Current correlations in the self-dual Interacting Resonant Level Model

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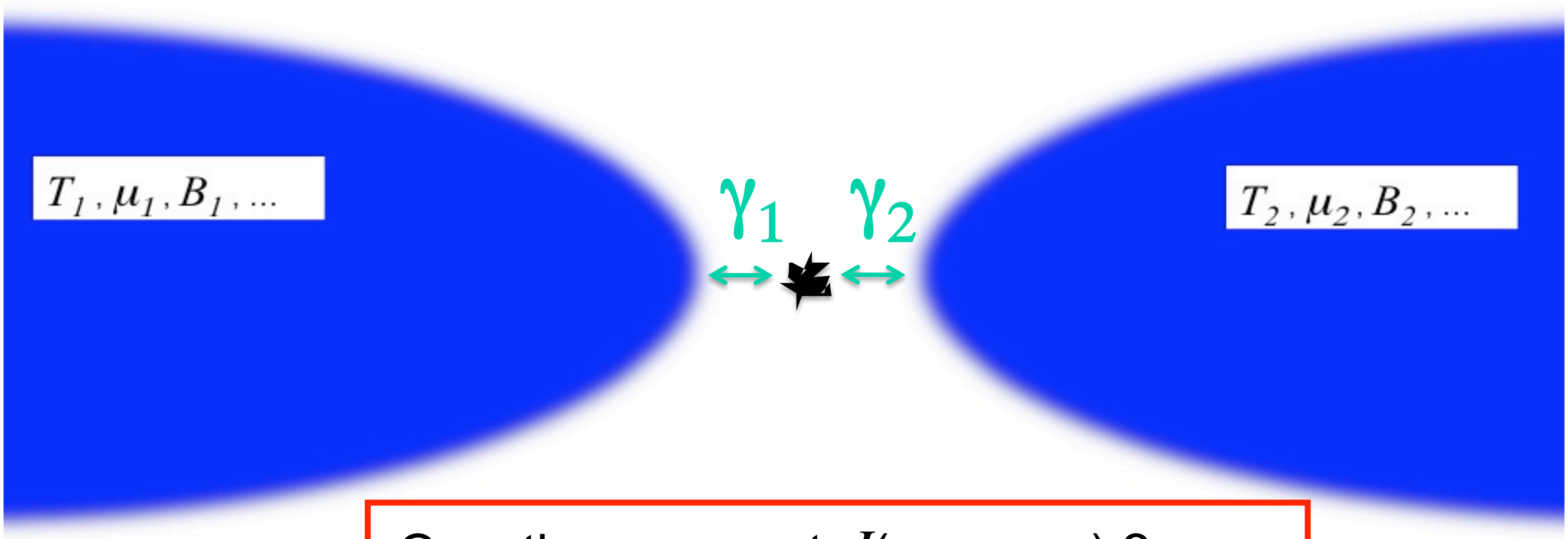
Impurity out-of-equilibrium

- Several baths (macroscopic, at equilibrium)
- Flow (of charge, spin, energy, …) through impurity
- Steady state



Questions: current $I(\mu_1,\mu_2,\ldots)$?
fluctuations $\Delta I(\mu_1,\mu_2,\ldots)$?
.....

Perturbative treatment

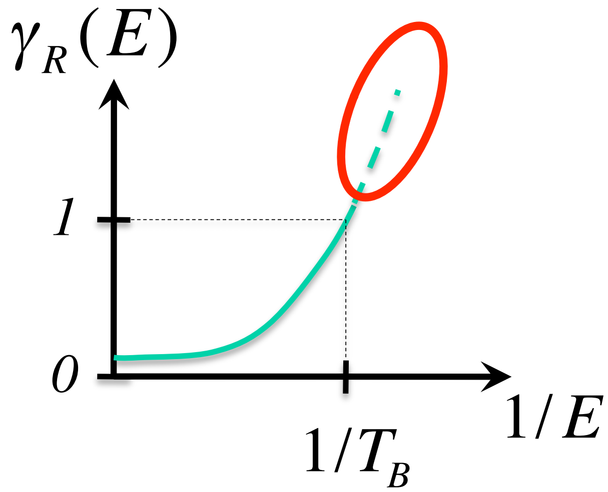
- Keldysh method:
  - allows for a formal expression of the out-of-equilibrium density matrix

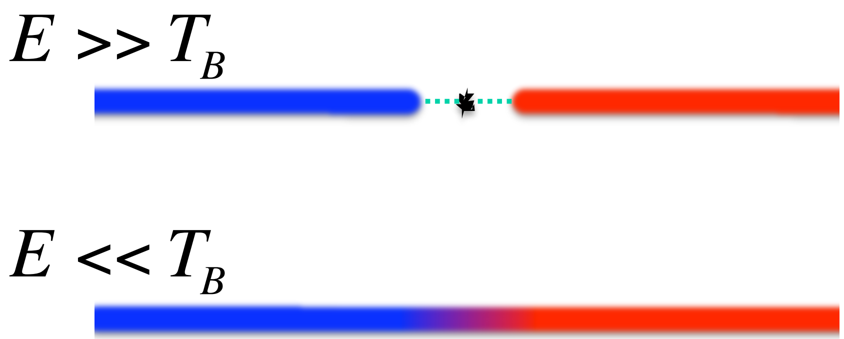
$$\hat{\rho}(t) = \mathcal{U}(0,t)\ \hat{\rho}(0)\ \mathcal{U}(0,t)^{-1}$$

$$\mathcal{U}(0,t) = \mathcal{P}\ e^{-i\gamma\int_0^t dt'\, H_I(t')}$$

  - but how to evaluate/resum the perturbative expansion?

    Crucial in the strong coupling regime





Perturbative treatment

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$$\hat{\rho}(t) = \mathcal{U}(0,t)\ \hat{\rho}(0)\ \mathcal{U}(0,t)^{-1}$$

$$\mathcal{U}(0,t) = \mathcal{P}\, e^{-i\,\gamma \int_0^t dt'\, H_I(t')}$$

  - but how to evaluate/resum the perturbative expansion?
    Crucial in the strong coupling regime

- How to control approximate methods?
  - non-equilibrium RG
  - truncated EOM
  - etc…

⇒ Need for non-perturbative methods

Non-perturbative approaches

- Dressed TBA (Quantum Hall edge states tunneling)
  (P.Fendley, A.W.W.Ludwig, H.Saleur 1995)

- Map to equilibrium problem (boundary sine Gordon model)
  (V.Bazhanov, S.Lukyanov, A.B.Zamolodchikov 1999)

- Effectively non-interacting systems (free fermions)
  - 1-ch Kondo (A. Schiller, U. Hershfield 1998) } Toulouse point
  - Lüttinger L. (A. Komnik, O. Gogolin 2003) } Toulouse point
  - 2-ch Kondo (E. Sela, I. Affleck 2009) QCP & vicinity

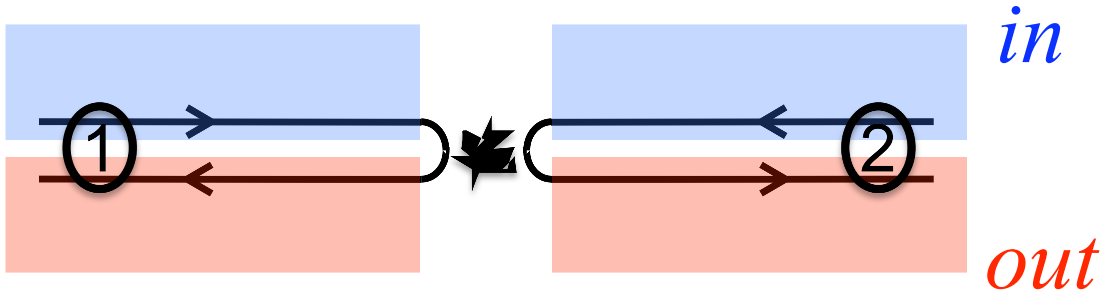
- Scattering Bethe Ansatz (IRLM)
  (P.Mehta, N.Andrei 2006)

Outline

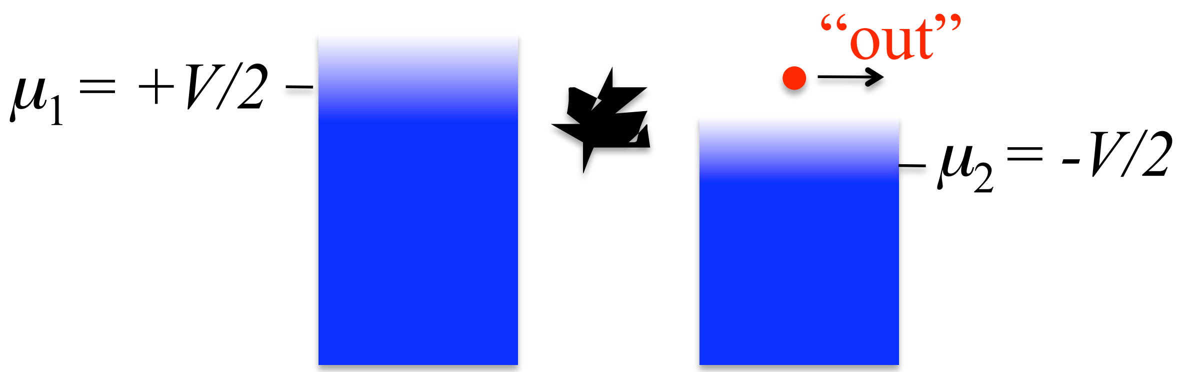
- Scattering approach (“dressed TBA”)
  - Why ?
  - When ?
- Interacting Resonant Level Model:
  - Self dual point
  - Current & Noise
- Time dependent DMRG

Modeling the electrodes

- Mapping to 1D
  - linearization around k_F
  - « ***in*** » and « ***out*** » modes

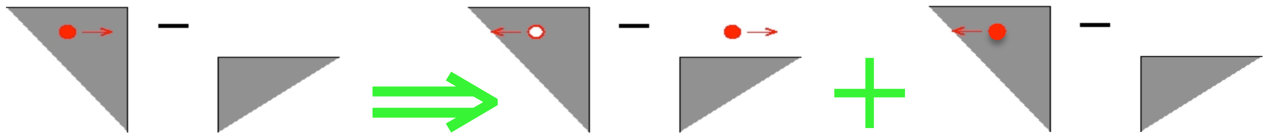


- What is a bath ?
  - A reservoir of **incoming** electrons with definite:
    - temperature
    - chemical potentials μ_1, μ_2
  - Outcoming electrons:
    - They are **not** thermalized right after the impurity.
    - Hypothesis (« good baths ») : their (non-equilibrium) distribution don't affect incoming electrons.



