RESIDUAL DECOHERENCE

AND

MANIPULATION

OF

PROTECTED QUBITS

With the collaboration of:

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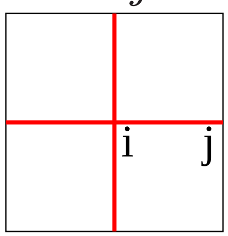
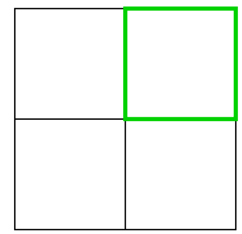
Lattice gauge theory in deconfined regime I

A. Kitaev, Ann. Phys. **303**, 2, (2003)

Z_2 charge $U_i = \prod_j^{(i)} \sigma_{ij}^x$



Z_2 flux $B_\square = \prod_{ij \in \square} \sigma_{ij}^z$



$$H_{\mathrm{Kitaev}} = -\frac{\Delta_{\mathrm{c}}}{2} \sum_i U_i - \frac{\Delta_{\mathrm{f}}}{2} \sum_\square B_\square$$

Localized excitations with finite energy gap

Ground-state degeneracy depends on global topology of the lattice.

Lattice gauge theory in deconfined regime II

Two-fold degenerate ground-state on a cylinder

Degeneracy enforced by non-local symmetries:

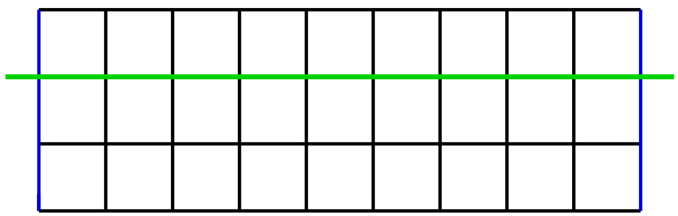
row operators:

$$P_i = \prod_j \sigma^z_{ij}$$

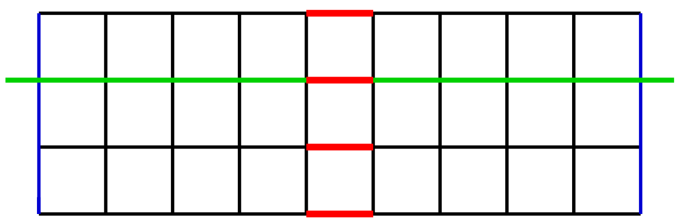
column operators:

$$Q_j = \prod_i \sigma^x_{ij}$$

$$\{P_i, Q_j\} = 0$$



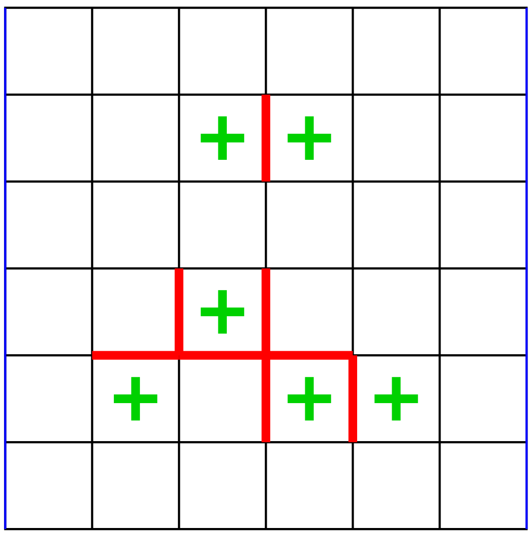
P=1



P=−1

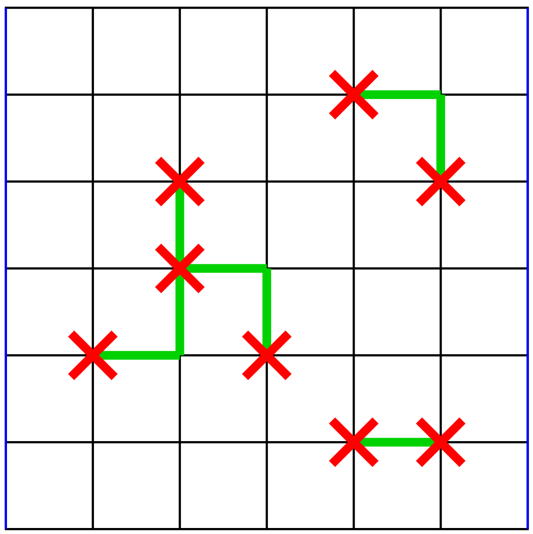
Local errors are harmless

Electric noise: $\prod_{ij \in \text{cluster}} \sigma_{ij}^x$



Creates localized Z_2 fluxes.

Magnetic noise: $\prod_{ij \in \text{cluster}} \sigma_{ij}^z$



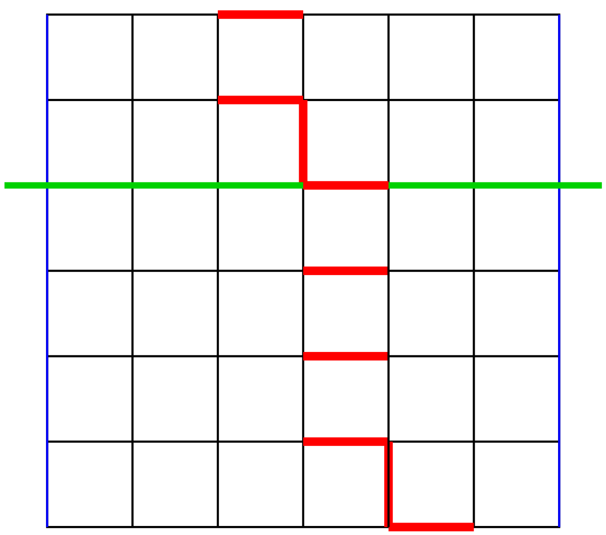
Creates local Z_2 charges.

These errors create only virtual states above finite energy gap.

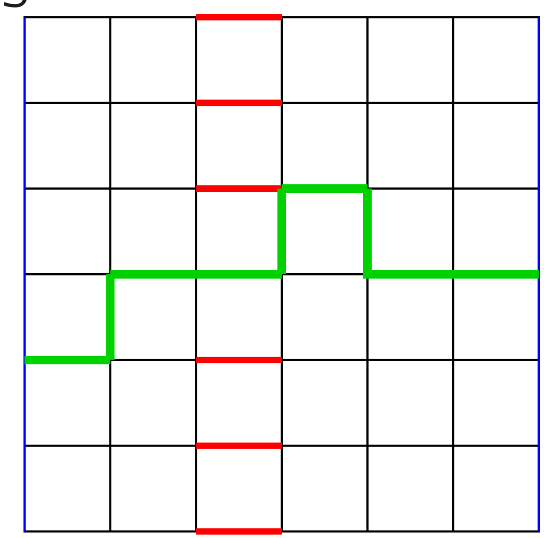
The only dangerous errors are non-local !

