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Competing orders in 1D multicomponent cold fermions at half filling

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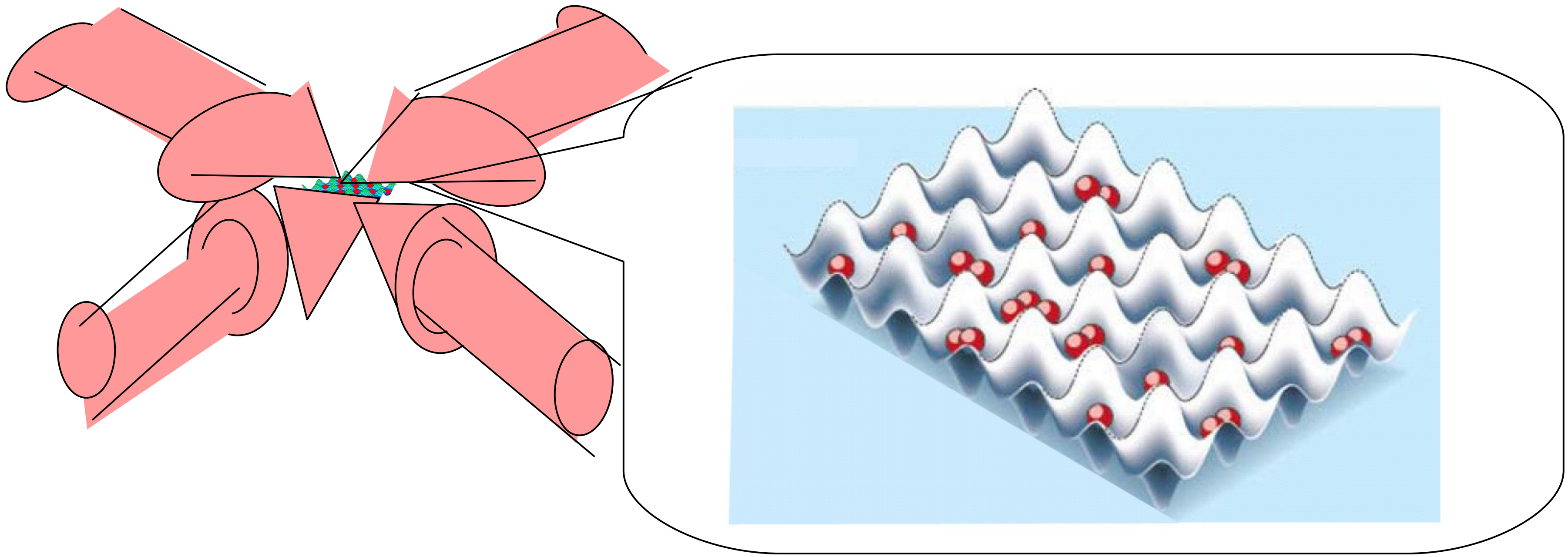
H. Nonne, P. Lecheminant, S. Capponi, G. Roux, E. Boulat, Phys. Rev. B 81, 020408(R) (2010)

Cold atoms in optical lattice : artificial solids



Theory: Jaksch et al. PRL (1998)

Experiments: Kasevich et al., Science (2001);
Greiner et al., Nature (2001);
Esslinger et al., PRL (2004)...

Effective Hamiltonian for multicomponent cold fermions

Effective Hamiltonian

Ho 1998

Fermionic cold atoms with spin F = N - 1/2

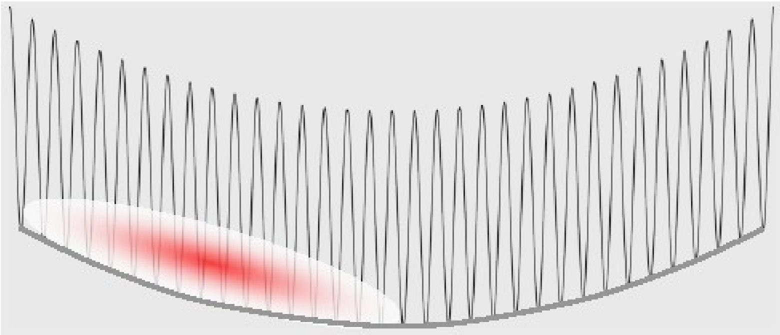
$$^{6}\,\mathrm{Li}\ (I=1,\ F=1/2),\ ^{40}\,\mathrm{K}\ (I=4,\ F=9/2)$$

Hyp: SU(2) invariance + contact interaction (s-wave)

$$\mathcal{H} = \sum_{\alpha=1}^{2N} \int d^d\vec{r}\, \Psi^\dagger_\alpha(\vec{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{lattice}}(\vec{r}) + V_T(\vec{r}) \right) \Psi_\alpha(\vec{r}) \qquad g_J = \frac{4\pi\hbar^2 a_J}{m}$$

$$+ \int d^d\vec{r} \sum_J g_J \sum_{M=-J}^{J} P^\dagger_{J,M}(\vec{r})\, P_{J,M}(\vec{r}) \qquad P^\dagger_{J,M}(\vec{r}) = \sum_{\alpha,\beta} \langle JM|F,F;\alpha,\beta\rangle\ \Psi^\dagger_\alpha(\vec{r})\,\Psi^\dagger_\beta(\vec{r})$$

Pauli's principle: J even J=0, 2,.., 2N-2



Wannier's approach:

$$\Psi_\alpha(\vec{r}) = \sum_i w^{(0)}(\vec{r}-\vec{r}_i)\ c_{\alpha,i}$$

Fermions on a lattice

Lattice effective Hamiltonian

F = N - 1/2 Fermions with 2N internal states

$$\mathcal{H} = -t\sum_{i,\alpha}\left(c^{\dagger}_{\alpha,i}c_{\alpha,i+1} + H.c.\right) - \mu\sum_{i} n_i + \sum_{i,J} U_J \sum_{M=-J}^{J} P^{\dagger}_{J,M,i}P_{J,M,i}$$

$\alpha = 1,\ldots,2N \quad J = 0,2,\ldots,2N-2$

N = 1 (F =1/2) Hubbard model: exactly solvable in1D

Attractive interaction (U < 0) + half-filling:

BCS Phase (charge 2e)

Metallic regime: Power-law decay $\langle P_{0,0,i}P^{\dagger}_{0,0,i+x}\rangle \sim \frac{1}{x}$

For arbitrary F in 1D: Phase diagram at half-filling?