No interactions

- Incoming particle can be either transmitted or reflected



- Generalizes to N-particle states (factorization)

➔ Landauer-Büttiker formula: $$I = \int dE \, \big(f_1(E) - f_2(E)\big) \, T(E)$$

Fermi functions
for **in-coming electrons**
in wires (1) and (2)

transmission probability

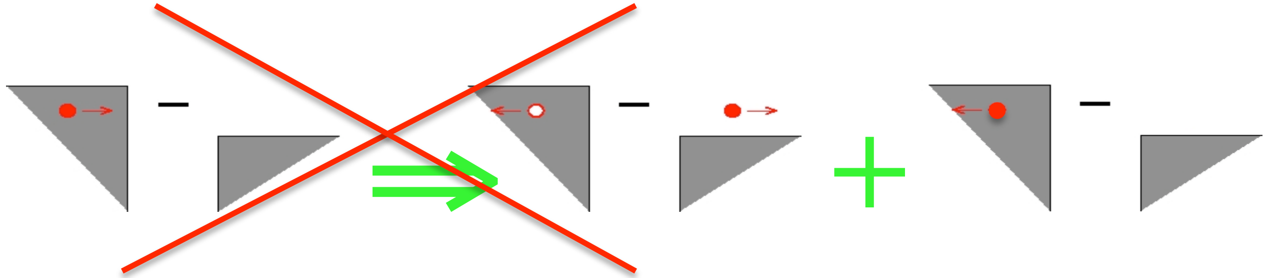
$$T = |R_{12}|^2$$

scattering states:

$$\|1\rangle = |1\rangle^{in} + R_{11}|1\rangle^{out} + R_{12}|2\rangle^{out}$$

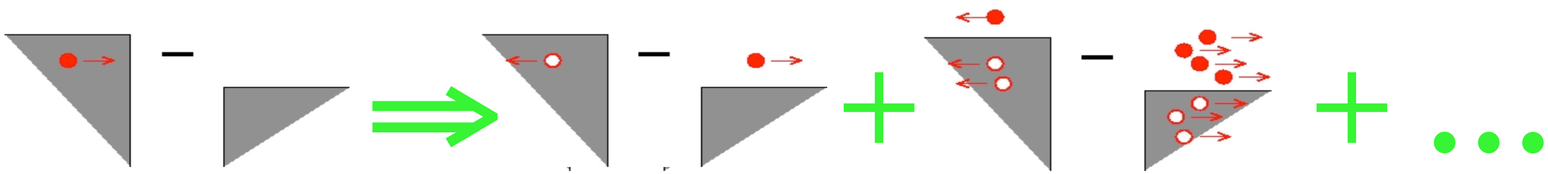
$$\|2\rangle = |2\rangle^{in} + R_{21}|1\rangle^{out} + R_{22}|2\rangle^{out}$$

With interactions



- There is **particle production**
- N-particle state ….

$$I = \int dE \left(f_1(E) - f_2(E)\right) T(E)$$

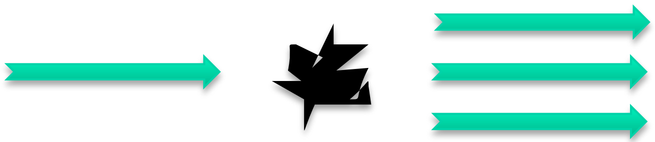


Scattering approach

1. Build the scattering states:

- N-body electronic states diagonalizing $H = H_0 + H_I$
- Relies on integrability of H ("**equilibrium property**")
- No particle production
- Factorization

$|\alpha,p\rangle^{in}$



$R_{\alpha\beta}(p)\,|\beta,p\rangle^{out}$

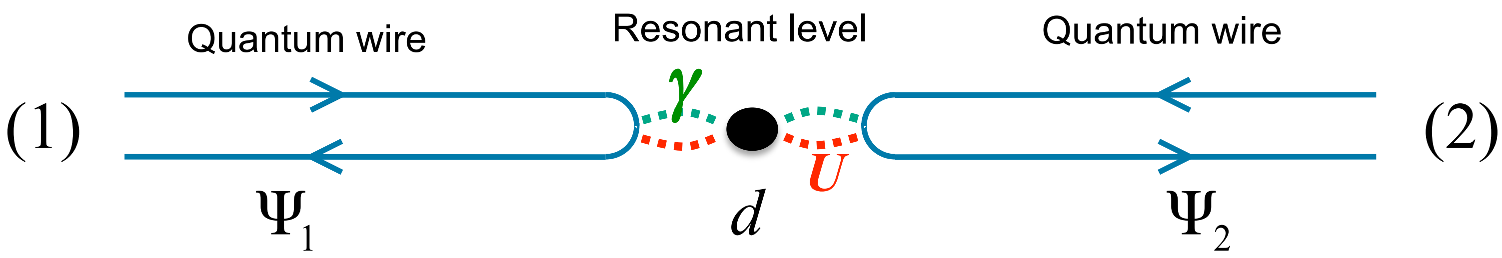
$$\|\alpha,p\rangle\rangle = |\alpha,p\rangle^{in} + R_{\alpha\beta}(p)\,|\beta,p\rangle^{out}$$

2. Impose a voltage

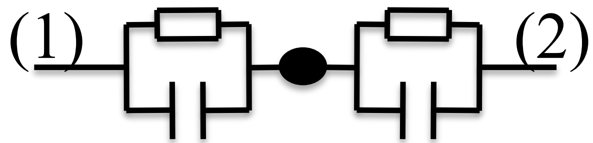
- Represent incoming Fermi seas in the basis of scattering states
- Determine the proper distribution for incoming quasiparticles $\rho_\alpha(p,V)$
- Non-trivial condition !!! (severe requirement)

IRLM

Interacting Resonant Level Model



Hamiltonian: $H = H_0 + H_I$



$$H_0 = -iv_{\mathrm{F}} \sum_{a=1,2} \int_{-\infty}^{\infty} dx \;\; \Psi_a^\dagger \partial_x \Psi_a(x)$$

electrodes

$$H_I = \left(\gamma_1 \Psi_1^\dagger(0) + \gamma_2 \Psi_2^\dagger(0) \right) d + \mathrm{h.c.} + U \left(:\Psi_1^\dagger \Psi_1:(0) + :\Psi_2^\dagger \Psi_2:(0) \right) \left(d^\dagger d - \tfrac{1}{2} \right) + \varepsilon_d \, d^\dagger d$$

tunnelling

Coulombic repulsion

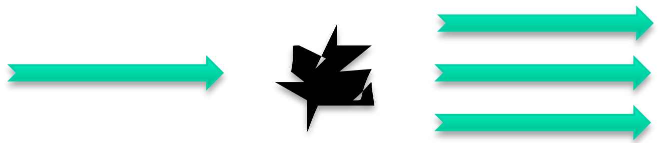
Gate voltage

$\varepsilon_d = 0$ at resonance

Route

1. Build the scattering states

$$|\alpha,p\rangle^{in}$$



$$R_{\alpha\beta}(p)\,|\beta,p\rangle^{out}$$

2. Impose a voltage ←→ represent biased Fermi seas

Scattering states

$H(\Psi)$

even/odd basis

$$\Psi_e = \tfrac{1}{\gamma}(\gamma_1\Psi_1 + \gamma_2\Psi_2)$$
$$\Psi_o = \tfrac{1}{\gamma}(\gamma_2\Psi_1 - \gamma_1\Psi_2)$$

$$\Psi_a \propto e^{i\sqrt{4\pi}\,\varphi_a}$$

bosonization

Unitary transformation: cancels interaction with d^+d

$$\mathcal{U} = e^{i(U/\sqrt{\pi})\, d^+ d(\varphi_e + \varphi_o)(0)}$$

Change of basis

$$\phi_+ = \frac{1}{\sqrt{8\pi^2 D}}\left((2\pi - U)\varphi_e - U\varphi_o\right)$$
$$\phi_- = \frac{1}{\sqrt{8\pi^2 D}}\left((2\pi - U)\varphi_o + U\varphi_e\right)$$

anisotropic Kondo model

$$H = H_0(\phi_+) + H_0(\phi_-) + H_I$$
$$H_I = \gamma e^{i\sqrt{8\pi D}\,\phi_+(0)} d^+ + \text{h.c.}$$

Self dual point $U=U^*$

→ Scaling dimension $D(U^*)=1/4$

→ Enhanced SU(2) symmetry in the bulk

$$J^z = \tfrac{1}{2}(\Psi_e^\dagger\Psi_e - \Psi_o^\dagger\Psi_o) \qquad J^+ = \Psi_e^\dagger\Psi_o$$

Scattering modes $A_\alpha(p)$ (sine Gordon $\beta = \sqrt{2\pi}$):

