Quench and Thermalization in integrable and non-integrable systems

Alessandro Silva

ICTP Trieste, Italy

Collaborators: Elena Canovi (SISSA), Davide Rossini (SNS)
Giuseppe Mussardo (SISSA)
Giuseppe Santoro (SISSA)
Rosario Fazio (SNS)

INSTANS Conference, Stockholm, September 2010

Thermalization in quantum systems

?

1) The system acts as its own thermal bath ?

2) Asymptotics depends on details of the initial state or just on its energy ?

3) Time average = microcanonical average ?

ERGODICITY

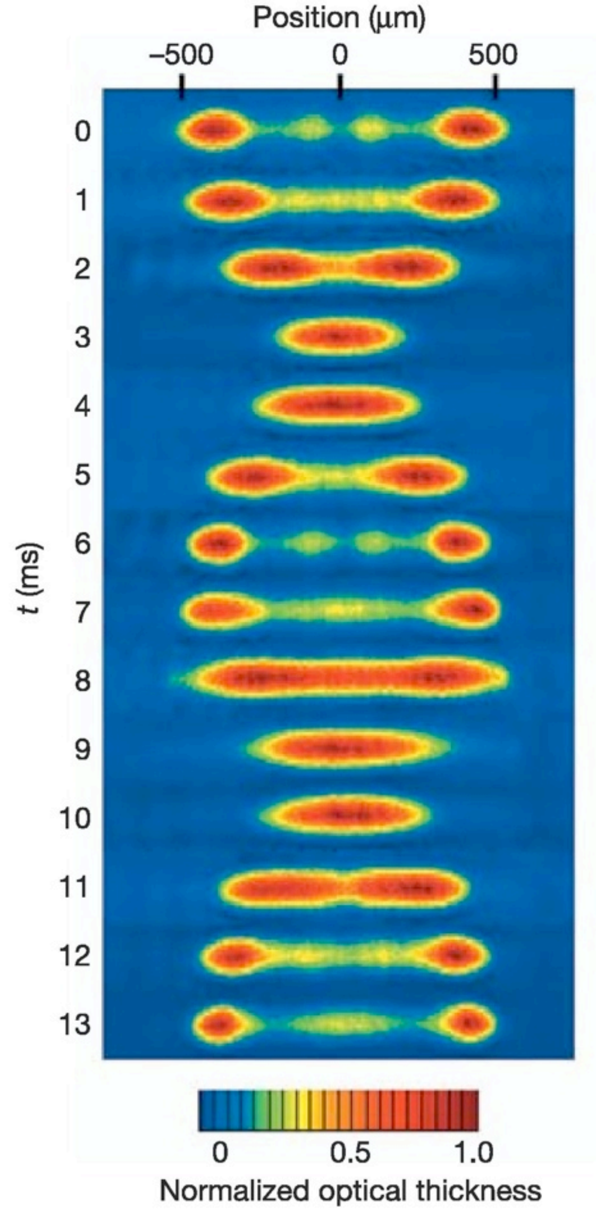
Von Neumann ('29), Mazur, Girardeau, Baruch, McCoy ('70), Peres ('80) Srednicki, Deutsch ('90)

see also **A. Polkovnikov, K. Sengupta, A.S., M. Vengalattore, arxiv: 1007.5331**

Motivations: experiments with cold atoms

✓Lack of thermalization in 1D condensates

Figure 2 | Absorption images in the first oscillation cycle for initial average peak coupling strength $\gamma_0 = 1$. Atoms are always confined to one dimension, in this case in 3,000 parallel tubes, with a weighted average of 110 atoms per tube. After grating pulses put each atom in a superposition of $\pm 2\hbar k$ momentum, they are allowed to evolve for a variable time t in the anharmonic 1D trap (crossed dipole trap), before being released and photographed 27 ms later. The false colour in each image is rescaled to show detail. These pictures are used to determine $f(p_{\mathrm{ex}})$. The first image shows that some atoms remain near $p_{\mathrm{ex}} = 0$ at $t = 0$. How many remain there depends on $n_{1\mathrm{D}}$, implying that these remnant atoms do not result from an imperfect pulse sequence, but rather from interactions during the grating pulses or evolution of the momentum distribution during expansion. The relative narrowness of the peaks in the last image compared to the first is indicative of the reduction in spatial density that results from dephasing (Fig. 1b). The transverse spatial width of each of the 14 image frames is 70 µm. Horizontal in the figure corresponds to vertical in the experiment, a minor distinction because a magnetic field gradient cancels gravity for the atoms.

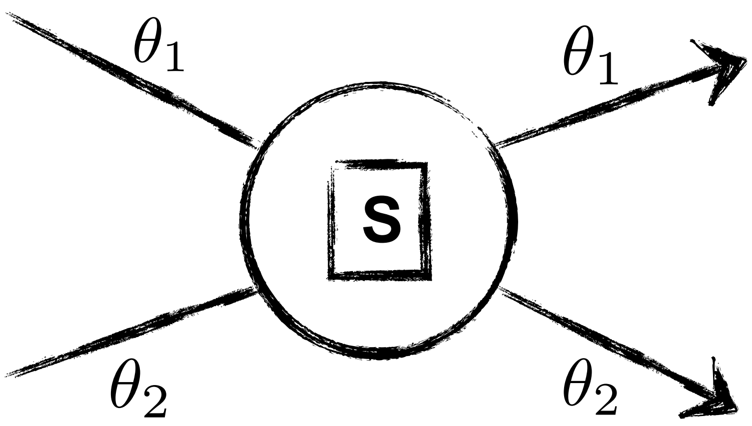


Kinoshita et al, *Nature* ***440***, *900 (2006)*

Integrable systems

Thermalization should **not** occur:
steady states remembers the
initial conditions (as in classical physics)

Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)*
Rigol, Muramatsu, & Olshanii, *PRA (2006)*
Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006)*, *JSTAT (2007)*
Gangardt & Pustilnik, *PRA (2008)*
Eckstein *& Kollar, PRL (2008)*, *PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
Barther, Schollwoeck, *PRL (2008)*

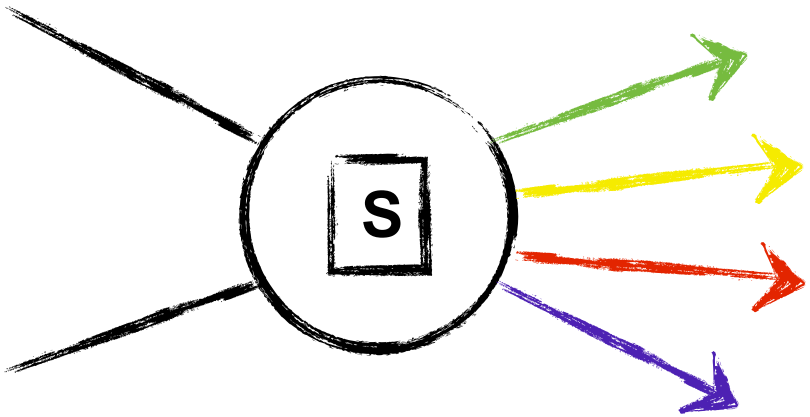


$$\hat{\rho}_G = \frac{e^{-\int d\theta \lambda(\theta) A^\dagger(\theta) A(\theta)}}{Z}$$

Ok for certain initial states and certain operators (see Fioretto, Mussardo ('10))

Breaking integrability

.... ergodicity ?



Gas of N particles in a box .... eigenstates = *pseudo-random superpositions of plane waves* (Berry's conjecture) .... (Srednicki '94)

$$\int d\mathbf{p}_2 d\mathbf{p}_3 \ \ldots \langle | \ \Psi_\alpha(\mathbf{p}, \mathbf{p_2}, \ldots) \ | \rangle^2 = \frac{e^{-\frac{\mathbf{p}^2}{2mkT}}}{(2\pi mkT)^{3/2}}$$

Eigenstate thermalization hypothesis

Deutsch, *PRA (1991)*
Srednicki, *PRE (1994)*
Rigol, Dunjko, & Olshanii, *Nature (2008)*
Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009)*, *PRA (2009)*
Biroli et al. *arXiv 0907.3731*
.....

Outline

– Integrable systems: thermal or non–thermal ?

it depends on the observable !!

Thermal or **non-thermal** = **non-local** or **local** in **qp space**

– Non–integrable system: thermal or non–thermal ?

