Quantum Magnetism and Topological Insulators (w and w/o Interactions)

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Ruegg, Wen, and Fiete, Phys. Rev. B 81, 205115 (2010)
Kargarian and Fiete, Phys. Rev. B 82, 085106 (2010)
Wen, Ruegg, Wang, Fiete, Phys. Rev. B 82, 075125 (2010)

Outline

- Brief review of topological phases and experimental support for topological insulators.
- Non-interacting lattice models with topological order.
- Interacting lattice models with topological order.
- Summary and future directions.

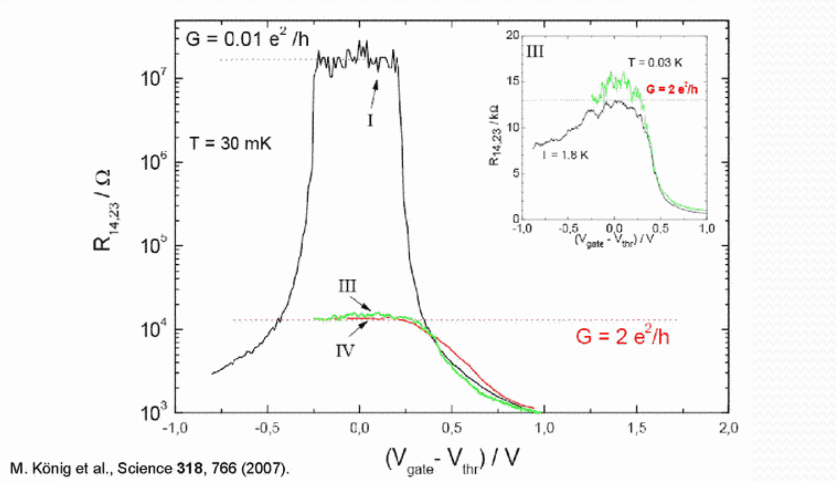
“Common” Topological Phases

	Th	**Exp**
Integer and Fractional Quantum Hall effects	✔	✔
Spin liquid ground states with gapped excitation spectra	✔	?
Time-reversal invariant topological band insulators	✔	✔

Experimental Evidence for Topological Band Insulators

- HgCdTe quantum wells

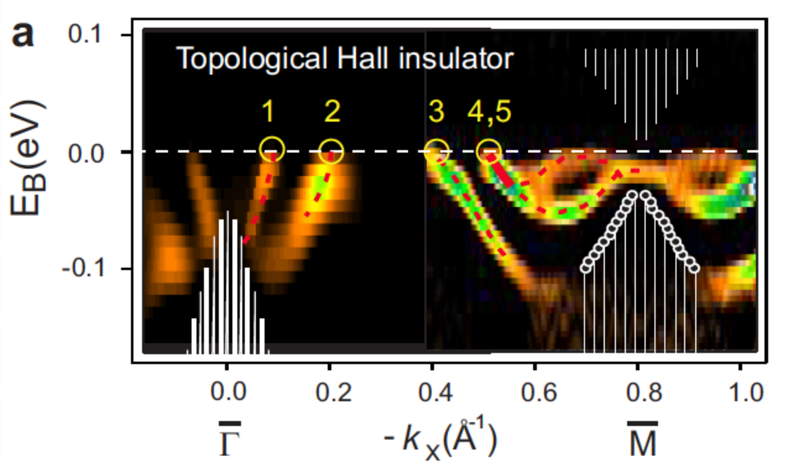
  M. Konig *et al*. Science **318**, 766 (2007)



M. König et al., Science **318**, 766 (2007).

- $Bi_{1-x}Sb_x$

  D. Hsieh *et al*. Nature **452**, 970 (2008)



- Also:

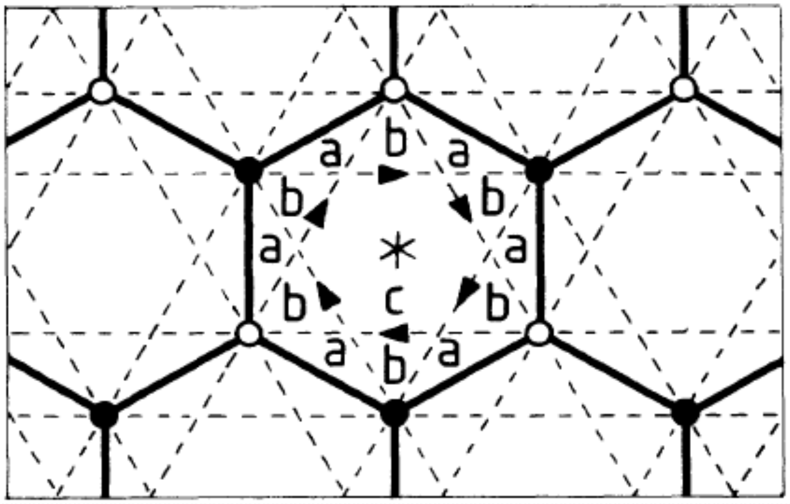
  Bi_2Se_3, Bi_2Te_3, $TlBiSe_2$, etc.

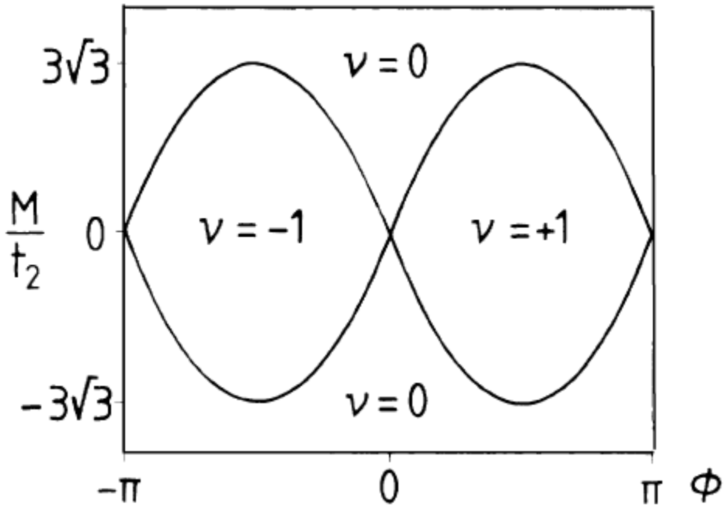
Key TBI precedent: Haldane model

F. D. M. Haldane, PRL (1988)

$$\mathcal{H} = -t_1 \sum_{<ij>} c_i^\dagger c_j - t_2 \sum_{<<ij>>} e^{i\phi} c_i^\dagger c_j + H.c. + M \sum_i \xi_i c_i^\dagger c_i$$