2 solitons $A_\pm(p)$ charge ±2

2 breathers $A_{0,1}(p)$ charge 0

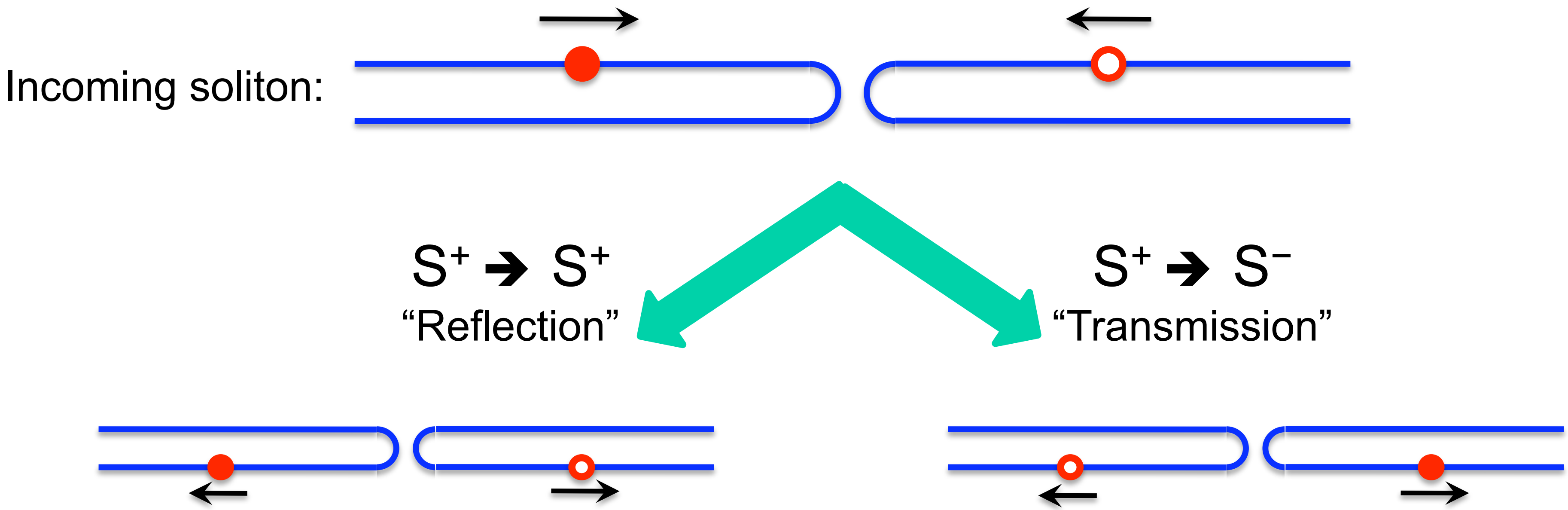
$$R_{\alpha\beta}(p) = \begin{pmatrix} \frac{1}{2}(R_1^K + P^K) & \frac{1}{2}(R_1^K - P^K) & 0 & 0 \\ \frac{1}{2}(R_1^K - P^K) & \frac{1}{2}(R_1^K + P^K) & 0 & 0 \\ 0 & 0 & P^K & 0 \\ 0 & 0 & 0 & R_0^K \end{pmatrix} \begin{matrix} + \\ - \\ 1 \\ 0 \end{matrix}$$

$$P^K = \tanh\left(\ln\sqrt{\tfrac{p}{T_B}} - \tfrac{i\pi}{4}\right)$$

$$R_\alpha^K = \frac{\tanh\left(\ln\sqrt{\frac{p}{T_B}} - \frac{i\pi(2-\alpha)}{12}\right)}{\tanh\left(\ln\sqrt{\frac{p}{T_B}} + \frac{i\pi(2-\alpha)}{12}\right)}$$

A caricature of solitons

- Charge carriers: solitons (S^+) and antisolitons (S^-)
- Charges:
  - Globally neutral: $Q_1 + Q_2 = 0$
  - Relative charge: $Q_1 - Q_2 = \pm 2e$



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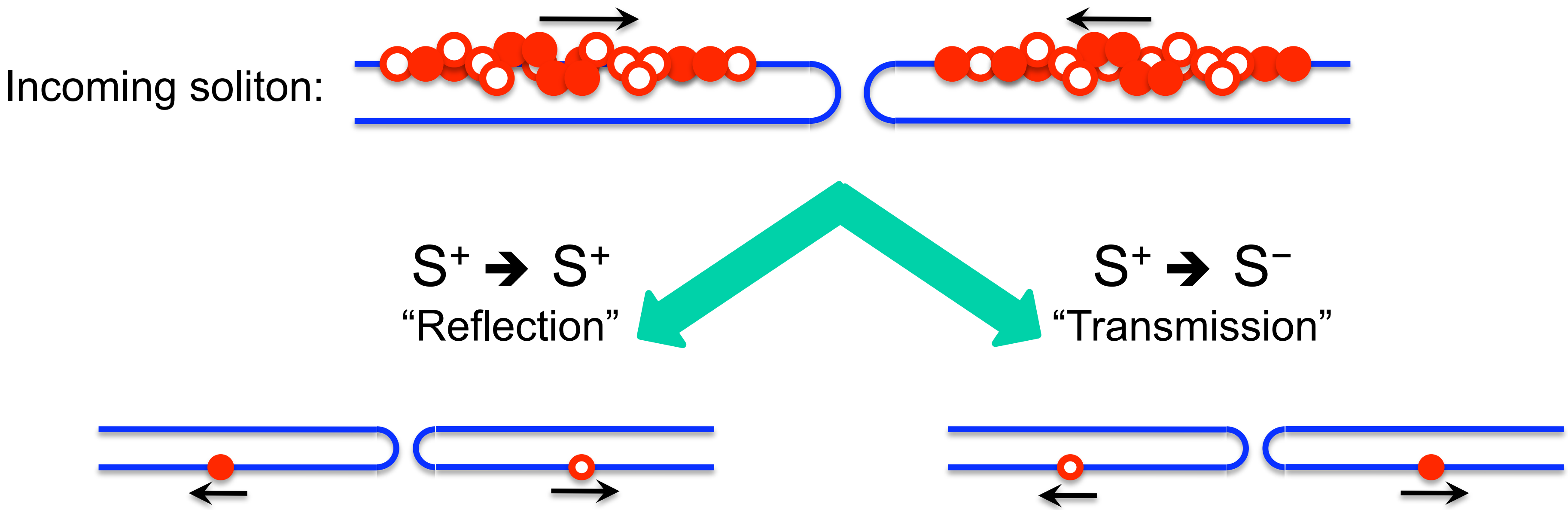
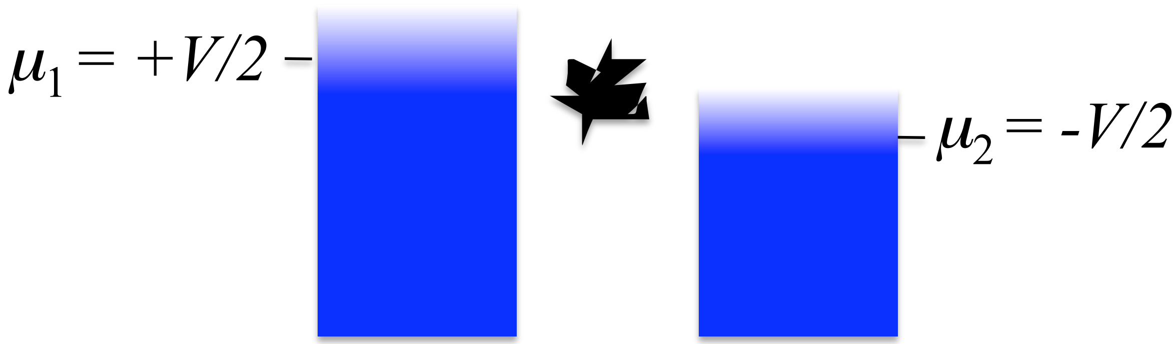


Route

1. Build the scattering states

2. Impose a voltage ←→ represent biased Fermi seas



Density matrix

2. Represent **incoming** Fermi seas in the basis of scattering states:

$$\hat{\rho} \propto \exp[-\tfrac{1}{k_B T}(H + Y)]$$

Hersfield's operator

Incoming fields: don't feel the impurity

$$Y = \frac{V}{2}\int_{-\infty}^{0} dx\,(\Psi_1^+\Psi_1 - \Psi_2^+\Psi_2) = \frac{V}{2}(N_1^{in} - N_2^{in})$$

Represent Y in the q.p. basis: in general: particle production !

Sufficient condition to avoid this: it is a one-particle operator:

$$Y.A_\alpha^{+,in}(p) = Y_{\alpha\beta}(p)\,A_\beta^{+,in}(p)$$

→ At the self-dual point, Y is the **charge :** diagonal !

$$Y.A_\alpha^{+,in}(p) = \frac{q_\alpha V}{2}\,A_\alpha^{+,in}(p) \qquad q_\alpha = 0, \pm 2$$

Y operator

- Closed expressions for the Y operator:

  – In terms of the "*in*" algebra:

$$\mathcal{J}^a(p) = \sum_{\alpha,\beta=0,\pm} A_\alpha^+(p)\ [J^a]_{\alpha\beta}\ A_\beta(p)$$

$$\mathcal{J}^z(p) = A_+^+(p)A_+(p) - A_-^+(p)A_-(p)$$

$$\hat{Y} = V\,\mathcal{J}^{z,in} = V\int dp\ \mathcal{J}^{z,in}(p)$$

$$\Rightarrow \quad \left\langle A_\alpha^{+,in}(p)A_\beta^{in}(p')\right\rangle = \delta_{\alpha\beta}\ \delta(p-p')\rho_\alpha(p,V)$$

  – In terms of the "*out*" algebra: NON DIAGONAL CORRELATIONS!
  – In terms of electrons: NON LOCAL!

$$\hat{Y} = V\int dp\ \ h^a(p)\ \mathcal{J}^{a,out}(p)$$

$$\Rightarrow \quad \left\langle A_\alpha^{+,out}(p)A_\beta^{out}(p')\right\rangle = \delta(p-p')\ R_{\alpha\gamma}^+(p)\rho_\gamma(p,V)R_{\gamma\beta}(p)$$

Observables

- Current:

$$\hat{I} = Q^{in} - Q^{out} = \int dp \sum_{\alpha} q_\alpha (A_\alpha^{+,in}(p) A_\alpha^{in}(p) - A_\alpha^{+,out}(p) A_\alpha^{out}(p))$$

→ recover Landauer Buttiker formula:

$$\langle I \rangle = \sum_\alpha \int dp \;\; \rho_\alpha(p) \;\; \tau_\alpha^{-1}(p)$$