Spin F = 3/2 (N=2) case

$$\mathcal{H} = \mathcal{H}_0 - \mu \sum_i c^\dagger_{\alpha,i} c_{\alpha,i} + U_0 \sum_i P^\dagger_{0,0,i} P_{0,0,i} + U_2 \sum_i \sum_{m=-2}^{2} P^\dagger_{2,m,i} P_{2,m,i}$$

Naive symmetry of $\mathcal{H} : \mathrm{U}(2) = \mathrm{U}(1) \times \mathrm{SU}(2)$

SO(5) symmetry without fine tuning!! Wu, Ping, Zhang 2003

$$\mathcal{H} = \mathcal{H}_0 - \mu \sum_i c^\dagger_{\alpha,i} c_{\alpha,i} + \frac{U}{2} \sum_i n_i^2 + V \sum_i P^\dagger_{0,0,i} P_{0,0,i}$$

$U = 2U_2,\ V = U_0 - U_2$ $\qquad 2P^\dagger_{0,0,i} = c^\dagger_{\alpha,i} \mathcal{J}_{\alpha\beta} c^\dagger_{\beta,i},\quad \mathcal{J} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$

$P^\dagger_{0,0,i}$ invariant sous $U \in U(4),\ \ {}^{t}U J U = J$

Symmetry group: $Sp(4) \sim SO(5)$

Sp(2N) symmetry spin F = N -1/2

Special Fine tuning: $U_2 = U_4 = \ldots = U_{2N-2}$

$$\mathcal{H} = \mathcal{H}_0 - \mu \sum_i c^\dagger_{\alpha,i} c_{\alpha,i} + \frac{U}{2} \sum_i n_i^2 + V \sum_i P^\dagger_{0,0,i} P_{0,0,i}$$

$U = 2U_2$, $V = U_0 - U_2$

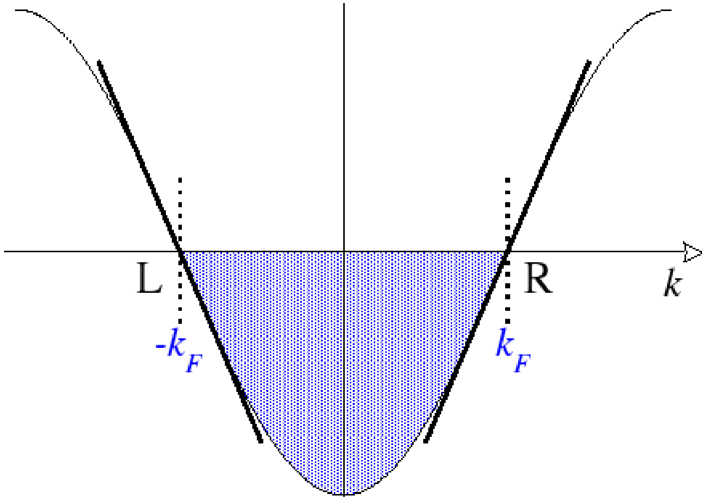
$V = 0$ U(2N) Hubbard model

$V \neq 0$ U(1)xSp(2N) symmetry

Low-energy approach

Low-energy Approach

Continuum limit:



Linearisation of the dispersion relation

2N Dirac fermions

Spin-charge separation: For N=1

$$\mathcal{H} = \mathcal{H}_c + \mathcal{H}_s,\ [\mathcal{H}_c, \mathcal{H}_s] = 0$$

For N>1: no spin-charge separation at half filling:

charge and spin modes are strongly coupled

CFT approach

Symmetry of the non-interacting fixed point: $SO(4N)_1$ CFT

Global symmetry group of the interaction: U(1) x Sp(2N)

Conformal embedding:

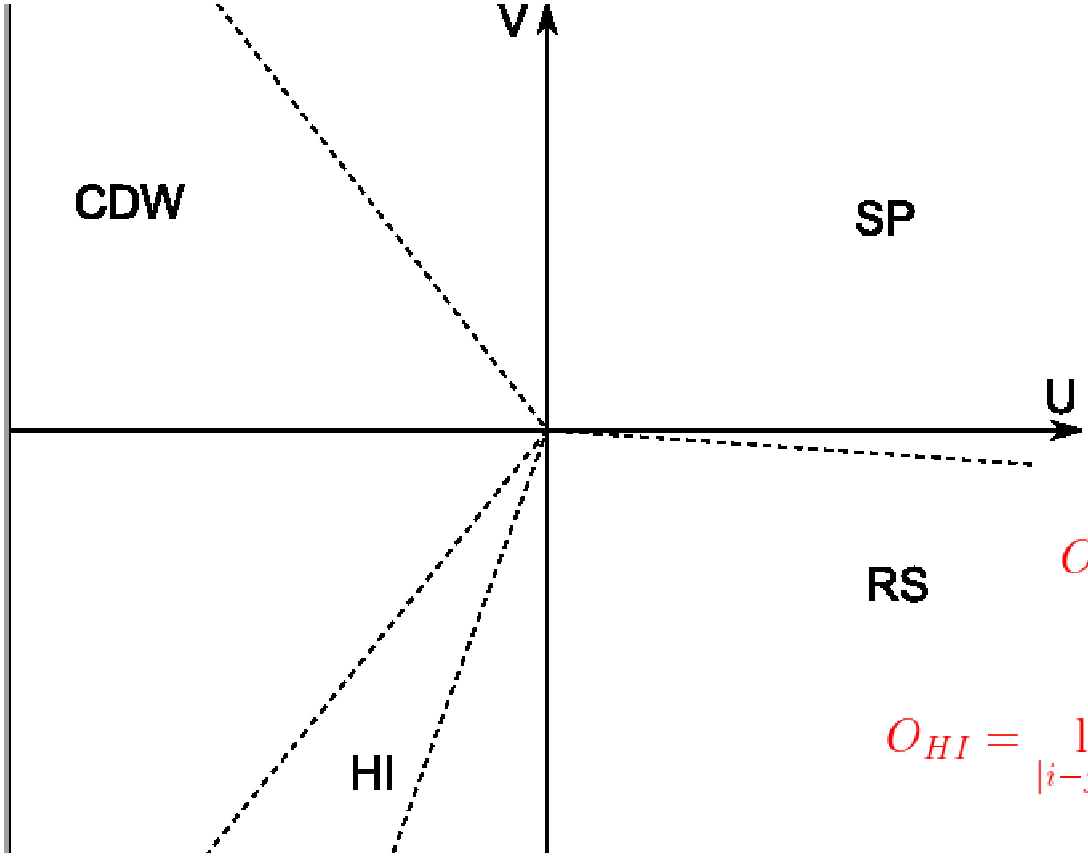
$SO(4N)_1 \sim SU(2)_N \times Sp(2N)_1 \sim U(1) \times Z_N \times Sp(2N)_1$