$\xi_i = \pm 1$ on a, b sublattices

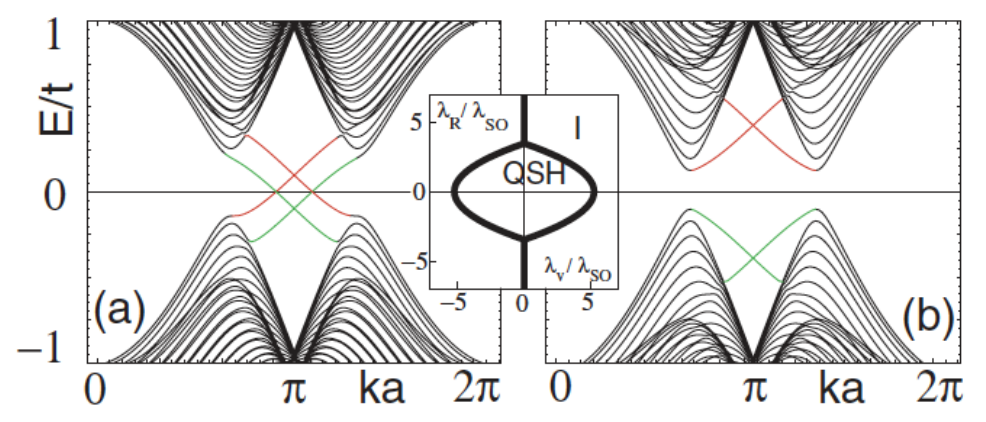




First TRS TBI model: Kane-Mele

$$\mathcal{H} = t_1 \sum_{<ij>} c_i^\dagger c_j + i\lambda_{\mathrm{SO}} \sum_{<<ij>>} \nu_{ij} c_i^\dagger s^z c_j + H.c.$$

$$+ i\lambda_R \sum_{<ij>} c_i^\dagger (\mathbf{s} \times \hat{\mathbf{d}}_{ij}) c_j + \lambda_v \sum_i \xi_i c_i^\dagger c_i$$



- Two "copies" of Haldane's model

Kane and Mele, PRL (2005)

Classification of TBI: Z_2 invariant

- Takes place of Chern number.

$$\delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i)$$

(Inversion symmetry)

TRI momentum

$$\xi_m(\Gamma_i)$$

Parity eigenvalue

$$\boxed{(-1)^\nu = \prod_i \delta_i}$$

Z_2 invariant

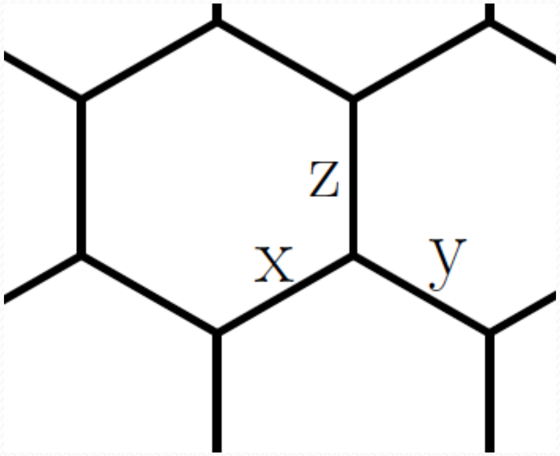
- Three-dimensional invariants as well
  - Fu and Kane, PRL 2007
  - Moore and Balents, PRB 2007
  - Roy, PRB 2009

Kane and Mele, PRL (2005)

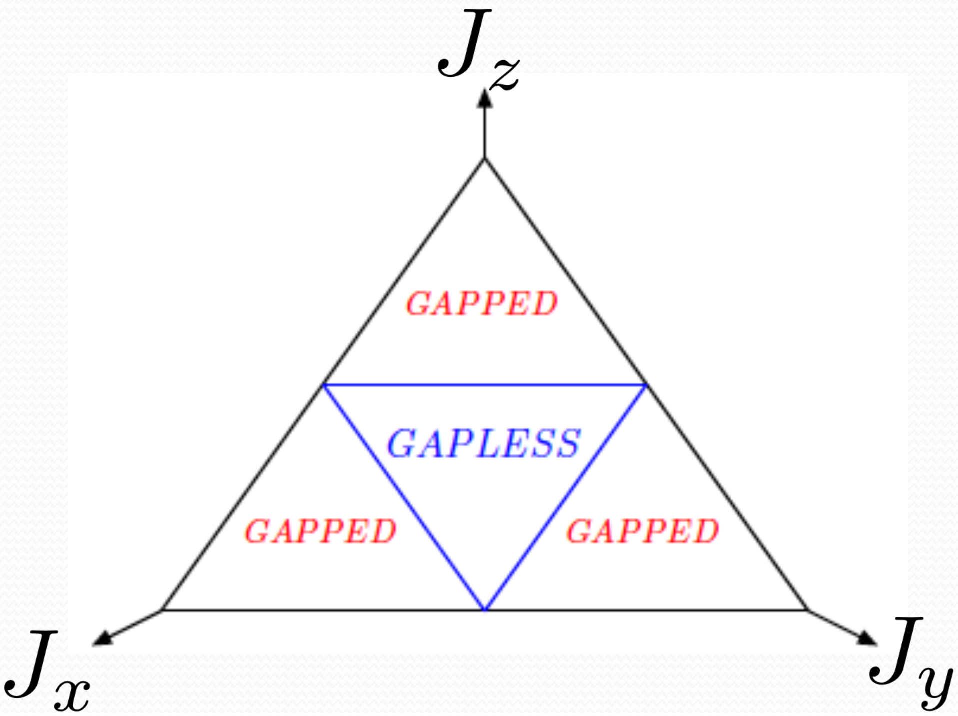
Related Spin Model: Kitaev model

- Lives on honeycomb lattice

$$\mathcal{H} = \sum_{x-link} J_x \sigma_i^x \sigma_j^x + \sum_{y-link} J_y \sigma_i^y \sigma_j^y + \sum_{z-link} J_z \sigma_i^z \sigma_j^z$$



Kitaev Ann. Phys. (2006)



Solving the Kitaev model

- Use Majorana representation of spins and Clifford algebra:

$$\sigma_j^x = ic_j^x c_j,\ \sigma_j^y = ic_j^y c_j,\ \sigma_j^z = ic_j^z c_j \qquad \{c_i, c_j\} = 2\delta_{ij}$$

$$\boxed{\mathcal{H} = \sum_{x-link} iJ_x u_{ij} c_i c_j + \sum_{y-link} iJ_y u_{ij} c_i c_j + \sum_{z-link} iJ_z u_{ij} c_i c_j}$$

Free fermion model!

$$[u_{ij}, \mathcal{H}] = [u_{ij}, u_{i'j'}] = 0$$

Non-dynamical Z_2 gauge fields

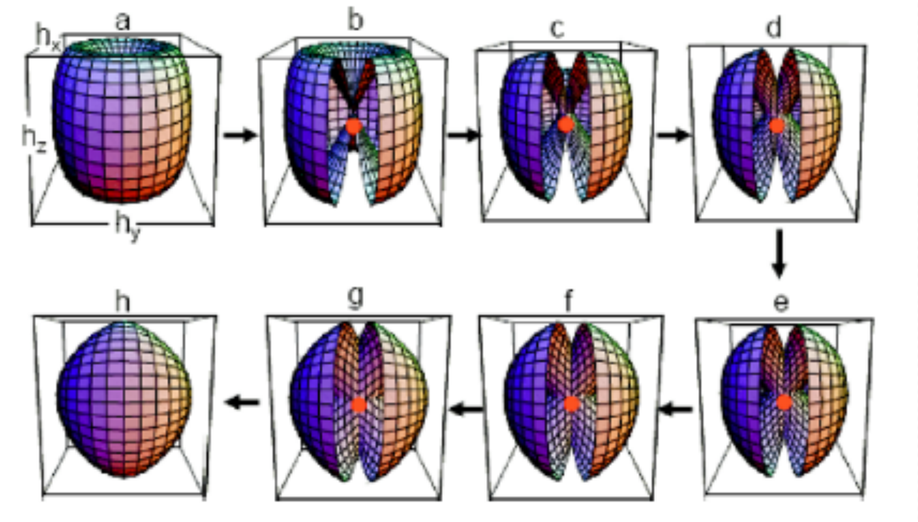
Kitaev Ann. Phys. (2006)