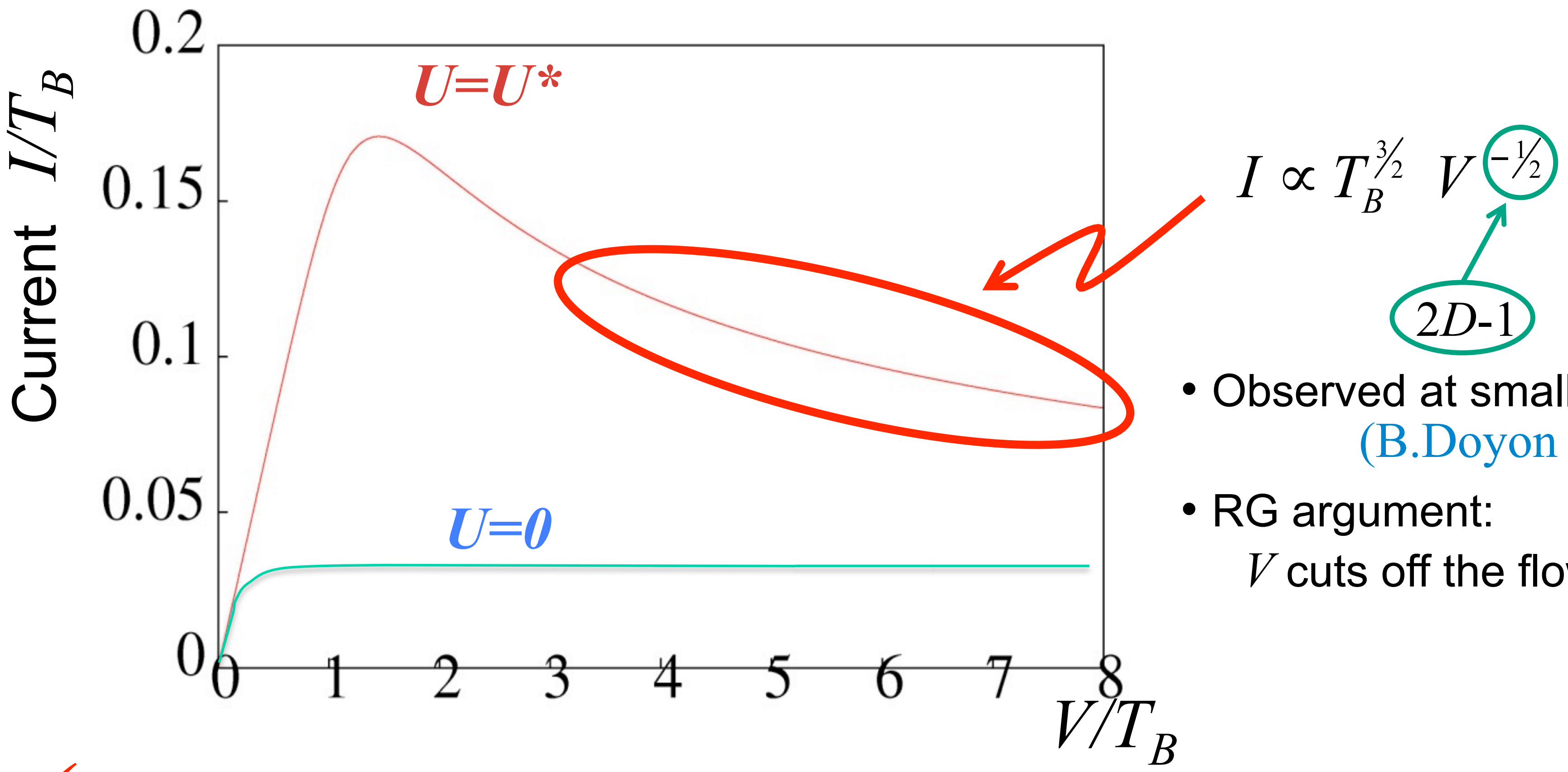
Charge transfer rate

$$\tau_\alpha^{-1}(p) = \sum_\beta (q_\alpha - q_\beta) \left| R_{\alpha\beta}(p) \right|^2$$

- DC Noise:

$$\hat{S} = \left(\hat{I} - \langle \hat{I} \rangle \right)^2 = \ldots$$

I(V) curve (T=0)



- Observed at small U (B.Doyon 2007)
- RG argument: V cuts off the flow

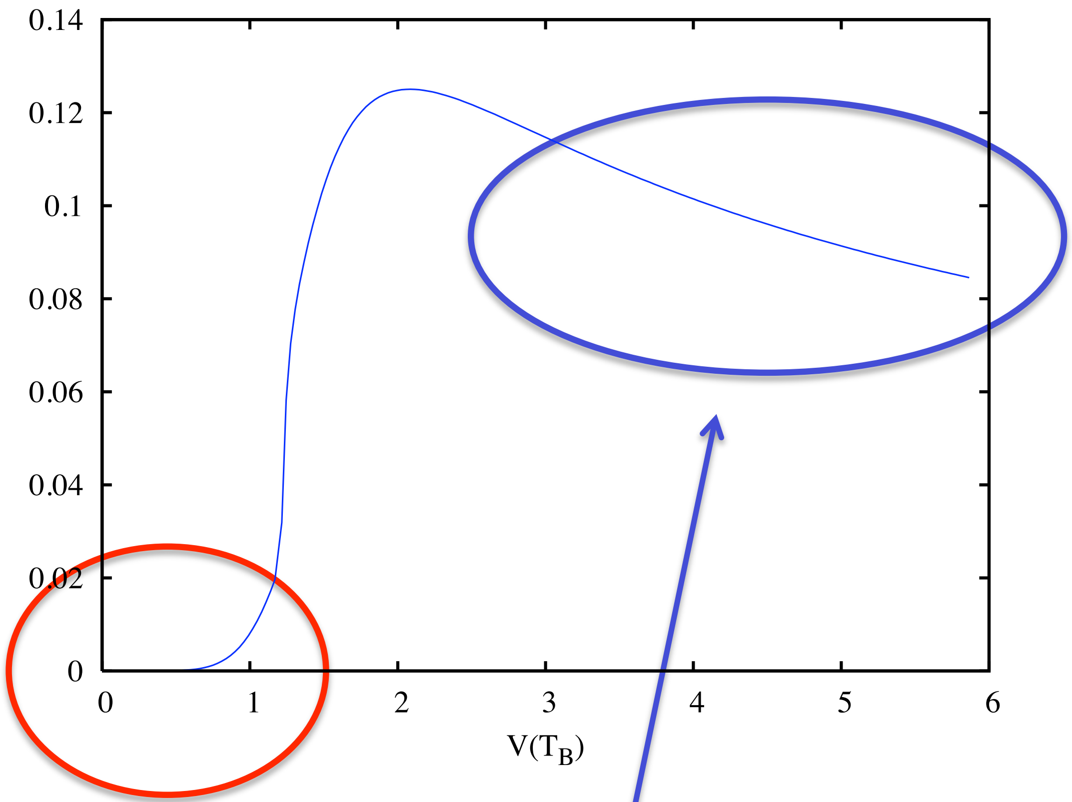
✓ **Universal** curve: $I/T_B = f\,(V/T_B)$

✓ Current enhancement for $U>0$ → **"Coulomb deblocking"**

✓ Large V decrease of the current → **effect of correlations of q.p.'s**

DC noise (T=0)

$$S_{\omega=0} = \int dt \left\langle I(t)I(0) \right\rangle - \left\langle I^2 \right\rangle$$



