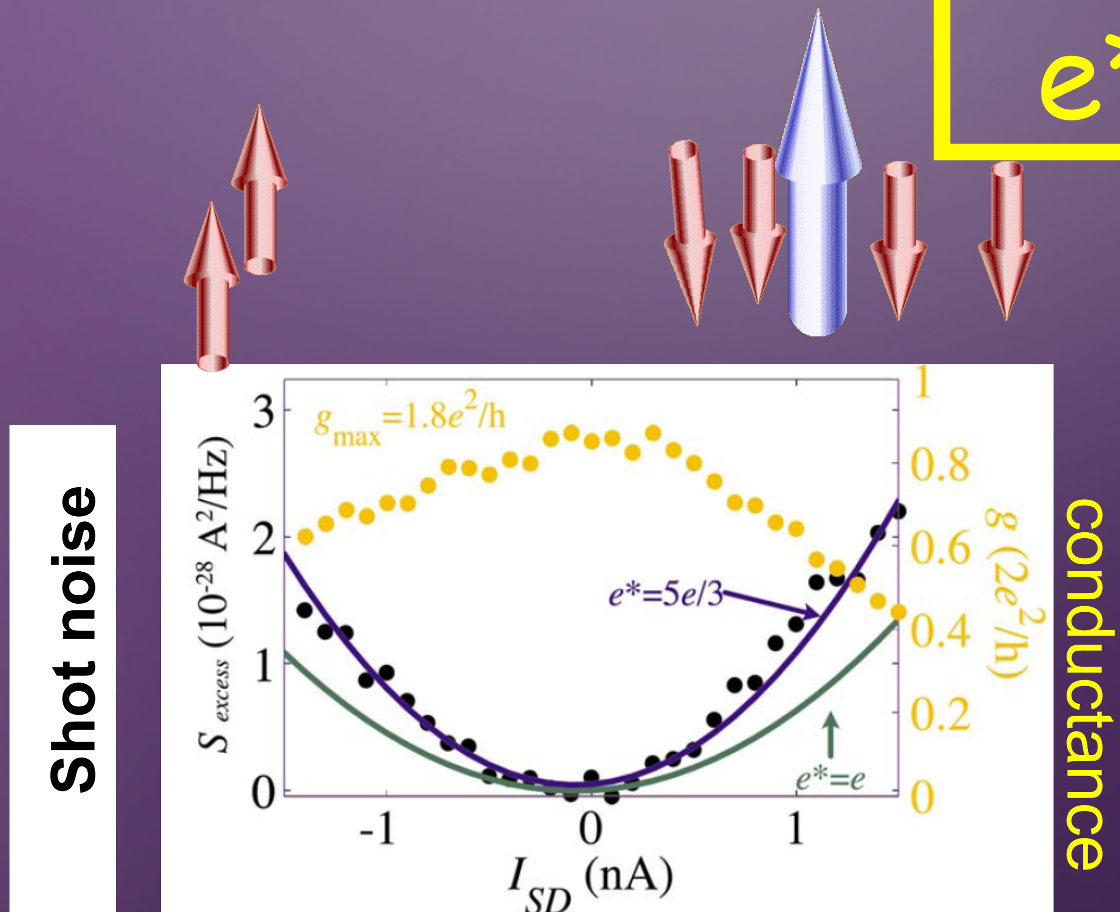
Sela, Affleck PRL 103, 087204 (2009)

PRL 97, 086601 (2006) PHYSICAL REVIEW LETTERS week ending 25 AUGUST 2006

Fractional Shot Noise in the Kondo Regime

Eran Sela,[1] Yuval Oreg,[1] Felix von Oppen,[2] and Jens Koch[2]

$e^*=5/3e$



Zarchin, Zaffalon, Heiblum,Mahalu, Umansky PRB 2007

SU(N) Generalizations: Mora, Leyronas, Regnault PRL 2008;
Vitushinsky, Clerk, Lehur PRL 2008
Delattre et. Al. Nature Physics 2009

Conclusions

- Universal behavior of 2 impurity Kondo model can be described exactly by mapping to free Majorana fermions
- Condition for critical behavior: $T_K >> T^*$

- Charge tunneling relevant operator is equally well treated within this formulation

- The voltage-biased double dot in series can be solved exactly

- Method can be used to calculate the Green functions at equilibrium with applications to two channel Kondo experiment