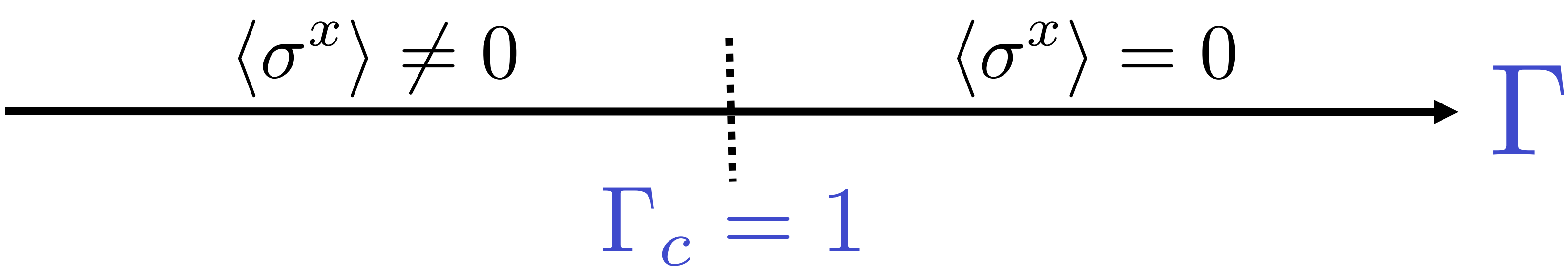
non-thermal --> Thermal = **localized/delocalized** in **qp space**

The simplest protocol

$$H(g_0) \to_\gamma H(g_1)$$

Quantum Ising model

$$\mathcal{H}(\Gamma) = -J \sum_j \left[\sigma_j^x \sigma_{j+1}^x + \Gamma \sigma_j^z \right]$$



- This model is INTEGRABLE
- The quasiparticles are FREE FERMIONS

$$\begin{cases} \sigma_j^z = & 2c_j^\dagger c_j - 1 \\ \sigma_j^x = & e^{i\pi \sum_{l<j} n_l} c_j + h.c. \end{cases}$$

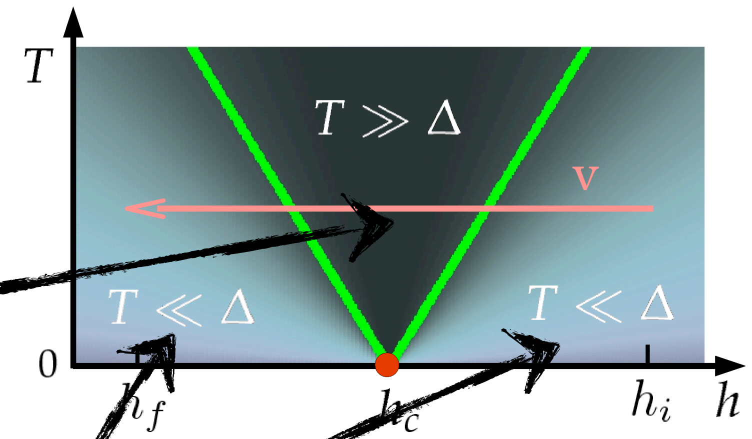
Correlators

$$\mathcal{H}(\Gamma) = -J \sum_j \left[\sigma_j^x \sigma_{j+1}^x + \Gamma \sigma_j^z\right]$$

$$\langle \sigma_i^x(t)\sigma_i^x(0)\rangle \approx e^{-t/\tau^\varphi}$$

$$\langle \sigma_{i+r}^x(t)\sigma_i^x(t)\rangle \approx e^{-r/\xi}$$

EQUILIBRIUM



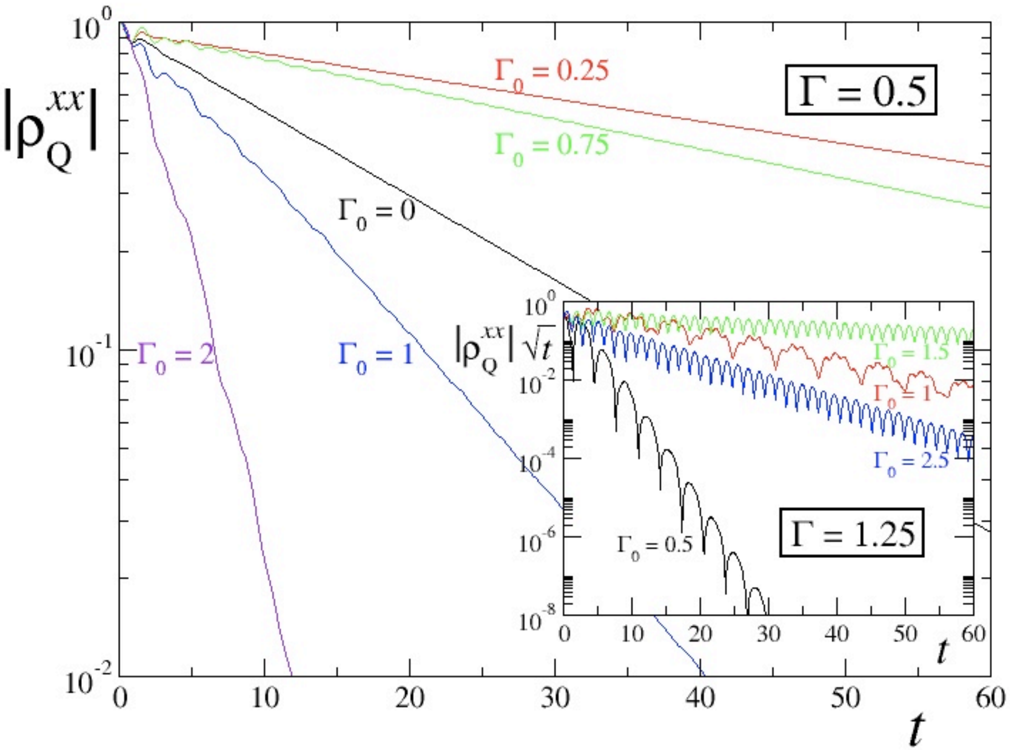
$$\tau_T^\varphi \simeq \frac{8}{\pi T}$$

$$\tau_T^\varphi \simeq \frac{\pi}{2T} e^{\Delta/T}$$

Order parameter correlation function

$$\rho_Q^{xx}(t) = \langle \psi(\Gamma_0) | \sigma_j^x(t) \sigma_j^x(0) | \psi(\Gamma_0) \rangle$$

Behavior after a quench (T=0):



Always exponential!

$$\rho_Q^{xx} \sim e^{-t/\tau_Q^{\varphi}}$$

like in equilibrium with T>0...

$$\rho_T^{xx} \sim e^{-t/\tau_T^{\varphi}}$$

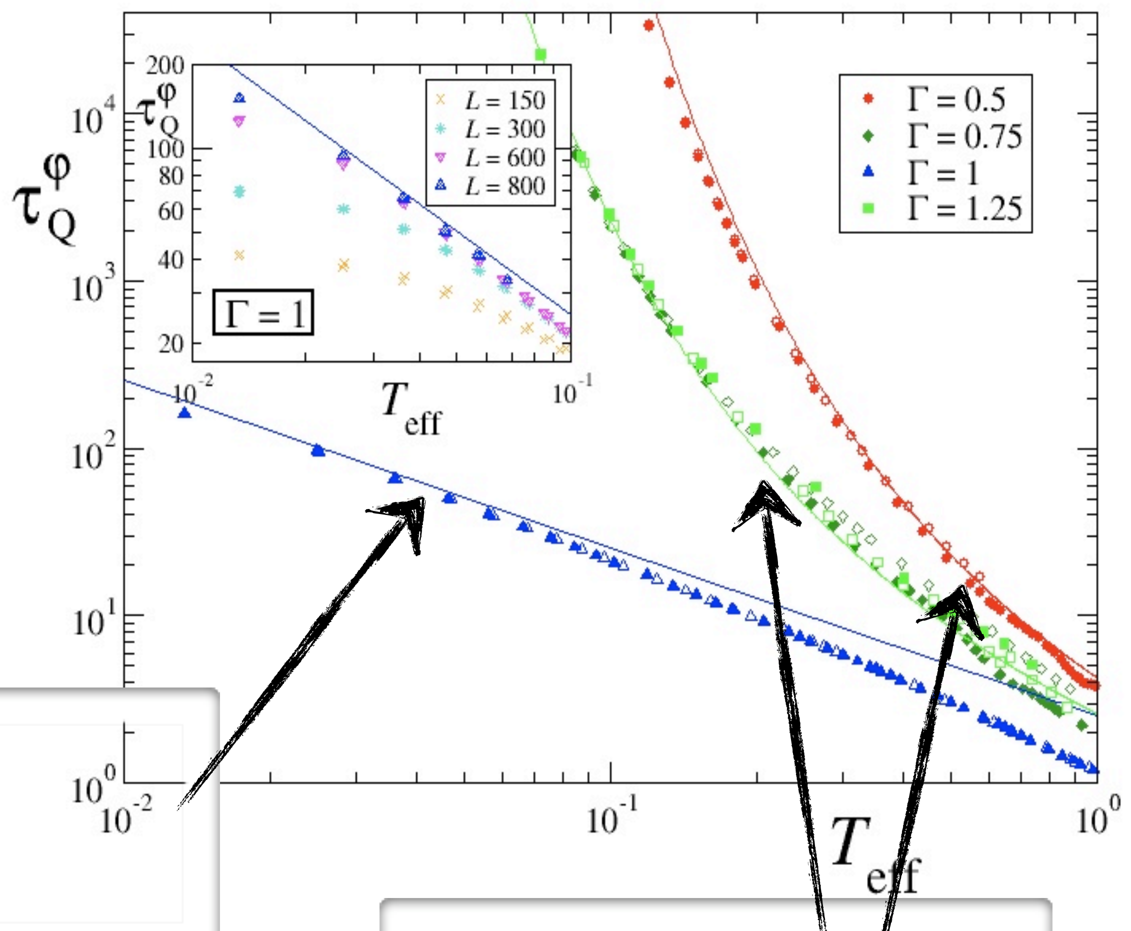
? IS THERE A LINK?