Simplification for N=2 (free-field fermionic representation):

$$\mathcal{H}_{\rm int} = \frac{g_1}{2}\left(\sum_{a=1}^{5} \xi_R^a \xi_L^a\right)^2 + g_2\, \xi_R^6 \xi_L^6 \sum_{a=1}^{5} \xi_R^a \xi_L^a + \frac{g_3}{2}\left(\xi_R^7 \xi_L^7 + \xi_R^8 \xi_L^8\right)^2 + \left(\xi_R^7 \xi_L^7 + \xi_R^8 \xi_L^8\right)\left(g_4 \sum_{a=1}^{5} \xi_R^a \xi_L^a + g_5\, \xi_R^6 \xi_L^6\right)$$

Low-energy approach (N=2) at half-filling

Four fully gapped insulating phases



$$O_{SP} = \langle (-1)^i \sum_\alpha c^\dagger_{\alpha,i+1} c_{\alpha,i} \rangle \neq 0$$

$$O_{CDW} = \langle (-1)^i \delta n_i \rangle \neq 0$$

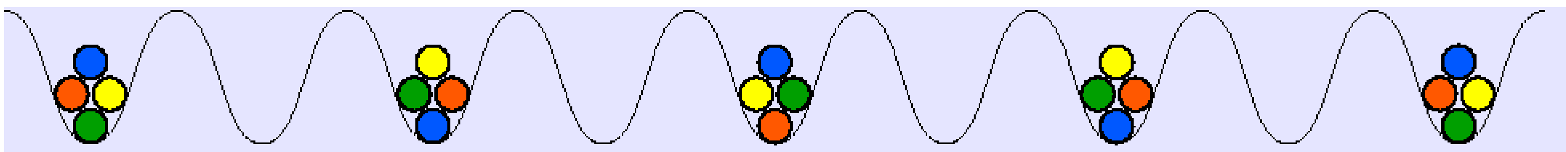
$$\delta n_i = n_i - \langle n_i \rangle = \sum_\alpha c^\dagger_{\alpha,i} c_{\alpha,i} - \langle n_i \rangle$$

$$O_{RS} = \lim_{|i-j|\to\infty} \langle \cos\left(\pi/2 \sum_{k=i+1}^{j-1} \delta n_k \right) \rangle \neq 0$$

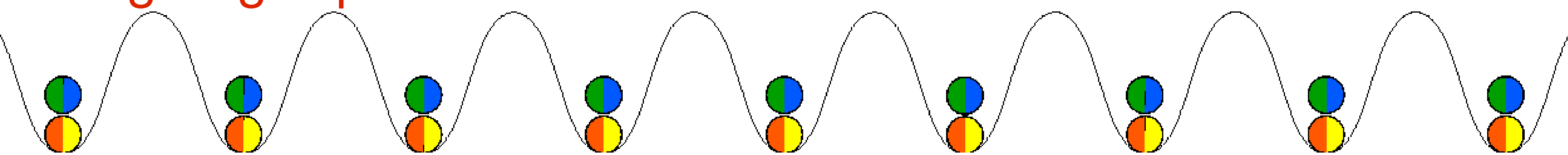
$$O_{HI} = \lim_{|i-j|\to\infty} \langle \delta n_i \cos\left(\pi/2 \sum_{k=i+1}^{j-1} \delta n_k \right) \delta n_j \rangle \neq 0$$

Physical interpretation of the insulating phases (2 atoms/site)

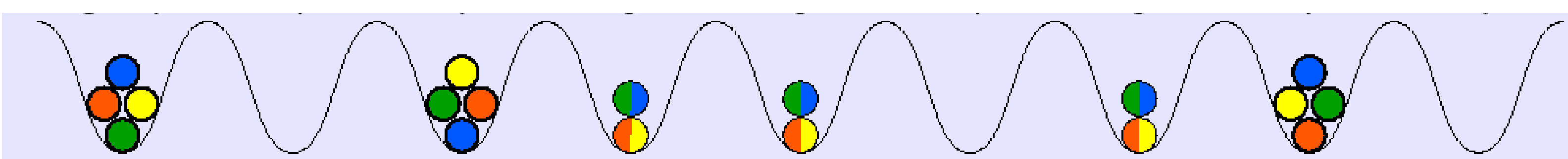
CDW phase



Rung singlet phase



Haldane insulating phase



Strong-coupling argument

Hidden SU(2) charge symmetry

2N component fermions $c_{\alpha,i}$

Pseudo-spin operators:

$$\mathcal{S}_i^{\dagger} = \frac{1}{2} c^{\dagger}_{\alpha,i} \mathcal{J}_{\alpha\beta} c^{\dagger}_{\beta,i}$$

SU(2) commutation relations

$$\mathcal{S}_i^{z} = \frac{1}{2}(n_i - N)$$

Sp(2N) spin-singlet

Along the special line V = N U, at half-filling

the model has an extended SU(2)c x Sp(2N)s symmetry

N=1 i.e. F=1/2

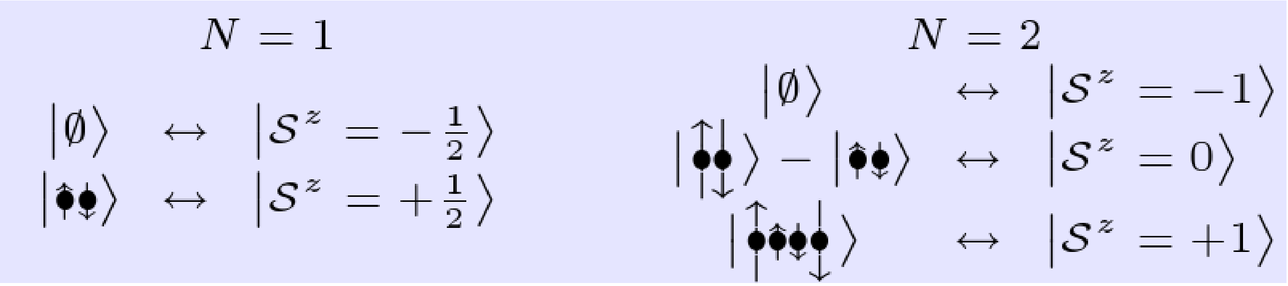
C. N. Yang 1989

Hubbard chain at half-filling has an SU(2) x SU(2) ~ SO(4) symmetry

Strong coupling analysis

Along the V=NU line $$\mathcal{H}_{t=0} = 2U \sum_i \left(\vec{\mathcal{S}}_i^2 - \frac{N}{2}\left(\frac{N}{2}+1\right)\right)$$

Lowest energy states for U<0: N+1 states, pseudo spin N/2



Strong-coupling Hamiltonian: $|U| \gg t$

Heisenberg Hamiltonian $$\mathcal{H}_{\text{eff}} = J \sum_i \vec{\mathcal{S}}_i \cdot \vec{\mathcal{S}}_{i+1} \quad J = \frac{4t^2}{N(2N+1)|U|}$$

Heisenberg Hamiltonian for spin-singlet charged states!