They are suppressed by a factor $(\mathsf{noise}/\Delta_{c,f})^L$

Electric noise transfers one Z_2 flux along v-path and flips P_i: Relaxation in *flux* basis or dephasing in *charge* basis.



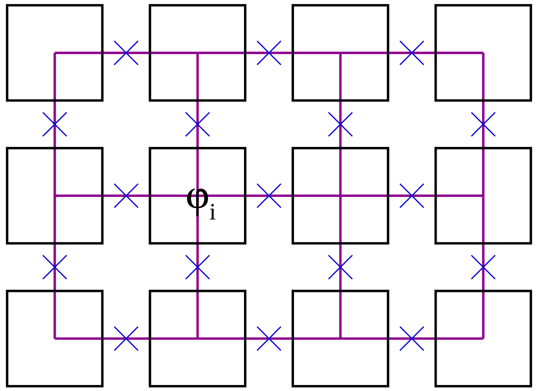
Magnetic noise transfers one Z_2 charge along h-path and flips Q_j: Relaxation in *charge* basis or dephasing in *flux* basis.



Basics of Josephson junction arrays

ϕ_j; local phase of Cooper pair condensate

$\widehat{n}_j = \frac{\partial}{i\partial\phi_j}$: number of Cooper pairs on island j



$$\Delta\phi_j \Delta n_j \simeq 2\pi$$

$$A_{ij} = \frac{2\pi}{\Phi_0}\int_i^j \vec{A}_{ij}.d\vec{r}$$

$$H = -E_{\mathrm{J}}\sum_{\langle ij\rangle}\cos(\phi_i - \phi_j - A_{ij}) + \frac{E_{\mathrm{C}}}{2}\sum_{ij}(C_{ij}^{-1})\widehat{n}_i\widehat{n}_j$$

E_{J}: Josephson coupling energy

E_{C}: Charging energy

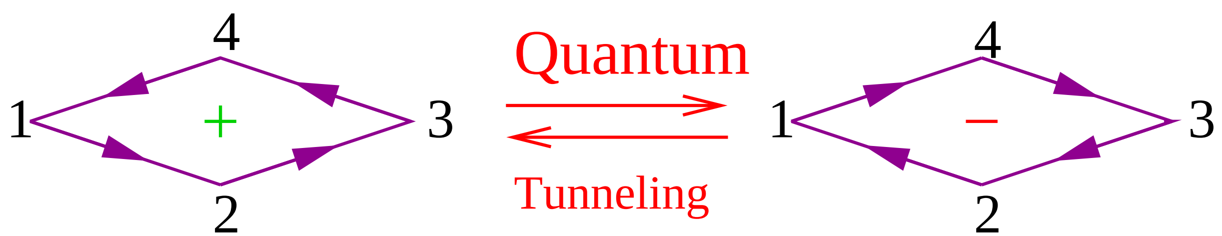
A rhombus with half a flux quantum

Define $\theta_{ij} = \phi_i - \phi_j - A_{ij}$, then:

$$\theta_{12} + \theta_{23} + \theta_{34} + \theta_{41} \equiv \pi, \text{ mod } 2\pi$$

$\longrightarrow$ Get two-fold degenerate classical ground-state, with $\theta_{ij} = \pm\frac{\pi}{4}$

$\longrightarrow$ Quantum fluctuations ($E_C \neq 0$) of phases lift this degeneracy



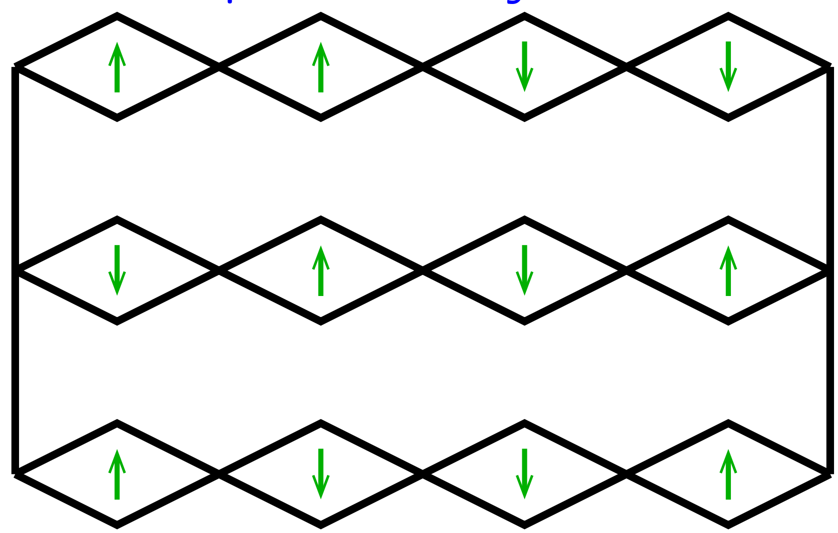
Principle of a protected qubit

$$H = -\frac{\Delta_f}{2} \sum_{i=1}^{M} \tau_j^z \tau_{j+1}^z - \frac{\Delta_c}{2} \sum_{i,j,j'} \sigma_{i,j}^x \sigma_{i,j'}^x$$

Very similar to the Kitaev model:

$$\tau_i^z = \prod_{j=1}^{N} \sigma_{i,j}^z = P_i$$

Josephson array version



$\sigma_{i,j}^z$: chirality of rhombus j
$(1 \leq j \leq N)$ in chain i
$(1 \leq i \leq M)$

Degenerate ground-states of a single chain

$$H_i = -\frac{\Delta_c}{2}\sum_{j,j'} \sigma^x_{i,j}\sigma^x_{i,j'}$$

Define $|\pm\rangle_{i,j} = \left(|\uparrow\rangle_{i,j} \pm |\downarrow\rangle_{i,j}\right)/\sqrt{2}$.