$$S_{\omega=0} = 2e\,(G_0 V - I)\ \ (1 + \mathcal{O}(V^6)) = \mathcal{O}(V^7)$$

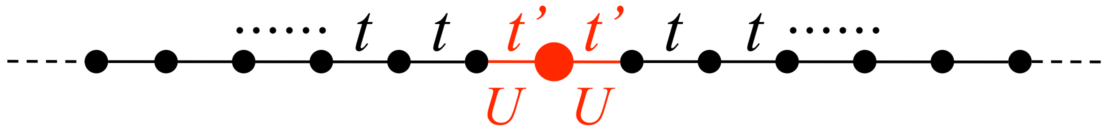
Back-scattered solitons

$$S_{\omega=0} = \frac{e}{2} I\ \ (1 + \mathcal{O}(V^{-3/2}))$$

Non-interacting electrons through double barrier

Numerical approach

- Lattice model



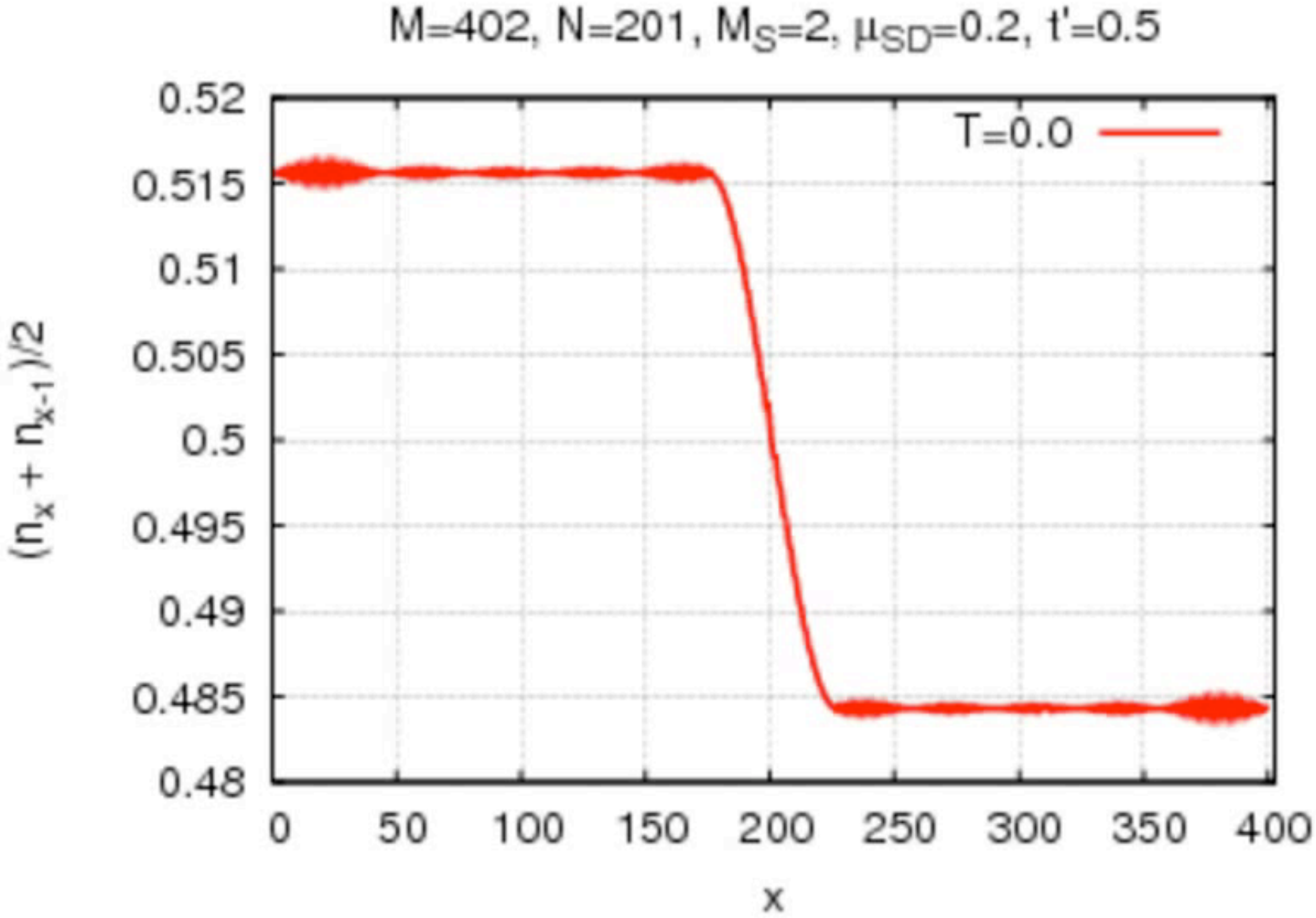
- Time-dependent DMRG

  (S.White, A.Feiguin 2004, P.Schmitteckert 2004)

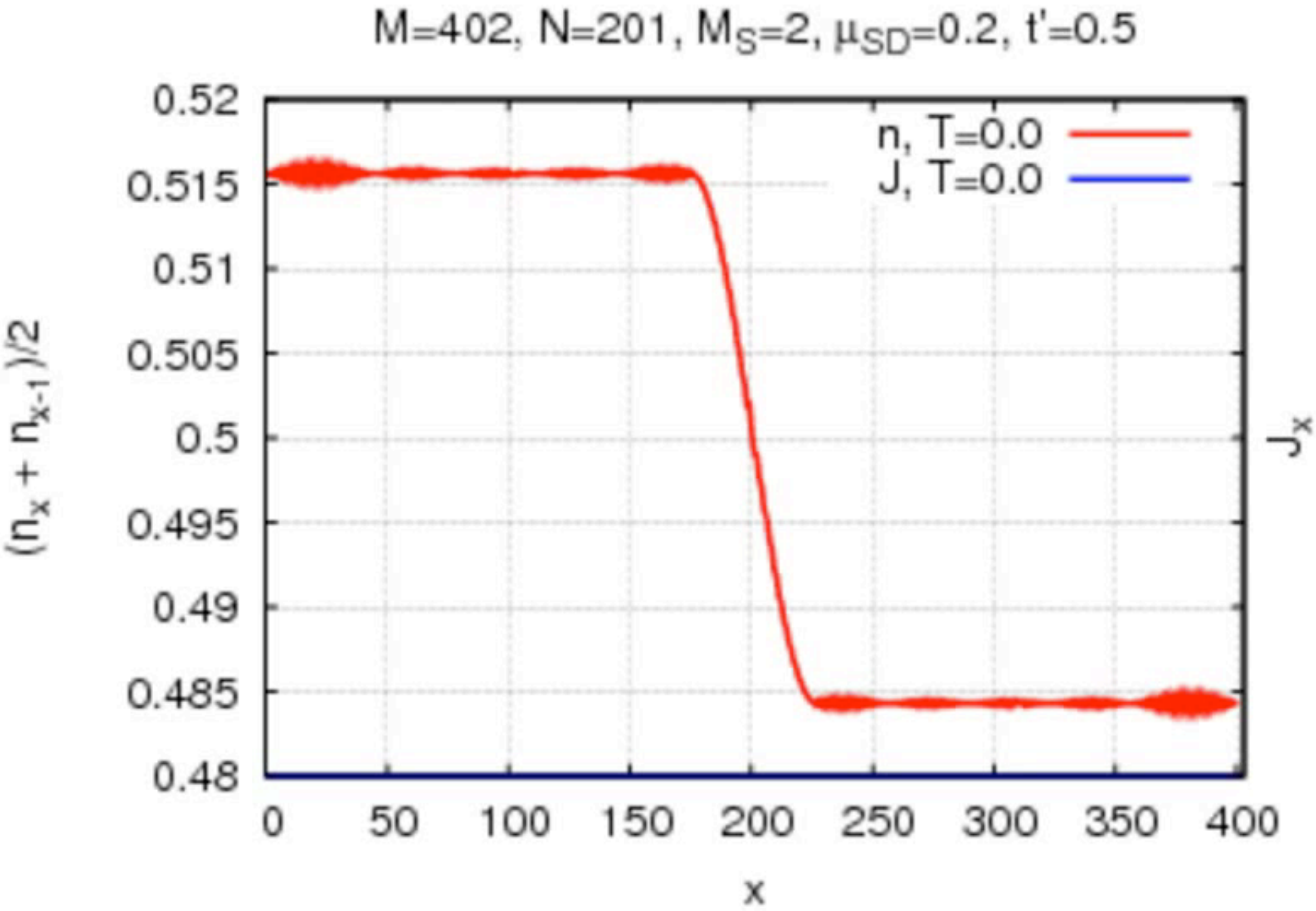
  – Initial state ($t<0$): prepare the electrodes at different chemical potentials $\pm V/2$
  – Switch off the voltage at $t=0$
  – Time-evolve using the full interacting Hamiltonian H_0+H_I

    (duration $\Delta t < L_{\mathrm{lead}} / v_{\mathrm{F}}$)
  – Extrapolate to infinite size

Electronic density



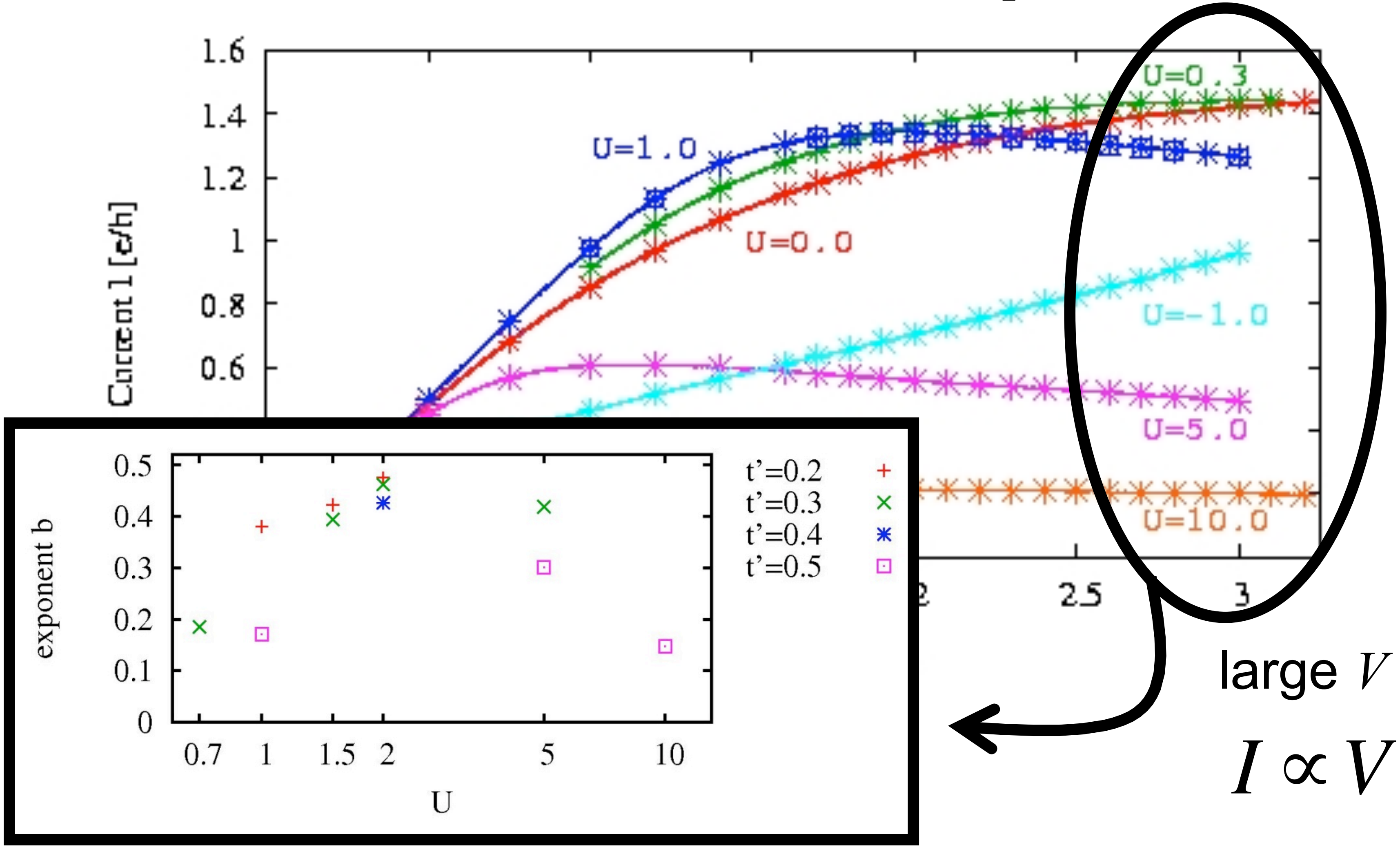
Current



M=402, N=201, M_S=2, μ_SD=0.2, t'=0.5
n, T=0.0
J, T=0.0
(n_x + n_x-1)/2
J_x
x
0.52
0.515
0.51
0.505
0.5
0.495
0.49
0.485
0.48
0
50
100
150
200
250
300
350
400

I(V) (T=0)