An interesting twist to Kitaev model

- Add 3-spin interaction (2nd neighbor hopping in Maj.)

$$\mathcal{H}' = J' \sum_{(ijk)\in\triangle} \sigma_i^y \sigma_j^z \sigma_k^x + J' \sum_{(ijk)\in\nabla} \sigma_i^x \sigma_j^z \sigma_k^y$$

- Adiabatically connected to Haldane's model



Haldane

Kitaev

D.-H. Lee, G.-H. Zhang, T. Xiang, Phys. Rev. Lett. 99, 196805 (2007)

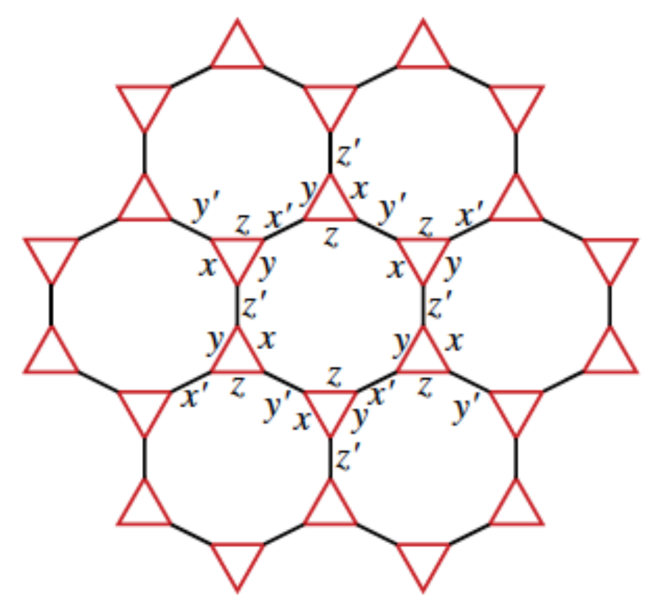
Kitaev model on the decorated honeycomb lattice

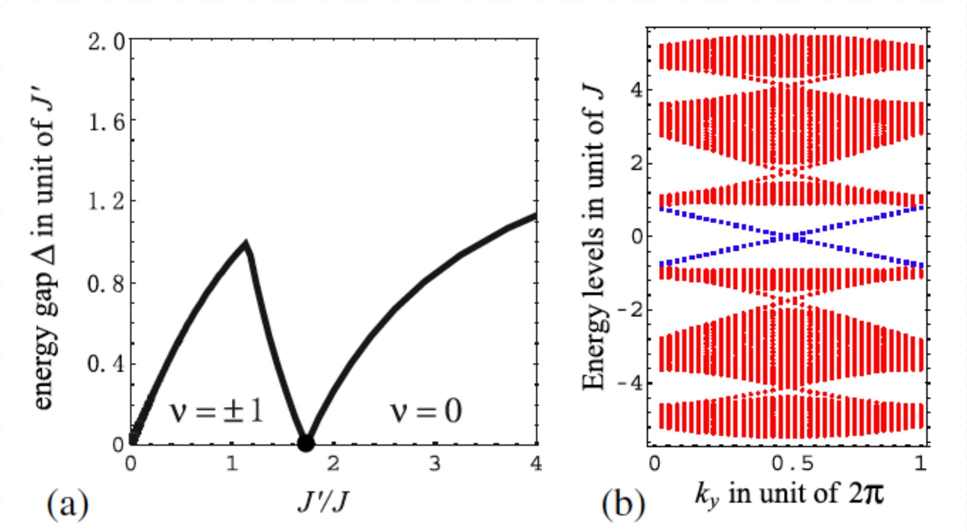
- Exact chiral spin liquid state with non-Abelian anyons

Yao and Kivelson, PRL (2007)

$$\mathcal{H} = \sum_{x-link} J\sigma_i^x\sigma_j^x + \sum_{y-link} J\sigma_i^y\sigma_j^y + \sum_{z-link} J\sigma_i^z\sigma_j^z$$

$$+ \sum_{x'-link} J'\sigma_i^x\sigma_j^x + \sum_{y'-link} J'\sigma_i^y\sigma_j^y + \sum_{z'-link} J'\sigma_i^z\sigma_j^z$$



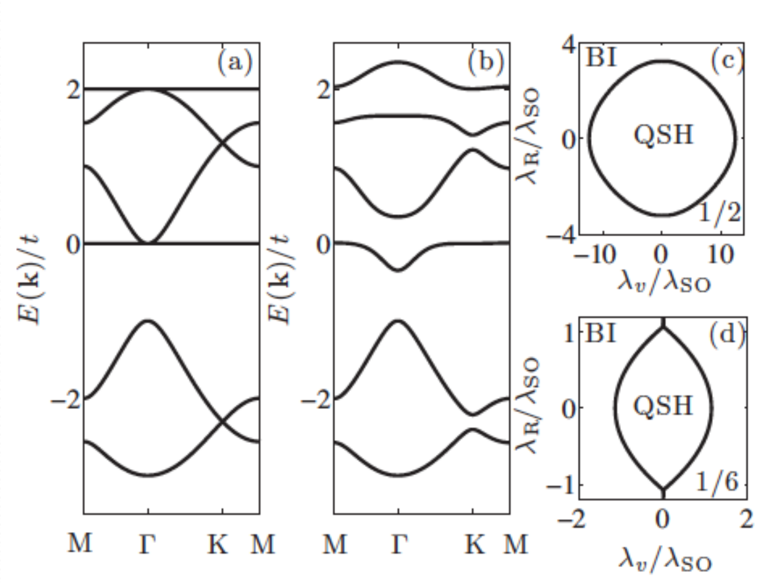


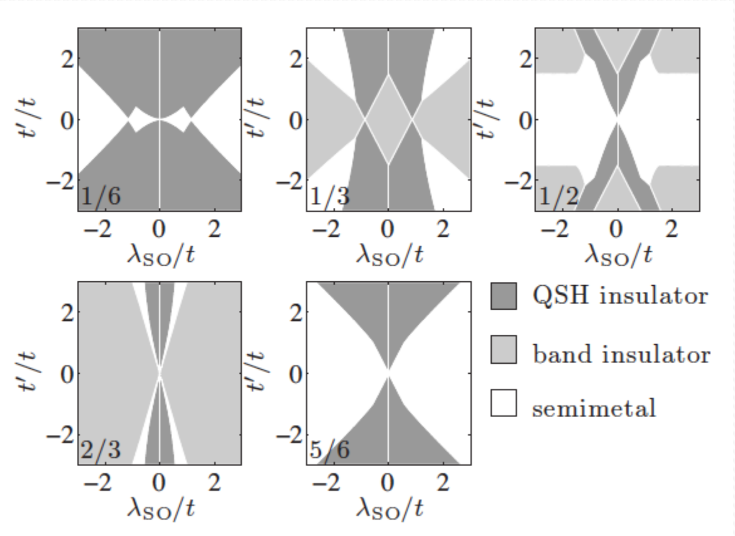
TBI on the decorated honeycomb lattice

Ruegg, Wen, Fiete PRB (2010)

$$\mathcal{H} = -t\sum_{<ij>,\triangle} c^{\dagger}_{i\sigma}c_{j\sigma} - t'\sum_{<ij>,\triangle\to\triangle} c^{\dagger}_{i\sigma}c_{j\sigma} + i\lambda_{\mathrm{SO}}\sum_{<<ij>>} \vec{e}_{ij}\cdot\vec{s}_{ij}c^{\dagger}_{i\alpha}c_{j\beta} + H.c.$$