The two ground-states of H_i are $|+,+,...,+\rangle_i$ and $|-,-,...,-\rangle_i$

$$[H_i, \prod_{j=1}^{N} \sigma^z_{i,j}] = 0$$

Recall the definition: $\tau^z_i = \prod_{j=1}^{N} \sigma^z_{i,j}$

$$|\tau^z_i = \uparrow\rangle_i = \left(|+,+,...,+\rangle_i + |-,-,...,-\rangle_i\right)/\sqrt{2}$$
$$|\tau^z_i = \downarrow\rangle_i = \left(|+,+,...,+\rangle_i - |-,-,...,-\rangle_i\right)/\sqrt{2}$$

Ground-states of a single chain($N = 4$)

$$|\tau_i^z = \uparrow\rangle_i \propto |+,+,+,+\rangle_i + |-,-,-,-\rangle_i \;\; \propto \;\; |\uparrow\uparrow\uparrow\uparrow\rangle_i + |\uparrow\uparrow\downarrow\downarrow\rangle_i + |\uparrow\downarrow\uparrow\downarrow\rangle_i + |\uparrow\downarrow\downarrow\uparrow\rangle_i + |\downarrow\uparrow\uparrow\downarrow\rangle_i + |\downarrow\uparrow\downarrow\uparrow\rangle_i + |\downarrow\downarrow\uparrow\uparrow\rangle_i + |\downarrow\downarrow\downarrow\downarrow\rangle_i$$

$$|\tau_i^z = \downarrow\rangle_i \propto |+,+,+,+\rangle_i - |-,-,-,-\rangle_i \;\; \propto \;\; |\uparrow\uparrow\uparrow\downarrow\rangle_i + |\uparrow\uparrow\downarrow\uparrow\rangle_i + |\uparrow\downarrow\uparrow\uparrow\rangle_i + |\downarrow\uparrow\uparrow\uparrow\rangle_i + |\uparrow\downarrow\downarrow\downarrow\rangle_i + |\downarrow\uparrow\downarrow\downarrow\rangle_i + |\downarrow\downarrow\uparrow\downarrow\rangle_i + |\downarrow\downarrow\downarrow\uparrow\rangle_j$$

τ_i^z is associated to the superconducting phase difference across line i of the array, which is twofold degenerate.

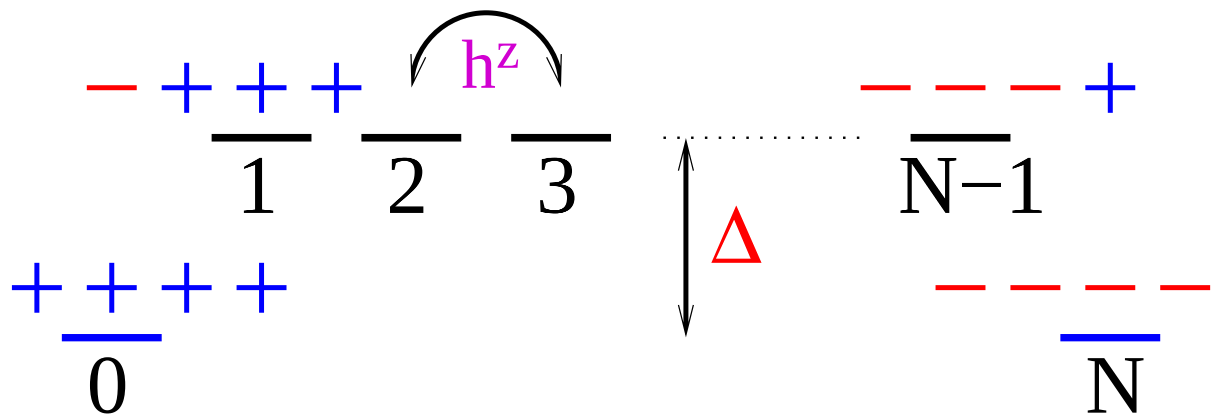
Protection of local τ_i^z against decoherence

Noise of the form $\sum_{i,j} h^x_{i,j}(t)\sigma^x_{i,j}$ flips τ_i^z, but the strongly coupled M parallel chains correct it efficiently: such error costs an energy of the order E_{J}/N.

Noise of the form: $\sum_{i,j} h^z_{i,j}(t)\sigma^z_{i,j}$ commutes with τ_i^z, or equivalently, induces a term of the form:
$|+,+,...,+\rangle_i\langle -,-,...,-|_i + h.c.$

But this decoherence can be made very small !

Assume static and uniform noise $h^z_{i,j}(t) = h$



Ground-state doublet splitting $\simeq h^z \left(\frac{h^z}{\Delta_c}\right)^{N-1}$

So, if $h^z/\Delta_c < 1$, spin flip induces only exponentially small dephasing when N is large !

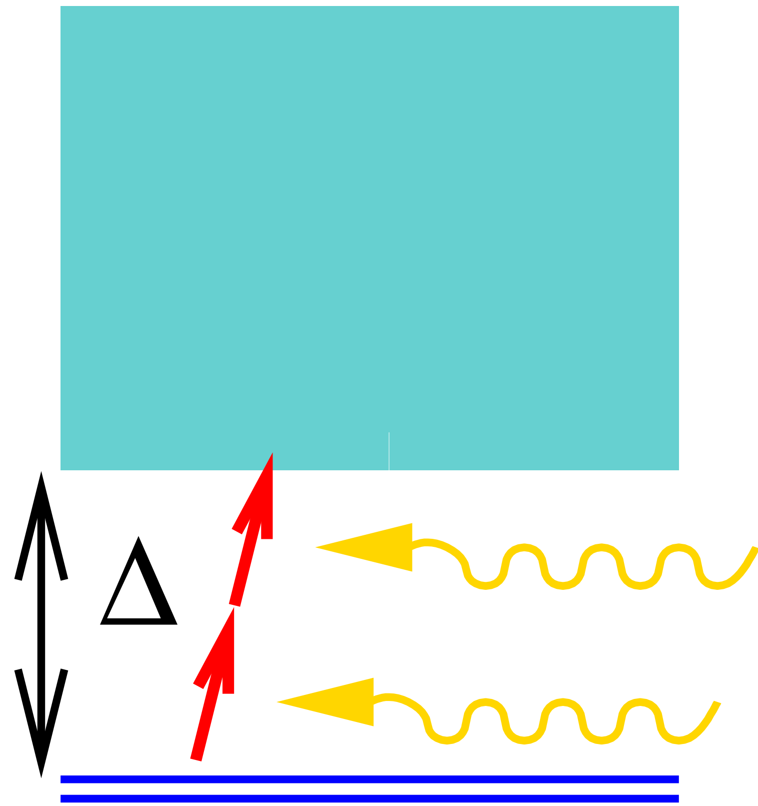
Same conclusion with non-uniform and dynamical noise $h^z_{i,j}(t)$, provided $\omega_{\text{noise}} \ll \Delta_c$