Deviation from the V=NU line

Strong coupling analysis: Heisenberg model with a single-ion anisotropy

$$\mathcal{H}_{\text{eff}} = J \sum_i \vec{\mathcal{S}}_i \cdot \vec{\mathcal{S}}_{i+1} + D \sum_i \left(\mathcal{S}_i^z\right)^2 \qquad D = \frac{2(NU - V)}{N}$$

Phase diagram is known from H. Schulz 1986

N even three fully gapped phases:

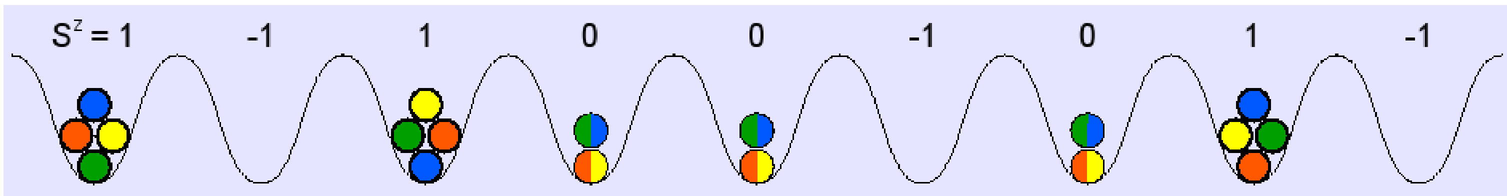
Haldane, Ising, large-D (singlet) phases

Ising ⟷ CDW Large-D phase ⟷ Rung singlet

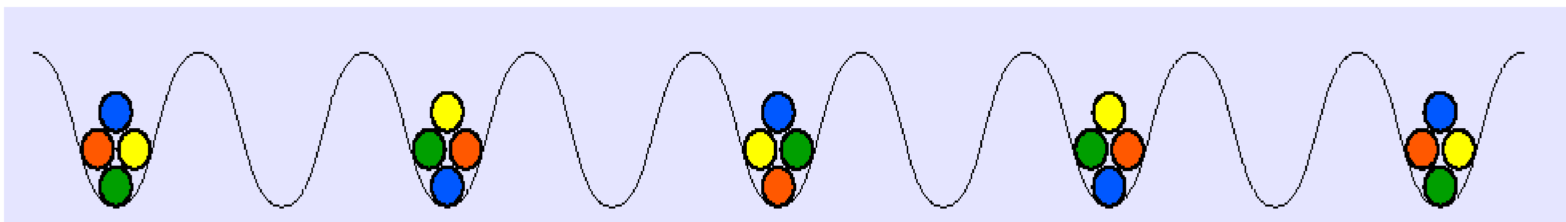
N odd: Ising and XY (gapless) phases

Summary: phases for N=2

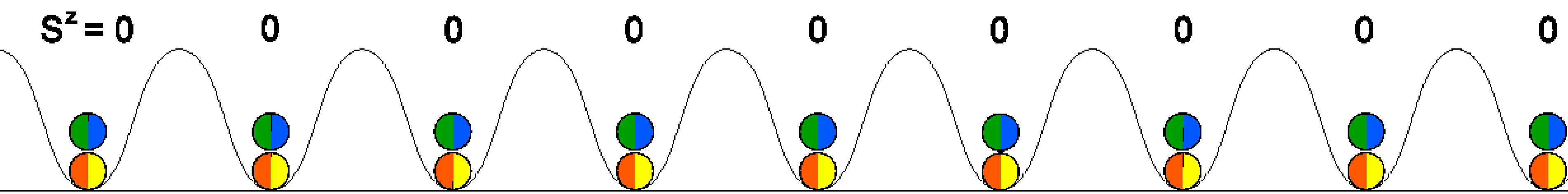
Haldane



Ising (CDW)



Large-D (rung singlet)



Haldane charge conjecture

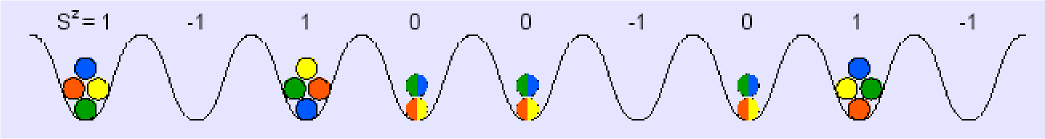
Along V=NU and attractive U, strong coupling approach gives:

N even Haldane insulating phase

$$\langle \mathcal{S}_i^{\dagger} \mathcal{S}_i^{-} \rangle = \langle P_{0,0,i} P^{\dagger}_{0,0,i+x} \rangle \sim \frac{1}{x}$$

N odd Metallic phase (Power-law decay)

Haldane insulating phase for N=2



Hidden ordering

However the generic cases are: N = 3 (F=5/2) and N=4 (F=7/2) and not N =1 (F=1/2) and N=2 (F=3/2)