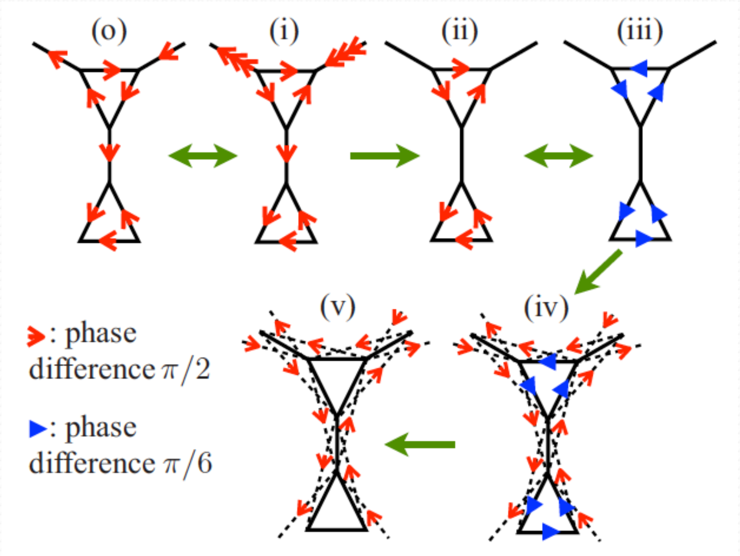
$$+\sum_{i\sigma}\lambda_{vi}c^{\dagger}_{i\sigma}c_{i\sigma} + i\lambda_{\mathrm{R}}\sum_{<ij>} c^{\dagger}_{i\alpha}(\vec{s}_{\alpha\beta}\times\hat{d}_{ij})_{z}c_{j\beta} + H.c$$

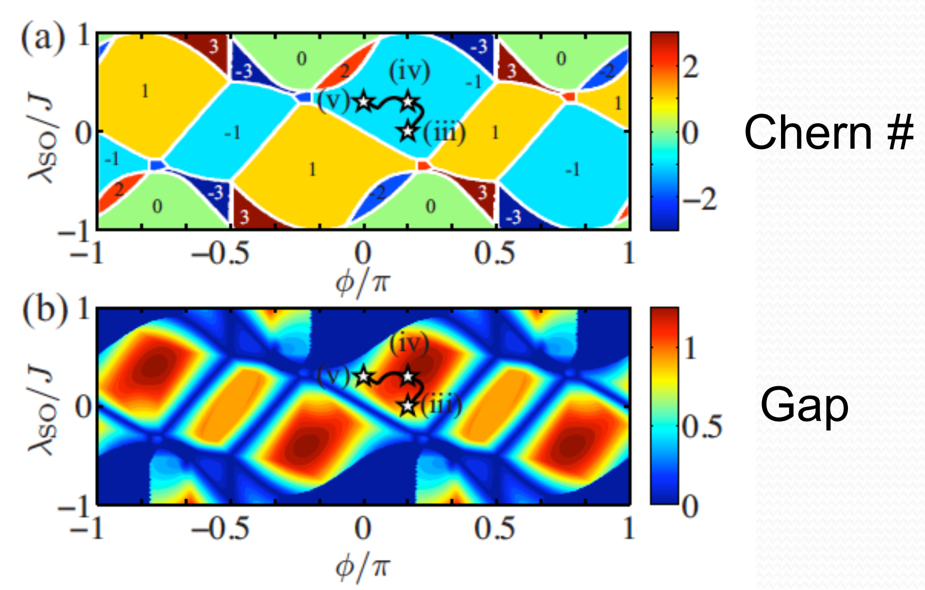




Adiabatic connection of chiral spin liquid to “1/2” of TBI

- Free fermion rep of CSL can be deformed to model of QSH (real 1st, imaginary 2nd neighbor hoppings) for one spin orientation (*i.e.* a QAH state)

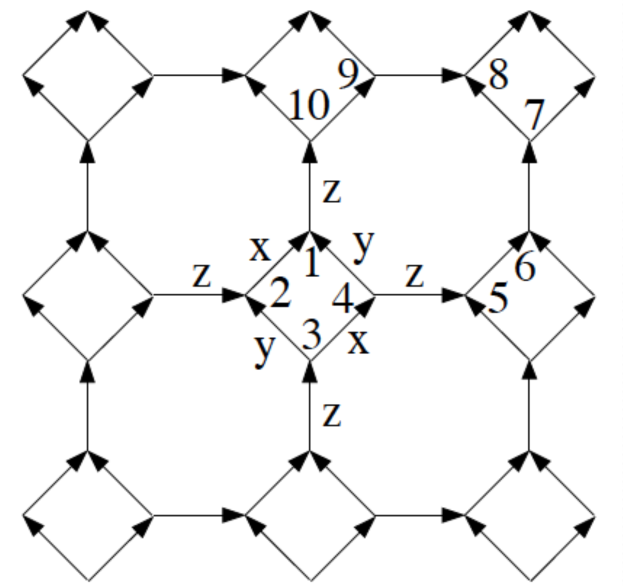




Ruegg, Wen, Fiete PRB (2010)

Kitaev model on decorated square lattice

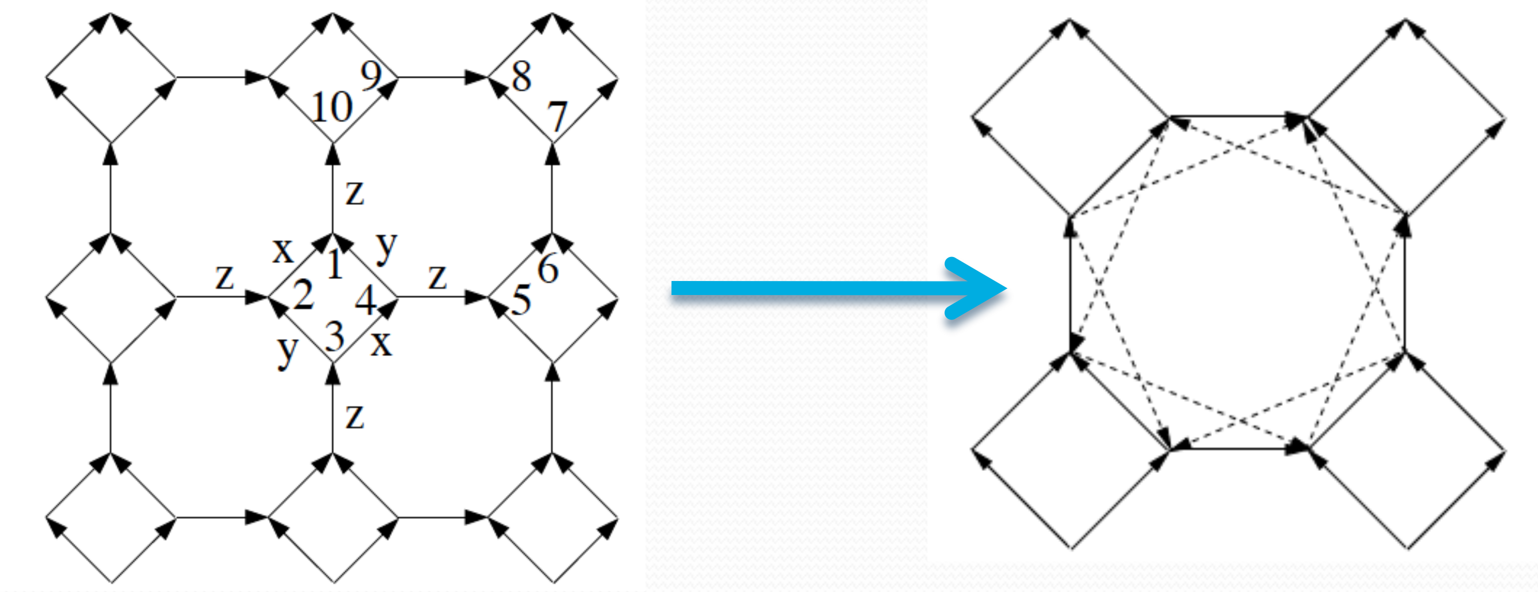
- Model reduces to free Majorana representation