Decoherence induced by finite frequency fluctuations

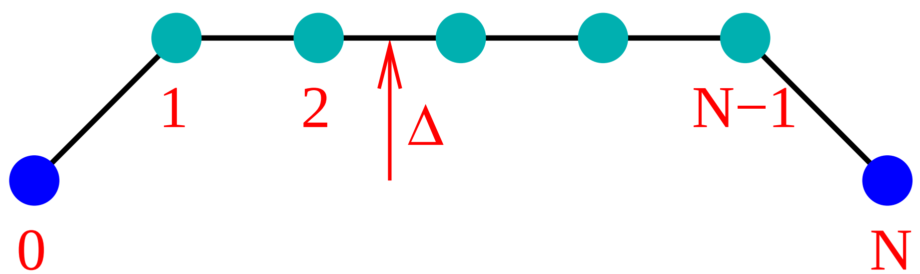
So far, we have considered only *virtual* transitions to excited states.
But the bath may provide some energy: problem of *real* transitions.
Spectral width of bath: D

$$D_{\text{eff}} = \text{Min}(k_{\text{B}}T, D)$$

$$n = \Delta / D_{\text{eff}}$$



Toy model



$$H = H_{\text{syst}} + H_{\text{bath}} + H_{\text{c}}$$

$$H_{\text{syst}} = \Delta \sum_{j=1}^{N-1} |j\rangle\langle j|$$

$$H_{\text{c}} = -\sum_{j=0}^{N-1} |j+1\rangle\langle j| \otimes X_{j+1/2} + h.c.$$

Tree approximation

Density of states at generation p:

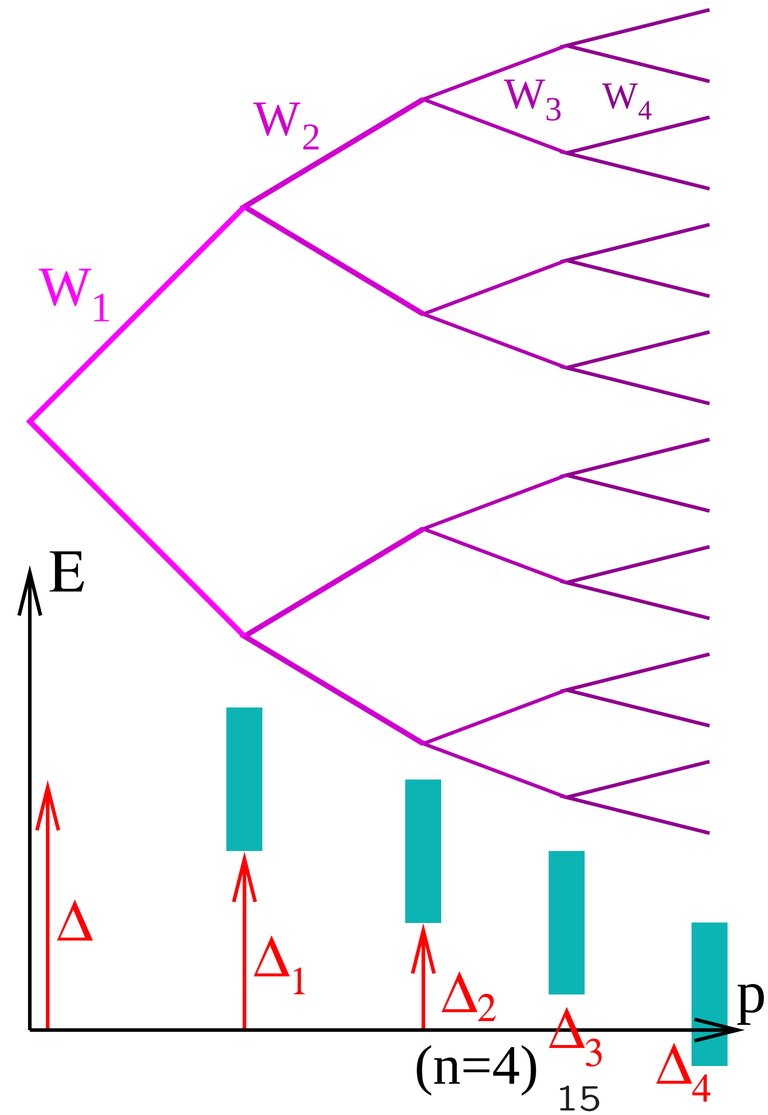
$\rho_p(\omega)$ is restricted to $\omega > \Delta_p$

Typically, $\Delta_{p+1} \simeq \Delta_p - k_{\mathrm{B}}T$

$$R(z) = \langle \mathrm{in}|(z-H)^{-1}|\mathrm{in}\rangle$$

$$R(z) = \frac{1}{z - \Sigma_0(z)}$$

$$\Sigma_p(z) = \int_{\Delta_{p+1}}^{\infty} d\omega \frac{W_{p+1}^2(\omega)\rho_{p+1}(\omega)}{z - \omega - \Sigma_{p+1}(z)}$$



Weak coupling analysis I

$$\Sigma_p(z) = \int_{\Delta_{p+1}}^{\infty} d\omega \frac{W_{p+1}^2(\omega)\rho_{p+1}(\omega)}{z - \omega - \Sigma_{p+1}(z)}$$

Let us assume that $\Re\Sigma_{p+1}$ can be neglected, then:

$$\Im\Sigma_p(z) = \Im\Sigma_{p+1}(z) \int d\omega \frac{W_{p+1}^2(\omega)\rho_{p+1}(\omega)}{(z-\omega)^2 + (\Im\Sigma_{p+1}(z))^2}$$

At generation n, the small energy z is embedded in the n^{th} continuum, so:

$$\Im\Sigma_{n-1}(z) \simeq -\pi W_n^2(z)\rho_n(z)$$

To lowest order in the couplings to the bath, this gives:

$$\Im\Sigma_0(z) \simeq -\pi W_n^2(z)\rho_n(z) \prod_{p=1}^{n-1} \int d\omega \frac{W_p^2(\omega)\rho_p(\omega)}{(z-\omega)^2}$$

Weak coupling analysis II

$$\Im\Sigma_0(z) \simeq -\pi W_n^2(z)\rho_n(z) \prod_{p=1}^{n-1} \int d\omega \frac{W_p^2(\omega)\rho_p(\omega)}{(z-\omega)^2}$$

$\longrightarrow$ Generalization of Fermi golden rule to n^{th} order.

$\longrightarrow$ Master equation, with rates appearing at order $2n$ in perturbative expansion (U. Gavish).

$\longrightarrow$ No use to make systems of size N with $N > n$.