Effective temperature!

$$E_0 \equiv \langle \psi(\Gamma_0) | \mathcal{H}(\Gamma) | \psi(\Gamma_0) \rangle = \langle \mathcal{H}(\Gamma) \rangle_{T_{\mathrm{eff}}}$$

Order parameter correlation function

Coherence time as a function of $T_{\rm eff}$



$$\tau_T^\varphi \simeq \frac{8}{\pi T}$$

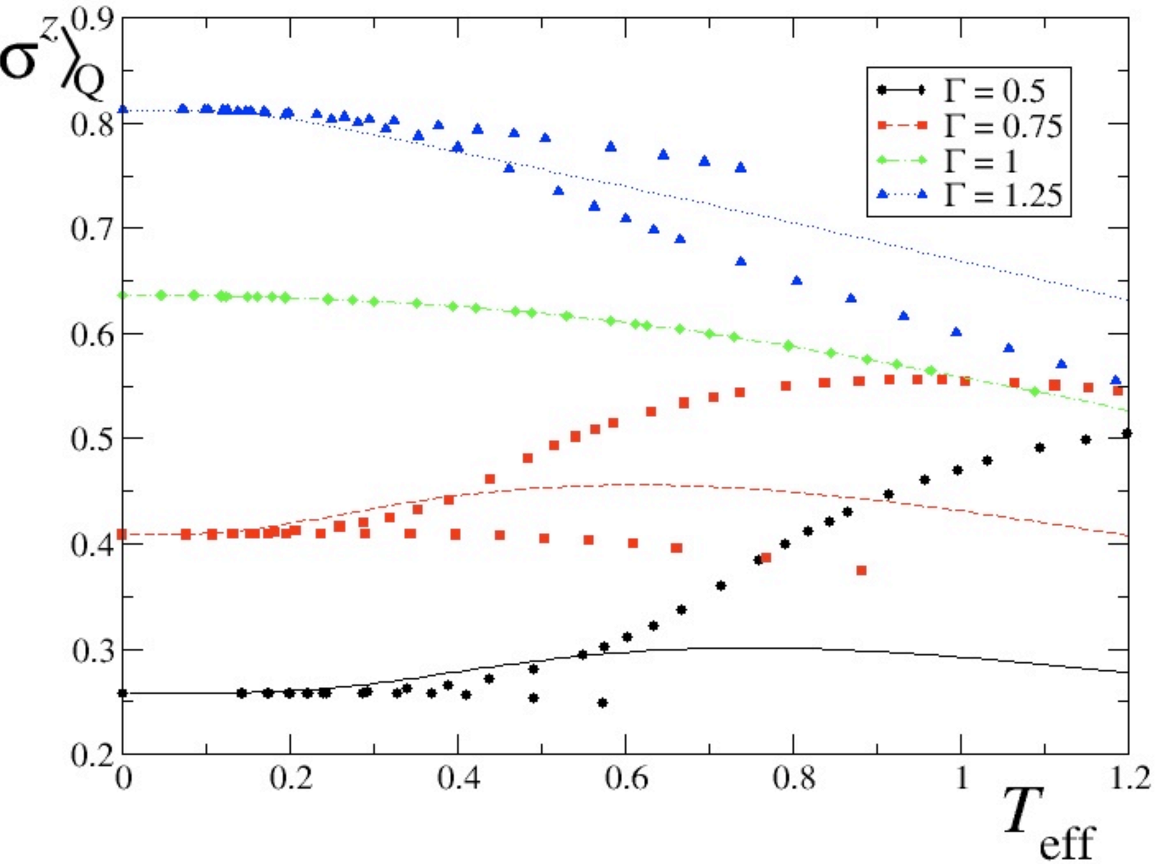
$$\tau_T^\varphi \simeq \frac{\pi}{2T} e^{\Delta/T}$$

Transverse field correlator

$$\rho_Q^{zz}(t,r) = \langle\psi(\Gamma_0)|\sigma^z_{j+r}(t_0+t)\sigma^z_j(t_0)|\psi(\Gamma_0)\rangle$$

$$\rho^{zz}(t,r) \xrightarrow{r,t\to\infty} \langle\sigma^z\rangle^2$$

BUT: $\langle\sigma^z\rangle_Q \neq \langle\sigma^z\rangle_{T\text{eff}}$!!!



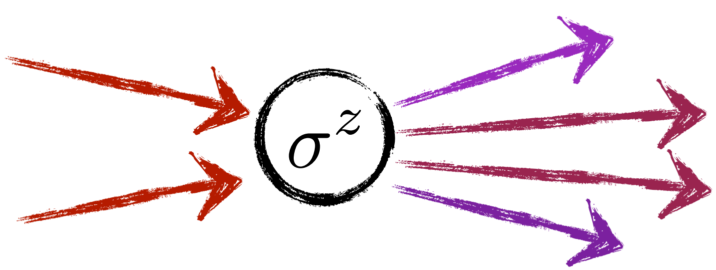
$$\overline{\langle\sigma^z(t)\rangle} = -\frac{2}{L}\sum_{k>0} A_k(\langle\gamma_k^\dagger\gamma_k\rangle - \langle\gamma_{-k}\gamma_{-k}^\dagger\rangle)$$

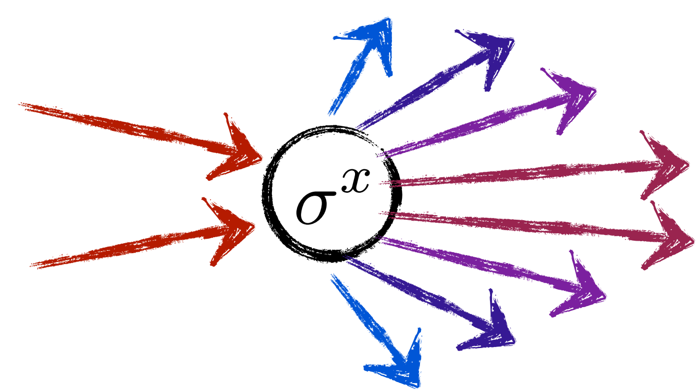
Locality vs. nonlocality

Local (integrabilty IS important)

$$\begin{cases} \sigma_j^z = & 2c_j^\dagger c_j - 1 \\ \sigma_j^x = & e^{i\pi \sum_{l<j} n_l} c_j + h.c. \end{cases}$$

Nonlocal (integrability IS NOT important)