Yang, Zhou, and Sun PRB (2007)
Baskaran, Santhosh, and Shankar, arXiv:0908.1614

Kitaev model on decorated square lattice → TBI

- Model reduces to free Majorana representation

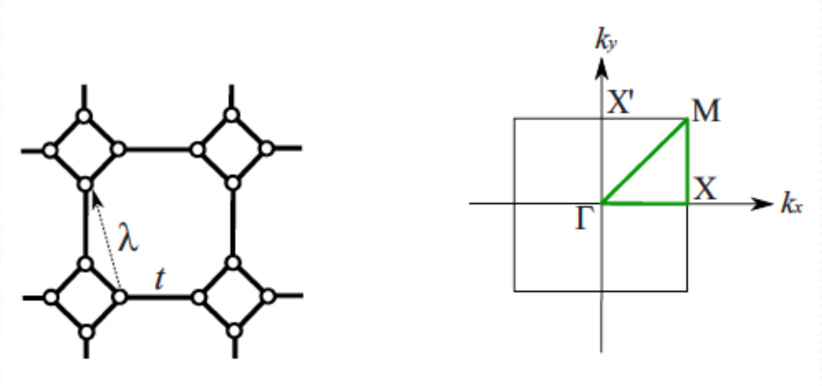


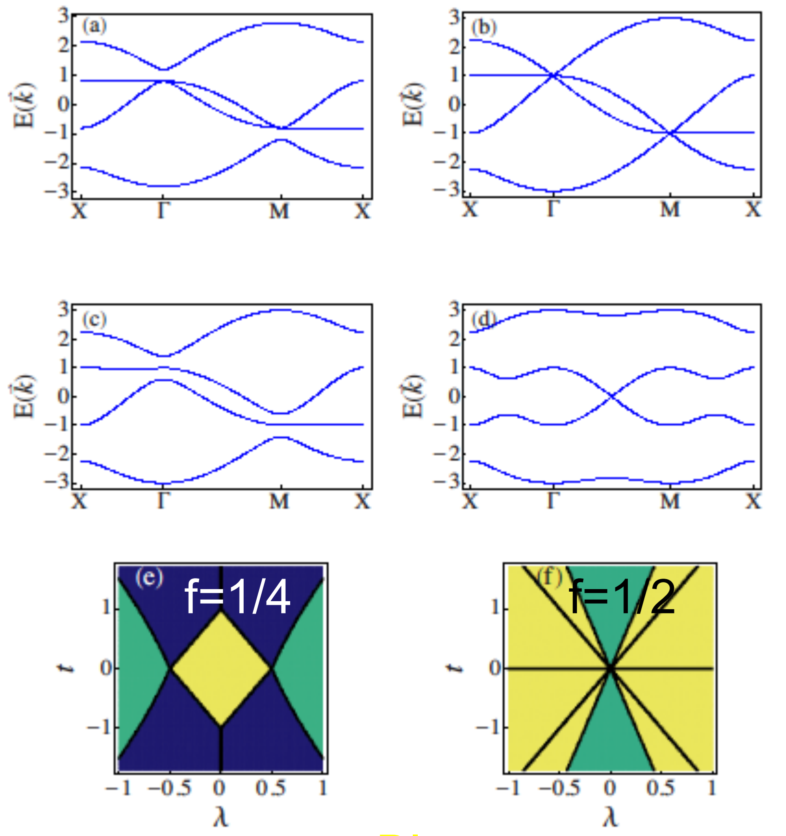
Yang, Zhou, and Sun PRB (2007)
Baskaran, Santhosh, and Shankar, arXiv:0908.1614

Properties of a TBI on decorated square lattice

Kargarian and Fiete PRB (2010)

$$\mathcal{H} = -\sum_{<ij>\in\diamond} c^{\dagger}_{i\sigma}c_{j\sigma} - t\sum_{<ij>\in\diamond\to\diamond} c^{\dagger}_{i\sigma}c_{j\sigma} + i\lambda\sum_{<<ij>>} c^{\dagger}_{i\alpha}(\vec{e}_{ij}\cdot\vec{\sigma})_{\alpha\beta}c_{j\beta}$$