Residual decoherence when $k_{\mathrm{B}}T < \Delta/N$ $(n > N)$

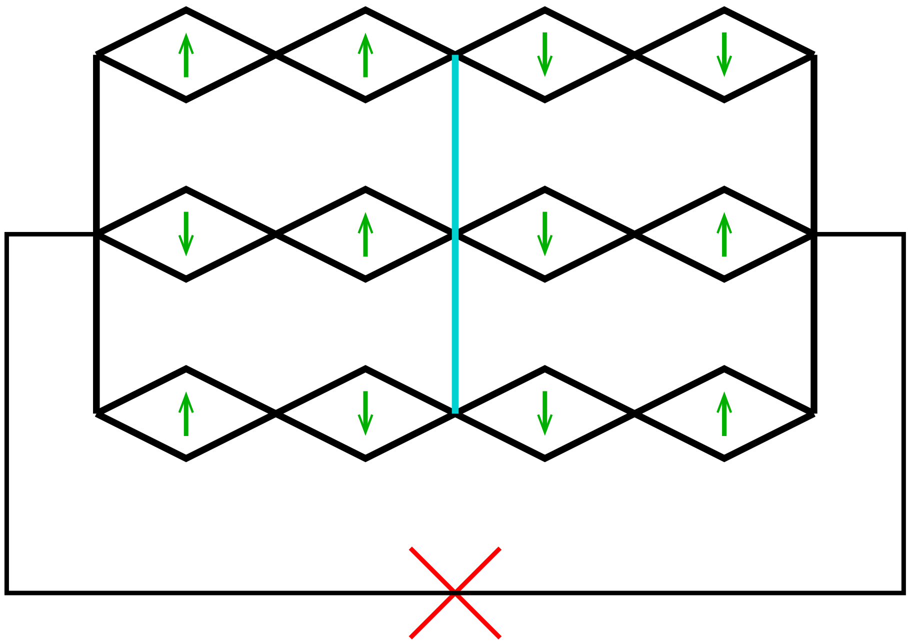
$$\Im\Sigma_0(z \to 0) \simeq -\int \frac{d\omega_1}{2\pi} ... \int \frac{d\omega_N}{2\pi} \pi\delta(\omega_1 + ... + \omega_N) C(\omega_1)...C(\omega_N) \times \frac{1}{(\omega_1 + \Delta)^2(\omega_1 + \omega_2 + \Delta)^2...(\omega_1 + ... + \omega_{N-1} + \Delta)^2}$$

where $\delta_{ij}C(\omega) = \int dt\, e^{i\omega t}\langle X_i(t)X_j(0)\rangle$.

The connection between the tree model and the bath model is: $C(\omega) = 2\pi W_j(\omega)^2\rho_j(\omega)$.

Note that $n > N$ implies that energy denominators do not vanish.

Experimental design



Total phase γ across the array is fixed by the combined effect of the large junction and the flux through the big loop.
Qubit encoded in the phase (0 or π) of the central island.

Experimental diagnosis of topological protection

$$E_{\mathrm{array}} = -E_1(\Phi_{\mathrm{Rh}})\cos\gamma - E_2(\Phi_{\mathrm{Rh}})\cos 2\gamma$$

If $\Phi_{\mathrm{Rh}} \to \Phi_0(n+1/2), n$ integer, then $E_1 \to 0$:

$\Phi_0/2$ periodicity,

else, $E_1 > E_2$: Φ_0 periodicity

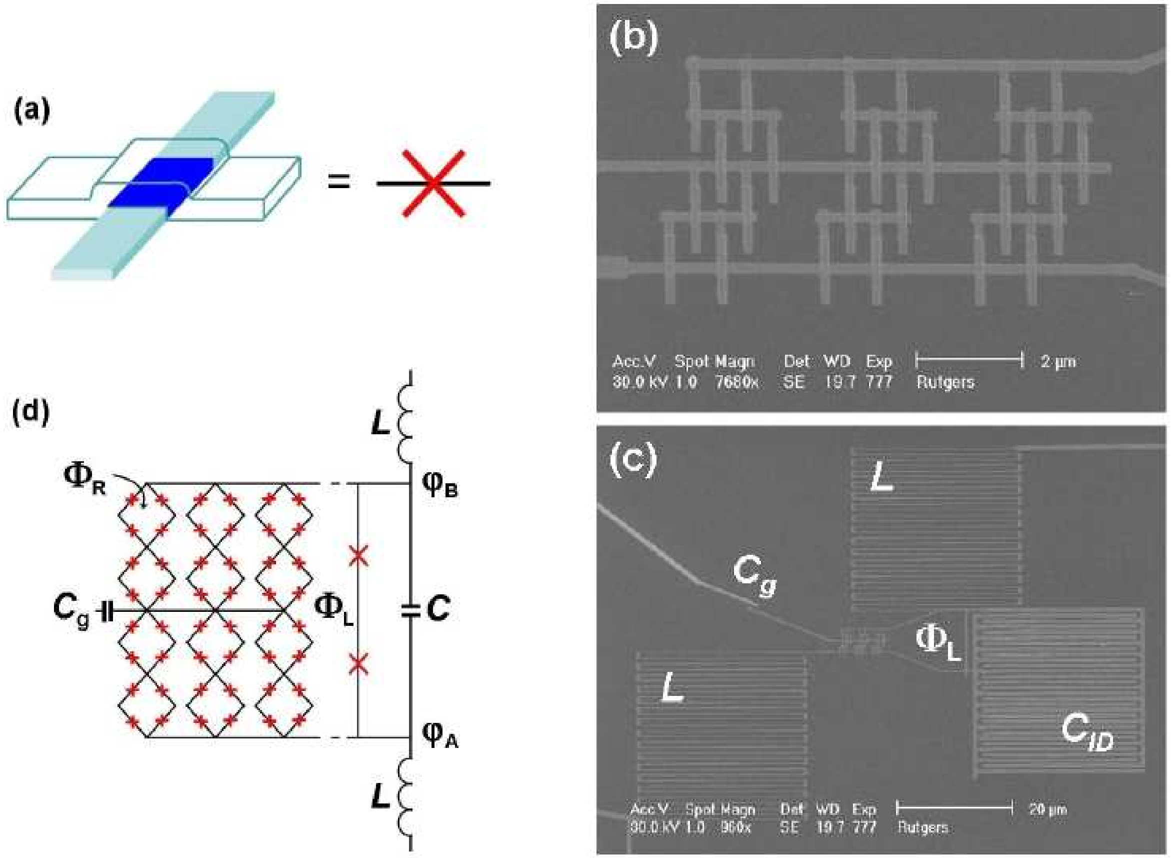
$$\Delta I_{\mathrm{c}} = \frac{2e}{\hbar}E_1 \sin\gamma + \frac{4e}{\hbar}\sin 2\gamma$$

Manifestation of protection: for $\Phi_{\mathrm{Rh}} = \Phi_0(n+1/2) + \Delta\Phi_{\mathrm{Rh}}$,