Low-energy approach for N>2

Low-energy approach N>2 at half-filling

Conformal embedding

$SO(4N)_1 \sim SU(2)_N \times Sp(2N)_1 \sim U(1) \times Z_N \times Sp(2N)_1$

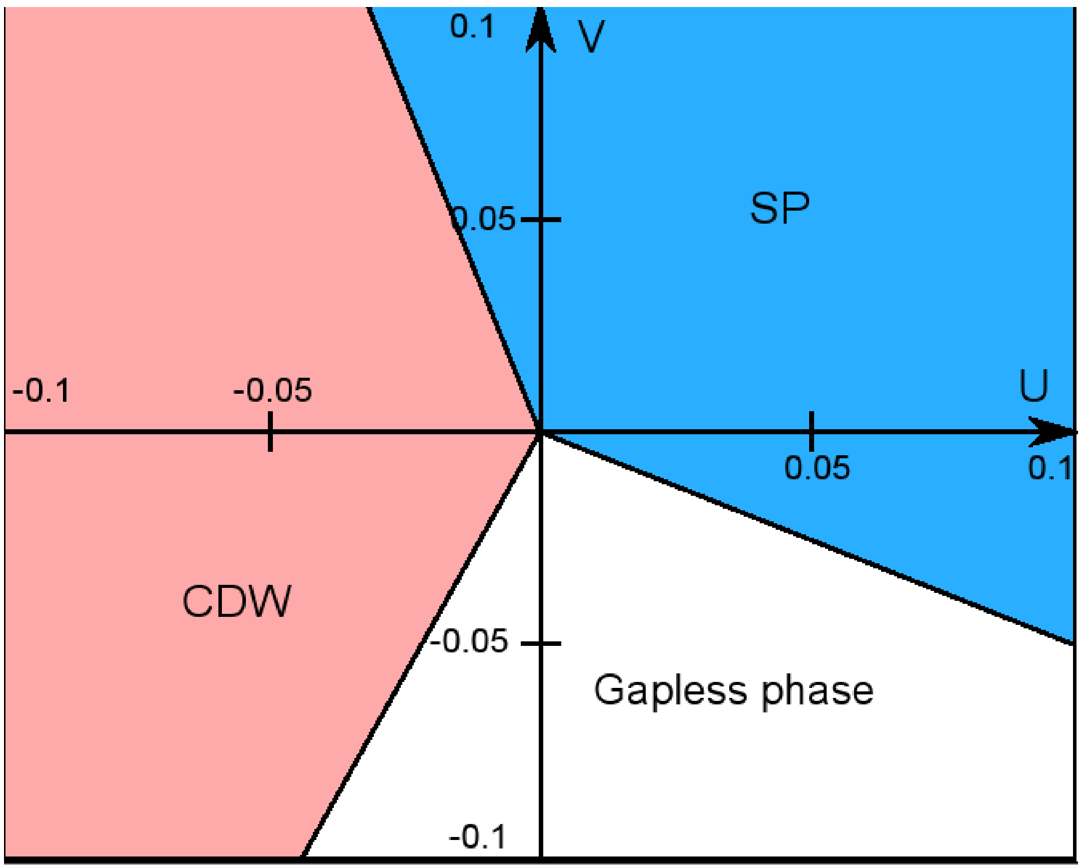
RG approach: always 2 doubly degenerate phases (CDW,SP)
+ others phases for V<0 depending on the parity N

Effective theory for V <0 (Integration of the
gapped Sp(2N) degrees of freedom)

$$\mathcal{H}_{\rm int} = g_2\, \epsilon_1 + g_3\, \left(\partial_x \Phi_c\right)^2 + g_4\, \mu_2 \exp\left(i\sqrt{8\pi/N}\Phi_c\right) + H.c.$$

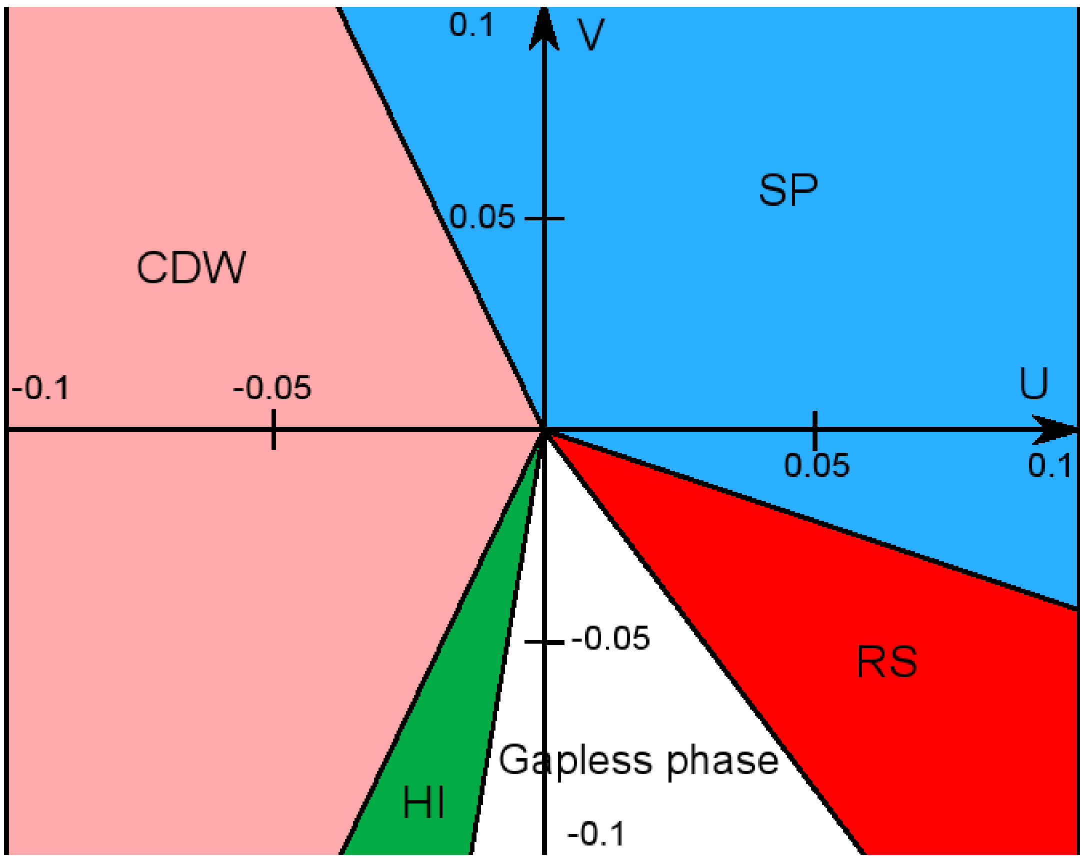
Phase diagram for N odd > 1

RG approach:



Phase diagram for N even > 2

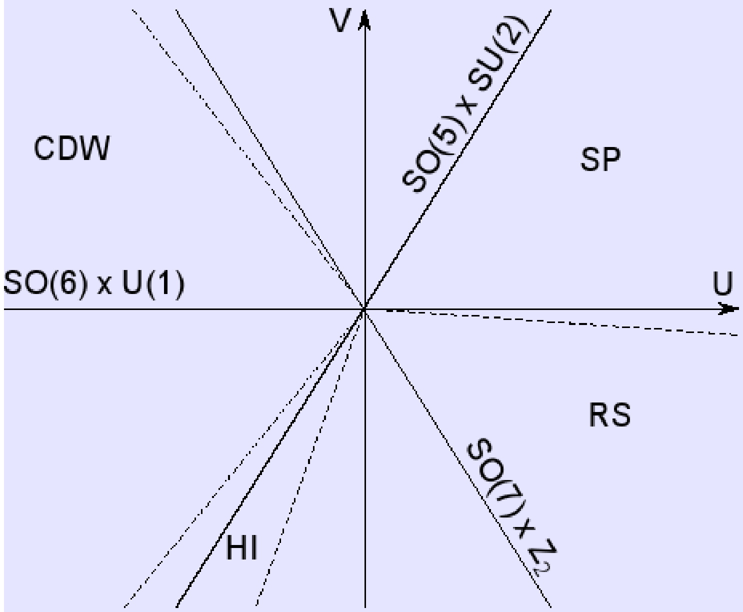
RG approach



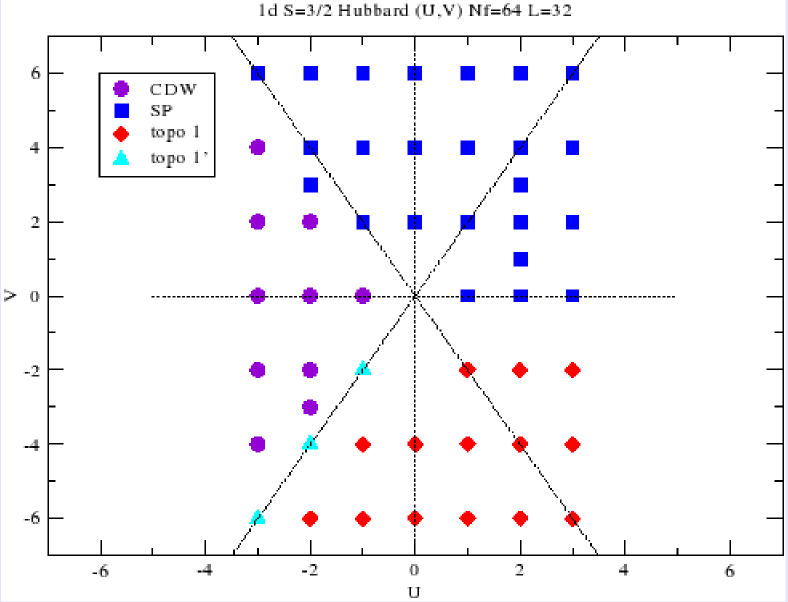
DMRG results for N=2,3

DMRG N=2 (F=3/2)

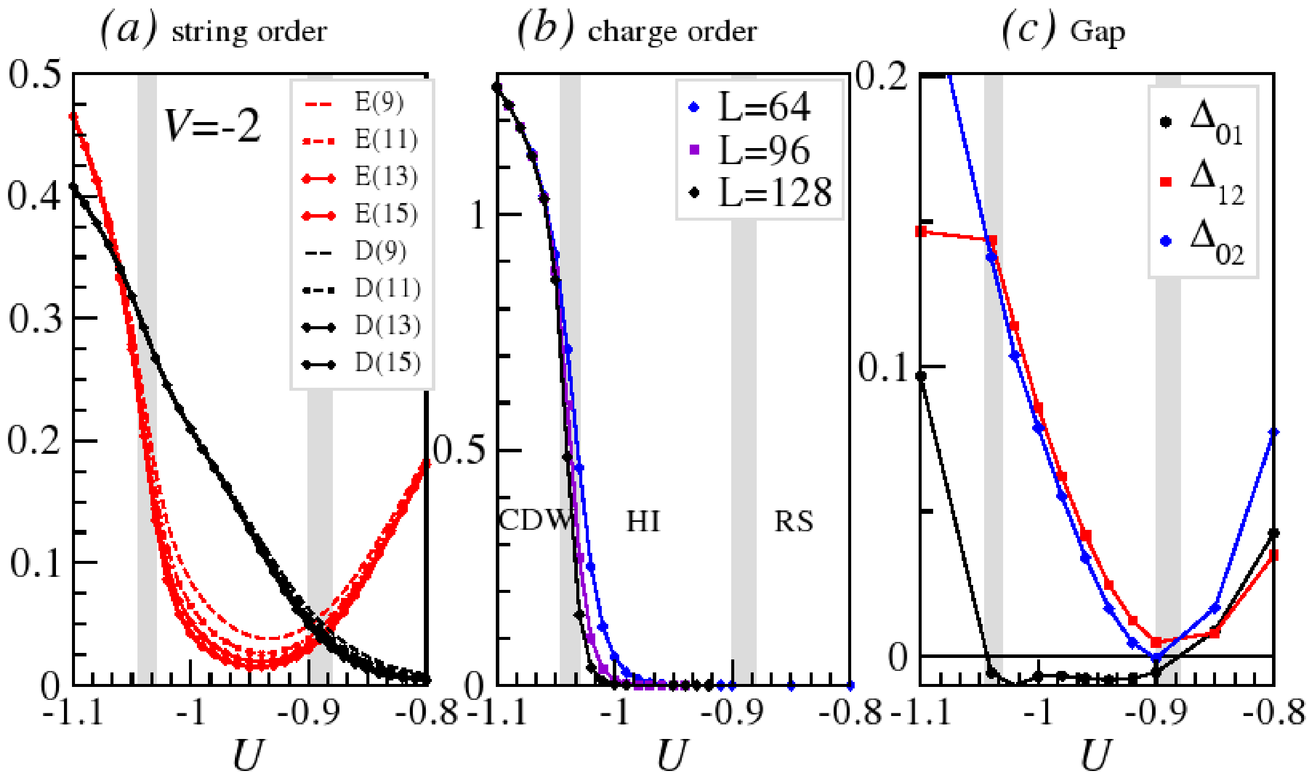
Low-energy approach



DMRG



DMRG results (N=2)