TBI

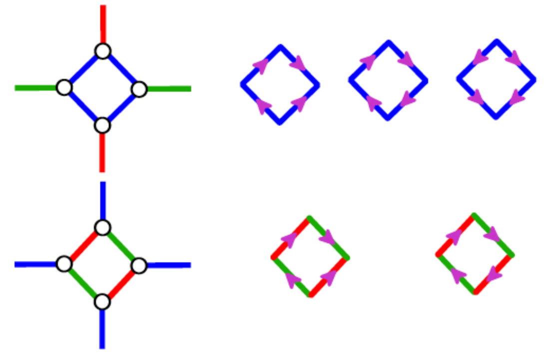
BI

SM

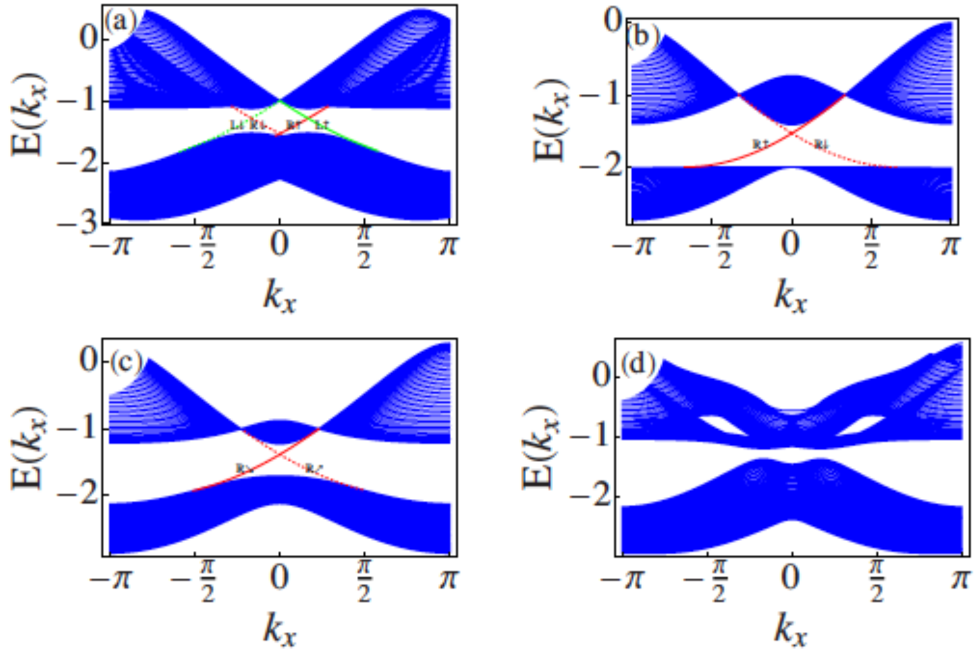
TRS TBI with only NN hopping

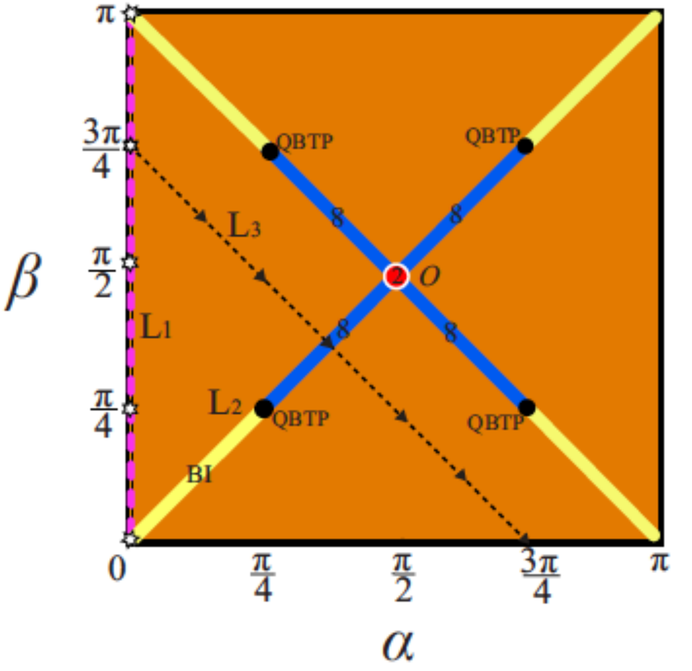
$$\mathcal{H} = -\sum_{<ij>} [U_{ij}]_{\sigma\sigma'} c^{\dagger}_{i\sigma} c_{j\sigma'} + H.c.$$

$$U_x = e^{i\alpha\sigma^x},\ U_y = e^{i\beta\sigma^y},\ U_z = e^{i\gamma\sigma^z}$$



U_x U_y U_z





Kargarian and Fiete PRB (2010)

Other known lattices supporting a TBI

- Kagome: Guo and Franz, PRB (2009)
- Pyrochlore: Guo and Franz, PRL (2009)
- Diamond: Fu, Kane, and Mele PRL (2007)
- Decorated square lattice: Sun *et al*. PRL (2009)
- Lieb and Perovskite: Weeks and Franz PRB (2010)

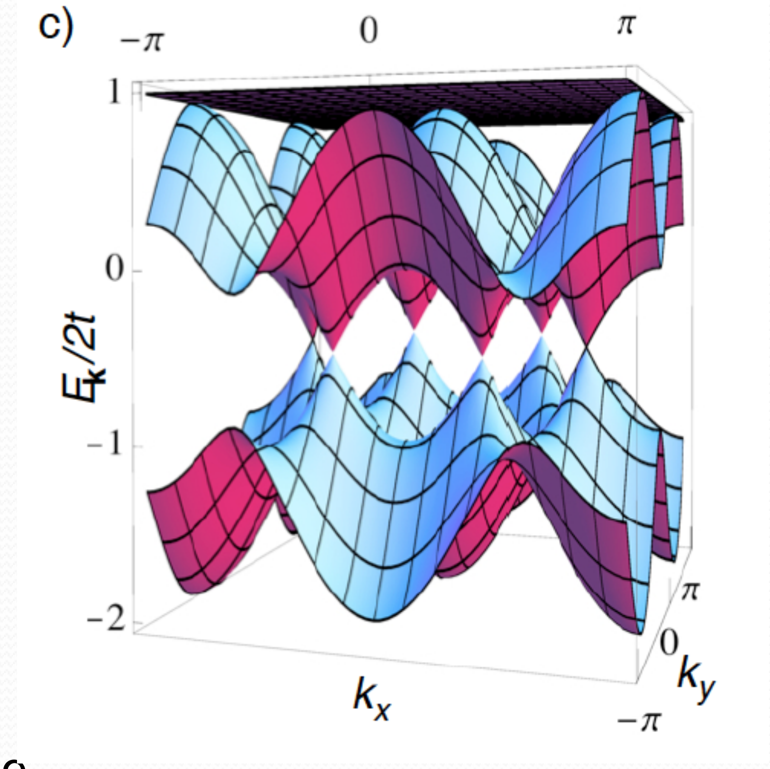
DFT calculations for ~100 real materials!

TBI are literally all over the place!!

What about interactions?

- Dirac band touching points vs. quadratic band touching points:
- Dirac points perturbatively stable against interactions
- QBTP perturbatively unstable to interactions

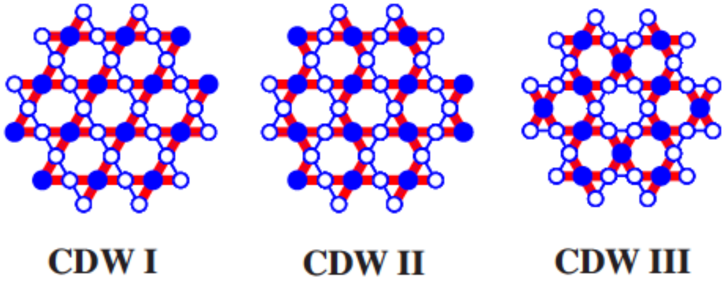
kagome band structure

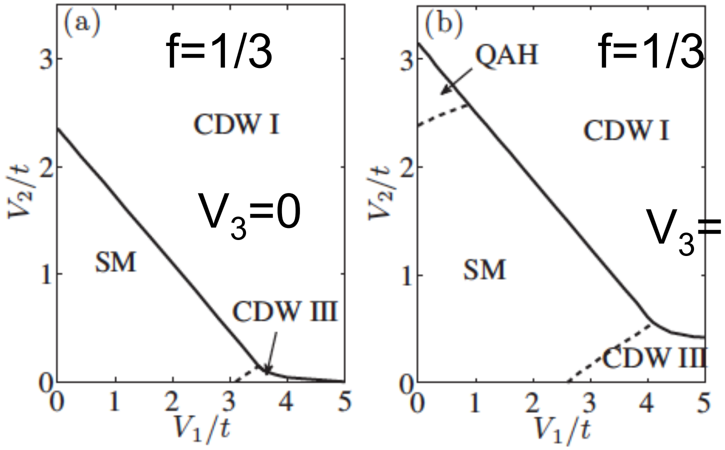


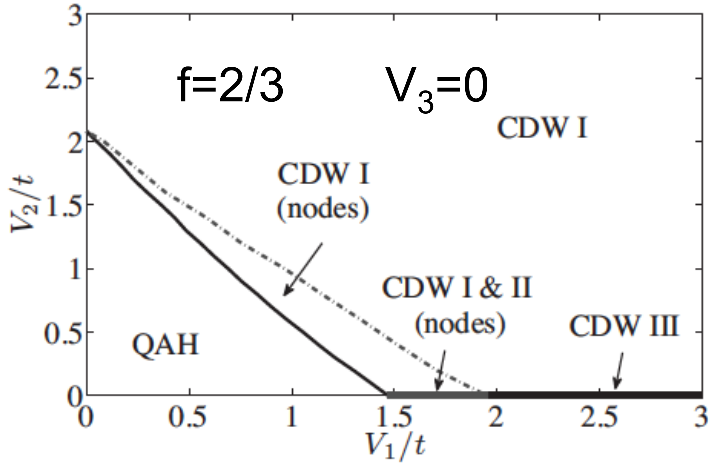
Sun, Yao, Fradkin, and Kivelson, PRL 2009