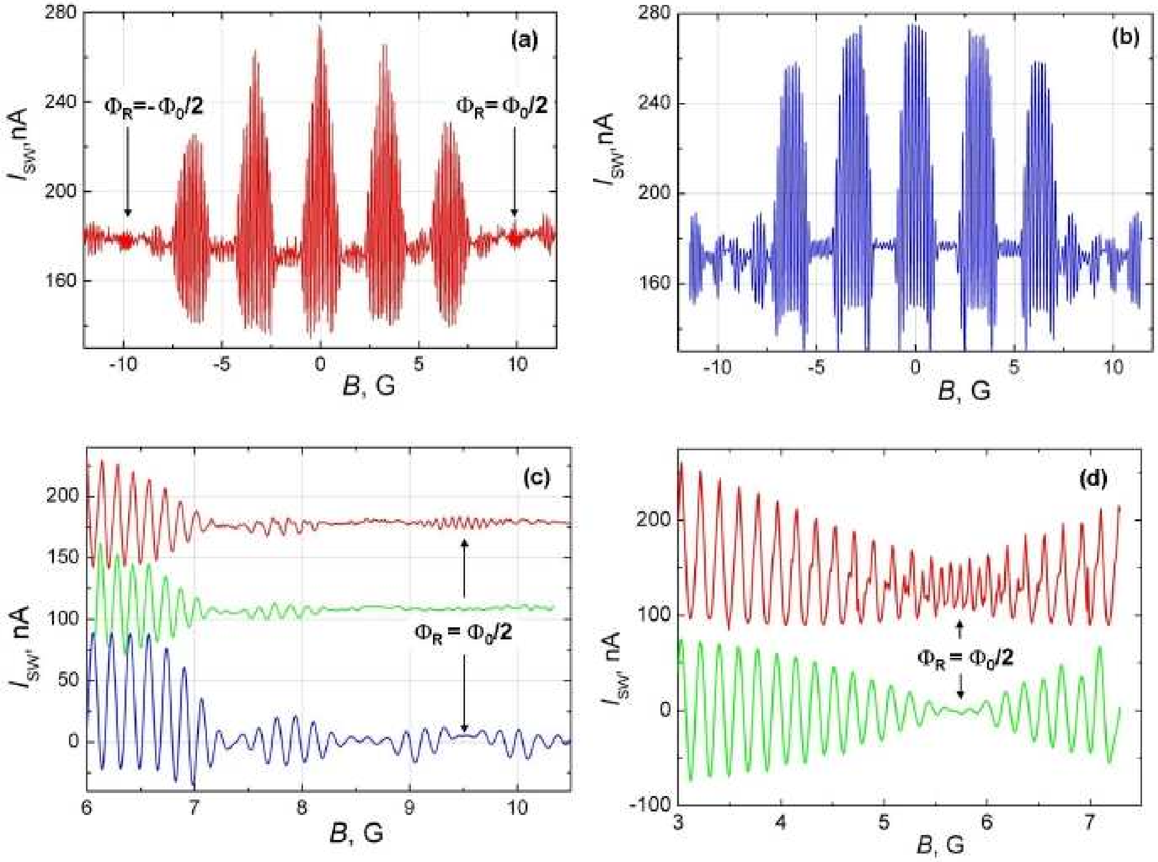
$$E_1 \simeq E_{\mathrm{J}}\left(\frac{E_{\mathrm{J}}}{\Delta_c}\right)^{N-1}(\Delta\Phi_{\mathrm{Rh}})^N$$

Exponential improvement with system length !

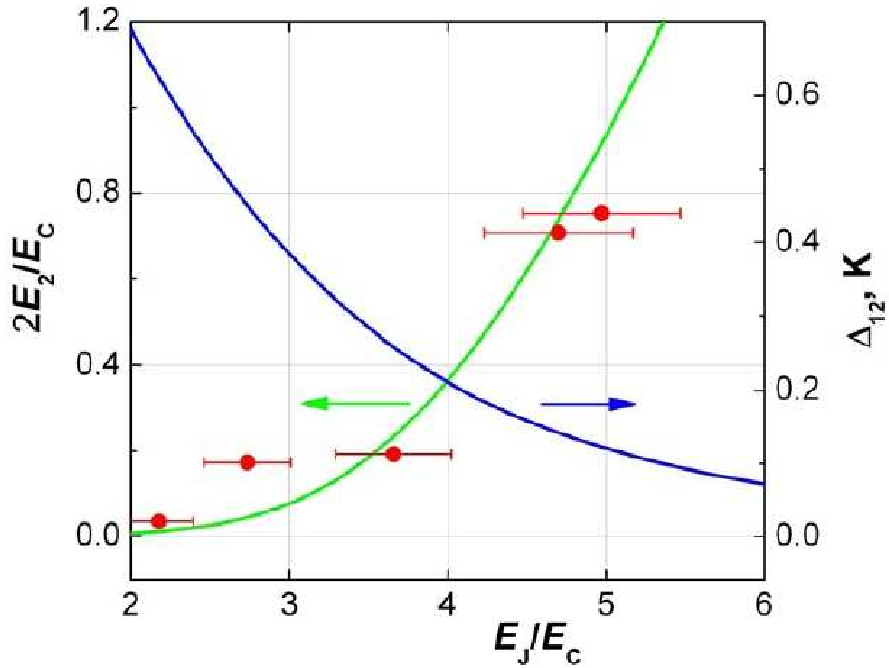
Experimental realization: M. Gershenson et al. (2007)



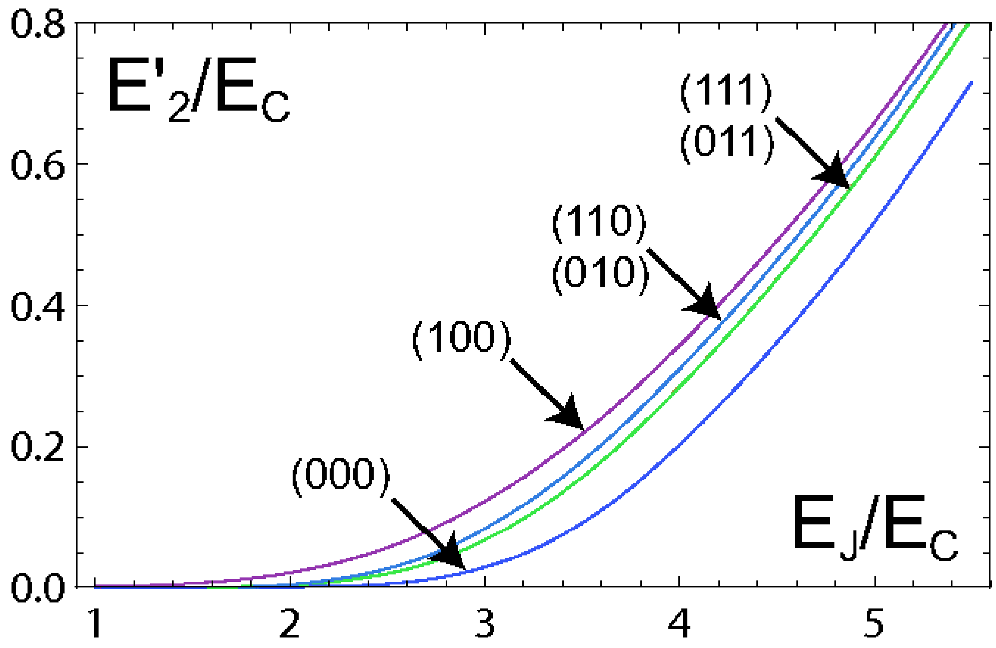
Evidence for finite Δ_C and charge $4e$ condensate