M=96 sites ; N=2000 states kept ; t'=0.5

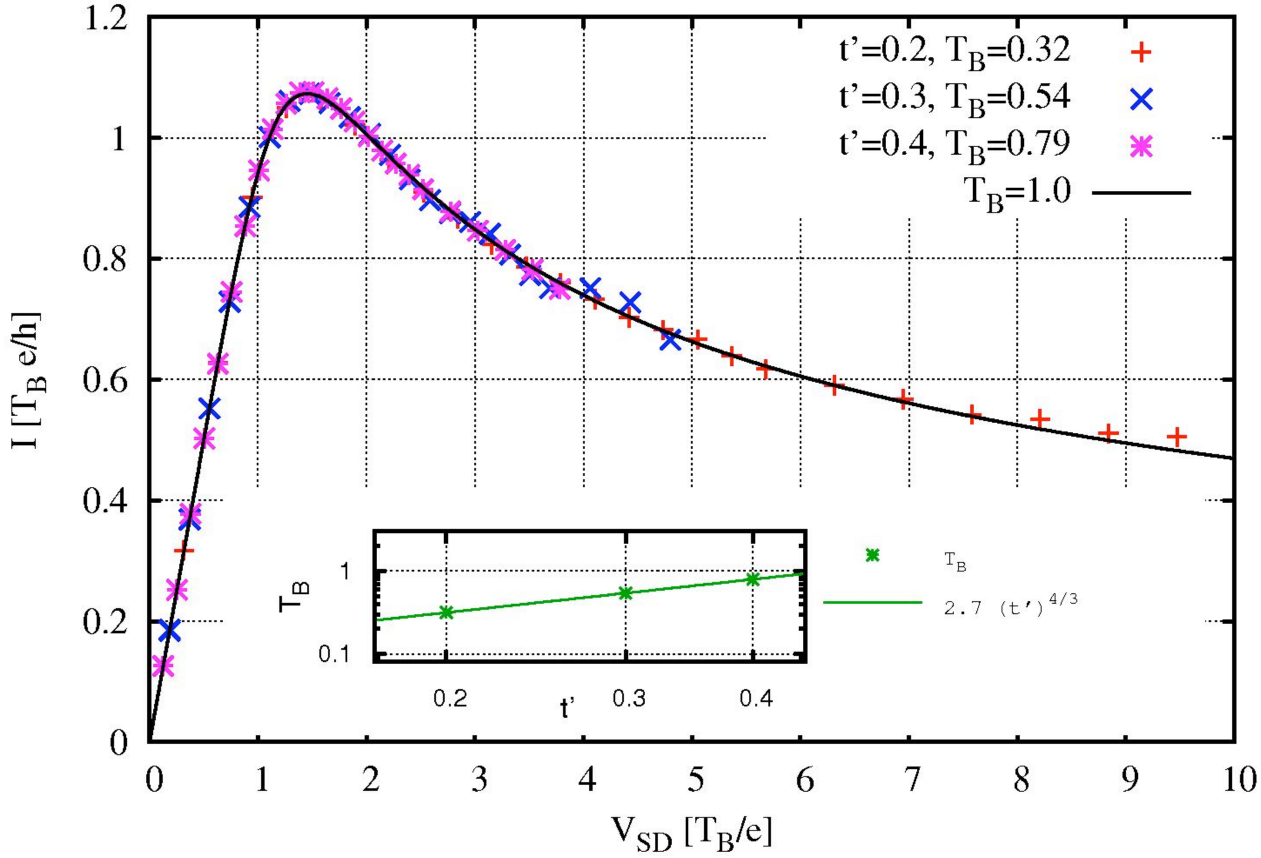


large V fit:

$$I \propto V^{-b}$$

Self-dual point

Fitting parameter: hybridization temperature T_B



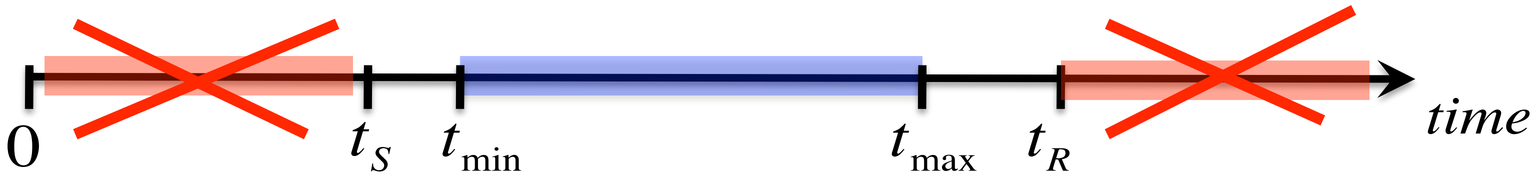
M=96 sites ; N=2000 states kept ; *U=2*

Noise - tdDMRG

- Need two-point function

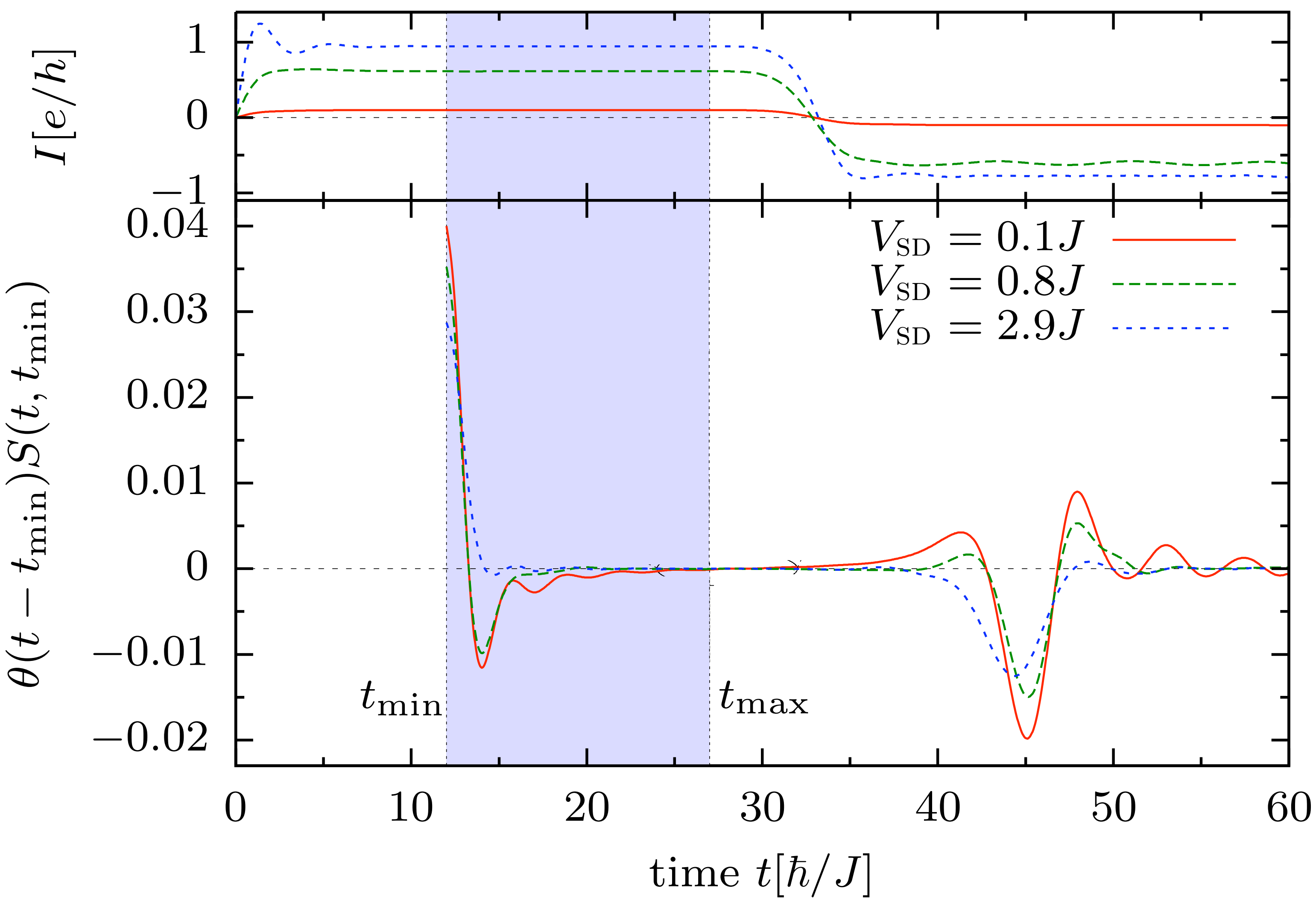
$$S(t_1,t_2) = \left\langle \hat{I}(t_1)\hat{I}(t_2) \right\rangle - \left\langle \hat{I}(t_1) \right\rangle \left\langle \hat{I}(t_2) \right\rangle$$

- Quasi-stationnary state:
  - No time translation invariance!
  - Restriction on time-window:
    - Settling time t_S (transient regime, damped oscillation of the current)
    - Recurrence time $t_R \propto L$ (finite size, wave packet bounces on the wall)