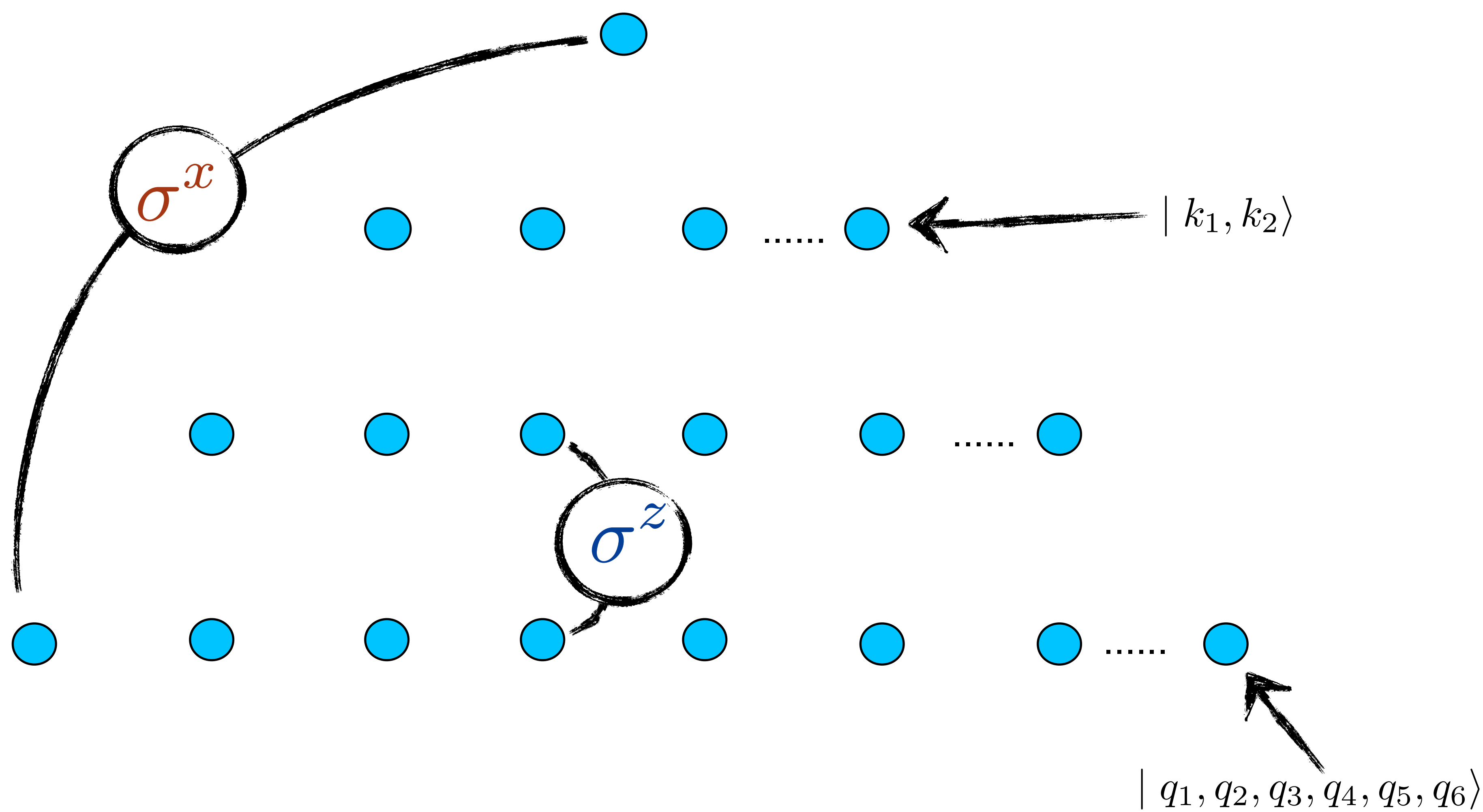




$n \rightarrow n \pm 2$

$n \rightarrow n \pm (\text{as many as you wish})$

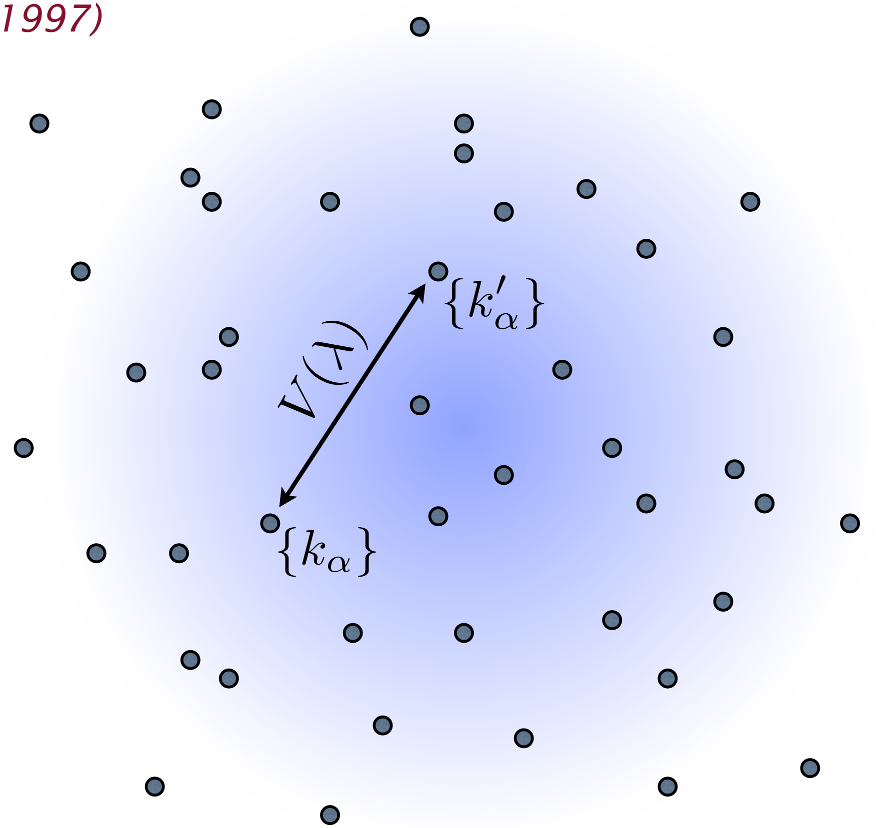
Locality vs. nonlocality



A simple scenario

Many Body Localization (Localization in qp space)

Altshuler, Gefen, Kamenev, Levitov, *Phys. Rev. Lett.* ***78***, *2803 (1997)*
Basko, Aleiner, Altshuler, *Ann. Phys. (NY)* ***321***, *1126 (2006)*



Break integrability = connect lattice points

localization/delocalization transition

Visible only in local observables (in qp)

Elena Canovi, Davide Rossini, Rosario Fazio, Giuseppe Santoro, AS, arXiv:1006.1634

The model: XXZ spin chain with random field

Uniform $\in [-1, 1]$

$$\mathcal{H} = \sum_{i=1}^{L-1} \left[\left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y \right) + J_z \sigma_i^z \sigma_{i+1}^z \right] + B_z \sum_{i=1}^{L} h_i \sigma_i^z$$

INTEGRABLE via Bethe Ansatz

NON INTEGRABLE

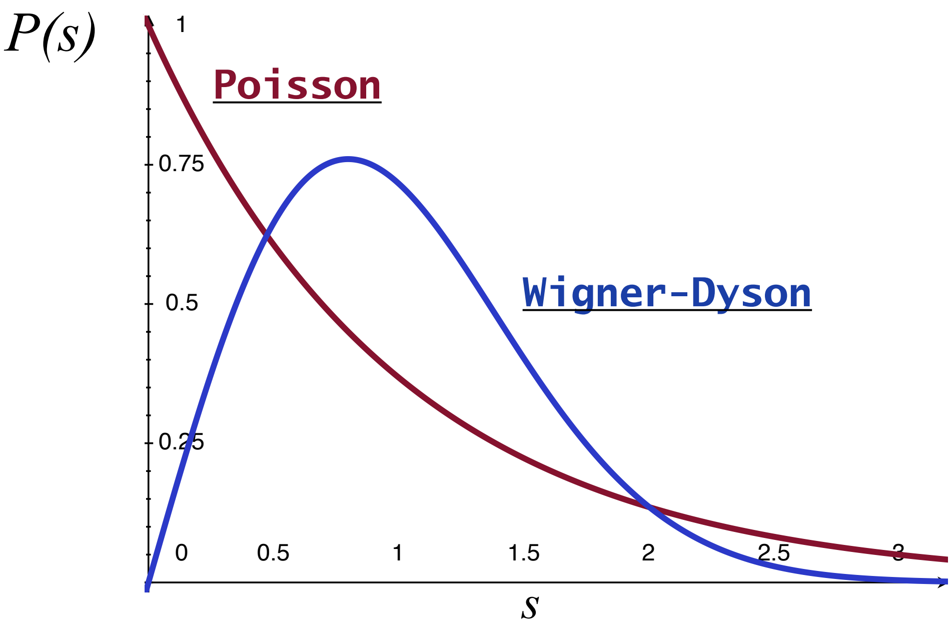
(!) For large B_z the system tends to a classical trivially integrable limit!

(!!) Magnetization is conserved, we work in the $S^z = \sum_i \sigma_i^z = 0$ sector.

(!!!) Exact diagonalization of small systems (L=14, 16).

Breaking integrability

- Spectral properties: level spacing



LEVEL STATISTICS INDICATOR:

$$\eta \equiv \frac{\int_0^\infty |P(s) - P_{\mathrm{P}}(s)| ds}{\int_0^\infty |P_{\mathrm{WD}}(s) - P_{\mathrm{P}}(s)| ds}$$

Breaking integrability

- **Eigenstates relative delocalization: <u>inverse participation ratio</u> (IPR)**

$|\psi\rangle$ **: arbitrary state,** $\{|n\rangle\}$ **: arbitrary basis**

IPR: $\xi(|\psi\rangle) \equiv \left(\sum_n |\langle n|\psi\rangle|^4 \right)^{-1}$ **: integrable basis (I)**

What do we expect in the integrable basis?

$\xi \sim 1$

in the LOCALIZED PHASE;