Meanfield study of kagome lattice model with spinless fermions & 2nd, 3rd neighbor interactions but no intrinsic spin-orbit coupling

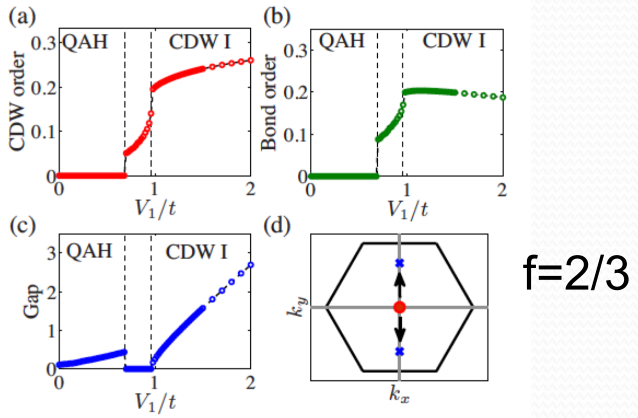
- Earlier studies: Raghu *et al.* PRL (2008), Yang et al. PRB (2009)



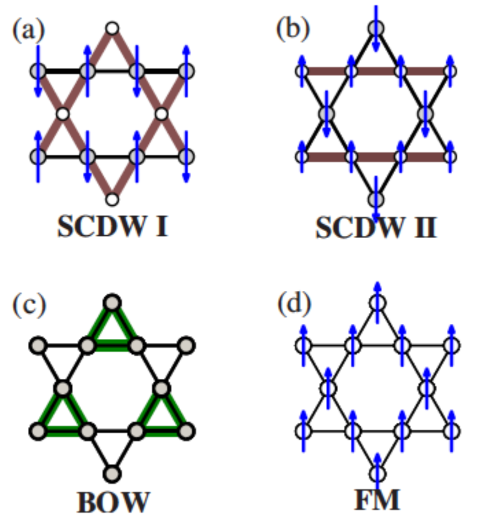


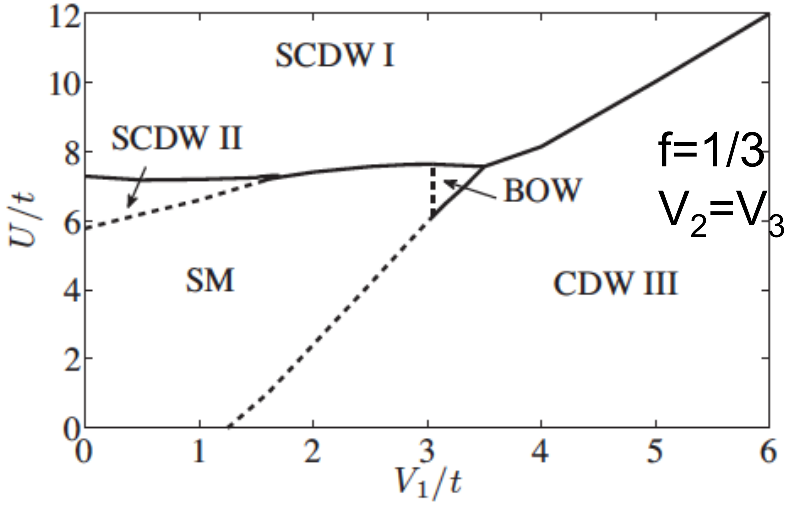


Wen, Ruegg, Wang, Fiete PRB (2010)