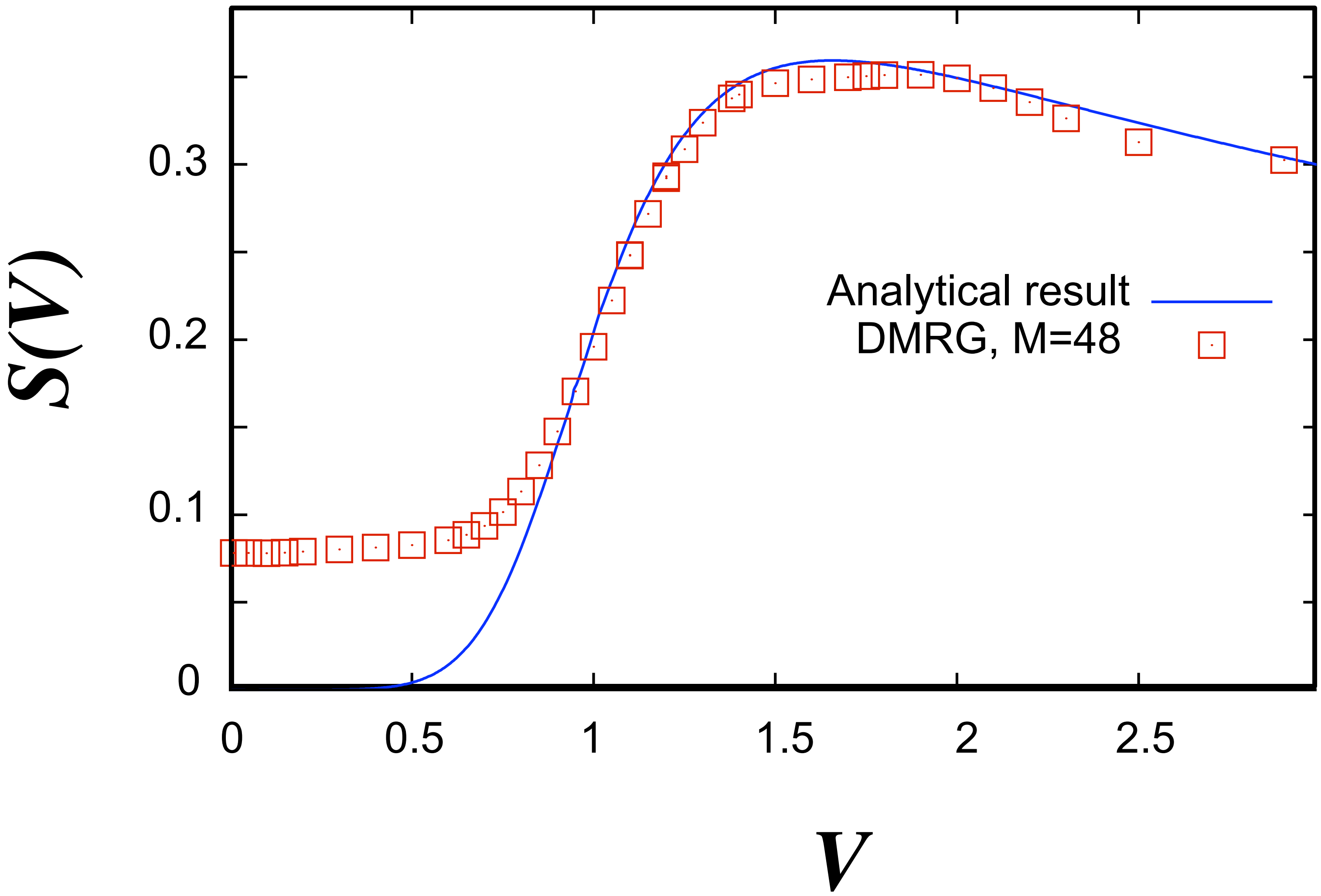
- Approximation of the noise: $S_{num}(\omega) = 4 \ \mathrm{Re} \int_{t_{\min}}^{t_{\max}} dt \ e^{i\omega t} S(t_{\min}, t)$
  - Neglects correlations outside window
  - need large sizes !

Noise - tdDMRG

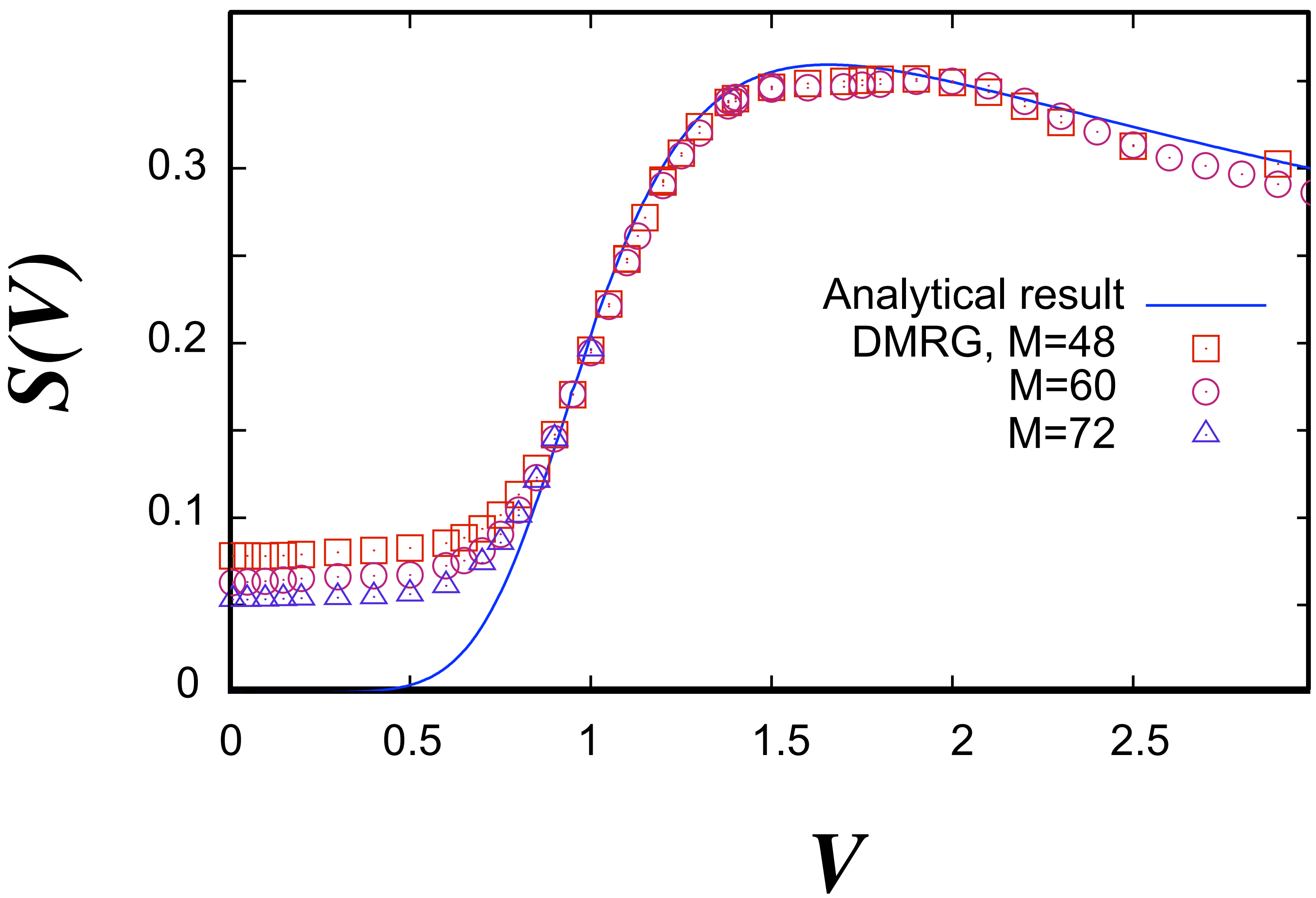


Noise - tdDMRG

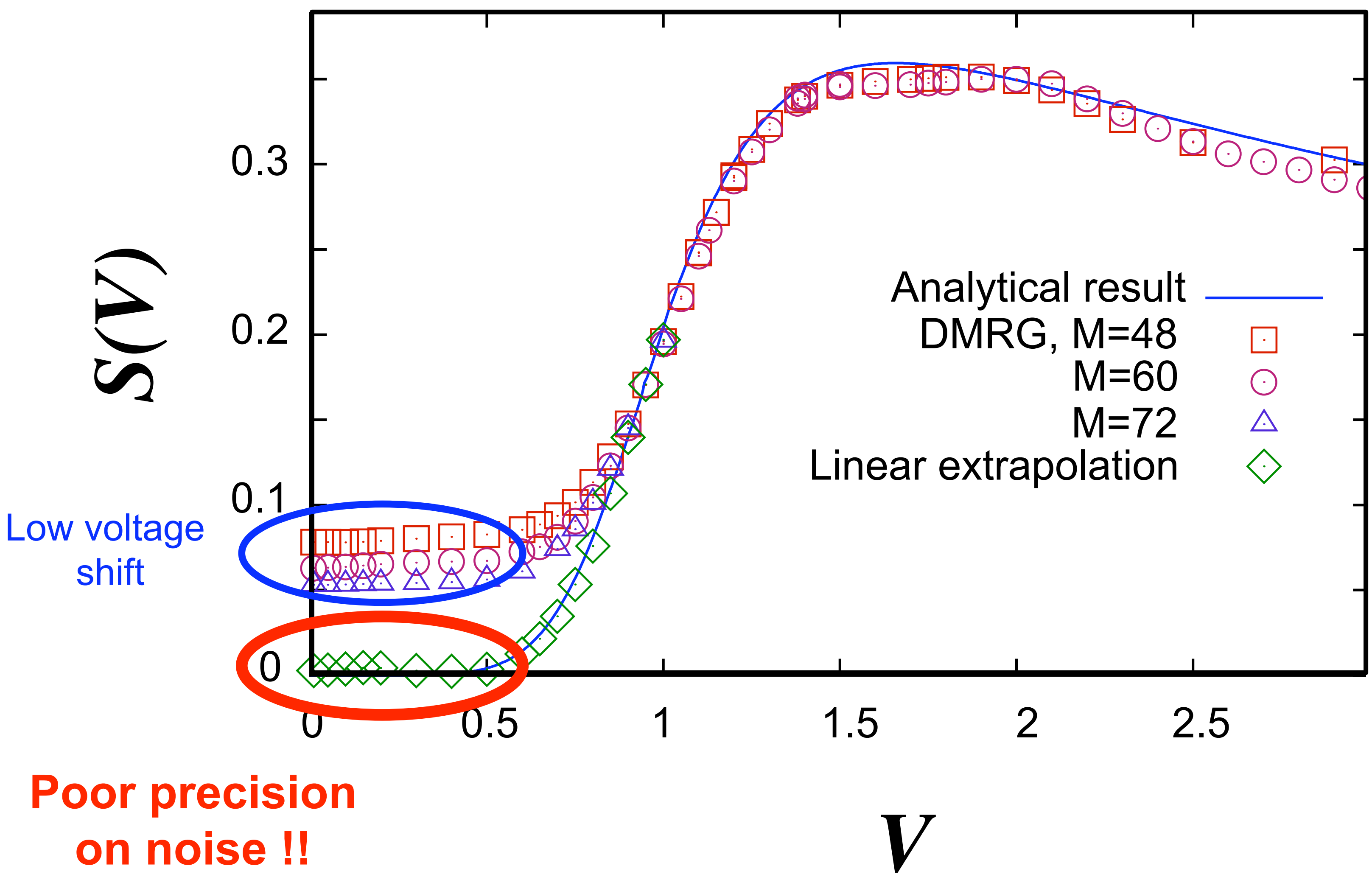
No fitting parameter !