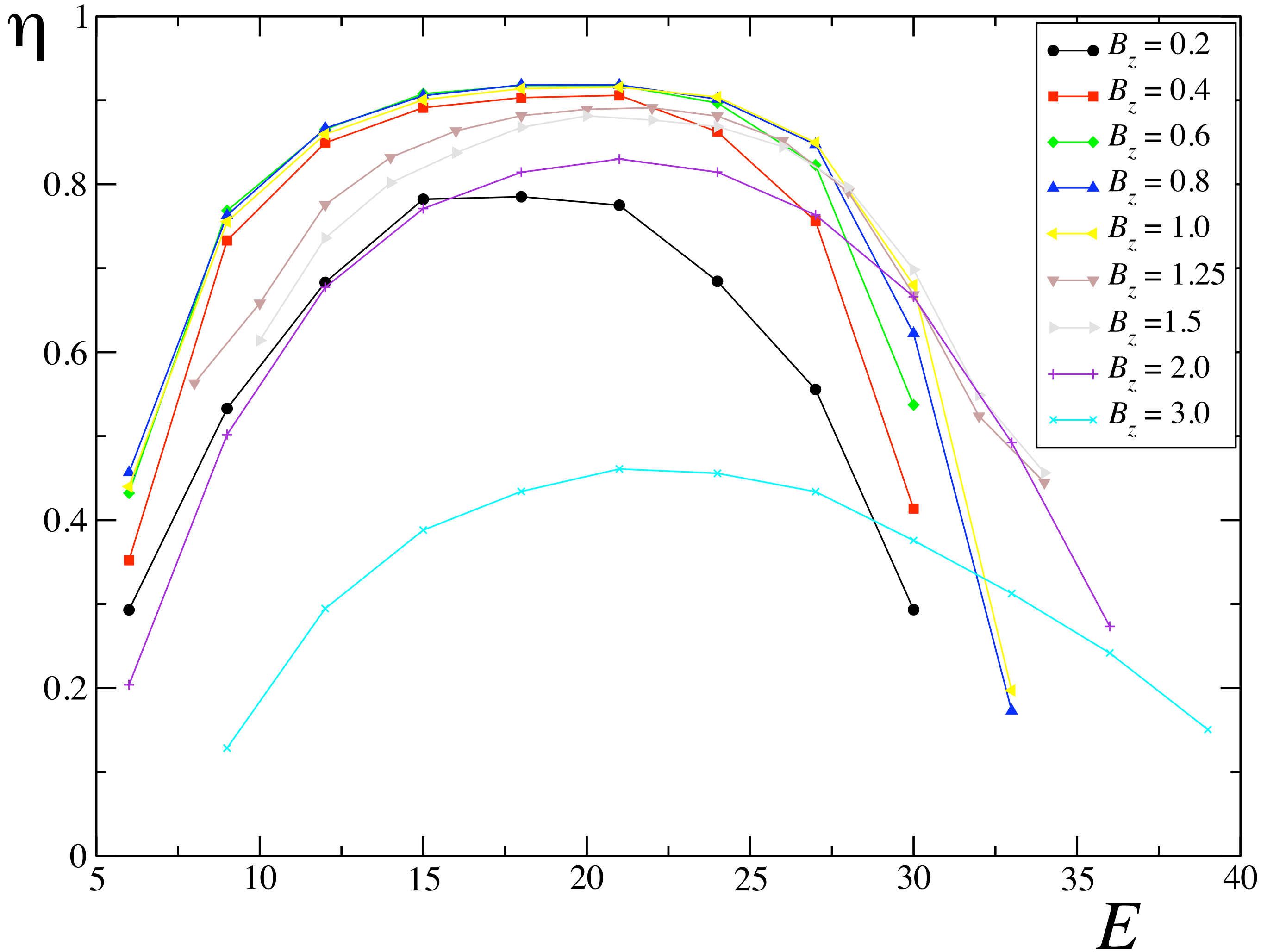
$\xi \sim \#\ \mathrm{states\ coupled\ by\ V}(\lambda)$

in the DELOCALIZED PHASE.

Breaking integrability:

<u>Level statistics indicator</u> in energy windows

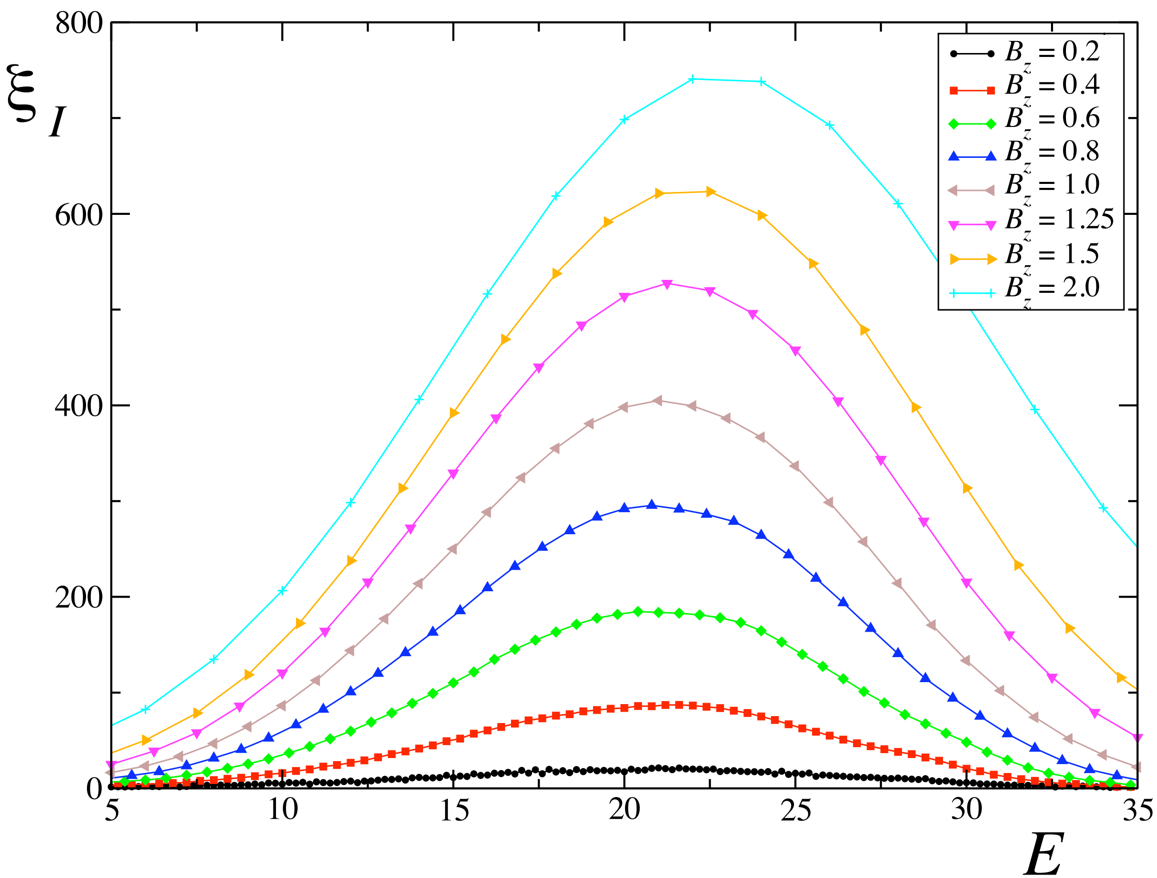
$$\eta(E) \equiv \frac{\int_0^\infty |P_{[E,E+W]}(s) - P_{\mathrm{P}}(s)|ds}{\int_0^\infty |P_{\mathrm{WD}}(s) - P_{\mathrm{P}}(s)|ds}$$



Breaking integrability:

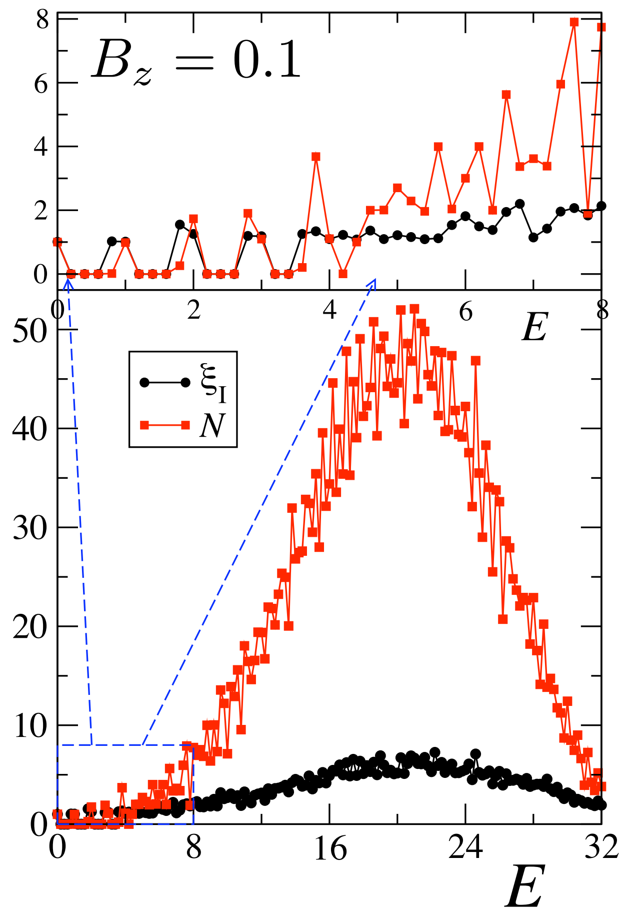
$$\xi(|\psi\rangle) \equiv \left(\sum_n |\langle n|\psi\rangle|^4\right)^{-1}$$