Phase stiffness E_2 of charge $4e$ condensate



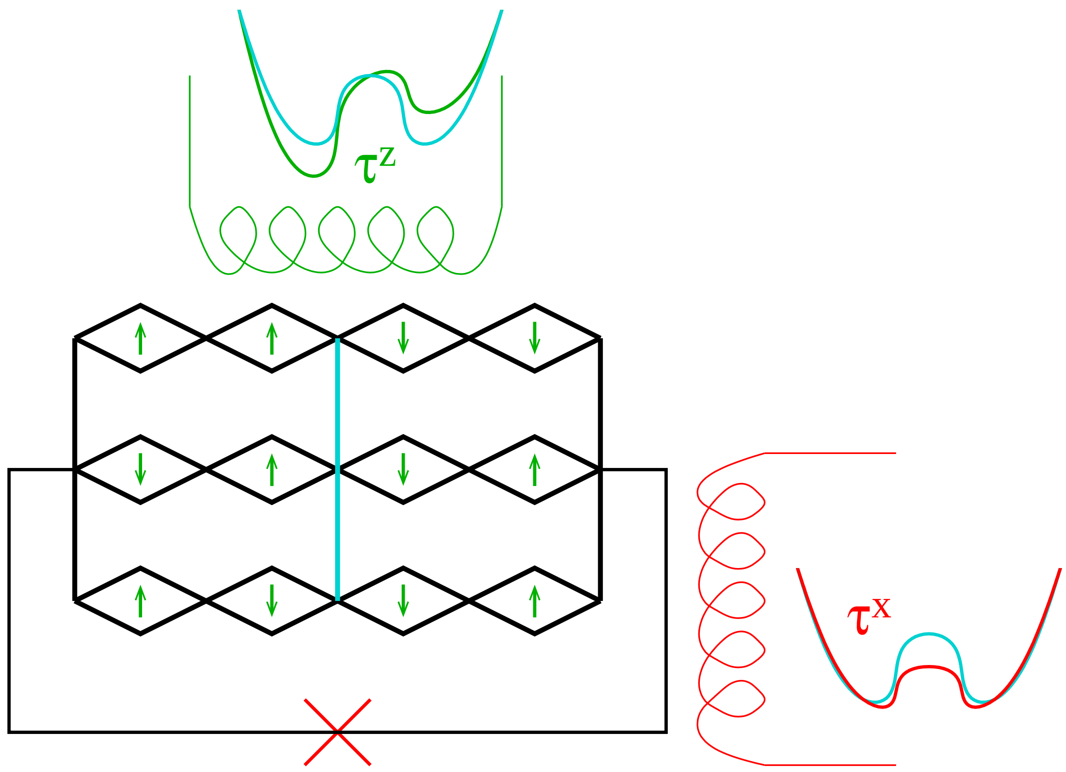
Measuring the Z_2 charge of the central island



Critical current of the array depends on the Z_2 charge of the central island:

Provides a way to measure τ^x

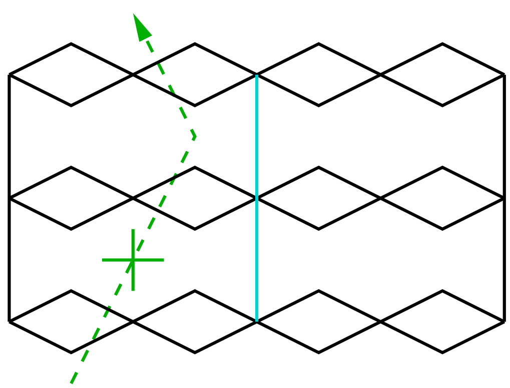
Manipulating the protected qubit



Remarks about conserved quantities in the real physical system

If local magnetic fluxes are exactly $\Phi_0/2$ for each rhombus: local Z_2 charge is conserved, which implies $[Q_j, H_{\text{array}}] = 0$.

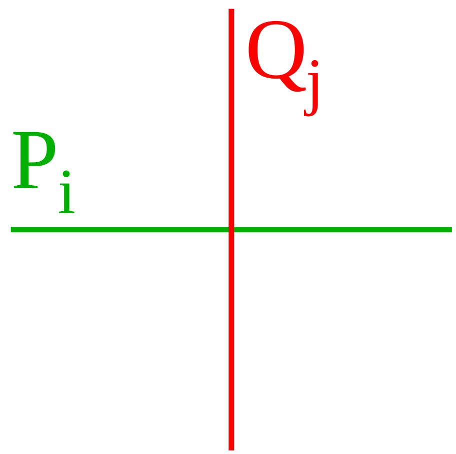
The array model allows for quantum tunneling processes where Z_2 vortices are created in pairs, which lead to $[P_i, H_{\text{array}}] \neq 0$. If we can neglect the processes where a single Z_2 vortex crosses the whole system, we can map the array into an effective Kitaev model where $[P_i, H_{\text{effective}}] = 0$.



Effect of noise during the τ^z manipulation

$$\Delta H(t) = f(t) \sum_{i,j} \sigma^z_{ij} + \sum_a \sum_{i,j} \epsilon^a_{ij}(t) \sigma^a_{ij}$$

σ^z_{ij} flips Q_j
σ^x_{ij} flips P_i
σ^y_{ij} flips P_i and Q_j



$$P_i = \prod_j \sigma^z_{ij}$$

$$Q_j = \prod_i \sigma^x_{ij}$$

$$\{P_i, Q_j\} = 0$$

Effect of various perturbations in qubit space

	Q_j's conserved	Q_j's flip
P_i's conserved	$\mathbf{1}$	τ^z
P_i's flip	τ^x	τ^y

Noise coupled to σ_{ij}^z

	Q_j's conserved	Q_j's flip	
P_i's conserved	$(f+\epsilon^z)^n \mathbf{1}$	$(f+\epsilon^z)^{N+n}\tau^z$	n even
P_i's flip	$0\tau^x$	$0\tau^y$	

Relative effect is of order $\frac{\epsilon^z}{f}$: the angle of the qubit rotation is not very robust.