Noise - tdDMRG



Noise - tdDMRG



Poor precision on noise !!

Low voltage

- At “zero” voltage, perfectly transparent structure: $I(V) \underset{V \to 0}{\longrightarrow} G_0 V$
- Analyse departure from perfect transparency by backscattering.

  Backscattered current:

$$I_{BS} = G_0 V - I(V)$$

  Backscattered Fano factor:

$$F_{BS}(V) = S(V)/I_{BS} \underset{V \to 0}{\longrightarrow} e^*$$

- TBA approach predicts $e^* = 2e$

Low voltage shift

Low voltage shift of noise:

$$\lim_{\omega,V\to 0} S_{num}(\omega,V) \neq 0 \quad !!!$$

Finite size (recurrence time):

➔ Finite lower frequency cut-off

$$\omega_{cut} \approx 1/t_R \approx 1/L$$

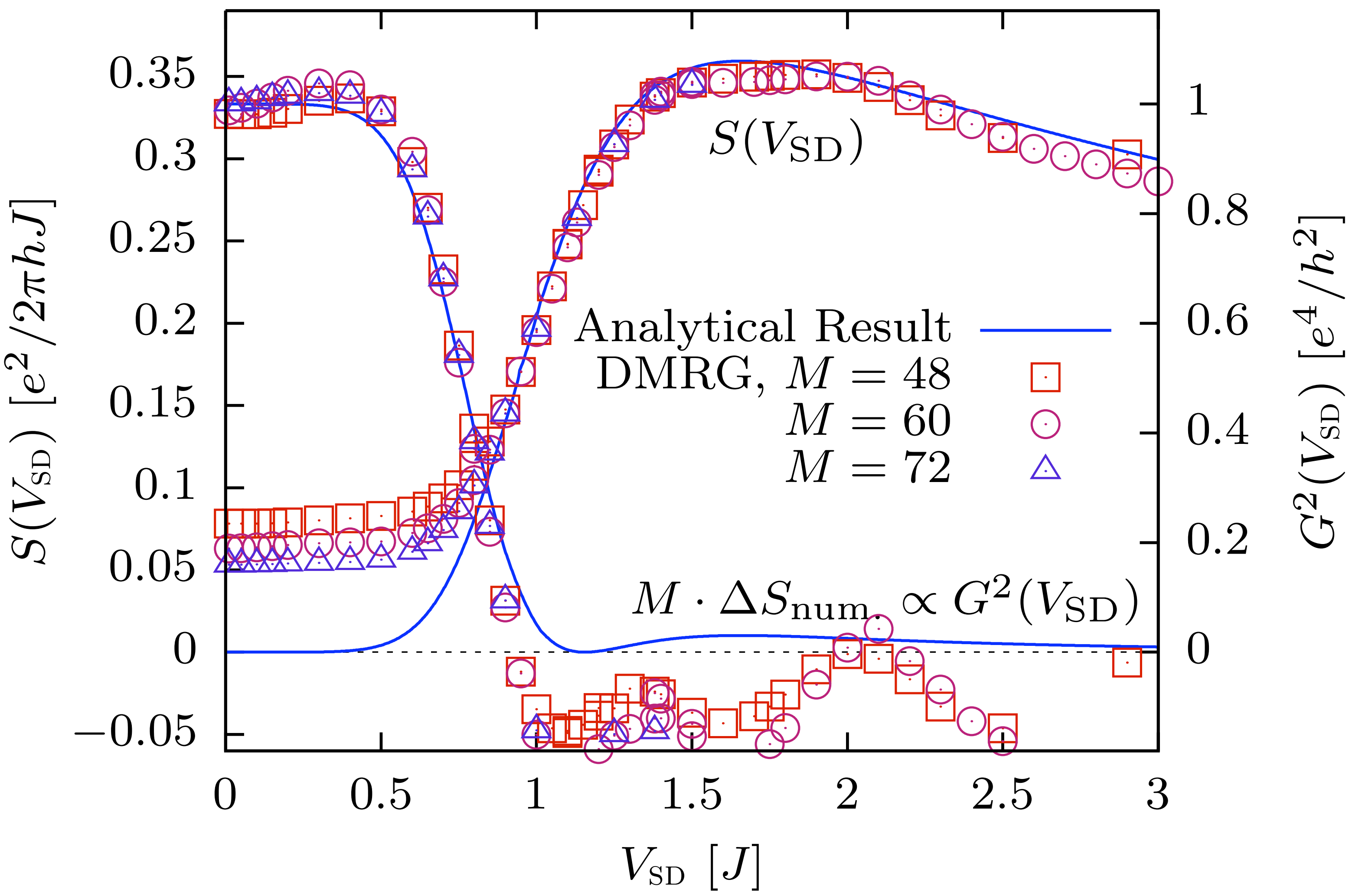
➔ Finite correction of order 1/L

$$S_{num}(0,V) \approx S(\omega_{cut},V) = S(0,V) + \boxed{\omega_{cut}\ \partial_\omega S(0,V)} + \ldots$$

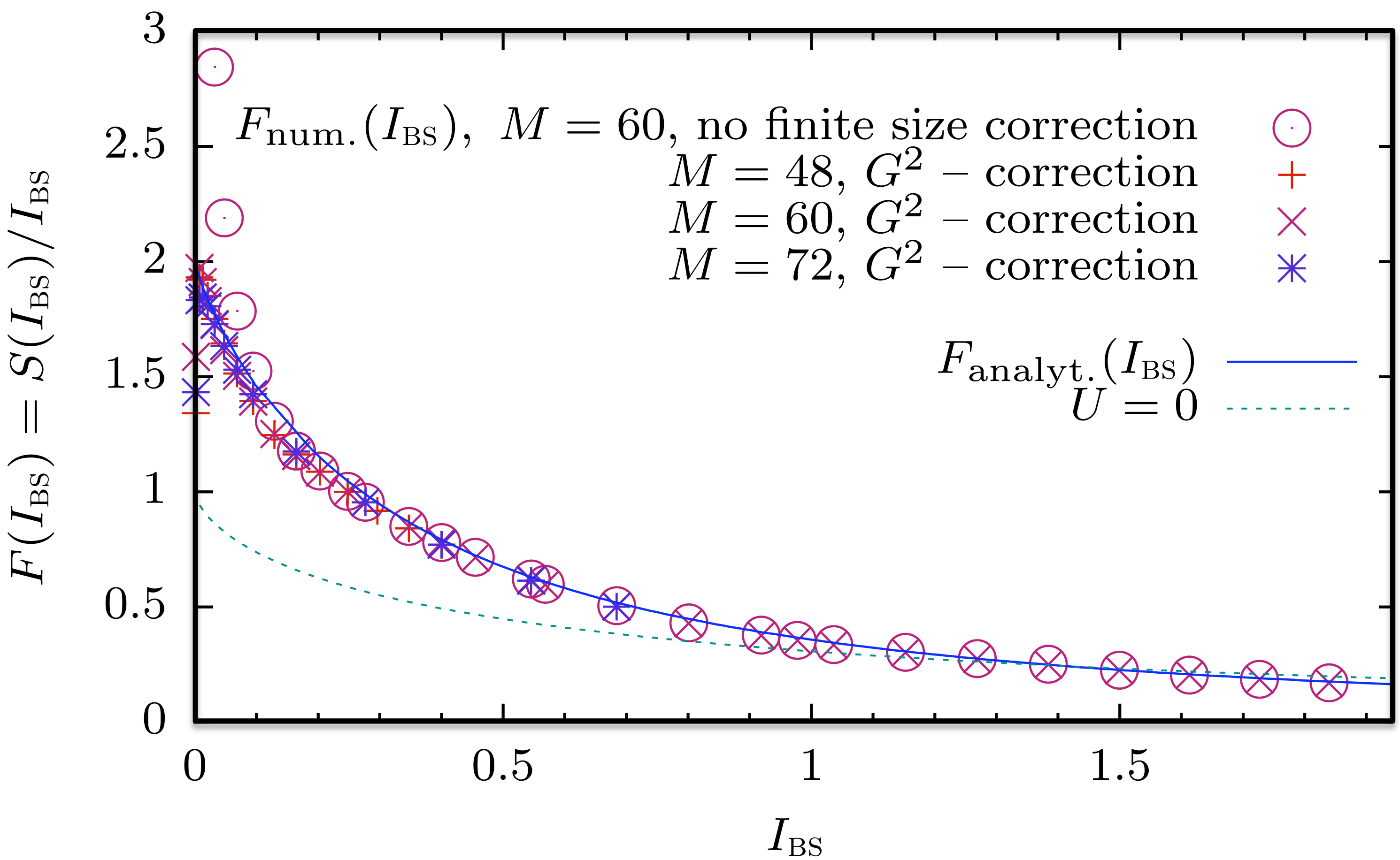
➔ Use analytical estimate of correction (from free case):

$$\partial_\omega S(0,V) \propto G^2(V)$$

1/L correction



Fano factor



3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
$F_{\mathrm{num.}}(I_{\mathrm{BS}})$, $M = 60$, no finite size correction
$M = 48$, G^2 – correction
$M = 60$, G^2 – correction
$M = 72$, G^2 – correction
$F_{\mathrm{analyt.}}(I_{\mathrm{BS}})$
$U = 0$
$F(I_{\mathrm{BS}}) = S(I_{\mathrm{BS}})/I_{\mathrm{BS}}$
I_{BS}

Conclusions

- Dressed TBA approach gives exact density matrix out-of-equilibrium
- Valid when *both* boundary scattering *and* Hershfield Y operator are diagonal (*sufficient* condition)
- td-DMRG gives current & noise, with remarquable accuracy

IRLM:

- Negative differential conductance
- Solitons have a clear signature in the low-V noise