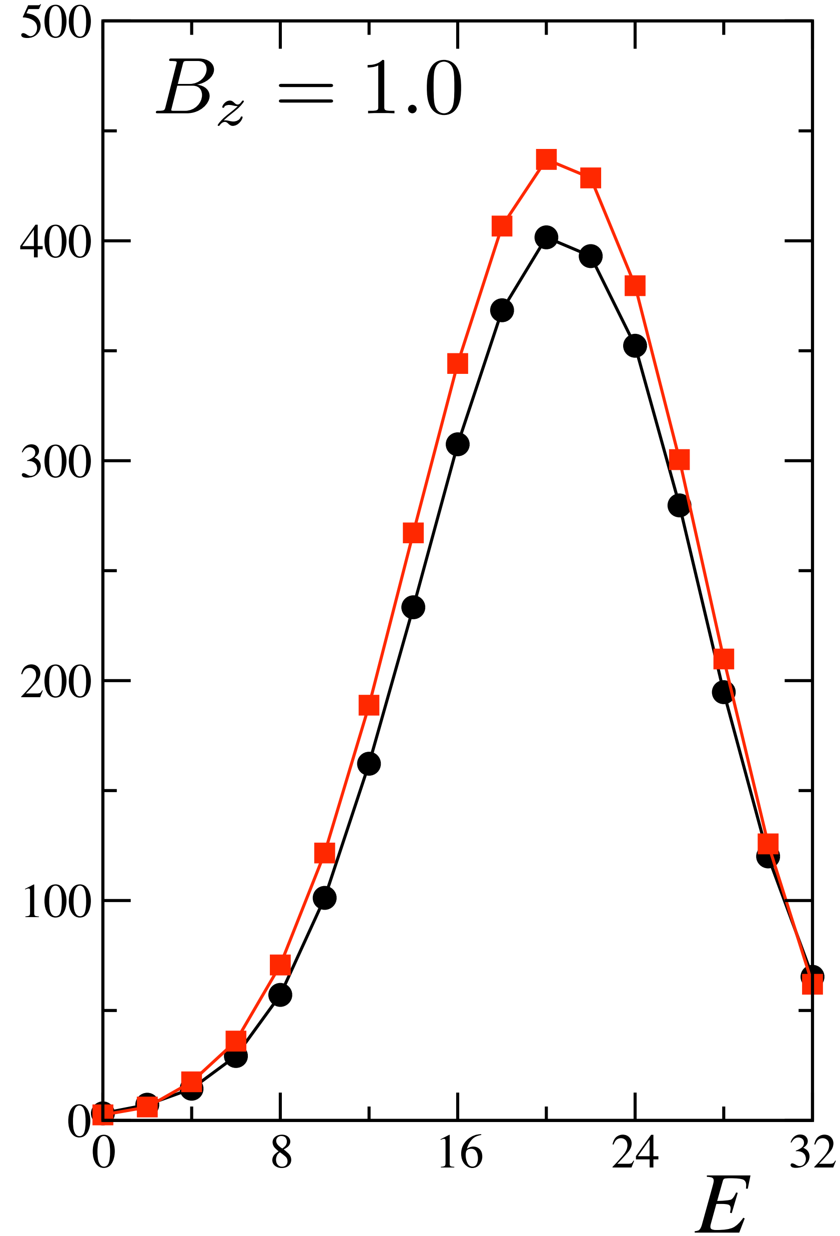
*Inverse participation ratio in the **integrable** basis*



Delocalization in qp space

$$\Delta E = 2B_z$$





Dynamics after a quantum quench

$$n_k^\alpha \equiv \frac{1}{L}\sum_{j,l=1}^{L} e^{2\pi i(j-l)k/L}\sigma_j^\alpha\sigma_l^\alpha \qquad \alpha = x, z$$

n_k^z **LOCAL**

n_k^x **NON LOCAL**

We compare the expectation values relative to:

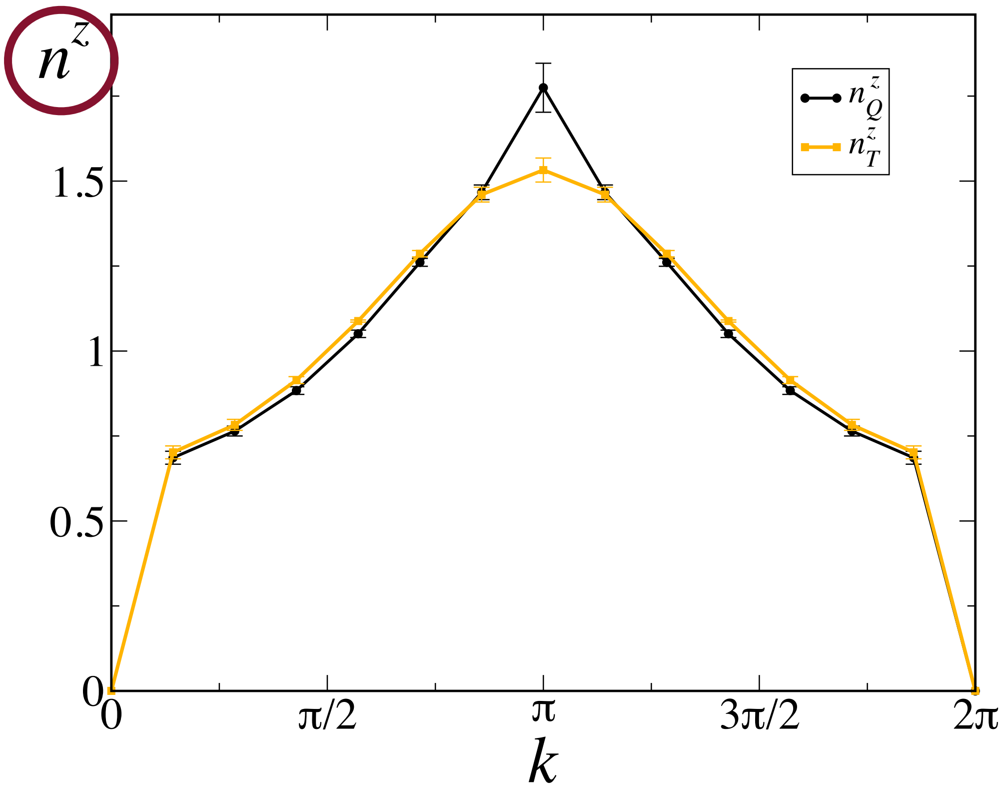
DIAGONAL ENSEMBLE

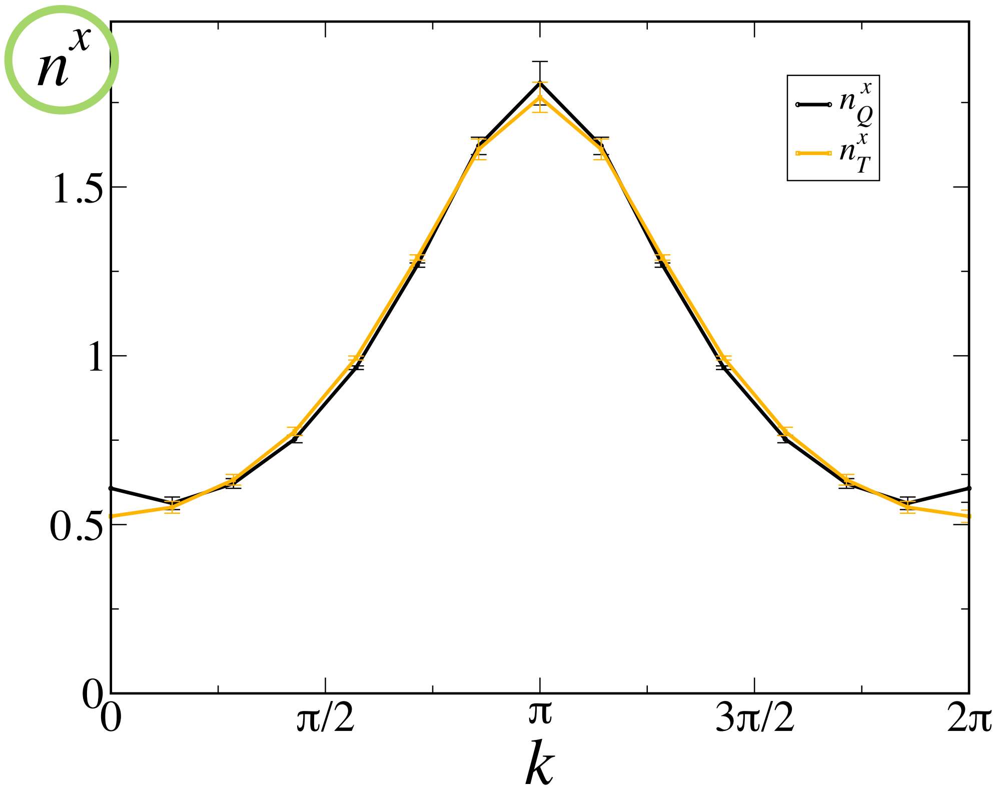
$$n_Q^\alpha(k) \equiv \lim_{t\to\infty}\langle\psi(t)|n_k^\alpha|\psi(t)\rangle$$
$$= \sum_i |c_i|^2\langle\phi_i|n_k^\alpha|\phi_i\rangle$$

CANONICAL ENSEMBLE

$$n_{T_{\mathrm{eff}}}^\alpha(k) \equiv \langle n_k^\alpha\rangle_{T_{\mathrm{eff}}}$$