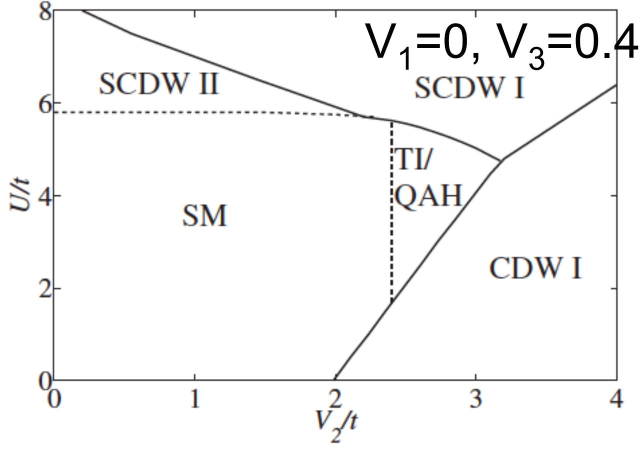
kagome lattice with spinful fermions

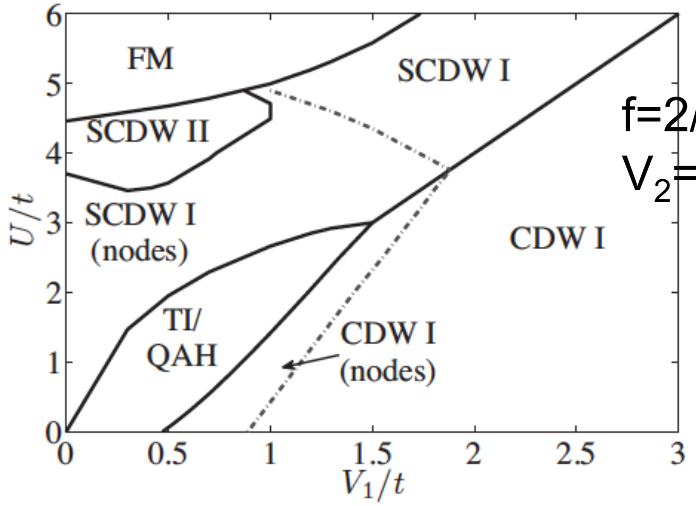




f=1/3
V_2=V_3=0

f=1/3
V_1=0, V_3=0.4t

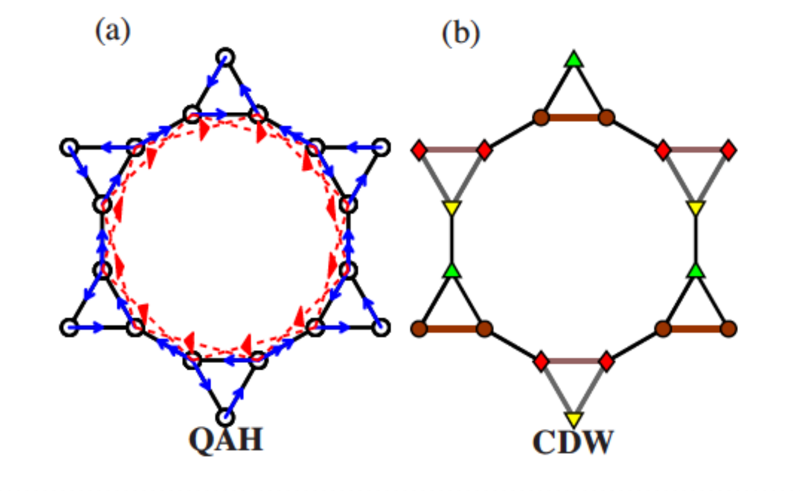


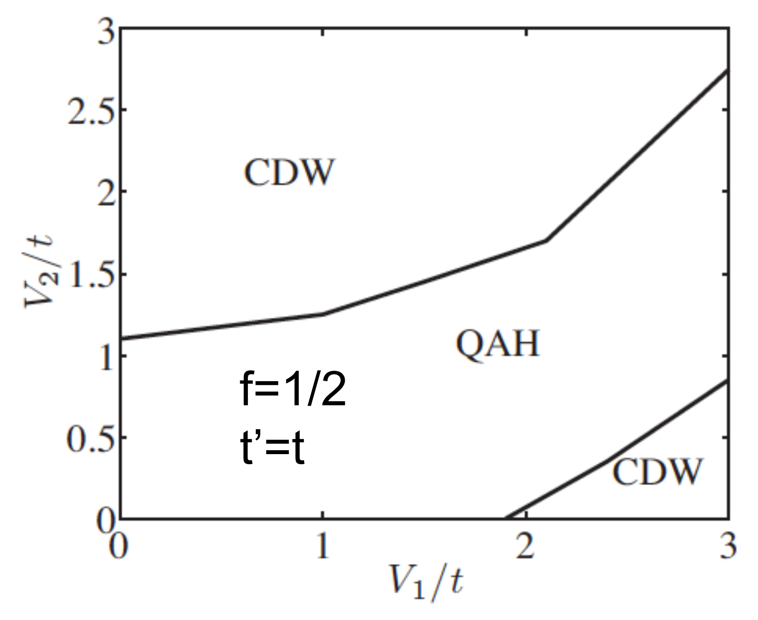


f=2/3
V_2=V_3=0

Wen, Ruegg, Wang, and Fiete PRB (2010)

Decorated honeycomb lattice with spinless fermions

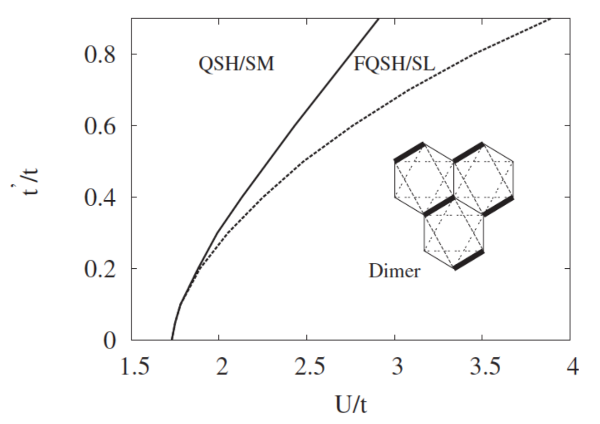




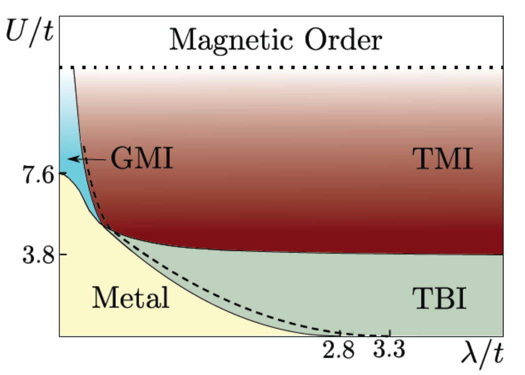
Wen, Ruegg, Wang, and Fiete PRB (2010)

Other key interaction studies allowing for exotic phases

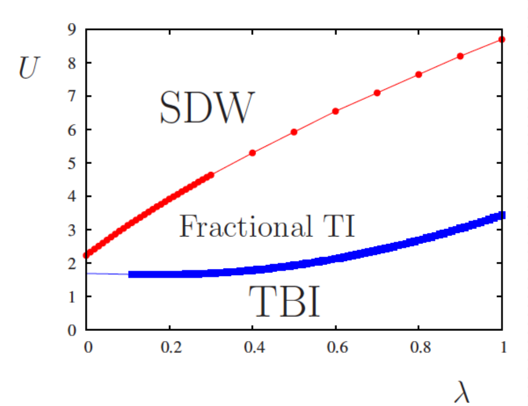
$$\boxed{c_{i\sigma} = e^{i\theta_i} f_{i\sigma}}$$



Young, Lee, Kallin PRB (2008)



Pesin, Balents Nat. Phys. (2010)

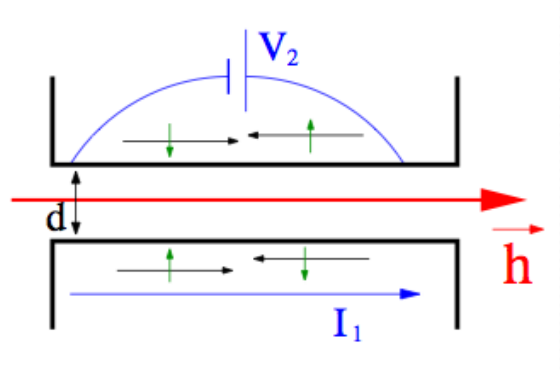


Rachel, Le Hur PRB (2010)

Even more exotic: Maciejko, Qi, Karch, Zhang arXiv:1004.3628
Swingle, Barkeshli, McGreevy, Senthil arXiv:1005.1076
“Fractional TI” with non-trivial ground state degeneracy.

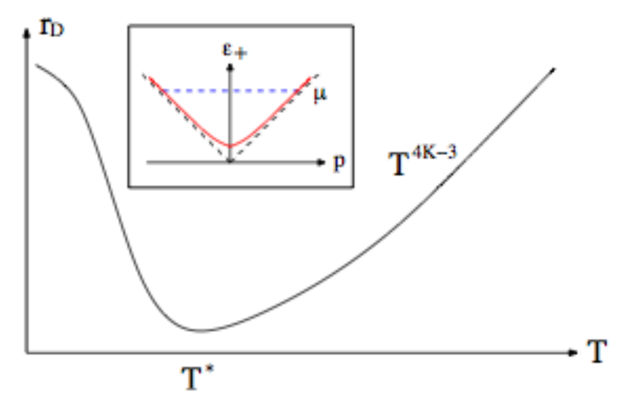
Advertisement

- Drag between helical edges follow behavior



$$r_D = -\lim_{I_1 \to 0} \frac{e^2}{h} \frac{1}{L} \frac{dV_2}{dI_1}$$
$$\propto h^4 T^{4K-3}$$

Zyuzin and Fiete, PRB (2010)



Summary

- We have studied a number of lattice models that realize topological phases.

- We have highlighted the relationship between a class of exactly solvable spin liquid models and topological band insulators.

- We have treated interaction effects at the meanfield level and focused on the important differences between Dirac and quadratic band touching points.