Noise coupled to σ^x_{ij}

	Q_j's conserved	Q_j's flip	
P_i's conserved	$f^n(\epsilon^x)^m\mathbf{1}$	$f^{N+n}(\epsilon^x)^m\tau^z$	m, n even
P_i's flip	$f^n(\epsilon^x)^{M+m}\tau^x$	$f^{N+n}(\epsilon^x)^{M+m}\tau^y$	

Relative effect on rotation angle is of order $(\frac{\epsilon^x}{\Delta})^2$.

Relative size of τ^x term is of order $(\frac{\epsilon^x}{\Delta})^M(\frac{\Delta}{f})^N$.

Relative size of τ^y term is of order $(\frac{\epsilon^x}{\Delta})^M$.

Noise coupled to σ^y_{ij}

	Q_j's conserved	Q_j's flip
P_i's conserved	$f^p(\epsilon^y)^m \mathbf{1}$	$f^{N-m+p}(\epsilon^y)^m \tau^z$
P_i's flip	$f^n(\epsilon^y)^{M+m}\tau^x$	$f^{N-M+p}(\epsilon^y)^{M+m}\tau^y$

m, p, $n-M$, even

Relative effect on rotation angle is of order $(\frac{\epsilon^x}{f})^2$.
Relative size of τ^x term is of order $(\frac{\epsilon^x}{f})^N$ (if $M=N$).
Relative size of τ^y term is of order $(\frac{\epsilon^x}{f})^N$ (if $M=N$).

Remarks about the τ^x manipulation

$$\Delta H(t) = g(t) \sum_{i,j} \sigma^z_{ij} + \sum_a \sum_{i,j} \epsilon^a_{ij}(t) \sigma^a_{ij}$$

Formally similar to the τ^z manipulation !

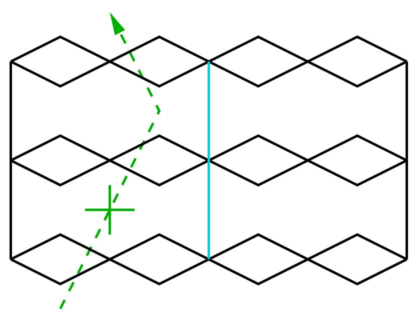
Summary: effect of noise during qubit manipulation

Angle of rotation is no longer robust: the strength of the manipulation term should be larger than the noise which acts in the same channel.

Provided this condition is satisfied, there is still some form of protection for the direction of the rotation applied in qubit space, in the sense that a larger size helps.

Open questions on the effect of noise during qubit manipulation

i)Putting back the true array Hamiltonian, for which strictly speaking, $[P_i, H_{\text{array}}] \neq 0$. As we have seen, these terms are associated to processes of the form:



which lift the degeneracy of the central island, whose phase tunnels between 0 and π, thus selecting its Z_2 charge as the good quantum number of the qubit.

ii)Include explicitely dynamical noise during manipulation

Conclusions

1) Kitaev's Z_2 lattice model implemented in the low energy sector of some Josephson junction arrays.
2) These arrays are composed of fully frustrated rhombi.
3) Experimental evidence for this phase: observation of enhanced immunity against static flux fluctuations, evidence of a finite Δ_C.
4) Residual decoherence induced by dynamical fluctuations is expected to be exponentially small with the system size, provided the noise temperature is much smaller than the energy gap.
5) Some form of protection is still expected to hold during qubit manipulation.

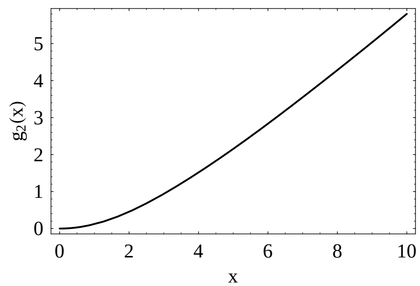
Computational issues I: hierarchical approximation

Series composition of Z_2 junctions

$$V(\phi) = -E_2 \cos(2\phi)$$

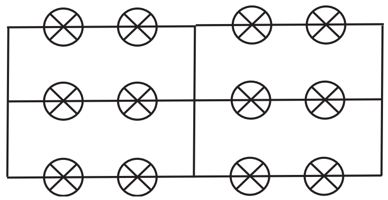
$$E'_2 = \left[1 - \frac{7}{256}\left(\frac{E_2}{E_C}\right)^2\right]\frac{1}{8}\frac{E_2^2}{E_C}$$

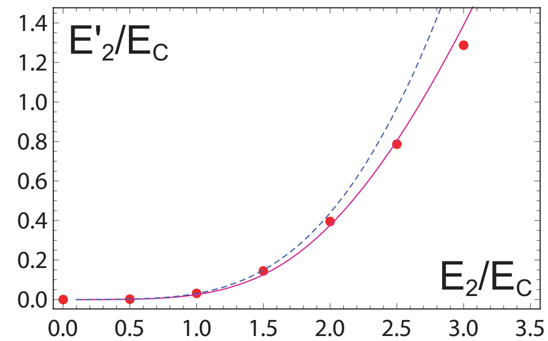
$$E'_C = \left[1 - \frac{1}{16}\left(\frac{E_2}{E_C}\right)^2\right] 2E_C$$



Parallel composition

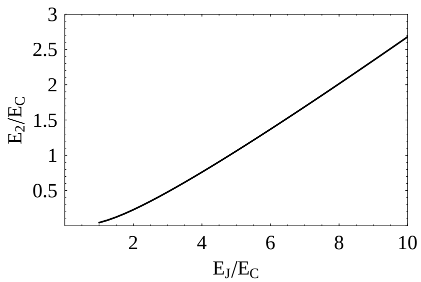
$$E'_2 = K E_2$$
$$E'_C = K^{-1} E_C$$



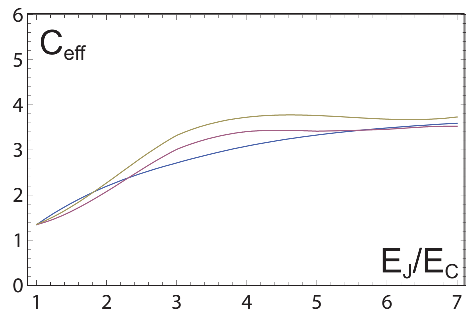


Computational issues II: single rhombus as Z_2 junction

Effective E_2



Effective capacitance



Test of coarse graining: 2 and 3 rhombi in series