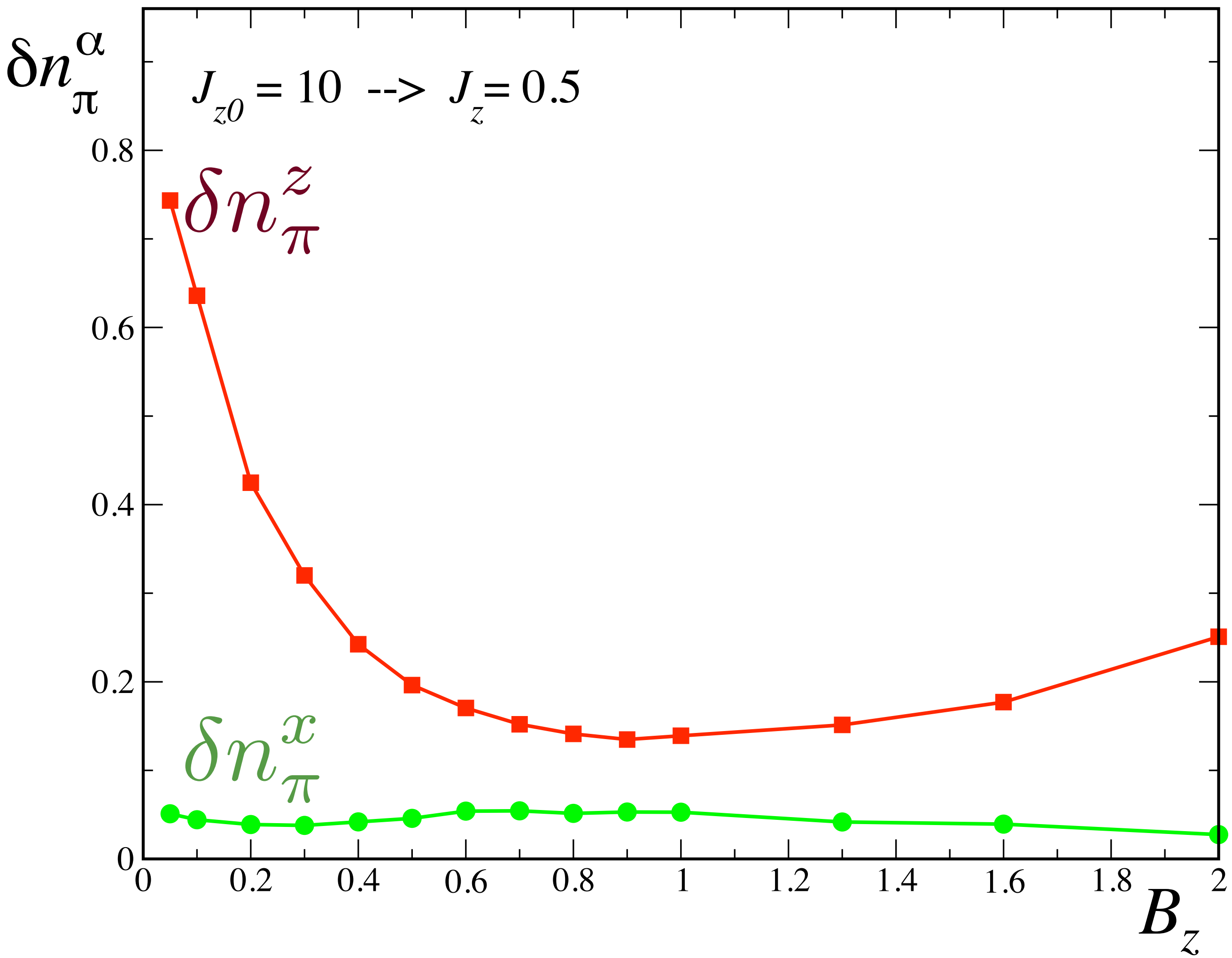
Thermalization?





$$J_{z0} = 10 \longrightarrow J_z = 0.5,\ B_z = 0.4$$

Thermalization?



Different behavior of the observables!

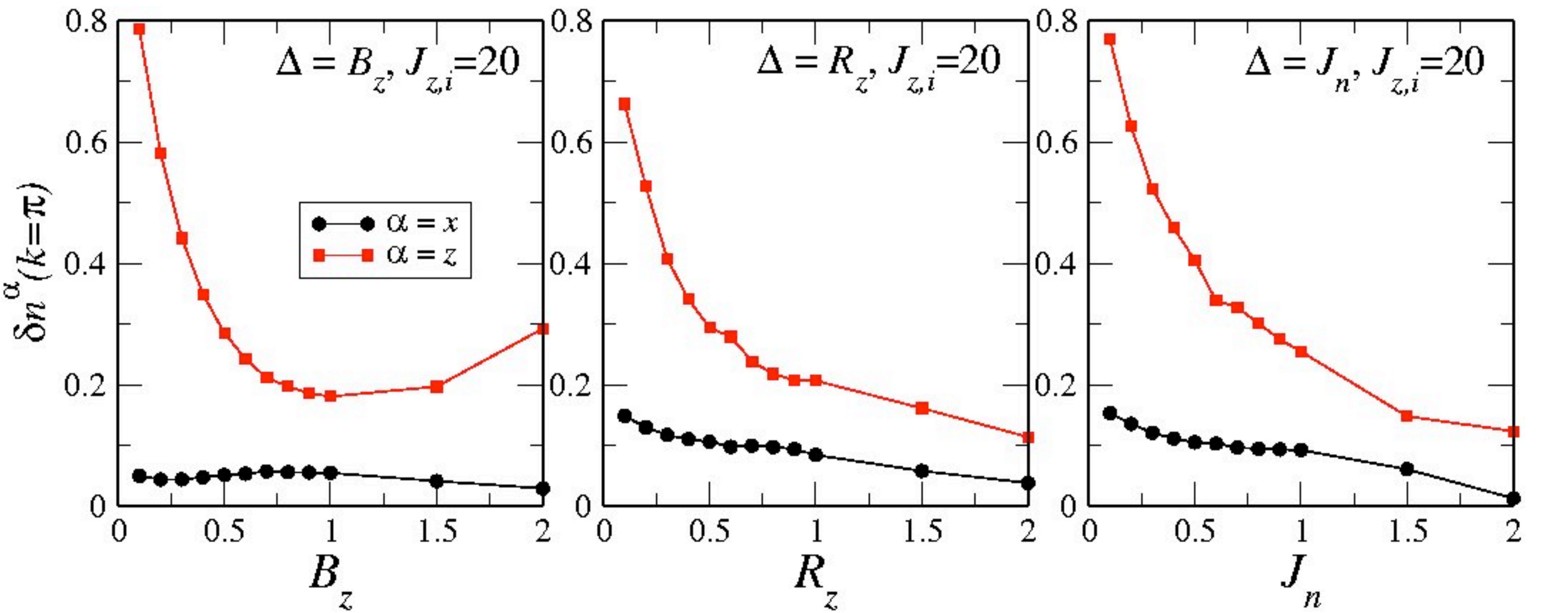
NON LOCAL in QP

- **Not sensitive to integrability.**
- **Always thermalize.**

LOCAL in QP

- **Sensitive to integrability.**
- **Thermalization with broken integrability.**

Does it depend on the integrability breaking ? Hardly



Random z-field

Random z-z Interaction

Random n.n.n. interaction

Conclusions

– Integrable systems: thermal or non–thermal ?

it depends on the observable !!

Thermal or **non-thermal** = **non-local** or **local** in **qp space**

– Non–integrable system: thermal or non–thermal ?

it depends on the eigenstates:

Thermal or **non-thermal** = **delocalized/localized** in **qp space**

Conclusions

- ☑ **Thermalization is in correspondence with the localization/delocalization transition in quasi-particle space.**

- ☑ **In order to observe this transition one has to study a <u>local</u> observable in quasi-particle space.**

Thank you for your attention!

Quantum Ising model

Sudden quench of the transverse field: $\Gamma(t) = \begin{cases} \Gamma_0 & \text{for } t < 0 \\ \Gamma & \text{for } t \geq 0 \end{cases}$

from the ground state: $|\psi(\Gamma_0)\rangle \xrightarrow{t} |\psi_t\rangle \equiv e^{-i\mathcal{H}(\Gamma)}|\psi(\Gamma_0)\rangle$

OBSERVABLES:

- Autocorrelation function of the order parameter:

$$\rho^{xx}(t) = \langle\psi(\Gamma_0)|\sigma_j^x(t)\sigma_j^x(0)|\psi(\Gamma_0)\rangle$$

NON LOCAL in QP!

- Transverse field correlator:

$$\rho^{zz}(t,r) = \langle\psi(\Gamma_0)|\sigma_{j+r}^z(t_0+t)\sigma_j^z(t)|\psi(\Gamma_0)\rangle$$

LOCAL in QP!

Rossini, Silva, Mussardo & Santoro, *PRL* ***102****, 127204 (2009);* *arXiv:1002.2842*

Not really !!

Classical

$$\overline{\delta(X - X(t))} \equiv \lim_{T\to\infty} \frac{1}{T}\int_0^T dt\, \delta(X - X(t)) = \rho_{\rm mc}(E)$$

Quantum

$$\overline{\mid \Psi(t)\rangle\langle\Psi(t)\mid} = \sum_\alpha \mid c_\alpha \mid^2 \mid \Psi_\alpha\rangle\langle\Psi_\alpha \mid = \hat{\rho}_{\rm diag} \neq \hat{\rho}_{\rm mc}(E)$$

Look at observables ... correlators, expectation values, etc ... !!!!

$$\overline{\langle\Psi(t)\mid M(t)\mid\Psi(t)\rangle} = \langle M\rangle_{\rm mc}$$