$$\Delta_{ab} = E_0(N_f = 2L + 2b) - E_0(N_f = 2L + 2a)$$

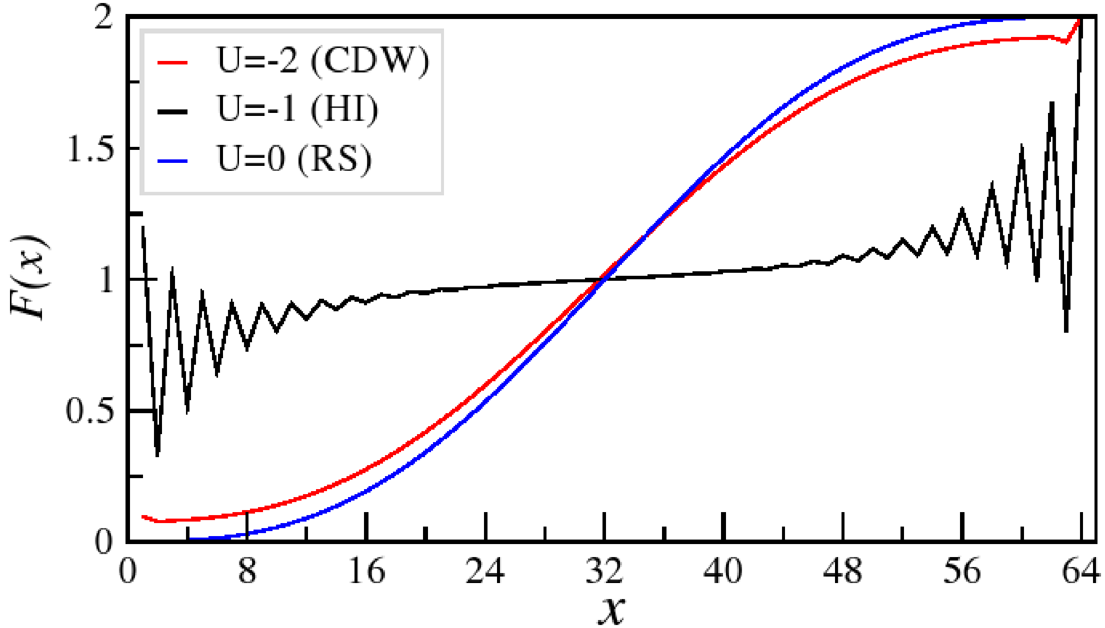
$$E(|i - j|) = |\langle \exp\left(i\pi \sum_{i<k<j} \frac{\delta n_k}{2}\right)\rangle|$$

$$D(|i - j|) = |\langle \frac{\delta n_i}{2} \exp\left(i\pi \sum_{i<k<j} \frac{\delta n_k}{2}\right) \frac{\delta n_j}{2}\rangle|$$

Edge states

Haldane insulating phases: edge states (spin-singlet)