Dynamics after a quantum quench

Quench of the anisotropy parameter starting from the ground state $|\psi(J_{z0})\rangle \equiv |\psi_0\rangle$:

$$\begin{cases} J_{z0}, & t < 0 \\ J_z, & t \geq 0 \end{cases}$$

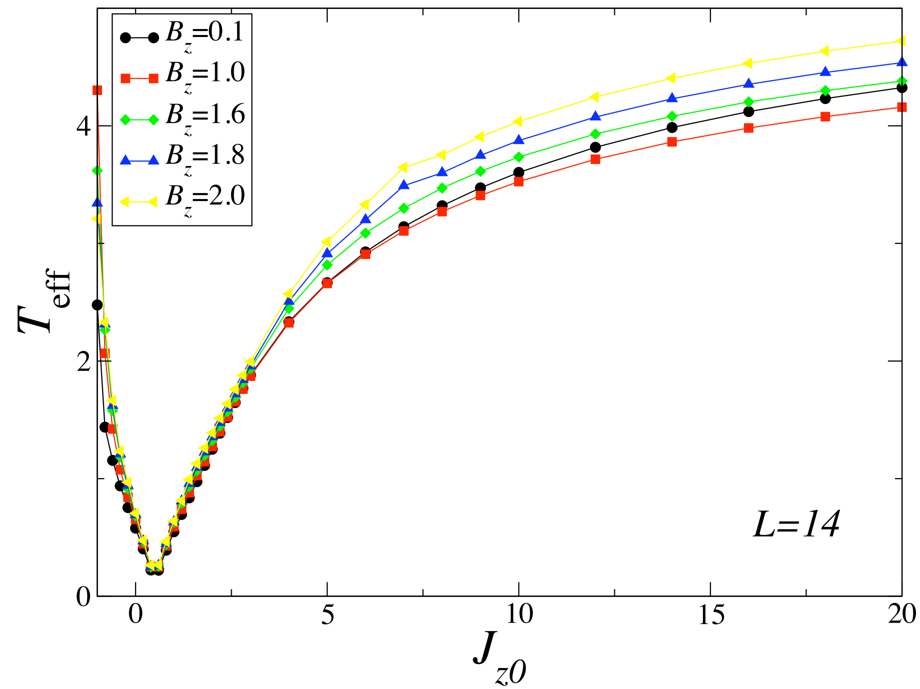
INITIAL ENERGY

$$E_0 = \langle \psi_0 | \mathcal{H}(J_z) | \psi_0 \rangle$$

$$\Rightarrow$$

EFFECTIVE TEMPERATURE

$$E_0 = \langle \mathcal{H}(J_z) \rangle_{T_{\rm eff}}$$



Integrability vs Non Integrability

Locality vs Non Locality

⇩

Many-Body Localization

Altshuler, Gefen, Kamenev, Levitov, *Phys. Rev. Lett.* ***78***, *2803 (1997)*
Basko, Aleiner, Altshuler, *Ann. Phys. (NY)* ***321***, *1126 (2006)*

Yes, but ...

---> is it true that integrable systems never thermalize ?

---> why the eigenstate thermalization hypothesis ?
what is the physics behind it ?

Generalized Gibbs Ensemble

A generic way of describing asymptotics ??

---> it keeps track of the initial values of all the constants of motion
---> it correctly reproduces the steady state in many situations
---> but it does not always work (see Gangardt and Pustilnik '08)

Rigol, Dunjko, Yurovsky, & Olshanii, *PRL (2007)*
Rigol, Muramatsu, & Olshanii, *PRA (2006)*
Cazalilla, *PRL (2006)*
Calabrese & Cardy, *PRL (2006)*, *JSTAT (2007)*
Gangardt & Pustilnik, *PRA (2008)*
Eckstein *& Kollar, PRL (2008)*, *PRA (2008)*
Iucci & Cazalilla, *arXiv (2009)*
Barther, Schollwoeck, *PRL (2008)*
Fioretto, Mussardo, *NJP (2010)*

Outline

- **Thermalization is in correspondence with the localization/ delocalization transition in quasi-particle space.**
- **In order to observe this transition one has to study a <u>local</u> observable in quasi-particle space.**

...which can be studied through...

- Model: XXZ chain with random field
- Breaking integrability: level statistics and eigenvectors
- Delocalization in Fock space
- Dynamics after a quantum quench: effective temperature
- Asymptotic behavior of observables: thermalization?

Why ??

γ_k = **quasiparticles**

$$\overline{\langle \sigma^z(t) \rangle} = -\frac{2}{L} \sum_{k>0} A_k \left(\langle \gamma_k^\dagger \gamma_k \rangle - \langle \gamma_{-k} \gamma_{-k}^\dagger \rangle \right)$$

memory through constants of motion: **integrability IS important**

$$\langle \sigma^x(x, t+T) \sigma^x(0, T) \rangle \simeq e^{-\int \frac{dk}{\pi} |x - v_k t| \langle \gamma_k^\dagger \gamma_k \rangle}$$

through semiclassics: **integrability IS NOT important** (Sachdev, Young ('95)), only universality class.

Eigenstate thermalization hypothesis

Thermalization at the level of individual eigenstates:

---> expectation value of few-body observable in a given eigenstate with energy E equals the *microcanonical average* at the mean energy E

Deutsch, *PRA (1991)*
Srednicki, *PRE (1994)*
Rigol, Dunjko, & Olshanii, *Nature (2008)*
Kollath, Lauchli & Altman, *PRL (2007)*
Manmana, Wessel, Noack, & Muramatsu, *PRL (2007)*
Rigol *PRL (2009)*, *PRA (2009)*
Biroli et al. *arXiv 0907.3731*
.....

Motivations: experiments with cold atoms

✓Collapse and revival of the order parameter

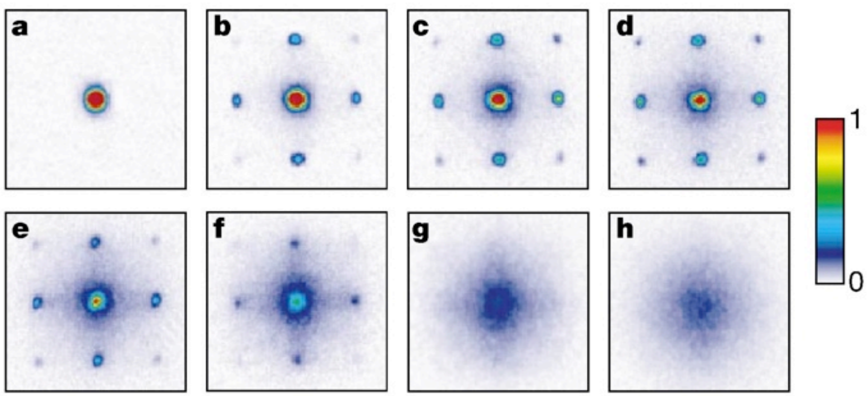
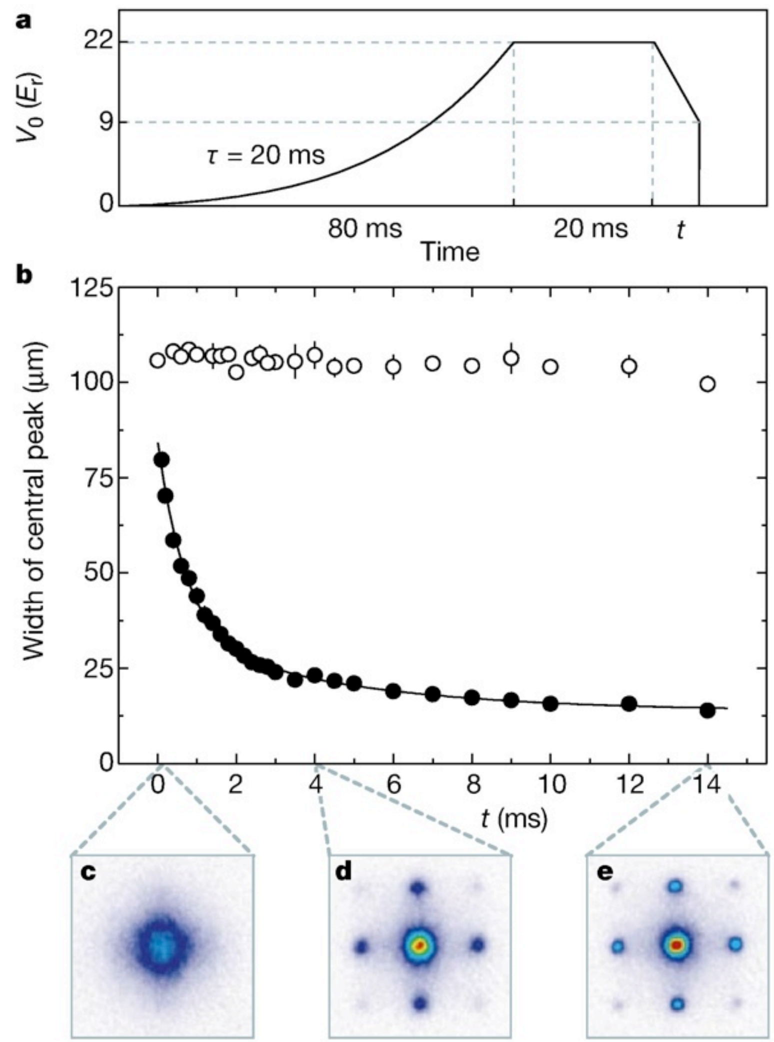


Figure 2 Absorption images of multiple matter wave interference patterns. These were obtained after suddenly releasing the atoms from an optical lattice potential with different potential depths V_0 after a time of flight of 15 ms. Values of V_0 were: **a**, 0 E_r; **b**, 3 E_r; **c**, 7 E_r; **d**, 10 E_r; **e**, 13 E_r; **f**, 14 E_r; **g**, 16 E_r; and **h**, 20 E_r.



Greiner et al, *Nature* ***415***, *39 (2002)*