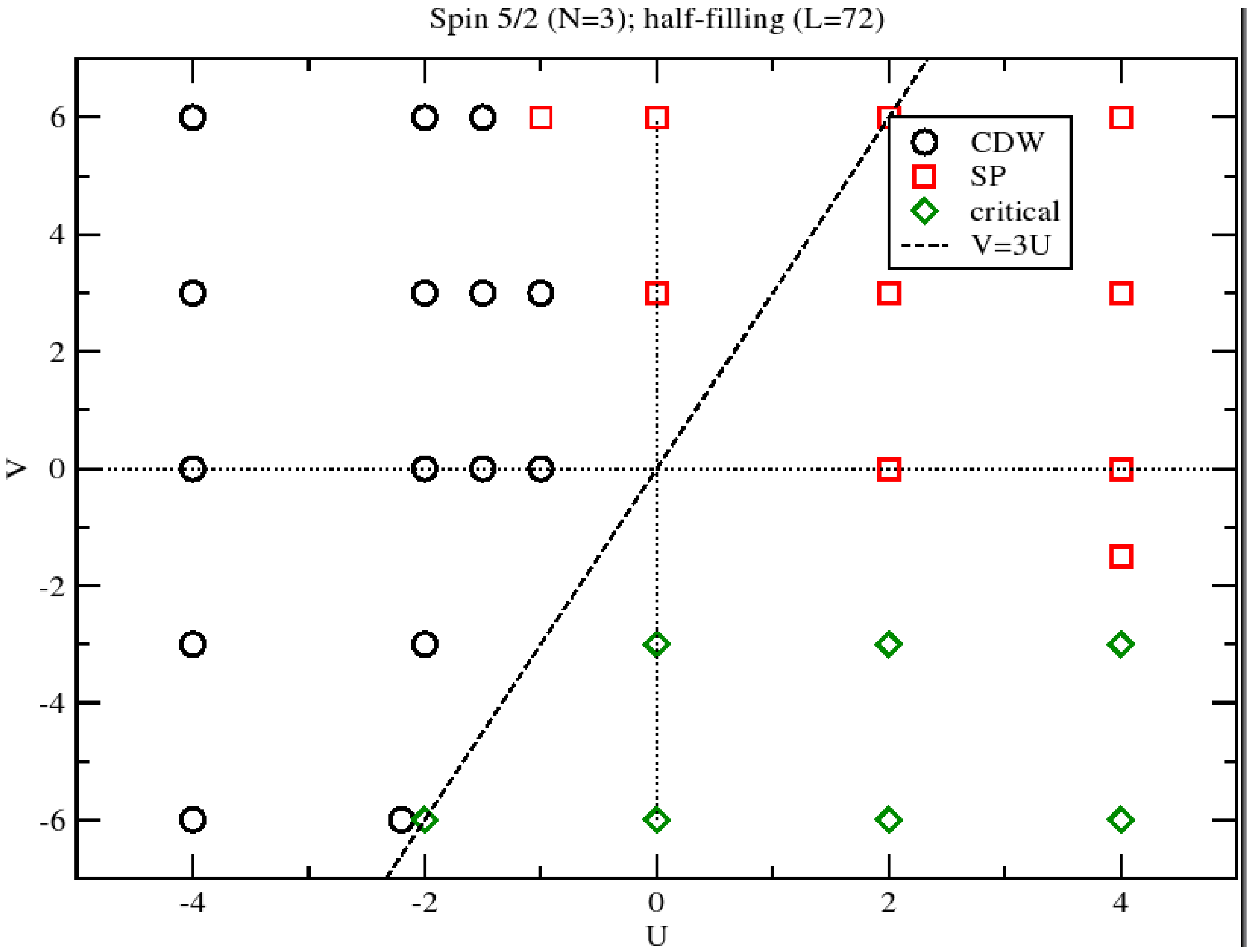
Rung singlet phase: no edge state



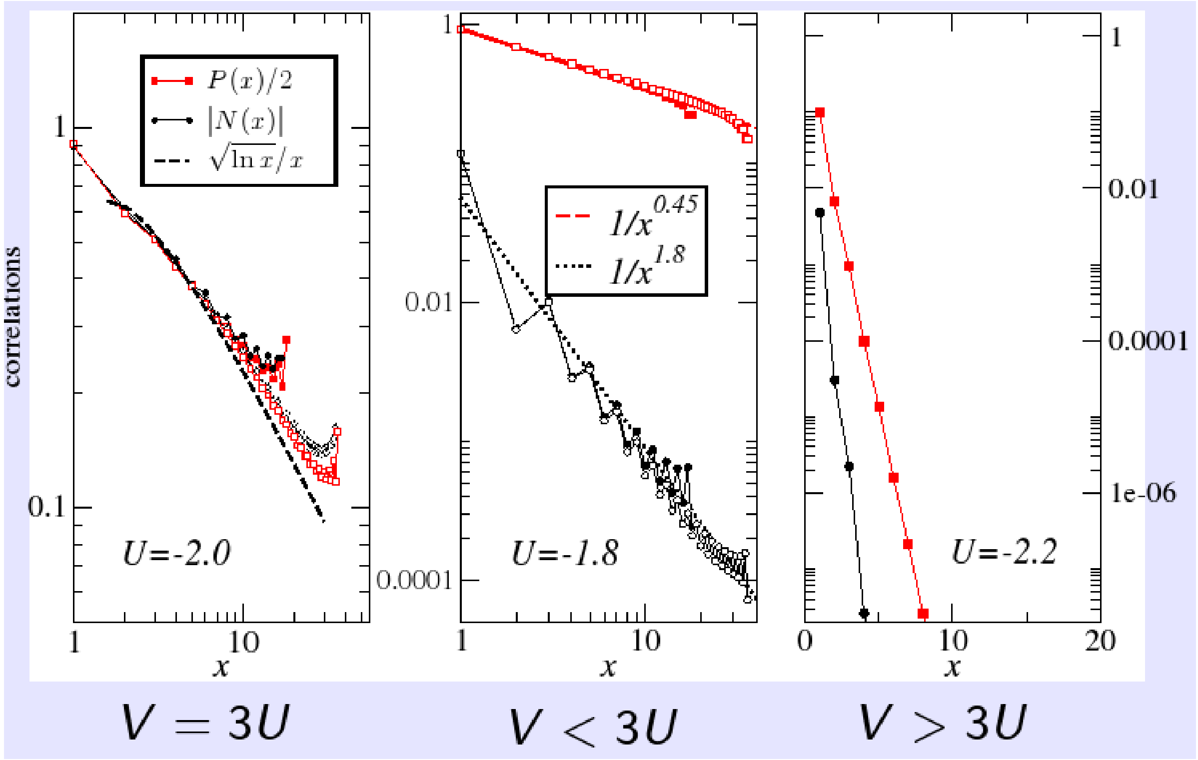
$$F(x) = \int^x dx(n(x) - 2)$$

Haldane insulating phase: the two added fermions are localized at the boundaries

DMRG results (N=3)



DMRG results (N=3)



$$P(x) = \langle P^{\dagger}_{00,L/2+x} P_{00,L/2} \rangle$$

$$N(x) = \langle \delta n_{L/2+x} \delta n_{L/2} \rangle$$

Criticality (SU(2))

XY phase

Ising (gapped) phase

Full agreement with the strong coupling analysis

Conclusion

Haldane's conjecture for attractive half-filled 2N components cold fermions:

N even: Insulating Haldane phase with hidden ordering

N odd: Metallic phase

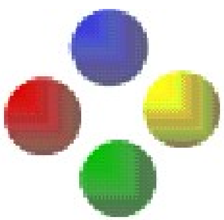
Charged spin-singlet modes and not neutral spin degrees of freedom as in the standard Haldane's conjecture

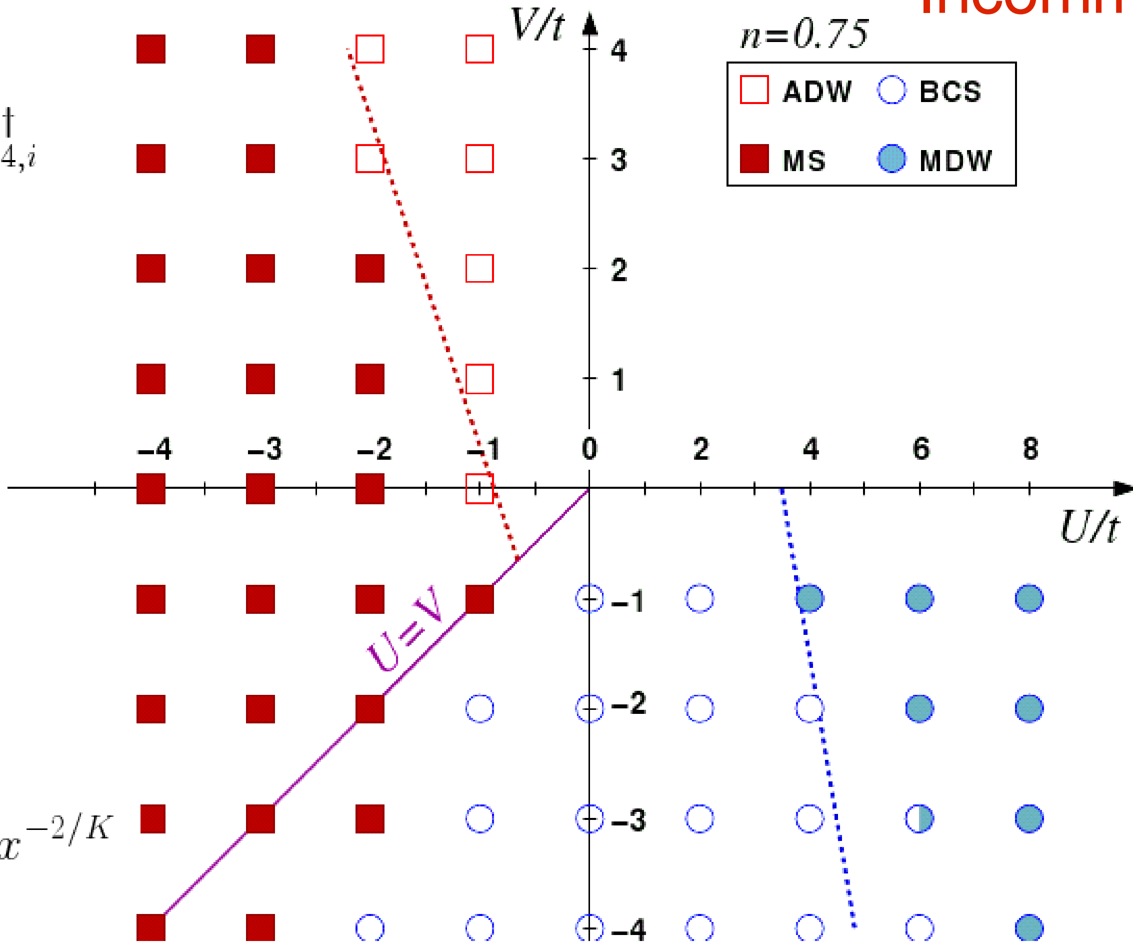
The generic cases are for each family are: N=3 and N=4 and not N=1, N=2

Phase diagram F=3/2

Incommensurate filling

$M_i = c^\dagger_{1,i} \dots c^\dagger_{4,i}$





DMRG

L=140 sites

Roux et al. 2008

Lecheminant et al 2005

Quartet phase

(charge 4e)

$\langle M_i M^\dagger_{i+x} \rangle \sim x^{-2/K}$

BCS phase (charge 2e)